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Conteúdo

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ON A SOBOLEV-TYPE INEQUALITY AND ITS MINIMIZERS

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Abstract

A continuous embedding between spaces $\mathcal{D}_R^{m,p}(\alpha)$ and L_θ^p was established. Furthermore, we examine the existence of extremal functions corresponding to the associated variational problem. As an application, we demonstrate the existence of weak solutions for a broad class of critical semilinear elliptic equations involving the polyharmonic operator.

1 Introduction

For $0 < R \leq \infty$, $p \geq 1$ and $\theta \in \mathbb{R}$, we denote by $L_\theta^p = L_\theta^p(0,R)$ the weighted Lebesgue space of the Lebesgue measurable functions $u : (0,R) \to \mathbb{R}$ endowed with the norm

$$\|u\|_{L_\theta^p} = \left(\int_0^R |u(r)|^p r^\theta \mathrm{d}r \right)^{\frac{1}{p}} < \infty.$$

For each $\alpha > -1$, we define the α-generalized Laplacian operator in the radial form $\Delta_\alpha u := r^{-\alpha}(r^\alpha u')'$ where $u \in C^2(0,R)$. The operator Δ_α appears in [3] and we observe that, for $\alpha = N-1$ positive integer number, $\Delta_\alpha u$ agrees precisely with the Laplacian operator acting on radial symmetric functions u defined on a ball centered at the origin $B(0,R) \subset \mathbb{R}^N, R > 0$. To deal with the α-generalized Laplacian operator, we introduce the set

$$D_{0,R}(\alpha) = D_{0,R}(\alpha, m, p) = \left\{ u \in AC_{\mathrm{R}}^{m-1}(0,R) \ : \ u^{(m)} \in L_\alpha^p \right\},$$

where $u^{(\ell)} = d^\ell u/dr^\ell$, $AC_{\mathrm{R}}^{m-1}(0,R)$ set of functions whose derivatives up to order $m-1$ are locally absolutely continuous functions on interval $(0,R)$, and such that $\lim_{r\to R} u^{(j)}(r) = 0$ for $j = 0, \cdots, m-1$. Finally, we define the new space $\mathcal{D}_R^{m,p}(\alpha)$ by completion of $D_{0,R}(\alpha,m,p)$ under the norm $\|u\| := \|u^{(m)}\|_{L_\alpha^p}$ An alternative standard we can consider on $\mathcal{D}_R^{m,p}(\alpha)$ is the *Navier norm* $\|u\|_{\nabla_\alpha^m} := \|\nabla_\alpha^m u\|_{L_\alpha^p}$ where, for $u \in D_{0,R}(\alpha,m,p)$

$$\nabla_\alpha^m u = \begin{cases} \Delta_\alpha^{\frac{m}{2}} u, & \text{if} \quad m \text{ is even}, \\ \left(\Delta_\alpha^{\frac{m-1}{2}} u \right)', & \text{if} \quad m \text{ is odd}. \end{cases}$$

is the α-generalized m-th order gradient.

It is worth noting that Sobolev weighted spaces hold surprises, and some interesting points have attracted attention since the pioneering work of E. Mitidieri et al. [4]. In this work, we are mainly interested in the *Sobolev* case for unbounded domain $R = \infty$.

2 Main Results

Our first result reads below.

Theorem 2.1 (Theorem 1.1, [2]). *Let $p \geq 2$, $\alpha, \theta > -1$, $0 < R \leq \infty$ and let $m \in \mathbb{Z}_+$ such that $\theta \geq \alpha - mp$ and $\alpha - mp + 1 > 0$. If $p^* = (\theta+1)p/(\alpha - mp + 1)$, then there exists $C > 0$ such that*

$$\left(\int_0^R |u(r)|^{p^*} r^\theta \, dr\right)^{\frac{1}{p^*}} \leq C \left(\int_0^R |\nabla_\alpha^m u(r)|^p r^\alpha \, dr\right)^{\frac{1}{p}}, \quad \textit{for all } u \in \mathcal{D}_R^{m,p}(\alpha).$$

In particular, we obtain the continuous embedding

$$\mathcal{D}_R^{m,p}(\alpha) \hookrightarrow L_\theta^{p^*}. \tag{1}$$

For each $0 < R \leq \infty$ we can define

$$\mathcal{S} = \mathcal{S}(m,p,\alpha,\theta,R) = \inf\left\{ \|\nabla_\alpha^m u\|_{L_\alpha^p}^p \; : \; u \in \mathcal{D}_R^{m,p}(\alpha), \; \|u\|_{L_\theta^{p^*}}^p = 1 \right\}.$$

We note that $\mathcal{S} > 0$ and $\mathcal{S}^{-\frac{1}{p}}$ is the *best constant* for the Sobolev type embedding (1).

Theorem 2.2 (Theorem 1.2, [2]). *Let $\alpha, \theta > -1$ and let $m \in \mathbb{Z}_+$ such that $\theta \geq \alpha - 2m$ and $\alpha - 2m + 1 > 0$. Then, there exists $z \in \mathcal{D}_\infty^{m,2}(\alpha)$ such that*

$$\|z\|_{L_\theta^{2^*}} = 1 \textit{ and } \|\nabla_\alpha^m z\|_{L_\alpha^2}^2 = \mathcal{S}(m,2,\alpha,\theta,\infty).$$

Here, the critical exponent $2^* = 2(\theta+1)/(\alpha - 2m + 1)$.

As application of Theorem 2.1 and Theorem 2.2 we provide the following existence result for a general class of elliptic equations driven by polyharmonic operator.

Corollary 2.1 (Corollary 1.3, [2]). *For α, θ and m under the assumptions of the Theorem 2.2, there exists a weak solution $z \in \mathcal{D}_\infty^{m,2}(\alpha)$ to the critical semilinear equation*

$$(-\Delta_\alpha)^m u = r^{\theta-\alpha}|u|^{2^*-2}u \textit{ in } (0,\infty).$$

We emphasize that if $\alpha = N - 1$ is a positive integer number, then the operator $(\Delta_\alpha)^m u$ agrees precisely with the polyharmonic operator acting on radially symmetric functions defined on balls B_R of $\mathbb{R}^N$. Furthermore, it is well-known that the polyharmonic operator is related to higher-order elliptic equations of reaction-diffusion type, see [1].

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ON THE ADJOINT OF MULTILINEAR OPERATORS BETWEEN RIESZ SPACES

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Abstract

In this work, we show that, for a multilinear operator of order bounded variation between Riesz spaces $A\colon E_1 \times \cdots \times E_n \to F$, its adjoint $A^*\colon F^\sim \to L_r(E_1, \ldots, E_n; \mathbb{R})$ is well-defined, order bounded, and order continuous. Moreover, when F is Dedekind complete, we have $|A^*|(f) = |A|^*(f)$ whenever f is a regular σ-order continuous functional.

1 Introduction

The theory of multilinear operators between Riesz spaces was initially developed by Fremlin in the 1970s. However, many definitions and known results from the linear case still lack their multilinear counterparts. In this work, we study adjoints of multilinear operators between Riesz spaces and generalize some well-known results for order bounded linear operators that can be found, e.g., in [1].

Let $X_1, \ldots, X_n, Y$ be real vector spaces. We denote by $L(X_1, \ldots, X_n; Y)$ the vector space of n-linear operators from $X_1 \times \cdots \times X_n$ into Y. The algebraic dual of Y is denoted by Y^*. Given an operator $A \in L(X_1, \ldots, X_n; Y)$, following the classical approach of Aron and Schottenloher [2], we define its adjoint as the linear operator

$$A^*\colon Y^* \to L(X_1, \ldots, X_n; \mathbb{R}) \ , \ A^*(y^*)(x_1, \ldots, x_n) = y^*(A(x_1, \ldots, x_n)).$$

Here, all Riesz spaces are Archimedean. For Riesz spaces $E_1, \ldots, E_n, F$, the space $L(E_1, \ldots, E_n; F)$ is not necessarily a Riesz space under the usual order, even when F is Dedekind complete. For this reason, multilinear operators of order bounded variation are studied. An operator $A \in L(E_1, \ldots, E_n; F)$:

• Has *order bounded variation*, in symbols $A \in L_{bv}(E_1, \ldots, E_n; F)$, if for all $0 \leq x_1 \in E_1, \ldots, 0 \leq x_n \in E_n$, the following set is order bounded in F:

$$\left\{ \sum_{i_1, \ldots, i_n = 1}^{N_1, \ldots, N_n} |A(x_{i_1}, \ldots, x_{i_n})| : N_1, \ldots, N_n \in \mathbb{N}, x_{i_1}, \ldots, x_{i_n} \geq 0, \ \sum_{i_1=1}^{N_1} x_{i_1} = x_1, \ldots, \sum_{i_n=1}^{N_n} x_{i_n} = x_n \right\}.$$

A linear operator has order bounded variation if and only if it is order bounded.

• Is *positive* if $A(x_1, \ldots, x_n) \geq 0$ for all $x_1 \geq 0, \ldots, x_n \geq 0$.

• Is *regular*, in symbols $A \in L_r(E_1, \ldots, E_n; F)$, if $A = A_1 - A_2$ with A_1 and A_2 positive.

2 Main Results

By $F^\sim$ we denote the order dual of a Riesz space F, that is, the space of order bounded (or, equivalently, regular) linear functionals. The order dual is the natural domain of the adjoint operator in the context of Riesz spaces.

Proposition 2.1. *Let $E_1, \ldots, E_n, F$ be Riesz spaces and let $A \in L_{bv}(E_1, \ldots, E_n; F)$ be given. Then:*

(a) $A^*(y^*)$ is a regular n-linear form for every functional $y^* \in F^\sim$. In particular, the adjoint $A^*\colon F^\sim \to L_r(E_1, \ldots, E_n; \mathbb{R})$ is a well-defined linear operator.

(b) A^* is an order bounded and order continuous linear operator.

(c) For all $0 \leq y^* \in F^\sim$ and all $0 \leq x_1 \in E_1, \ldots, 0 \leq x_n \in E_n$ we have $y^*(|A(x_1, \ldots, x_n)|) \leq [|A^*|(y^*)](x_1, \ldots, x_n)$.

For F Dedekind complete, every multilinear operator $A \in L_{bv}(E_1, \ldots, E_n; F)$ has a modulus $|A| \in L_{bv}(E_1, \ldots, E_n; F)$ (see [3]). It is easy to see that $|A^*| \leq |A|^*$, but in general, $|A^*| = |A|^*$ does not hold. We now show that the equality holds for regular σ-order continuous functionals.

Proposition 2.2. *Let $A \in L_{bv}(E_1, \ldots, E_n; F)$ be given, where $E_1, \ldots, E_n, F$ be Riesz spaces with F Dedekind complete. Then $|A^*|(y^*) = |A|^*(y^*)$ for every regular σ-order continuous functional $y^* \in F^\sim$.*

Proof We prove the case $n = 2$; the general argument is analogous. Let $A \in L_{bv}(E_1, E_2; F)$ be given. Fix a positive σ-order continuous functinal $y^* \in F^\sim$. As mentioned above $|A^*|(y^*) \leq |A|^*(y^*)$ holds. On the other hand, for $0 \leq a \in E_1$ and $0 \leq b \in E_2$, we have

$$\begin{aligned}
[|A|^*(y^*)](a, b) &= y^*(|A|(a, b)) \\
&= y^* \left(\sup \left\{ \sum_{i,j=1}^{N,M} |A(a_i, b_j)| : a_i \geq 0, b_j \geq 0, \sum_{i=1}^{N} a_i = a, \sum_{j=1}^{M} b_j = b \right\} \right) \\
&= \sup \left\{ \sum_{i,j=1}^{N,M} y^* \left(|A(a_i, b_j)| \right) : a_i \geq 0, b_j \geq 0, \sum_{i=1}^{N} a_i = a, \sum_{j=1}^{M} b_j = b \right\} \\
&\leq \sup \left\{ \sum_{i,j=1}^{N,M} [|A^*|(y^*)](a_i, b_j) : a_i \geq 0, b_j \geq 0, \sum_{i=1}^{N} a_i = a, \sum_{j=1}^{M} b_j = b \right\} \\
&= [|A^*|(y^*)](a, b).
\end{aligned}$$

Above, the second and the last equalities follow from [3, Theorem 4]; the third one holds because y^* is σ-order continuous; and the inequality follows from Proposition 2.1(c). Therefore, $|A|^*(y^*) \leq |A^*|(y^*)$, so $|A^*|(y^*) = |A|^*(y^*)$ for every positive σ-order continuous functional $y^* \in F^\sim$. Now, for a regular σ-order continuous functional $y^* \in F^\sim$, its modulus $|y^*|$ and $|y^*| - y^*$ are both positive σ-order continuous functionals (see [1, Theorem 1.56]). To complete the proof, note that

$$\begin{aligned}
|A^*|(y^*) &= |A^*|(|y^*| - (|y^*| - y^*)) = |A^*|(|y^*|) - |A^*|(|y^*| - y^*) \\
&= |A|^*(|y^*|) - |A|^*(|y^*| - y^*) = |A|^*(|y^*|) - |A|^*(|y^*|) + |A|^*(y^*) = |A|^*(y^*). \qquad \blacksquare
\end{aligned}$$

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NUMERICAL SIMULATION OF OF A POROUS ELASTIC SYSTEM WITH KELVIN-VOIGT DAMPING FROM THE SECOND SPECTRUM PERSPECTIVE

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Abstract

In this work, a numerical simulation is shown for a one-dimensional porous elastic system free of the second spectrum with Kelvin-Voigt damping. A numerical scheme is proposed and analyzed. Furthermore, under additional regularity assumptions on the solution, we establish numerical results demonstrating the exponential behavior of the energy.

1 Introduction

We consider the following porous system with second spectrum free and partial Kelvin-Voigt damping:

$$\begin{cases} \rho u_{tt} - \mu u_{xx} - b\phi_x = 0, \quad \text{in} \quad (0,l) \times (0.\infty), \\ -Ju_{ttx} - \delta\phi_{xx} + bu_x + \xi\phi - \gamma\phi_{txx} = 0, \quad \text{in} \quad (0,l) \times (0.\infty), \end{cases} \tag{1}$$

where l represents the distance along the center line of the beam, with respective Dirichlet boundary and initial conditions

$$\begin{aligned} &u(0,t) = u(l,t) = \phi(0,t) = \phi(l,t) = 0, \quad t > 0, \\ &u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \phi(x,0) = \phi_0(x), \quad x \in (0,l), \end{aligned} \tag{2}$$

where $\mu\xi - b^2 > 0$ and $\gamma > 0$ is the damping coefficient.

Previous works proved that the property of exponential decay holds for any coefficient values in the system. In general, numerous publications, including [1, 2, 3], have focused on determining the rate of decay in solutions for elastic systems from the second spectrum perspective.

2 Main Results

2.1 Assumptions

In this work, the existence and uniqueness of the weak solution of the system (1.1)-(1) were shown. The classical Faedo-Galerkin approximation was used together with a priori estimates and then passing through the limits using compactness arguments. We define $V = H_0^1(0,L)$, $H = L^2(0,L)$ and the Hilbert space $\mathcal{H} := H_0^1(0,l) \times H_0^1(0,l) \times L^2(0,l) \times H_0^1(0,l)$. Therefore, the following definition and theorem are necessary.

Definition 2.1. *Let the Hilbert space* $\mathcal{H} := H_0^1(0,l) \times H_0^1(0,l) \times L^2(0,l) \times H_0^1(0,l)$ *and the initial data* $(u_0, u_1, u_2, \phi_0) \in \mathcal{H}$*. Then, a function* $V = (u, u_t, u_{tt}, \phi) \in C(0,T;\mathcal{H})$ *is said to be a weak solution of* (1.1)-(1) *if it is a solution of the weak problem for almost* $t \in [0,T]$.

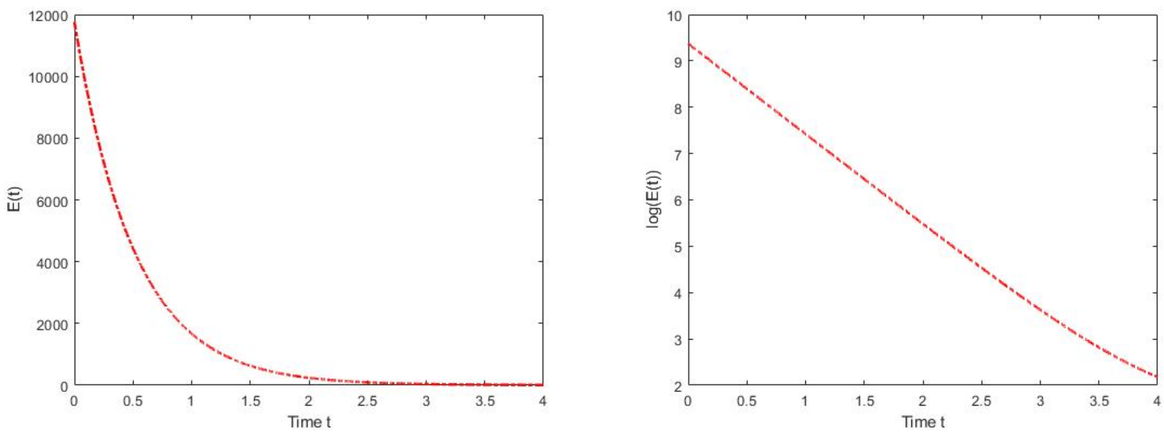
Theorem 2.1. *Suppose that the initial data* $(u_0, u_1, u_2, \phi_0) \in \mathcal{H}$ *then system* (1.1)-(1) *have a weak solution satisfying* $u, u_t, \phi, \phi_t \in L^\infty\left(0,T;H_0^1(0,l)\right), u_{tt} \in L^\infty\left(0,T;L^2(0,l)\right)$*, where the solution* $V = (u, u_t, u_{tt}, \phi)$ *depends continuously on the initial data in* $\mathcal{H}$*. In particular* V *is unique solution of system* (1.1)-(1).

2.2 Numerical simulations

Example: Let the initial conditions given respectively by

$$\begin{aligned} u(x,0) &= \frac{1}{256}\left[\frac{L}{2}\Big(\cos\Big(\frac{\nu\pi x}{L}\Big)-1\Big)+x\right]; \quad u_t(x,0)=-\frac{1}{256}\left[\frac{L}{2}\Big(\cos\Big(\frac{\nu\pi x}{L}\Big)-1\Big)+x\right]; \\ \phi(x,0) &= \frac{1}{1024}\left[\sin\Big(\frac{\nu\pi x}{L}\Big)\right]. \end{aligned} \tag{3}$$

We use the numerical coefficients $\rho = 10^5$, $J = 1$, $b = 0.1$, $\mu = 10^4$, $\xi = 10^6$, $\gamma = 10^8$, $\delta = 10^{10}$. The Figure 1 shows the energy decay and its logarithm for $T = 4$, $h = 2^{-5}$ and $\Delta t = 2^{-10}$. This experiment corroborates exponential stability without assuming the condition of equal wave speeds.



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EXISTENCE OF A POSITIVE SOLUTION TO A SECOND-ORDER NONLINEAR PROBLEM WITH MIXED BOUNDARY CONDITIONS: A SUPERLINEAR CASE

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Abstract

In this work, we present the existence of a positive solution to a second-order nonlinear problem with mixed boundary conditions. The proofs of the main results are based on the Mawhin's coincidence degree.

1 Introduction

The goal is to prove the existence of positive solutions for the problem

$$\begin{cases} u'' + b(t)g(u) = 0, \ 0 < t < T \\ u(0) = u(T) \text{ and } u'(0) = 0, \end{cases} \qquad (\mathcal{E})$$

where $g : [0, +\infty) \to [0, +\infty)$ is a continuous function such that

(g_1) $$g(0) = 0, \quad g(s) > 0 \text{ for } s > 0.$$

The weight coefficient $b : [0, T] \to \mathbb{R}$ is a L^1-function such that

(b_1) there exists $\delta > 0$ such that $b(t)$ is essentially negative on $[0, \delta]$ and also on $[T - \delta, T]$;

(b_2) there exist $m \geq 1$ intervals $I_1, \dots, I_m$, closed and pairwise disjoint, such that

$$b(t) \geq 0, \text{ for a.e. } t \in I_i, \text{ with } b(t) \not\equiv 0 \text{ on } I_i \quad (i = 1, \dots, m);$$

$$b(t) \leq 0, \text{ for a.e. } t \in [0, T] \setminus \bigcup_{i=1}^{m} I_i;$$

(b_3) $$\int_0^s b(t)dt < 0 \text{ for all } 0 < s < T.$$

Let $\lambda_1^i, i = 1, \dots, m$, be the first eigenvalue of the eigenvalue problem

$$\varphi'' + \lambda b(t)\varphi = 0, \quad \varphi|_{\partial I_i} = 0.$$

A function $g : [0, +\infty) \to [0, +\infty)$ satisfying (g_1) is *regularly oscillating* at zero if

$$\lim_{\substack{s \to 0^+ \\ \omega \to 1}} \frac{g(\omega s)}{g(s)} = 1.$$

Before proving the existence of a positive solution to problem $(\mathcal{E})$, we study the more general problem

$$\begin{cases} u'' + f(t, u, u') = 0, \quad 0 < t < T \\ u(0) = u(T) \text{ and } u'(0) = 0 \end{cases} \qquad (1)$$

where $f : [0, T] \times [0, +\infty) \times \mathbb{R} \to \mathbb{R}$ be an L^p-Carathéodory function, for some $1 \leq p \leq \infty$, satisfying certain conditions named $(f_1), (f_2)$ and (f_3).

2 Main Results

Theorem 2.1. *Assume* (f_1), (f_2), *and* (f_3), *and suppose that there exist two constants* $r, R > 0$, *with* $r \neq R$, *such that the following hypotheses are true.*

(H_1) *The condition are satisfied:*

$$\int_0^T \left(\int_0^s f(t, r, 0) dt \right) ds < 0.$$

are satisfied. Moreover, any solution $u(t)$ *of the problem*

$$\begin{cases} u'' + \vartheta f(t, u, u') = 0, \;\; 0 < t < T \\ u(0) = u(T) \text{ and } u'(0) = 0, \end{cases} \tag{2}$$

for $0 < \vartheta \leq 1$, *such that* $u(t) > 0$ *in* $[0, T]$, *satisfies* $\|u\|_\infty \neq r$.

(H_2) *There exist a non-negative function* $v \in L^p([0, T], \mathbb{R})$ *with* $v \not\equiv 0$ *and a constant* $\alpha_0 > 0$, *such that every solution* $u(t) \geq 0$ *of the problem*

$$\begin{cases} u'' + f(t, u, u') + \alpha v(t) = 0, \;\; 0 < t < T \\ u(0) = u(T) \text{ and } u'(0) = 0, \end{cases} \tag{3}$$

for $\alpha \in [0, \alpha_0]$, *satisfies* $\|u\|_\infty \neq R$.

(H_3) *There are no solutions* $u(t)$ *of* (3) *for* $\alpha = \alpha_0$ *with* $0 \leq u(t) \leq R$, *for every* $t \in [0, T]$.

Then the problem (1) has at least one positive solution $u(t)$ *with*

$$\min\{r, R\} < \max_{t \in [0,T]} u(t) < \max\{r, R\}.$$

Proof. The proof is given by a topological approach based on the Mawhin's coincidence degree. Furthermore, to ensure that the found solution is positive, we employ a maximum principle. ■

Theorem 2.2. *Let* $g(s)$ *and* $b(t)$ *be as in the introduction. Suppose also that* $g(s)$ *is regularly oscillating at zero and satisfies*

$$(g_2) \qquad \lim_{s \to 0^+} \frac{g(s)}{s} = 0 \quad \text{ and } \quad g_\infty := \liminf_{s \to +\infty} \frac{g(s)}{s} > \max_{i=1,\ldots,m} \lambda_1^i.$$

Then problem ($\mathcal{E}$) *has at least one positive solution.*

Proof.

The proof is based on showing that the hypotheses about $g(u)$ and $b(t)$ allow us to apply Theorem 2.1. ■

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KIRCHHOFF-TYPE THIN PLATES WITH MIXED BOUNDARY CONDITIONS

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Abstract

This paper deals with initial-boundary value problems for a thin quasilinear plate equation. It is shown the global existence and uniqueness of strong solutions and that the strong solution is uniformly stable (i.e., the solution's behaviour changes continuously with the data) and consequently it is unique.

1 Introduction

Our interest in this work is to study the initial-boundary value problem

$$\begin{aligned} \partial_t^2 u - \alpha\Delta\partial_t^2 u + \Delta^2 u - M\left(\cdot,\cdot,|\nabla u|^2\right)\Delta u = 0 \ &\text{ in } \ \Omega\times(0,\infty),\\ u = \partial_\nu u = 0 \ &\text{ on } \ \Gamma_0\times(0,\infty),\\ \Delta u + (1-\mu)B_1 u = 0 \ &\text{ on } \ \Gamma_1\times(0,\infty),\\ \partial_\nu\Delta u + (1-\mu)\partial_\tau B_2 u - \alpha\partial_\nu\partial_t^2 u - M\left(\cdot,\cdot,|\nabla u|^2\right)\partial_\nu u = 0 \ &\text{ on } \ \Gamma_1\times(0,\infty),\\ u(x,0) = u_0(x) \ \text{ and } \ \partial_t u(x,0) = u_1(x) \ &\text{ in } \ \Omega, \end{aligned} \tag{1}$$

where $u = u(x,t)$ is the instantaneous transverse deflection of a bar, $\alpha > 0$ is a constant, M is a real function defined in $\Omega\times[0,\infty[\times[0,\infty[$, $|\cdot|$ is the $L^2(\Omega)$ norm, $\Omega\subset\mathbb{R}^2$ is a open bounded set, and

$$B_1 u = 2\nu_1\nu_2\partial^2_{x_1x_2}u - \nu_1^2\partial^2_{x_2}u - \nu_2^2\partial^2_{x_1}u \ \text{ and } \ B_2 u = \left(\nu_1^2-\nu_2^2\right)\partial^2_{x_1x_2}u + \nu_1\nu_2\left[\partial^2_{x_2}u - \partial^2_{x_1}u\right].$$

1.1 Preliminaries

1. $V = \left\{v\in H^2(\Omega);\ v = \partial_\nu v = 0 \text{ on } \Gamma_0\right\}$ is a Hilbert space with inner product of $H^2(\Omega)$.

2. Due to the problem (1.1) the space V will be equipped with inner product and norm,

$$\begin{aligned} ((v,\varphi)) &= \int_\Omega \left[\Delta v\Delta\varphi - (1-\mu)\left(\partial^2_{x_1}v\partial^2_{x_2}\varphi + \partial^2_{x_2}v\partial^2_{x_1}\varphi\right) + 2(1-\mu)\partial^2_{x_1x_2}v\partial^2_{x_1x_2}\varphi\right]dx,\\ ||v||^2 &= ((v,v)) = |\Delta v|^2 + 2(1-\mu)\int_\Omega\left((\partial^2_{x_1x_2}v)^2 - \partial^2_{x_2x_2}v\partial^2_{x_1x_1}v\right)dx. \end{aligned}$$

3. $\left(V,((\cdot,\cdot))\right)$ is a Hilbert space, since that $||v||$ and $||v||_{H^2(\Omega)}$ are equivalent in V.

4. Let W be the vector space $W = \left\{w\in V;\ \Delta^2 w\in L^2(\Omega)\right\}$, which is a Hilbert space with the inner product $(w,\varphi)_W = ((w,\varphi)) + (\Delta^2 w,\Delta^2\varphi)$ for all $w,\varphi\in W$.

5. The total energy of problem (1.1) is the function

$$E(t) = \frac{1}{2}\left\{|u'(t)|^2 + \alpha|\nabla u'(t)|^2 + ||u(t)||^2 + \int_\Omega M\left(\cdot,t,|\nabla u(t)|^2\right)|\nabla u(t)|^2_{\mathbb{R}}\,dx\right\}.$$

6. Assumptions on M and initial conditions: there exist positive real constants d_0 and C_i, for $i = 1, 2, 3$, and an increasing function $g \in C^0((0,\infty);\mathbb{R})$ such that for all $(x,t,\lambda) \in \overline{\Omega} \times [0,\infty) \times (0,\infty)$,

$$\begin{aligned}
&M \in C^1\left(\overline{\Omega} \times [0,\infty) \times [0,\infty)\right),\ 0 \leq M(x,t,\lambda) \ \text{ where } \ \lambda = |\nabla u(t)|^2 > 0,\\
&M(x,t,\lambda) = M_1(x,t) + M_2(x,t,\lambda),\ \ \partial_t M_1(x,t) \leq -d_0,\ \ |\partial_t M_2(x,t,\lambda)|_{\mathbb{R}} \leq C_1\, g(\lambda),\\
&|\partial_\lambda M(x,t,\lambda)|_{\mathbb{R}} \leq C_2 \frac{g(\lambda)}{\sqrt{\lambda}}, \quad |\nabla M(x,t,\lambda)|_{\mathbb{R}} \leq C_3\, g(\lambda)\sqrt{\lambda},\\
&(\partial_t \nabla) M \text{ and } (\partial_\lambda) M \text{ belong to } C^0\left(\overline{\Omega} \times [0,\infty) \times [0,\infty)\right),\\
&u_0 \in W \text{ and } u_1 \in V.
\end{aligned} \tag{2}$$

2 Main Results

Theorem 2.1 (Existence of global strong solutions)**.** *Suppose that all hypotheses in (2) hold. Then there is a unique function u solution of problem (1.1) in the class*

$$u \in L^\infty(0,\infty;V) \cap L^\infty_{loc}(0,\infty;W),\ \ \partial_t u \in L^\infty\left(0,\infty;H^1(\Omega)\right) \cap L^\infty_{loc}(0,\infty;V),\ \ \partial_t^2 u \in L^\infty_{loc}\left(0,\infty;H^1(\Omega)\right),$$

$$\begin{aligned}
&\partial_t^2 u - \alpha\Delta\partial_t^2 u + \Delta^2 u - M\left(\cdot,\cdot,|\nabla u|^2\right)\Delta u = 0 \ \text{ in } \ L^2_{loc}(0,\infty;L^2(\Omega)),\\
&\widetilde{\gamma}_0 u = 0 \ \text{ in } \ L^2_{loc}(0,\infty;H^{-1/2}(\Gamma_1)),\\
&\widetilde{\gamma}_1 u - \alpha\partial_\nu\partial_t^2 u - M\left(\cdot,\cdot,|\nabla u|^2\right)\partial_\nu u = 0 \ \text{ in } \ L^2_{loc}(0,\infty;H^{-3/2}(\Gamma_1)),\\
&u(x,0) = u_0(x) \ \text{ and } \ \partial_t u(x,0) = u_1(x) \ \text{ in } \ \Omega.
\end{aligned}$$

Suppose the initial data u_0 and u_1 are perturbed by the functions w_0 and w_1 respectively. Then we get the so-called perturbed problem associated with (1.1) whose solution will be denoted by $\widehat{u}$, and has the same regularity as Theorem (2.1).

The goal is to obtain a global solution $\widehat{u}$ of the perturbed system, such that $\|u(t) - \widehat{u}(t)\| \leq C\delta$ for all $t > 0$, where $C > 0$ and $\delta > 0$ are constants and δ is arbitrary, sufficiently small, and dependent on the data.

Theorem 2.2 (The uniform stability of strong solution for problem (1.1))**.** *Under the same hypotheses of Theorem 2.1, if u and $\widehat{u}$ are global strong solutions of systems (1.1) and of the perturbed system respectively, and $(\widehat{u}_0, \widehat{u}_1)$ belongs to $\mathcal{O}_\delta\big((u_0,u_1)\big) = \big\{(\varphi,\phi) \in H^1(\Omega) \times V;\ |\nabla u_1 - \nabla\varphi|^2 + \|u_0 - \phi\|^2 < \delta\big\}$ then there exists a positive real constant $C > 0$, dependent of the initial data and independent of δ such that*

$$|\nabla u'(t) - \nabla\widehat{u}'(t)|^2 + \|u(t) - \widehat{u}(t)\|^2 \leq C\delta \ \text{ for all } \ t \geq 0. \tag{1}$$

Remark 2.1. *The uniqueness of the solution of problem (1.1) is an immediate consequence of (1).*

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THE GENERALIZED FRACTIONAL KDV EQUATION IN WEIGHTED SOBOLEV SPACES

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Abstract

This work concerns the study of persistence property in polynomial weighted spaces for solutions of the generalized fractional KdV equation in any spatial dimension $d \geq 1$. By establishing well-posedness results in conjunction with some asymptotic at infinity unique continuation principles, it is verified that dispersive effects and dimensionality mainly determine the maximum spatial decay allowed by solutions of this model. In particular, we recover and extend some known results on weighted spaces for different models such as the Benjamin-Ono equation, and the dispersion generalized Benjamin-Ono equation. The estimates obtained for the linear equation seem to be of independent interest, and they are useful to obtain persistence properties in weighted spaces for models with different nonlinearities as the fractional KdV equation with combined nonlinearities.

1 Introduction

We consider the initial value problem (IVP) associated to the generalized fractional Korteweg-de Vries equation (fKdV)

$$\begin{cases} \partial_t u - \partial_{x_1} D^a u + \nu u^{k-1} \partial_{x_1} u = 0, \quad x = (x_1, \dots, x_d) \in \mathbb{R}^d, \ t \in \mathbb{R}, \ a \in (0,2), \\ u(x,0) = u_0(x), \end{cases} \tag{1}$$

where $u = u(x,t)$ is a real-valued function, $k \geq 2$ is an integer, $\nu \in \{1, -1\}$, $D^s f = (-\Delta)^{\frac{s}{2}} f$, $s \in \mathbb{R}$ denotes the Riesz potential of negative order, which is defined through the Fourier transform as $D^s f(x) = \mathcal{F}^{-1}\big(|\xi|^s \widehat{f}(\xi)\big)(x)$, $\quad |\xi| = \sqrt{\xi_1^2 + \cdots + \xi_d^2}$. For several values of dispersion $0 < a \leq 2$, and nonlinearity $k \geq 2$, the equation in (1) has been used to model physical situations. Setting $a = 1$, and using the fact that $D^2 = -\Delta$, we have that fKdV coincides with the k-generalized d-dimensional Benjamin-Ono equation

$$\partial_t u - \mathcal{R}_1 \Delta u + \nu u^{k-1} \partial_{x_1} u = 0, \quad x \in \mathbb{R}^d, \ t \in \mathbb{R}, \tag{2}$$

where $\mathcal{R}_1 = -\partial_{x_1} D^{-1}$ denotes the Riesz transform operator in the x_1 variable. Here, we adopt the approach presented in [2, 3, 4].

2 Main Results

To state our main results, we impose certain regularity conditions, which are interconnected through the following parameters:

$$s_{d,k,1} := \frac{1}{2}\Big(\frac{d}{2} + \frac{k}{k-1}\Big) + \Big(\frac{1}{4}\Big(\frac{d}{2} + \frac{k}{k-1}\Big)^2 - \frac{d}{2}\Big)^{\frac{1}{2}}, \tag{3}$$

and

$$s_{d,k,2} := \frac{1}{2}\Big(\frac{d}{2} + \frac{k}{k-1}\Big) - \Big(\frac{1}{4}\Big(\frac{d}{2} + \frac{k}{k-1}\Big)^2 - \frac{d}{2}\Big)^{\frac{1}{2}}. \tag{4}$$

Our first result establishes local well-posedness for the Cauchy problem (1) in weighted spaces, see [1].

Theorem 2.1. *Let $d \geq 1$ be integer, $0 < a < 2$, $k \geq 2$ be integer, and $\nu \in \{1, -1\}$.*

(i) *If* $0 < r < 1$, $s > \frac{d}{2} + 1$, *then the Cauchy problem* (1) *is locally well-posed in* $H^s(\mathbb{R}^d) \cap L^2(|x|^{2r}\,dx)$.

(ii) *If* $1 \leq r < a + 1 + \frac{d}{2}$, $s \geq \{(\frac{d}{2}+1)^+, ar+1\}$ *with* $s \in (0, s_{d,k,2}) \cup (s_{d,k,1}, \infty)$, *then the Cauchy problem* (1) *is locally well-posed in* $H^s(\mathbb{R}^d) \cap L^2(|x|^{2r}\,dx)$.

(iii) *If* $a + 1 + \frac{d}{2} \leq r < a + 2 + \frac{d}{2}$, $s \geq \{(\frac{d}{2}+1)^+, ar+1\}$ *with* $s \in (0, s_{d,k,2}) \cup (s_{d,k,1}, \infty)$, *then the Cauchy problem* (1) *is locally well-posed in*

$$H^s(\mathbb{R}^d) \cap L^2(|x|^{2r}\,dx) \cap \{f \in H^s(\mathbb{R}^d) : \int_{\mathbb{R}^d} f(x)\,dx = 0\}.$$

Our next result establishes a unique continuation principle for solutions of the IVP (1), see [1].

Theorem 2.2. *Let* $d \geq 1$ *be integer,* $k \geq 2$ *be an even integer,* $0 < a < 2$, $s \geq \{(\frac{d}{2}+1)^+, a(a+2+\frac{d}{2})+1\}$ *with* $s \in (0, s_{d,k,2}) \cup (s_{d,k,1}, \infty)$, *and* $\nu \in \{1, -1\}$. *Let* $u \in C([0,T]; H^s(\mathbb{R}^d) \cap L^2(|x|^{2(a+2+\frac{d}{2})^-}\,dx))$ *be a solution of* (1) *with* $\int_{\mathbb{R}^d} u_0(x)\,dx = 0$.

(i) *If there exist three different times* $t_1, t_2, t_3 \in [0,T]$ *such that*

$$u(\cdot, t_1), u(\cdot, t_2), u(\cdot, t_3) \in L^2(|x|^{2(a+2+\frac{d}{2})}\,dx), \tag{5}$$

then $u \equiv 0$.

(ii) *If there exist two times such that*

$$u(\cdot, t_1), u(\cdot, t_2) \in L^2(|x|^{2(a+2+\frac{d}{2})}\,dx), \tag{6}$$

and

$$\int_{\mathbb{R}^d} x_1 u(x, t_1)\,dx = 0, \qquad \text{or} \qquad \int_{\mathbb{R}^d} x_1 u(x, t_2)\,dx = 0. \tag{7}$$

Then $u \equiv 0$.

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ON THE ORBITAL STABILITY OF PERIODIC SNOIDAL WAVES FOR THE ϕ^4−EQUATION

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Abstract

The main purpose of this work is to investigate the global well-posedness and orbital stability of odd periodic traveling waves for the ϕ^4-equation in the Sobolev space of periodic functions with zero mean. We establish the existence of a global weak solution by combining semigroup methods with energy estimates. The orbital stability is then obtained via a Morse index theorem applied to the linearized operator.

1 Introduction

We consider the well-known ϕ^4−equation

$$\phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0, \tag{1}$$

where $\phi : \mathbb{R} \times \mathbb{R}_+ \to \mathbb{R}$ is an L-periodic function at the spatial variable. This model arises in particle and nuclear physics and admits kinks, anti-kinks, and periodic solutions. Our focus is on the odd periodic traveling waves of the form $\phi(x,t) = h(x - ct)$, where $c \in \mathbb{R}$ is the wave speed, and h satisfies the ODE

$$-\omega h'' - h + h^3 = 0, \quad \omega = 1 - c^2. \tag{2}$$

A family of odd periodic solutions is explicitly expressed in terms of the Jacobi elliptic function as

$$h(x) = \frac{\sqrt{2k}}{\sqrt{k^2+1}} sn\left(\frac{4K(k)}{L}x; k\right), \quad k \in (0,1), \tag{3}$$

satisfying the relation $\omega = \frac{L^2}{16K^2(k)(1+k^2)}$, where $K(k)$ is the complete elliptic integral of the first kind. This guarantees that the snoidal-type solution h possesses the zero-mean property.

In this context, one of the most important features of our work is the proof of orbital stability in the space $Y = H^1_{per,m} \times L^2_{per,m}$, which lies between $H^1_{per,odd} \times L^2_{per,odd}$ and the full space $H^1_{per} \times L^2_{per}$ (see [2, 3]).

2 Local and global well-posedness

Let us consider the well-known Cauchy problem associated with the evolution equation (1)

$$\begin{cases} \phi_{tt} - \phi_{xx} - \phi + \phi^3 = 0, & \text{in } [0,L] \times (0,+\infty), \\ \phi(x,0) = \phi_0(x), \quad \phi_t(x,0) = \phi_1(x), & \text{in } [0,L]. \end{cases} \tag{4}$$

It is not possible to guarantee, using the standard semigroup approach, that (4) is locally well-posed in $H^2_{per,m} \times H^1_{per,m}$, since it is not natural to expect that $H(\phi,\psi) = \int_0^L \phi(x,t)dx$ is a conserved quantity for all $t > 0$. To resolve this challenge, let us define $\phi_t = \partial_x^{-1}\psi$. It is necessary to examine the auxiliary Cauchy problem related to the equation in (4) expressed by

$$\begin{cases} \begin{pmatrix} \phi \\ \psi \end{pmatrix}_t = \begin{pmatrix} \partial_x^{-1} & 0 \\ 0 & \partial_x \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \partial_x^2 & 0 \end{pmatrix} \begin{pmatrix} \phi \\ \psi \end{pmatrix} + \begin{pmatrix} 0 \\ \partial_x(\phi - \phi^3) \end{pmatrix}, & \text{in } [0,L] \times (0,+\infty), \\ \begin{pmatrix} \phi(0) \\ \psi(0) \end{pmatrix} = \begin{pmatrix} \phi_0 \\ \psi_0 \end{pmatrix}, & \text{in } [0,L]. \end{cases} \tag{5}$$

We first need to obtain local strong solutions to the auxiliary problem in (5) by applying the abstract semigroup theory. To this end, we prove that the linear (unbounded) operator $A = \begin{pmatrix} \partial_x^{-1} & 0 \\ 0 & \partial_x \end{pmatrix} \begin{pmatrix} 0 & 1 \\ \partial_x^2 & 0 \end{pmatrix}$ defined in $X = H^2_{per,m} \times L^2_{per,m}$ with domain $D(A) = H^3_{per,m} \times H^1_{per,m}$ is a generator of a contraction semigroup on X.

Based on Banach's Fixed Point Theorem, we establish the existence of a local (strong) solution of the Cauchy problem (5) for initial data $(\phi_0, \psi_0) \in H^3_{per,m} \times H^1_{per,m}$. This result is sufficient to prove the well-posedness of the Cauchy problem (4), considering strong data $(\phi_0, \phi_1) \in H^3_{per,m} \times H^2_{per,m}$.

We also show that the local (strong) solution (ϕ, ϕ_t) is global in time in $H^1_{per,m} \times L^2_{per,m}$. Finally, for $L \in (0, 2\pi)$, we prove the existence and uniqueness of a weak solution $(\phi, \phi_t) \in C([0,+\infty), Y)$ of (4).

3 Spectral analysis and orbital stability for the ϕ^4−equation

Let $L \in (0, 2\pi)$ be fixed and consider $c \in (-1, 1)$. We begin by considering the operators defined by

$$\mathcal{L} = \begin{pmatrix} -\partial_x^2 - 1 + 3h^2 & c\partial_x \\ -c\partial_x & 1 \end{pmatrix}, \qquad \mathcal{L}_\Pi = \mathcal{L} - \begin{pmatrix} \frac{3}{L}\int_0^L h^2 \cdot \, dx & 0 \\ 0 & 0 \end{pmatrix}. \tag{6}$$

As shown in [2, Proposition 3.8], the operator $\mathcal{L}$ has exactly one simple negative eigenvalue, and zero is a simple eigenvalue with eigenfunction (h', ch''). Additionally, the remaining spectrum consists of a discrete set of eigenvalues. Next, we prove that the linearized constraint operator $\mathcal{L}_\Pi$ has no negative eigenvalues and a simple zero eigenvalue with eigenfunction (h', ch'').

Based on these facts, we assert the existence of a constant $C > 0$ satisfying

$$(\mathcal{L}(p,q),(p,q))_{\mathbb{L}^2_{per,m}} = (\mathcal{L}_\Pi(p,q),(p,q))_{\mathbb{L}^2_{per,m}} \geq C||(p,q)||^2_{\mathbb{L}^2_{per,m}}, \quad ((p,q),(h',ch''))_{\mathbb{L}^2_{per}} = 0. \tag{7}$$

As established in [1], the condition in (7) is sufficient to prove that the periodic wave (h, ch') is orbitally stable in Y, recalling that the instability is known in the full energy space. Thus, our results offer a new perspective on periodic waves with zero mean in the context of Klein-Gordon-type equations.

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SEMILINEAR REACTION-DIFFUSION EQUATIONS WITH MEMORY

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Abstract

In this work, we study the initial value problem associated with semilinear reaction-diffusion equations with memory and critical nonlinearity. We prove local-in-time existence, uniqueness, continuation, and blow-up alternative of regular mild solutions that satisfy a specific condition of controlled behavior at $t = 0$.

1 Introduction

To problems of heat flow in homogeneous isotropic rigid heat conductors $\Omega \subset \mathbb{R}^N$ subject to hereditary memory, Gurtin and Pipkin [2] proposed to replace the Fourier's constitutive law for the heat flux, which leads to the classical heat equation, by

$$q(x,t) = -\int_0^\infty g(t-s)\nabla u(x,s)ds.$$

This model for the constitutive law for the flux has been well accepted in modeling heat conduction phenomena in materials with memory, and leads us to equations of the form

$$\begin{cases} u_t = \displaystyle\int_0^t g(t-s)\Delta u(x,s)\,ds + f(u), & \text{in } \Omega \times (0,\infty), \\ u(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,\infty), \\ u(x,0) = u_0(x), \quad x \in \Omega. \end{cases} \tag{1}$$

The scalar kernel g, also called *material function*, is associated with the physical properties (more precisely, to the *shear modulus*) of the type of material under consideration. Here, we consider a general scalar kernel of the form

$$g(t) = \sum_{i=1}^{n} k_i t^{\alpha_i - 1} e^{c_i t}, \quad t > 0, \tag{2}$$

where $k_i > 0, \alpha_i > 0, c_i \in \mathbb{R}$. We mention that many types of materials, as the *Hookean solid*, the *Maxwell fluid*, the *Poynting-Thompson solid*, and the *power type materials*, are described by material functions of the form (2), see [1, 3]. Note that $g \in L^1_{loc}([0,\infty);\mathbb{C})$ is a non identically zero Laplace transformable function such that $\widehat{g}(\cdot)$ admits meromorphic extension to some sector $\Sigma(\omega_0, \eta_0 + \pi/2)$, with $\omega_0 \geq 0$ and $\eta_0 \in (0, \pi/2]$. Also $\widehat{g}(\lambda) \neq 0$, for all $\lambda \in \Sigma(\omega_0, \eta_0 + \pi/2)$, and, for each $\omega_1 > \omega_0$ and $\eta_1 \in (0, \eta_0)$, there exists $\psi_1 \in (0, \pi/2)$ with $-\lambda/\widehat{g}(\lambda) \in \mathbb{C}\backslash\Sigma[0, \psi_1]$, for all $\lambda \in \Sigma(\omega_1, \eta_1 + \pi/2)$. Furthermore, for some $\omega > \omega_0$ and $\eta \in (0, \eta_0)$, there is $\zeta_g > 1$ so that

$$\limsup_{\substack{|\lambda|\to\infty,\\ \lambda\in\Sigma(\omega,\eta+\pi/2)}} \frac{1}{|\widehat{g}(\lambda)||\lambda|^{\zeta_g - 1}} < \infty.$$

Our main result deals with well-posedness and regularity theory to (1) when the initial data u_0 belongs to $L^q(\Omega)$, $1 < q < \infty$, and the nonlinear term f has a critical behavior.

2 Main Results

Theorem 2.1. *Let* $\{F^\alpha\}_{\alpha\in\mathbb{R}}$ *be the fractional power scale associated with the operator* $-\Delta$ *on* $L^q(\Omega)$ *with domain* $W^{2,q}(\Omega)\cap W_0^{1,q}(\Omega)$ *and let* $1<\rho<1+\frac{2q}{N}$. *Suppose* $g(\cdot)$ *satisfies (H1)-(H4) with* $1<\zeta_g\leq\frac{2q}{N(\rho-1)}$ *and let* ϵ *such that*

$$\max\left\{0,\frac{1}{\rho}\left(\frac{1}{\zeta_g}-\frac{N}{2q'}\right)\right\}<\epsilon<\min\left\{\frac{1}{\rho\zeta_g},\frac{N}{2q}\right\}.$$

Then, for all $w\in L^q(\Omega)$, *there exist* $r=r(w)>0$ *and* $\tau_0=\tau_0(w)>0$ *such that the problem* (1) *has an* ϵ*-regular mild solution* $u(\,\cdot\,,u_0)$ *defined on* $[0,\tau_0]$ *for all* u_0 *in the closed ball* $B_{L^q(\Omega)}[w,r]$. *Moreover, the following statements hold.*

(A) For all $\theta\in(0,\rho\epsilon)$, *we have*

$$u(\,\cdot\,,u_0)\in C\left((0,\tau_0];F^\theta\right)$$

and, if $J\subset B_{L^q(\Omega)}[w,r]$ *is compact, then*

$$\lim_{t\to0^+}t^{\zeta_g\theta}\sup_{u_0\in J}\|u(t;u_0)\|_{F^\theta}=0.$$

(B) For each $\theta\in[0,\rho\epsilon)$, *there exists a constant* $L=L(\theta,w)$ *such that*

$$t^{\zeta_g\theta}\|u(t;u_0)-u(t,u_1)\|_{F^\theta}\leq L\|u_0-u_1\|_{L^q(\Omega)},\quad t\in(0,\tau_0],\,u_0,u_1\in B_{L^q(\Omega)}[w,r].$$

(C) The ϵ*-regular mild solution* $u(\cdot;u_0)$ *can be continued on an interval* $[0,\tau_{\max})$, *where* $\tau_{\max}\in(\tau_0,\infty]$. *If* $\tau_{\max}<\infty$, *then*

$$\limsup_{t\to\tau_{max}{}^-}\|u(t;u_0)\|_{F^\epsilon}=\infty.$$

Moreover, if v *is an* ϵ*-regular mild solution on some interval* $[0,\tau_1]$ *for the problem* (1) *satisfying*

$$\lim_{t\to0^+}t^{\zeta_g\epsilon}\|v(t)\|_{F^\epsilon}=0,$$

then $\tau_1<\tau_{\max}$ *and* $v(t)=u(t;u_0)$ *for all* $t\in[0,\tau_1]$.

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HYPERBOLIC-ELLIPTIC SHEAR BEAM MODEL WITH KELVIN-VOIGT DAMPING

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Abstract

This study investigates the dynamic behavior of a shear beam model incorporating Kelvin-Voigt type damping. The well-posedness of the system is established via the Faedo-Galerkin method, ensuring the existence and uniqueness of weak solutions. For the stabilization analysis, an energy perturbation technique is employed. It is shown that the system exhibits exponential stability when the Kelvin-Voigt mechanism acts either in both the shear force and the bending moment, or in the shear force alone. In contrast, if the damping is applied solely to the bending moment, the system exhibits only polynomial decay. Owing to the absence of rotational inertia in the shear model, the governing equation for the angle rotation becomes elliptic in nature, thereby precluding exponential decay under such partial damping. This structural feature is briefly discussed from a physical perspective.

1 Introduction

The shear model for wave propagation in elastic beam-like structures defines an important class of coupled partial differential equations with wide applications in engineering and, more recently, in mathematical analysis. It can be regarded as a Timoshenko-type system [3], but without rotational inertia as presented by Han *et al.* [1] and given by

$$\rho_1\varphi_{tt} - \kappa(\varphi_x + \psi)_x = 0, \tag{1}$$
$$-\psi_{xx} + \kappa(\varphi_x + \psi) = 0. \tag{2}$$

Chronologically [1], the shear beam appears as an intermediate model between the hyperbolic Rayleigh beam model

$$\rho_1\varphi_{tt} + b\varphi_{xxxx} - \rho_2\varphi_{xxtt} = 0, \tag{3}$$

and the coupled hyperbolic Timoshenko beam equations [3]

$$\rho_1\varphi_{tt} - \kappa(\varphi_x + \psi)_x = 0, \tag{4}$$
$$\rho_2\psi_{tt} - b\psi_{xx} + \kappa(\varphi_x + \psi) = 0. \tag{5}$$

Therefore, the shear beam model stands as the first coupled system addressing wave propagation in elastic prismatic beam structures.

While the Rayleigh model incorporates rotary inertia but neglects shear deformation, and the classical Timoshenko model includes both effects, the shear beam retains shear deformation while neglecting rotary inertia. This simplification results in a system that is still rich in dynamic behavior but mathematically more tractable in certain settings. Moreover, the absence of rotational inertia leads to an elliptic-type equation for the rotational angle variable, which significantly affects the stability and dispersive properties of the model. These features make the shear beam model particularly appealing for analytical studies focused on well-posedness, asymptotic behavior, and control as well.

In this work, we are concerned with the well-posedness and stabilization studies for the Shear beam model subject to Kelvin-Voigt. Our investigations are motivated by the analysis of the dissipative counterpart of the classical Timoshenko beam model, carried out by Malacarne and Rivera [2], we are concerned the study of the following model:

$$\rho_1\varphi_{tt} - \kappa(\varphi_x + \psi)_x - \mu_1(\varphi_x + \psi)_{xt} = 0, \tag{6}$$

$$-b\psi_{xx} - \mu_2\psi_{xxt} + \kappa(\varphi_x + \psi) + \mu_1(\varphi_x + \psi)_t = 0, \tag{7}$$

$$\varphi(x,0) = \varphi_0(x),\ \varphi_t(x,0) = \varphi_1(x),\ \psi(x,0) = \psi_0(x),\ x \in (0,L), \tag{8}$$

$$\varphi(0,t) = \varphi(L,t) = \psi_x(0,t) = \psi_x(L,t) = 0,\ t \geq 0. \tag{9}$$

2 Main Results

Theorem 2.1. *The following statements regarding the system (6)-(9) are valid:*

(i) If the initial data $W_0 = (\varphi_0, \varphi_1, \psi_0) \in H_0^1(0,L) \times L^2(0,L) \times H_\star^1(0,L)$, then system (6)-(9) possesses a unique weak global solution

$$\varphi \in L^\infty\left(0,T;H_0^1(0,L)\right),\ \varphi_t \in L^\infty\left(0,T;L^2(0,L)\right), \psi \in L^\infty\left(0,T;H_\star^1(0,L)\right),\ \psi_t \in L^2\left(0,T;H_\star^1(0,L)\right).$$

(ii) If the initial data $W_0 = (\varphi_0, \varphi_1, \psi_0) \in H^2(0,L) \cap H_0^1(0,L) \times H_0^1(0,L) \times H_\star^2(0,L)$, then system (6)-(9) possesses a unique weak global solution

$$\varphi \in L^\infty\left(0,T;H^2(0,L)\cap H_0^1(0,L)\right), \varphi_t \in L^\infty\left(0,T;H_0^1(0,L)\right), \psi \in L^\infty\left(0,T;H_\star^2(0,L)\right), \psi_t \in L^2\left(0,T;H_\star^2(0,L)\right).$$

(iii) In both cases: (i) and (ii), the solution $(\varphi, \varphi_t, \psi)$ of the system (6)-(9) depends continuously on the initial data $(\varphi_0, \varphi_1, \psi_0)$.

Theorem 2.2. *The energy functional $E(t)$ of the system (6)-(9) decays exponentially as time t goes to infinity if $\mu_1, \mu_2 > 0$ or $\mu_1 > 0, \mu_2 = 0$, and decays polinomially as time t goes to infinity if $\mu_1 = 0, \mu_2 > 0$.*

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EXISTENCE AND STABILITY OF PULLBACK EXPONENTIAL ATTRACTORS FOR A CLASS OF NON-AUTONOMOUS REACTION-DIFFUSION EQUATIONS

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Abstract

This work establishes the existence and stability of pullback exponential attractors for a class of non-autonomous reaction-diffusion equations. For this purpose, an abstract theoretical result is developed, which applies in particular to heat-type equations. Furthermore, the upper and lower semicontinuity of the family of pullback exponential attractors is proved with respect to a parameter in the system. Additionally, the existence of a pullback attractor in a suitable space is obtained, along with its upper semicontinuity.

1 Introduction

The aim of this work is to investigate and prove the existence and stability of a family of pullback exponential attractors for the class of non-autonomous reaction-diffusion equations described by:

$$\begin{cases} \dfrac{\partial v}{\partial t} + \lambda_\epsilon(t)v - \Delta v = f(v) + \sigma(x,t), \quad x \in \Omega,\ t \geq \tau, \\ v(x,t)\Big|_{t=\tau} = v_\tau(x), \quad v(x,t)\Big|_{\partial\Omega} = 0, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, $N \geq 3$, $\epsilon \in [0,1]$ is a parameter, and $\lambda_\epsilon : \mathbb{R} \longrightarrow \mathbb{R}$ is a function in $\mathcal{C}^1(\mathbb{R})$ satisfying $0 < a_0 \leq \lambda_\epsilon(t) \leq a_1$ for all $t \in \mathbb{R}$ and $\epsilon \in [0,1]$. The nonlinear term $f(\cdot) \in \mathcal{C}^1(\mathbb{R})$ satisfies certain growth conditions, and the external force σ belongs to the space $L^2_b(\mathbb{R}; L^2(\Omega))$, the space of translation bounded functions. In general, this class of nonlinear parabolic equations (reaction-diffusion) models chemical reactions in spatial domains where diffusion plays a significant role. Our results establish that this class of equations admits pullback exponential attractors that are stable with respect to the parameter ϵ, contributing to a deeper understanding of the asymptotic behavior of solutions to the model. Furthermore, we prove the upper and lower semicontinuity of this family of pullback exponential attractors at time zero. As a particular case, we also obtain the existence of a pullback attractor in a appropriate space and establish its upper semicontinuity.

2 Main Results

To establish existence and uniqueness of solutions, we employ the Galerkin method, followed by extension arguments to obtain global-in-time solutions. As a consequence, we ensure the existence and stability-under suitable conditions-of a pullback exponential attractor. More precisely, for each $\theta \in (\lambda, 1)$ and $\epsilon \in [0,1]$, the evolution process $\{S^{(\epsilon)}_{t\geq s}\}$ admits a pullback exponential attractor $\{\mathcal{M}^\epsilon_\theta(t) : t \in \mathbb{R}\}$.

With the following theorem, we show the existence of a pullback attractor for the evolution process $\{S^{(\epsilon)}(t,s)\}_{t\geq s}$.

Theorem 2.1. *For each $\epsilon \in [0,1]$, the evolution process $\{S^{(\epsilon)}(t,s)\}_{t\geq s}$ admits a pullback attractor $\{\mathcal{A}^\epsilon(t)\}_{t\in\mathbb{R}}$ in $L^2(\Omega)$. Furthermore, for every $\theta \in (\lambda, 1)$, with $\lambda \in (0,1)$ suitably chosen, it holds that $\mathcal{A}^\epsilon(t) \subset \mathcal{M}^\epsilon_\theta(t)$ for all $t \in \mathbb{R}$.*

With the following theorems, we further prove the upper and lower semicontinuity of this family of pullback exponential attractors with respect to a parameter in the system. As a particular case, we also establish the existence of a pullback attractor in a suitable space and prove its upper semicontinuity.

Theorem 2.2. *For any $\theta \in (\lambda, 1)$, the family of pullback exponential attractors $\{\mathcal{M}_\theta^\epsilon(t) : t \in \mathbb{R}\}_{\epsilon\in[0,1]}$ is such that there exists $\epsilon_1 \in [0,1]$ satisfying*

$$\sup_{t\in\mathbb{R}} \left\{ \mathrm{dist}_H^{symm} \left(\mathcal{M}_\theta^\epsilon(t), \mathcal{M}_\theta^0(t) \right) \right\} \leq C \left\| \lambda_\epsilon - \lambda_0 \right\|_{L^\infty(\mathbb{R})}^\zeta, \quad 0 \leq \epsilon < \epsilon_1,$$

for some $0 < \zeta < 1$ and $C > 0$ independent of ϵ. In particular, $\{\mathcal{M}_\theta^\epsilon(t) : t \in \mathbb{R}\}_{\epsilon\in[0,1]}$ is upper and lower semicontinuous at $\epsilon_0 = 0$ for each choice of $\theta \in (\lambda, 1)$, that is, given $\theta \in (\lambda, 1)$, we have

$$\lim_{\epsilon\to 0} \left[\sup_{t\in\mathbb{R}} \left\{ \mathrm{dist}_H \left(\mathcal{M}_\theta^\epsilon(t), \mathcal{M}_\theta^0(t) \right) \right\} \right] = \lim_{\epsilon\to 0} \left[\sup_{t\in\mathbb{R}} \left\{ \mathrm{dist}_H \left(\mathcal{M}_\theta^0(t), \mathcal{M}_\theta^\epsilon(t) \right) \right\} \right] = 0.$$

Theorem 2.3. *The family of pullback attractors $\{\mathcal{A}^\epsilon(t) : t \in \mathbb{R}\}_{\epsilon\in[0,1]}$ is upper-semicontinuous at $\epsilon_0 = 0$, that is, for each $t \in \mathbb{R}$ it holds*

$$\lim_{\epsilon\to 0} \left[\mathrm{dist}_H \left(\mathcal{A}^\epsilon(t), \mathcal{A}^0(t) \right) \right] = 0.$$

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EXISTENCE OF SINGULAR SOLUTIONS FOR NONLINEAR WAVE EQUATIONS

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Abstract

We derive new results about existence and uniqueness of local solutions for the nonlinear wave equation. Our analysis is performed in the framework of Marcinkiewicz spaces.

1 Introduction

We consider the nonlinear wave equation

$$\partial_{tt}u - \Delta u = |u|^{\rho}u,\ x \in \mathbb{R}^n,\ t \in \mathbb{R}, \tag{1}$$

$$u(0,x) = f(x),\ x \in \mathbb{R}^n, \tag{2}$$

$$u_t(0,x) = g(x) \tag{3}$$

where $u = u(t,x)$ is a real valued function, $1 < \rho < \infty$ and $f : \mathbb{R}^n \to \mathbb{R}$, $g : \mathbb{R}^n \to \mathbb{R}$ are a given initial value. As is well known, the Cauchy problem (1)-(2) is formally equivalent to the integral equation

$$u(t) = \partial_t W(t)f + W(t)g - \int_0^t W(t-s)(|u(s)|^{\rho}u(s))ds, \tag{4}$$

where $W(t)$ is the unitary group determined by the linear wave equation

$$\partial_{tt}u - \Delta u = 0,\ x \in \mathbb{R}^n,\ t \in \mathbb{R}.$$

We begin by recalling some facts about the Lorentz spaces. For more details see, for instance, [?] and [?].

Let $1 < p \leq \infty$ and $1 \leq q \leq \infty$. A measurable function f defined on $\mathbb{R}^n$ belongs to Lorentz space $L^{(p,q)}(\mathbb{R}^n)$ if the quantity

$$\|f\|_{(p,q)} = \begin{cases} \left(\frac{p}{q}\int_0^{\infty}\left[t^{\frac{1}{p}}f^{**}(t)\right]^q \frac{dt}{t}\right)^{\frac{1}{q}} & , \text{if} \quad 1 < p < \infty,\ 1 \leq q < \infty, \\ \sup_{t>0} t^{\frac{1}{p}} f^{**}(t) & , \text{if} \quad 1 < p \leq \infty,\ q = \infty, \end{cases}$$

is finite, where

$$f^{**}(t) = \frac{1}{t}\int_0^t f^*(s)\ ds, \quad f^*(t) = \inf\big\{s > 0 : m\{x \in \mathbb{R}^n : |f(x)| > s\} \leq t\big\}, t > 0.$$

Note that $L^p(\mathbb{R}^n) = L^{(p,p)}(\mathbb{R}^n)$. The spaces $L^{(p,\infty)}(\mathbb{R}^n)$ are called weak-L^p spaces or Marcinkiewicz spaces.

Definition 1.1. *We define the following Banach spaces*

$$E \equiv BC\left((0,\infty), L^{(\rho_0,\infty)}\right)$$

$$E_{q_1}^T \equiv BC\left((0,T), L^{(q_1,\infty)}\right)$$

with the norms defined, respectively, as

$$\|u\|_E = \sup_{t>0}\|u(t)\|_{(\rho_0,\infty)},$$

$$\|u\|_{E_{q_1}^T} = \sup_{0<t<T}\|u(t)\|_{(q_1,\infty)}$$

2 Main Results

Definition 2.1. *A global mild solution of the initial value Problem (1)-(3) in E and E_q is a function $u(t)$ in the corresponding space satisfying*

$$u(t) = \partial_t W(t)f + W(t)g + B(u)(t) \equiv \partial_t W(t)f + W(t)g - \int_0^t W(t-s)|u|^\rho u\,(s)\,ds, \tag{1}$$

Our main results are

Lemma 2.1. *Let $1 \leq d \leq \infty$, $1 < p < 2$ and $p_1 < q < \frac{2p}{2-p}$. There exists a constant $C = C(n, p_1, q) > 0$ such that*

$$\|W(t)\varphi\|_{(q,d)} \leq C t^{-n(\frac{1}{p_1}-\frac{1}{q})+1} \|\varphi\|_{(p_1,d)}\,, \tag{2}$$

for all $\varphi \in L^{(p_1,d)}$ and $t > 0$.

Theorem 2.1. *(Local in time solutions, $n = 2$) Let $0 < \rho < \infty$ and $\max\{2\rho, \rho+1\} < q_1$.*

- *If f, g is a distribution such that $\sup\limits_{0<t<T} \|\partial W(t)f\|_{(q_1,\infty)} < \dfrac{\varepsilon}{2}$, $\sup\limits_{0<t<T} \|W(t)g\|_{(q_1,\infty)} < \dfrac{\varepsilon}{2}$ small enough, then there exists $0 < T < \infty$ such that the initial value problem (1)-(2) has a unique mild solution $u(t,x) \in E^T_{q_1}$.*

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ON INTEGRODIFFERENTIAL $P(X)$-KIRCHHOFF TYPE EQUATION WITH CONVECTION AND LAPLACIAN DEPENDENCE

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Abstract

This paper deals with the existence of global solutions to nonlinear integrodifferential equations involving a Kirchhof type $p(x)$-biharmonic operator, under Dirichlet boundary conditions, with a $(\nabla u, \triangle u)$-dependent nonlinearity f. Our method is mainly based on the topological degree theory for compact perturbations of monotone type operators, within the context of Sobolev spaces with variable exponents.

1 Introduction

In this research, we focus on the following nonlinear integrodifferential problem

$$\begin{aligned} \frac{\partial u}{\partial t} - M\Big(L(u)\Big)\triangle\Big(a(|\triangle u|^{p(x)})|\triangle u|^{p(x)-2}\triangle u\Big) + a\frac{\partial}{\partial t}\int_\Omega u\,dx + f(x,t,u,\nabla u,\triangle u)& \\ = h(x,t) \ \text{ in } \Omega,& \qquad (1)\\ u = 0 = \frac{\partial u}{\partial \nu} \qquad\qquad \text{on } \partial\Omega.& \\ u(x,0) = u_0(x), \qquad\qquad x \in \Omega.& \end{aligned}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $N \geq 2$, with a smooth boundary $\partial\Omega$, T is a fixed positive number, $p(x) \in C(\overline{\Omega})$ with $p(x) > 1$ for any $x \in \overline{\Omega}$, $L(u) = \int_\Omega \frac{1}{p(x)}|\triangle u|^{p(x)}\,dx$ is a $p(x)$-biharmonic type functional, e, M, f and h are functions that satisfy conditions which will be stated later.

Recently, higher-order equations have gained much importance in studies. In particular, fourth order parabolic equation are widely used to model a variety of phenomena in science and engineering, for instance inverse problem involving the identifcation of a space dependent source term [3] and edge preservation in image processing [4]. Also, due to the simultaneous involvement of the $p(x)$-biharmonic operator and the nonlocal Kirchhoff function $M\Big(L(u)\Big)$, problems as (1) are of interest to several fields of application of the study of partial differential equations (see [?, 2]). Here, our work covers a wide class of nonlocal parabolic problems and the main tool to tackle it, is the topological degree theory for compact perturbations of monotone type operators (See [1]).

2 Notations and Main Results

Throughout this paper we assume that $1 < p(x < \infty)$, $p(x) \in C(\overline{\Omega})$.

Set $C_+(\overline{\Omega}) = \{p(x) \in C(\overline{\Omega}) : p(x) > 1, \forall x \in \overline{\Omega}\}$; $p^+ = max\{p(x); x \in C(\overline{\Omega})\}$,
$p^- = min\{p(x); x \in C(\overline{\Omega})\}$, and define $L^{p(x)}(\Omega) = \{u : \Omega \to \mathbb{R}$ measurable and $\int_\Omega |u(x)|^{p(x)}dx < \infty\}$, endowed with the norm $|u|_{p(x)} = inf\{\lambda > 0 : \int_\Omega |\frac{u(x)}{\lambda}|^{p(x)}dx \leq 1\}$. So $L^{p(x)}(\Omega)$ is separable and reflexive. For any positive integer k, let $W^{k,p(x)}(\Omega) = \{u \in L^{p(x)}(\Omega) : D^\alpha u \in L^{p(x)}(\Omega), |\alpha| \leq k\}$.

Then $W^{k,p(x)}(\Omega)$ is a separable and reflexive Banach space equipped with the norm

$$\|u\|_{k,p(x)} = \sum_{|\alpha|\leq k} |D^\alpha u|_{p(x)}.$$

The space $W_0^{k,p(x)}(\Omega)$ is the closure of $C_0^\infty(\Omega)$ in $W^{k,p(x)}(\Omega)$.
Now, we set $\gamma(x) = (1 - H(k_3))p(x) + H(k_3))q(x)$, where $H(k) = 1$ if $k > 0$ and $H(k) = 0$ if $k = 0$.
Let the space $X = W_0^{2,p(x)}(\Omega) \cap W_0^{2,\gamma(x)}(\Omega)$ endowed with the norm $\|u\| = \|u\|_{2,p(x)} + H(k_3)\|u\|_{2,q(x)}$.

We give the following hypotheses.

(M_0) $M : [0, +\infty[\to]m_0, m_1[$ is a continuous and nondecreasing function with $m_0, m_1 > 0$.

(a_0) The function $a : \mathbb{R}^+ \to \mathbb{R}^+$ is of class C^1 and there exist positive constants $k_0, k_1 > 0$, $k_2, k_3 \geq 0$, $q^- > p^+$ such that

$$k_0 + H(k_3)k_2 t^{(q(x)-p(x))/p(x)} \leq a(t) \leq k_1 + k_3 t^{(q(x)-p(x))/p(x)} \text{ for all } \ t \geq 0,$$

(a_1) There exists $c > 0$ such that $\min\{a(t^{p(x)})t^{p(x)-2}, a(t^{p(x)})t^{p(x)-2} + t\frac{\partial(a(t^{p(x)})t^{p(x)-2})}{\partial t}\} \geq ct^{p(x)-2}$, for all $t \geq 0$ and for all $x \in \bar{\Omega}$.

(F_1) $f : \Omega \times (0,T) \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \longrightarrow \mathbb{R}$ is a Carathêodory function such that
i)$|f(x,t,\eta,\xi,\zeta)| \leq c_1(|\eta| + |\xi| + |\zeta|) + k(x,t), \quad k \in L^\infty(0,T; L^{p(x)}(\Omega))$;
ii) $f(x,t,\eta,\xi,\zeta)\eta \geq |\eta|^{r(x)}$, for all $(x,t) \in \Omega \times (0,T), \eta, \zeta \in \mathbb{R}$ and $\xi \in \mathbb{R}^n$, $\quad 1 \leq r(x) \leq 2$.

(H) $h \in \left(L^{p(x)}(0,T;X)\right)'$

Theorem 2.1. *Assume that $(M_0), (a_0), (a_1)), (F_1)$ and (H) hold. Then (1) has a weak solution.*

Proof We construct operators T, S, and C, so that we can write system (1) in the form $T + S + C = h$, then we verify that these operators satisfy the conditions of Theorem 12 in [1], to conclude the proof. ■

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DOUBLE PHASE PROBLEMS INVOLVING SANDWICH TYPE AND CRITICAL GROWTH

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Abstract

In this talk, we discuss about double phase problems involving critical Sobolev terms and sandwich type nonlinear perturbations. More precisely, our problems are driven by the so-called *double phase operator* given by

$$u \in W_0^{1,\mathcal{H}}(\Omega) \mapsto \operatorname{div}\left(|\nabla u|^{p-2}\nabla u + a(x)|\nabla u|^{p-2}\nabla u\right),$$

set on an appropriate Musielak-Orlicz-Sobolev space $W_0^{1,\mathcal{H}}(\Omega)$, with $1 < p < q < \infty$ and $a \in C^{0,1}(\overline{\Omega})$ such that $a(x) \geq 0$ a.e. in Ω.

Under different values of the parameters involved, we prove the existence and multiplicity of solutions to our problems. For this, we mainly exploit different variational methods combined with topological tools, like a new concentration-compactness principle, a suitable truncation argument and the Krasnoselskii's genus theory, by considering very mild assumptions on the data.

1 Introduction

In this talk, we present the following problem

$$\begin{cases} -\operatorname{div}\big(A\,(x,\nabla u)\,\big) = \lambda w(x)|u|^{s-2}u + \theta B\,(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with Lipschitz boundary $\partial\Omega$, λ and θ are real parameters, w is a suitable weight, while $A : \overline{\Omega} \times \mathbb{R}^N \to \mathbb{R}^N$ and $B : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ are given by

$$A(x,\xi) := |\xi|^{p-2}\xi + a(x)|\xi|^{q-2}\xi, \qquad B(x,t) := b_0(x)|t|^{p^*-2}t + b(x)|t|^{q^*-2}t, \tag{2}$$

where $r^* := Nr/(N-r)$ for $r \in \{p,q\}$. Denoting

$$\Omega_+ := \{x \in \Omega :\ a(x) > 0\},$$

we suppose the following structure conditions on the data of problem (1):

(H_1) $1 < p < s < q < N$, $\dfrac{q}{p} < 1 + \dfrac{1}{N}$, $0 \leq a(\cdot) \in C^{0,1}(\overline{\Omega})$ and $\Omega_+ \neq \emptyset$.

(H_2) $0 < b_0(\cdot) \in L^\infty(\Omega)$ and $0 \leq b(\cdot) \in L^\infty(\Omega)$ such that $b(x) \leq Ca(x)^{\frac{q^*}{q}}$ a.e. in Ω and

$$\left\|b^{\frac{1}{q^*}}u\right\|_{q^*} \leq C\left\|a^{\frac{1}{q}}\nabla u\right\|_q, \quad \text{for any } u \in C_c^\infty(\Omega),$$

with some $C > 0$.

(H_3) $w:\Omega\to\mathbb{R}$ is a measurable function such that $|\{x\in\Omega_+ :\ w(x)>0\}|>0$, $w\chi_{\{b=0\}}b_0^{-\frac{s}{p^*}}\in L^{\frac{p^*}{p^*-s}}(\Omega)$, $w\chi_{\{b>0\}}b^{-\frac{s}{q^*}}\in L^{\frac{q^*}{q^*-s}}(\Omega)$ and

$$\int_\Omega w(x)|u|^s \,\mathrm{d}x \leq C_w \left(\int_\Omega a(x)|\nabla u|^q \,\mathrm{d}x\right)^{\frac{s}{q}},$$

for any $u\in C_c^\infty(\Omega)$ with some $C_w>0$. Here, χ_E denotes the characteristic function of E and $\chi_{\{b>0\}}b^{-\frac{s}{q^*}}:=0$ on the set $\{b=0\}$.

2 Main Results

Our first result is devoted to the multiplicity of solutions to problem (1), proved in [2, Theorem 1.3] and stated below.

Theorem 2.1. *Let hypotheses (H_1) – (H_3) be satisfied and assume that there exists a ball $B\subset\Omega_+$ such that $w(x)>0$ a.e. in B. Then, there exists $\{\theta_j\}_{j\in\mathbb{N}}$ with $0<\theta_j<\theta_{j+1}$, such that for any $j\in\mathbb{N}$ and with $\theta\in(0,\theta_j)$, there exist $\lambda_\star$, $\lambda^\star>0$ with $\lambda_\star<\lambda^\star$ and possibly depending on θ, such that for any $\lambda\in(\lambda_\star,\lambda^\star)$, problem (1) admits at least j pairs of distinct solutions with negative energy.*

The proof of Theorem 2.1 relies on a careful combination of variational and topological tools, such as truncation techniques and Krasnoselskii's genus theory, similar to the work in [1, Theorem 1.1] under the sublinear situation, namely with $s\in(1,p)$. However, unlike in [1, Theorem 1.1], the sandwich perturbation with exponent $s\in(p,q)$ does not allow us to provide the existence of infinitely many solutions for (1).

We also present a second independent result, which provides the existence of at least one solution to problem (1), as proved in [2, Theorem 1.3].

Theorem 2.2. *Let hypotheses (H_1) – (H_3) be satisfied. Then, there exists $\lambda_0>0$ such that for any given $\lambda>\lambda_0$, there exists $\theta_*=\theta_*(\lambda)>0$ such that for any $\theta\in(0,\theta_*)$, problem (1) has a nontrivial nonnegative solution with negative energy.*

Theorem 2.2 generalizes the (p,q)-Laplacian situation studied by [3, Theorem 1.1] in a nontrivial way. Indeed, in [3, Theorem 1.1] they set their problem in the trivial intersection space given by $W_0^{1,p}(\Omega)\cap W_0^{1,q}(\Omega)=W_0^{1,q}(\Omega)$ with $p<q$. In this way, they minimize their energy functional on the ball $B_r=\left\{u\in W_0^{1,q}(\Omega):\ \|\nabla u\|_q\leq r\right\}$, that is disregarding the norm $\|\nabla u\|_p$. In Theorem 2.2, we still apply a minimization argument, combined with Ekeland's variational principle, but taking care of the Luxemburg type norm of the Musielak-Orlicz-Sobolev space $W_0^{1,\mathcal{H}}(\Omega)$.

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EFFECT FOR A NONLOCAL MAXWELL-SCHRÖDINGER SYSTEM

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Abstract

In this paper we prove existence and regularity of weak solutions for the following system

$$\begin{cases} -\operatorname{div}\left(\left(\|\nabla u\|_{L^p}^p + \|\nabla v\|_{L^p}^p\right)|\nabla u|^{p-2}\nabla u\right) + g(x,u,v) = f \quad \text{in } \Omega; \\ -\operatorname{div}\left(\left(\|\nabla u\|_{L^p}^p + \|\nabla v\|_{L^p}^p\right)|\nabla v|^{p-2}\nabla v\right) = h(x,u,v) \quad \text{in } \Omega; \\ u = v = 0 \text{ on } \partial\Omega. \end{cases}$$

where Ω is an open bounded subset of $\mathbb{R}^N$, $N > 2$, $f \in L^m(\Omega)$, where $m > 1$ and g, h are two Carathéodory functions. We prove that under appropriate conditions on g and h, there is gain of Sobolev and Lebesgue regularity for the solutions of this system.

1 Introduction

In this paper, we investigate the existence and regularity of positive solutions for the following class of Maxwell-Schrödinger systems of Kirchhoff

$$\begin{cases} -\operatorname{div}\left(\left(\|\nabla u\|_{L^p}^p + \|\nabla v\|_{L^p}^p\right)|\nabla u|^{p-2}\nabla u\right) + g(x,u,v) = f \quad \text{in } \Omega; \\ -\operatorname{div}\left(\left(\|\nabla u\|_{L^p}^p + \|\nabla v\|_{L^p}^p\right)|\nabla v|^{p-2}\nabla v\right) = h(x,u,v) \quad \text{in } \Omega; \\ u = v = 0 \text{ on } \partial\Omega, \end{cases} \tag{P}$$

where, from now on, $\Omega \subset \mathbb{R}^N$, is an open bounded subset, $N > 2$, $1 < p < N$ and $f \in L^m(\Omega)$ for $m > 1$. Moreover, g and $h : \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are both Carathéodory functions responsible for the coupling of our system, with following properties:

(i) there exist constants $c_1 > 0$, $c_2 > 0$, such that

$$c_1|s|^r|t|^{\theta+1} \leqslant g(x,s,t)s \tag{H$_1$}$$
$$|g(x,s,t)| \leqslant c_2|s|^{r-1}|t|^{\theta+1} \tag{H$_2$}$$

for all $s, t \in \mathbb{R}$ a.e. x in Ω;

(ii) there exist constants $d_1 > 0$, $d_2 > 0$ such that

$$d_1|s|^r|t|^{\theta+1} \leqslant h(x,s,t)t \tag{H$_3$}$$
$$|h(x,s,t)| \leqslant d_2|s|^r|t|^{\theta} \tag{H$_4$}$$

for all $s, t \in \mathbb{R}$ a.e. x in Ω.

In recent years, there have been several results investigating the gain of regularity of solutions of Maxwell-Schrödinger systems. As it turned out, even in rough regimes where the datum is not regular, the coupling in these systems allows the existence of solutions which somehow are more regular than expected, the so-called Regularizing Effect. By now, it is known that this is caused by the very interesting nature of the coupling between the equations of systems of Maxwell-Schrödinger type. Without the intention of being complete, the interested reader is invited to see [1, 2, 3, 4, 3].

2 Main Results

Our main results concern existence of solutions, and also the investigation of regularizing properties of system (1). For this, following [3], we consider the idea o Sobolev and Lebesgue regularized solutions.

Theorem 2.1. *Let $f \in L^m(\Omega)$, where $f \geqslant 0$ a.e in Ω, $\min\{(r+\theta+1)', (p^*)'\} < m < \frac{N}{p}$ and $0 < \theta < \min\{p-1, \frac{p^2}{N-p}\}$. Then there exists a distributional solution (u, v) for (1), with $u \in W_0^{1,p}(\Omega) \cap L^{r+\theta+1}(\Omega)$, $u \geqslant 0$ a.e in Ω and $v \in W_0^{1,p}(\Omega)$, $v \geqslant 0$ a.e. in Ω.*

Corollary 2.1. *Consider (u, v) a solution for (1), given by Theorem (2.1).*

(I) If $r + \theta > p^* - 1$ and $(r + \theta + 1)' < m < (p^*)'$, then u is Sobolev regularized.

(II) If $r + \theta > p^* - 1$ and $(p^*)' \leqslant m < \frac{N(r+\theta+1)}{N(p-1)+p(r+\theta+1)}$, then u is Lebesgue regularized.

(III) If $r + \theta > p^* - 1$, then v is Sobolev regularized.

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MULTIPLICITY OF SOLUTIONS FOR A CLASS OF LOGISTIC PROBLEMS

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Abstract

This work employs variational methods to prove the existence of at least two positive solutions for a logistic problem. We consider the energy functional restricted to the Nehari manifold $\mathcal{N}_\lambda$. The first solution is obtained by combining the Nehari manifold technique with the generalized Rayleigh quotient method. The second solution follows from the Mountain Pass Theorem applied to a suitable subset of $\mathcal{N}_\lambda$.

1 Introduction

The purpose of this work is to obtain multiple solutions for the following logistic problem:

$$\begin{cases} -\Delta u = \lambda u - b(x)(|u|^{p-2}u - |u|^{q-2}u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a smooth bounded domain, $\lambda > 0$ a parameter, and $2 < q < p < 2^* = \frac{2N}{N-2}$ (with $2^* = \infty$ if $N = 2$). The function b is bounded, nonnegative, and vanishes on a connected subset $\Omega_0 \subset \Omega$ with positive measure and smooth boundary.

Motivated by the seminal works of López-Gómez [5], Fernandes and Maia [3], and Cardoso, Furtado, and Maia [1], which analyze the existence of solutions in logistic-type problems, we study Problem (1) by combining the Nehari manifold technique with the generalized nonlinear Rayleigh quotient method. Our approach is inspired by recent advances where nonlinear Rayleigh quotients have been extensively investigated, with pioneering contributions in [1] and more recent developments in [2, 7].

To our knowledge, this is the first work to comprehensively combine these methods to investigate multiplicity of positive solutions in this framework.

Weak solutions correspond to critical points of the energy functional

$$E_\lambda(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 dx - \frac{\lambda}{2}\int_\Omega |u|^2 dx + \int_\Omega b(x)\left(\frac{|u|^p}{p} - \frac{|u|^q}{q}\right) dx, \quad u \in H_0^1(\Omega).$$

The associated Nehari manifold is

$$\mathcal{N}_\lambda = \left\{u \in H_0^1(\Omega) \setminus \{0\} : \|u\|^2 - \lambda\|u\|_2^2 = -\int_\Omega b(x)(|u|^p - |u|^q)dx\right\},$$

which is decomposed into

$$\mathcal{N}_\lambda^+ = \{u \in \mathcal{N}_\lambda : E_\lambda''(u)(u,u) > 0\},$$
$$\mathcal{N}_\lambda^- = \{u \in \mathcal{N}_\lambda : E_\lambda''(u)(u,u) < 0\},$$
$$\mathcal{N}_\lambda^0 = \{u \in \mathcal{N}_\lambda : E_\lambda''(u)(u,u) = 0\}.$$

2 Main Results

We also assume the function b satisfies the following hypothesis:

(b_0) [$b(x) = 0$] in a connected set $\Omega_0 \subset\subset \Omega$ with positive Lebesgue measure and smooth boundary. Moreover, we suppose $\|b\|_\infty < \frac{\lambda_1(\Omega)}{C}$, for some positive constant C to be determined throughout the work.

We denote by $\lambda_1(\Omega)$ the first eigenvalue of the operator $-\Delta$ in $H_0^1(\Omega)$ and by $\phi_1 > 0$ its corresponding eigenfunction.

Now, we are stay in position to state our main result as follows:

Theorem 2.1. *Assume $2 < q < p < 2^*$ and (b_0). Then, we obtain, $0 < \lambda_* < \overline{\lambda} < \lambda^* < \lambda_1(\Omega)$. Moreover, for each $\lambda > \overline{\lambda}$, problem (1) has a positive solution $u_\lambda \in \mathcal{N}_\lambda^+$ satisfying: (i) $E_\lambda(u_\lambda) > 0$ for $\lambda \in (\overline{\lambda}, \lambda^*)$; (ii) $E_\lambda(u_{\lambda^*}) = 0$; (iii) $E_\lambda(u_\lambda) < 0$ for $\lambda > \lambda^*$; (iv) $c_\lambda^+ \to -\infty$ and $\|u_\lambda\| \to \infty$ as $\lambda \to \infty$.*

The existence of a solution in $\mathcal{N}_\lambda^+$, provided by Theorem 2.1, guarantees us that the functional E_λ satisfies the Mountain Pass geometry, which allows us to show the existence of another solution to the Problem (1).

Theorem 2.2. *Suppose that assumptions $2 < q < p < 2^*$ and (b_0) are satisfied. Then, Problem (1) admits at least one mountain pass solution for each $\lambda \in (\overline{\lambda}, \lambda_1(\Omega))$. In addition, there are $\delta_\lambda > 0$ and $A_{\delta_\lambda} > 0$ such that $||v_\lambda|| \geq \delta_\lambda$ and $E_\lambda(v_\lambda) \geq A_{\delta_\lambda}$. Moreover, as $\lambda \to \lambda_1(\Omega)^-$, we have $c_\lambda^m \to 0$ and $\|v_\lambda\| \to 0$.*

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NODAL SOLUTIONS FOR FRACTIONAL KIRCHHOFF PROBLEMS INVOLVING CRITICAL EXPONENTIAL GROWTH

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Abstract

In this paper we discuss the existence of least energy nodal solutions for a class of fractional Kirchhoff problems

$$\left(a+b[u]_{1/2}^2\right)(-\Delta)^{1/2}u+V(x)u=f(x,u) \quad \text{in} \quad \mathbb{R},$$

where $a>0$, $b\geq 0$ and $f(x,u)$ is a nonlinear term with critical exponential growth. By using the deformation lemma, we obtain a least energy nodal solution u_b for this class of problems. Furthermore, the study of the asymptotic behavior of u_b as $b\to 0$ allows us to prove the existence of nodal solutions for the equation in the absence of the Kirchhoff term. To the best of our knowledge, this is the first result proving the existence of nodal solutions for this type of equations.

1 Introduction

This paper is concerned with the existence of least energy nodal solutions for the following class of stationary Kirchhoff problem

$$\left(a+b\int_{\mathbb{R}^2}\frac{|u(x)-u(y)|^2}{|x-y|^2}\,\mathrm{d}x\,\mathrm{d}y\right)(-\Delta)^{1/2}u+V(x)u=f(x,u) \quad \text{in} \quad \mathbb{R}, \tag{$\mathcal{P}$}$$

where a,b are positive parameters and the potential $V(x)$ satisfies the following Bartsch-Wang assumptions

(V_1) $V\in C(\mathbb{R})$ and there exists a positive constant V_0 such that $\inf\limits_{x\in\mathbb{R}}V(x)\geq V_0>0$ for all $x\in\mathbb{R}$,

(V_2) There exists $r>0$ such that $\lim\limits_{|y|\to+\infty}\text{meas}\,\{x\in B_r(y):\,V(x)\leq c\}=0$ for any $c>0$.

We are assuming the nonlinearity $f(x,t)$ is a Carathéodory function satisfying the following hypotheses:

(f_1) $|f(x,t)|\leq Ce^{\pi t^2}$ for all $(x,t)\in\mathbb{R}^2$,

(f_2) $f(x,t)=o(|t|^3)$ as $t\to 0$ uniformly in $x\in\mathbb{R}$;

(f_3) $\dfrac{f(x,t)}{|t|^3}$ is nondecreasing in t for $t\in\mathbb{R}\setminus\{0\}$.

(f_4) There exist constants $p>3$ and $C_p>0$ such that $\text{sign}(t)f(x,t)\geq C_p|t|^p$, for all $(x,t)\in\mathbb{R}^2$, with

$$C_p\geq\max\left\{(4d)^{\frac{p-1}{2}},(4d)^{\frac{p-3}{4}}\right\},$$

where d is the least energy nodal level of the subcritical Kirchhoff problem

$$\left(a+b[u]_{1/2}^2\right)(-\Delta)^{1/2}u+V(x)u=|u|^{p-1}u \quad \text{in} \quad \mathbb{R}. \tag{$\mathcal{A}$}$$

1.1 Motivation and related results

Nonlinear elliptic equations involving nonlocal operators have been widely studied since they naturally arise in many different contexts, such as, obstacle problems, flame propagation, minimal surfaces, conservation laws, financial market, optimization, crystal dislocation, phase transition and water waves (for an example, see [1] and references therein). There is extensive literature on fractional Kirchhoff-type equations

$$\left(a+b\int_{\mathbb{R}^N}\left|(-\Delta)^{\frac{s}{2}}u\right|^2\,\mathrm{d}x\right)(-\Delta)^s u+W(x)u=g(x,u)\quad\text{in}\quad\mathbb{R}^N. \tag{1}$$

When $a=1$ and $b=0$, equation (1) simplifies to the fractional Schrö dinger equation:

$$(-\Delta)^s u+W(x)u=g(x,u)\quad\text{in}\quad\mathbb{R}^N, \tag{2}$$

This equation was originally proposed by [4] in the context of fractional quantum mechanics, extending Feynman integrals from Brownian-like to Lévy-like quantum paths.

In turn, K. Cheng and Q. Gao [2] obtained a least energy nodal solution u_b for (1) with a convergence property in which $b\to 0^+$. For this purpose they assumed $s\in(0,1)$, $N>2s$, a and b are positive constants, the nonlinear term g satisfies polinomial subcritical growth.

2 Main Results

Now we state our main results related to Problem ($\mathcal{P}$).

Theorem 2.1. *Assume that* (V_1)-(V_2) *and* (f_1)-(f_4) *hold. Then Problem* ($\mathcal{P}$) *has a least energy nodal solution* $u_b\in E$.

Theorem 2.2. *Assume that* (V_1)-(V_2) *and* (f_1)-(f_4) *hold. Then, for any sequence* $b_n\to 0$, *there exists a subsequence, still denoted by* (b_n), *such that* $u_{b_n}\to u_0$ *in* E *as* $n\to+\infty$, *where* u_0 *is a least energy nodal solution for Problem*

$$5(-\Delta)^{1/2}u+V(x)u=f(x,u),\quad x\in\mathbb{R}. \tag{$\mathcal{L}$}$$

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POSITIVE GROUND STATES FOR INTEGRODIFFERENTIAL SCHRÖDINGER-POISSON SYSTEMS

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Abstract

This work is dedicated to the study of existence of solutions for a Schrödinger-Poisson type system involving possible different integrodifferential operators in the presence of a nonnegative but not bounded away from zero potential. We consider a pure power nonlinearity $u \mapsto |u|^{p-2}u$, with $4-\beta < p < 6/(3-2\alpha)$, where α and β are positive parameters described in each equation. Our argument is variational in nature and involves the use of a deformation type lemma on a minimization over a Nehari-Pohozaev manifold. Some regularity results are provided and a maximum type principle for integrodifferential operators is proved.

1 Introduction

In this work, we are interested in the following class of *Schrödinger-Poisson type system* involving integrodifferential operators

$$\begin{cases} -\mathcal{L}_{A_\alpha} u + V(x)u + \phi u = |u|^{p-2}u & \text{in} \quad \mathbb{R}^3, \\ -\mathcal{L}_{B_\beta}\phi = u^2 & \text{in} \quad \mathbb{R}^3, \end{cases} \tag{1}$$

where $0 < \alpha, \beta < 1$, $4\alpha + 2\beta > 3$, $4 - \beta < p < 2^*_\alpha := 6/(3-2\alpha)$ and V is a nonnegative continuous potential. The operators $-\mathcal{L}_{A_\alpha}$ and $-\mathcal{L}_{B_\beta}$ are given by

$$-\mathcal{L}_K w(x) = \lim_{\varepsilon \to 0^+} \int_{\mathbb{R}^3 \setminus B_\varepsilon(x)} (w(x) - w(y))K(x-y)\mathrm{d}y, \tag{2}$$

with $K = A_\alpha$ or $K = B_\beta$. The measurable kernels A_α and B_β satisfy, among others, the structural condition of being even and homogeneous, for instance $A_\alpha(\lambda x) = \lambda^{-(3+2\alpha)}A_\alpha(x)$ for all $\lambda > 0$ and $x \in \mathbb{R}^3$. A particular case is the fractional Laplacian operator $(-\Delta)^s$, which leads to the fractional Schrödinger-Poisson System studied in several recent works [2] and [3]. A distinctive feature of our work is the generality on the parameters α and β. Unlike several results found in the literature, which often assume restrictive conditions such as $0 < \alpha \le \beta < 1$, our main theorem only requires $\alpha, \beta \in (0,1)$ with $4\alpha + 2\beta > 3$. This approach allows us to complement existing results by addressing different configurations of the integrodifferential operators involved in the system. Our approach is variational, and solutions are sought in a suitable functional space. A weak solution $(u, \phi) \in X \times Y$ of (1) is a pair such that $u \in X$ satisfies the first equation and $\phi = \phi_u \in Y$ is the unique weak solution of the second equation. The energy functional $I : X \to \mathbb{R}$ associated with the system is given by

$$I(u) = \frac{1}{2}\|u\|_X^2 + \frac{1}{4}\int_{\mathbb{R}^3} \phi_u u^2 \mathrm{d}x - \frac{1}{p}\int_{\mathbb{R}^3} |u|^p \mathrm{d}x.$$

2 Main Results

The functional framework is built upon Sobolev-type spaces. The space for the first variable, X, is the completion of $C_0^\infty(\mathbb{R}^3)$ under the norm

$$\|u\|_X := \left(\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} (u(x) - u(y))^2 A_\alpha(x-y)\mathrm{d}x\mathrm{d}y + \int_{\mathbb{R}^3} V(x)u^2 \mathrm{d}x \right)^{1/2}.$$

The space for the potential, Y, is the completion of $C_0^\infty(\mathbb{R}^3)$ with respect to the norm

$$\|\phi\|_Y := \left(\int_{\mathbb{R}^3}\int_{\mathbb{R}^3}(\phi(x)-\phi(y))^2 B_\beta(x-y)\mathrm{d}x\mathrm{d}y\right)^{1/2}.$$

Our main result establishes the existence of a positive solution under certain conditions on the kernels A_α, B_β and the potential $V(x)$, which we highlight **(V_1)** $V(x)\geq 0$ for all $x\in\mathbb{R}^3$ and

$$d_1 := \inf\left\{\int_{\mathbb{R}^3}\int_{\mathbb{R}^3}(u(x)-u(y))^2 A_\alpha(x-y)\mathrm{d}x\mathrm{d}y + \int_{\mathbb{R}^3}V(x)u^2\mathrm{d}x : u\in C_0^\infty(\mathbb{R}^3),\ \int_{\mathbb{R}^3}u^2\mathrm{d}x = 1\right\} > 0.$$

Theorem 2.1 ([1]-Theorem 1.1)**.** *Assume the structural hypotheses on the kernels A_α, B_β and (V_1) on the potential V hold, with $0<\alpha,\beta<1$, $4\alpha+2\beta>3$, and $4-\beta<p<2_\alpha^*$. If either:*

i) A_α, B_β, and V are radially symmetric,

ii) A specific coercive type condition on V holds.

Then, System (1) has a positive weak solution $(u,\phi_u)\in X\times Y$. When $A_\alpha(x)=|x|^{-(3+2\alpha)}$, $B_\beta(x)=|x|^{-(3+2\beta)}$ and $V\in C^1(\mathbb{R}^3)$, u is also a ground state solution.

The hypotheses on the potential $V(x)$ are general, accommodating a class of nonnegative potentials that are not necessarily bounded away from zero and ensure the compactness of certain embeddings. A notable example that satisfies our conditions is the power function $V(x)=|x|^\sigma$ for $\sigma\in(0,1)$. A similar statement can be said about the measurable kernels, which in our main result can include $A_\alpha(z)=a(z/|z|)|z|^{-(3+2\alpha)}$ and $B_\beta(z)=b(z/|z|)|z|^{-(3+2\beta)}$, with $4\alpha+2\beta>3$ and α, $\beta\in(0,1)$, where the functions a and b are uniformly bounded away from zero defined in $\mathbb{S}^2$. The proof of the theorem is based on showing the existence of a minimizer for the energy functional I on a Nehari-Pohozaev manifold. A further novel aspect of the work is establishing a maximum principle for general integrodifferential operators. This ensures the positivity of the solution found and represents a result not previously available in the literature for this class of operators.

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EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR FRACTIONAL P-LAPLACIAN WITH CRITICAL NONLINEARITY IN $\mathbb{R}^N$ VIA NONLINEAR RAYLEIGH QUOTIENT

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Abstract

In this talk, we establish the existence and multiplicity of solutions for the nonlocal problem defined by

$$\begin{cases} (-\Delta)_p^s u \ + \ V(x)\,|u|^{p-2}\,u \ = \ \lambda a(x)\,|u|^{q-2}\,u \ + \ \gamma b(x)\,|u|^{p_s^*-2}\,u \text{ in } \mathbb{R}^N, \\ u \in W^{s,p}(\mathbb{R}^N), \end{cases} \tag{P_λ}$$

where $\gamma > 0$, $\lambda \in (0, \lambda^*), \lambda^* > 0$ and $N > ps$ with $s \in (0, 1)$ fixed, $1 < q < p < p_s^* = Np/(N - ps)$.

1 Introduction

In this talk, we establish the existence and multiplicity of solutions for the nonlocal problem defined by

$$\begin{cases} (-\Delta)_p^s u \ + \ V(x)\,|u|^{p-2}\,u \ = \ \lambda a(x)\,|u|^{q-2}\,u \ + \ \gamma b(x)\,|u|^{p_s^*-2}\,u \text{ in } \mathbb{R}^N, \\ u \in W^{s,p}(\mathbb{R}^N), \end{cases} \tag{P_λ}$$

where $\gamma > 0$, $\lambda \in (0, \lambda^*), \lambda^* > 0$ and $N > ps$ with $s \in (0, 1)$ fixed, $1 < q < p < p_s^* = Np/(N - ps)$. It is worthwhile to mention that these values are optimal in the sense that we can use the Nehari method in order to find existence and multiplicity of solutions for nonlocal elliptic problems. For the interested reader we refer to the work [3] and references therein. The nonlinear Rayleigh quotient method have been widely studied in the last years, see [1].

2 Main Results

The main idea here is to ensure sharp conditions for the Problem (P_λ) such that the Nehari method can be applied. Throughout this work we assume the following assumptions:

(A) There holds $a \in L^{\tilde{q}}(\mathbb{R}^N) \cap L^\infty_{loc}(\mathbb{R}^N)$ where $\tilde{q} = p_s^*/(p_s^* - q)$. Furthermore $a(x) > 0$ for all $x \in \mathbb{R}^N$;

(B) Assume that $b \in L^\infty(\mathbb{R}^N)$. Moreover, suppose $b(x) > 0$ for all $x \in \mathbb{R}^N$ and $\left[\frac{a(x)}{(b(x))^{\frac{q}{p_s^*}}}\right] \in L^{\tilde{q}}(\mathbb{R}^N)$,

(V_1) $V \in C(\mathbb{R}^N, \mathbb{R})$ and there exists a constant $V_0 > 0$ such that $V_0 = \inf_{x\in\mathbb{R}^N} V(x)$;

The hypotheses (V_1) and (V_2) are usual in order to consider nonlocal or local elliptic problems. Namely, hypotheses (V_1) and (V_2) are used in order to recover continuous and compact embedding from Sobolev spaces into Lebesgue spaces. Now, we define the working space for the Problem (P_λ) as follows:

$$X = \left\{ u \in W^{s,p}(\mathbb{R}^N) : \int_{\mathbb{R}^N} V(x)|u|^p dx < \infty \right\},$$

where $W^{s,p}(\mathbb{R}^N)$ is the fractional Sobolev space, see [2]. Furthermore, we observe that X is equipped with the norm:

$$\|u\|^p = [u]_{s,p}^p + \int_{\mathbb{R}^N} V(x)|u|^p dx, u \in X.$$

Recall that the notation $[u]_{s,p}$ represents the well-known Gagliardo semi norm of function u given by

$$[u]_{s,p}^p = \int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}} dxdy.$$

Define the energy functional $J : X \to \mathbb{R}$ given by

$$J_\lambda(u) = \frac{1}{p}\|u\|^p - \frac{\lambda}{q}\int_{\mathbb{R}^N} a(x)|u|^q dx - \frac{\gamma}{p_s^*}\int_{\mathbb{R}^N} b(x)|u|^{p_s^*} dx = \frac{1}{p}\|u\|^p - \frac{\lambda}{q}\|u\|_{q,a}^q - \frac{\gamma}{p_s^*}\|u\|_{p_s^*,b}^{p_s^*}.$$

Under our hypotheses we observe that J_λ is well defined and belongs to $C^1(X, \mathbb{R})$. In this case, given a function $u \in X$ we deduce that u is a critical point of J_λ if and only if u is a weak solution for Problem (P_λ).

Under certain conditions, we establish the existence of parameters $\lambda^* > 0$ and $\lambda_* > 0$ given by

$$\lambda^* \quad = \inf_{u \in X\setminus\{0\}} \Lambda_n(u);\ \lambda_* = \inf_{u \in X\setminus\{0\}} \Lambda_e(u). \tag{1}$$

It is worth emphasizing that the parameter $\lambda^* > 0$ represents the smallest positive value of λ for which $\mathcal{N}_\lambda^0 \neq \emptyset$. Under these conditions, we are able to state our main first results as follows:

Theorem 2.1. *Suppose $(A), (B)$, and (V). Then, $0 < \lambda_* < \lambda^* < \infty$. Furthermore, we prove the following statements:*

i) For each $\lambda \in (0, \lambda_*)$ and γ sufficiently large, the Problem (P_λ) admits at least one solution v in $\mathcal{N}_\lambda^-$, such that $J_\lambda(v) > 0$.

ii) For $\lambda = \lambda_*$ and $\gamma > 0$, the Problem (P_λ) admits at least one solution $v \in \mathcal{N}_\lambda^-$, such that $J_\lambda(v) = 0$.

iii) For each $\lambda \in (\lambda_*, \lambda^*)$ and $\gamma > \gamma_1$, the Problem (P_λ) admits at least one solution $v \in \mathcal{N}_\lambda^-$ such that $J_\lambda(v) < 0$ where $\gamma_1 > 0$.

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POSITIVE SOLUTIONS FOR A KIRCHHOFF-TYPE PROBLEM WITH CRITICAL GROWTH IN $\mathbb{R}^N$

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Abstract

In the present work we establish the existence and multiplicity of positive solutions for a critical elliptic problem in $\mathbb{R}^N$. The main feature here is to treat a Kirchhoff-type elliptic problem where the nonlinearity is critical and defines a sign-changing function. Our approach relies on the minimization method applied to the Nehari method together with the nonlinear Rayleigh quotient method. Here, we use the fibering map associated with the energy functional which exhibits degenerate points under suitable values on the two parameters within the nonlinearity. This difficulty does not allow us to apply the Lagrange Multipliers Theorem in general. Our main contribution relies on restoring the strong convergence and compactness results. Furthermore, we establish nonexistence results under specific assumptions on the nonlinear term by using a Pohozaev identity.

1 Introduction

The main objective of this work is to investigate the existence and multiplicity of positive solutions for the elliptic problem of the Kirchhoff-type with a critical nonlinearity given by

$$\begin{cases} -m\left(\|\nabla u\|_2^2\right)\Delta u + V(x)u = \lambda f(x)|u|^{q-2}u - \theta|u|^{2^*-2}u \text{ in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{$\mathcal{P}_c$}$$

where $\lambda, \theta > 0$ and $N \geq 3$. Initially, we assume the following hypotheses on m, f and V:

(M_1) The function m is defined by $m(t) = \alpha_1 + \alpha_2 t^\sigma$ for all $t \in \mathbb{R}^+$ with $\alpha_1, \alpha_2 > 0$ and $0 < \sigma < \frac{2}{N-2}$;

(A_1) $2 < q < 2(\sigma+1) < 2^* := \frac{2N}{N-2}$;

(A_2) The function $f : \mathbb{R}^N \to \mathbb{R}$ satisfies $f \in L^{\frac{2^*}{2^*-q}}(\mathbb{R}^N) \cap L^\infty_{\text{loc}}(\mathbb{R}^N)$ with $f(x) > 0$ almost everywhere in $\mathbb{R}^N$;

(V_1) The potential $V : \mathbb{R}^N \to \mathbb{R}$ satisfies $V \in L^\infty_{\text{loc}}(\mathbb{R}^N)$ and $V(x) \geq V_0 > 0$ for all $x \in \mathbb{R}^N$.

Throughout the work, we use X as a proper subspace of $H^1(\mathbb{R}^N)$ defined by

$$X = \left\{ u \in H^1(\mathbb{R}^N) : \int_{\mathbb{R}^N} V(x)u^2 dx < +\infty \right\}.$$

It is well known that X is a Hilbert space equipped with the following norm and inner product, denoted by

$$\|u\|^2 = \int_{\mathbb{R}^N} \left[\alpha_1|\nabla u|^2 + V(x)u^2\right] dx \quad \text{and} \quad \langle u, \varphi\rangle = \int_{\mathbb{R}^N} \left[\alpha_1 \nabla u \nabla\varphi + V(x)u\varphi\right] dx, \ \varphi \in X.$$

Here, we define the energy functional $J : X \to \mathbb{R}$ associated with Problem ($\mathcal{P}_c$) as follows:

$$J(u) = \frac{1}{2}\|u\|^2 + \frac{\alpha_2}{2(\sigma+1)}\|\nabla u\|_2^{2(\sigma+1)} - \frac{\lambda}{q}\int_{\mathbb{R}^N} f(x)|u|^q dx + \frac{\theta}{2^*}\int_{\mathbb{R}^N} |u|^{2^*} dx, \ u \in X.$$

According to our assumptions, we have that a function $u \in X$ is a critical point for the functional J if, and only if, u is a weak solution for Problem ($\mathcal{P}_c$). Moreover, we define a set called *Nehari manifold* given by

$$\mathcal{N} = \{u \in X\backslash\{0\} : J'(u)u = 0\} = \left\{u \in X\backslash\{0\} : \lambda\|u\|_{q,f}^q = \|u\|^2 + \alpha_2\|\nabla u\|_2^{2(\sigma+1)} + \theta\|u\|_{2^*}^{2^*}\right\}.$$

As a consequence, by using [3] and [4], we consider the nonlinear Rayleigh quotients $R_n, R_e : X\backslash\{0\} \to \mathbb{R}$ as

$$R_n(u) = \frac{\|u\|^2 + \alpha_2\|\nabla u\|_2^{2(\sigma+1)} + \theta\|u\|_{2^*}^{2^*}}{\|u\|_{q,f}^q} \quad \text{and} \quad R_e(u) = \frac{\frac{1}{2}\|u\|^2 + \frac{\alpha_2}{2(\sigma+1)}\|\nabla u\|_2^{2(\sigma+1)} + \frac{\theta}{2^*}\|u\|_{2^*}^{2^*}}{\frac{1}{q}\|u\|_{q,f}^q}, \ u \in X\backslash\{0\}.$$

Definition 1.1. *Consider the Nehari set $\mathcal{N}$. Define the following energy levels:*

$$c_{\mathcal{N}^+} = \inf_{u\in\mathcal{N}^+} J(u), \ \ c_{\mathcal{N}^-} = \inf_{u\in\mathcal{N}^-} J(u) \ \ \textit{and} \ \ c_{\mathcal{N}^0} = \inf_{u\in\mathcal{N}^0} J(u).$$

Definition 1.2. *Consider $S_n, S_e : X\backslash\{0\} \to \mathbb{R}$ given by $S_n(u) = \inf_{t>0} R_n(tu)$ and $S_e(u) = \inf_{t>0} R_e(tu)$. Consider also the following extremal values:*

$$\lambda^* = \inf_{u\in X\backslash\{0\}} S_n(u) \quad \textit{and} \quad \lambda_* = \inf_{u\in X\backslash\{0\}} S_e(u).$$

2 Main Results

At this moment, we present the main results of the work.

Theorem 2.1. *Suppose (M_1), (A_1), (A_2), (V_1) and $c_{\mathcal{N}^-} < c_{\mathcal{N}^0}$. Then for each $\lambda \in (\lambda^*, +\infty)$, there exists $\theta_2 > 0$ small such that Problem ($\mathcal{P}_c$) admits at least one weak solution $u \in \mathcal{N}^-$ whenever $\theta \in (0, \theta_2)$. Furthermore, assume that V and f are Hölder continuous functions, then the solution u is strictly positive in $\mathbb{R}^N$.*

Theorem 2.2. *Suppose (M_1), (A_1), (A_2), (V_1) and $c_{\mathcal{N}^+} < c_{\mathcal{N}^0}$. Then for each $\lambda \in (\lambda^*, +\infty)$, there exists $\theta_0 > 0$ small such that Problem ($\mathcal{P}_c$) admits at least one weak solution $v \in \mathcal{N}^+$ whenever $\theta \in (0, \theta_0)$. Furthermore, assume that V and f are Hölder continuous functions, then the ground state solution v is strictly positive in $\mathbb{R}^N$.*

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GLOBAL LIPSCHITZ ESTIMATES FOR FULLY NON-LINEAR SINGULAR PERTURBATON PROBLEMS WITH NON-HOMOGENEOUS DEGENERACY

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Abstract

This manuscript is dedicated to investigating the global regularity of singularly perturbed unbalanced models with variable exponents. In this context, we aim to find a non-negative function u^ϵ that satisfies the following equation for each fixed $\epsilon > 0$

$$\begin{cases} \mathcal{H}(x, \nabla u^\epsilon(x))[\Delta u^\epsilon(x) + b(x) \cdot \nabla u^\epsilon(x)] & = & \zeta_\epsilon(u^\epsilon) + f_\epsilon(x) & \text{in} & \Omega, \\ u^\epsilon(x) & = & g(x) & \text{on} & \partial\Omega, \end{cases}$$

where Ω denotes a bounded regular domain in $\mathbb{R}^n$, and $\mathcal{H}$ represents a function exhibiting a variable degeneracy signature. Additionally, ζ_ϵ approaches the Dirac measure δ_0 as ϵ tends to zero, and f_ϵ remains bounded away from zero and infinity. We aim to establish global gradient bounds that are unaffected by the parameter ϵ. Specifically, this family of solutions converges uniformly (as $\epsilon \to 0$) to a Lipschitz limiting profile associated with a one-phase Bernoulli-type free boundary problem.

1 Introduction

In this work, we study the global Lipschitz regularity of one-phase solutions to singularly perturbed problems exhibiting non-homogeneous variable double degeneracy. The mathematical model is given by fixing a parameter $\epsilon \in (0, 1)$, where we seek a non-negative solution to

$$\begin{cases} \mathcal{H}(x, \nabla u^\epsilon)[F(x, \nabla u^\epsilon, D^2 u^\epsilon) + b(x) \cdot \nabla u^\epsilon] & = & \zeta_\epsilon(x, u^\epsilon) + f_\epsilon(x) & \text{in} & \Omega, \\ u^\epsilon(x) & = & g(x) & \text{on} & \partial\Omega, \end{cases} \tag{1}$$

for a bounded regular domain $\Omega \subset \mathbb{R}^n$, where $0 \leq g \in C^{1,\beta_g}(\partial\Omega)$, and F is a second-order, fully non-linear (uniformly elliptic) operator, i.e., non-linear in its highest derivatives. Our focus is on reaction-diffusion models with a singular term ζ_ϵ of order $\mathrm{O}(\epsilon)$ and a source term f_ϵ of order $\mathrm{O}(1)$.

We must also stress that the vector function $\mathcal{H}$ described above is given by:

$$(\Omega, \mathbb{R}^n) \ni (x, \xi) \mapsto \mathcal{H}(x, \xi) = \left(|\xi|^{p(x)} + \mathfrak{a}(x)|\xi|^{q(x)} \right),$$

for appropriate functions $p, q : \Omega \to \mathbb{R}$ and $\mathfrak{a} : \Omega \to \mathbb{R}_+$. Throughout the paper, we use this identity to simplify our notation, thus facilitating the treatment of our theorems.

In summary, under appropriate hypotheses on the data, we demonstrate that as $\epsilon \to 0^+$, the family of solutions $\{u^\epsilon\}_{\epsilon>0}$ serves as asymptotic approximations to a one-phase solution u_0 of an inhomogeneous non-linear free boundary problem (abbreviated as FBP) of Bernoulli type, achieved by fixing prescribed boundary value data.

2 Main Result

The main result is

Theorem 2.1 (Optimal Lipschitz estimates). *Let* $\{u^\epsilon\}_{\epsilon>0} > 0$ *be a family of solutions to* (1.1). *Given* $\Omega' \Subset \Omega$, *there exists a constant* $\mathrm{C}_0 > 0$, *depending on the dimension, the ellipticity constants,* $\|b\|_\infty$, p_{min}, q_{max} *and on* Ω', *but independent of* $\epsilon > 0$, *such that*

$$\|\nabla u^\epsilon\|_{L^\infty(\Omega')} \leq \mathrm{C}_0.$$

Moreover, when this family is equibounded there is a subsequence $\{u_{\epsilon_{k_j}}\}_{j\in\mathbb{N}}$ *and* $u_0 \in C^{0,1}(\overline{\Omega})$ *such that*

- $u^{\epsilon_{k_j}} \to u_0$ *when* $\epsilon_{k_j} \to 0$ *uniformly in* $\overline{\Omega}$;
- $\mathcal{H}(x, \nabla u_0)F(x, \nabla u_0, D^2u_0) = f_0$ *in* $\Omega \cap \{u_0 > 0\}$ *in the viscosity sense,* $f_0 \in L^\infty(\Omega) \cap C^0(\Omega)$.

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RETHINKING INTEGRABILITY IN STOCHASTIC CALCULUS: A NEW PERSPECTIVE WITH NON-ABSOLUTE INTEGRALS

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Abstract

In this work, we propose a novel approach to stochastic integration based on the non-absolute Kurzweil-Henstock integral. In contrast to Itô integration, this new approach does not require absolute (or quadratic) integrability, thereby encompassing a broader class of stochastic integrands.

1 Introduction

Classical stochastic calculus, as built upon the Itô integral, fundamentally relies on mean square integrability. This requirement is analogous to the condition of absolute integrability in the Lebesgue framework, which restricts the class of admissible integrands. However, many stochastic models - particularly those involving irregular behavior - demand a more flexible notion of integration.

In this work, we introduce the Kurzweil belated integral, a specific adaptation of the Kurzweil-Henstock integral tailored for stochastic settings. By dispensing with the need for absolute or mean-square integrability, this approach provides a more general framework for integrating stochastic processes.

We present the theoretical foundations of the Kurzweil belated integral, outline its construction and key properties, and compare it to classical Itô integration. Applications and illustrative examples are discussed, emphasizing contexts where the Kurzweil belated integral yields analytical advantages. This framework opens new perspectives for stochastic analysis in settings where standard integrability conditions may not hold.

2 Main Results

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, $I \subset \mathbb{R}$ be a non-degenerated interval, $\{\mathcal{F}_t\}_{t\in I}$ be an increasing family of sub-σ-algebras of $\mathcal{F}$ (i.e., $\mathcal{F}_t \subset \mathcal{F}_s$ for all $t, s \in I$ with $t < s$) and $\mathbb{E}$ be the mathematical expectation. Such an increasing family $\{\mathcal{F}_t\}_{t\in I}$ is also known as a *filtration* in $(\Omega, \mathcal{F}, \mathbb{P})$. The space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in I}, \mathbb{P})$ is called a *filtering probability space*, and a stochastic process $X = \{X_t : t \in I\}$ is said to be $\{\mathcal{F}_t\}$-*adapted*, if X_t is $\mathcal{F}_t$-measurable for all $t \in I$. Moreover, for every two real separable Hilbert spaces, $(U, \langle\cdot,\cdot\rangle_U)$ and $(V, \langle\cdot,\cdot\rangle_V)$, we denote by $L(U, V)$ the space of all bounded linear operators from U to V and by $L^p(\Omega, V)$, $1 \le p < \infty$, the space of all $\mathcal{F}$-measurable random variables $Z \colon \Omega \to V$ endowed with the norm

$$\|Z\|_{L^p} = \left(\int_\Omega \|Z(\omega)\|_V^p d\mathbb{P}\right)^{\frac{1}{p}}.$$

In the next lines, we recall the definition of a (δ, η)-fine belated partial division of an interval $[a, b] \subset \mathbb{R}$.

Definition 2.1. *Let $\delta \colon [a, b] \to [0, +\infty)$ be a non-negative function (called gauge on $[a, b]$). A δ-belated partial division of $[a, b]$ is any finite collection of point-interval pairs, $D = \{(\xi_i, (\xi_i, \nu_i]) : i = 1, 2, \ldots, |D|\}$, such that $(\xi_i, \nu_i]$, $i = 1, 2 \ldots, |D|$, are disjoint left-open subintervals of $[a, b]$ and*

$$(\xi_i, \nu_i] \subset (\xi_i, \xi_i + \delta(\xi_i)), \quad \textit{for all } i = 1, 2 \ldots, |D|.$$

If, in addition, for a given $\eta > 0$,

$$\left| b - a - \sum_{i-1}^{|D|} (\nu_i - \xi_i) \right| \leq \eta,$$

then $D = \{(\xi_i, (\xi_i, \nu_i]) : i = 1, 2, \ldots, |D|\}$*, is called a* (δ, η)*-belated partial division of* $[a, b]$*, that is,* D *fails to cover* $[a, b]$ *by at most a set o Lebesgue measure* η*.*

In order to present the definition of the Kurzweil-belated integral, introduced in [2], we denote by $\mathfrak{F}(\Omega, V)$ the space of all operators from Ω to a Hilbert space V.

Definition 2.2. *Let* $I \subset \mathbb{R}$ *be a non-degenerate interval and* $[a, b] \subset I$*. Assume that* $G\colon [a, b] \times [a, b] \to \mathfrak{F}(\Omega, V)$ *is a* $\{\mathcal{F}_t\}$*-adapted process on a filtering probability space* $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \in I}, \mathbb{P})$*. We say that* G *is Kurzweil-belated integrable over* $[a, b]$*, if for every* $\epsilon > 0$*, there exist an element* $K \in L^p(\Omega, V)$*, a gauge* δ *on* $[a, b]$ *and* $\eta > 0$ *such that*

$$\mathbb{E} = \left[\left\| \sum_{i=1}^{|D|} G(\xi_i, \nu_i) - G(\xi_i, \xi_i) - K \right\|_V^p \right] < \epsilon. \qquad (1)$$

for every (δ, η)*-fine belated partial division* $D = \{(\xi_i, (\xi_i, s\nu_i]) : i = 1, 2 \ldots, |D|\}$ *of* $[a, b]$*. In this case, we write* $K = \int_a^b G(\tau, s)$ *and, we use the convention* $\int_a^b G(\tau, s) = -\int_b^a G(\tau, s)$*.*

Theorem 2.1. *The Kurzweil belated integral generalizes the Itô integral.*

Proof For every function $f : [a, b] \to L(U, V)$ and every Brownian motion $B = \{B_t\colon a \leq t \leq b\}$, we can define $G(\tau, s) = f_\tau B_s$ and obtain $\int_a^b G(\tau, s) = (IH) \int_a^b f dB$, that is, the delayed Kurzweil integral extends the ItÃ´-Henstock integral. Moreover, consider a fixed random variable $X \sim \mathcal{N}(0, 1)$ independent of B, and fix $\beta \in \left(\frac{1}{2}, 1\right)$. Define the stochastic process $f(\tau, \omega) := \frac{X(\omega)}{(\tau - a)^\beta}$, $\tau \in (a, b]$, and $f(a) = 0$. Then f is not ItÃ´ integrable over $[a, b]$. However, $G(\tau, s)(\omega) := f(\tau, \omega) B_s(\omega)$ is Kurzweil-belated integrable with $\int_a^b G(\tau, s) = g$, where g is the null-operator. ∎

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PULLBACK OSCILLATION FOR LINEAR GENERALIZED ODES

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Abstract

In this work, we investigate the oscillatory behavior of solutions to linear generalized ordinary differential equations (linear generalized ODEs) where the functions involved take values in Banach spaces. To this end, we introduce the theory of pullback oscillation for linear evolution processes. This notion extends classical concepts of oscillation to non-autonomous systems in infinite-dimensional settings. Necessary and sufficient conditions are presented to obtain pullback oscillation via geometric interpretation of the closed conic hull.

1 Introduction

In most of the classical literature on oscillation theory, the notion of oscillation is associated particularly with functions taking values in $\mathbb{R}$. These notions have been extensively studied in the context of differential equations and are typically characterized by the frequent sign changes of the solution over time.

However, in recent searches, such as those presented in [1], oscillatory behavior has been extended to vector-valued functions in $\mathbb{R}^n$, motivated by geometric examples like Lissajous curves, which illustrate oscillation in multiple directions. These curves provide a natural inspiration for studying multidimensional oscillatory phenomena in dynamical systems.

In [1], the authors present conditions for oscillation of solutions of the following linear generalized ODE

$$\frac{dx}{d\tau} = D[A(t)x],$$

for the n-dimensional case.

A central notion introduced in that context is that of *strong oscillation*, which we adopt and generalize in this work. Let X be a Banach space and X^* its dual. A function $f : \mathbb{R} \to X$ is said to be *strongly oscillatory* if, for every $\zeta \in X^*$, the real-valued function $\zeta \circ f$ oscillates in the classical sense, i.e., changes sign infinitely often.

In this work, we study the oscillatory behavior of solutions to linear generalized ODEs where the unknown functions take values in abstract Banach spaces. To do so, we develop an oscillation theory for *linear evolution processes*. These evolution processes, which generalize strongly continuous semigroups, allow for a robust formulation of oscillation, including the introduction of *pullback oscillation*.

Using this framework, we establish necessary and sufficient conditions for pullback oscillation, expressed via geometric properties of the solution's trajectory-specifically, through the closed conic hull of its pullback orbit.

The results presented in this work can be found in [2].

2 Main Results

Let $(X, \|\cdot\|)$ be a Banach space and let $\mathcal{L}(X)$ denote the space of bounded linear operators from X to itself. Let $A : \mathbb{R} \to \mathcal{L}(X)$ be an operator which fulfills the following properties:

(H1) $A \in BV_{\text{loc}}(\mathbb{R}, \mathcal{L}(X))$, i.e., A has bounded variation on every compact interval;

(H2) For all $t \in \mathbb{R}$, both $(I + [A(t^+) - A(t)])^{-1}$ and $(I - [A(t) - A(t^-)])^{-1}$ exist and belong to $\mathcal{L}(X)$.

We consider the linear generalized ODE

$$\frac{dx}{d\tau} = D[A(t)x]. \tag{1}$$

If the operator A satisfies the above conditions and $x(t^*) = \tilde{x}$, then the equation (1) admits a unique global solution $x : \mathbb{R} \to X$ given by $x(t) = U(t, t^*)\tilde{x}$, where $U : \mathbb{R} \times \mathbb{R} \to \mathcal{L}(X)$ is the *fundamental operator* satisfying:

$$U(t, s) = I + \int_s^t d[A(r)]U(r, s).$$

The family $\{U(t, s) : t \geq s\}$ defines a **linear evolution process**.

Definition (Closed conic hull). The closed conic hull of a set $C \subset X$ is:

$$\overline{\text{con}}(C) = \overline{\left\{ \sum_{i=1}^{k} \alpha_i x_i : \alpha_i \geq 0,\ x_i \in C,\ k \in \mathbb{N} \right\}}.$$

Definition (Pullback oscillation). A solution $x : \mathbb{R} \to X$ of the equation (1) pullback oscillates at time $t_0 \in \mathbb{R}$ if for every $\zeta \in X^*$ and each $s \leq t_0$, there exist $s_1 < s_2 < s$ such that:

$$\zeta(U(t_0, s_1)x(0)) \cdot \zeta(U(t_0, s_2)x(0)) < 0.$$

The following theorem summarize the main results concerning pullback oscillation:

Theorem 2.1. *Let $x : \mathbb{R} \to X$ be a solution of (1) with $x(0) \neq 0$, and let $U(t, s)$ be the associated fundamental operator. Fix $t_0 \in \mathbb{R}$. Then the following statements hold:*

(i) *If x pullback oscillates at time t_0, then $-x(0) \in \overline{\text{con}} \left(\bigcup_{s \leq t_0} U(t_0, s)x(0) \right)$.*

(ii) *If $-x(0) \in \overline{\text{con}} \left(\bigcup_{s \leq \sigma} U(\sigma, s)x(0) \right)$ for all $\sigma \leq t_0$, then x pullback oscillates at time t_0.*

(iii) *If there exist $\rho(t_0) > -t_0$ and a sequence $\{\tau_n\}_{n \in \mathbb{N}} \subset \mathbb{R}_-$ with $\tau_n \to -\infty$ such that*

$$-x(0) \in \overline{\text{con}} \left(U(t_0 + \rho(t_0) + \tau_n,\ \tau_n)x(0) \right) \quad \text{for all } n \in \mathbb{N},$$

then x pullback oscillates at time t_0.

(iv) *If $-x(0) \in \bigcap_{\sigma \leq t_0} \omega(x(0), \sigma) \setminus \{0\}$, then x pullback oscillates at time t_0. Here, $\omega(x(0), \sigma)$ denotes the pullback omega limit set at time σ.*

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ON CONTINUOUSLY DIFFERENTIABLE VECTOR-VALUED FUNCTIONS OF NON-INTEGER ORDER

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Abstract

In this talk, we present the spaces of continuously fractional differentiable functions of order $\alpha > 0$, considering both the Riemann-Liouville and Caputo fractional derivatives. We start by exploring some fundamental properties of these spaces and, inspired by a result of Hardy and Littlewood, we compare them with the space of Hölder continuous functions. We provide a sharp characterization of fractional differentiability via Hölder-type regularity. Also, we establish optimal continuous embeddings between fractional spaces of different orders.

1 Introduction

We introduce the spaces of functions with continuous fractional derivatives, denoted by $RL^\alpha([t_0,t_1];X)$ and $C^\alpha([t_0,t_1];X)$, where X is a Banach space:

Definition 1.1. *Let $\alpha \in (0,\infty)$.*

(i) the Riemann-Liouville $\alpha-$times continuously differentiable functions in $[t_0,t_1]$ is the set

$$RL^\alpha\big([t_0,t_1],X\big) := \Big\{ f \in C^{[\alpha]-1}([t_0,t_1],X) : D^\alpha_{t_0,t} f \in C^0([t_0,t_1],X) \Big\}.$$

(ii) the Caputo $\alpha-$times continuously differentiable functions in $[t_0,t_1]$ is the set

$$C^\alpha\big([t_0,t_1],X\big) := \Big\{ f \in C^{[\alpha]-1}([t_0,t_1],X) : cD^\alpha_{t_0,t} f \in C^0([t_0,t_1],X) \Big\}.$$

Here $D^\alpha_{t_0,t} f$ and $cD^\alpha_{t_0,t} f$ denotes, respectively, the Riemann-Liouville fractional derivative of order α of f and the Caputo fractional derivative of order α of f.

In the first part of this talk, we establish some fundamental properties of these spaces and explore the relationships between them. Next, we present the relation with the space of Hölder continuous functions.

2 Main Results

Let $\alpha, \beta \in (0,\infty)$ with $[\alpha] < \beta$. First, we stablish the following interactions between these spaces:

$$C^\beta\big([t_0,t_1];X\big) \subset C^{[\beta]-1}\big([t_0,t_1];X\big) \subset C^{[\alpha]}\big([t_0,t_1];X\big) \subset C^\alpha\big([t_0,t_1];X\big),$$

and

$$RL^\beta\big([t_0,t_1];X\big) \subset C_0^{[\beta]-1}\big([t_0,t_1];X\big) \subset C_0^{[\alpha]}\big([t_0,t_1];X\big) \subset RL^\alpha\big([t_0,t_1];X\big),$$

where, for $m \in \mathbb{N}$,

$$C_0^m([t_0,t_1];X) := \Big\{ f \in C^m\big([t_0,t_1];X\big) \mid f^{(j)}(t_0) = 0,\ \forall j \in \{0,1,\ldots,m\} \Big\}.$$

In the sequence, we present the main results related to space of Hölder continuous functions. First, we give a characterization of the fractional derivative for Hölder continuous functions. As a consequence, we generalize a result by Hard and Littlewood (see [2]):

Theorem 2.1. *Let* $f \in H^{0,\beta}_{t_0}([t_0,t_1];X)$ *with* $0 < \alpha < \beta < 1$. *Then, the fractional derivative* $D^{\alpha}_{t_0,t}f(t)$ *exists for all* $t \in [t_0,t_1]$ *and satisfies*

$$D^{\alpha}_{t_0,t}f(t) = \frac{-\alpha}{\Gamma(1-\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}[f(s)-f(t)]\,ds + \frac{(t-t_0)^{-\alpha}f(t)}{\Gamma(1-\alpha)},$$

for every $t \in (t_0,t_1]$, *while for* $t = t_0$, *both sides vanish. Consequently, the inclusion*

$$H^{0,\beta}_{t_0}([t_0,t_1];X) \subsetneq RL^{\alpha}([t_0,t_1];X)$$

holds and is continuous.

Corollary 2.1. *Let* $0 < \alpha < \beta \leq 1$ *and* $n \in \mathbb{N}$. *Then* $H^{n,\beta}_{t_0}([t_0,t_1],X) \subsetneq RL^{n+\alpha}([t_0,t_1],X)$ *and* $H^{n,\beta}([t_0,t_1],X) \subsetneq C^{n+\alpha}([t_0,t_1],X)$ *continuously.*

To complete the analysis of the relationship between these spaces, we now consider the remaining cases:

Theorem 2.2. *Let* $0 < \alpha \leq 1$ *and* $n \in \mathbb{N}$. *Then* $RL^{n+\alpha}([t_0,t_1],X) \subsetneq H^{n,\alpha}_{t_0}([t_0,t_1],X)$ *continuously.*

Corollary 2.2. *Let* $0 < \alpha \leq 1$ *and* $n \in \mathbb{N}$. *Then* $C^{n+\alpha}([t_0,t_1],X) \subsetneq H^{n,\alpha}([t_0,t_1],X)$ *continuously.*

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A GENERAL VERSION OF CARATHÉODORY'S THEOREM FOR FRACTIONAL DIFFERENTIAL SYSTEMS

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Abstract

We establish a general version of Carathéodory's existence and uniqueness theorem for semilinear systems of fractional ordinary differential equations involving Caputo derivatives of possibly distinct orders. The main result shows that if the nonlinear term $f \in L^p(0,T;\mathbb{R}^n)$ is a Carathéodory function and the integrability exponent p satisfies $p > \max\{1/\alpha_j : j = 1,\ldots,n\}$, then the system admits a unique continuous solution.

1 Introduction

Recall that the Riemann-Liouville fractional integral of order $\alpha > 0$ for $f \in L^1(0,T;\mathbb{R}^n)$ is defined by

$$J_t^\alpha f(t) := \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)\, ds.$$

The Caputo derivative of order $\alpha \in (0,1)$ for $f \in C([0,T];\mathbb{R}^n)$, with $J_t^{1-\alpha} f \in W^{1,1}(0,T;\mathbb{R}^n)$, is given by

$$cD_t^\alpha f(t) := \frac{d}{dt}\left[J_t^{1-\alpha}\left(f(t) - f(0)\right)\right].$$

In this work, we study the initial value problem

$$\begin{cases} cD_t^{\alpha_1}\varphi_1(t) = f_1(\varphi_1(t),\ldots,\varphi_n(t),t), & \text{a.e. } t \in [0,T], \\ \qquad\vdots & \qquad\vdots \\ cD_t^{\alpha_n}\varphi_n(t) = f_n(\varphi_1(t),\ldots,\varphi_n(t),t), & \text{a.e. } t \in [0,T], \\ \varphi_j(0) = \xi_j, & j = 1,\ldots,n, \end{cases} \tag{1}$$

where $cD_t^{\alpha_j}$ denotes the Caputo derivative of order $\alpha_j \in (0,1]$ and $f = (f_1,\ldots,f_n)$ is a nonlinear Carathéodory-type function.

This type of system arises naturally in the modeling of processes with memory and nonlocal dynamics, and the presence of multiple distinct fractional orders introduces significant analytical challenges. While the classical case $\alpha_j = 1$ is well understood under the framework introduced by Carathéodory, the fractional setting; especially when the orders differ, demands more refined integrability and regularity assumptions.

2 Main Results

We begin by recalling the standard definition of a Carathéodory function, which allows for discontinuities in time while preserving sufficient regularity for meaningful solution theory.

Definition 2.1. *A function $f : \mathbb{R}^n \times [0,T] \to \mathbb{R}^n$ is called a Carathéodory function if:*

(i) for a.e. $t \in [0,T]$, the map $x \mapsto f(x,t)$ is continuous;

(ii) for every $x \in \mathbb{R}^n$, the map $t \mapsto f(x,t)$ is Lebesgue measurable.

Our main existence and uniqueness theorem relies on additional structural conditions on f, which we formulate in terms of growth and Lipschitz-type bounds in time-dependent spaces. These are natural generalizations of the classical assumptions from Carathéodory theory, adapted to the fractional setting.

Theorem 2.1. *Suppose $\{\alpha_j\}_{j=1}^n \subset (0,1]$ and let $p > \max\{1/\alpha_j : j \in \{1,\dots,n\}\}$. Assume $f : \mathbb{R}^n \times [0,T] \to \mathbb{R}^n$ is a Carathéodory function satisfying:*

(C_1) There exists $\ell \in L^1(0,T)$ such that

$$\|f(x,t) - f(y,t)\| \leq \ell(t)\,\|x-y\|\,, \quad \text{for all } x, y \in \mathbb{R}^n;$$

(C_2) There exist $\gamma \in L^p(0,T)$ and $C > 0$ such that

$$\|f(x,t)\| \leq C\,\|x\| + \gamma(t), \quad \text{for all } x \in \mathbb{R}^n.$$

Then, for each $\xi \in \mathbb{R}^n$, there exists a unique continuous solution $\varphi : [0,T] \to \mathbb{R}^n$ to the problem (1).

It is important to emphasize that, contrary to the classical case, the assumption $p > 1/\alpha_j$ is not merely technical, since it plays a fundamental role in ensuring that the fractional integral operator yields continuous functions. The next result shows that this threshold is, in fact, optimal.

Theorem 2.2. *Let $p > 1$ and suppose f satisfies the growth condition (C_2) with $\gamma \in L^p(0,T)$. If there exists $j_0 \in \{1,\dots,n\}$ such that $\alpha_{j_0} = 1/p$, then the system (1) may fail to admit a solution.*

This nonexistence result demonstrates the sharpness of Theorem 2.1, and clarifies the delicate interplay between fractional regularity and integrability conditions in the study of multi-order fractional systems. In particular, it shows that the threshold $p > 1/\alpha_{\min}$ is critical for the well-posedness of such problems, and cannot be relaxed.

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ALMOST PERIODIC FUNCTIONS ON ISOLATED TIME SCALES

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Abstract

In this work, we introduce a general concept of almost periodicity for functions defined on isolated time scales. Our concept is consistent with the existing concepts of almost periodicity on quantum calculus and on $\mathbb{Z}$. Also, we prove important properties such as the equivalence between different definitions for almost periodic functions, as well as results ensuring the existence of almost periodic solutions for dynamic equations on time scales under certain properties. We present several examples to illustrate our definition and main results. All the results can be found in [5].

1 Introduction

Our primary objective in this work is to generalize the concept of almost periodicity from the quantum time scale to any isolated time scale, in order to generalize the results found in [3]. The two classical definitions of almost periodicity (the Bochner and Bohr ones) presented here for isolated time scales are consistent with the known ones for the discrete and quantum calculus settings. Establishing those definitions, we prove that any Bohr almost periodic function is also Bochner almost periodic and the reciprocal also remains true under a weak condition, allowing us to call those functions just as almost periodic. Moreover, we prove many properties for this class of almost periodic functions. Also, we show that the set of Bochner almost periodic functions with operation $\oplus$ is a subgroup of $(\mathcal{R}, \oplus)$, which shows that our definition makes sense in the time scale environment. In addition, we explicit the relation between the exponential function and the almost periodicity concept, and we state some others equivalences for this class of functions, using almost periodic functions defined on $\mathbb{Z}$ and $\mathbb{R}$. Furthermore, we stablish the hypothesis for the first order linear dynamic equation $X^{\Delta}(t) = A(t)X(t) + f(t)$ to have a almost periodic solution.

2 Main Results

The main definitions and results are listed bellow:

Definition 2.1. *A function $f\colon \mathbb{T} \to \mathbb{R}$ is called **Bochner almost periodic** on $\mathbb{T}$ if for every sequence $\{\alpha'_n\} \subset D$, there exists a subsequence $\{\alpha_n\} \subset \{\alpha'_n\}$ such that $\lim_{n\to\infty} \nu^{\Delta}_{\alpha_n}(t) f(\nu_{\alpha_n}(t))$ exists uniformly on $\mathbb{T}$, for D defined by*

$$D = \begin{cases} \mathbb{Z}, & \text{if } \sup \mathbb{T} = +\infty \text{ and } \inf \mathbb{T} = -\infty, \\ \mathbb{N}_0, & \text{if } \sup \mathbb{T} = +\infty \text{ and } \inf \mathbb{T} > -\infty, \\ -\mathbb{N}_0, & \text{if } \sup \mathbb{T} < +\infty \text{ and } \inf \mathbb{T} = -\infty, \end{cases}$$

where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $-\mathbb{N}_0 = \{-n \ : \ n \in \mathbb{N}_0\}$. We will denote the set of all Bochner almost periodic and regressive functions $f\colon \mathbb{T} \to \mathbb{R}$ by $AP_B(\mathbb{T}, \mathbb{R})$ or simply AP_B.

Definition 2.2. *A function $f\colon \mathbb{T}\times\mathbb{R}\to\mathbb{R}$ is called **Bochner almost periodic** on $t\in\mathbb{T}$ for each $x\in\mathbb{R}$, if for every sequence $\{\alpha'_n\}\subset D$, there exists a subsequence $\{\alpha_n\}\subset\{\alpha'_n\}$ such that*

$$\lim_{n\to\infty}\nu^{\Delta}_{\alpha_n}(t)f(\nu_{\alpha_n}(t),x)$$

exists uniformly on $\mathbb{T}$ for each $x\in\mathbb{R}$.

Definition 2.3. *Let $A(t)$ be a $n\times n$ rd-continuous matrix-valued function on $\mathbb{T}$. We say that the linear system*

$$X^{\Delta}(t)=A(t)X(t) \tag{1}$$

*has an μ-**exponential dichotomy** on $\mathbb{T}$ if there exist positive constants K and γ, and a projection P, which commutes with $X(t)$, $t\in\mathbb{T}$, where $X(t)$ is a fundamental matrix of (1) satisfying*

$$|X(t)PX^{-1}(s)|\le Ke_{\ominus\frac{\gamma}{\mu}}(t,s) \text{ for all } s,t\in\mathbb{T}, t\ge s,$$

and

$$|X(t)(I-P)X^{-1}(s)|\le Ke_{\ominus\frac{\gamma}{\mu}}(s,t) \text{ for all } s,t\in\mathbb{T}, s\ge t.$$

Theorem 2.1. *Let $A\in\mathcal{R}(\mathbb{T},\mathbb{R}^{n\times n})$ be almost periodic and nonsingular on $\mathbb{T}$. Suppose that (1) admits an μ-exponential dichotomy with positive constants K and γ and for every sequence $\{\alpha'_n\}\subset\mathbb{Z}$, there exists a subsequence $\{\alpha_n\}\subset\{\alpha'_n\}$ such that*

$$\lim_{n\to+\infty}\nu^{\Delta}_{\alpha_n}(t)\nu^{\Delta\sigma}_{\alpha_n}(t)f(\nu_{\alpha_n}(t))=\overline{f}(t).$$

exists uniformly for all $t\in\mathbb{T}$. Also, assume that $\frac{\nu^{\Delta}_{\alpha_n}(t)}{\nu^{\Delta}_{\alpha_n}(\sigma(s))}$, for $t,s\in\mathbb{T}$, is bounded for each $n\in\mathbb{N}$. Then, the equation

$$X^{\Delta}(t)=A(t)X(t)+f(t) \tag{2}$$

has an Bochner almost periodic solution.

Theorem 2.2. *Suppose $f\colon\mathbb{T}\to\mathbb{R}$ is Bochner almost periodic and there exists $M>0$ such that*

$$\frac{\mu^{\nu_k}(t)}{\mu(t)}>M \text{ for all } t\in\mathbb{T} \text{ and } k\in\mathbb{Z}. \tag{3}$$

Then f is Bohr almost periodic. Conversely, if f is Bohr almost periodic, then f is Bochner almost periodic.

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AN INTEGRATOR FOR SIMULATING THE DYNAMICS OF STOCHASTIC URBAN NETWORK MODELS

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Abstract

We introduce a stochastic spatial-interaction model of urban structures, examining its main properties in detail. Additionally, we develop an effective time-stepping integrator that enables us to analyze the evolution of this stochastic system over large time intervals. This integrator provides valuable insights into the dynamic behavior of the system as it reaches equilibrium. We also carried out numerical simulation studies to demonstrate the practical effectiveness of the proposed methodology.

1 Introduction and the Stochastic Urban Network Model

Urban network models consist in analytical tools and frameworks used to represent and analyze the complex interconnections within urban systems. These models are essential to help urban planners and researchers understand how the systems function, evolve, and respond to different interventions. The focus is on ways in which the elements comprising the system interact with one another and the long-term evolution of the urban structures. In the present work, random fluctuation are incorporated to the core spatial-interaction model developed and studied by Harris and Wilson (HW) (see [2, 3, 4]) in order to built a more realistic model capable to better reflect the often unpredictable nature of urban systems. We consider the stochastic differential equation:

$$dX^j(t) = \epsilon X^j(t) \left(\sum_{i=1}^{N} S_{ij}(t) - \kappa_j X^j(t) \right) dt + \sigma_j \; X^j(t) \circ dW_t^j, \quad j = 1, \dots M$$

where $S_{ij}(t)$ is the flow (of demand) from origin i to destination j at time t, given by

$$S_{ij}(t) = O_i \frac{\left(X^j(t)\right)^{\alpha} e^{-\beta c_{ij}}}{\sum\limits_{j=1}^{M} \left(X^j(t)\right)^{\alpha} e^{-\beta c_{ij}}},$$

O_i is the total outflow (*e.g., demand*) from origin i; c_{ij} is the cost of carrying out an activity in j from i (*e.g., travel cost*); α regulates the influence of attractiveness on flows, and β controls sensitivity to costs.

Concerning this model, we prove the following proposition:

Proposition 1.1. *For any initial value $X_0 \in \mathbb{R}_+^M$, the system has a **unique positive solution** $X(t) \in \mathbb{R}_+^M$ for any $t \geq 0$, **almost surely**. Also, there exists a **stationary distribution** ρ_∞ for the system. (i.e., the distribution describing **the system's state reaches equilibrium**) which is **ergodic**. That is, **time averages converge a.s** to **expectations under the invariant measure** ρ_∞.*

We now propose a numerical method for the long-term simulation of the model.

2 Proposed Method and Convergence Result

For the computer simulation of the stochastic differential equation above, we propose the integrator (see details in [1]):

$$X_{n+1}^j = e^{-\left(U_{n+1}^j - U_n^j\right)} X_n^j + e^{-U_{n+1}^j} Z_n^j$$

where

$$\begin{bmatrix} * & * & Z_n \\ \mathbf{0} & 1 & * \\ \mathbf{0} & 0 & 1 \end{bmatrix} = \exp(h\mathbf{C}),$$

with

$$\mathbf{C} = \begin{bmatrix} J_Y G(e^{U_n^1} X_n^1(t), \dots, U_n) & J_U G(e^{U_n^1} X_n^1(t), \dots, U_n) \frac{U_{n+1} - U_n}{h} & G(e^{U_n^1} X_n^1(t), \dots, U_n) \\ \mathbf{0} & 0 & 1 \\ \mathbf{0} & 0 & 0 \end{bmatrix}$$

Concerning the convergence of the integrator, we have the following important result

Proposition 2.1. $\{X_n\}_{n=1,\dots}$ *is* ***pathwise convergent*** *if* $U^j(t)$ *is the stationary Ornstein–Uhlenbeck process (i.e. if* $U^j(0) \sim \mathcal{N}(0, \sigma_j^2/2)$*)*

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NUMERICAL ANALYSIS OF THE DISCONTINUOUS GALERKIN METHOD(SIPG)

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Abstract

In this work, we show error estimates for the Symmetric Interior Penalty Galerkin Method (SIPG) studied at [1] applied to a diffusion-absorption problem in one dimension with Dirichlet boundary conditions, equations modeling dissipative heat transfer, chemical reactions, drug diffusion, etc. We obtain the convergence rate in the energy norm and in the L^2 norm, for which we use Cea's Lemma, interpolation results, regularity results, and the Aubin-Nitsche duality argument. Our main interest here is to find a relationship between the parameters involved, such as the diffusion parameter, the absorption parameter, the degrees of the polynomials used for the approximations along with the penalty parameter σ^0.

1 Introduction

We will study the behavior of the solutions and numerically analyze the error obtained from a Diffusion-Absorption problem with α and γ are positive constants:

$$\begin{cases} -\alpha p''(x) + \gamma p(x) &= f(x), \quad \forall x \in (0,1) \\ p(0) = 1, \ \ p(1) &= 0 \end{cases} \tag{1}$$

The broken Sobolev spaces $H^s(\mathcal{E}_h)$, are natural spaces to study DG methods, $\forall s \in \mathbb{R}$, defined by,

$$H^s(\mathcal{E}_h) = \left\{ v \in L^2(\Omega) : \forall I_n \in \mathcal{E}_h,\ v|_{I_n} \in H^s(I_n) \right\}, \tag{2}$$

equipped with the norm and seminorm,

$$\|v\|_{H^s(\mathcal{E}_h)} = \left(\sum_{I_n \in \mathcal{E}_h} \|v\|^2_{H^s(I_n)} \right)^{1/2} \quad \text{and} \quad \|v'\|_{H^0(\mathcal{E}_h)} = \left(\sum_{I_n \in \mathcal{E}_h} \|v'\|^2_{L^2(I_n)} \right)^{1/2}$$

respectively. Let us consider the finite-dimensional subspaces $D_l(\mathcal{E}_h)$ of the broken Sobolev spaces $H^s(\mathcal{E}_h)$ for $s > 3/2$ (see [1]),

$$D_l(\mathcal{E}_h) = \{ q \in H^s(\mathcal{E}_h) : q|_{I_n} \in \mathbb{P}_l(I_n),\ \forall I_n \in \mathcal{E}_h \}, \tag{3}$$

called the space of piecewise discontinuous polynomials of degree l, where $\mathbb{P}_l(I_n)$ is the space of polynomials of degree l in the interval $I_n = (x_n, x_{n+1})$. The formulation of the Discontinuous Galerkin SIPG method for the problem (1) is as follows (see [3]),

Find $P_h \in \mathcal{D}_l(\mathcal{E}_h)$ such that

$$\mathcal{A}(P_h, q_h) = \mathcal{L}(q_h), \quad \forall q_h \in \mathcal{D}_l(\mathcal{E}_h) \tag{4}$$

with,

$$\begin{aligned} \mathcal{A}(P_h, q_h) \quad = \quad & \sum_{n=0}^{N-1} \int_{x_n}^{x_{n+1}} \left[\alpha P_h{}'(x) q_h{}'(x) + \gamma P_h(x) q_h(x) \right] dx - \sum_{n=0}^{N} [\![\alpha P_h{}'(x_n)]\!] [\![q_h(x_n)]\!] \\ & - \sum_{n=0}^{N} [\![\alpha q_h'(x_n)]\!] [\![P_h(x_n)]\!] + \sum_{n=0}^{N} \frac{\sigma^0}{h} [\![P_h(x_n)]\!] [\![q_h(x_n)]\!], \end{aligned}$$

$$\mathcal{L}(q_h) \quad = \quad \int_{x_0}^{x_N} f(x) q_h(x) dx - \alpha q_h'(0) + \frac{\sigma^0}{h} q_h(0),$$

where the bilinear form $\mathcal{A} : \mathcal{D}_l(\mathcal{E}_h) \times \mathcal{D}_l(\mathcal{E}_h) \to \mathbb{R}$ y $\mathcal{L} : \mathcal{D}_l(\mathcal{E}_h) \to \mathbb{R}$.

2 Main Results

The main results of our work are

Theorem 2.1 (Coercivity, see [3]). *If* $\sigma^0 \geq \frac{8}{3}\alpha l^2$ *then* $\mathcal{A}(P_h, P_h) \geq \frac{1}{4}\|P_h\|_{\mathcal{E}}^2, \quad \forall P_h \in \mathcal{D}_l(\mathcal{E}_h)$.

Theorem 2.2 (Continuity, see [3]). *There is the constant* $M = \max\left\{1 + \frac{3l^2\alpha}{2h}, 1, \frac{\alpha h}{\sigma^0} + 1\right\} > 0$ *such that*

$$|\mathcal{A}(P_h, q_h)| \quad \leq \quad M\|P_h\|_{\varepsilon}^2 \|q_h\|_{\varepsilon}^2, \quad \forall P_h, q_h \in \mathcal{D}_l(\mathcal{E}_h).$$

Theorem 2.3 (Error estimate in the energy norm). *Assume that the exact solution* p *to* (1) *belongs to* $H^s(\mathcal{E}_h)$ *for* $s > 3/2$. *If in addition* $\sigma^0 \geq \frac{8}{3}\alpha l^2$ *then for the SIPG method there exists a constant* C *independent of* h *such that we have the following estimate of the error in the energy norm:*

$$\|p - P_h\|_{\mathcal{E}_h} \leq C h^{\min\{l+1,s\}-1} \|p\|_{H^s(\mathcal{E}_h)}$$

Proof This Theorem was obtained using Theorems 2.1 and 2.2, Young's inequality and the Trace theorem for polynomials. ■

Theorem 2.4 (Estimate of the error in the L^2 norm). *Suppose that Theorem 2.3 holds. There exists an independent constant* C *independent of* h *such that*

$$\|p - P_h\|_{L^2(\Omega)} \leq C h^{\min\{l+1,s\}-1/2} \|p\|_{H^s(\mathcal{E}_h)} \tag{1}$$

Proof The above theorem was generated by first obtaining the dual problem associated to the equation (1), with source term $p(x) - P_h(x)$. Then, the Theorem 2.3, the error orthogonality equation, the Cauchy-Schwarz inequality, among others, were used. ■

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SPACES OF SEQUENCES NOT CONVERGING TO ZERO

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Abstract

Let E be a Banach space, let τ be a vector topology on E and let $\mathbf{x}$ be a sequence in E not converging to zero with respect to τ . We show how to construct infinite dimensional Banach spaces consisting, up to the origin, of sequences in E not converging to zero with respect to τ and containing a subsequence of $\mathbf{x}$. The applications we provide encompass the improvement of known results, as well as new results, concerning Banach spaces not satisfying classical properties and linear/nonlinear maps not belonging to well studied classes.

1 Introduction

One of the problems in the field of lineability is the following: given a subset A of a Banach space (or a topological vector space) E, is there a closed infinite dimensional subspace W of E contained in $A \cup \{0\}$? If yes, the set A is said to be *spaceable*. Many proofs start with a vector $x \in A$ and then manipulate x conveniently to construct the subspace W. Sometimes, the mother vector x does not belong to W. In [3], the set A is said to be *pointwise spaceable* if, regardless of the vector $x \in A$, there is a closed infinite dimensional subspace W of E contained in $A \cup \{0\}$ and containing x. In the setting of sequence spaces, sometimes one can get a bit more than spaceability and a bit less than pointwise lineability: maybe that, given a sequence $x \in A$, one can find a closed infinite dimensional subspace W of E contained in $A \cup \{0\}$ and containing *a subsequence* of x. In this case, we shall say that the set A is *almost pointwise spaceable*. This notion is motivated by the observation that, although many sets are known to be spaceable, their (almost) pointwise spaceability may not be evident.

The purpose of this work is to prove a general theorem that gives conditions on a (non necessarily linear) map $f\colon E \longrightarrow F$ between Banach spaces, on a subset A of $\ell_\infty(E)$ and on a vector topology τ on F so that the set of E-valued sequences $(x_j)_{j=1}^\infty$ belonging to A such that $(f(x_j))_{j=1}^\infty$ does not converge to zero with respect to τ is almost pointwise spaceable in $\ell_\infty(E)$. The proof of our general theorem is a refinement of a technique introduced by Jiménez-Rodríguez [3].

The results of this work can be found in [2].

2 Main Results

The following definition was motivated in the introduction.

Definition 2.1. A nonvoid subset A of a Banach sequence space X is said to be *almost pointwise spaceable* if, for every sequence $\mathbf{x} \in A$, there exists a closed infinite dimensional subspace of X contained in $A \cup \{0\}$ and containing a subsequence of $\mathbf{x}$.

Definition 2.2. A map $f\colon E \longrightarrow F$ between topological vector spaces is said to be of *homogeneous type* if f is continuous at 0 and there exists a nonzero integer number n such that $f(\lambda x) = \lambda^n f(x)$ for every scalar $\lambda \neq 0$ and every $x \in E$.

It is easy to see that $f(0) = 0$ in this case.

Definition 2.3. Let E be a Banach space. A subset A of $\ell_\infty(E)$ is said to be:
(i) *Subsequence invariant* if subsequences of sequences belonging to A belong to A as well.
(ii) ℓ_∞*-complete* if, for $(x_j)_{j=1}^\infty \in A$ and $(\alpha_j)_{j=1}^\infty \in \ell_\infty$, it holds $(\alpha_j x_j)_{j=1}^\infty \in A$.

We now present the main result of this work.

Theorem 2.1. *Let $f\colon E \longrightarrow F$ be a map of homogeneous type between Banach spaces, let $A \subseteq \ell_\infty(E)$ be a subsequence invariant and ℓ_∞-complete, and let τ be a vector topology on F weaker than the norm topology. Then the subset $\mathcal{C}$ of sequences $(x_j)_{j=1}^\infty$ in A for which $(f(x_j))_{j=1}^\infty$ is non-τ-null is empty or almost pointwise spaceable in $\ell_\infty(E)$. Moreover, $\mathcal{C} \cap c_0^w(E)$ is empty or almost pointwise spaceable in $c_0^w(E)$.*

We next present applications of the main result to Banach spaces without well studied properties and to operators and homogeneous polynomials that do not belong to classical classes.

Corollary 2.1. (a) *Let $T\colon E \longrightarrow F$ be a non-completely continuous operator between Banach spaces. Then the set of E-valued weakly null sequences $(x_j)_{j=1}^\infty$ such that $T(x_j) \not\longrightarrow 0$ is almost pointwise spaceable in $c_0^w(E)$.*
(b) *Let $P\colon E \longrightarrow F$ be a non-weakly sequentially continuous homogeneous polynomial. Then the set of E-valued weakly null sequences $(x_j)_{j=1}^\infty$ such that $P(x_j) \not\longrightarrow 0$ is almost pointwise spaceable in $c_0^w(E)$.*
(c) *Let E be a Banach space failing the polynomial Schur property. Then the set of E-valued polynomially null non-norm null sequences is almost pointwise spaceable in $c_0^w(E)$.*

Corollary 2.2. *Let E be an infinite dimensional Banach space. Then the set of weak*-null non-norm null sequences in E^* is almost pointwise spaceable in $\ell_\infty(E^*)$.*

Corollary 2.3. *Let E be Banach space containing no copy of ℓ_1 and let F be a closed infinite dimensional subspace of E^*. Then the set $\mathcal{C} = \{(x_j^*)_{j=1}^\infty \in \ell_\infty(F)\colon x_j^* \xrightarrow{\omega^*} 0$ in E^* and $x_j^* \not\longrightarrow 0\}$ is almost pointwise spaceable in $\ell_\infty(F)$.*

Corollary 2.4. *Let $T\colon E \longrightarrow F$ be a non-p-convergent operator, $1 \le p < \infty$. Then the set of weakly p-summable sequences in E such that $(T(x_j))_{j=1}^\infty$ is non-norm null in F is almost pointwise spaceable in $c_0^w(E^*)$.*

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LINEAR DYNAMICS FOR AFFINE COMPOSITION OPERATORS ON A WEIGHTED BERGMAN SPACE

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Abstract

In this work, we present a complete characterization of positive expansiveness and absolutely CÃ¨saro bounded operators among composition operators induced by affine self-maps ϕ of the right half-plane $\mathbb{C}_+$ on the weighted Bergman space $\mathcal{A}^2_\alpha(\mathbb{C}_+)$. Furthermore, we also present which of these operators have the positive shadowing property.

1 Introduction

Let $\mathbb{C}_+ = \{z \in \mathbb{C} : \mathrm{Re}(z) > 0\}$ denote the open right half-plane. We aim to generalize the work of Álvarez and Henríquez-Amador [1] for composition operators over the Hardy space of the right half-plane $H^2(\mathbb{C}_+)$. For $\alpha > -1$, the weighted Bergman space over $\mathbb{C}_+$, $\mathcal{A}^2_\alpha(\mathbb{C}_+)$, consists of all holomorphic functions $f : \mathbb{C}_+ \to \mathbb{C}$ for which

$$\|f\| = \left(\frac{1}{\pi}\int_{\mathbb{C}_+} |f(x+iy)|^2 x^\alpha dxdy\right)^{1/2} < \infty. \tag{1}$$

Note that $H^2(\mathbb{C}_+)$ can be formally interpreted as the limit case $\alpha \to -1$, that is, we have $\mathcal{A}^2_{-1}(\mathbb{C}_+) = H^2(\mathbb{C}_+)$.

Consider ϕ a self-map of $\mathbb{C}_+$, we define the composition operator C_ϕ on $\mathcal{A}^2_\alpha(\mathbb{C}_+)$ by $C_\phi f = f \circ \phi,\ \ f \in \mathcal{A}^2_\alpha(\mathbb{C}_+)$.

Our interest are in composition operators symbols of the form $\phi(w) = aw + b$, where $a > 0$ and $\mathrm{Re}(b) \geq 0$, and we will call operators induced by this type of symbols by affine composition operators.

Now we focus on the linear dynamics. Let X denote a complex Banach space and $\mathcal{B}(X)$ denote the space of all bounded linear operators on X. We are interested in studying

Definition 1.1. *We say that a operator $T \in \mathcal{B}(X)$ is called Li-Yorke chaotic if there exists a vector $x \in X$ such that*

$$\liminf_{n\to\infty} \|T^n x\| = 0 \quad \textit{and} \quad \limsup_{n\to\infty} \|T^n x\| = \infty.$$

In this case, we say that x is an irregular vector.

Another important concept in dynamics is of expansiveness

Definition 1.2. *Let $T \in \mathcal{B}(X)$ be an invertible operator, then we say T is:*

(i) expansive if for each $x \in X$ with $\|x\| = 1$, there exists $n \in \mathbb{Z}$ such that $\|T^n x\| \geq 2$.

(ii) uniformly expansive, if there exists $n \in \mathbb{N}$ such that for each $x \in X$ with $\|x\| = 1$, we have $\|T^n x\| \geq 2$ or $\|T^{-n} x\| \geq 2$.

We also have a similar definition for expansiveness for non-invertible operators, so there exists the notions of positive expansive and uniformly positive expansive by replacing $\mathbb{Z}$ for $\mathbb{N}$. In the study of orbit for operators we have the concepts of pseudotrajectory and shadowing which we define as follows

Definition 1.3. *Let $T \in \mathcal{B}(X)$. For $\delta > 0$, we say that a sequence $(x_n)_{n\in\mathbb{N}} \subset X$ is a δ-pseudotrajectory of T, if $\|Tx_n - x_{n+1}\| \leq \delta$ for all $n \in \mathbb{N}$.*

Definition 1.4. *Let $T \in \mathcal{B}(X)$. We say that T has the positive shadowing property if for every $\varepsilon > 0$, there exists $\delta > 0$ such that every δ-pseudotrajectory $(x_n)_{n\in\mathbb{N}}$ of T there exists $x \in X$ such that $\|T^n x - x_n\| < \varepsilon$ for all $n \in \mathbb{N}$.*

2 Main Results

The main results obtained in [2] are the following

Proposition 2.1. *Let $\phi(w) = aw + b$ with $a > 0$ and $Re(b) \geq 0$, then C_ϕ is not Li-Yorke chaotic on $\mathcal{A}^2_\alpha(\mathbb{C}_+)$.*

In particular, this also show that there is no hypercyclic affine composition operator. Now, we characterize expansiveness for this class of composition operators

Proposition 2.2. *Let $\phi(w) = aw + b$ with $a > 0$ and $Re(b) = 0$. Then the following statements are equivalent*

(i) C_ϕ *is uniformly expansive on* $\mathcal{A}^2_\alpha(\mathbb{C}_+)$;

(ii) C_ϕ *is expansive on* $\mathcal{A}^2_\alpha(\mathbb{C}_+)$;

(iii) $a \neq 1$.

Similarly we may prove the following for positive expansiveness:

Proposition 2.3. *Let $\phi(w) = aw + b$ with $a > 0$ and $Re(b) \geq 0$. Then the following statements are equivalent*

(i) C_ϕ *is uniformly positive expansive on* $\mathcal{A}^2_\alpha(\mathbb{C}_+)$;

(ii) C_ϕ *is positive expansive on* $\mathcal{A}^2_\alpha(\mathbb{C}_+)$;

(iii) $a \in (0, 1)$.

For the positive shadowing property, we have also a total characterization:

Theorem 2.1. *Let $\phi(w) = aw + b$ with $a > 0$ and $Re(b) \geq 0$. Then C_ϕ has the positive shadowing property on $\mathcal{A}^2_\alpha(\mathbb{C}_+)$ if and only if one of the following cases occurs:*

(i) $a \in (0, 1)$ *and* $Re(b) = 0$;

(ii) $a > 1$.

Finally, we have the characterization for absolutely CÃ¨saro bounded:

Proposition 2.4. *Let $\phi(w) = aw + b$ with $a > 0$ and $Re(b) \geq 0$. Then C_ϕ is an absolutely CÃ¨saro bounded operator on $\mathcal{A}^2_\alpha(\mathbb{C}_+)$ if and only if $a \geq 1$.*

Interestingly, absolutely CÃ¨saro boundedness implies mean ergodicity, so the above class are also mean ergodic operators.

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FUSED GROTHENDIECK'S AND KWAPIEŃ'S THEOREMS FOR NONLINEAR MAPS

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Abstract

Kwapień's theorem asserts that every continuous linear operator from ℓ_1 to ℓ_p is absolutely $(r,1)$-summing for $1/r = 1 - |1/p - 1/2|$. When $p = 2$ it recovers the famous Grothendieck's theorem. In this talk, concerning the multilinear variants of these results, we present a version of Kwapień's and Grothendieck's theorems that encompasses the cases of multiple summing and absolutely summing multilinear operators.

1 Introduction

The theory of absolutely summing operators was initially introduced and developed by A. Pietsch, B. Mitiagin and A. Pełczyński and J. Lindenstrauss and A. Pełczyński (see the book [3]), inspired by previous works of A. Grothendieck and it plays an important role in the study of Banach Spaces and Operator Theory, with important connections with other fields of mathematics.

Let E, F be Banach spaces and $r \geq s \geq 1$ be real numbers. A continuous linear operator $T : E \to F$ is absolutely (r,s)-summing if $(T(x_j))_{j=1}^{\infty} \in \ell_r(F)$ whenever $(x_j)_{j=1}^{\infty} \in \ell_s^w(E)$, where $\ell_s^w(E)$ denotes the space of weakly s-summable sequences in E.

One of the cornerstones of the theory of absolutely summing operators is Grothendieck's theorem, which asserts that every continuous linear operator from ℓ_1 to ℓ_2 is absolutely $(1,1)$-summing. Kwapień extended Grothendieck's theorem replacing ℓ_2 by ℓ_p, for $p \geq 1$, as follows: every continuous linear operator from ℓ_1 to ℓ_p is absolutely $(r,1)$-summing, with

$$1/r = 1 - |1/p - 1/2|, \tag{1}$$

and this result is optimal. In the last decades the notion of absolutely summing operators was extended to the multilinear setting in several different lines of research. In this work we shall be interested in the notions of multiple summing and absolutely summing multilinear operators (see [1], for the definitions).

The extension of Kwapień's theorem to multilinear operators is a natural problem to be investigated. For multiple summing operators, an immediate consequence of [2, Corollary 4.3] is that every continuous m-linear operator from ℓ_1 to ℓ_p is multiple $(r,1)$-summing, with r as in (1) and the parameter r is sharp. For absolutely summing multilinear operators, was proved in [1] that every continuous m-linear operator from ℓ_1 to ℓ_p is absolutely $(r,1)$-summing for

$$r = \begin{cases} \frac{2p}{mp+2p-2}, & \text{if } 1 \leq p \leq 2 \\ \frac{2p}{mp+2}, & \text{if } 2 \leq p \leq \infty. \end{cases}$$

The aim in this work is to present a version of Kwapień's theorem (see Theorem 2.1) that encompasses, as extreme cases, the cited results concerning the notion of multiple summing and absolutely summing multilinear operators, in the following terms:

Let m be a positive integer and let $1 \leq k \leq m$ and $\mathcal{I} = \{I_1, \ldots, I_k\}$ be a family of pairwise disjoint non-void subsets of $\{1, \ldots, m\}$ such that $\cup_{j=1}^{k} I_j = \{1, \ldots, m\}$. Let $\Omega_{I_j} \subset \mathbb{N}^{|I_j|}$ defined by

$$\Omega_{I_j} = Diag\left(\mathbb{N}^{|I_j|}\right) = \left\{(i, \ldots, i) \in \mathbb{N}^{|I_j|}\right\}$$

for all $j \in \{1, ..., k\}$. Let us also define $\Omega_{\mathcal{I}} \subset \mathbb{N}^m$ as the product of diagonals

$$\Omega_{\mathcal{I}} = \Omega_{I_1} \times \cdots \times \Omega_{I_k}.$$

We will denote by

$$\mathcal{L}_{\mathcal{I},as(r;1)}\left({}^m\ell_1; F\right)$$

the space of all $T \in \mathcal{L}\left({}^m\ell_1; F\right)$, for which there is a constant $C > 0$ be such that

$$\sum_{(j_1,...,j_m)\in\Omega_{\mathcal{I}}} \left\|T\left(x_{j_1}^{(1)}, ..., x_{j_m}^{(m)}\right)\right\|_F^r \leq C \prod_{i=1}^m \left\|\left(x_{j_i}^{(i)}\right)_{j_i=1}^\infty\right\|_{w,1}$$

for every

$$\left(x_{j_i}^{(i)}\right)_{j_i=1}^\infty \in \ell_1^w\left(\ell_1\right).$$

We stress that, when $k = m$ (or $k = 1$, respectively) this notion recovers the definition of the multiple summing operators (absolutely summing multilinear operators, respectively).

2 Main Result

Theorem 2.1. *[4, Theorem 4.3] Let $\mathcal{I} = \{I_1, \ldots, I_k\}$ be a family of pairwise disjoint non-void subsets of $\{1, \ldots, m\}$ such that $\cup_{j=1}^k I_j = \{1, \ldots, m\}$. If $n = \min\{|I_1|, \ldots, |I_k|\}$, then*

$$\mathcal{L}\left({}^m\ell_1; \ell_p\right) = \mathcal{L}_{\mathcal{I},as(t;1)}\left({}^m\ell_1; \ell_p\right).$$

with

$$t = \begin{cases} \frac{2p}{np+2p-2}, & \text{if } 1 \leq p \leq 2 \\ \frac{2p}{np+2}, & \text{if } 2 \leq p \leq \infty. \end{cases}$$

Moreover, the parameter t is optimal when $2 \leq p \leq \infty$.

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DYNAMICS OF COMPOSITION OPERATORS ON SPACES OF MEASURABLE FUNCTIONS

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Abstract

We study composition operators acting on L^p from a dynamical point of view, introduced by Bayart, Darji and Pires in 2018. We give characterizations for composition operators being hypercyclic, mixing and satisfying Kitai's Criterion. We then show that the notions of weak mixing and hypercyclicity coincide in this context, and give an example of a mixing composition operator that does not satisfy Kitai's Criterion.

1 Introduction

Motivated by the study of weighted shifts (a very important class of operators in linear dynamics), Bayart, Darji and Pires have studied in [1] the dynamics of composition operators on spaces $\mathcal{X}$ of (equivalence classes of) measurable functions that generalize the spaces L^p. We say that a continuous linear operator $T : \mathcal{X} \to \mathcal{X}$ acting on a Banach space is:

- *topologically transitive* if for every pair $U, V \subseteq \mathcal{X}$ of non-empty open sets, there exists $k \geq 0$ such that $T^k(U) \cap V \neq \varnothing$;
- *weakly mixing* if $T \times T$ is topologically transitive;
- *topologically mixing* if for every pair $U, V \subseteq \mathcal{X}$ of non-empty open sets, there exists $k_0 \geq 1$ such that $T^k(U) \cap V \neq \varnothing$ for every $k \geq k_0$.

By the Birkhoff transitivity theorem (see [4, Theorem 1.16]), topological transitivity is equivalent to *hypercyclicity* (i.e. the operator admits a dense orbit).

A very useful tool to show that an operator is mixing is to show that it satisfies Kitai's Criterion, that it, it satisfies the hypotheses of the following theorem:

Theorem 1.1 (Kitai's Criterion, [5])**.** *Let $T : \mathcal{X} \to \mathcal{X}$ be a continuous linear operator. If there are dense subsets $\mathcal{X}_0, \mathcal{Y}_0 \subseteq \mathcal{X}$ and a map $S : \mathcal{Y}_0 \to \mathcal{Y}_0$ such that, for any $x \in \mathcal{X}_0$ and $y \in \mathcal{Y}_0$*

1. $T^n x \to 0$,
2. $S^n y \to 0$,
3. $TSy = y$,

then T is mixing.

The question regarding if every mixing operator satisfy Kitai's Criterion was solved by S. Grivaux in [3], where it is shown that every infinite dimension separable Banach space admits a mixing operator that does not satisfy Kitai's Criterion.

We assume that $(X, \mathcal{B}, \mu)$ is a σ-finite measure space and that $(\mathcal{X}, \|\cdot\|) \subseteq L^0_\mu(X)$ is a Banach space of measurable functions. Given a measurable transformation $f : X \to X$, we consider the operator $T_f : \mathcal{X} \to \mathcal{X}$ given by $\varphi \mapsto \varphi \circ f$, where we assume that f is a non-singular transformation, i.e. for every $B \in \mathcal{B}$, $\mu(B) = 0$ implies $\mu(f^{-1}(B)) = 0$. In [1, Theorems 1.1 and 1.2], Bayart et al. characterize operators $T_f : \mathcal{X} \to \mathcal{X}$ that are hypercyclic or mixing. We are able to also characterize composition operators that are weakly mixing or satisfy Kitai's Criterion.

2 Main Results

We are able to show that the notions of hypercyclicity and weak mixing coincide for composition operators:

Theorem 2.1 ([2]). *The following are equivalent:*

1. *T_f is hypercyclic;*
2. *T_f is weakly mixing.*

By characterizing composition operators satisfying Kitai's Criterion and using the characterization for mixing composition operators from [1], we are able to construct an example of a mixing composition operator not satisfying Kitai's Criterion.

Theorem 2.2 ([2]). *Suppose that f is invertible and bi-measurable. Then T_f satisfies Kitai's Criterion if, and only if, for every $\varepsilon > 0$ and every measurable set A of finite measure, there exists a measurable set $B \subseteq A$ such that*

$$\mu(A \setminus B) < \varepsilon, \quad \mu(f^{-n}(B)) \to 0 \quad \text{and} \quad \mu(f^{n}(B)) \to 0.$$

Theorem 2.3 ([2]). *There exists an invertible measurable system $(X, \mathcal{B}, \mu, f)$ such that $T_f : L^p(X) \to L^p(X)$, $1 \leq p < \infty$, is mixing and does not satisfy Kitai's Criterion.*

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STABILITY RESULTS FOR THE BISHOP-PHELPS-BOLLOBÁS PROPERTY

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Abstract

In this talk, we present results concerning the stability of the Bishop-Phelps-Bollobás property for spaces of continuous functions and uniform algebras.

1 Introduction

Let X and Y be Banach spaces over $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$). We write B_X, S_X, and X^* for the closed unit ball, the unit sphere, and the topological dual of X. We denote by $\mathcal{L}(X,Y)$ the Banach space of all bounded linear operators from X into Y and by NA (X,Y) the set of all norm attaining operators (recall that $T \in \mathcal{L}(X,Y)$ is said to be *norm attaining* if there exists $x \in S_X$ such that $\|Tx\| = \|T\|$).

In 1961, Bishop and Phelps [2] presented the famous Bishop-Phelps theorem which states that the set of norm attaining functionals on a Banach space X is dense in the dual space, that is, NA $(X,\mathbb{K})$ is dense in X^*. This result raised a natural question whether the set of norm-attaining linear operators is dense in $\mathcal{L}(X,Y)$ for all Banach spaces X and Y. Lindenstrauss [7] gave a negative answer to this question. In 1970, Bollobás [3] proved a numerical version of the Bishop-Phelps theorem, which is known as the Bishop-Phelps-Bollobás theorem and can be stated as follows.

Theorem 1.1. *Let X be a Banach space and $0 < \varepsilon < 1$. Then for any $x \in B_X$ and $x^* \in S_{X^*}$ satisfying $|1 - x^*(x)| < \frac{\varepsilon^2}{4}$, there exist $y \in S_X$ and $y^* \in S_{X^*}$ such that $y^*(y) = 1$, $\|x - y\| < \varepsilon$ and $\|x^* - y^*\| < \varepsilon$.*

In 2008, Acosta et al. [1] considered this theorem for bounded linear operators between Banach spaces and introduced the Bishop-Phelps-Bollobás property for operators.

Definition 1.1. *A pair of Banach spaces (X,Y) is said to have the* ***Bishop-Phelps-Bollobás property for operators*** *(BPBp for short) if, given $\varepsilon > 0$, there is $\eta(\varepsilon) > 0$ such that for every $T \in S_{\mathcal{L}(X,Y)}$ and $x_0 \in S_X$ satisfying $\|Tx_0\| > 1 - \eta(\varepsilon)$, there exist an operator $S \in S_{\mathcal{L}(X,Y)}$ and a point $u_0 \in S_X$ such that $\|Su_0\| = 1$, $\|u_0 - x_0\| < \varepsilon$ and $\|S - T\| < \varepsilon$.*

We refer the reader to [5] for further results on the Bishop-Phelps-Bollobás property for operators.

2 Main Results

For a compact Hausdorff space K and a locally compact Hausdorff space L, we denote by $\mathcal{C}(K,X)$ the space of all continuous functions from K into X and by $\mathcal{C}_0(L,X)$ for the Banach space of all continuous functions from L into X that vanish at infinity.

Let $\mathcal{C}(K)$ denote the space of complex-valued continuous functions on a compact Hausdorff space K. A *uniform algebra* is a closed subalgebra $\mathcal{A} \subset \mathcal{C}(K)$ that separates the points of K. The uniform algebra $\mathcal{A}$ is said to be *unital* if the constant function $\mathbf{1}$ belongs to $\mathcal{A}$. For a uniform algebra $\mathcal{A}$, we denote by $\mathcal{A}^X$ the subspace of $\mathcal{C}(K,X)$ defined by $\mathcal{A}^X = \{f \in \mathcal{C}(K,X) : x^* \circ f \in \mathcal{A} \text{ for all } x^* \in X^*\}$.

In 2015, Aron et al. [2] proved the following result.

Theorem 2.1. [2, Proposition 2.8] *Let X, Y be Banach spaces and let K be a compact Hausdorff space. If the pair $(X, \mathcal{C}(K, Y))$ has the BPBp, then the pair (X, Y) has the BPBp.*

The converse of this theorem is false. Indeed, when $K = [0, 1]$ and $Y = \mathbb{K}$, there exists a Banach space X such that $(X, \mathcal{C}(K, Y))$ does not have the BPBp, while the pair (X, Y) has the BPBp, as ensured by the Bishop-Phelps theorem. However, the converse does hold for the generalized AHSP (see [6, Definition 1.3]) when X is finite-dimensional by [6, Theorem 2.2].

Motivated by the above theorem, we present analogous results for the case of $(X, \mathcal{C}_0(L, Y))$ and $(X, \mathcal{A}^Y)$. Proofs of these results are available in the preprint [6].

Theorem 2.2. *Let X, Y be Banach spaces and let L be a locally compact Hausdorff space. If the pair $(X, \mathcal{C}_0(L, Y))$ has the BPBp, then the pair (X, Y) has the BPBp.*

Theorem 2.3. *Let X, Y be complex Banach spaces and let $\mathcal{A}$ be a unital uniform algebra. If the pair $(X, \mathcal{A}^Y)$ has the BPBp, then the pair (X, Y) has the BPBp.*

As in the case of Theorem 2.1, the converses of Theorems 2.2 and 2.3 are false, as shown by the argument given after Theorem 2.1. Nevertheless, positive results hold for the generalized AHSP. We refer the reader to [6] for more details.

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SPACEABILITY IN HYPERCYCLIC AND SUPERCYCLIC OPERATORS

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Abstract

We investigate the linear and topological structure of hypercyclic and supercyclic operators on Banach spaces. Positive and negative results are established concerning the lineability of the set of hypercyclic operators, as well as the spaceability of these sets with respect to both the strong operator topology (SOT) and the uniform topology. Moreover, we prove that the set of supercyclic operators that are not hypercyclic is lineable in any separable Fréchet space not isomorphic to $\mathbb{K}^{\mathbb{N}}$. Finally, we show that the set of non-supercyclic operators on ℓ_p contains isometric copies of ℓ_q.

1 Introduction

The theory of hypercyclic and supercyclic operators has gained prominence in Linear Dynamics due to the complex topological and algebraic structures these sets exhibit. An operator T on a separable infinite-dimensional Banach space X is hypercyclic if there exists a vector $x \in X$ whose orbit $\{T^n x : n \in \mathbb{N}\}$ is dense in X; if the projective orbit $\{\lambda T^n x : \lambda \in \mathbb{K}, n \in \mathbb{N}\}$ is dense, the operator is called supercyclic. While every hypercyclic operator is supercyclic, the converse does not generally hold.

In recent years, concepts such as lineability and spaceability, which seek to identify linear substructures or infinite-dimensional closed subspaces within sets that are a priori nonlinear, have been widely applied to the study of these operators. In this work, we present new results on the linear structure of the sets of hypercyclic and supercyclic operators, proving both lineability and non-lineability properties. Additionally, we investigate the spaceability of these sets under different topologies. Although it is known that the set of hypercyclic operators is dense in the Strong Operator Topology (SOT), we show that this set is also spaceable in this topology. On the other hand, we demonstrate that even in the uniform operator topology, where there are relatively few hypercyclic operators, the complement set still possesses a spaceable structure,, i.e., a closed infinite-dimensional subspace up to the origin.

These results deepen the understanding of the interplay between dynamical properties, algebraic structure, and functional topology in operator theory.

2 Main Results

This section presents the main result concerning properties of lineability and spaceability of hypercyclic and supercyclic operators, highlighting their relevance in linear dynamics and functional analysis. Throughout this section, all Banach and Fréchet spaces considered are assumed to be infinite-dimensional and separable.

Theorem 2.1. *Let X be a Banach space. Then $\mathcal{L}(X) \setminus \mathcal{L}_{HC}(X)$ is SOT-spaceable.*

This theorem establishes the spaceability of the complement of hypercyclic operators under the SOT. Building upon this, the following proposition extends the analysis to supercyclic operators, revealing additional structural properties and specific embeddings in classical sequence spaces.

Proposition 2.1. *Let X be a Banach space. Then the following holds.*

a) The set $\mathcal{L}(X) \setminus \mathcal{L}_{SC}(X)$ of bounded non-supercyclic operators in X is spaceable.

b) For $X = \ell_p$ and $p > 0$, the set $\mathcal{L}(\ell_p) \setminus \mathcal{L}_{SC}(\ell_p)$ contains an isometric copy of ℓ_p.

Finally, we highlight the most significant result of this work, which reveals the lineability of the set of supercyclic operators that are not hypercyclic in separable Fréchet spaces, substantially broadening the understanding of the algebraic structure of these operators.

Theorem 2.2. *Let X be a Fréchet space that is not isomorphic to $\omega = \mathbb{K}^{\mathbb{N}}$. Then $\mathcal{L}_{SC}(X) \setminus \mathcal{L}_{HC}(X)$ is lineable.*

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ON LINEABILITY OF THE SET OF HYPERCYCLIC VECTORS

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Abstract

We investigate the linear structure of the set of hypercyclic vectors associated with a bounded linear operator T on a Banach space X. Motivated by classical problems such as the Invariant Subspace Problem, we study conditions under which the set $\mathrm{HC}(T)$ of hypercyclic vectors contains large linear subspaces. While this set is generally not a vector space, we prove that, under Kitai's Criterion, any hypercyclic subspace satisfying a uniform decay condition can be extended to a hypercyclic subspace of dimension equal to the cardinality of the continuum. The proof combines techniques from linear dynamics with a refinement of an ℓ_∞-independence lemma, allowing us to embed ℓ_∞ into $\mathrm{HC}(T)$. This contributes to the ongoing program of understanding lineability phenomena in operator theory and functional analysis.

1 Introduction

Linear dynamics is a field of mathematics that has seen significant growth in recent decades, closely intertwined with classical problems such as the Invariant Subspace Problem. In fact, this problem can be reformulated in terms of linear dynamics as follows: a bounded linear operator $T : X \to X$ on a Banach space has no nontrivial closed invariant subspace if and only if every nonzero vector $v \in X$ is hypercyclic, that is,

$$\overline{\{v, Tv, T^2v, \dots\}} = X,$$

as stated in [1, p.38].

In this context, understanding the size and structure of the set of hypercyclic vectors is of great interest, when this set is nonempty. One way to quantify this is through the notion of lineability.

Definition 1.1. *A subset A of a vector space is said to be lineable if there exists an infinite-dimensional linear subspace W such that $W \subset A \cup \{0\}$.*

More generally, we may seek even richer linear structures within such sets.

Definition 1.2. *A subset A is said to be (α, β)-lineable if, given any linear subspace $M \subset A \cup \{0\}$ with $\dim(M) = \alpha$, there exists a subspace $W \subset A \cup \{0\}$ with $\dim(W) = \beta$ such that $M \subset W$.*

Determining whether a set is lineable or (α, β)-lineable amounts to identifying when a linear structure can be found within sets that are typically not linear in nature, such as the set of hypercyclic vectors, $\mathrm{HC}(T)$. A classical result asserts that if X is a Banach space and T is a bounded linear operator on X that admits hypercyclic vectors, then every vector in X can be written as the sum of two hypercyclic vectors, i.e.,

$$X = \mathrm{HC}(T) + \mathrm{HC}(T).$$

As a consequence, the only situation in which $\mathrm{HC}(T) \cup \{0\}$ forms a vector space is when it equals the entire space, $\mathrm{HC}(T) \cup \{0\} = X$.

In this work, we extend the current understanding of the structure of the set of hypercyclic vectors by showing that, under suitable conditions, any linear subspace of hypercyclic vectors can be extended to a subspace of hypercyclic vectors of dimension equal to the cardinality of the continuum.

2 Main Results

For the main result, we assume hypotheses 1-3, which are known in Linear Dynamics as the Kitai Criterion, [1, p.71].

Theorem 2.1. *Let X be a separable Banach space and let $T : X \to X$ be an operator. Suppose that there exists a dense subset $X_0 \subset X$ and a map $S : X_0 \to X_0$ such that, for every $x \in X_0$, the following hold:*

1. $T^n x \to 0$;
2. $S^n x \to 0$;
3. $TSx = x$.

Given a cardinal $\alpha \leq \aleph_0$ and a hypercyclic subspace $[\{y_i \mid i \in \alpha\}] \subset HC(T) \cup \{0\}$ that admits a sequence $\{n_k\}_{k\in\mathbb{N}}$ such that $T^{n_k} y_i \to 0$ uniformly, for every $i \in \alpha$, then there exists a hypercyclic subspace W such that $[\{y_i \mid i \in \alpha\}] \subset W \subset HC(T) \cup \{0\}$ and $\dim W = \mathfrak{c}$.

Given any two vectors v, w the existence of a right inverse for the operator T, as required by the criterion, allows us to add small perturbations to the vector v in order to construct a new vector whose orbit approximates w. This technique plays a central role in the construction of infinitely many hypercyclic vector. Together with the following lemma:

Lemma 2.1 ([2]). *Let (x_n) be an linearly independent sequence in X. Then, there exist $\gamma_n > 0$ such that $(\gamma_n x_n)$ is ℓ_∞-independent. That is, we call a sequence (y_n) ℓ_∞-independent, if for every bounded sequence $(\lambda_n) \in \ell_\infty$ satisfying*

$$\sum \lambda_n y_n = 0,$$

it follows that $(\lambda_n) = 0$, for all n.

That way, we can find a continuum-dimensional subspace of hypercyclic vectors that extends a given subspace, by embedding ℓ_∞ into HC(T).

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ORTHOGONAL PROJECTIONS ONTO INVARIANT SUBSPACES OF THE HARDY SPACE OVER THE BIDISC

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Abstract

The Invariant Subspace Problem (ISP) remains unsolved for infinite-dimensional separable Hilbert spaces and asks if every bounded operator has a non-trivial closed invariant subspace. Due to the existence of universal operators (in the sense of Rota [3]), the ISP can be solved by proving that every invariant subspace of a universal operator is non-minimal (see [1]). In these notes we study the behavior of restricted orthogonal projections to invariant subspaces of universal Toeplitz operators on the Hardy space over the bidisc.

1 Introduction

Let $L^2(\mathbb{T}^2)$ denote the usual Lebesgue space and $L^\infty(\mathbb{T}^2)$ the essentially bounded functions with respect to σ. The Hardy space $H^2(\mathbb{D}^2)$ is the Hilbert space of holomorphic functions f on $\mathbb{D}^2$ for which

$$\|f\|^2 := \sup_{0<r<1} \int_{\mathbb{T}^2} |f(r\zeta)|^2 d\sigma(\zeta) < \infty.$$

Let P denote the orthogonal projection from $L^2(\mathbb{T}^2)$ onto $H^2(\mathbb{D}^2)$. For a function $\varphi \in L^\infty(\mathbb{T}^2)$, the Toeplitz operator T_φ with symbol φ is defined by

$$T_\varphi f = P(\varphi f)$$

for $f \in H^2(\mathbb{D}^2)$. Under analogous conditions, the 1-dimensional Toeplitz operator with symbol φ will be denoted by t_φ.

Let $H^2(z)$ and $H^2(w)$ denote the classical Hardy spaces over $\mathbb{D}$ in the variables z and w respectively. Then $H^2(\mathbb{D}^2)$ may be defined as the $H^2(z)$-valued Hardy space

$$H^2(\mathbb{D}^2) = \left\{ g(z,w) = \sum_{n=0}^{\infty} g_n(z) w^n : \sum_{n=0}^{\infty} \|g_n\|^2_{H^2(z)} < \infty \right\}.$$

Thus, considering $H_n = H^2(z) w^n$ for each $n \in \mathbb{N}$, we have

$$H^2(\mathbb{D}^2) = \bigoplus_{n=0}^{\infty} H_n.$$

For each $n \in \mathbb{N}$, we denote by P_n the orthogonal projection of $H^2(\mathbb{D}^2)$ onto H_n.

Consider the following classes of invariant subspaces

$$\mathcal{M} = \{M \subset H^2(\mathbb{D}^2) : M \text{ is a minimal invariant subspace for } T_z^*\}$$

and

$$\mathcal{M}_0 = \{M \in \mathcal{M} : \dim(M) = 1\}.$$

Since T_z^* is an universal operator for $H^2(\mathbb{D}^2)$ if we have $\mathcal{M} \subset \mathcal{M}_0$, then the ISP is true. Note that to show that $\mathcal{M} \subset \mathcal{M}_0$ it is enough to show that if M is an T_z^*-invariant subspace such that $\dim(M) \neq 1$ then M is non minimal. The next result suggests that, under certain conditions for the projections P_n, the invariant subspaces of T_z^* are either minimal of dimension 1 or are non-minimal.

Proposition 1.1. *Let $M \subset H^2(\mathbb{D}^2)$ an invariant subspace of T_z^*. If there are $n, m \in \mathbb{N}$ such that $\overline{P_n(M)} \neq \overline{P_m(M)}$ where both are non-trivial, then either $M \in \mathcal{M}_0$ or $M \notin \mathcal{M}$.*

Considering that the ranges of projection by M play an important role in the structure of the minimality of M (see [2]), it is necessary to better understand their behavior. The next corollary helps us with this task.

Corollary 1.1. *If $M \in \mathcal{M}$ and $\dim(M) = \infty$, then there exists $V \subset H^2(\mathbb{D})$ such that $\dim(V) = \infty$. Moreover for all $n \in \mathbb{N}$, either $P_n(M) = \{0\}$ or $P_n(M)$ dense in V.*

2 Main Results

For each non-zero $g \in H^2(w)$, let $P_g : H^2(\mathbb{D}^2) \to H^2(z)$ be the operator defined by

$$P_g(f)(z) = \int_{\mathbb{T}} f(z,w)\overline{g}(w)dw = \langle f, g\rangle_w(z)$$

for $f \in H^2(\mathbb{D}^2)$, where the subscript under the inner product denotes the variable of integration. The following lemma relates the operator P_g with the projections P_n.

Lemma 2.1. *Let $g(w) = \sum_{i=0}^{\infty} a_i w^i \in H^2(w)$. For all $f(z,w) \in H^2(\mathbb{D}^2)$ we have*

$$P_g(f)(z) = \sum_{i=0}^{\infty} \overline{a_i} P_i(f)(z).$$

Lemma 2.2. *Let $g, h \in H^2(w)$ with $\langle g, h\rangle = 0$ and $M \subset H^2(\mathbb{D}^2)$ an invariant subspace of T_z^*. If both $P_g(M)$ and $P_h(M)$ are nonzero and $\overline{P_g(M)} \neq \overline{P_h(M)}$, then either $\dim(M) = 1$ or $M \notin \mathcal{M}$.*

Corollary 2.1. *Let $f, g, h \in H^2(w)$ with $g \perp h$ and $f \perp h$. If $M \in \mathcal{M}$ and $P_f(M), P_g(M)$ and $P_h(M)$ are nonzero, then $\overline{P_f(M)} = \overline{P_g(M)} = \overline{P_h(M)}$.*

Theorem 2.1. *If $M \in \mathcal{M}$, then for each $g \in H^2(w)$ there is an invariant subspace $V \subset H^2(\mathbb{D})$ for t_z^* such that either $P_g(M) = \{0\}$ or $\overline{P_g(M)} = V$.*

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EXISTENCE RESULTS FOR SOME ELLIPTIC PROBLEMS IN $\mathbb{R}^N$ INCLUDING VARIABLE EXPONENTS ABOVE THE CRITICAL GROWTH

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Abstract

Basede on [1], in this work, we establish existence results for the following class of equations involving variable exponents

$$-\Delta u + u = |u(x)|^{p(|x|)-1}u(x) + \lambda |u(x)|^{q(|x|)-1}u(x), \quad x \in \mathbb{R}^N,$$

where $\lambda \geq 0$, $N \geq 3$ and $p, q : [0, +\infty) \to (1, +\infty)$ are radial continuous functions which satisfy suitable conditions. For this purpose, it is sufficient to consider either subcriticality or criticality within a small region near the origin. Surprisingly, outside this region, the nonlinearity may oscillate between subcritical, critical, and supercritical growth in the Sobolev sense. Our approach enables the use of the variational methods to tackle problems with variable exponents in $\mathbb{R}^N$ without imposing restrictions outside of a neighborhood of zero.

1 Introduction

The present work builds upon the results established in [1] and draws inspiration from the celebrated work of Berestycki and Lions [2], where it is noted that the equation

$$-\Delta u + u = |u|^{p-1}u, \quad x \in \mathbb{R}^N,$$

does not admit solution, whenever $p \geq \frac{N+2}{N-2}$, see [2, Examples 1,2]. This result is sharp in the sense that if $p < \frac{N+2}{N-2}$, then the equation admits positive solution, see for instance [5, 2, 4]. Inspired by this classical nonexistence result, we introduce a new class of equations that involves variable exponents that may have a supercritical growth, except near to the origin. Precisely, we consider the following class of equations:

$$-\Delta u + u = |u(x)|^{p(|x|)-1}u(x), \quad x \in \mathbb{R}^N, \tag{$\mathcal{P}_1$}$$

where p satisfies the following assumption:

(p_1) $p : [0, +\infty) \to (1, +\infty)$ is a continuous function such that $p(0) < 2^* - 1$, where $2^* := 2N/(N-2)$ is the critical Sobolev exponent.

It is important to emphasize that condition (p_1) implies that $p(|x|)$ should have subcritical growth near to the origin and the nonlinearity has a superlinear behavior. Our first main result can be stated as follows.

Theorem 1.1. *If (p_1) holds, then Problem $(\mathcal{P}_1)$ admits a nontrivial radial weak solution.*

Inspired by the ideas introduced in the renowned work of H. Brezis and L. Nirenberg [3], many authors have studied existence of solutions for the following class of equations

$$-\Delta u + u = |u|^{p-1}u + \lambda |u|^{q-1}u, \quad x \in \mathbb{R}^N, \tag{1}$$

where $1 < p \leq 2^* - 1$ and $1 < q < 2^* - 1$. In this line, we refer the readers to [4, Theorem 1.1]. In view of the Pohozaev identity, Equation (1) does not admit nontrivial solution when $p = \frac{N+2}{N-2}$ and $\frac{N+2}{N-2} \leq q$. For $N \geq 3$, it is noteworthy that compared to the subcritical Sobolev case (see [2]), the existence of positive solutions of (1) becomes significantly more complex in the critical Sobolev case. Motivated by the preceding discussion, we study the existence of weak solutions for the following class of equations

$$-\Delta u + u = |u(x)|^{p(|x|)-1}u(x) + \lambda|u(x)|^{q(|x|)-1}u(x), \quad x \in \mathbb{R}^N, \tag{$\mathcal{P}_2$}$$

where $p, q : [0, +\infty) \to (1, +\infty)$ are continuous functions satisfying the following hypothesis:

(p_2) There exists $r_0 > 0$ such that $q(|x|) < p(|x|) = 2^* - 1$ in $B := B_{r_0}(0)$.

Theorem 1.2. *Suppose that* (p_2) *holds and one of the following conditions is satisfied:*

(a) $N \geq 4$ *and* $\lambda > 0$*;*

(b) $N = 3$, $3 < q(|x|) < 5$ *and* $\lambda > 0$*;*

(c) $N = 3$, $1 < q(|x|) \leq 3$ *and* $\lambda > 0$ *sufficiently large.*

Then Problem $(\mathcal{P}_2)$ *admits a nontrivial radial weak solution.*

Our approach relies on variational methods, particularly the Mountain Pass Theorem, with the Radial Lemma being essential. For Theorem 1.1, we verify the $(PS)_c$ condition at all levels to establish the existence of a solution. Theorem 1.2 is more delicate due to critical behavior near the origin, where the $(PS)_c$ condition may fail. To address this, we follow an alternative strategy inspired by Brezis and Nirenberg [3], employing Talenti functions to ensure the minimax level stays below a critical threshold, thereby recovering compactness.

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EXISTENCE AND NONEXISTENCE RESULTS FOR A SEMILINEAR ELLIPTIC EQUATION WITH CRITICAL GROWTH

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Abstract

In this work, we present, via variational methods, existence and nonexistence results for the semilinear elliptic equation with a critical term.

$$-\Delta u = \phi(|x'|)|u|^{p-2}u + \lambda|u|^{2^*-2}u \quad \text{in} \quad \mathbb{R}^N,$$

where $N \geq 3$, λ is a real parameter, and the exponent p and the weight function ϕ satisfy certain assumptions.e

1 Introduction

In this work, we study the following semilinear elliptic equation in the zero mass case and involving critical growth

$$-\Delta u = \phi(|x'|)|u|^{p-2}u + \lambda|u|^{2^*-2}u \quad \text{in} \quad \mathbb{R}^N, \qquad (\mathcal{P}_\lambda)$$

where $x = (x', z) \in \mathbb{R}^N = \mathbb{R}^k \times \mathbb{R}^{N-k}$, $N \geq 3$, $k \geq 2$, λ is a real parameter, $p \in [2_*(s), 2_*(0))$ for some $s \in (0, 2)$, where $2_*(s) = \frac{2(N-s)}{N-2}$, and the weight function $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ satisfies the following assumptions:

(ϕ_1) $\phi \in C^0(\mathbb{R}_+, \mathbb{R}_+)$;

(ϕ_2) $\phi(r)r^2 \in L^\infty(\mathbb{R}_+)$;

(ϕ_3) $\phi(r) = 0$ if, and only if, $r = 0$.

A typical example of function satisfying our assumptions is $\phi(r) := \frac{r^{2\alpha}}{(1+r^2)^{1+\alpha}}$, with $\alpha > 0$.

In this work, we extend the result of M. Badiale and G. Tarantello [1], in which the authors obtained the existence of a ground state solution for equation $(\mathcal{P}_\lambda)$ in the case $\lambda = 0$.

Definition 1.1. *By a weak solution of* $(\mathcal{P}_\lambda)$*, we mean a function* $u \in \mathcal{D}^{1,2}(\mathbb{R}^N)$ *that satisfies*

$$\int_{\mathbb{R}^N} \nabla u \nabla \varphi \, dx - \int_{\mathbb{R}^N} \phi(|x'|)|u|^{p-2}u\varphi \, dx - \lambda \int_{\mathbb{R}^N} |u|^{2^*-2}u\varphi \, dx = 0 \, , \quad \forall \varphi \in \mathcal{D}^{1,2}(\mathbb{R}^N).$$

We observe that the energy functional $J : \mathcal{D}^{1,2}(\mathbb{R}^N) \to \mathbb{R}$ associated to equation $(\mathcal{P}_\lambda)$ is given by

$$J(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 \, \mathrm{d}x - \frac{1}{p}\int_{\mathbb{R}^N} \phi(|x'|)|u|^p \, \mathrm{d}x - \frac{\lambda}{2^*}\int_{\mathbb{R}^N} |u|^{2^*} \, \mathrm{d}x.$$

To prove our results, the following Hardy-Sobolev type inequality plays a fundamental role (see [1]).

Theorem 1.1. *Suppose that* $1 < q < N$ *and* $2 \leq k \leq N$*. For each* $0 \leq s \leq q$ *with* $s < k$*, there exists a constant* $C = C(s, q, N, k) > 0$ *such that, for every function* $u \in \mathcal{D}^{1,q}(\mathbb{R}^N)$*,*

$$\int_{\mathbb{R}^N} \frac{|u|^{q_*(s)}}{|x'|^s} \, dx \leq C \left(\int_{\mathbb{R}^N} |\nabla u|^q \, dx \right)^{\frac{q_*(s)}{q}},$$

where $q_*(s) = \frac{q(N-s)}{N-q}$ *and* $x = (x', z) \in \mathbb{R}^N = \mathbb{R}^k \times \mathbb{R}^{N-k}$.

2 Main Results

Using arguments from [2, 3], we will prove the following existence result for equation ($\mathcal{P}_\lambda$) in the case where the parameter $\lambda \geq 0$.

Theorem 2.1 (Existence)**.** *Suppose that assumptions* $(\phi_1)-(\phi_3)$ *hold, with* $2_*(s) < p < 2_*(0)$ *and* $N \geq 3$. *Then, for every* $\lambda \geq 0$, *problem* ($\mathcal{P}_\lambda$) *admits a nontrivial weak solution.*

In the case where $\lambda < 0$, we obtain a nonexistence result, also known in the literature as Liouville type theorem. For this, we assume that $N \geq 4$ and $k > 2$. Considering the constants defined by

$$C_{p,N} := \frac{(p-2)^{\frac{p-2}{2^*-p}}(2^*-p)}{(2^*-2)^{\frac{2^*-2}{2^*-p}}} \quad \text{and} \quad C_0 := \|\phi\|^{\frac{p-2}{2^*-p}}_{L^\infty(\mathbb{R}^N)} \|\phi(|x'|)|x'|^2\|_{L^\infty(\mathbb{R}^N)} \left(\frac{2}{k-2}\right)^2,$$

we have the following result:

Theorem 2.2 (Liouville)**.** *Suppose that assumptions* $(\phi_1)-(\phi_3)$ *are satisfied,* $N \geq 4$, *and* $k \geq 3$ *is an integer. Then, for every*

$$\lambda < \bar{\lambda} = -\left(C_{p,N} C_0\right)^{\frac{2^*-p}{p-2}} < 0,$$

problem ($\mathcal{P}_\lambda$) *admits only the trivial solution.*

Remark 2.1. *An open problem is to investigate the existence or nonexistence of solutions when* $\lambda \in (\bar{\lambda}, 0)$.

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EXISTENCE OF SOLUTIONS FOR A CLASS OF $P(B(U))\&Q(B(U))$ ELLIPTIC PROBLEMS

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Abstract

In this work, we study the existence of weak solutions for a class of quasilinear elliptic problems, involving nonhomogeneous operators of $p(b(u))\&q(b(u))$-Laplace type. Under suitable assumptions, we prove our results by using the Galerkin method together with the technique of Zhikov for passing to the limit in a sequence of p_ν-Laplacian problems, then we finish by applying a fixed point theorem

1 Introduction

The purpose of this work is to investigate the existence of weak solutions for the following quasilinear elliptic problems operator

$$\begin{aligned} -\operatorname{div}\left(a(|\nabla u|^{p(b(u))})|\nabla u|^{p(b(u))-2}\nabla u\right) &= f(x,u) \text{ in } \Omega, \\ u = 0 \quad &\text{on} \quad \partial\Omega, \end{aligned} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, a, p, b and f are functions that satisfy conditions which will be stated later.

In recent years, quasilinear elliptic problems involving the $p(b(u))$-Laplacian have attracted an increasing attention and many results have been obtained (see for example [2, 3]), they showed existence of solutions via the theory of monotone operators and a fixed-point theorem. However, there are few contributions to the study of the $p(b(u)))\&q(b(u))$-Laplace problems with reaction term $f(x,u)$. Recently, Fadil et al. [1] considered the existence and multiplicity of solutions for the problem (1), with $p(u)\&q(u)$ instead of $p(b(u))\&q(b(u))$, $a(t) = 1 + t^{\frac{q-p}{p}}$ and $f(x,u) := f(x)$. Due to the presence of the source term $f(x,u)$ we cannot directly apply the theory of monotone operators, so we first solve an auxiliary problem with the aforementioned term via the Galerkin method, then we use a approximation procedure employed by Zhikov [4]. Next, the the Brouwer fixed-point theorem allows us to conclude our result.

2 Notations and Main Results

Let $p : \mathbb{R} \to [1, +\infty[$ be the nonlinear exponent function such that

$$p \quad \text{is a Lipschitz-continuous function}, \quad \text{and } 1 < \alpha < p(x) \leq \beta < +\infty \quad \text{for a.e. } x \in \Omega. \tag{2}$$

We consider a mapping $b : W_0^{1,\alpha}(\Omega) \to \mathbb{R}$ such that b is continuous and bounded .

We note that $p(b(u))$ is here a real number and not a function, then the Sobolev spaces involved in this work are the classical ones. We will consider the well-known Sobolev space $W_0^{1,p}(\Omega)$ with the norm $\|u\|_{1,p} = \|\nabla u\|_p$. Also , we set $\gamma(b(u)) = (1 - H(k_3))p(b(u)) + H(k_3)q(b(u))$, where $H(k) = 1$ if $k > 0$ and $H(k) = 0$ if $k = 0$.

We will need the spaces

$$W_0^{1,p(b(u))}(\Omega) = \{u \in W^{1,1}(\Omega)\} : \int_\Omega |\nabla u|^{p(b(u))}\, dx < +\infty\},$$

and $X = W_0^{1,p(b(u))}(\Omega) \cap W_0^{1,\gamma(b(u))}(\Omega)$ endowed with the norm

$$\|u\| = \|u\|_{1,p(b(u))} + H(k_3)\|u\|_{1,q(b(u))}$$

Assume that the following assumptions hold:

(a_0) The function $a : \mathbb{R}^+ \to \mathbb{R}^+$ is of class C^1 and there exist positive constants $k_0, k_1 > 0$, $k_2, k_3 \geq 0$, $q > p$ such that

$$k_0 + H(k_3)k_2 t^{(q-p)/p} \leq a(t) \leq k_1 + k_3 t^{(q-p)/p} \text{ for all } \; t \geq 0,$$

(a_1) The function $g : \mathbb{R}^+ \to \mathbb{R}^+$ given by $g(t) = a(t^p)t^{p-2}$ is increasing.

(F_1) $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function such that

$$|f(x,u)| \leq c_1 + c_2|u|^{\alpha-1} \quad \text{a.e. } x \in \Omega, \text{ all } u \in \mathbb{R}, c_1, c_2 > 0,$$

with $1 \leq \alpha < p*$ where $p* = Np/(N-p)$, if $N > p$.

Theorem 2.1. *If (a_0), (a_1) and (F_1) hold, then (1) has a weak solution in X.*

Proof We employ the Galerkin method combined with the technique of Zhikov, and the Brouwer fixed-point theorem. ■

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CONVERGENCE OF SOLUTIONS TO P-LAPLACIAN PROBLEMS PERTURBED WITH A CONVECTIVE TERM

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Abstract

We study the behavior, as $p \to \infty$, of solutions to the Dirichlet problem

$$\begin{cases} -\Delta_p u = u^{q(p)-1} + \mu_p u^{a(p)-1} |\nabla u|^{r(p)-a(p)} & \text{in} \quad \Omega \\ u > 0 & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

under asymptotic hypotheses on the parameter μ_p and the functions $q(p)$, $a(p)$ and $r(p)$.

1 Introduction

Let us consider the p-family of problems

$$\begin{cases} -\Delta_p u = u^{q(p)-1} + \mu_p u^{a(p)-1} |\nabla u|^{r(p)-a(p)} & \text{in} \quad \Omega \\ u > 0 & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ is the p-Laplacian operator and Ω is a bounded, smooth domain of $\mathbb{R}^N$, $N \geq 2$.

We assume that

$$1 \leq q(p) < p \quad \text{and} \quad 1 \leq a(p) \leq r(p), \tag{2}$$

for every p sufficiently large, and

$$0 < Q < 1 \quad \text{and} \quad 0 \leq A \leq R < \infty, \tag{3}$$

where

$$Q := \lim_{p \to \infty} \frac{q(p)}{p}, \quad A := \lim_{p \to \infty} \frac{a(p)}{p} \quad \text{and} \quad R := \lim_{p \to \infty} \frac{r(p)}{p}.$$

As for the parameter μ_p we assume that

$$0 < \Lambda := \lim_{p \to \infty} (\mu_p)^{1/p} < \Lambda_\infty^{A + \frac{Q(R-1)}{1-Q}} \tag{4}$$

where $\Lambda_\infty := \|d_\Omega\|_\infty^{-1}$ and d_Ω denotes the distance function to the boundary.

The p-family (1) can be considered as a perturbation of the p-family of problems

$$\begin{cases} -\Delta_p v = v^{q(p)-1} & \text{in} \quad \Omega \\ v > 0 & \text{in} \quad \Omega \\ v = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{5}$$

As it is well known, the Dirichlet problem (2) has a unique weak solution $v_p \in C_0^1(\overline{\Omega})$. In [1], Charro and Peral proved that $\lim_{p \to \infty} v_p = v_Q$, uniformly in $\overline{\Omega}$, where $v_Q \in W^{1,\infty}(\Omega) \cap C_0(\overline{\Omega})$ is the only viscosity solution to the problem

$$\begin{cases} \min\left\{|\nabla v| - v^Q, -\Delta_\infty v\right\} = 0 & \text{in} \quad \Omega \\ v > 0 & \text{in} \quad \Omega, \\ v = 0 & \text{on} \quad \partial\Omega. \end{cases}$$

One of the our main results is stated as follows and its proof is obtained from a gradient estimate we have derived in [3].

Theorem 1.1. *Let $u_p \in C_0^1(\overline{\Omega})$ be an arbitrary weak solution to (1), for all p sufficiently large. If $R \leq 1^-$ and $0 < Q \leq A$, then u_p converges uniformly in $\overline{\Omega}$ to v_Q, as $p \to \infty$.*

The next result is obtained by applying an existence theorem proved by de Araujo, Ercole and Lanazca Vargas (see [2]).

Theorem 1.2. *If $0 < R < \infty$ and $0 < Q \leq A$, then (1) admits, for all p sufficiently large, a solution $u_p \in C_0^1(\overline{\Omega})$ converging uniformly in $\overline{\Omega}$ to v_Q, as $p \to \infty$.*

It is also showed that the assumption $0 < Q \leq A$ can be removed in both theorems if Ω enjoys the property

$$\{x \in \Omega : d_\Omega \text{ is not differentiable at } x\} = \{x \in \Omega : d_\Omega(x) = \|d_\Omega\|_\infty\}.$$

In this case, u_p converges uniformly in $\overline{\Omega}$ to $\Lambda_\infty^{-\frac{Q}{1-Q}} d_\Omega$, as $p \to \infty$.

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GROUND STATES OF NORMALIZED NONLOCAL NONLINEAR PROBLEMS WITH A POINT INTERACTION

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Abstract

In physics, a point defect at $0 \in \mathbb{R}^N$ is often modeled by an operator of the form $-\Delta + \frac{1}{\alpha}\delta_0$, where $\alpha \in \mathbb{R}\setminus\{0\}$. More precisely, we understand this operator as taking $u \in C^\infty(\mathbb{R}^N)$ to the distribution $-\Delta u + \frac{1}{\alpha}\delta_0 u$ that acts as

$$\Big(-\Delta u + \frac{1}{\alpha}\delta_0 u\Big)[v] = \int \nabla u(x) \cdot \nabla v(x) \mathrm{d}x + \frac{1}{\alpha}u(0)v(0).$$

The *Laplacian of point interaction* $-\Delta_\alpha$ is the L^2-self-adjoint operator that, in a certain sense, best mimics the behavior of $-\Delta + \frac{1}{\alpha}\delta_0$ in $\mathbb{R}^N$. Even though its definition was introduced by Berezin & Faddeev in the 1960s and a comprehensive treatise about this operator was already avaiable by the late 1980s, nonlinear elliptic equations involving this operator have only started being considered very recently.

In this talk, we briefly describe the structure of the Sobolev spaces $W^{1,2}_\alpha(\mathbb{R}^N)$ induced by $-\Delta_\alpha$ and we communicate recent results proposed by the author concerned with the existence of ground states to normalized nonlocal nonlinear problems involving $-\Delta_\alpha$.

1 Ground states of the Kirchhoff equation and the Schrödinger-Poisson system in $\mathbb{R}^3$

In [1], we considered the following *abstract minimization problem*:

$$\begin{cases} I_\alpha(u) = \inf\{I_\alpha(v) : v \in W^{1,2}_\alpha(\mathbb{R}^3) \quad \text{and} \quad \|v\|^2_{L^2} = c\}; \\ v \in W^{1,2}_\alpha(\mathbb{R}^3) \quad \text{and} \quad \|v\|^2_{L^2} = c, \end{cases}$$

where $I_\alpha \colon W^{1,2}_\alpha(\mathbb{R}^3) \to \mathbb{R}$ denotes an abstract energy functional. Our main results [1, Lemmas 1.4 and 1.6] state sufficient conditions for the existence of solution to this problem. As proofs of concept, we proved the existence of ground states to the following normalized problems with a point interaction:

- The Schrödinger–Poisson system

$$\begin{cases} -\Delta_\alpha u + \lambda u + \phi u = u|u|^{p-2} & \text{in } \mathbb{R}^3; \\ -\Delta\phi = |u|^2 & \text{in } \mathbb{R}^3; \\ \|u\|^2_{L^2} = c \end{cases} \tag{1}$$

  and

- the Kirchhoff-type equation

$$\begin{cases} -\big(1 + H_\alpha(u)\big)\Delta_\alpha u + \lambda u = u|u|^{p-2} & \text{in } \mathbb{R}^3; \\ \|u\|^2_{L^2} = c, \end{cases} \tag{2}$$

  where H_α denotes the quadratic form associated with $-\Delta_\alpha$.

Let us state these existence results more precisely.

Theorem 1.1 ([1, Theorems 1.10 and 1.12]). *Suppose that $0 \leq \alpha < \infty$ and $2 < p < \frac{5}{2}$.*

1. *If $c > 0$ is sufficiently small, then (1) and (2) admit a ground state.*
2. *If u is a ground state of (1) or (2), then $u \in W^{1,2}_{\alpha}(\mathbb{R}^3) \setminus W^{1,2}(\mathbb{R}^3)$.*

2 Ground states of the Schrödinger-Newton system on $\mathbb{R}^2$

In [2], we considered the following normalized Schrödinger-Newton system on $\mathbb{R}^2$ with a point interaction:

$$\begin{cases} -\Delta_\alpha u + \lambda u = \phi u + \beta u|u|^{p-2} & \text{on } \mathbb{R}^2; \\ -\Delta\phi = 2\pi|u|^2 & \text{on } \mathbb{R}^2; \\ \|u\|_{L^2}^2 = c. \end{cases} \tag{3}$$

Our main result establishes the existence of ground state in function of the considered mass c.

Theorem 2.1 ([2, Theorem 1.5]). *Suppose that $\alpha \in \mathbb{R}$, $c > 0$ and one of the following conditions is satisfied: (i) $\beta \leq 0$ and $p > 2$; (ii) $\beta > 0$ and $2 < p < 4$; (iii) $\beta > 0$, $p = 4$ and $c < 2/\big(\beta K(p)\big)$ for a certain positive constant $K(p)$. The following implications are satisfied.*

1. *If $(u_n)_{n\in\mathbb{N}}$ is a minimizing sequence of the associated energy functional on the L^2-sphere with radius c^2, then it admits a subsequence that converges to a ground state of (3).*
2. *If u is a ground state of (3), then $u \in W^{1,2}_{\alpha}(\mathbb{R}^2) \setminus W^{1,2}(\mathbb{R}^2)$.*

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A CLASS OF HARTREE-FOCK SYSTEMS WITH NULL MASS VIA NEHARI-POHOZAEV IN DIMENSION TWO

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Abstract

It is establish existence of nontrivial solutions for planar nonlocal elliptic systems defined in the whole space $\mathbb{R}^2$. The main point is to consider a energy functional with logarithmic kernel for an auxiliary elliptic problem. Using a suitable Nehari-Pohozaev manifold we prove existence and asymptotic behavior of solutions depending on the parameter $\beta > 0$. Furthermore, we prove that our main problem has a vector ground state solution or a semitrivial ground state solution which depends on the size of parameter $\beta > 0$.

1 Introduction

In the present work, we consider the following class of planar elliptic systems:

$$\begin{cases} -\Delta u + \phi_{u,v} u = |u|^{2p-2}u + \beta |v|^p |u|^{p-2}u, & \text{in } \mathbb{R}^2, \\ -\Delta v + \phi_{u,v} v = |v|^{2p-2}v + \beta |u|^p |v|^{p-2}v, & \text{in } \mathbb{R}^2, \end{cases} \tag{$\mathcal{S}_\beta$}$$

where $2 \leq p < \infty$, $\beta \geq 0$, and

$$\phi_{u,v}(x) := \int_{\mathbb{R}^2} \log(|x-y|) \left(u^2(y) + v^2(y)\right) \, \mathrm{d}y. \tag{1}$$

Due to the presence of the term $\phi_{u,v}$, the appropriate workspace for the system given above can be written in the following form:

$$W^\lambda := \left\{ u \in H^1(\mathbb{R}^2) : \|u\|_{\lambda,*}^2 := \int_{\mathbb{R}^2} \log(\lambda + |x|) u^2 \, \mathrm{d}x < \infty \right\},$$

where the norm is defined by

$$\|u\|_{W^\lambda} := \left(\|\nabla u\|_2^2 + \|u\|_2^2 + \|u\|_{\lambda,*}^2 \right)^{1/2},$$

for $\lambda > 0$. Naturally, the energy functional $I_\beta : W^\lambda \times W^\lambda \to \mathbb{R}$ associated to this system is given by

$$I_\beta(u,v) = \frac{1}{2}\left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2 \right) + \frac{1}{4}\mathcal{V}(u,v) - \frac{1}{2p}\psi_\beta(u,v),$$

where

$$\mathcal{V}(u,v) := \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \log(|x-y|) \left(u^2(y) + v^2(y)\right) \left(u^2(x) + v^2(x)\right) \, \mathrm{d}y \, \mathrm{d}x$$

and

$$\psi_\beta(u,v) := \|u\|_{2p}^{2p} + \|v\|_{2p}^{2p} + 2\beta \int_{\mathbb{R}^2} |uv|^p \, \mathrm{d}x. \tag{2}$$

2 Main Results

The main results are:

Theorem 2.1 (Existence and asymptotic behavior of solutions)**.** *Assume that $2 \leq p < \infty$ and $\beta \geq 0$. Then, the system ($\mathcal{S}_\beta$) possesses a weak solution $(u_\beta^\lambda, v_\beta^\lambda) \in \mathcal{M}_\beta$ such that*

$$I_\beta(u_\beta^\lambda, v_\beta^\lambda) = c_\beta := \inf_{(u,v)\in\mathcal{M}_\beta} I_\beta(u,v),$$

for each $\lambda > e^{1/4}$, with $u_\beta^\lambda, v_\beta^\lambda \geq 0$. Here the set $\mathcal{M}_\beta$ represents the Nehari-Pohozaev. Furthermore,

(a) *for $\lambda > e^{1/4}$ fixed, $u_\beta^\lambda \rightharpoonup u^\lambda$ and $v_\beta^\lambda \rightharpoonup v^\lambda$ weakly in W^λ, as $\beta \to 0^+$, such that*

$$I_0(u^\lambda, v^\lambda) = c_0;$$

(b) *for $\lambda > e^{1/4}$ fixed, we obtain that $\|u_\beta^\lambda\|_2 \to 0$ and $\|v_\beta^\lambda\|_2 \to 0$, as $\beta \to \infty$.*

Theorem 2.2 (Vector and semi-trivial solutions)**.** *Considering the pair $(u_\beta^\lambda, v_\beta^\lambda) = (u_\beta, v_\beta)$ (with $\lambda > e^{1/4}$ fixed) obtained in Theorem 2.1, it follows that:*

(i) *for any $\beta > 2^{p-1} - 1$, the pair (u_β, v_β) is a vector solution, that is, $u_\beta \neq 0$ and $v_\beta \neq 0$;*

(ii) *for any $0 \leq \beta < 2^{p-1} - 1$, the pair (u_β, v_β) is a semi-trivial solution, that is, $u_\beta = 0$ or $v_\beta = 0$;*

(iii) *$\beta = 2^{p-1} - 1$, if and only if, $(u/\sqrt{2}, u/\sqrt{2})$ is a solution of ($\mathcal{S}_\beta$), where u is a solution of the scalar equation given in the problem*

$$-\Delta u + \phi_u u = |u|^{2p-2}u, \quad in\ \mathbb{R}^2.$$

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WEIGHTED ORLICZ ESTIMATES FOR FULLY NONLINEAR ELLIPTIC MODELS WITH OBLIQUE TANGENTIAL DERIVATIVE CONDITION

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Abstract

In this manuscript, we present some of the main results from [1], in which the author studies the weighted Orlicz-Sobolev regularity of viscosity solutions to fully nonlinear elliptic equations with oblique boundary conditions, using an asymptotic approach to the governing operator. As a byproduct of the developed results, we also investigate the regularity of the obstacle problem and other related topics.

1 Introduction

In this work we deal with global Calderón-Zygmund type estimates for the following problem with oblique derivative

$$\begin{cases} F(D^2u, Du, u, x) &= f(x) \quad \text{in} \quad \Omega \\ \beta(x)\cdot Du + \gamma(x)u &= g(x) \quad \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $F : \text{Sym}(n)\times\mathbb{R}^n\times\mathbb{R}\times\Omega\to\mathbb{R}$ is a uniformly elliptic operator, $\Omega\subset\mathbb{R}^n$ $(n\geq 2)$ a bounded domain with $\partial\Omega\in C^{2,\alpha}$ for some $\alpha\in(0,1)$, the boundary data $\beta,\gamma,g\in C^{1,\alpha}(\partial\Omega)$ under the oblique condition , i. e., there exists a constant $\mu_0>0$ such that $\beta\cdot\vec{\mathbf{n}}\geq\mu_0$, where $\vec{\mathbf{n}}$ is the outward normal vector of Ω. We study the problem (1.1) when the source term belongs to the *weighted Orlicz space* $L^{\Phi}_{\omega}(\Omega)$, where Φ is an N-function in the class $\Delta_2\cap\nabla_2$, and ω is a weight in the Muckenhoupt class $\mathfrak{A}_q$, defined as the set of measurable functions h in Ω such that

$$\rho_{\Phi,\omega}(h) =: \int_{\Omega}\Phi(|h(x)|)\omega(x)dx < +\infty.$$

Here, we can equip $L^{\Phi}_{\omega}(\Omega)$ with the following Luxemburg norm

$$\|h|_{L^{\Phi}_{\omega}(\Omega)} =: \inf\left\{t>0 : \rho_{\Phi,\omega}\left(\frac{h}{t}\right)\leq 1\right\}.$$

Moreover, the *weighted Orlicz-Sobolev space* $W^{k,\Phi}_{\omega}(\Omega)$ $(k\geq 0)$ is the set of all measurable functions h in Ω such that all weak derivatives $D^{\alpha}h$ with $|\alpha|\leq k$ belong to $L^{\Phi}_{\omega}(\Omega)$ with the following norm

$$\|h\|_{W^{2,\Phi}_{\omega}(\Omega)} = \sum_{|\alpha|\leq k}\|D^{\alpha}h\|_{L^{\Phi}_{\omega}(\Omega)}.$$

We observe that, given the above definitions, the space $L^{\Phi}_{\omega}(\Omega)$ can be continuously embedded into a certain Lebesgue space $L^{p_0}(\Omega)$, for some $p_0\in(1,\infty)$.

Our approach makes use of geometric tangential methods, which consist of importing "fine regularity estimates" from a limiting profile, i.e., the Recession operator associated to F, that is defined by

$$F^{\sharp}(\mathrm{X},\varsigma,s,x) =: \lim_{\tau\to 0^+}\tau\cdot F\left(\tau^{-1}\mathrm{X},\varsigma,s,x\right),\ (\mathrm{X},\varsigma,s,x)\in\text{Sym}(n)\times\mathbb{R}^n\times\mathbb{R}\times\Omega$$

and we use the compactness and stability procedures.

2 Main Results

Now, we present our main results obtained in [1]. The first one provides global weighted Orlicz-Sobolev estimates for the solutions, under an asymptotic regime, of (1.1).

Theorem 2.1 (Weighted Orlicz Estimates). *Let u be an L^p- viscosity solution of (1.1) where $p = p_0 n$ for the previous constant p_0. Under the previous conditions, there exists a universal constant $\mathrm{C} > 0$ such that $u \in W^{2,\Upsilon}_{\omega}(\Omega)$ where $\Upsilon(t) = \Phi(t^n)$ and the following estimate holds*

$$\|u\|_{W^{2,\Upsilon}_{\omega}(\Omega)} \leq \mathrm{C} \cdot \left(\|u\|^n_{L^\infty(\Omega)} + \|f\|_{L^{\Upsilon}_{\omega}(\Omega)} + \|g\|_{C^{1,\alpha}(\partial\Omega)} \right).$$

An important application that is linked to free boundary problems (see [1, Theorem 1.6])

Theorem 2.2 ($W^{2,\Phi}_{\omega}$-estimates for obstacle problem). *Let u be an L^p-viscosity solution of*

$$\begin{cases} F(D^2u, Du, u, x) & \leq & f(x) & in & \Omega \\ (F(D^2u, Du, u, x) - f)(u - \psi) & = & 0 & in & \Omega \\ u(x) & \geq & \psi(x) & in & \Omega \\ \beta \cdot Du + \gamma u & = & g(x) & on & \partial\Omega, \end{cases}$$

with $p = p_0 n$, where F and f satisfy the structural described above, $\partial\Omega \in C^3$, $\beta, \gamma \in C^2(\partial\Omega)$, $\psi \in W^{2,\Upsilon}_{\omega}(\Omega)$, where $\Upsilon(t) = \Phi(t^n)$ and satisfies $\beta \cdot D\psi + \gamma\psi \geq g$ a.e. on $\partial\Omega$. Then, $u \in W^{2,\Upsilon}(\Omega)$ and

$$\|u\|_{W^{2,\Upsilon}_{\omega}(\Omega)} \leq \mathrm{C} \cdot \left(\|f\|_{L^{\Upsilon}_{\omega}(\Omega)} + \|g\|_{C^{1,\alpha}(\partial\Omega)} + \|\psi\|_{W^{2,\Upsilon}_{\omega}(\Omega)} \right),$$

where $\mathrm{C} > 0$ is a universal constant.

Other interesting applications include Weighted Orlicz-BMO type estimates (cf. [1, Theorem 5.1]) and the density of $W^{2,\Upsilon}_{\omega}$ in the fundamental class of solutions of (1.1)(cf. [1, Remark 3.4]).

Finally, these results constitute a natural generalization of [2, 3, 4].

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SCHAUDER ESTIMATES FOR NON-CONVEX FULLY NONLINEAR ELLIPTIC PDES WITH DINI CONTINUOUS DATA

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Abstract

In this talk, we establish local Schauder estimates for flat viscosity solutions, that is, solutions with sufficiently small norms, to a class of fully nonlinear elliptic partial differential equations of the form

$$F(D^2u, x) + \langle \mathfrak{B}(x), Du \rangle = f(x) \quad \text{in} \quad \mathrm{B}_1 \subset \mathbb{R}^n, \tag{1}$$

where the operator F is differentiable, though not necessarily convex or concave. In addition, we impose suitable Dini-type continuity assumptions on the data. Our methodology is based on geometric tangential techniques, combined with compactness and perturbative arguments.
Keywords: Fully nonlinear elliptic PDEs, Schauder estimates, Dini continuity condition.

1 Introduction

Caffarelli (see e.g. [1, Theorem 8.1]) established Schauder estimates for the inhomogeneous problem $F(D^2u, x) = f(x)$ via a perturbation and compactness technique under suitable Hölder assumptions on data, and concavity or convexity assumption on the operator F.

For nearly twenty years, the question of whether *arbitrary* fully nonlinear elliptic equations $F(D^2\mathfrak{h}) = 0$ in B_1 (resp. $F(D^2\mathfrak{h}) = f(x)$) admit a general C^2 *a priori* regularity theory remained unresolved. This problem was finally settled by Nadirashvili and Vlăduţ's counterexamples to $C^{1,1}$ regularity in [4], which concluded this line of investigation. Their works, however, stimulated new research directions. In light of the inherent obstacles to developing a universal existence theory for classical solutions to fully nonlinear equations, a major focus of current research involves identifying supplementary structural or qualitative conditions on $F(\mathrm{X}, \cdot), F(\cdot, x), f$, and even on u that enable C^2 estimates.

2 Main Results

We make the following structural assumptions:

(A1) (**Uniform Ellipticity**) The operator $F : \mathrm{Sym}(n) \times \Omega \to \mathbb{R}$ is fully nonlinear and uniformly elliptic, with ellipticity constants $0 < \lambda \leq \Lambda$. Specifically, we require

$$\mathscr{P}^-_{\lambda,\Lambda}(\mathrm{N}) \leq F(\mathrm{M}+\mathrm{N}, x) - F(\mathrm{M}, x) \leq \mathscr{P}^+_{\lambda,\Lambda}(\mathrm{N}), \ \forall\ x \in \Omega, \ \forall\ \mathrm{M}, \mathrm{N} \in \mathrm{Sym}(n), \ \ \text{whith}, \ \ \mathrm{N} \geq 0. \tag{2}$$

(A2) (**Differentiability of the Nonlinearity**) We assume that $F \in C^1(\mathrm{Sym}(n))$, and there exists a modulus of continuity $\omega : [0, \infty) \to [0, \infty)$ such that

$$\|\mathrm{D_M}F(\mathrm{X}, x) - \mathrm{D_M}F(\mathrm{Y}, x)\| \leq \omega(\|\mathrm{X} - \mathrm{Y}\|), \qquad \forall\ x \in \Omega \text{ and } \forall\ \mathrm{X}, \mathrm{Y} \in \mathrm{Sym}(n).$$

(A3) (**Dini continuity in the L^n-sense**) There exists a modulus of continuity $\tau : [0,\infty) \to [0,\infty)$, and non-negative constants $\mathrm{C}_f, \mathrm{C}_{\theta_F}, \mathrm{C}_{\mathfrak{B}}$, such that

$$\left(\int_{\mathrm{B}_r(x_0)} |f(x)-f(x_0)|^n\,dx\right)^{\frac{1}{n}} \le \mathrm{C}_f\tau(r), \quad \left(\int_{\mathrm{B}_r(x_0)} |\theta_F(x,x_0)|^n\,dx\right)^{\frac{1}{n}} \le \mathrm{C}_{\theta_F}\tau(r)), \quad \left(\int_{\mathrm{B}_r(x_0)} |\mathfrak{B}(x)-\mathfrak{B}(x_0)|^n\,dx\right)^{\frac{1}{n}} \le \mathrm{C}_{\theta_F}\tau(r))$$

where τ satisfies the Dini condition $\displaystyle\int_0^1 \frac{\tau(s)}{s}ds < \infty$.

(A4) (**Compatibility condition**) We further suppose the **nullity conditions**:

$$\liminf_{s\to 0^+} \sup_{0<r\le 1} \frac{\tau(rs)}{\tau(r)} = 0 \quad \text{and} \quad \liminf_{s\to 0^+} \sup_{k\in\mathbb{N}} \frac{s^{\alpha_0}\tau(s^k)}{\tau(s^{k+1})} = 0 \text{ for some } \alpha_0 \in (0,1].$$

Within this framework, we are able to derive the following local Schauder estimates.

Theorem 2.1 (**Local $C^{2,\mathbf{Dini}}$ regularity**). *Let $u \in C^0(B_1)$ be a viscosity solution to (1), where the assumptions (A1) – (A4) are in force. There exists $\delta_0 > 0$, depending only upon n, λ, Λ, ω, and $\tau(1)$, such that if*

$$\|u\|_{L^\infty(\boldsymbol{B}_1)} \le \delta_0,$$

then $u \in C^{2,\psi}_{loc}(\boldsymbol{B}_1)$ and

$$\|u\|_{C^{2,\psi}(\boldsymbol{B}_{1/2})} \le \boldsymbol{C}_0 \cdot \delta_0,$$

where $\boldsymbol{C}_0 > 0$ depends only upon n, λ, Λ, α_0, ω, and τ and $\psi(t) = \tau(t) + \displaystyle\int_0^t \frac{\tau(s)}{s}ds$.

Our results can be viewed as an extension of the work by dos Prazeres and Teixeira [2, Theorem 2.2] and Kovats [3, Theorem 1], now within the framework of linear drift terms, non-convexity, and Dini continuity assumptions.

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NONLINEAR PERTURBATIONS OF A PERIODIC MAGNETIC CHOQUARD EQUATION WITH HARDY-LITTLEWOOD-SOBOLEV CRITICAL EXPONENT

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Abstract

In this work, we consider the following magnetic nonlinear Choquard equation

$$-(\nabla + iA(x))^2 u + V(x)u = \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha}\right)|u|^{2^*_\alpha - 2}u + \lambda f(u) \ \text{ in } \ \mathbb{R}^N,$$

where $2^*_\alpha = \frac{2N-\alpha}{N-2}$ is the critical exponent in the sense of the Hardy-Littlewood-Sobolev inequality, $\lambda > 0$, $N \geq 3$, $0 < \alpha < N$, $A : \mathbb{R}^N \to \mathbb{R}^N$ is an C^1, $\mathbb{Z}^N$-periodic vector potential and V is a continuous scalar potential given as a perturbation of a periodic potential. Under suitable assumptions on different types of nonlinearities f, namely, $f(x,u) = \left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u$ for $(2N-\alpha)/N < p < 2^*_\alpha$, then $f(u) = |u|^{p-1}u$ for $1 < p < 2^* - 1$ and $f(u) = |u|^{2^*-2}u$ (where $2^* = 2N/(N-2)$), we prove the existence of at least one ground state solution for this equation by variational methods if p belongs to some intervals depending on N and λ.

1 Introduction

We consider the problem

$$-(\nabla + iA(x))^2 u + V(x)u = \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha}\right)|u|^{2^*_\alpha - 2}u + \lambda f(u) \ \text{ in } \ \mathbb{R}^N, \tag{1}$$

where $\nabla + iA(x)$ is the covariant derivative with respect to the C^1, $\mathbb{Z}^N$-periodic vector potential $A\colon \mathbb{R}^N \to \mathbb{R}^N$, i.e,

$$A(x+y) = A(x), \ \forall\, x \in \mathbb{R}^N, \ \forall\, y \in \mathbb{Z}^N.$$

The exponent $2^*_\alpha = \frac{2N-\alpha}{N-2}$ is critical, in the sense of the Hardy-Littlewood-Sobolev inequality, $\lambda > 0$, $N \geq 3$, $0 < \alpha < N$, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous scalar potential and f stands for different types of nonlinearities. Namely, we first consider $f(x,u) = \left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u$ for $(2N-\alpha)/N < p < 2^*_\alpha$, then $f(u) = |u|^{p-1}u$ for $1 < p < 2^* - 1$, where 2^* is the critical exponent of immersion $D^{1,2}(\mathbb{R}^N) \hookrightarrow L^{2^*}(\mathbb{R}^N)$, and finally we examine $f(u) = |u|^{2^*-2}u$.

Inspired by the seminal work of Coti Zelati and Rabinowitz [1], but also by Alves, CarriÃ£o and Miyagaki [3] and by Alves and Figueiredo [4], we assume that there is a continuous, $\mathbb{Z}^N$-periodic potential $V_{\mathcal{P}} : \mathbb{R}^N \to \mathbb{R}$, constants $V_0, W_0 > 0$ and $W \in L^{\frac{N}{2}}(\mathbb{R}^N)$ with $W(x) \geq 0$ such that

(V_1) $V_{\mathcal{P}}(x) \geq V_0, \quad \forall\, x \in \mathbb{R}^N$;

(V_2) $V(x) = V_{\mathcal{P}}(x) - W(x) \geq W_0, \quad \forall\, x \in \mathbb{R}^N$,

where the last inequality is strict on a subset of positive measure in $\mathbb{R}^N$.

2 Main Results

The main results of this work are the following theorems.

Theorem 2.1. *For* $\frac{2N-\alpha}{N} < p < 2^*_\alpha$*, under the hypotheses already stated on* A*,* V *and* α*, problem*

$$-(\nabla + iA(x))^2 u + V(x)u = \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha}\right)|u|^{2^*_\alpha - 2}u + \lambda\left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u \quad in \quad \mathbb{R}^N \tag{1}$$

has at least one ground state solution if either

(i) $\frac{N+2-\alpha}{N-2} < p < 2^*_\alpha$*,* $N = 3, 4$ *and* $\lambda > 0$*;*

(ii) $\frac{2N-\alpha}{N} < p \leq \frac{N+2-\alpha}{N-2}$*,* $N = 3, 4$ *and* λ *sufficiently large;*

(iii) $\frac{2N-2-\alpha}{N-2} < p < 2^*_\alpha$*,* $N \geq 5$ *and* $\lambda > 0$*;*

(iv) $\frac{2N-\alpha}{N} < p \leq \frac{2N-2-\alpha}{N-2}$*,* $N \geq 5$ *and* λ *sufficiently large.*

Theorem 2.2. *For* $1 < p < 2^* - 1$*, under the hypotheses already stated on* A*,* V *and* α*, problem*

$$-(\nabla + iA(x))^2 u + V(x)u = \left(\frac{1}{|x|^\alpha} * |u|^{2^*_\alpha}\right)|u|^{2^*_\alpha - 2}u + \lambda|u|^{p-1}u \quad in \quad \mathbb{R}^N. \tag{2}$$

has at least one ground state solution if either

(i) $3 < p < 5$*,* $N = 3$ *and* $\lambda > 0$;

(ii) $p > 1$*,* $N \geq 4$ *and* $\lambda > 0$*;*

(iii) $1 < p \leq 3$*,* $N = 3$ *and* λ *sufficiently large.*

Theorem 2.3. *Under the hypotheses already stated on* A*,* V *and* α*, the problem*

$$-(\nabla + iA(x))^2 u + V(x)u = \lambda\left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u + |u|^{2^*-2}u \quad in \quad \mathbb{R}^N, \tag{3}$$

has at least one ground state solution in the intervals already described in Theorem 2.1.

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GLOBAL EXISTENCE AND REGULARITY RESULTS FOR A CLASS QUASILINEAR NON-UNIFORMLY ELLIPTIC PROBLEMS WITH FAST DIFFUSION

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Abstract

In this work, we investigate the existence, nonexistence, and regularity of solutions for a class of quasilinear non-uniformly elliptic problems characterized by a fast diffusion term at infinity. We establish Schauder and Calderón-Zygmund type regularity results, providing new insights into the smoothness of weak solutions associated with the operator

$$\mathcal{L}_p u = -\operatorname{div}\Big(p|\nabla u|^{p-2}e^{|\nabla u|^p}\nabla u\Big)$$

for $p > 1$. By employing variational techniques, sub and supersolution methods, and advanced functional analysis in Orlicz-Sobolev spaces, we prove that weak solutions belong to $C^1(\Omega)$ and, under certain conditions, are strong solutions in $W^{2,2}_{\text{loc}}(\Omega)$. Moreover, we establish an optimal threshold Λ_p determining the existence and nonexistence of positive strong solutions for a class of quasilinear elliptic problems.

1 Introduction

In this work, we explore the existence, nonexistence, and regularity of solutions to a class of nonlinear problems governed by the operator

$$\mathcal{L}_p u = -\operatorname{div}\left(p|\nabla u|^{p-2}e^{|\nabla u|^p}\nabla u\right), \tag{1}$$

which exemplifies the intricate behavior introduced by exponential-type terms in the gradient. We focus on the boundary value problem

$$\begin{cases} \mathcal{L}_p u = \lambda f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{$P_{p,\lambda}$}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain and f allows subcritical exponential growth.

Our approach combines variational methods with sub and supersolution techniques, within the framework of Orlicz-Sobolev spaces, to prove the existence of positive strong solutions and establish their regularity. We identify a threshold parameter $\Lambda_p > 0$ such that solutions exist for $\lambda \geq \Lambda_p$, and do not exist for $\lambda < \Lambda_p$.

Moreover, by considering the identity relating the operator $\mathcal{L}_p$ with the non-divergence form operator

$$\mathcal{L}_\infty w = -\text{tr}\left[(I_N + 2\nabla w \otimes \nabla w)D^2 w\right],$$

we extend the obtained results to non-divergence form problems, concluding existence and nonexistence of classical solutions in more general settings.

2 Main Results

We consider the N-function $\Phi_p : \mathbb{R} \to [0,\infty)$ defined by

$$\Phi_p(t) = \int_0^{|t|} s\,\phi_p(s)\,ds = e^{|t|^p} - 1, \quad \forall\, t \in \mathbb{R}. \tag{1}$$

To state the main results of this work and investigate the existence and nonexistence of positive strong solutions to the class of quasilinear problems $(P_{p,\lambda})$, we assume that $p > 1$, $\lambda > 0$, and $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, with $N \geq 2$. The nonlinearity $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is assumed to satisfy the following conditions:

(f_1) $f \in C(\overline{\Omega} \times \mathbb{R}; \mathbb{R})$, with $f(x,t) = 0$ for $t \leq 0$ and $x \in \Omega$;

(f_2) There is $C_0 > 0$ such that $f(x,t)t \leq C_0 \Phi_p(t/d)$, for all $t \geq 0$, where d is twice the diameter of Ω;

(f_3) $\displaystyle\lim_{t \to +\infty} \frac{f(x,t)t}{\Phi_p(t/d)} = 0$ e $\displaystyle\lim_{t \to 0} \frac{f(x,t)t}{\Phi_p(t/d)} = 0$ uniformly in $\overline{\Omega}$;

(f_4) $f(x,t) > 0,\ t > 0$ and $x \in \Omega$.

Theorem 2.1. *Let Φ_p be the $\mathcal{N}$-function defined in (2.1) with $p > 1$, and assume that $(f_1) - (f_4)$ are satisfied. Then, there exists a constant $\Lambda_p > 0$ such that, for every $\lambda \geq \Lambda_p$ the problem $(P_{p,\lambda})$ has a positive strong solution $u_\lambda \in C^1(\Omega) \cap W^{2,2}_{loc}(\Omega)$, i.e., $\mathcal{L}_p u = \lambda f(x,u)$, a.e. in Ω. Moreover, for every $0 < \lambda < \Lambda_p$, the problem $(P_{p,\lambda})$ has no positive strong solution.*

Corollary 2.1. *Let Φ_p be the $\mathcal{N}$-function defined in (2.1). Assume $p = 2$ and $f \in C^1(\overline{\Omega} \times \mathbb{R}; \mathbb{R})$ satisfying $(f_1) - (f_4)$. Then, there exist positive constant $\Lambda_2 > 0$, such that, for every $\lambda \geq \Lambda_2$, the problem $(P_{2,\lambda})$ has at least one positive classical solution. Moreover, for every $0 < \lambda < \Lambda_2$, the problem $(P_{2,\lambda})$ has no positive classical solution.*

Given $w \in C^2(\Omega)$, we have

$$-\Delta w - 2\Delta_\infty w = -\mathrm{tr}\big[(I_N + 2\nabla w \otimes \nabla w)D^2 w\big] = e^{-|\nabla w|^2}\mathcal{L}_2 w, \tag{2}$$

where $\mathcal{L}_2$ is the operator defined in (1), I_N is the identity matrix of order N, and $\nabla w \otimes \nabla w$ denotes the $N \times N$ matrix with entries $\left(\frac{\partial w}{\partial x_i}\frac{\partial w}{\partial x_j}\right)$ for $i,j = 1,\ldots,N$. Corollary 2.1 thus provides a criterion for the existence and nonexistence of positive classical solutions to the non-uniformly elliptic quasilinear problem of the type:

$$(P_{\infty,\lambda}) \qquad \begin{cases} -\Delta w - 2\Delta_\infty w = \lambda e^{-|\nabla w|^2} f(x,w) & \text{in } \Omega, \\ w = 0 & \text{on } \partial\Omega. \end{cases}$$

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A LOCAL APPROACH TO RESONANT ELLIPTIC EQUATIONS WITH DEPENDENCE ON THE GRADIENT

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Abstract

This talk is concerned with the study of solutions for a class of semilinear elliptic problem with dependence on the gradient when the nonlinear term satisfies a local Landesman-Lazer condition. The method used to establish the existence of solution combines the Lyapunov-Schmidt reduction method, minimax critical points theory and a fixed point theorem with appropriated truncation of the nonlinear term and an approximation argument. Results on the existence of multiple solutions are also provided. The results do not impose growth restriction at infinity on the nonlinear term and it may change sign.

1 Introduction

Motivated by our earlier results for semilinear problems under a local Landesman-Lazer condition [3] (see also [1] for corresponding problems involving the p-Laplacian operator), we study the existence, non existence and multiplicity of weak solutions for the following problem:

$$\begin{cases} -\Delta u = & \lambda u + \mu h_\mu(x,u,\nabla u) \text{ in } \Omega, \\ \quad u = & 0 \qquad\qquad\qquad\qquad \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\lambda > 0$ and $\mu \in (-\mu_1, \mu_1)$, $\mu_1 > 0$, are real parameters and $\hat{h}(x,\mu,s,\xi) := h_\mu(x,s,\xi) : \Omega \times (-\mu_1,\mu_1) \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}$ is a Carathéodory function satisfying:

(H_1) there exists $\sigma > N$ such that $\hat{h}$ is locally $L^\sigma(\Omega)$-Lipschitz with respect to the variables s and ξ and locally $L^\sigma(\Omega)$-bounded;

and the following version of the local Landesman-Lazer condition:

$(LL)_{loc}$ there exist real numbers t_1 and t_2, with $t_1 < t_2$, such that $\ell_1\ell_2 < 0$, where ℓ_i is defined by

$$\ell_i := \int_\Omega h_0(x, t_i\varphi_k, t_i\nabla\varphi_k)\varphi_k dx, \ i \in \{1,2\},$$

being φ_k a unitary eigenfunction associated with λ_k, a simple eigenvalue of the operator $-\triangle$ in $H^1_0(\Omega)$.

2 Main Results

Our main result provides a solution for Problem (1) when the parameters μ and λ are close to zero and λ_k, respectively:

Theorem 2.1. *Suppose h_μ satisfies (H_1) and $(LL)_{loc}$. Then there exist positive constants μ^* and ν^* such that, for every $0 \le |\mu| < \mu^*$ and $|\lambda - \lambda_k| \le |\mu|\nu^*$, Problem (1) has a weak solution $u_\mu = t\varphi_k + v$, with $t \in (t_1, t_2)$ and $v \in \langle\varphi_k\rangle^\perp$.*

Note that in Theorem 2.1 the nonlinear term may be indefinite and it is not imposed any global growth restriction on the nonlinear term with respect to the second and third variables. The fact that the projection of the solution u_μ on the direction of φ_k is located between $t_1\varphi_k$ and $t_2\varphi_k$ allows us to derive the existence of multiple solutions.

We also present a version of Theorem 2.1 when $h_\mu(x,0,0) \equiv 0$. Noting that $u \equiv 0$ is a trivial solution for Problem (1), we derive the existence of a nontrivial solution solution by supposing

(LL_0) there exists $t_0 > 0$ such that $\ell_0 := \int_\Omega h_0(x, t_0\varphi_k, t_0\nabla\varphi_k)\varphi_k dx \neq 0$, and

(H_0) given $\mu \in (0, \mu_1)$, there exists $f_\mu \in L^{\sigma_\mu}(\Omega), \sigma_\mu > N$, such that

$$|h_\mu(x,s,\xi)| \leq f_\mu(x) o(|(s,\xi)|), \text{ when } (s,\xi) \to (0,0), \text{ a.e. in } \Omega.$$

Theorem 2.2. *Suppose h_μ satisfies (H_1), (H_0) and (LL_0). Then there exist constants $\mu^*, \nu^* > 0$ such that, for every $0 < \mu < \mu^*$, Problem (1) has a solution $u = t\varphi_k + v$, with $t \in (0, t_0)$ and $v \in \langle\varphi_k\rangle^\perp$ if either*

(i) $\ell_0 > 0$ and $\lambda_k - \nu^*\mu < \lambda < \lambda_k$, or

(ii) $\ell_0 < 0$ and $\lambda_k < \lambda < \lambda_k + \nu^*\mu$.

As a direct consequence of Theorems 2.1 and 2.2, we obtain:

Corollary 2.1. *Suppose h_μ satisfies (H_1), (H_0) and $(LL)_{loc}$, with $t_1 > 0$. Then there exist constants $\mu^*, \nu^* > 0$ such that, if $0 < \mu < \mu^*$,*

(a) Problem (1) has a solution $u_1 = \tau_1\varphi_k + v_1$, with $t_1 < \tau_1 < t_2$ and $v_1 \in \langle\varphi_k\rangle^\perp$ if $\lambda_k - \nu^*\mu < \lambda < \lambda_k + \nu^*\mu$, or

(b) Problem (1) has a second solution $u_2 = \tau_2\varphi_k + v_2$, with $0 < \tau_2 < t_1$ and $v_2 \in \langle\varphi_k\rangle^\perp$ if either

(i) $\ell_1 > 0$ and $\lambda_k - \nu^*\mu < \lambda < \lambda_k$, or

(ii) $\ell_1 < 0$ and $\lambda_k < \lambda < \lambda_k + \nu^*\mu$.

We also present some applications of our results for the existence and multiplicity of solutions for indefinite problems.

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ASYMPTOTIC BEHAVIOR OF FRACTIONAL MUSIELAK-ORLICZ-SOBOLEV MODULARS

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Abstract

In this work we study the asymptotic behavior of anisotropic nonlocal nonstandard growth seminorms and modulars as the fractional parameter goes to 1. This gives a so-called Bourgain-Brezis-Mironescu type formula for a very general family of functionals. In the particular case of fractional Sobolev spaces with variable exponent, we point out that our proof asks for a weaker regularity of the exponent than the considered in previous articles.

1 Introduction

Our main goal is to study the asymptotic behavior of the following energy functionals (modular functions) as $s \uparrow 1$.

In this work, $G\colon \mathbb{R}^n \times \mathbb{R}^n \times [0,\infty) \to [0,\infty)$ is a generalized Young function, namely,

$$G(x,y,t) = \int_0^t g(x,y,s)\,ds,$$

where $g\colon \mathbb{R}^n \times \mathbb{R}^n \times [0,\infty) \to [0,\infty)$ is a function satisfying

(i) $g(x,y,0) = 0$ and $g(x,y,t) > 0$ for any $(x,y) \in \mathbb{R}^n \times \mathbb{R}^n, t > 0$;

(ii) $g(\cdot,\cdot,t)$ is nondecreasing in $[0,\infty)$;

(iii) $g(\cdot,\cdot,t)$ is right continuous in $[0,\infty)$ and $\lim_{t\to\infty} g(\cdot,\cdot,t) = \infty$.

Moreover, we suppose that there exist constants $0 < C_1 \le C_2 < \infty$ such that

$$C_1 \le \inf_{x,y\in\mathbb{R}^n} G(x,y,1) \le G(x,y,1) \le \sup_{x,y\in\mathbb{R}^n} G(x,y,1) \le C_2,\ \forall (x,y) \in \mathbb{R}^n \times \mathbb{R}^n. \tag{H_1}$$

$$y \mapsto G(x,y,t) \ \text{ is continuous.} \tag{H_2}$$

2 Main Results

Our main results are

Theorem 2.1. *Let $u \in C_c^2(\mathbb{R}^n)$. Then, there exists λ_0 such that*

$$\lim_{s\uparrow 1}(1-s)\int_{\mathbb{R}^n}\int_{\mathbb{R}^n} G\left(x,y,\frac{|u(x)-u(y)|}{\lambda|x-y|^s}\right)\frac{dxdy}{|x-y|^n} = \int_{\mathbb{R}^n} H_0\left(x,\frac{|\nabla u(x)|}{\lambda}\right)dx, \tag{1}$$

for all $\lambda \ge \lambda_0$, where the function H_0 is a generalized Young function given by

$$H_0(x,t) = \int_0^1 \int_{\mathbb{S}^{n-1}} G(x,x,t|w_n|r)\,d\mathcal{H}^{n-1}(w)\frac{dr}{r} \tag{2}$$

and w_n is the $n-$th coordinate of any point in $\mathbb{S}^{n-1}$. In particular, if G satisfies Δ_2-condition, then equation (1) holds for every $\lambda > 0$.

As a direct consequence of the Theorem 2.1 in the usual fractional Sobolev spaces, we obtain the following:

Corollary 2.1. *Let* $u \in W^{1,p^-}(\mathbb{R}^n) \cap W^{1,p^+}(\mathbb{R}^n)$. *Then* (1) *holds.*

Next, we consider its equivalent, defined as

$$[[u]]_{s,G} := \inf \left\{ \lambda > 0 \colon (1-s)\mathcal{J}\left(\frac{u}{\lambda}\right)\, dx \leq 1 \right\}.$$

Corollary 2.2. *Let* $u \in C_c^2(\mathbb{R}^n)$. *Then,*

$$\limsup_{s \uparrow 1} [[u]]_{s,G} \leq \|\nabla u\|_{H_0},$$

where H_0 *was defined in* (2).

With the same technique than in Theorem **??**, we prove a BBM result for smooth functions.

Theorem 2.2. *Let* $u \in C_c^2(\mathbb{R}^n)$ *and* $k \in \{1, \ldots, n\}$. *Then,*

$$\lim_{s \uparrow 1}(1-s) \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} G\left(x, x - he_k, \frac{|u(x-he_k) - u(x)|}{|h|^s}\right) \frac{dhdx}{|h|} = \int_{\mathbb{R}^n} H_0\left(x, \left|\frac{\partial u(x)}{\partial x_k}\right|\right) dx,$$

where

$$H_0(x,t) = 2\int_0^1 G(x,x,tr)\frac{dr}{r}.$$

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CHEBYSHEV POLYNOMIALS, DIFFERENTIAL EQUATIONS AND FRACTIONAL POWERS

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Abstract

In this work we show the characterization of the fractional powers of a class of positive operators by Chebyshev polynomials of the second kind. We consider the following higher order abstract Cauchy problems $\dfrac{d^n u}{dt^n} + Au = 0, \quad t > 0$, with initial conditions given by $\dfrac{d^i u}{dt^i}(0) = u_i \in X^{\frac{n-(i+1)}{n}}, \quad i \in \{0, 1, \ldots, n-1\},\ n \geqslant 1$, where X be a separable Hilbert space and $A : D(A) \subset X \to X$ is an unbounded linear, closed, densely defined, self-adjoint and positive definite operator, and its fractional counterpart. Here, X^α $(0 \leqslant \alpha \leqslant 1)$ denotes the domain of the fractional powers A^α endowed with graphic norm.

1 Introduction

In this work we consider the following abstract linear problem of n-th order in time

$$\frac{d^n u}{dt^n} + Au = 0, \quad t > 0, \qquad \frac{d^i u}{dt^i}(0) = u_i \in X^{\frac{n-(i+1)}{n}}, \quad i \in \{0, 1, \ldots, n-1\},\ n \geqslant 3, \tag{1}$$

where X is a separable Hilbert space and $A : D(A) \subset X \to X$ is an unbounded linear, closed, densely defined, self-adjoint and positive definite operator, and therefore, A is a sectorial operator in the sense of Henry [3, Definition 1.3.1]. This allows us to define the fractional power A^α of order $0 < \alpha < 1$ according to Amann [1], as a closed linear operator. Denote by $X^\alpha = D(A^\alpha)$ for $0 \leqslant \alpha \leqslant 1$ (taking $A^0 := I$ on $X^0 := X$ when $\alpha = 0$). Recall that X^α is dense in X for all $0 \leqslant \alpha \leqslant 1$. The fractional power space X^α endowed with the norm $\| \cdot \|_{X^\alpha} := \|A^\alpha \cdot \|_X$ is a Banach space. It is not difficult to show that A^α is the generator of a strongly continuous analytic semigroup on X, see Henry [3] for any $\alpha \in [0, 1]$. With this notation, we have $X^{-\alpha} = (X^\alpha)'$ for all $\alpha > 0$, see Amann [1] for the characterization of the negative scale.

We will rewrite (1) as a first order abstract system, we will calculate the fractional powers of order $\alpha \in (0, 1)$ of the main part of the differential system and we are interested in finding for which values of $\alpha \in [0, 1]$ the fractional first order system is well-posed in some sense; namely, we will consider the phase space $Y = X^{\frac{n-1}{n}} \times X^{\frac{n-2}{n}} \times X^{\frac{n-3}{n}} \times \cdots \times X$ which is a Banach space equipped with the norm given by $\| \cdot \|^2_Y = \| \cdot \|^2_{X^{\frac{n-1}{n}}} + \| \cdot \|^2_{X^{\frac{n-2}{n}}} + \| \cdot \|^2_{X^{\frac{n-3}{n}}} + \cdots + \| \cdot \|^2_X$.

We can write the problem (1) as a Cauchy problem on Y, letting $v_1 = u$, $v_2 = \frac{du}{dt}$, $v_3 = \frac{d^2u}{dt^2}$, $\ldots$, $v_n = \frac{d^{n-1}u}{dt^{n-1}}$, $\mathbf{u} = [v_1\ v_2\ v_3\ \ldots\ v_n]^T$, and $\mathbf{u}_0 = [u\ u_1\ u_2\ \ldots\ u_{n-1}]^T$, and the initial value problem

$$\frac{d\mathbf{u}}{dt} + \Lambda_n \mathbf{u} = 0,\ t > 0, \qquad \mathbf{u}(0) = \mathbf{u}_0, \tag{2}$$

where the unbounded linear operator $\Lambda_n : D(\Lambda_n) \subset Y \to Y$ is defined by

$$D(\Lambda_n) = X^1 \times X^{\frac{n-1}{n}} \times X^{\frac{n-2}{n}} \times \cdots \times X^{\frac{1}{n}}, \tag{3}$$

and

$$\Lambda_n \mathbf{u} = \begin{bmatrix} 0 & -I & 0 & \cdots & 0 & 0 \\ 0 & 0 & -I & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -I \\ A & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{n-1} \\ v_n \end{bmatrix} := \begin{bmatrix} -v_2 \\ -v_3 \\ -v_4 \\ \vdots \\ -v_n \\ Av_1 \end{bmatrix}, \ \forall \mathbf{u} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} \in D(\Lambda_n). \tag{4}$$

2 Main Results

The main results are as follows.

Theorem 2.1. *Let Λ_n be the unbounded linear operator defined in (3)-(4) with $n \geqslant 3$. Then $-\Lambda_n$ is not the infinitesimal generator of a strongly continuous semigroup on Y.*

Theorem 2.2. *Let Λ_n be the unbounded linear operator defined in (3)-(4). Then*

i) $0 \in \rho(\Lambda_n)$. *Moreover, if A has compact resolvent on X, then Λ_n has compact resolvent on Y;*

ii) *Fractional powers Λ_n^α can be defined for $0 < \alpha < 1$ by* $\Lambda_n^\alpha = \dfrac{\sin(\alpha\pi)}{\pi} \int_0^\infty \lambda^{\alpha-1} \Lambda_n (\lambda I + \Lambda_n)^{-1} d\lambda$;

iii) *Given any $0 \leqslant \alpha \leqslant 1$ we have the unbounded linear operator $\Lambda_n^\alpha : D(\Lambda_n^\alpha) \subset Y \to Y$ defined by*

$$D(\Lambda_n^\alpha) = X^{\frac{\alpha+n-1}{3}} \times X^{\frac{\alpha+n-2}{3}} \times \cdots \times X^{\frac{\alpha}{3}}, \quad \Lambda_n^\alpha = \left[\frac{(-1)^{i-j}}{n} U_{n-1} \left(cos \left(\frac{(\alpha + i - j)\pi}{n} \right) \right) A^{\frac{\alpha+i-j}{n}} \right]_{ij}, \tag{1}$$

where $U_n : \mathbb{C} \to \mathbb{C}$ is the nth degree Chebyshev polynomial of second kind defined by the recurrence relation $U_0(x) = 1$, $U_1(x) = 2x$, and $U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x)$, for all $x \in \mathbb{C}$ and $n \geqslant 1$.

From now on we will assume that A has compact resolvent on X.

Theorem 2.3. *Let Λ_n be the unbounded linear operator defined in (3)-(4). For each $0 < \alpha \leqslant 1$ the spectrum of $-\Lambda_n^\alpha$ is such that the point spectrum consisting of eigenvalues*

$$\bigcup_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \left(\left\{ \mu_j^{\frac{\alpha}{n}} e^{i\frac{\pi(n-(n-2k-1)\alpha)}{n}} \ : \ j \in \mathbb{N} \right\} \cup \left\{ \mu_j^{\frac{\alpha}{n}} e^{i\frac{\pi(n+(n-2k-1)\alpha)}{n}} \ : \ j \in \mathbb{N} \right\} \right) \tag{2}$$

where $\{\mu_j\}_{j \in \mathbb{N}}$ denotes the ordered sequence of eigenvalues of A including their multiplicity.

Finally, we present some observations on the differential equation $\dfrac{d\mathbf{u}^\alpha}{dt} + \Lambda_n^\alpha \mathbf{u}^\alpha = 0, \quad 0 < \alpha < 1$ in Y.

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DECAY RATES GIVEN BY REGULARLY VARYING FUNCTIONS FOR C_0-SEMIGROUPS ON BANACH SPACES

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Abstract

We study rates of decay for (possibly unbounded) C_0-semigroups on Banach spaces under the assumption that the norm of the resolvent of the respective semigroup generator grows as a regularly varying function of type $\beta > 0$, that is, as $|s|^\beta \ell(1 + |s|)$ or $|s|^\beta/\kappa(1 + |s|)$, where ℓ, κ are arbitrary monotone and slowly varying functions. The main result extends the estimates obtained by Deng, Rozendaal and Veraar (J. Evol. Equ. 24, 99 (2024)) to this setting of regularly varying functions and improves the estimates obtained by Santana and Carvalho (J. Evol. Equ. 24, 28 (2024)) in case $|s|^\beta \log(1 + |s|)^b$, with $b \geq 0$.

1 Introduction

The asymptotic theory of semigroups provides tools for investigating the convergence to zero of mild and classical solutions to the abstract Cauchy problem

$$\begin{cases} u'(t) + Au(t) = 0, & t \geq 0, \\ u(0) = x. \end{cases} \tag{1}$$

It is known that (1) has a unique mild solution for every $x \in X$ and that the solution depends continuously on x if, and only if, $-A$ generates a C_0-semigroup $(T(t))_{t\geq 0}$ on X.

In the last two decades, there have been published a great number of works in semigroup theory devoted to the study of decay rates for classical solutions (assuming some regularity for the initial data) and some spectral properties for the infinitesimal generator. Such results arise from concrete problems, for example, in the study of the damped wave equation. However, in many concrete problems, the associated semigroup $(T(t))_{t\geq 0}$ is not bounded, or at least one cannot prove such boundedness. Hence, the problem of proving polynomial (and other rates) decay for not necessarily bounded semigroups is of great importance. In this vein, we highlight the pioneering work of BÃ¡tkai, Engel, Prüss and Schnaubelt [1]. In 2018, Rozendaal and Veraar [4] obtained new estimates that improved the estimates obtained so far for the decay rates of unbounded semigroups. These new estimates, take into account geometric properties of the underlying Banach space (namely, its Fourier type). More precisely, let $(T(t))_{t\geq 0}$ be a C_0-semigroup with generator $-A$ defined in a Banach space X with Fourier type $p \in [1, 2]$, and let $\frac{1}{r} = \frac{1}{p} - \frac{1}{p'}$ (where $\frac{1}{p} + \frac{1}{p'} = 1$) and suppose that $\overline{\mathbb{C}_-} \subset \rho(A)$, there exist $\beta, C \geq 0$ such that $\|(\lambda + A)^{-1}\|_{\mathcal{L}(X)} \leq C(1 + |\lambda|)^\beta$ for each $\lambda \in \overline{\mathbb{C}_-}$. Let $\tau > \beta + 1/r$; then, for each $\varepsilon > 0$, there exists $C_\varepsilon \geq 0$ such that for each $t \geq 1$, $\|T(t)(1 + A)^{-\tau}\|_{\mathcal{L}(X)} \leq C_\varepsilon t^{1-\frac{\tau-1/r}{\beta}+\varepsilon}$. In 2024, in our work [5] we went a step further by considering that $\|(\lambda + A)^{-1}\|_{\mathcal{L}(X)} \lesssim (1 + |\lambda|)^\beta \log(2 + |\lambda|)^b$, with $b \geq 0$ (a particular example of a *regularly varying function of index* β), obtaining, for each $\delta > 0$, a constants $c_{\delta,\tau} \geq 0$ and $t_0 > 1$ such that for each $t \geq t_0$, $\|T(t)(1 + A)^{-\tau}\|_{\mathcal{L}(X)} \leq c_{\delta,\tau} t^{1-\frac{\tau-r^{-1}}{\beta}} \log(t)^{\frac{b(\tau-r^{-1})}{\beta}+\frac{1+\delta}{r}}$ (see Theorem 1.13 in [5]). In particular, if one let $b = 0$, then for each $t \geq t_0 > 1$, $\|T(t)(1 + A)^{-\tau}\|_{\mathcal{L}(X)} \lesssim t^{1-\frac{\tau-r^{-1}}{\beta}} \log(t)^{\frac{1+\delta}{r}}$, a result that improves the estimate obtained by Rozendaal and Veraar [4]. Recently, Deng, Rozendaal and Veraar [3], by using the boundedness of Fourier multipliers defined in some specific Besov spaces, improving, in the case $b = 0$, the estimate we obtained, by removing the logarithm factor $\log(t)^{\frac{1+\delta}{r}}$.

2 Main Results

By considering this new result, one may ask if the logarithmic factor $\log(t)^{\frac{1+\delta}{r}}$ if it can also be dropped in case $b > 0$. Our main goal is to address this question not only for the result presented in Theorem 1.13 [5], but also for the case where the growth of the norm of the resolvent is given by a more general regularly varying function (as originally discussed in [2] for bounded C_0-semigroups). Note that the new estimates (see the following theorem) improve on the previous ones we obtained(in case $\ell(t) = \log(2+t)$, with $t \geq 1$). The proofs of the following results can be found in our work [6].

Theorem 2.1. *Let $\beta > 0$, $b \geq 0$ and let $(T(t))_{t\geq 0}$ be a C_0-semigroup with generator $-A$ defined in a Banach space X with Fourier type $p \in [1,2]$. Suppose that $\overline{\mathbb{C}_-} \subset \rho(A)$ and let $r \geq 0$ be such that $\frac{1}{r} = \frac{1}{p} - \frac{1}{p'}$.*

(i) Assume that for each $\lambda \in \mathbb{C}$ with $Re(\lambda) \leq 0$, $\|(\lambda + A)^{-1}\|_{\mathcal{L}(X)} \lesssim (1+|\lambda|)^{\beta}\ell(1+|\lambda|)$, where ℓ is an increasing and slowly varying function. Then, for each $\tau > 0$ such that $\tau > \beta + \frac{1}{r}$, there exist $c_\tau > 0$ and $t_0 \geq 1$ so that for each $t \geq t_0$, $\|T(t)(1+A)^{-\tau}\|_{\mathcal{L}(X)} \leq c_\tau t^{1-\frac{\tau-r^{-1}}{\beta}}(\ell(t^{1/\beta}))^{\frac{\tau-r^{-1}}{\beta}}$.

(ii) Assume that for each $\lambda \in \mathbb{C}$ with $Re(\lambda) \leq 0$, $\|(\lambda + A)^{-1}\|_{\mathcal{L}(X)} \lesssim (1+|\lambda|)^{\beta}(1/\kappa(1+|\lambda|))$, where κ is an increasing and slowly varying function. Then, for each $\tau > 0$ such that $\tau > \beta + \frac{1}{r}$, there exist $\tilde{c}_\tau > 0$ and $t_0 \geq 1$ so that for each $t \geq t_0$, $\|T(t)(1+A)^{-\tau}\|_{\mathcal{L}(X)} \leq \tilde{c}_\tau t \left(\frac{1}{t\kappa(t^{1/\beta})}\right)^{\frac{\tau-r^{-1}}{\beta}}$.

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APPROXIMATION OF A NAVER-STOKES-VOIGT TYPE SYSTEM BY A CAUCHY-KOWALESKA TYPE SYSTEM

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Abstract

In this paper we investigate a problem for a model of a Navier-Stokes-Voigt fluid. The problem is considered in a bounded domain of $\mathbb{R}^n$ with Dirichlet boundary conditions. We proved existence of weak solutions when $n \leq 4$ by using the method of approximations by a of Cauchy-Kowaleska type system. Uniqueness is also investigated for $n \leq 4$.

1 Introduction

The unsteady flows of incompressible fluids in a boundary domain $\Omega \in \mathbb{R}^n$, $n > 1$, are described by the system of equations, called the Navier-Stokes equations

$$\begin{aligned} u_t - \nu\Delta u \quad &+ \quad (u \cdot \nabla)\, u + \nabla p = f \qquad \text{in } Q_T, \\ \nabla \cdot u \quad &= \quad 0 \qquad \text{in } Q_T, u = 0 \text{ on } \Sigma_T, u(0) = u_0 \qquad \text{in } \Omega, \end{aligned} \tag{1}$$

where $u = (u_1, u_2, ..., u_n)$ is the velocity, p represents the pressure, $f = (f_1, f_2, ..., f_n)$ stands for the given external and $\nu > 0$ is the constant kinematic viscosity of the fluid. In [1], the author first (see p.p 75) uses a special basis of the eigenvectors of the spectral problem $((u, v)) = \lambda(u, v)$ to pass the limit in the nonlinear term (we observe that the problem loses generalization), obtaining existence for any n and uniqueness if n=2. Later, (see p.p. 77) the author uses another method to solve problem 1, he uses fractional derivative for the passage to the limit in the nonlinear term to obtain a solution when $n \leq 4$ (see also [2], page 144 and [5]), now using any Hilbert basis (we observe that in this case, dimension is lost). Another technique used by the author (see p.p. 465-470) to solve the system 1 is approximating it by a Cauchy-Kowaleska type system, in which case the solutions are found in more general spaces ($H_0^1(\Omega)$ and $L^2(\Omega)$), obtaining existence for $n \leq 4$.
The system below describe the motion of non-Newtonian fluid to which a small quantity of polymers is added:

$$\begin{aligned} u_t \quad &- \quad \alpha\Delta u_t - \nu\Delta u + (u \cdot \nabla)(u - \alpha\Delta u) + \nabla p = f \qquad \text{in } Q_T, \\ \nabla \cdot u \quad &= \quad 0 \text{ in } Q_T, u = 0 \text{ on } \Sigma_T, u(0) = u_0 \text{ in } \Omega. \end{aligned} \tag{2}$$

where α is called the relaxation coefficient of fluid. The system (2) is known in the literature as Navier-Stokes-Voigt, as already been studied by A. P. Oskolkov [3]. We observe that if $\alpha = 0$ in (2), we find the Navier-Stokes equations. In [4] the author investigates the existence of strong solutions for the following variation of the system (2) using a special basis and Galerkin's approximations:

$$\begin{aligned} u_t \quad &- \quad \alpha^2\Delta u_t - \nu\Delta u + (u \cdot \nabla)u + \nabla p = f \qquad \text{in } Q_T, \\ \nabla \cdot u \quad &= \quad 0 \text{ in } Q_T, u = 0 \text{ on } \Sigma_T, u(0) = u_0 \qquad \text{in } \Omega. \end{aligned} \tag{3}$$

In this work we analyze the existence of weak solutions to system (3) by approximating it by a system of Cauchy-Kowaleska type as in Lions [1] pp 466-470, without using fractional derivatives. The method consists in to consider the following system of Cauchy-Kowaleska type:

$$\begin{aligned} u_t^\epsilon \quad &- \quad \alpha^2\Delta u_t^\epsilon - \nu\Delta u^\epsilon + (u^\epsilon \cdot \nabla)u^\epsilon + \frac{1}{2}(\nabla \cdot u^\epsilon)u^\epsilon + \nabla p^\epsilon = f \qquad \text{in } Q_T, \\ \epsilon p_t^\epsilon + \nabla \cdot u^\epsilon \quad &= \quad 0 \text{ in } Q_T, u^\epsilon = 0 \text{ on } \Sigma_T, u^\epsilon(0) = u_0^\epsilon \text{ in } \Omega, p^\epsilon(0) = p_0^\epsilon \text{ in } \Omega. \end{aligned} \tag{4}$$

After we employing the method of Faedo-Galerking, we proved that (4) has a weak solution $\{u^\epsilon, p^\epsilon\}$, for each $\epsilon > 0$, which converges as $\epsilon \to 0$ to a weak solution to the problem (3) in a certain sense.

2 Main Results

Definition 2.1. *We suppose $n \leq 4, u_0 \in H,\ f, f_t \in L^2(0,T;H)$. A weak solution to the boundary value problem (3) is a function u, such that $u \in L^\infty(0,T;V)$, for $T > 0$, satisfying the identity*

$$\begin{gathered}(u_t,\varphi) + \alpha^2 a(u_t,\varphi) + \nu a(u,\varphi) + \langle B_u u, \varphi\rangle = (f,\varphi),\ \text{in } \mathcal{D}'(\Omega)\ \forall\, \varphi \in V,\\ \nabla\cdot u = 0 \ \text{in } Q_T, u = 0 \ \text{on } \Sigma_T, u(0) = u_0 \ \text{in } \Omega.\end{gathered} \tag{1}$$

Definition 2.2. *We suppose $n \leq 4, u_0^\epsilon \in L^2(\Omega),\ f, f_t \in L^2(0,T;L^2(\Omega))$ and $p_0^\epsilon \in L^2(\Omega)$. A weak solution to the boundary value problem (4) is a pair of functions $\{u^\epsilon, p^\epsilon\}$, such that $u^\epsilon \in L^\infty(0,T;H_0^1(\Omega)^n), u_t^\epsilon \in L^\infty(0,T;H_0^1(\Omega)^n)$, and $p^\epsilon \in L^\infty(0,T;L^2(\Omega))$, for $T > 0$, satisfying the identity*

$$(u_t^\epsilon(t),v) + \alpha^2 a(u_t^\epsilon(t),v) + \nu a(u^\epsilon(t),v) + \langle \widetilde{B}_{u^\epsilon} u^\epsilon(t), v\rangle \tag{2}$$

$$+ (\nabla p^\epsilon, v) = \langle f, v\rangle \ \text{in } \mathcal{D}'(\Omega)\ \forall v \in H_0^1(\Omega)^n, \epsilon(p_t^\epsilon, q) + (\nabla\cdot u^\epsilon, q) = 0 \ \text{in } \mathcal{D}'(\Omega)\ \forall q \in L^2(\Omega), \tag{3}$$

$$u^\epsilon \;=\; 0 \ \text{on } \Sigma_T, u^\epsilon(0) = u_0^\epsilon \ \text{in } \Omega, p^\epsilon(0) = p_0^\epsilon \ \text{in } \Omega. \tag{4}$$

Theorem 2.1. *If $f \in L^2(0,T;H)$, and $u_0 \in V$, then there exists a function u defined for $(x,t) \in Q_T$, solution to the boundary value problem (3) in the sense of Definition 2.1.The solution is unique if $n \leq 4$.*

Theorem 2.2. *If $f \in L^2(0,T;L^2(\Omega)^n)$, $u_0^\epsilon \in H_0^1(\Omega)^n$ and $p_0^\epsilon \in L^2(\Omega)$, then there exists a pair of functions $\{u^\epsilon, p^\epsilon\}$ defined for $(x,t) \in Q_T$, solution to the boundary value problem (4) in the sense of Definition 2.2.. The solution is unique if $n \leq 4$.*

Proof First, we prove Theorem 2.2 using Galerkin's method. The first estimate is obtained in the usual way: by multiplying the approximated equation by u^ϵ. For the second estimate, we differentiate the approximated equation with respect to t and multiply it by u_t^ϵ, em seguida usamos um argumento de compacidade. To prove Theorem 2.1, we use the first estimate obtained in Theorem 2 and to get the second estimate we use fractional derivatives and a compactness argument, then we do $\epsilon \longrightarrow 0$ to show that $u^\epsilon \to u$ is the solution of 1. Uniqueness is obtained by the energy method. ■

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PULLBACK ATTRACTORS FOR NON-AUTONOMOUS HEAT EQUATIONS ON TIME-VARYING DOMAINS WITH NEUMANN BOUNDARY CONDITIONS

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Abstract

We are interested in studying a non-autonomous semilinear heat equation on time-varying domains with Neumann boundary conditions. Using a differential geometry approach with coordinate transformations technique, we will show that the non-autonomous problem on a time-varying domain is equivalent, in some sense, to a non-autonomous problem on a fixed domain. Furthermore, we intend to show the local existence and uniqueness of solutions to this problem, as well as, to extend these solutions globally. Finally, we will show the existence of pullback attractors.

1 Introduction

In the works [1, 2] we are interested in the study of the pullback attractors for non-autonomous semilinear parabolic equations on time-varying domains with Neumann boundary conditions. Semilinear parabolic problems with time-varying domains are intrinsically non-autonomous, even if the terms in the problem do not explicitly depend on time. In [3] the existence of pullback attractors was established for a semilinear heat equation on time-varying domains with homogeneous Dirichlet boundary conditions. Therefore, the works [1, 2] extend the results obtained in [3] to the case of Neumann boundary conditions and still using other techniques.

Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be a nonempty bounded open set with C^2 boundary $\partial\Omega$. We consider the function $r : \mathbb{R} \times \overline{\Omega} \to \mathbb{R}^n$ such that $r \in C^1(\mathbb{R} \times \overline{\Omega}, \mathbb{R}^n)$, $r(t, \cdot) : \overline{\Omega} \to \overline{\Omega}_t$ is a diffeomorphism of class C^2 for all $t \in \mathbb{R}$, with $\Omega_t = r(t, \Omega)$, and $r^{-1}(\cdot, x) \in C^1(\mathbb{R}, \mathbb{R}^n)$ for all $x \in \overline{\Omega}_t$. We define

$$Q_{\tau,T} = \bigcup_{t\in(\tau,T)} \{t\} \times \Omega_t \quad \text{and} \quad \Sigma_{\tau,T} = \bigcup_{t\in(\tau,T)} \{t\} \times \partial\Omega_t, \quad \text{for all } T > \tau.$$

We are particularly interested in studying the following non-autonomous semilinear heat equation with nonlinear Neumann boundary conditions

$$\begin{cases} \dfrac{\partial u}{\partial t} - \Delta u + \lambda u = f(t, u), & (t, x) \in Q_{\tau,T} \\[2ex] \dfrac{\partial u}{\partial n_t} = g(t, u), & (t, x) \in \Sigma_{\tau,T} \\[2ex] u(\tau, x) = u_\tau(x), & x \in \Omega_\tau \end{cases} \tag{1}$$

where $\lambda > 0$, $n_t(x)$ is the unit outward normal vector at $x \in \partial\Omega_t$, $u_\tau : \Omega_\tau \to \mathbb{R}$ and f, $g : \mathbb{R}^2 \to \mathbb{R}$ are nonlinear functions satisfying certain conditions.

2 Main Results

We consider the following coordinate transformation

$$v(t,y) = u(t, r(t,y)), \quad \text{for any } (t,y) \in [\tau, T] \times \Omega,$$

or equivalently,

$$u(t,x) = v(t, r^{-1}(t,x)), \quad \text{for any } (t,x) \in [\tau, T] \times \Omega_t.$$

Using this coordinate transformations technique, we can rewrite the original equation (1) as the following auxiliary non-autonomous heat equation on a fixed domain

$$\begin{cases} \dfrac{\partial v}{\partial t} - \displaystyle\sum_{k,j=1}^{n} \frac{\partial}{\partial y_j}\left(a_{jk}(t,y)\frac{\partial v}{\partial y_k}\right) + \sum_{k=1}^{n} b_k(t,y)\frac{\partial v}{\partial y_k} + \lambda v = f(t,v), & (t,y) \in (\tau, T] \times \Omega \\ \Gamma(t,v) = g(t,v), & (t,y) \in (\tau, T] \times \partial\Omega \\ v(\tau, y) = u_\tau(r(\tau, y)) = v_\tau(y), & y \in \Omega \end{cases} \tag{2}$$

where the coefficients and the boundary condition are given by

$$\begin{aligned} a_{jk}(t,y) &= \sum_{i=1}^{n} \frac{\partial r_j^{-1}}{\partial x_i}(t, r(t,y)) \frac{\partial r_k^{-1}}{\partial x_i}(t, r(t,y)), \quad \text{for } j,k = 1, \ldots, n; \\ b_k(t,y) &= \frac{\partial r_k^{-1}}{\partial t}(t, r(t,y)) - \Delta_x r_k^{-1}(t, r(t,y)) + \sum_{j=1}^{n} \frac{\partial a_{jk}}{\partial y_j}(t,y), \quad \text{for } k = 1, ..., n; \\ \Gamma(t,v) &= K(t,y)\sum_{k=1}^{n} n_k(y) \sum_{j=1}^{n} a_{jk}(t,y)\frac{\partial v}{\partial y_k}, \quad \text{for some function } K(t,y) > 0. \end{aligned}$$

We will show the local existence and uniqueness of solutions to the problem (2), as well as extend these solutions globally, and then we will prove the existence of pullback attractors.

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RATES OF DECAY OF MGT THEMOELASTIC PLATES

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Abstract

In this paper we investigate two examples of thermoelastic plates free of the paradox of instantaneous propagation of thermal or mechanical waves when only one of them is dissipative and the other is conservative. To be precise we consider a Moore-Gibson-Thompson plate with type II heat conduction and a conservative elastic plate with Moore-ibson-hompson heat conduction. In both cases we prove the well-posedness. In the first case we also show the exponential decay of the solutions, but in the second we obtain the lack of the exponential decay and the optimality of the polynomial decay for the associated semigroup.

1 Introduction

The first system we will consider corresponds to assume that the equation determined for the displacement is of the Moore-Gibson-Thompson type and the heat equation is of conservative kind. This is he system in given the following form

$$\begin{aligned} \rho(\tau u_{ttt} + u_{tt}) + K_1\Delta^2 u + \gamma\Delta^2 u_t + m(\Delta\alpha_t + \tau\Delta\alpha_{tt}) &= 0, \\ c\alpha_{tt} - \beta\Delta\alpha - m\Delta u_t &= 0, \end{aligned}$$

with corresponding physical conditions

$$\rho > 0, \quad \tau > 0, \quad K_1 > 0, \quad \gamma > 0 \quad c > 0, \quad \beta > 0, \quad m \neq 0 \quad \text{and} \quad \gamma > K_1\tau.$$

The second system we consider describes a Moore-Gibson-Thompson equation for the heat and a conservative equation for the displacement.

$$\begin{aligned} \rho\hat{u}_{tt} + K_1\Delta^2\hat{u} + m\Delta(\alpha_t + \tau\alpha_{tt}) &= 0, \\ c(\tau\alpha_{ttt} + \alpha_{tt}) - \beta\Delta\alpha - \beta_1\Delta\alpha_t - m\Delta\hat{u}_t &= 0, \end{aligned}$$

with conditions

$$\rho > 0, \quad \tau > 0, \quad K_1 > 0, \quad c > 0, \quad \beta > 0, \quad \beta_1 > 0, \quad m \neq 0 \quad \text{and} \quad \beta_1 > \beta\tau.$$

These two models are compatible with the finite velocity propagation of thermal and mechanical waves.

The main results for these models are the following: We will prove the well-posedness of the first system as well as the exponential decay of the solutions. Secondly, we study the second model (elastic plate coupled with a Moore-Gibson-Thompson heat equation), showing the well-posedness of the problem, but in this case we see that the decay is not of exponential kind. Additionally, we obtain optimal polynomial rates of decay of type $O(t^{-1})$ for the associated semigroup, implying the optimal rate of decay $O(t^{-2})$ for the first order energy of the system, which is unusual and difficult to obtain by multiplier techniques. To our best knowledge, we can cite [2] where the same rate of decay is obtained for a thermoelastic plate with second sound.

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UNIFORM DECAY FOR THE WAVE EQUATION ON COMPACT RIEMANNIAN MANIFOLD WITH EXPONENTIAL SOURCE TERM

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Abstract

We investigate well-posedness and stability of the wave equation on a 2D compact Riemannian manifold without boundary subject to nonlinearities of arbitrary growth and localized nonlinear dissipation. Employing a novel truncation and limit-based approach, we simultaneously establish the energy identity and the Observability Inequality - both of which are essential for deriving uniform energy decay.

1 Introduction

In this work, we investigate the decay properties of solutions to the wave equation involving nonlinearities of arbitrary growth and locally distributed nonlinear dissipation. More precisely, we analyze the associated initial value problem defined by:

$$\begin{cases} \partial_t^2 u - \Delta_{\mathbf{g}} u + f(u) + a(x) g(\partial_t u) = 0 \text{ in } \mathcal{M} \times (0, +\infty), \\ u(x,0) = u_0(x),\, \partial_t u(x,0) = u_1(x) \text{ in } \mathcal{M}, \end{cases} \tag{1}$$

where $\mathcal{M}$ is a 2-dimensional compact Riemannian manifold without boundary, endowed with the Riemannian metric $\mathbf{g}$. Let $\Delta_{\mathbf{g}}$ denote the Laplace-Beltrami operator on $(\mathcal{M}, \mathbf{g})$, and $\nabla_{\mathbf{g}}$ its corresponding Riemannian connection.

The nonlinearity f is taken as a function in $C^1(\mathbb{R})$, satisfying the following growth condition: for each $\beta > 0$, there exists a constant $C_\beta > 0$ such that$|f^{(i)}(t)| \leq C_\beta e^{\beta t^2}$, for all $t \in \mathbb{R}$ and $i = 0, 1$. Moreover, we assume $\frac{f(t)}{t} \to 0$ as $t \to 0$, and that $\frac{f(t)}{t}$ is strictly increasing in $(0, +\infty)$.

The feedback function $g : \mathbb{R} \to \mathbb{R}$ is continuous and monotonically increasing, satisfying $g(s)s > 0$ for all $s \neq 0$. Moreover, there exist constants $\widetilde{m}, \widetilde{M} > 0$ such that $\widetilde{m} s^2 \leq g(s)s \leq \widetilde{M} s^2$, for all $|s| > 1$.

The non-negative function $a \in C(\mathcal{M})$, modeling localized dissipation, satisfies $a(x) \geq a_0 > 0$ on an open set $\omega \subset \mathcal{M}$ that geometrically controls $\mathcal{M}$; i.e., there exists $T_0 > 0$ such that every unit-speed geodesic intersects ω within time $t < T_0$.

Let $V = \left\{ u \in H^1(\mathcal{M}) \,\middle|\, \int_{\mathcal{M}} u \, d\mathcal{M} = 0 \right\}$. Then V is a Hilbert space endowed with the topology induced by $H^1(\mathcal{M})$. Moreover, the Trudinger–Moser inequality for 2-dimensional compact Riemannian manifolds without boundary (see [4]) holds in V and plays a crucial role in establishing key estimates for the solutions.

Theorem 1.1 (Trudinger–Moser)**.** *Let $\mathcal{M}$ be a 2-dimensional compact Riemannian manifold without boundary. Then*

$$\sup_{\substack{u \in V \\ \|\nabla_{\mathbf{g}} u\|_{L^2(\mathcal{M})} \leq 1}} \int_{\mathcal{M}} e^{\alpha \pi u^2} \, d\mathcal{M} \leq C(\mathcal{M}, \mathbf{g}), \ \forall \alpha \leq 4\pi. \tag{2}$$

2 Main Results

By employing the Trudinger-Moser inequality, the Ambrosetti-Rabinowitz condition, and the solutions to truncated problems, we establish the well-posedness of the problem through semigroup theory.

Theorem 2.1 (Global in time)**.** *Under the Assumptions for f, g and a, then given $(u_0, u_1) \in V \times L^2(\mathcal{M})$, problem* (1) *has a unique global solution $(u, \partial_t u) \in C([0,T]; V \times L^2(\mathcal{M}))$ and $\partial_t^2 u \in L^2(0,T;V')$, for all $T > 0$. Moreover, if we define $F(t) = \int_0^t f(\tau)\, d\tau$ and $E_u(t) = \frac{1}{2}\left[\|\partial_t u(x,t)\|^2_{L^2(\mathcal{M})} + \|\nabla_{\mathbf{g}} u(x,t)\|^2_{L^2(\mathcal{M})}\right] + \int_{\mathcal{M}} F(u(x,t))\, d\mathcal{M}$, then the following energy identity holds for all $0 \le t_1 < t_2 \le T$,*

$$E_u(t_2) + \int_{t_1}^{t_2} \int_{\mathcal{M}} a(x) g(\partial_t u(x,t)) \partial_t u(x,t)\, d\mathcal{M}\, dt = E_u(t_1). \tag{1}$$

Based on the assumptions on ω, the energy identity, the observability inequality for truncated problems, and the unique continuation principle, one can derive the following lemma.

Lemma 2.1 (Observability Inequality)**.** *Let $R > 0$ and $T \ge T_0$. Assuming the structural conditions on f, g, a, and the damping region ω, stated in Theorem 2.1, there exists a constant $C = C(R,T) > 0$ such that, provided that $E_u(0) < R$, every solution u to the problem* (1) *satisfies*

$$E_u(0) \le C \int_0^T \int_{\mathcal{M}} a(x) \left[|\partial_t u(x,t)|^2 + |g(\partial_t u(x,t))|^2\right] d\mathcal{M}\, dt. \tag{2}$$

The principal achievement of this work is the Uniform Energy Decay Theorem, which emerges from the essential roles of the energy identity and the observability inequality.

Theorem 2.2 (Uniform decay)**.** *Let $T_0 > 0$ be as in Lemma 2.1, and let u be the solution to problem* (1)*. Then, for all $t \ge T_0$, the energy satisfies $E_u(t) \le S(\frac{t}{T_0} - 1)$, where $S(t)$ is the solution to the differential equation $\frac{d}{dt} S(t) + q(S(t)) = 0$, $S(0) = E_u(0)$, with q constructed from the properties of the feedback function g, and satisfying the decay condition $\lim_{t \to +\infty} S(t) = 0$.*

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TWO STABILITY RESULTS FOR THE KAWAHARA EQUATION WITH A TIME-DELAYED BOUNDARY CONTROL

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Abstract

In this paper, we consider the Kawahara equation in a bounded interval and with a delay term in one of the boundary conditions. Using two different approaches, we prove that this system is exponentially stable under a condition on the length of the spatial domain. Specifically, the first result is obtained by introducing a suitable energy and using the Lyapunov approach, to ensure that the unique solution of the Kawahara system exponentially decays. The second result is achieved by means of a compactness-uniqueness argument, which reduces our study to prove an observability inequality. Furthermore, the main novelty of this work is to characterize the critical set phenomenon for this equation by showing that the stability results hold whenever the spatial length is related to the Mübius transformations.

1 Introduction

The main concern of this paper is to deal with the Kawahara equation in a bounded domain under the action of a time-delayed boundary control, namely

$$\begin{cases} \partial_t u(t,x) + a\partial_x u(t,x) + b\partial_x^3 u(t,x) - \partial_x^5 u(t,x) + u(t,x)^p \partial_x u(t,x) = 0, & (t,x) \in \mathbb{R}^+ \times \Omega, \\ u(t,0) = u(t,L) = \partial_x u(t,0) = \partial_x u(t,L) = 0, & t > 0, \\ \partial_x^2 u(t,L) = \mathcal{F}(t,h), & t > 0, \\ \partial_x^2 u(t,0) = z_0(t), & t \in \mathcal{T}, \\ u(0,x) = u_0(x), & x \in \Omega. \end{cases} \tag{1}$$

In (1), $\Omega = (0,L)$, where $L > 0$, while $a > 0$ and $b > 0$ are physical parameters. Moreover, $p \in [1,2]$ and $\mathcal{F}(t,h)$ is the delayed control given by

$$\mathcal{F}(t) = \alpha\partial_x^2 u(t,0) + \beta\partial_x^2 u(t-h,0), \tag{2}$$

in which $h > 0$ is the time-delay, α and β are two feedback gains satisfying the restriction

$$|\alpha| + |\beta| < 1. \tag{3}$$

Finally, $\mathcal{T} = (-h,0)$, while u_0 and z_0 are initial conditions.

Thereafter, the functional energy associated to the system (1)-(2) is

$$E(t) = \int_0^L u^2(t,x)dx + h|\beta| \int_0^1 (\partial_x^2 u(t-h\rho,0))^2 d\rho, \ t \geq 0. \tag{4}$$

Now, recall that if $\alpha = \beta = 0$, then the term $\partial_x^2 u(t,0)$ represents a feedback damping mechanism but an extra internal damping is required to achieve the stability of the solutions. Therefore, taking into account the action of the time-delayed boundary control (2) in (1), the following issue will be dealt with in this article:

Does $E(t) \longrightarrow 0$, as $t \to \infty$? If it is the case, can we provide a decay rate?

2 Main Results

Theorem 2.1. *Let $\alpha \neq 0$ and $\beta \neq 0$ be two real constants satisfying (3) and suppose that the spatial length L fulfills*

$$0 < L < \sqrt{\frac{3b}{a}}\pi. \tag{5}$$

Then, there exists $r > 0$ sufficiently small, such that for every $(u_0, z_0) \in H$ with $\|(u_0, z_0)\|_H < r$, the energy of the system (1)-(2), denoted by E and defined by (4) exponentially decays, that is, there exist two positive constants κ and λ such that

$$E(t) \leq \kappa E(0)e^{-2\lambda t},\ t > 0. \tag{6}$$

Here,

$$\lambda \leq \min\left\{\frac{\mu_2}{2h(\mu_2 + |\beta|)}, \frac{3b\pi^2 - r^2 L - L^2 a}{2L^2(1 + L\mu_1)}\mu_1\right\} \tag{7}$$

and

$$\kappa \leq \left(1 + \max\left\{L\mu_1, \frac{\mu_2}{|\beta|}\right\}\right),$$

for $\mu_1, \mu_2 \in (0, 1)$ sufficiently small.

Theorem 2.2. *Assume that α and β satisfy (3), whereas $L > 0$ is taken so that the problem $(\mathcal{N})$ (see [1]) has only the trivial solution. Then, there exists $r > 0$ such that for every $(u_0, z_0) \in H$ satisfying*

$$\|(u_0, z_0)\|_H \leq r,$$

the energy of system (1)-(2), denoted by E and defined by (4), decays exponentially. More precisely, there exist two positive constants ν and κ such that

$$E(t) \leq \kappa E(0)e^{-\nu t}, \quad t > 0.$$

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CONTROL OF A LOTKA-VOLTERRA SYSTEM WITH WEAK COMPETITION

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Abstract

In this talk, we study the controllability of a Lotka-Volterra system with weak competition between two species. Using boundary controls, we identify conditions for steering the system towards coexistence, single-species states, or extinction, depending on domain size, diffusion, and competition. We also show when control is not possible by constructing barrier solutions, and support our results with numerical experiments.

1 Introduction

Our problem is described below

$$\begin{cases} u_t = d_1 u_{xx} + u(1 - u - k_1 v), & (x,t) \in (0,L) \times \mathbb{R}^+ \\ v_t = d_2 v_{xx} + v(a - v - k_2 u), & (x,t) \in (0,L) \times \mathbb{R}^+ \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), & x \in (0,L) \end{cases} \tag{1}$$

where u and v are the population densities of the two species competing in $(0,L)$ and $(u(x,t), v(x,t))$ is the state to be controlled. Note that 1 and a are the carrying capacities of u and v, respectively, and therefore it is natural to constrain the solutions by these values, i.e.

$$0 \le u(x,t) \le 1 \text{ and } 0 \le v(x,t) \le a \text{ for all } (x,t) \in (0,L) \times \mathbb{R}^+. \tag{2}$$

Moreover, we assume $k_1, k_2 < 1$ and this condition results in a *weak competition system.*

We suppose *boundary controls constraints* $c_u(x,t), c_v(x,t) \in L^\infty(\{0,L\} \times \mathbb{R}^+)$,

$$\begin{cases} u(x,t) = c_u(x,t) & (x,t) \in \{0,L\} \times \mathbb{R}^+ \\ v(x,t) = c_v(x,t) & (x,t) \in \{0,L\} \times \mathbb{R}^+ \end{cases} \tag{3}$$

satisfying $0 \le c_u \le 1$ and $0 \le c_v \le a$.

We are interested in controlling system (1) towards the steady-states of (1). The targets to be considered in this work are: a homogeneous state of species coexistence

$$(u^*, v^*) = \left(\frac{1 - k_1 a}{1 - k_1 k_2}, \frac{a - k_2}{1 - k_1 k_2} \right), \tag{4}$$

which only makes sense to us when $k_2 < a < 1/k_1$; the extinction of the species $(0,0)$; the survival of one of the species $(1,0)$ and $(0,a)$; a heterogeneous state of species coexistence for the case $d_1 = d_2 = d$ and $a = 1$,

$$(u^{**}(x), v^{**}(x)) = \left(\left(\frac{1 - k_1}{1 - k_1 k_2} \right) \theta(x), \left(\frac{1 - k_2}{1 - k_1 k_2} \right) \theta(x) \right) \tag{5}$$

where $\theta(x)$ is a specific smooth function.

2 Main Results

The main results are stated below.

Theorem 2.1. *If* $k_2 < a < \dfrac{1}{k_1}$ *then* (1) *is controllable towards* (u^*, v^*) *defined in* (4).

Controllability towards the target (u^*, v^*), regardless of the domain size or parameters, shows that species competition plays a crucial role. Even with high diffusion in a large region, weak competition allows boundary control to reach a coexistence state.

Theorem 2.2. (*i*) *If* $L \le \sqrt{\dfrac{d_2}{a}}\pi$ *or* $k_2 > a$ *then* (1) *is controllable towards* $(1, 0)$.

(*ii*) *If* $L \le \sqrt{d_1}\pi$ *or* $k_1 > \dfrac{1}{a}$ *then* (1) *is controllable towards* $(0, a)$.

This theorem shows how diffusion, competition, and domain size affect species dominance. For fixed parameters, the system can be controlled towards $(1, 0)$ or $(0, a)$ if L is small enough, using static controls. Moreover, if $k_2 > a$ or $k_1 > 1/a$, making coexistence impossible, control towards these states is still achievable regardless of L.

Theorem 2.3. *If* $k_2 < a < 1/k_1$ *and* $L > \max\left\{\sqrt{\dfrac{d_1}{1 - ak_1}}\pi, \sqrt{\dfrac{d_2}{a - k_2}}\pi\right\}$ *then* (1) *is not controllable towards either* $(1, 0)$ *or* $(0, a)$.

Theorem 2.4. (*i*) *If* $L \le \min\left\{\sqrt{d_1}\pi, \sqrt{\frac{d_2}{a}}\pi\right\}$ *then the system* (1) *is controllable towards* $(0, 0)$*;*

(*ii*) *if* $a < \dfrac{1}{k_1}$ *and* $L > \sqrt{\dfrac{d_1}{1 - ak_1}}\pi$ *or* $a > k_2$ *and* $L > \sqrt{\dfrac{d_2}{a - k_2}}\pi$, *then the system* (1) *is not controllable towards* $(0, 0)$.

Theorem 2.5. *If* $a = 1$, $d_1 = d_2 = d > 0$ *and* $L > \sqrt{d}\pi$, *then* (1) *is controllable towards an specific heterogeneous coexistence state* (u^{**}, v^{**}) *defined in* (5).

Biologically, if species share reproduction and diffusion traits, and interact weakly, a large enough L ensures a unique and stable coexistence state vanishing at the boundary. In this controlled case, extinction of either species is prevented, regardless of initial conditions.

This talk is based on reference [1], which was recently submitted for publication.

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LOCAL NULL CONTROLLABILITY FOR THE NAVIER–STOKES SYSTEM IN A NON-CYLINDRICAL DOMAIN

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Abstract

This paper addresses the n–dimensional Navier–Stokes systems in a non-cylindrical domain. The main results establish null controllability with internal controls supported by potentially small subdomains. Firstly, we study the systems by transforming them into an equivalent system defined in a cylinder and solve the control problems within this cylinder (null controllability is achieved under a restriction on the mentioned transformation). In our approach, we employ a Carleman inequality for the Stokes system with variable coefficients. We solve the nonlinear case, we also utilize Liusternik's Inverse Mapping Theorem.

1 Introduction

Let Ω be a bounded open set in $\mathbb{R}^n$ with a boundary Γ of class C^2, and let $\eta(x)$ represent the outer unit normal vector at $x \in \Gamma$. Denote $\mathcal{O}$ as a nonempty open subset of $\mathbb{R}^n$, strictly contained in Ω. Fix an arbitrary real number $T > 0$. Define Σ as the lateral boundary $\Gamma \times (0,T)$ of Q.

Let $k \in C^2([0,\infty))$ be a function such that $k(t) \geq k_0 > 0$ for all $t \geq 0$ and M be a $n \times n$ invertible matrix with real number entries. Consider the family of matrices

$$K(t) = k(t)M, \qquad t \geq 0.$$

For all times $t > 0$, introduce the following sets

$$\Omega_t = \{x = K(t)y \ : \ y \in \Omega\}, \quad \Gamma_t = \partial\Omega_t \quad \text{and} \quad \mathcal{O}_t = \{x = K(t)y \ : \ y \in \mathcal{O}\}.$$

Now, we define the non-cylindrical domain $\widehat{Q}$ contained in $\mathbb{R}^n$

$$\widehat{Q} = \bigcup_{0<t<T} \Omega_t \times \{t\}, \quad \widehat{\Sigma} = \bigcup_{0<t<T} \Gamma_t \times \{t\} \quad \text{and} \quad \widehat{\mathcal{O}} = \bigcup_{0<t<T} \mathcal{O}_t \times \{t\}.$$

We consider the Navier–Stokes system

$$\left| \begin{array}{lll} \widehat{\mathcal{S}}\{\hat{u}, \hat{p}\} = \hat{v}\, 1_{\widehat{\mathcal{O}}} & \text{in} & \widehat{Q}, \\ \text{div}\hat{u} = 0 & \text{in} & \widehat{Q}, \\ \hat{u} = 0 & \text{on} & \widehat{\Sigma}, \\ \hat{u}(0) = \widehat{u}^0 & \text{in} & \Omega_0, \end{array} \right. \tag{1}$$

where

$$\widehat{\mathcal{S}}\{\hat{u}, \hat{p}\} = \hat{u}' + \widehat{u} \cdot \nabla\widehat{u} - \nu_0(||\widehat{u}||^2)\Delta\widehat{u} + \nabla\widehat{p}$$

In the system (1), $\hat{u}$ is the vector field representing fluid velocity, $\nu > 0$ is a constant, Δ is the Laplace operator, $\nabla \hat{p}$ is the pressure gradient, $\hat{v}$ is the control supported in $\widehat{\mathcal{O}} \Subset \widehat{Q}$ and $1_{\widehat{\mathcal{O}}}$ denotes the characteristic function of the set $\widehat{\mathcal{O}}$ and ν_0 is a real function continuously differentiable, with $0 \leq \nu_0 \leq C$ and $|\nu_0'| \leq C$, for some constant $C > 0$. We can consider the system (1) as a simplified model of the Ladyzhenskaya-Smagorinsky system for the turbulence of a fluid, as discussed in [3, **?**].

Definition 1.1. *The system* (1) *is locally null controllable at time* $T > 0$ *if, there exists a* $\delta > 0$ *such that, for any initial data* $\hat{u}^0 \in L^2(\Omega_0)^n$ *with*

$$||\hat{u}^0||_{H_0^1(\Omega_0)^n} \leq \delta$$

there exists a control $\widehat{v} \in L^2(\widehat{\mathcal{O}})^n$ *such that the corresponding solution* $\{\hat{u}, \hat{p}\}$ *of* (1) *satisfies*

$$\hat{u}(T) = 0 \quad in \quad \Omega_T.$$

2 Main Result

The main result is as follows.

Theorem 2.1. *Let* $T > 0$ *be a given time and assume that*

$$k'(t) \geq -\frac{C}{k(t)} \quad \forall t \in [0, T], \tag{2}$$

where the constant $C > 0$ *is independent of* $t \in [0, T]$*. Then, there exists a* $\delta > 0$ *such that, for any initial data* $\hat{u}^0 \in L^2(\Omega_0)^n$ *with*

$$||\hat{u}^0||_{H_0^1(\Omega_0)^n} \leq \delta$$

there exists a control $\widehat{v} \in L^2(\widehat{\mathcal{O}})^n$ *such that the corresponding solution* $\{\hat{u}, \hat{p}\}$ *of* (1) *satisfies*

$$\hat{u}(T) = 0 \quad in \quad \Omega_T.$$

The demonstration of this result is done in [1]. We prove the (2.1) applying Liusternik's Inverse Mapping Theorem in Hilbert spaces, as outlined in [4]. This technique aligns with the approach used in [2, **?**]

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COMMON HYPERCYCLICITY FOR SHIFTS ON THE TREE

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Abstract

In this work, we explore a property very important in the field of Linear Dynamics, the hypercyclicity - in our context for weighted shifts on trees. Throught the Costakis-Sambarino Criterion, we observe what happens when we set some conditions of symmetry on the tree and weight. This criterion is important because it guarantees not only the existence of hypercyclic vectors, but the existence of a set topologically "large" of common hypercyclic vectors for a family of operators. Finally, we observe in the examples studied that the symmetry together with Lipschitz and increasing conditions can result in interesting applications of that criterion.

1 Introduction

In the field of Linear Dynamics, we study the dynamical properties satisfied by a linear map $T : X \to X$ acting on a topological vector space X. One of these notions is the following definition.

Definition 1.1. *An operator $T : X \to X$ is called hypercyclic if there is some $x \in X$ such that the set $\{T^n x, n \in \mathbb{N}\}$ is dense in X. In such a case, x is called a hypercyclic vector for T. The set of hypercyclic vectors for T is denoted by $HC(T)$.*

In 1929, G. D. Birkhoff showed that for a continuous map T on a separable complete metric space X without isolated points, the set $HC(T) \neq \emptyset$ is a dense G_δ subset of X, what became known as Birkhoff transitivity theorem [2, p. 111, 112]. This result is interesting because the existence of even a single element in $HC(T)$ guarantees that this set is very "large" in the topological sense.

Futhermore, on separable Frechét spaces, we can consider a family of hypercyclic operators $(T_\lambda)_{\lambda\in\Lambda}$ and try to ensure that there exists a *common hypercyclic vector* to any T_λ. When Λ is countable, it follows from the Baire category theorem that $\bigcap_{\lambda\in\Lambda} HC(T_\lambda)$ is a residual subset of X, that is, there are many common hypercyclic vectors. However, if Λ is uncountable, the problem is non-trivial. Nevertheless, under some conditions - such as in the Costakis-Sambarino Criterion - the existence of common hypercyclic vectors can still be ensured. Before stating it, it is important to consider for the family of operators that $(\lambda, x) \to T_\lambda(x)$ is continuous and the parameter space Λ is a coutable union of compacts sets.

Theorem 1.1 (Costakis-Sambarino)**.** *Let $\mathcal{D}$ be a set dense in X and assume that, for each $f \in \mathcal{D}$ and each compact interval $K \subset \Lambda$ and each $\lambda, \alpha, \mu \in K$, the following properties hold true:*

(1) There exist $\kappa \in \mathbb{N}$ and a sequence of positive numbers $(c_k)_{k\geq\kappa}$ such that

- *(a)* $\sum_{k=\kappa}^{\infty} c_k < \infty$;
- *(b)* $\|T_\lambda^{n+k} S_\alpha^n(f)\| \leq c_k$, $n \in \mathbb{N}$, $k \geq \kappa$, $\alpha \leq \lambda$;
- *(c)* $\|T_\lambda^n S_\alpha^{n+k}(f)\| \leq c_k$, $n \in \mathbb{N}$, $k \geq \kappa$, $\lambda \leq \alpha$.

(2) Given $\eta > 0$, one can find $\tau > 0$ such that, for all $n \geq 1$,

$$0 \leq \mu - \lambda < \frac{\tau}{n} \Rightarrow \| T_\lambda^n S_\mu^n(f) - f\| < \eta.$$

Then $\bigcap_{\lambda\in\Lambda} HC(T_\lambda)$ *is a dense* G_δ *subset of* X*.*

In this work we study conditions on the tree and weight and we provide a criterion on the existence of common hypercyclic vector for a family of weighted backward shifts on the sequence space of a tree.

introduced by [1, p.12]. Existence and uniqueness results can be found in [2, 3].

2 Main Results

We say that (V, E) is a *directed graph* if V is a nonemppty contable set, whose elements are called *vertices* and $E \subset \{(u, v) \in V \times V : u \neq v\}$ represents the set of *edges*. A *directed tree*, which we denote only by V, is a connected directed graph such that there are no paths that lead back to a vertex already visited, and each vertex $v \in V$ has at most one parent. The *parent* of a vertex $v \in V$, denoted by $par(v)$, is a vertex $u \in V$ for which $(u, v) \in E$. We also define $Chi(u) = \{v \in V : (u, v) \in E\}$ the set of *children* of u. In general, $par^k(v) = par(par^{k-1}(v))$ and $Chi^k(v) = Chi(Chi^{k-1}(v))$, for all $k \geq 2$. Futhermore, if V has at most one vertex without a parent called *root*, denoted by $\mathbf{r}$, V is a *rooted tree*.

Let V be a rooted tree, we enumerate the *generations* with respect to $\mathbf{r}$ by defining $Gen_n = Chi^n(\mathbf{r})$, $n \in \mathbb{N}$. When $u, v \in Gen_n$ we say that u and v are in the same generation. Let V be a rooted directed tree and $(\lambda(v))_{v\in V}$ be a family of nonzero scalars, called *weight*. This weight $(\lambda(v))_{v\in V}$ will be called *symmetric* if $\lambda_u = \lambda_v$ whenever u and v are in the same generation. In addition we will consider the *weighted* c_0-*space* of V, which is $c_0(V) = \{f \in \mathbb{K}^V : \forall\varepsilon > 0, \exists F \subset V \text{finite}, \forall v \in V\backslash F, |f(v)| < \varepsilon\}$. The *backward shift* $B : c_0(V) \to c_0(V)$ is defined on $\mathbb{K}^V$ by

$$(Bf)(v) = \sum_{u\in Chi(v)} f(u), \quad v \in V.$$

Suppose that V is a rooted tree in which each vertex has exactly $N \geq 1$ children, let $X = c_0(V)$ and $\Lambda = (1/N, \infty)$. Assume that the family $(T_\lambda)_{\lambda\in\Lambda}$ is defined, for each λ, by the Rolewicz Operator $T_\lambda = \lambda B$. In this work, we verify that the conclusion of the Costakis-Sambarino Criterion is true for this family. Note that λB is a particular case of a family of operators, where $(\lambda_v)_{v\in V}$ is symmetric. More generally, when considering $(\lambda_v(x))_{v\in V}$ admissible, symmetric and $x \to \lambda_v(x)$ increasing, and satisfies some special Lipschitz condition, it is possible to obtain more a general result, which we explore in this project.

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STRUCTURALLY SPACEABLE SETS AND LINEAR DYNAMICS: GENERAL CRITERION AND RICH SUBSETS BEYOND ε-HYPERCYCLICITY

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Abstract

We prove that the difference set ε-$HC(T) \setminus \delta$-$HC(T)$ is *spaceable* whenever T satisfies the ε-hypercyclicity criterion. Motivated by this, we introduce and characterize the concept of *structurally spaceable* subsets in topological vector spaces in terms of $[\ell_\infty]$-lineability.

1 Introduction

In the context of linear dynamics, the study of large linear or topological structures inside dynamically meaningful sets has attracted considerable attention (see [1] and references therein). In this work, we first show that the set of vectors that are ε-hypercyclic but not δ-hypercyclic is rich from a spaceability perspective. This naturally motivates a refinement of classical spaceability that incorporates convergence structure.

We introduce the concept of *structurally spaceable* subsets, which refines classical spaceability by additionally requiring the existence of a basic sequence.

Definition 1.1. *A subset $A \subset X$ of a topological vector space is* structurally spaceable *if there exists a closed infinite-dimensional subspace $F \subset A \cup \{0\}$ such that F contains a basic sequence.*

Definition 1.2. *Let $T \in \mathcal{L}(X)$ and $\varepsilon \in (0,1)$. A vector $x \in X$ is ε-hypercyclic for T if for every $y \neq 0$, there exists $n \in \mathbb{N}$ such that $\|T^n x - y\| \leq \varepsilon \|y\|$.*

Theorem 1.1 (ε-hypercyclicity criterion [1])**.** *Let X be separable Banach, $T \in \mathcal{L}(X)$, and $\varepsilon \in (0,1)$. Suppose there exist:*

- *a dense set $D \subset X$;*
- *a dense sequence $u(k) \in X$, repeated infinitely often;*
- *a sequence $v(k) \to 0$ in X;*
- *integers $n_k \uparrow \infty$;*

with

- $\|T^{n_k} x\| \to 0$ *for all $x \in D$;*
- $\|T^{n_k} v(k) - u(k)\| \leq \varepsilon \|u(k)\|$.

Then T is δ-hypercyclic for every $\delta > \varepsilon$.

2 Main Results

The first main result: Structurally spaceable sets and dynamic richness

Our first main result highlights the richness of the set of ε-hypercyclic vectors that fail to be δ-hypercyclic for $0 < \delta < \varepsilon$. Such sets naturally emerge in the context of the ε-hypercyclicity criterion, in [1]. What we show below is that these sets are not only nontrivial, but also exhibit deep structural properties in the sense of spaceability.

Theorem 2.1. *Let $T \in \mathcal{L}(X)$ be an operator satisfying the ε-hypercyclicity criterion for some $\varepsilon \in (0,1)$. Fix $0 < \delta < \varepsilon$. Then the set*

$$A := \varepsilon\text{-}HC(T) \setminus \delta\text{-}HC(T)$$

is structurally spaceable.

The second main result: A characterization via ℓ_∞-lineability

We now relate this to a refined form of lineability proposed in [2] and [4], capturing convergence constraints via sequences in ℓ_∞.

Definition 2.1 (see [2, 4])**.** *Let $\mathcal{S} \subset \mathbb{K}^{\mathbb{N}}$. A subset A of a topological vector space X is $[(u_n), \mathcal{S}]$-lineable if for every sequence $(c_n) \in \mathcal{S}$ the series $\sum c_n u_n$ converges in X to an element of $A \cup \{0\}$, where (u_n) is a sequence of linearly independent vectors in X. We say A is $[\mathcal{S}]$-lineable if this holds for some (u_n).*

Theorem 2.2. *Let $X \neq \{0\}$ be a topological vector space. A subset $A \subset X$ is structurally spaceable if and only if there exists a basic sequence $(u_n) \subset X$ such that A is $[(u_n), \ell_\infty]$-lineable.*

Corollary 2.1. *Let X be a Banach space. Then a subset $A \subset X$ is spaceable if and only if there exists a basic sequence $(u_n) \subset X$ such that A is $[(u_n), \ell_\infty]$-lineable.*

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ALMOST POSITIVE POINTWISE LATTICEABILITY IN SEQUENCE SPACES

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Abstract

We introduce the notion of almost positive pointwise latticeable sets in Banach sequence lattices, which is the natural lattice counterpart of some lineability-tye concepts that have been studied recently. General conditions on a set to be almost positive pointwise latticeable are proved. Applications to several classical topics in Banach lattice theory are provided.

1 Introduction

Let X be a Banach sequence space, that is, a Banach space whose elements are sequences, either scalar sequences or sequences belonging to a given Banach space. According to [1], a nonvoid subset A of X is *almost pointwise spaceable* if, for every sequence $\mathbf{x} \in A$, there exists a closed infinite dimensional subspace of X contained in $A \cup \{0\}$ and containing a subsequence of $\mathbf{x}$. When X is a Banach lattice, it is necessary to replace *subspace of* X with *sublattice of* X. Many well studied properties of Banach lattices, as well many relevant classes of operators between Banach lattices, deal with positive sequences instead of arbitrary sequences. So, in the definition above, it is natural to replace *for every sequence* $\mathbf{x} \in A$ with *for every positive sequence* $\mathbf{x} \in A$. In summary, the lattice counterpart of the notion of almost pointwise spaceability is the following:

Definition 1.1. A nonvoid subset A of a Banach sequence lattice X is said to be *almost positive pointwise latticeable* if, for every positive sequence $\mathbf{x} \in A$, there exists a closed infinite dimensional sublattice of X contained in $A \cup \{0\}$ and containing a subsequence of $\mathbf{x}$.

Of course, by a Banach sequence lattice we mean a Banach lattice whose elements are sequences endowed with the coordinatewise ordering. In this work we shall work with the Banach sequence lattice $\ell_\infty(E)$ of bounded E-valued sequences, where E is a Banach lattice, endowed with the coordinatewise ordering and the supremum norm.

In this work, we first prove a general criterion for a subset of $\ell_\infty(E)$ to be almost positive pointwise latticeable. Then we apply this abstract result to concrete situations concerning lattices failing classical properties in Banach lattice theory and operators not belonging to some well studied classes.

Non explained concepts can be found in [1, 2].

2 Main Results

A refinement of the proof of [1, Theorem 2.1], which is, in its turn, a refinement of the proof of the main result of [3], gives the following:

Theorem 2.1. *Let $f\colon E \longrightarrow F$ be a map of homogeneous type from a Banach lattice E to a Banach space F, let A be a subsequence invariant and ℓ_∞-complete subset of $\ell_\infty(E)$, and let τ be a vector topology on F weaker than the norm topology. Consider the sets*

$$\mathcal{C}_1 = \{(x_j)_{j=1}^\infty \in A \colon f(x_j) \overset{\tau}{\not\longrightarrow} 0\}\ ,\ \mathcal{C}_2 = \{(x_j)_{j=1}^\infty \in A \colon (x_j)_{j=1}^\infty \text{ is disjoint and } f(x_j) \overset{\tau}{\not\longrightarrow} 0\}.$$

For $i = 1, 2$, $\mathcal{C}_i \cap \ell_\infty(E)^+ = \emptyset$ or C_i is almost positive pointwise latticeable in $\ell_\infty(E)$.

Corollary 2.1. *If the Banach lattice E fails the positive Schur property, then the sets*

$$\mathcal{C}_1 = \{(x_j)_{j=1}^\infty \in \ell_\infty(E) \colon x_j \xrightarrow{|\sigma|} 0 \text{ and } x_j \not\longrightarrow 0\} \text{ and}$$

$$\mathcal{C}_2 = \{(x_j)_{j=1}^\infty \in \ell_\infty(E) \colon (x_j)_{j=1}^\infty \text{ is disjoint, } x_j \xrightarrow{|\sigma|} 0 \text{ and } x_j \not\longrightarrow 0\}$$

are almost positive pointwise latticeable.

Corollary 2.2. *Let E be a Banach lattice.*
(a) *If E fails the positive Grothendieck property, then the set of weak* null non weakly null E^*-valued sequences is almost positive pointwise latticeable in $\ell_\infty(E^*)$.*
(b) *If E fails the dual positive Schur property, then the set of weak* null (or absolutely weak*-null) disjoint non-norm null E^*-valued sequences is almost positive pointwise latticeable in $\ell_\infty(E^*)$.*

Corollary 2.3. *Let $T \colon E \longrightarrow F$ be an operator from a Banach lattice E to a Banach space F.*
(a) *If T fails to be order weakly compact, then the set of order bounded weakly null sequences $(x_j)_{j=1}^\infty$ in E such that $T(x_j) \not\longrightarrow 0$ in F is almost positive pointwise latticeable in $\ell_\infty(E)$.*
(b) *If T fails to be almost Dunford-Pettis, then the set of disjoint weakly null sequences $(x_j)_{j=1}^\infty$ in E such that $T(x_j) \not\longrightarrow 0$ in F is almost positive pointwise latticeable in $\ell_\infty(E)$.*

Corollary 2.4. *Let $T \colon E \longrightarrow F$ be a positive operator between Banach lattices.*
(a) *If T fails to be M-weakly compact, then the set of norm bounded disjoint sequences $(x_j)_{j=1}^\infty$ in E such that $T(x_j) \not\longrightarrow 0$ in F is almost positive pointwise latticeable in $\ell_\infty(E)$.*
(b) *If T fails to be weak M weakly compact, then the set of disjoint bounded $(x_j)_{j=1}^\infty$ in E such that $T(x_j) \overset{\omega}{\not\longrightarrow} 0$ in F is almost positive pointwise latticeable in $\ell_\infty(E)$.*

Some of the results above improve upon known results and some are the first latticeability-type results in each specific situation.

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ON ORLICZ ℓ_M-SUMS THAT CONTAIN A SUBSPACE WITHOUT THE AP AND SOME PROBLEMS ABOUT THE RELATION OF LIPSCHITZ ISOMORPHISM BETWEEN SUBSPACES IN A SEPARABLE BANACH SPACE

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Abstract

We study the problem of determining whether there exists a nontrivial twisted Hilbert space that contains a subspace without the approximation property (AP). The most important nontrivial twisted Hilbert space is the Kalton-a Peck space Z_2, of which the Orlicz sequence space ℓ_{M_2} is known to be a subspace. Our goal is to verify whether ℓ_{M_2} contains a subspace without the AP. In this direction, we show that for suitable choices of $k_n \to \infty$, if $|\alpha| > 2$, the sum $\left(\bigoplus_{n=1}^{\infty} \ell_{M_\alpha}^{k_n}\right)_{M\alpha}$ contains a subspace without the AP, where M_α denotes the Orlicz function $t \mapsto t^2|\ln t|^\alpha$. This is done by adapting the construction in [6], where a subspace of ℓ_p (with $p \neq 2$) without the AP is found. Independently, we investigate the classification of analytic equivalence relations modulo Borel reducibility among subspaces of a separable Banach space in the nonlinear setting. We formally show that certain ergodic-type results, such as those in [1], still hold in a Lipschitz context. We motivate the study of this topic by proving that, if $1 \leq p < \infty$, then the space ℓ_p contains continuum many pairwise non-Lipschitz isomorphic subspaces.

1 Introduction

A Banach space has the *approximation property* (AP) if the identity operator on it can be uniformly approximated on compact subsets by linear operators of finite rank. The AP has proven to be a remarkable tool for solving classification problems in Banach space geometry and descriptive set theory in the separable setting. In 1978, using some combinatorial techniques, Szankowski [6] proved that ℓ_p, for $1 \leq p < 2$, contains a subspace without the AP. He generalized this result for *non-near Hilbert spaces*: that is, Banach spaces for which the supremum of its types coincides with the infimum of its cotypes and both are equal to 2. In the realm of near Hilbert spaces, we can distinguish *twisted Hilbert spaces*; that is, Banach spaces containing a copy of ℓ_2 such that the induced quotient is also isomorphic to ℓ_2.

Motivated by the classification of the structure of twisted Hilbert spaces, we study the problem of determining whether there exists a nontrivial twisted Hilbert space that contains a subspace without the AP. We begin by addressing the most important nontrivial twisted Hilbert space: the Kalton-Peck space Z_2 [5], of which the Orlicz sequence space ℓ_{M_2} is known to be a subspace, where M_2 denotes the Orlicz function $t \mapsto t^2|\ln t|^2$. Our goal is to verify whether ℓ_{M_2} contains a subspace without the AP. We then consider the family of examples of nontrivial twisted Hilbert spaces $\ell_2(\phi)$, constructed from certain unbounded Lipschitz functions ϕ, as well as twisted Hilbert spaces obtained through interpolation schemes [4]. This problem is related to the question raised by Johnson and Szankowski [3] about whether $d_n(X) = o(\log n)$ implies that X has a subspace without the AP, where $d_n(X)$ denotes the supremum of the distances between n-dimensional subspaces of X and the Euclidean space ℓ_2^n.

In a somewhat different, yet related, line of research, the solution to the homogeneous space problem, obtained from the combination of the works of Gowers, Maurey, Komorowski and Tomczak-Jaegermann, establishes that if the equivalence relation of isomorphism among infinite-dimensional subspaces of a Banach space X has only one class, then X must be isomorphic to ℓ_2. In this context, G. Godefroy posed the following question: What is the possible number of non-isomorphic subspaces of a Banach space that is not isomorphic to ℓ_2? This question can also be posed in the setting of classifying analytic equivalence relations modulo Borel reducibility. This was investigated in the context of descriptive set theory by Ferenczi and Rosendal [2] with the introduction of the notion of *ergodic* Banach spaces. Let E_0 be the equivalence relation in $2^{\mathbb{N}}$, where two sequences are related whenever they eventually coincide, i.e., xE_0y if and only if there exists $N \in \mathbb{N}$ such that $x(n) = y(n)$ for all $n > N$. A separable Banach space X is called ergodic if the relation E_0 is Borel reducible to the isomorphism relation between the subspaces of X. In particular, an ergodic space must contain continuum many pairwise non-isomorphic subspaces. In [2], the authors conjectured that ℓ_2 is the only separable Banach space that is not ergodic. In [1] a criterion for ergodic spaces was obtained via a "Cantorization" process of Banach spaces without the AP. The main result shows that every separable non-ergodic Banach space is near-Hilbert. In the nonlinear setting, we formally show that certain ergodic-type results, such as those in [1], still hold in a Lipschitz context.

2 Main Results

Our main results are the following.

Theorem 2.1. *Let $|\alpha| > 2$. For appropriately chosen $k_n \to \infty$, the space $\left(\bigoplus_{n=1}^{\infty} \ell_{M_\alpha}^{k_n}\right)_{M_\alpha}$ contains a subspace without the AP.*

Theorem 2.2. *Let $1 \leq p < \infty$, $p \neq 2$. The space ℓ_p contains continuum many pairwise non-Lipschitz isomorphic subspaces.*

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DISTRIBUTIONAL CHAOS FOR WEIGHTED COMPOSITION OPERATORS ON $L^P(\mu)$ SPACES

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Abstract

We study the property of distributional chaos for weighted composition operators on $L^p(\mu)$ spaces. As applications of the general theorem, we characterize the distributionally chaotic weighted shifts on the classical Banach sequence spaces $\ell^p(\mathbb{N})$ and $\ell^p(\mathbb{Z})$ $(1 \leq p < \infty)$.

1 Introduction

Throughout this text we fix a real number $p \in [1, \infty)$ and an arbitrary positive measure space $(X, \mathcal{B}, \mu)$. $L^p(\mu)$ denotes the classical Banach space over $\mathbb{K}$ $(= \mathbb{R}$ or $\mathbb{C})$ of all $\mathbb{K}$-valued p-integrable functions on $(X, \mathcal{B}, \mu)$ endowed with the p-norm. We also consider a measurable map $w : X \to \mathbb{K}$ such that

$$\varphi \cdot w \in L^p(\mu) \ \text{ for all } \varphi \in L^p(\mu).$$

Given a bimeasurable map $f : X \to X$ (i.e., $f(B) \in \mathcal{B}$ and $f^{-1}(B) \in \mathcal{B}$ whenever $B \in \mathcal{B}$), it is not difficult to show that the *weighted composition operator*

$$C_{w,f}(\varphi) = (\varphi \circ f) \cdot w$$

is a well-defined bounded linear operator on $L^p(\mu)$ if and only if there is a constant $c \in (0, \infty)$ such that

$$\int_B |w|^p d\mu \leq c\, \mu(f(B)) \ \text{ for every } B \in \mathcal{B}. \tag{1}$$

In this text we always assume (1). Moreover, we associate to w and f the following sequence of positive measures on $(X, \mathcal{B})$:

$$\mu_1(B) = \int_B |w|^p d\mu, \quad \mu_n(B) = \int_B |w \circ f^{n-1}|^p \cdots |w \circ f|^p |w|^p d\mu \quad (B \in \mathcal{B}, n \geq 2).$$

Definition 1.1. (Distributional chaos) Given a metric space M, a map $f : M \to M$ is said to be *distributionally chaotic* if there exist an uncountable set $\Gamma \subset M$ and $\varepsilon > 0$ such that each pair (x, y) of distinct points in Γ is a *distributionally chaotic pair for* f, in the sense that

$$\liminf_{k\to\infty} \frac{\text{card}\,(\{n \in \{1, \cdots, k\} : d(f^n(x), f^n(y)) < \varepsilon\})}{k} = 0$$

and

$$\limsup_{k\to\infty} \frac{\text{card}\,(\{n \in \{1, \cdots, k\} : d(f^n(x), f^n(y)) < \tau\})}{k} = 1 \ \text{ for all } \tau > 0,$$

where $\text{card}(A)$ denotes the cardinality of the set A.

This concept of chaotic behavior was introduced by Schweizer and Smítal [2] as a strengthening of the concept of Li-Yorke chaos and is closely related with the fundamental notion of topological entropy.

2 Main Results

In the results below, $\overline{\text{dens}}(D)$ denotes the *upper density* of a set $D \subset \mathbb{N}$, which is given by

$$\overline{\text{dens}}(D) = \limsup_{n\to\infty} \frac{\text{card}(D \cap [1,n])}{n}.$$

Theorem 2.1. *A weighted composition operator $C_{w,f}$ on $L^p(\mu)$ is distributionally chaotic if and only if there exists a nonempty countable family $(B_i)_{i\in I}$ of measurable sets of finite positive μ-measure such that the following properties hold:*

(a) *There exists a set $D \subset \mathbb{N}$ with $\overline{\text{dens}}(D) = 1$ such that*

$$\lim_{n\in D} \mu_n(f^{-n}(B_i)) = 0 \quad \text{for all } i \in I.$$

(b) *There exist $\varepsilon > 0$ and an increasing sequence $(N_k)_{k\in\mathbb{N}}$ of positive integers such that, for each $k \in \mathbb{N}$, there are $r \in \mathbb{N}$, $i_1, \dots, i_r \in I$ and $b_1, \dots, b_r \in (0,\infty)$ with*

$$\text{card}\Big\{1 \le n \le N_k : \frac{b_1\mu_n(f^{-n}(B_{i_1})) + \cdots + b_r\mu_n(f^{-n}(B_{i_r}))}{b_1\mu(B_{i_1}) + \cdots + b_r\mu(B_{i_r})} > k\Big\} \ge \varepsilon N_k.$$

As applications of the above theorem, we have the following characterizations of distributional chaos for unilateral and bilateral weighted shifts.

Corollary 2.1. *A weighted shift B_w on $\ell^p(\mathbb{N})$ is distributionally chaotic if and only if there exist $\varepsilon > 0$ and an increasing sequence $(N_k)_{k\in\mathbb{N}}$ of positive integers such that, for each $k \in \mathbb{N}$, there are $r \in \mathbb{N}$, $i_1, \dots, i_r \in \mathbb{N}$ and $b_1, \dots, b_r \in (0,\infty)$ with*

$$\text{card}\Big\{1 \le n \le N_k : \frac{1}{b_1 + \cdots + b_r} \sum_{1\le j\le r, i_j > n} b_j |w_{i_j-n} \cdots w_{i_j-1}|^p > k\Big\} \ge \varepsilon N_k.$$

Corollary 2.2. *A weighted shift B_w on $\ell^p(\mathbb{Z})$ is distributionally chaotic if and only if there exists a set $S \subset \mathbb{Z}$ such that the following properties hold:*

(a) *There exists $D \subset \mathbb{N}$ with $\overline{\text{dens}}(D) = 1$ and $\lim_{n\in D} |w_{i-n} \cdots w_{i-1}| = 0$ for all $i \in S$.*

(b) *There exist $\varepsilon > 0$ and an increasing sequence $(N_k)_{k\in\mathbb{N}}$ of positive integers such that, for each $k \in \mathbb{N}$, there are $r \in \mathbb{N}$, $i_1, \dots, i_r \in S$ and $b_1, \dots, b_r \in (0,\infty)$ with*

$$\text{card}\Big\{1 \le n \le N_k : \frac{b_1|w_{i_1-n} \cdots w_{i_1-1}|^p + \cdots + b_r|w_{i_r-n} \cdots w_{i_r-1}|^p}{b_1 + \cdots + b_r} > k\Big\} \ge \varepsilon N_k.$$

All the above-mentioned results are contained in Subsection 4.1 of [1].

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THE SHARP CONSTANTS IN THE REAL ANISOTROPIC LITTLEWOOD'S 4/3 INEQUALITY

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Abstract

The real anisotropic LITTLEWOOD's 4/3 inequality is an extension of a famous result obtained in 1930 by J. E. LITTLEWOOD. It asserts that, for $a, b \in (0, \infty)$, the following conditions are equivalent:

- There is an optimal constant $\mathsf{L}^{\mathbb{R}}_{a,b} \in [1, \infty)$ such that
$$\left(\sum_{k=1}^{\infty} \left(\sum_{j=1}^{\infty} \left| A\left(\mathbf{e}^{(k)}, \mathbf{e}^{(j)}\right) \right|^{a} \right)^{\frac{b}{a}} \right)^{\frac{1}{b}} \leq \mathsf{L}^{\mathbb{R}}_{a,b} \cdot \|A\|$$
for every continuous bilinear form $A\colon c_0 \times c_0 \to \mathbb{R}$.
- The values a, b satisfy $a, b \geq 1$ and $\frac{1}{a} + \frac{1}{b} \leq \frac{3}{2}$.

Several authors have obtained the values of $\mathsf{L}^{\mathbb{R}}_{a,b}$ for diverse pairs (a, b). In this talk I will show how to obtain the complete list of such optimal values, as well as new estimates for $\mathsf{L}^{\mathbb{C}}_{a,b}$ (the analog for continuous $\mathbb{C}$-bilinear forms), which are exact in several cases. If time permits, I will sketch the proof of a variant of KHINCHIN's inequality for STEINHAUS variables, which involves the values $\mathsf{L}^{\mathbb{C}}_{1,r}$, as well as some estimates for the (q, s)-cotype constants of the spaces $\ell_1(\mathbb{K})$ (with $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) in terms of the values $\mathsf{L}^{\mathbb{R}}_{1,q}$.

1 Introduction

Let $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, and let $c_0 = c_0(\mathbb{K})$. LITTLEWOOD's 4/3 inequality [1] asserts that there is a constant $\mathsf{L}_{\mathbb{K}} \in [1, \infty)$ such that
$$\left(\sum_{k,j=1}^{\infty} \left| A\left(\mathbf{e}^{(k)}, \mathbf{e}^{(j)}\right) \right|^{\frac{4}{3}} \right)^{\frac{3}{4}} \leq \mathsf{L}_{\mathbb{K}} \cdot \|A\|$$
for every continuous bilinear form $A\colon c_0 \times c_0 \to \mathbb{K}$, where $\|\cdot\|$ is the usual norm given by
$$\|A\| = \sup\left\{ |A(\mathbf{x}, \mathbf{y})| \colon \mathbf{x}, \mathbf{y} \in c_0 \;\; ; \;\; \|\mathbf{x}\|_{c_0}, \|\mathbf{y}\|_{c_0} \leq 1 \right\},$$
and $\left(\mathbf{e}^{(n)}\right)_{n \geq 1}$ is the sequence of canonical vectors of c_0. It is well known that the exponent 4/3 is optimal, as well as the value $\mathsf{L}_{\mathbb{R}} = \sqrt{2}$ (for the latter, see [2]). For complex scalars we only know that the corresponding optimal value satisfies $\mathsf{L}_{\mathbb{C}} \leq \frac{2}{\sqrt{\pi}}$.

However, the optimality feature of the exponent 4/3 is related to the specific configuration of this inequality. A key issue in the theory has been to investigate optimality ranges for summing inequalities involving the so-called *anisotropic exponents*. More specifically, given $a, b \in (0, \infty]$, our objective is to control the quantity
$$\left(\sum_{k=1}^{\infty} \left(\sum_{j=1}^{\infty} \left| A\left(\mathbf{e}^{(k)}, \mathbf{e}^{(j)}\right) \right|^{a} \right)^{\frac{b}{a}} \right)^{\frac{1}{b}} := \left(\sum_{k=1}^{\infty} \left[\left(\sum_{j=1}^{\infty} \left| A\left(\mathbf{e}^{(k)}, \mathbf{e}^{(j)}\right) \right|^{a} \right)^{\frac{1}{a}} \right]^{b} \right)^{\frac{1}{b}} \tag{✠}$$

for all norm-1 continuous bilinear form $A\colon c_0 \times c_0 \to \mathbb{K}$ (by convention, for $s = \infty$ we define

$$\Big(\sum_j |x_j|^s\Big)^{1/s} = \sup_j |x_j|).$$

Denoting the supremum of the values (✠) by $\mathsf{L}^{\mathbb{K}}_{a,b} \in [1,\infty]$, we have [3, Theorem 5.1]

$$\mathsf{L}^{\mathbb{K}}_{a,b} < \infty \iff a,b \geq 1 \quad \text{and} \quad \frac{1}{a} + \frac{1}{b} \leq \frac{3}{2}\,.$$

2 Main Results

Theorem 2.1. *For any $a,b \in [1,\infty]$ with $\frac{1}{a}+\frac{1}{b} \leq \frac{3}{2}$ we have $\mathsf{L}^{\mathbb{R}}_{a,b} = 2^{\max\{0,\frac{1}{a}+\frac{1}{b}-1\}}$ and $\mathsf{L}^{\mathbb{C}}_{a,b} \leq \big(\frac{4}{\pi}\big)^{\max\{0,\frac{1}{a}+\frac{1}{b}-1\}}$. In particular, $\mathsf{L}^{\mathbb{R}}_{a,b} = \mathsf{L}^{\mathbb{C}}_{a,b} = 1$ whenever $\frac{1}{a}+\frac{1}{b} \leq 1$.*

Proposition 2.1 (Khinchin-type inequality for Steinhaus variables)**.** *Let $r \in [2,\infty]$. For $N \geq 1$, let $(\varepsilon_1,\ldots,\varepsilon_N)$ be a tuple of independent and identically distributed random variables with uniform distribution on the complex unit circle. Then for any $a_1,\ldots,a_N \in \mathbb{C}$ we have*

$$\left(\sum_{j=1}^N |a_j|^r\right)^{\frac{1}{r}} \leq \mathsf{L}^{\mathbb{C}}_{1,r} \cdot \mathbb{E}\left|\sum_{j=1}^N a_j\varepsilon_j\right|.$$

Moreover, the value $\mathsf{L}^{\mathbb{C}}_{1,r}$ is optimal.

Proposition 2.2. *Given $q \in [2,\infty]$ and $s \in (0,\infty)$, let $\mathsf{C} \in [1,\infty)$ be the best constant such that, for all $N \geq 1$ and any $x_1,\ldots,x_N \in \ell_1 = \ell_1(\mathbb{K})$,*

$$\left(\sum_{k=1}^N \|x_k\|_{\ell_1}^q\right)^{\frac{1}{q}} \leq \mathsf{C}\cdot\left(\int_0^1 \left\|\sum_{j=1}^N r_j(t)\cdot x_j\right\|_{\ell_1}^s \mathrm{d}t\right)^{\frac{1}{s}}.$$

Then we have $2^{\frac{1}{q}} \leq \mathsf{C} \leq 2^{\frac{1}{q}+\max\{\frac{1}{s}-1,0\}}$. In particular, $\mathsf{C} = 2^{\frac{1}{q}}$ for $s \in [1,\infty)$.

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ENTROPY NUMBERS OF POLYNOMIALS AND HOLOMORPHIC FUNCTIONS

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Abstract

We study entropy numbers and box dimension of homogeneous polynomials and holomorphic functions between Banach spaces. First, we see that entropy numbers and box dimensions of subsets of Banach spaces are related. Furthermore, we relate the entropy numbers of a holomorphic function to those of the polynomials of its Taylor series expansion. As a consequence, if the box dimension of the image of a ball by a holomorphic function f is finite, then the entropy numbers of the polynomials in the Taylor series expansion of f at any point of the ball belong to ℓ_p for every $p > 1$.

1 Introduction

We study the *compactness* of the image of homogeneous polynomials and holomorphic functions between Banach spaces. Let E and F be Banach spaces, $U \subset E$ an open set and $f\colon U \to F$ a holomorphic mapping. Whenever f maps a ball $B \subset U$ onto a relatively compact set, we use entropy numbers or box dimension to *measure* the compactness of $f(B)$. For $x_0 \in U$, let $P_m f(x_0)$ be the m-homogeneous polynomial of the Taylor series expansion of f at x_0. This work was originally motivated by the following question: given $\varepsilon > 0$ and some ball $B \subset U$, can we relate the *degree of compactness* (in terms of entropy numbers or box dimension) of $f(B)$ and that of $P_m f(x_0)(B)$? Similar questions were addressed in [1, 2] for other ways of measuring the compactness of a set. Also, entropy numbers and, in general, the theory of s-numbers and quasi s-numbers of multilinear operators was treated in [3, 4].

We will see that entropy numbers and box dimension are closely related. Indeed, Proposition 2.1 essentially states that, for a connected set K, the box dimension of K is finite if and only if the entropy numbers of K decay exponentially.

Finally, for a given holomorphic function $f\colon U \to F$, $x_0 \in U$ and $\varepsilon > 0$ we obtain in Lemma 2.1 a relationship between the entropy numbers $e_n(f(x_0 + \varepsilon B_E))$ and $e_N(P_m f(x_0)(B_E))$, where n and N are related (here, B_E is the unit ball of E). As a consequence, we see in Theorem 2.1 that if the (upper) box dimension of $f(x_0 + \varepsilon B_E)$ is finite, then the sequence $(e_n(P_m f(x_0)(B_E)))_n$ belongs to ℓ_p for every $p > 1$.

2 Main Results

A possible way to refine the concept of compactness in a Banach space is via the so-called entropy numbers. Recall that the *n-th entropy number* $\mathcal{E}_n(K)$ of a set K of a metric space (X, d) is defined as

$$\mathcal{E}_n(K) \colon = \inf \left\{ \varepsilon > 0 \colon \exists\, x_1, \ldots, x_n \in X \colon K \subset \bigcup_{i=1}^{n} B_X(x_i, \varepsilon) \right\},$$

where $B_X(x,\varepsilon)=\{\widetilde{x}\in X\colon d(x,\widetilde{x})<\varepsilon\}$. The *n-th dyadic entropy numbers* of K are given by $e_n(K)=\mathcal{E}_{2^{n-1}}(K)$. A subset K of a Banach space is relatively compact if and only if the sequence $(e_n(K))_{n\in\mathbb{N}}\in c_0$. Stronger conditions on the decay rate of $(e_n(K))_{n\in\mathbb{N}}\in c_0$ lead to stronger versions of compactness.

The concept of covering numbers is closely related to that of entropy numbers. For a bounded set $L\subset X$ and $\varepsilon>0$, the *ε-covering number* $N(L,\varepsilon)$ is given by

$$N(L,\varepsilon)\colon =\min\left\{n\in\mathbb{N}\colon \exists\ x_1,\dots,x_n\in X\colon L\subset\bigcup_{i=1}^{n}B_X(x_i,\varepsilon)\right\}.$$

Also, the *upper* and *lower box counting dimension of* L are given by

$$\overline{\dim}_B L=\limsup_{\varepsilon\to 0^+}\frac{\log N(L,\varepsilon)}{-\log(\varepsilon)}\quad\text{and}\quad \underline{\dim}_B L=\liminf_{\varepsilon\to 0^+}\frac{\log N(L,\varepsilon)}{-\log(\varepsilon)}.$$

Proposition 2.1. *Let (X,d) be a metric space and $L\subset X$ a connected and totally bounded set. Then, $\overline{\dim}_B L<\infty$ if and only if $\limsup_{n\to\infty} e_n(L)^{1/n}<1$.*

For $a\in\mathbb{R}$, we write $\lceil a\rceil=\min\{k\in\mathbb{Z}|k\geq a\}$.

Lemma 2.1. *Let E and F be Banach spaces, $U\subset E$ be an open set, $x_0\in U$ and $\varepsilon>0$ be such that $x_0+\varepsilon B_E\subset U$. For an holomorphic function $f\colon U\to F$ and $n,m\in\mathbb{N}$, the following inequality holds*

$$e_{(n-1)C_n+1}(P_m f(x_0)(\varepsilon B_E))\leq 2e_n(f(x_0+\varepsilon B_E))$$

where $C_n=\left\lceil\dfrac{C}{e_n(f(x_0+\varepsilon B_E))}\right\rceil$ for some positive constant $C=C(f)$ which depends only on f.

Theorem 2.1. *Let E and F be Banach spaces, $U\subset E$ be an open set, $x_0\in U$ and $\varepsilon>0$ be such that $x_0+\varepsilon B_E\subset U$. Let $f\colon U\to F$ be a holomorphic function such that $\overline{\dim}_B f(x_0+\varepsilon B_E)<\infty$. Then, $(e_n(P_m f(x_0)(B_E))_{n\in\mathbb{N}}\in\ell_p$ for every $p>1$ and every $m\in\mathbb{N}$.*

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ARENS EXTENSIONS OF DISJOINTNESS PRESERVING MULTILINEAR OPERATORS ON RIESZ SPACES

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Abstract

Let $A : E_1 \times \cdots \times E_m \to F$ be a regular disjointness preserving m-linear operator. We prove that all Arens extensions of A are disjointness preserving if either A has finite rank or the spaces are Banach lattices and F^* has a Schauder basis consisting of disjointness preserving functionals.

1 Introduction

Recent studies on multilinear disjointness preserving operators were conducted in [4, 5]. Related issues concerning for Arens extensions, such as Riesz multimorphisms and order continuous multilinear operators were studied in [2, 3]. Given Riesz spaces $E_1, \ldots, E_m, F$, the space of all regular m-linear operators from $E_1 \times \cdots \times E_m$ to F is denoted by $\mathcal{L}_r(E_1, \ldots, E_m; F)$. When $F = \mathbb{R}$, we write $\mathcal{L}_r(E_1, \ldots, E_m)$. For a Riesz space E, we denoted $E^\sim$ the order dual of E and the topological dual of a Banach lattice E by E^*, thus, $E^{\sim\sim} = (E^\sim)^\sim$ and $E^{**} = (E^*)^*$.

The set of all permutations of $\{1, \ldots, m\}$ is denoted by S_m. Let $A \in \mathcal{L}_r(E_1, \ldots, E_m; F)$ disjointness preserving. In this work, we establish conditions on A or on the Banach lattice F^*, so that the Arens extensions of A, $AR_m^\rho(A) \colon E_1^{\sim\sim} \times \cdots \times E_m^{\sim\sim} \longrightarrow F^{\sim\sim}, \rho \in S_m$, are also disjointness preserving. It is know that if A is positive, then A is disjointness preserving if and only if it is a Riesz multimorphisms. The symbol $x \perp y$ means that the vectors x and y of a Riesz space are disjoint, that is, the infimum of $\{|x|, |y|\}$ is zero (in notation $|x| \wedge |y| = 0$). For more details, see [1].

2 Main Results

Let $m \in \mathbb{N}$ and $\rho \in S_m$ be given. For $i = 1, \ldots, m-1$, and $x''_{\rho(i)} \in E^{\sim\sim}_{\rho(i)}$, consider the operator

$$\overline{x''_{\rho(i)}} \colon \mathcal{L}_r(E_{\rho(i)}, \ldots, E_{\rho(m)}) \longrightarrow \mathcal{L}_r(E_{\rho(i+1)}, \ldots, E_{\rho(m)}) \ , \ \overline{x''_{\rho(i)}}(B) = x''_{\rho(i)} \circ B^i,$$

where $B^i \colon E_{\rho(i+1)} \times \cdots \times E_{\rho(m)} \longrightarrow E^\sim_{\rho(i)}$ is given by $B^i(x_{i+1}, \ldots, x_m)(x_i) = B(x_i, x_{i+1}, \ldots, x_m)$ for all $x_j \in E_{\rho(j)}, j = i, \ldots, m$. In the case $i = m$, we define $\overline{x''_{\rho(m)}} \colon E^\sim_{\rho(m)} \longrightarrow \mathbb{R}$ by $\overline{x''_{\rho(m)}} = x''_{\rho(m)}$.

Given an m-linear form $C \colon E_1 \times \cdots \times E_m \longrightarrow \mathbb{R}$, to each permutation $\rho \in S_m$ we consider the m-linear form $C_\rho \colon E_{\rho(1)} \times \cdots \times E_{\rho(m)} \longrightarrow \mathbb{R}\ ,\ C_\rho(x_1, \ldots, x_m) = C(x_{\rho^{-1}(1)}, \ldots, x_{\rho^{-1}(m)})$.

The Arens extension of a regular m-linear operator $A \colon E_1 \times \cdots \times E_m \longrightarrow F$ with respect to the permutation ρ is defined as the m-linear operator $AR_m^\rho(A) \colon E_1^{\sim\sim} \times \cdots \times E_m^{\sim\sim} \longrightarrow F^{\sim\sim}$ given by $AR_m^\rho(A)(x''_1, \ldots, x''_m)(y') = \left(\overline{x''_{\rho(m)}} \circ \cdots \circ \overline{x''_{\rho(1)}}\right)((y' \circ A)_\rho)$ for all $x''_1 \in E_1^{\sim\sim}, \ldots, x''_m \in E_m^{\sim\sim}$ and $y' \in F^\sim$. The first result on Arens extensions that preserve disjointness for bilinear operators is the following.

Definition 2.1. *An m-linear operator $A \colon E_1 \times \cdots \times E_m \to F$ is said to preserve disjointness if, for all $j \in \{1, \ldots, m\}$ and $x_i \in E_i^+$, $i \neq j$, the linear operator $A_j \colon E_j \longrightarrow F, A_j(x_j) = A(x_1 \ldots, x_m)$ preserves disjointness. That is, if $x_j \perp z_j$ in E_j then $A_j(x_j) \perp A_j(z_j)$ in F.*

Clearly, the vectors $x_i \in E^+$ in Definition 2.1, can be replaced by vectors not necessarily positive.

Proposition 2.1. *Let E_1, E_2, F be Riesz spaces with F Archimedean, and let $A\colon E_1 \times E_2 \longrightarrow F$ be a regular bilinear operator. If A is disjointness preserving, then, for $\rho \in S_2$ and $x_1 \in E_1, x_2 \in E_2$, the operators $AR_2^\rho(A)(J_{E_1}(x_1), \cdot)\colon E_2^{\sim\sim} \longrightarrow F^{\sim\sim}$, $AR_2^\rho(A)(J_{E_1}(x_1), \cdot)(y_2'') = AR_2^\rho(A)(J_{E_1}(x_1), y_2'')$ and $AR_2^\rho(A)(\cdot, J_{E_2}(x_2))\colon E_1^{\sim\sim} \longrightarrow F^{\sim\sim}$, $AR_2^\rho(A)(\cdot, J_{E_2}(x_2))(x_2'') = AR_2^\rho(A)(x_1'', J_{E_2}(x_2))$, are disjointness preserving.*

As in [2, p. 11], a map taking values in a Riesz space is said to have finite rank if the sublattice generated by its range is finite dimensional.

Theorem 2.1. *Let $E_1, \ldots, E_m, F$ be Riesz space with F Archimedean and let $A\colon E_1 \times \cdots \times E_m \longrightarrow F$ be a finite rank regular m-linear operator. If A is disjointness preserving, then all Arens extensions $AR_m^\rho(A)$ of A, $\rho \in S_m$, are disjointness preserving.*

Proposition 2.2. *Let $E_1, \ldots, E_m, F$ be Banach lattices, let $A \in \mathcal{L}_r(E_1, \ldots, E_m; F)$ be a disjointness preserving operator, and let $\rho \in S_m$ be given. Then:*
(a) *$y^{***} \circ AB_m^\rho(A)$ is a disjointness preserving m-linear operator for every weak*-continuous disjointness preserving functional $y^{***} \in F^{***}$.*
(b) *Fix $i \in \{1, \ldots, m\}$ and let $w_i^{**}, z_i^{**} \in E_i^{**}$ be such that $w_i^{**} \perp z_i^{**}$. Then*

$$AB_m^\rho(A)(x_1^{**}, \ldots, w_i^{**}, \ldots, x_m^{**})(y^*) \perp AB_m^\rho(A)(x_1^{**}, \ldots, z_i^{**}, \ldots, x_m^{**})|)(y^*) \text{ in } \mathbb{R}$$

for all $x_j^{**} \in E_j^{**}, j \neq i$ and any $y^* \in \overline{\text{span}}\{\varphi \in F^* : \varphi \text{ is disjointness preserving}\}$.

Corollary 2.1. *Let F be a Dedekind complete Banach lattice such that F^* has a Schauder basis consisting of disjointness preserving linear functionals. Then all Arens extensions of any F-valued disjointness preserving regular multilinear operator are disjointness preserving.*

Corollary 2.1 holds for F: c_0, ℓ_p with $1 < p < \infty$, Tsirelson's original space T^* and its dual T, Schreier's space S, and the predual $d_*(w, 1)$ of the Lorentz sequence space $d(w, 1)$

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AN ABSTRACT HARDY-LITTLEWOOD METHOD AND APPLICATIONS

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Abstract

A continuous linear operator between Banach spaces $u : X \to Y$ is absolutely summing when $(\|u(x_j)\|)_{j\in\mathbb{N}} \in \ell_1$ whenever $(x_j)_{j\in\mathbb{N}}$ is unconditionally summable. The theory of absolutely summing operators has its origins in the 50s with Grothendieck's resume, but it was only in 1966-67 that the class of summing operators was presented in its modern form.

The success of the linear theory of absolutely summing operators motivated the emergence of a non linear theory. In 1983 A. Pietsch initiated a research program sketching the roots of the multilinear theory. Now, the multilinear theory of absolutely summing operators is a very fruitful field of nonlinear functional analysis with important connections with other fields.

In this work, we present a Frederic Bayart's article [2], where he discusses the multiple summability of a multilinear map $T : X_1 \times \cdots \times X_m \to Y$ when we have information on the summability of the maps it induces on each coordinate. His methods have applications to inclusion theorems for multiple summing multilinear mappings.

1 Introduction

To prove their bilinear inequality on ℓ_p-spaces in [3], Hardy and Littlewood have introduced a method to go from ℓ_p to c_0 and back again. This was performed several times later (for instance in [1] or [4]). In [2] Frederic Bayart developed an abstract version of this machinery, first in the bilinear case and then in the m-linear one.

Remember that we will denote by $(e_i)_{i\in\mathbb{N}}$ the standard basis of ℓ_p and $e_{\mathbf{i}}$, $\mathbf{i} \in \mathbb{N}^m$, will mean $(e_{i_1}(1), \ldots, e_{i_m}(m))$ where $(e_i(j))_i$ is a copy of $(e_i)_i$. For $u \in \prod_{j=1}^m \ell_{p_j}$, $\mathbf{i} \in \mathbb{N}^m$ and $\alpha \in \mathbb{R}$, $u_{\mathbf{i}}$ will stand for $u_{i_1}(1) \times \cdots \times u_{i_m}(m)$ and $u_{\mathbf{i}}^\alpha$ for $u_{i_1}(1)^\alpha \times \cdots \times u_{i_m}(m)^\alpha$. Finally, if $(a_{\mathbf{i}})_{\mathbf{i}\in\mathbb{N}^m}$ is a sequence indexed by $\mathbb{N}^m$ and $C \subset \{1, \ldots, m\}$, we shall identify $\mathbf{i}$ with $\mathbf{j}, \mathbf{k}$ with $\mathbf{j} = \mathbf{i}(C)$, $\mathbf{k} = \mathbf{i}(\bar{C})$ so that we shall write $a_{\mathbf{i}} = a_{\mathbf{j},\mathbf{k}}$.

Bayart shows that:

Lemma 1.1. *Let $m_1, m_2 \geq 1$, $p_1, p_2, q \in [1, +\infty)$, $(a_{i,j})_{i\in\mathbb{N}^{m_1},\, j\in\mathbb{N}^{m_2}}$ a sequence of non-negative real numbers. Assume that there exists $\kappa > 0$ and $0 < \alpha, \beta \leq q$ such that*

- *for all $u \in \prod_{j=1}^{m_1} B_{\ell_{p_1}}$,*

$$\left(\sum_{j\in\mathbb{N}^{m_2}} \left(\sum_{i\in\mathbb{N}^{m_1}} u_i^q a_{i,j}^q \right)^{\alpha/q} \right)^{1/\alpha} \leq \kappa;$$

- *for all $v \in \prod_{j=1}^{m_2} B_{\ell_{p_2}}$,*

$$\left(\sum_{i\in\mathbb{N}^{m_1}} \left(\sum_{j\in\mathbb{N}^{m_2}} v_j^q a_{i,j}^q \right)^{\beta/q} \right)^{1/\beta} \leq \kappa.$$

Then

$$\left(\sum_{j\in\mathbb{N}^{m_2}} \left(\sum_{i\in\mathbb{N}^{m_1}} a_{i,j}^q \right)^{\gamma/q} \right)^{1/\gamma} \leq \kappa$$

where

$$\frac{1}{\gamma} = \frac{1}{\alpha} - \frac{m_1}{p_1}\left(\frac{1 - \frac{q}{\alpha} - \frac{m_2 q}{p_2}}{1 - \frac{q}{\beta} - \frac{m_1 q}{p_1}}\right)$$

provided $\gamma > 0$, $\frac{m_1\alpha}{p_1} \leq 1$ *and* $\frac{m_2\beta}{p_2} \leq 1$.

2 Main Result

The next result seems to be a natural multilinear analogue to the linear inclusion theorem and is an application of Lemma 1.1.

Theorem 2.1. *Let* $T : X_1 \times \cdots \times X_m \to Y$ *be* m*-linear, let* $r, s \in [1, +\infty)$, $\mathbf{p}, \mathbf{q} \in [1, +\infty)^m$. *Assume that* T *is multiple* $(r, \mathbf{p})$*-summing, that* $q_k \geq p_k$ *for all* $k = 1, \ldots, m$ *and that* $\frac{1}{r} - \sum_{j=1}^m \frac{1}{p_j} + \sum_{j=1}^m \frac{1}{q_j} > 0$. *Then* T *is multiple* $(s, \mathbf{q})$*-summing, with*

$$\frac{1}{s} - \sum_{j=1}^{m} \frac{1}{q_j} = \frac{1}{r} - \sum_{j=1}^{m} \frac{1}{p_j}.$$

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SPACEABILITY IN STANDARD SEQUENCE SPACES

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Abstract

We introduce and study the notion of standard sequence classes that generate (quasi)-Banach sequence spaces and investigate the spaceability of certain sets associated with these classes. As a consequence, we obtain a result on (α, β)-spaceability concerning specific differences between sequence spaces.

1 Introduction

The property known as *lineability*, formalized by Gurariy et al. in [1], refers to the search for linear structures within sets that, at first glance, appear to lack any linear character. More precisely, given a cardinal number μ and a linear space V, a subset $A \subset V$ is said to be μ*-lineable* if $A \cup 0$ contains a μ-dimensional linear subspace of V. When V is a topological vector space and the subspace is closed, A is called μ*-spaceable*. More refined notions of lineability have been developed, deepening the theory. Among these are *pointwise lineability*, introduced in [4], and (α, β)*-lineability*, introduced in [2].

In this work, we introduce and study the concept of standard sequence classes and present a related result involving (α, β)*-spaceability*. As consequence, we can particularize this analysis to the difference between certain quasi-Banach sequence spaces that satisfy the hypotheses of our main result.

The letters E and F will denote Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. The notation $E \overset{C}{\hookrightarrow} F$ means that E is a linear subspace of F and $\|x\|_F \leq C\|x\|_E$ for every $x \in E$ and some constant $C > 0$. For each $j \in \mathbb{N}$ and $x \in E$ the symbol $x \cdot e_j$ denotes the sequence $(0, \ldots, 0, x, 0, 0, \ldots) \in E^{\mathbb{N}}$, where x is placed at the j-th coordinate. When we refer to a partition $\mathbb{N} = \mathbb{N}_1 \cup \mathbb{N}_2$, we mean that $\mathbb{N}_1$ and $\mathbb{N}_2$ are infinite, ordered (the canonical order of $\mathbb{N}$) and disjoint subsets of $\mathbb{N}$.

2 Main Results

A *quasi-Banach standard sequence class* $X(\cdot)$ (or simply a *standard sequence class* X) is a rule that assigns to each Banach space E a quasi-Banach E-valued sequence space $X(E)$ (endowed with the usual coordinatewise algebraic operations) enjoying the following conditions:

i) There exists a constant $C > 0$ such that $c_{00}(E) \subseteq X(E) \overset{C}{\hookrightarrow} \ell_\infty(E)$.

ii) If $x = (x_j)_{j \in \mathbb{N}} \in X(E)$ and $(x_{n_k})_{k \in \mathbb{N}}$ is a subsequence of x, then $(x_{n_k})_{k \in \mathbb{N}} \in X(E)$ and $\|(x_{n_k})_{k \in \mathbb{N}}\|_X \leq \|x\|_X$.

iii) If $x = (x_j)_{j \in \mathbb{N}} \in X$ and $\mathbb{N}' = \{n_1 < n_2 < n_3 < \cdots\}$ is an infinite subset of $\mathbb{N}$, then the E-valued sequence y defined by $y = \sum_{i \in \mathbb{N}} x_i \cdot e_{n_i}$ belongs to $X(E)$ and $\|y\|_X \leq \|x\|_X$.

Spaces of the form $X(E)$, where E is a Banach space and X is a standard sequence class, will be called *standard sequence spaces*. Based on the definition available in [3, Definition 2.3], we present the following definitions.

Definition 2.1. *Let X be a standard sequence class. A map $f : E \to F$ with $f(0) = 0$ is said to be:*

a) Compatible with X *if for any* $(x_j)_{j=1}^{\infty} \in E^{\mathbb{N}}$ *and all* $\alpha \in \mathbb{K} \setminus \{0\}$*, we have* $(f(\alpha x_j))_{j=1}^{\infty} \notin X(F)$ *whenever* $(f(x_j))_{j=1}^{\infty} \notin X(F)$.

b) Linearly compatible with X *if it satisfies a) and for any* $(x_j)_{j=1}^{\infty}, (y_j)_{j=1}^{\infty} \in E^{\mathbb{N}}$*, we have* $(f(x_j+y_j))_{j=1}^{\infty} \in X(F)$ *whenever* $(f(x_j))_{j=1}^{\infty}, (f(y_j))_{j=1}^{\infty} \in X(F)$.

In what follows, for a standard sequence class X, a map $f : E \to F$, and a family of standard sequence classes $\{X_\lambda\}_{\lambda\in\Lambda}$, we define the following set:

$$G(X, f, \{X_\lambda\}_{\lambda\in\Lambda}) := \left\{ (x_j)_{j=1}^{\infty} \in X(E) : (f(x_j))_{j=1}^{\infty} \notin \bigcup_{\lambda\in\Lambda} X_\lambda(F) \right\}.$$

Definition 2.2. *Let* X *and* Y *be standard sequence classes and* $f : E \to F$ *be a map linearly compatible with* Y*. We say that a sequence* $(x_j)_{j\in\mathbb{N}} \in G(X, f, Y)$ *is* detachable *if there exists a partition* $\mathbb{N} = \mathbb{N}_1 \cup \mathbb{N}_2$*, such that* $(f(x_n))_{n\in\mathbb{N}_1} \in Y(F)$*. We say that the triple* (X, f, Y) *is detachable if all* $(x_j)_{j\in\mathbb{N}} \in G(X, f, Y)$ *is detachable. The triple* $(X, f, \{X_\lambda\}_{\lambda\in\Lambda})$ *is said detachable if* (X, f, X_λ) *is detachable for all* $\lambda \in \Lambda$.

Let E be a Banach space and $\{X_\lambda\}_{\lambda\in\Lambda}$ be a family of standard sequence classes. We say that the family $\{X_\lambda\}_{\lambda\in\Lambda}$ is *E-nested* if for all $\lambda_1, \lambda_2 \in \Lambda$ we have $X_{\lambda_1}(E) \subseteq X_{\lambda_2}(E)$ or $X_{\lambda_2}(E) \subseteq X_{\lambda_1}(E)$.

We are now able to present our main result.

Theorem 2.1. *Let* X *be a standard sequence class,* $\{X_\lambda\}_{\lambda\in\Lambda}$ *be an* F*-nested family of standard sequence classes and* $f : E \to F$ *be a map linearly compatible with* X_λ *for all* $\lambda \in \Lambda$*. If* $(X, f, \{X_\lambda\}_{\lambda\in\Lambda})$ *is detachable, then* $G(X, f, \{X_\lambda\}_{\lambda\in\Lambda})$ *is* $(\alpha, \mathfrak{c})$*-spaceable if and only if* $\alpha < \aleph_0$.

As $X(E) \setminus \bigcup_{\lambda\in\Lambda} X_\lambda(E) = G(X, Id_E, \{X_\lambda\}_{\lambda\in\Lambda})$, we obtain the following immediate application:

Corollary 2.1. *Let* X *be a standard sequence class,* $\{X_\lambda\}_{\lambda\in\Lambda}$ *be an* E*-nested family of standard sequence classes such that* $(X, Id_E, \{X_\lambda\}_{\lambda\in\Lambda})$ *is detachable. If* $X(E) \setminus \bigcup_{\lambda\in\Lambda} X_\lambda(E)$ *is a non-empty set, then it is* $(\alpha, \mathfrak{c})$*-spaceable if and only if* $\alpha < \aleph_0$.

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SHADING AND STRONG STRUCTURAL STABILITY IN LINEAR DYNAMICS

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Abstract

In this work, we study a characterization for weighted shifts on c_0 or ℓ_p spaces (with $1 \leq p < \infty$) to have the shadowing property, as estabilished by N. Bernardes Jr. and A. Messaoudi (2020). Finally, we discuss how F. Bayart (2021) proved that, in this same context, the shadowing property is equivalent to *strong structural stability*.

1 Introduction

The shadowing property guarantees that for every pseudotrajectory - a sequence of points where each iteration exhibits a small, controlled error - there exists a true trajectory of the system that uniformly approximates it. On the other hand, strong structural stability expands the scope of structural stability by requiring a strong type of conjugacy.

Let $X = c_0(\mathbb{Z})$ or $\ell_p(\mathbb{Z})$ (with $1 \leq p < \infty$), let $w = (w_n) \in \ell_\infty$ and consider the weighted backward shift $B_w : X \to X$, which is defined by $B_w e_n = w_n e_{n-1}$. In this work, we study a characterization for B_w to have shadowing, which was obtained by N. Bernardes Jr. and A. Messaoudi in [1] and consists of three possible properties on w. Later on, F. Bayart have shown in [3] that the same conditions are equivalent to strong structural stability (SSS), thus proving that these two notions are the same in the context of shifts. In [2], the two first conditions were shown to be equivalent to hiperbolicity. We provide a simple example of weighted shift which has shadowing/SSS but is not hiperbolic.

2 Basic definitions

Let us provide the basic definitions that are going to be used in the next section.

Definition 2.1. *Let X be a Banach space. A operator $T \in \mathcal{L}(X)$ is said to be hyperbolic if $\sigma(T) \cap \mathbb{T}$.*

Definition 2.2. *Let (M, d) be a metric space, and let $h : M \to M$ a homeomorphism. A sequence $(x_n)_{n\in\mathbb{Z}}$ is called a δ-pseudotrajectory of h, where $\delta > 0$, if $d(h(x_n), x_{n+1}) < \delta$, $\forall n \in \mathbb{Z}$.*

Definition 2.3. *Let (M, d) be a metric space. The homemorphism $h : X \to X$ is said to have the shadowing property if for very $\varepsilon > 0$ there exists $\delta > 0$ such that every δ-pseudotrajectory $(x_n)_{n\in\mathbb{Z}}$ of h is ε-shadowed by a real trajectory of h, that is, there exists $x \in M$ such that $d(x_n, h^n(x)) < \varepsilon$, $\forall n \in \mathbb{Z}$.*

Definition 2.4. *Let X be a Banach space. A operator $T \in \mathcal{L}(X)$ is said to be strongly structurally stable if for every $\varepsilon > 0$, there exists $\delta > 0$ such that, for any Lipschitz map $\alpha \in Lip(X)$, with $\|\alpha\|_\infty \leq \delta$ and $Lip(\alpha) \leq \delta$, there is a homeomorphism $\varphi : X \to X$ with $\|\alpha\|_\infty \leq \varepsilon$ such that $T \circ (I + \varphi) = (I + \varphi) \circ (T + \alpha)$.*

3 Main Results

The next result provides sufficient conditions on the weight for the corresponding backward shift to be hyperbolic.

Proposition 3.1 ([2, Remark 35]). *Let $X = \omega$, let $w = (w_n)n \in \mathbb{Z}$ be an admissible weight with $\inf_{n\in\mathbb{Z}} |w_n| > 0$, and let $B_w : X \to X$ be the backward shift. Then the following assertions are equivalent:*

(i) *B_w is hyperbolic ;*

(iii) $\lim_{n\to\infty} \sup_{k\in\mathbb{Z}} |w_k \dots w_{k+n-1}|^{\frac{1}{n}} < 1$ *or* $\lim_{n\to\infty} \inf_{k\in\mathbb{Z}} |w_k \dots w_{k+n-1}|^{\frac{1}{n}} > 1.$

It was shown in [2, Theorem A] and [4, Theorem 1] that any hyperbolic operator has the shadowing property and is strongly structurally stable (SSS). The following result was obtained by F. Bayart in [3].

Theorem 3.1 ([3, Theorem 1.1]). *Let $X = \ell_p(\mathbb{Z})$ $(1 \le p < \infty)$ or $X = c_0(\mathbb{Z})$, let $(w_k)k \in \mathbb{N}$ be an admissible weight with $\inf_{k\in\mathbb{Z}} |w_k| > 0$, and let B_w be the weighted backward shift. Then, B_w is strongly structurally stable if and only if one of the following conditions holds:*

(A) $\lim_{n\to\infty} \sup_{k\in\mathbb{Z}} |w_k w_{k+1} \cdots w_{k+n}|^{\frac{1}{n}} < 1;$

(B) $\lim_{n\to\infty} \inf_{k\in\mathbb{Z}} |w_k w_{k+1} \cdots w_{k+n-1}|^{\frac{1}{n}} > 1;$

(C) $\lim_{n\to\infty} \sup_{k\in\mathbb{N}} |w_{-k} w_{-k-1} \cdots w_{-k-n}|^{\frac{1}{n}} < 1$ *and* $\lim_{n\to\infty} \inf_{k\in\mathbb{N}} |w_k w_{k+1} \cdots w_{k+n}|^{\frac{1}{n}} > 1;$

For B_w, with $\inf_{k\in\mathbb{Z}} |w_k| > 0$, N. Bernardes Jr. and A. Messaoudi obtained in [1, Theorem 18] that satisfying one of the conditions $(A), (B)$ or (C) above is equivalent to stating that B_w has the shadowing property. Thus, combining with the previous result of F. Bayart, we immediately conclude the following.

Corollary 3.1. *Let $(w_k)_{k\in\mathbb{N}} \in \omega$ be an admissible weight with $\inf_{k\in\mathbb{Z}} |w_k| > 0$, and let $B_w : X \to X$ be the weighted shift operator. Then, B_w has SSS if and only if it has the shadowing property.*

Example. Consider $B_w : X \to X$, with $X = c_0$ or $X = \ell_p, 1 \le p < \infty$, where w is given by

$$w_k = \begin{cases} \lambda & \text{se } k \ge 0 \text{ Ã© par} \\ \lambda^{-1} & \text{se } k < 0 \text{ Ã© par} \\ 1 & \text{se } k \text{ Ã© impar.} \end{cases}$$

Then B_w does not satisfy (A) nor (B), so it is not hyperbolic. But it satisfies (C), so it has shadowing/SSS.

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A NEW APPROACH TO SPACEABILITY IN SEQUENCE AND FUNCTION SETS

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Abstract

Recently, in [4], the notion of (α, β)-spaceability (which is more restritive than spaceability), where α and β are cardinal numbers, were introduced and explored in many contexts. In this work we explore this notion in sequences and functions sets, in particular, we investigate this more restritive notion in certain sets of absolutely p-summable sequences and measurable functions.

1 Introduction

In the early 2000s, several researchers became interested in the study of linearity in *a priori* nonlinear structure, that is, the search for linear structures in mathematical objects that satisfy certain special properties. This approach gave rise to an area of intense research in mathematics called *lineability*, a term introduced by the mathematician V.I. Gurariy. More precisely, a subset A of an infinite-dimensional normed space E is called *lineable* (*spaceable*) if A contains, except possibly for the null vector, a subspace (*closed subspace*) of infinite-dimensional E. The following concepts are more restritive notions of lineability/spaceability.

Definition 1.1. *Let α, β, λ be cardinal numbers, and E be a vector space, with $\dim E = \lambda$ and $\alpha < \beta \leq \lambda$. A set $A \subset E$ is:*

i) *α-lineable (α-spaceable) if there is a vector subspace (closed subspace) $W_\alpha \subset E$ such that $W_\alpha \subset A \cup \{0\}$ with $\dim W_\alpha = \alpha$;*

ii) *(α, β)-spaceable if it is α-lineable and for every subspace $W_\alpha \subset E$ with $W_\alpha \subset A \cup \{0\}$ and $\dim W_\alpha = \alpha$, there is a closed subspace $W_\beta \subset E$ with $\dim W_\beta = \beta$ e $W_\alpha \subset W_\beta \subset A \cup \{0\}$.*

We are considering vector spaces over the field $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. Furthermore, we will denote $card(\mathbb{R}) = \mathfrak{c}$ and $card(\mathbb{N}) = \aleph_0$.

Definition 1.2. *Let $p \geq 1$. We define the space ℓ_p as the set of all sequences in $\mathbb{K}$ that are absolutely p-summable, that is,*

$$\ell_p = \left\{ x = (\xi_j)_{j=1}^{\infty} \subset \mathbb{K} : \sum_{j=1}^{\infty} |\xi_j|^p < \infty \right\},$$

which is a Banach space with the norm

$$\|x\|_p = \left(\sum_{j=1}^{\infty} |\xi_j|^p \right)^{\frac{1}{p}}.$$

Definition 1.3. *Let $p \geq 1$. We denote by $L_p[0,1]$ the set of all (equivalence class of) measurable functions $f : [0,1] \to \mathbb{R}$ such that*

$$\|f\|_p := \left(\int_0^1 |f|^p \right)^{\frac{1}{p}} < \infty.$$

Here we are considering the Borel σ-algebra and the Lebesgue measure. $L_p[0,1]$ is a Banach with the norm $\| \cdot \|_p$.

Both definitions (1.2 and 1.3) are valid for $0 < p < 1$. But in each case, $\|\cdot\|_p$ is a p-norm and the respective space is a p-Banach space.

2 Main Results

The following result was proved in [4].

Theorem 2.1. *For all $p > 0$, the set*

$$\ell_p \setminus \bigcup_{0<q<p} \ell_q$$

is $(\alpha, \mathfrak{c})$-spaceable in ℓ_p if and only if $\alpha < \aleph_0$.

In [3], it was shown that the set $L_p[0,1] \setminus \bigcup_{q>p} L_q[0,1]$ is spaceable. Later, in [4], the authors proved the $(1, \mathfrak{c})$-spaceability of this set. This result was improved recently, in [1], as follows:

Theorem 2.2. *For all $0 < p < \infty$, the set*

$$L_p[0,1] \setminus \bigcup_{q>p} L_q[0,1]$$

is $(\alpha, \mathfrak{c})$-spaceable in $L_p[0,1]$ if, and only if, $\alpha < \aleph_0$.

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A NEW APPROACH TO INSPECT WEAKLY COUPLED LOGISTIC SYSTEMS AND THEIR ASYMPTOTIC BEHAVIOR

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Abstract

We consider the weakly coupled elliptic system of logistic type,

$$\begin{cases} -\Delta u &= \lambda_1 u - |u|^{p-2}u + \beta|u|^{\frac{p}{2}-2}u|v|^{\frac{p}{2}-1}v \text{ in } \Omega, \\ -\Delta v &= \lambda_2 v - |v|^{p-2}v + \beta|u|^{\frac{p}{2}-1}u|v|^{\frac{p}{2}-2}v \text{ in } \Omega, \\ \quad u, v &\in H_0^1(\Omega), \end{cases} \tag{LS}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with $N \geq 2$. Denoting by $\lambda_1(\Omega)$ the principal eigenvalue of the Laplacian operator with Dirichlet conditions on Ω, we assume $\lambda_1(\Omega) < \lambda_1 \leq \lambda_2$, and prove the existence and multiplicity of solutions to the problem (LS) in alternative variational frameworks, depending on the range of β. In case $N \geq 2$ and the suitable values of $2 < p < \min\{4, 2^*\}$, we extend the existence results, for all β in the whole line. We prove the existence and multiplicity of solutions to the problem (LS) in alternative variational frameworks depending on the information we have about the parameter β. We do not rely on bifurcation or degree theory, which have been used in the literature for logistic-type problems. Instead, the novelty is to obtain min-max type solutions by exploring the different geometry of the functional associated with the logistic problem. Furthermore, we analyze the asymptotic behavior of such solutions as $\beta \to 0$ or $\beta \to \pm\infty$.

1 Introduction

We study the weakly coupled elliptic system of the logistic type (LS), whose motivation comes from the particular model,

$$\begin{cases} -\Delta u = \lambda_1 u - u^3 + \beta u v^2 \text{ in } \Omega, \\ -\Delta v = \lambda_2 v - v^3 + \beta u^2 v \text{ in } \Omega, \\ u, v \in H_0^1(\Omega), \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded domain. Such a system appears in a vast context of dynamic of populations, and hence has been extensively studied in the past decades. Many authors have been working on logistic problems in the last years. They have applied a variety of techniques involving topological methods to obtain positive solutions. Only recently the variational approach has begun to be applied. This has allowed to explore more information about the existence of solutions for the ranges of coupling parameters for which sign-changing solutions are detected and positive do not exist.

2 Main Results

Theorem 2.1. *Assume $\lambda_2 \geq \lambda_1 > \lambda_1(\Omega)$. There exists a constant δ_0 such that for any $\beta \in (-\delta_0, 1)$, the ground state m_β is attained by a non-negative vectorial solution (u_β, v_β) to problem (LS). Moreover, $m_\beta \to -\infty$ as $\beta \to 1^-$.*

For β negative, and sufficiently large in absolute value we prove the existence of vectorial solution as a mountain-pass solution. Moreover, we study the asymptotic behavior of these solutions as $\beta \to -\infty$, obtaining a segregation phenomena.

Theorem 2.2. *Assume $\lambda_2 \geq \lambda_1 > \lambda_1(\Omega)$. If $N \geq 2$ and $2 < p < \min\{4, 2^*\}$, then there exist a constant $\delta_\star > 0$ and a non-negative vectorial solution (u_β, v_β) to problem (LS) such that*

$$c_1 + \delta_\star < J_\beta(u_\beta, v_\beta) < -\delta_\star, \tag{2}$$

for all $\beta < 0$. Moreover, in the classical case $N = 3$ and $p = 4$, there exist constants $\beta_\star, \delta_\star > 0$ and a non-negative vectorial solution (u_β, v_β) to problem (LS) such that, for $\beta < -\beta_\star$, it holds (2).

In addition, the asymptotic limit of (u_β, v_β) as $\beta \to -\infty$ is (w_∞^+, w_∞^-), where $w_\infty = w_\infty^+ - w_\infty^-$ is a sign-changing solution to

$$\begin{cases} -\Delta u = \lambda_1 u^+ - \lambda_2 u^- - |u|^{p-2}u \text{ in } \Omega, \\ u^\pm \in H_0^1(\Omega). \end{cases} \tag{3}$$

Remark 2.1. *For $2 < p < \min\{4, 2^*\}$ and sufficiently small $|\beta|$, Theorems 2.1 and 2.2 guarantee the existence of two different non-negative vectorial solutions to Problem (LS). Indeed, Theorem 2.1 produces a ground state solution, whose energy m_β is below the minimum of the energies of positive semi-trivial solutions. On the other hand, Theorem 2.2 gives us a solution, whose energy is above the maximum of the energies of positive semi-trivial solutions.*

We present a non-existence result of positive solutions is obtained to problem (LS) with $\beta > 1$.

Theorem 2.3. *Assume that $\beta \geq 1$ and $\lambda_1(\Omega) < \lambda_1 \leq \lambda_2$. Problem (LS) does not admit a positive vectorial solution.*

Moreover, a vectorial solution is found, which in fact will have to change sign at least for one of the component functions.

Theorem 2.4. *Assume that $\beta \geq 1$ and $\lambda_1(\Omega) < \lambda_1 \leq \lambda_2$. Problem (LS) admits a vectorial solution (u, v), and at least one of the component u or v changes sign.*

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NONLOCAL QUAISLINEAR ELLIPTIC PROBLEMS ON IN BOUNDED DOMAINS

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Abstract

In this we establish existence of solutions for nonlocal elliptic problems driven by the fractional $(p, q)-$ Laplacian. More specifically, we shall consider the following nonlocal elliptic problem :

$$\begin{cases} (-\Delta)_p^{s_1} u - \mu(-\Delta)_q^{s_2} = \lambda |u|^{r-2}u \text{ in } \Omega \\ u = 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{P_μ}$$

where$N > s_1 p$, $N > s_2 q$, $s_1 > s_2$ and $r > p > q$. The main feature is to find sharp parameters $\lambda > 0$ and $\mu > 0$ where the Nehari method can be applied finding the largest positive number $\mu^* > 0$ such that our main problem admits at least two distinct solutions for each $\mu \in (0, \mu^*)$. A crucial part of this work is the fact that we consider the term $-\mu(-\Delta)_q^{s_2}$ in the problem (P_μ)

1 Introduction

In the present work we shall consider nonlocal elliptic problems driven by the fractional $(p, q)-$ Laplacian defined in bounded domanin. Namely, we shall consider the following nonlocal elliptic problem

$$\begin{cases} (-\Delta)_p^{s_1} u - \mu(-\Delta)_q^{s_2} = \lambda |u|^{r-2}u \text{ in } \Omega \\ u = 0 \text{ in } \mathbb{R}^N \setminus \Omega, \end{cases} \tag{P_μ}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N > s_1 p$, $N > s_2 q$, $s_1 > s_2$ and $r > p > q$.

In order to do that we employ the nonlinear Rayleigh quotient together a fine analysis on the fibering maps associated to the energy functional. It is important to mention also that for each parameters $\lambda > 0$ and $\mu > 0$ there exist degenerate points in the Nehari set which give serious difficulties.

2 Main Results

The working space is defined by $X = \{u \in W^{s,p}(\mathbb{R}^N);\ u = 0 \text{ in} \, \mathbb{R}^N \setminus \Omega\}$ and the energy functional $J : X \to \mathbb{R}$ associated to Problem (P_μ) is given by

$$J_{\lambda,\mu}(u) = \frac{1}{p}[u]_p^p - \frac{\mu}{q}[u]_q^q - \frac{\lambda}{r}\|u\|_r^r,$$

where

$$[u]_p^p := [u]_{s,p}^p = \int_\Omega \int_\Omega \frac{|u(x) - u(y)|^p}{|x - y|^{N+sp}}\, dxdy,\ u \in X.$$

In this work, we study the fibers maps of two functionals based on the parameter μ. The first defined for the case where $J'_{\lambda,\mu}(u) = 0$ and the second, considering μ, for which $J(u) = 0$. In short, we consider the functionals $R_n, R_e : X \setminus \{0\} \to \mathbb{R}$ associated with the parameter $\mu > 0$ in the following form

$$R_n(u) = \frac{[u]_p^p - \lambda\|u\|_r^r}{[u]_q^q} \quad \text{and} \quad R_e(u) = \frac{\frac{q}{p}[u]_p^p - \lambda\frac{q}{r}\|u\|_r^r}{[u]_q^q}, u \in X \setminus \{0\},$$

the nonlinear Rayleigh quotients. In association with these, based on work [1], we define the following coefficients

$$\mu^* := \inf_{u\in X\setminus\{0\}} \inf_{t>0} R_n(tu) \qquad \text{and} \qquad \mu^{**} := \inf_{u\in X\setminus\{0\}} \inf_{t>0} R_e(tu). \tag{1}$$

The subset of X, in which the $J_{\lambda,\mu}$ function will be minimized, well known and studied in recent years for Nehari is

$$\mathcal{N}_{\lambda,\mu} = \{u \in X,\, u \neq 0 \ :\ \langle J'_{\lambda,\mu}(u), u\rangle = 0\}.$$

Under these conditions, by using the same ideas considered in [2], we shall split the Nehari manifold $\mathcal{N}_{\lambda,\mu}$ into three disjoint subsets in the following way:

$$\begin{aligned} \mathcal{N}^+_{\lambda,\mu} &= \{u \in \mathcal{N}_{\lambda,\mu} : J''_{\lambda,\mu}(u)(u,u) > 0\}, \\ \mathcal{N}^-_{\lambda,\mu} &= \{u \in \mathcal{N}_{\lambda,\mu} : J''_{\lambda,\mu}(u)(u,u) < 0\}, \\ \mathcal{N}^0_{\lambda,\mu} &= \{u \in \mathcal{N}_{\lambda,\mu} : J''_{\lambda,\mu}(u)(u,u) = 0\}. \end{aligned}$$

We shall state our first main result as follows:

Theorem 2.1. *Suppose $\mu \in (0,\mu^*)$, where μ^* follows from (1). Then there are two solutions $u_1, u_2 \in X \setminus \{0\}$ that satisfy the following statements:*

i) $J''_{\lambda,\mu}(u_1,u_1) < 0$, *that is,* $u_1 \in \mathcal{N}^-_{\lambda,\mu}$;

ii) $J''_{\lambda,\mu}(u_2,u_2) > 0$, *that is,* $u_2 \in \mathcal{N}^+_{\lambda,\mu}$;

iii) $J_{\lambda,\mu}(u_2) < 0$, *for all* $\mu \in (0,\mu^*)$.

Moreover, the weak solution $u_2 \in X$ satisfies the following assertions:

a) For each $0 < \mu < \mu^{**}$, *we obtain* $J_{\lambda,\mu}(u_2) > 0$;

b) For $\mu = \mu^{**}$ *it follows that* $J_{\lambda,\mu}(u_2) = 0$

c) For each $\mu^{**} < \mu < \mu^*$ *we obtain also that* $J_{\lambda,\mu}(u_2) < 0$

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ASYMPTOTIC BEHAVIOUR OF SOLUTIONS TO A FAMILY OF P-LAPLACIAN PROBLEMS

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Abstract

The theory of viscosity solutions, introduced in the 1980s, revolutionized the study of nonlinear Partial Differential Equations by providing a robust framework for problems where classical and weak solution concepts were insufficient. In recent decades, this theory has proven particularly effective in studying the asymptotic behaviour of solutions to elliptic problems.

In this work, we investigate the asymptotic behaviour of weak solutions to a Dirichlet problem with a variable exponent $q(p)$ that enjoys a certain property. Problems of this nature were first introduced by Charro and Peral [1] and Charro and Parini [2]. However, our approach is based on the recent work by Ercole introduced in [3].

1 Introduction

Let Ω be a bounded domain of $\mathbb{R}^N$, $N \geq 2$. Consider the Dirichlet problem

$$\begin{cases} -\Delta_p u = \lambda_{p,q(p)}(\Omega)|u|^{q(p)-2}u & \text{in } \Omega, \\ \quad u > 0 & \text{in } \Omega, \\ \quad u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $q = q(p)$ satisfies

$$Q := \lim_{p \to +\infty} \frac{q(p)}{p} \in [0, \infty].$$

and $\lambda_{p,q(p)}(\Omega)$ is the best Sobolev constant of the embedding $W_0^{1,p}(\Omega) \hookrightarrow L^{q(p)}(\Omega)$.

In [1] and [2], the authors studied the asymptotic behaviour of the weak solutions to this problem with $0 < Q < 1$ and $1 < Q < \infty$. In both cases, they found that the sequence of weak solutions converges uniformly to a viscosity solution of the Dirichlet problem

$$\begin{cases} \min\left\{|\nabla u| - \Lambda_\infty(\Omega)u^Q, -\Delta_\infty u\right\} = 0 & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where $\Lambda_\infty(\Omega)$ is the first eigenvalue of the ∞-Laplacian in Ω.

In [3], Ercole introduced a theorem that allowed him to generalize this result for $0 \leq Q \leq \infty$.

2 Main Results

Theorem 2.1. *If*

$$\lim_{p \to +\infty} q(p) = +\infty,$$

then

$$\lim_{p \to +\infty} \lambda_{p,q(p)}^{\frac{1}{p}}(\Omega) = \Lambda_\infty(\Omega)$$

Furthermore, up to a subsequence,

$$\lim_{p\to+\infty} u_{p,q(p)} = u_\infty \text{ uniformly}$$

and $u_\infty \in W^{1,\infty}(\Omega)\cap C(\overline{\Omega})$ *satisfies*

1. $0 \le u_\infty \le d_\Omega \Lambda_\infty(\Omega)$ *in* Ω*;*
2. $\|u_\infty\|_\infty = 1$ *and* $\|\nabla u_\infty\|_\infty = \Lambda_\infty(\Omega)$*;*
3. $M := \{x \in \Omega : u_\infty(x) = 1\} \subseteq M_\Omega := \{x \in \Omega : d_\Omega(x) = \|d_\Omega\|_\infty\}$*;*
4. $u_\infty > 0$ *in* Ω*.*

This Theorem allows us to study the asymptotic behaviour of solutions to problem (1) not only in the cases $0 < Q < 1$ and $1 < Q < \infty$, as it was studied in [1] and [2], respectively, but also study the cases $Q = 1$ with $q(p) \to \infty$, $Q = 0$ and $Q = \infty$. We aim to conclude that, to each of these cases, the limit function u_∞ will be a viscosity solution of a Dirichlet problem similar to (2).

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ON A CLASS OF PRESCRIBED MEAN CURVATURE EQUATIONS WITH CRITICAL GROWTH IN THE WHOLE $\mathbb{R}^2$

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Abstract

In this work, we study the problem

$$-\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)+V(x)u=\lambda u^{p-1}+f(u), \quad u\in H^1(\mathbb{R}^2), \quad u>0,$$

where $p>2$, $\lambda>0$, $V:\mathbb{R}^2\to\mathbb{R}$ is a continuous potential bounded away from zero and $f:\mathbb{R}^2\to\mathbb{R}$ is continuous and has exponential critical growth without the assumption of monotonicity on $s\mapsto f(s)/s$. Firstly, we truncate the prescribed mean curvature operator and obtain a nonzero solution for an auxiliary problem. Next, we use the Moser iteration technique to get some uniform estimates of this solution. We finalize by proving that the solution of the auxiliary problem is positive and actually is a solution of the original problem when λ is large.

1 Introduction

Motivated by the work [2] that deals with exponential critical growth and the papers [1, 4, 5] on all $\mathbb{R}^N$, in the present work we are interested in the existence of solution for the following elliptic problem:

$$\begin{cases} -\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right)+V(x)u=\lambda u^{p-1}+f(u) \quad \text{in} \quad \mathbb{R}^2, \\ u\in H^1(\mathbb{R}^2), \quad u>0, \end{cases} \tag{P_λ}$$

where $p>2$ and $\lambda>0$ is a parameter. Here, the function $V:\mathbb{R}^2\to\mathbb{R}$ is a continuous potential bounded away from zero and $f:\mathbb{R}\to\mathbb{R}$ is a critical nonlinearity in the sense of the Trudinger-Moser inequality.

To state our main result, let us to introduce more precisely the hypotheses on V and f. The assumptions on the potential $V(x)$ are the following:

(V_1) $\inf_{x\in\mathbb{R}^2} V(x) =: V_0>0$;

(V_2) There exists $\lim_{|x|\to\infty} V(x)=: V_\infty$ and $V_\infty\geq V(x)$ for all $x\in\mathbb{R}^2$.

We also assume that the continuous nonlinearity $f:\mathbb{R}\to\mathbb{R}$ satisfies the following three conditions:

(f_1) $f(s)/s\to 0$ as $s\to 0^+$;

(f_2) there exists $\alpha_0>0$ such that

$$\lim_{s\to+\infty}\frac{f(s)}{e^{\alpha s^2}}=\begin{cases} 0, & \text{if } \alpha>\alpha_0, \\ +\infty, & \text{if } \alpha<\alpha_0. \end{cases}$$

(f_3) there exists $\mu>2$ so that

$$0<\mu F(s)\leq f(s)s, \quad \text{for all } s>0,$$

where $F(s):=\int_0^s f(t)dt$.

2 Main Results

A function $u : \mathbb{R}^2 \to \mathbb{R}$ is a *weak solution* for (1) if $u \in H^1(\mathbb{R}^2)$, $u > 0$ in $\mathbb{R}^2$ and it holds

$$\int_{\mathbb{R}^2} \frac{\nabla u \nabla v}{\sqrt{1+|\nabla u|^2}} dx + \int_{\mathbb{R}^2} V(x) uv\, dx = \lambda \int_{\mathbb{R}^2} u^{p-1} v\, dx + \int_{\mathbb{R}^2} f(u) v\, dx, \quad \text{for all } \ v \in H^1(\mathbb{R}^2). \tag{1}$$

For simplicity, we will denote $\nu_0 := \frac{7}{8\sqrt{2}}$ and $\Theta := \min\{p, \mu\}$. The main result of this work establishes the following:

Theorem 2.1. *Suppose that the conditions $(V_1) - (V_2)$ and $(f_1) - (f_3)$ are satisfied. Let*

$$\lambda_0 := \max \left\{ p \left[\frac{\max\{1, 1/V_0\} \alpha_0 \Theta q(p-2)}{3\nu_0(\Theta - 2)(q-1)} \right]^{\frac{p-2}{2}} \left[\frac{4(1+V_\infty)}{p} \right]^{\frac{p}{2}}, \left[\left(\sqrt{K_1} + 2K_2\sqrt{\pi} \right) K \right]^{p-2} \right\}, \tag{2}$$

where $q = q(p)$, $K_1 = K_1(V_0, V_\infty, \mu, p)$, $K_2 = K_2(V_0, V_\infty, \alpha_0, \mu, p, D_0)$ and K is an universal constant that appears in a version of the Trudinger-Moser inequality. Then (1) has a positive weak solution $u \in C^1(\mathbb{R}^2)$ for all $\lambda \geq \lambda_0$.

In order to prove our main result, we exploit minimax methods together with a version of the Trudinger-Moser inequality and elliptic regularization. First, we used arguments as in [3] to truncate the prescribed mean curvature operator and obtain a nonzero solution for an auxiliary problem. After some arguments and by using the condition $\lambda \geq \lambda_0$, we can conclude that the solution to the auxiliary problem is indeed a solution to the original problem (1).

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METRIC PERTURBATION THEORY FOR THE $\mathcal{L}_{\mathcal{T}}$ GRUSHIN OPERATOR

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Abstract

In the present work, we are interested in the Metric Perturbation Theory Theory for the differential operators that generalize the Grushin Laplacian considering both Neumann and Dirichlet boundary conditions.

1 Introduction

Let (M, g) be an n-dimensional compact Riemannian manifold with boundary ∂M. Consider a symmetric positive semidefinite $(0, 2)$-tensor T, and a smooth function $\eta : M \to \mathbb{R}$. The Grushin-type Laplacian operator is defined by

$$\mathcal{L}_{T,\eta} f := div_\eta(T\nabla f) = div_g(T\nabla f) - g(\nabla\eta, T\nabla f).$$

Note that $\mathcal{L}_{T,\eta}$ is a degenerate operator since T is positive semidefinite. Here, div stands for the divergence of smooth vector fields and ∇ for the gradient of smooth functions.

The existence of eigenvalues and eigenfunctions for the operator $\mathcal{L}_{T,\eta}$ with Dirichlet or Neumann condition defined in M depends on suitable assumptions over T. There is a sequence of eigenvalues $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_n \le \cdots \longrightarrow +\infty$ and $\phi_i : M \to \mathbb{R}$ associated eigenfunctions, for the eigenvalues problem given by

$$\begin{cases} -\mathcal{L}_{T,\eta}\phi_i & = \lambda\phi_i \text{ in } M \\ \mathcal{B}_\alpha(u) & = 0 \text{ on } \partial M \end{cases} \tag{1}$$

where $\mathcal{B}_\alpha(u) = \alpha\langle\nabla u, T\nu\rangle + (1-\alpha)u$, for $\alpha \in \{0, 1\}$, in other words, when $\alpha = 0$ the Dirichlet boundary condition is met, and when $\alpha = 1$, it satisfies the Neumann boundary condition. The operator $\mathcal{L}_{T,\eta}$ generalizes the Grushin operator [3].

2 Main Results

Let Q be a symmetric, semidefinite, non-negative $(0, 2)-$tensor on M. We considering the weighted degenerate Sobolev space $W^{1,2}_Q(M)$ is defined as the completion of the vector space

$$Lip_Q(M) = \{w \in Lip(M) : \|w\|_Q < \infty\}$$

with respect to the norm $\|w\|_Q = \left\{ \int_M |w|^2 dm + \int_M |\nabla w|^2 dm \right\}^{\frac{1}{2}}$.

$Lip_0(M)$ is the set of functions with compact support that belong to Lip_Q. We denote by $W^{1,2}_{Q,0}(M)$ the closure of $Lip_0(M)$ in $W^{1,2}_Q(M)$.

Definition 2.1. *A function $u \in W^{1,2}_{Q,0}(M)$ (resp $(W^{1,2}_Q(M)$) and $\lambda \in \mathbb{R}$ is called a weak solution to the problem (1) if it satisfies*

$$\int_M \langle T\nabla u, \nabla v\rangle\, dm = \int_M \lambda uv\, dm, \quad \textit{for all } v \in W^{1,2}_{Q,0}(M)(respW^{1,2}_Q(M)). \tag{2}$$

Note that $W^{1,2}_{Q,0}(M)$ is the space for solutions of the Dirichlet boundary condition and $W^{1,2}_Q(M)$ for Neumann boundary condition.

Let $\mathcal{M}^k$ be the set of all C^k- Riemannian metrics on M. In general, we cannot expect the $\lambda_n : \mathcal{M}^k \to \mathbb{R}$ that associates each metric g to the nth eigenvalue of $\mathcal{L}_{T,\eta}$ to be differentiable. Instead, we will consider symmetric functions of the eigenvalues defined in the following way. Let F be a non-empty finite subset of $\mathbb{N}$ and $\tau \in 1, \cdots, |F|$. Consider the map $\Lambda_{F,\tau} : \mathcal{M}^k \longrightarrow \mathbb{R}$ given by

$$\Lambda_{F,\tau}(g) := \sum_{\substack{j_1,\ldots,j_\tau \in F \\ j_1 < \cdots < j_\tau}} \lambda_{j_1}(g) \cdots \lambda_{j_\tau}(g) \quad \text{for all } g \in \mathcal{M}^k.$$

Theorem 2.1. *Let $A^F := \{g \in M | \lambda_n(g) = \lambda_m(g)\ \forall n \in F, \forall m \in \mathbb{N} \setminus F\}$. Then $\mathcal{A}^F$ is open in $\mathcal{M}^k$, and the map $\Lambda_{F,\tau} : \mathcal{A}^F \longrightarrow \mathbb{R}$ is a real-analytic function.*

Proposition 2.1. *Let M be a differential manifold, F a non-empty finite subset of $\mathbb{N}$, $\tau \in 1, \cdots, |F|$, and λ_F the common value of all the eigenvalues λ_{j_i}. The FrÃ©chet differential of the map $\Lambda_{F,\tau}$ is given by*

$$d_{g=\bar{g}}\Lambda_{F,\tau} \quad = \quad \lambda_F^{\tau-1}\binom{|F|-1}{\tau-1}\sum_{j=1}^{|F|}\int_M \langle H, div_\eta(T\nabla\phi_i^2)\bar{g} + 2\nabla\phi_i \otimes \tilde{T}\nabla\phi_i\rangle + \frac{1}{2}c\int_M \langle H, \bar{g}\rangle dm$$

The volume functional $\mathcal{V}(\cdot)$ is the map from $\mathcal{A}^F$ to $\mathbb{R}$ defined by

$$\mathcal{V}(g) := \int_M 1 dm_g \text{ where } \mathcal{V}[v] := \{g \in \mathcal{A}^F | \mathcal{V}(g) = v\}.$$

Definition 2.2. *We say that $\bar{g}$ is a critical metric for $\Lambda_{F,\tau}$ under the volume constraint $\bar{g} \in \mathcal{V}[v]$ if it satisfies*

$$d_{g=\bar{g}}\Lambda_{F,\tau}[H] + \bar{c}d_{g=\bar{g}}\mathcal{V}[H] = 0 \quad \textit{for some } \bar{c} \in \mathbb{R} \textit{ and for all } H \in S^2(\mathcal{M}).$$

Theorem 2.2. *If $\bar{g}$ is a critical metric for $\Lambda_{F,\tau}$ under the volume constraint $\bar{g} \in \mathcal{V}[v]$, then there exists a constant $c \in \mathbb{R}$ such that*

$$\sum_{i\in F} div_\eta(T\nabla\phi_i^2)\bar{g} + 2Sym\nabla\phi_i \otimes T\nabla\phi_i = c\bar{g}.$$

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ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS FOR A DEGENERATE LOGISTIC EQUATION WITH MIXED LOCAL AND NON-LOCAL DIFFUSION

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Abstract

In this work, we analyze a stationary degenerate logistic equation with both local and non-local diffusion. Primarily employing bifurcation results, sub- and supersolution methods, and maximum principles, we establish results regarding the existence, non-existence, and uniqueness of positive solutions. Additionally, using appropriate large solutions, we conduct a detailed study of the asymptotic behavior of the solutions with respect to one of the equation's parameters, showing that the presence of the non-local diffusion can drastically change this pointwise behavior when compared with the local case.

1 Introduction

Reaction-diffusion models have been used to study the behavior of a population living in a habitat. In order to obtain more realistic models, several authors have considered the inclusion of non-local terms, crucial to understand many biological processes, by their ability to take into account the effect of the surrounding environment to describe what happens at a certain point, in contrast to local differential terms. See, for instance, [2], [3]

Motivated by this, in this work, we studied the following stationary logistic equation that combines local and non-local diffusion:

$$\begin{cases} -\Delta u - \displaystyle\int_\Omega K(x,y)u(y)dy = \lambda u - a(x)u^2 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $K \in C(\overline{\Omega} \times \overline{\Omega})$ is a non-negative and nonzero function, $\lambda \in \mathbb{R}$, and $a \in C(\overline{\Omega})$ is a function that can vanish on some subset of Ω. From the point of the view of population dynamics, the right side is the well-known degenerate logistic reaction term, where λ is the intrinsic rate of natural increase of the species and $C(x) := \lambda/a(x)$ denotes the maximum density supported locally by resources available, that is, the carrying capacity. Thus, the region where $a(x) = 0$ can be understood as a refuge area for the species, where the carrying capacity is infinite. On the left side we have the diffusion term that combines local and non-local movement. Thus, $-\Delta u$ describes the random diffusion, and $\int_\Omega K(x,y)u(y)dy$ measures the dispersion of individuals from y to x.

Our main goal is to establish results concerning the existence, nonexistence, and uniqueness of positive solutions, as well as to determine the asymptotic behavior of these solutions with respect to the parameter λ, and to understand how the combination of local and nonlocal diffusion influences the structure of the solutions to this model.

2 Main Results

To state the main results of this work, we first introduce some notation and assumptions. We define $\Omega_0 := \text{int}\{x \in \Omega; a(x) = 0\}$ and assume that it is a regular subdomain of Ω. We denote by λ_1 and λ_1^0 the principal eigenvalue of

$$\begin{cases} -\Delta u + m(x)u - \displaystyle\int_D K(x,y)u(y)dy = \lambda u & \text{in } D, \\ u = 0 & \text{on } \partial D, \end{cases}$$

with $D = \Omega$ and $D = \Omega_0$, respectively. Finally, we define $\mathcal{K}_+ = \{(x, y) \in \Omega \times \Omega; K(x, y) > 0\}$.

Theorem 2.1. *The problem* (1) *possesses a positive solution in* $W^{2,p}(\Omega)$ $(p > 1)$ *if, and only if,* $\lambda \in (\lambda_1, \lambda_1^0)$. *Moreover, it is unique if it exists, and it will be denoted by* u_λ.

a) For each $x \in \Omega_0$, *we have*

$$\lim_{\lambda \uparrow \lambda_1^0} u_\lambda(x) = +\infty.$$

b) Assume $\overline{\mathcal{K}}_+ \cap (\Omega \times \overline{\Omega}_0) = \emptyset$. *Then, for each compact* $D \subset \Omega \setminus \overline{\Omega}_0$, *there exists a constant* $M = M(D) > 0$ *such that*

$$\|u_\lambda\|_{C(\overline{D})} \leq M \quad \forall \lambda \in (\lambda_1, \lambda_1^0).$$

c) Assume $W := [\mathcal{K}_+ \cap (\Omega \times \Omega_0)] \neq \emptyset$.

c.1) For each $x \in W_x := \mathrm{Proj}_x W$,

$$\lim_{\lambda \uparrow \lambda_1^0} u_\lambda(x) = +\infty.$$

c.2) If $\Omega \setminus \overline{\mathrm{Proj}_x(\mathcal{K}_+) \cup \Omega_0}$ *is nonempty, then for each compact* $D \subset \Omega \setminus \overline{\mathrm{Proj}_x(\mathcal{K}_+) \cup \Omega_0}$, *there exists a constant* $M = M(D) > 0$ *such that*

$$\|u_\lambda\|_{C(\overline{D})} \leq M \quad \forall \lambda \in (\lambda_1, \lambda_1^0).$$

Corollary 2.1. *Assume* $\overline{\mathcal{K}}_+ \cap (\Omega \times \overline{\Omega}_0) = \emptyset$. *Then the pointwise limit*

$$U(x) := \lim_{\lambda \uparrow \lambda_1^0} u_\lambda(x), \quad x \in \Omega \setminus \overline{\Omega}_0 \tag{1}$$

is finite and provides us with a positive solution for the singular boundary value problem:

$$\begin{cases} -\Delta u - \displaystyle\int_{\Omega \setminus \Omega_0} K(x, y) u(y) dy = \lambda_1^0 u - a(x) u^2 & \text{in } \Omega \setminus \overline{\Omega}_0, \\ u = +\infty & \text{on } \partial\Omega_0, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{2}$$

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GROUND STATE CRITICAL POINTS OF FUNCTIONALS ON CONES

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Abstract

In this work, we study C^1 functionals Φ defined on a reflexive Banach space X, for which the fiber map $t \mapsto \Phi(tu)$ does not, in general, have the same geometry for every $u \in X \setminus \{0\}$. Instead, we assume a uniform behavior for vectors u belonging to an open cone $Y \subset X \setminus \{0\}$. Under additional assumptions, we show that solutions of the Euler-Lagrange equation associated with the functional Φ that are energy minimizers with respect to the cone Y can also be global energy minimizers in the whole space X. Finally, we present a class of functionals that includes energy functionals associated with problems involving the (p, q)-Laplacian operator.

1 Introduction

Given a reflexive Banach space X and a C^1 functional Φ, the critical point equation is given by $\Phi'(u) = 0$, where Φ' denotes the Fréchet derivative of the functional Φ, also known as the energy functional. Motivated by physical applications, minimal energy solutions of this equation have been widely studied that is, solutions that minimize Φ over the set of nontrivial solutions. A classical approach to obtaining such solutions consists in minimizing the functional Φ over the set

$$\mathcal{N} = \mathcal{N}_\Phi := \{u \in X \setminus \{0\} \, ; \, \Phi'(u)u = 0\},$$

which, under certain additional assumptions on Φ, forms a C^1 manifold, known as the Nehari manifold.

In most of the results studied, the functional Φ has a local minimum, usually assumed to be $\Phi(0) = 0$, and is assumed to have the so-called mountain pass geometry. However, in this work we consider functionals that do not have a uniform geometry throughout the space X. Thus, we introduce certain types of assumptions on an open cone in X.

This work is based on my master's dissertation, which consists of a bibliographic study of reference [2]. We also present abstract results that will be applied to two distinct classes of functionals. Finally, we address an application involving one of these classes of functionals, namely the (p, q)-Laplacian problem.

2 Main Results

We consider an open, nonempty cone $Y \subset X \setminus \{0\}$, where X is a reflexive Banach space, and a functional $\Phi \in C^1(X, \mathbb{R})$ such that $\Phi(0) = 0$. We define the map $J : X \to \mathbb{R}$ by $J(u) = \Phi'(u)u$ and, for each $u \in X$, we define the fiber map $\varphi_u : [0, \infty) \to \mathbb{R}$ by $\varphi_u(t) = \Phi(tu)$. We also assume that both Φ and J are weakly lower semicontinuous in X. In general, we consider the following property:

$(H1)$ The map $t \mapsto \Phi(tu)$, defined on $(0, \infty)$, has a unique critical point t_u, which is a global maximum.

In most cases, we will find conditions on Φ such that

$$c^* := \inf_{\mathcal{N} \cap Y} \Phi \tag{1}$$

is attained, and provides a minimal energy level for Φ. An important property in the minimization process is the following compactness condition:

$(HY)_{c^*}$ If $(u_n) \subset \mathcal{N} \cap Y$ and $\Phi(u_n) \to d \in \mathbb{R}$, then (u_n) admits a subsequence that converges weakly in Y.

Theorem 2.1. *Assume that $(HY)_{c^*}$ holds for the value $c^* \in \mathbb{R}$ given in (1), and that $(H1)$ holds for every $u \in Y$. Then $c^* > 0$ and is attained by a critical point of Φ. Moreover, if $J(u) \neq 0$ for every $u \in X \setminus Y$ with $u \neq 0$, then c^* is a minimal energy level of Φ.*

As the main application of the theory, we consider problems involving operators of (p, q)-Laplacian type. Let $A(t) := \int_0^t a(s)\, ds$ for $t \geq 0$, where $a : [0, \infty) \to [0, \infty)$ is a C^1 function satisfying the following conditions:

(A1) $k_0 \left(1 + t^{\frac{q-p}{p}}\right) \leq a(t) \leq k_1 \left(1 + t^{\frac{q-p}{p}}\right)$ for all $t > 0$, where $k_0, k_1 > 0$ and $p \geq q > 1$ are constants;

(A2) a is non-increasing;

(A3) The maps $t \mapsto a(t^p)t^p$ and $t \mapsto A(t^p)$ are convex on $(0, \infty)$.

We consider the following problem:

$$-\operatorname{div}(a(|\nabla u|^p)|\nabla u|^{p-2}\nabla u) = g(x, u), \quad u \in W_0^{1,p}(\Omega), \tag{2}$$

which is a generalization of the (p, q)-Laplacian problem. Indeed, if we take $a(t) = 1+t^{\frac{q-p}{p}}$, the conditions (A1)-(A3) are satisfied.

Assume that $g(x, t) = 0$ for all $t \in \mathbb{R}$ and $x \in \Omega \setminus \Omega'$, where $\Omega' \subset \Omega$ is an open subset. Under the following assumptions:

(a) $\displaystyle\lim_{|t|\to 0} \frac{g(x,t)}{|t|^{q-1}} = 0$ uniformly for $x \in \Omega'$;

(b) There exists $r \in (p, p^*)$ such that $\displaystyle\lim_{|t|\to 0} \frac{g(x,t)}{|t|^{r-1}} = 0$ uniformly for $x \in \Omega'$;

(c) For all $x \in \Omega'$, the map $t \mapsto \dfrac{g(x,t)}{|t|^{p-1}}$ is increasing on $\mathbb{R} \setminus \{0\}$;

(d) $\displaystyle\lim_{|t|\to\infty} \frac{G(x,t)}{|t|^p} = \infty$ uniformly for $x \in \Omega'$, where $G(x, t) = \int_0^t g(x, s)\, ds$,

we can show that problem (2) admits a positive critical energy level.

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EXISTENCE RESULTS FOR THE 1−LAPLACIAN PROBLEM WITH A CRITICAL CONCAVE-CONVEX NONLINEARITY

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Abstract

In this work, we study a critical concave-convex type problem involving the 1−Laplacian operator in a general Lipschitz-continuous domain. We establish an existence result using an approximation method, where the solution is obtained as the limit of solutions to p-Laplacian type problems. To overcome the lack of compactness, a version of the well known Concentration Compactness Principle of Lions is used.

1 Introduction

Problems involving 1−Laplacian operator has received special attention in recent years, starting with the pioneering works written by F. Andreu, C. Ballesteler, V. Caselles and J.M. Mazón in a series of papers (among them [1, 2, 3]). Indeed, in [2], the authors characterize the imprecise quotient $\dfrac{Du}{|Du|}$ (when Du is just a Radon measure, rather than an L^1 function), through the Pairing Theory of G. Anzellotti [4]. This theory allows them to introduce a vector field $\mathbf{z} \in L^\infty(\Omega, \mathbb{R}^N)$ which plays the role of $\dfrac{Du}{|Du|}$.

In the last years, problems with critical growth have been extensively studied, beginning with the celebrate paper of Brezis and Nirenberg [5]. In [9] Garcia Azorero and Peral Alonso established an immediate extension of this problem for the p−Laplacian operator and Degiovanni and Magrone studied a version of the Brezis-Nirenberg problem to the 1-Laplacian operator, through a linking theorem [7]. On the other hand, in [8] the result of Brezis and Nirenberg has been extended by Demengel, who used the symmetry of the domain to get nodal solutions to problems involving the 1−Laplacian operator.

Recently, De Cicco, Giachetti and Segura [6] and Ortiz, Pimenta and Segura [12] established existence of solution for elliptic problem involving 1-Laplacian operator and a singular term.

Inspired by [12] and [6], we shall extend those results to the critical case. Indeed, the purpose of this work is to find a positive solution to the following elliptic equation involving 1-Laplacian operator which combines a singular term and a critical one

$$\begin{cases} -\mathrm{div}\left(\dfrac{Du}{|Du|}\right) &= \dfrac{\lambda}{u^\gamma} + |u|^{1^*-2}u \quad \text{in } \Omega \\ \qquad\qquad u &= 0 \quad \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open set having Lipschitz–continuous boundary, $N \geq 2$, $\lambda > 0$ and $0 < \gamma < 1$.

2 Main Results

Our main result is the following.

Theorem 2.1. *There exists a positive solution of* $(\mathcal{P}_\lambda)$ *for each* λ *small enough.*

The proof of Theorem 2.1 follows the ideas of [6], [12] and [9]. It is based on an approximation method. We emphasize that the proof presents an additional challenge due to the presence of a critical nonlinearity. To address this difficulty, we employ a result from [8], which is a version of Lions' Concentration-Compactness Principle [10].
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A MINIMUM PROBLEM ASSOCIATED WITH SCALAR GINZBURG-LANDAU EQUATION AND FREE BOUNDARY

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Abstract

Let $N > 2$, $p \in \left(\frac{2N}{N+2}, +\infty\right)$, and Ω be an open bounded domain in $\mathbb{R}^N$. We consider the minimum problem $\mathcal{J}(u) := \int_\Omega \left(\frac{1}{p}|\nabla u|^p + \lambda_1 \left(1-(u^+)^2\right)^2 + \lambda_2 u^+\right) \mathrm{d}x \to \min$ over a certain class $\mathcal{K}$, where $\lambda_1 \geq 0$ and $\lambda_2 \in \mathbb{R}$ are constants, and $u^+ := \max\{u, 0\}$. The corresponding Euler-Lagrange equation is related to the Ginzburg-Landau equation and involves a subcritical exponent when $\lambda_1 > 0$. For $\lambda_1 \geq 0$ and $\lambda_2 \in \mathbb{R}$, we prove the existence, non-negativity, and uniform boundedness of minimizers of $\mathcal{J}(u)$. Then, we show that any minimizer is locally $C^{1,\alpha}$-continuous with some $\alpha \in (0,1)$ and admits the optimal growth $\frac{p}{p-1}$ near the free boundary. Finally, under the additional assumption that $\lambda_2 > 0$, we establish non-degeneracy for minimizers near the free boundary and show that there exists at least one minimizer for which the corresponding free boundary has finite $(N-1)$-dimensional Hausdorff measure.

1 Introduction

Let Ω be an open bounded domain in $\mathbb{R}^N (N > 2)$ and $p \in \left(\frac{2N}{N+2}, +\infty\right)$. Let $\lambda_1 \geq 0$ and $\lambda_2 \in \mathbb{R}$ be constants. Given $g \in W^{1,p}(\Omega) \cap L^\infty(\Omega)$ with $g \geq 0$ on $\partial\Omega$, we consider the following minimum problem

$$\mathcal{J}(u)=\int_\Omega \left(\frac{1}{p}|\nabla u|^p + \lambda_1 \left(1-(u^+)^2\right)^2 + \lambda_2 u^+\right) \mathrm{d}x \to \min \tag{1}$$

over the set $\mathcal{K} := \left\{u \in W^{1,p}(\Omega);\ u - g \in W_0^{1,p}(\Omega)\right\}$.

The minimum problem of the type of (1) is known as the free boundary problem with free boundary $\Gamma^+ := (\partial\{x \in \Omega; u(x) > 0\}) \cap \Omega$, and has a wide range of applications in various fields.

In the past few decades, great efforts have been devoted to investigating the existence and regularities of minimizers of $\mathcal{J}(u)$ when $\lambda_1 = 0$ and $\lambda_2 > 0$, see for instance [1, 2, 3]. For example in [1], the authors considered the regularity properties for an obstacle problem for a quasilinear elliptic equation of p-Laplacian type. In [2], a complete description of the regularity was provided for a family of heterogeneous, two-phase variational free boundary problems. In the reference [3], a free boundary problem was studied in the Orlicz-Sobolev spaces setting. Further geometric properties of the free boundary can be found in [4, 5, 6]. In particular, it was proved in [4,5] that the free boundary has finite $(N-1)$-dimensional Hausdorff measure when $\lambda_1 = 0$, $\lambda_2 > 0$, and $p \in [2, +\infty)$. This result was included in [6] under the framework of Orlicz-Sobolev spaces. Nevertheless, property of the free boundary in the minimum problem of the type of (1) with $\lambda_1 \neq 0$ is less studied, except the work presented in [7], where finite $(N-1)$-dimensional Hausdorff measure of the free boundary was proved in the non-zero obstacle problem for the Ginzburg-Landau equation when $\lambda_1 > 0$, $\lambda_2 > 0$, and $p = 2$.

In this paper, we study the minimum problem (1) with $\lambda_1 \geq 0$, $\lambda_2 \in \mathbb{R}$, and p in a general case. It is worth noting that, compared with the work in the existing literature, both the higher order of u^+ and nonlinearity of the

p-Laplacian bring more intricacies when proving the existence and regularity of minimizers, as well as Hausdorff measure for the free boundary. This puts forward a constraint on p, namely, only the case of $p \in \left(\frac{2N}{N+2}, +\infty\right)$ is considered in this paper.

2 Main Results

First, we show the free boundary is locally porous.

Theorem 2.1. *Let u be a minimizer of $\mathcal{J}(u)$ with $\lambda_2 > 0$. Then, for every compact set $K \subset \Omega$, the intersection $K \cap \partial\{u > 0\}$ is porous with porosity constant σ depending only on N, p, λ_1, λ_2, $\|g\|_{L^\infty(\Omega)}$, $\|g\|_{W^{1,p}(\Omega)}$, and the diameter of Ω.*

Then, we show that there exists at least one minimizer with finite $(N-1)$-dimensional Hausdorff measure for the corresponding free boundary when $\lambda_2 > 0$.

Theorem 2.2. *Assume that $\lambda_2 > 0$. Let r_2 be a positive constant. Then, there exists at least one minimizer of $\mathcal{J}(u)$ over the set $\mathcal{K}$ such that, for any $x_0 \in \partial\{u > 0\} \cap B_{r_2}$ and $r \in (0, \frac{r_2}{2})$ there holds*

$$\mathcal{H}^{N-1}(B_r(x_0) \cap \Gamma^+) \leq Cr^{N-1},$$

where C is a positive constant depending only on r_2, N, p, λ_1, λ_2, $\|g\|_{L^\infty(\Omega)}$, $\|g\|_{W^{1,p}(\Omega)}$, and the diameter of Ω.

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SYNCHRONIZATION IN A QUASILINEAR SYSTEM

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Abstract

In this work, we analyze the $\varepsilon-$synchronization phenomenon in a class of quasilinear systems governed by the $p-$Laplacian operator. The main result is derived through the application of upper semicontinuity of the family of global attractors corresponding to the perturbed systems, combined with the synchronization behavior exhibited by the limiting system.

1 Introduction

In the early 1970s, the mathematicians Chafee and Infante completely described in [2] the bifurcation scheme and the stability framework of the equilibria of the semilinear problem

$$(CI)\begin{cases} u_t - \epsilon u_{xx} = u - u^3, & (x,t)\in(0,\pi)\times(0,+\infty)\\ u(0,t)=u(\pi,t)=0, & 0<t<+\infty\\ u(x,0)=u_0(x), & x\in(0,\pi).\end{cases}$$

More recently, the mathematicians Takeuchi and Yamada provided in [5] an equally detailed description of the bifurcation scheme and the stability framework of the equilibria of the quasilinear problem.

$$(P_\epsilon)\begin{cases} u_t - \epsilon(|u_x|^{p-2}u_x)_x = |u|^{q-2}u(1-|u|^r), & (x,t)\in(0,1)\times(0,+\infty)\\ u(0,t)=u(1,t)=0, & 0<t<+\infty\\ u(x,0)=u_0(x), & x\in(0,1)\end{cases}$$

where $p>2$, $q\geq 2$, $r>0$ and $\epsilon>0$.

In this work, we are interested in analyzing a system involving two equations of the type previously considered in [5], but coupled through an interaction term. More precisely, we consider a system of the form

$$(S_n)\begin{cases} u_t^n = \epsilon_1^n(|u_x^n|^{p_1^n-2}u_x^n)_x + |u^n|^{q_1^n-2}u^n(1-|u^n|^{r_1^n}) - k(u^n-v^n)\\ v_t^n = \epsilon_2^n(|v_x^n|^{p_2^n-2}v_x^n)_x + |v^n|^{q_2^n-2}v^n(1-|v^n|^{r_2^n}) + k(u^n-v^n)\end{cases}$$

in $W_0^{1,p_1^n}(0,1)\times W_0^{1,p_2^n}(0,1)$, where ϵ_i^n, k are positive constants, $q_i^n\geq 2$, $r_i^n\geq q_i^n-2$ e $p_i^n>2$, for $i=1,2$ and for any $n\in\mathbb{N}$. Our concern in analyzing the sequence (S_n) was to determine whether the system exhibits the phenomenon known as ϵ-synchronization as in [1]. That is, whether under the convergence $(\epsilon_i^n,p_i^n,q_i^n,r_i^n)\to(\epsilon,p,q,r)$, as $n\to\infty$ for $i=1,2$, given any $\eta>0$, there exists $n_0\in\mathbb{N}$ such that

$$\limsup_{t\to\infty}\|u^n(t)-v^n(t)\|_{L^2(0,1)}<\eta \qquad \forall n\geq n_0$$

which indeed occurs. Such time adjustment in coupled systems has been described in a large number of works, among which we cite [1, 3, 4].

2 Main Results

The following result guarantees the upper semicontinuity of the family of global attractors $\{\mathcal{A}_n,\ n \in \mathbb{N}\}$, that is, when $n \to \infty$, the attractors $\mathcal{A}_n$, of systems (S_n), approach $\mathcal{A}$ with respect to the Hausdorff distance, where $\mathcal{A}$ is the attractor associated with the limit system

$$(SL)\begin{cases} u_t = \epsilon(|u_x|^{p-2}\, u_x)_x + |u|^{q-2}\, u(1-|u|^r) - k(u-v) \\ v_t = \epsilon(|v_x|^{p-2}\, v_x)_x + |v|^{q-2}\, v(1-|v|^r) + k(u-v). \end{cases}$$

Theorem 2.1. *Consider the family $\{\mathcal{A}_n\}_{n\in\mathbb{N}}$ of attractors associated with the semigroups $\{S_n(t);\, t \geq 0\}_{n\in\mathbb{N}}$. As $n \to \infty$,*

$$d(\mathcal{A}_n, \mathcal{A}_L) \to 0,$$

where $\mathcal{A}_L$ is the attractor associated with the limit system (SL).

When analyzing the system (SL), it is possible to observe that if (u, v) is a solution of (SL) with initial data (u_0, v_0), then $u(t)$ and $v(t)$ become arbitrarily close as $t \to +\infty$, for each sufficiently large k, independently of the initial data. This property is known as "Synchronization" (see [1]).

Theorem 2.2. *Let (u, v) be a solution of the system (SL). Then, for all sufficiently large $k > 0$, we have*

$$\|u(t) - v(t)\|_2 \to 0 \quad as\ t \to \infty.$$

The following theorem is the main result of this paper. In this theorem, we prove that the system (S_n) achieves ϵ-synchronization.

Theorem 2.3. *Consider the system (S_n) with $(u^n(0,t), v^n(0,t)) = (0,0)$ and $(u^n(1,t), v^n(1,t)) = (0,0)$ for all $t \in (0, +\infty)$, and $(u^n(x,0), v^n(x,0)) = (u_0(x), v_0(x)) \in L^2(0,1) \times L^2(0,1)$. The parameters ϵ_i^n and k are positive constants, with $q_i^n \geq 2$, $p_i^n > 2$, and $r_i^n \geq q_i^n - 2$, for $i = 1,2$ and for all $n \in \mathbb{N}$. If*

$$(\epsilon_i^n, p_i^n, q_i^n, r_i^n) \to (\epsilon, p, q, r) \quad as\ n \to +\infty,$$

then, for sufficiently large k, the solution of (S_n) achieves ϵ-synchronization.

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ON THE CONTROLLABILITY OF PARABOLIC EQUATIONS WITH LARGE PARAMETERS IN SMALL TIME

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Abstract

We investigate the null controllability of the heat equation perturbed by a large anomalous diffusion term. We prove that the cost of control decays exponentially as the parameter grows, thus confirming a conjecture by J.-L. Lions and E. Zuazua. Furthermore, by combining the source term method with an inverse mapping theorem, we extend our results to certain nonlinear systems, showing that their controllability cost also vanishes in the large parameter regime.

1 Introduction

Let $\Omega \subset \mathbb{R}^N$, with $N \geq 1$, be a bounded connected open set whose boundary $\partial\Omega$ is regular enough. Let $T > 0$, and let ω be a nonempty subset of Ω, which will usually be referred to as the *control domain*.

We deal with the heat equation with an anomalous diffusion term given by

$$\left| \begin{array}{lll} u_t - \Delta u + k(-\Delta)^\theta u = f_k 1_\omega & \text{in} & Q, \\ u = 0 & \text{on} & \Sigma, \\ u(0) = u_0 & \text{in} & \Omega, \end{array} \right. \tag{1}$$

where $0 \leq \theta < 1$, $k \in \mathbb{R}$, and $(-\Delta)^\theta$ denotes the spectral fractional Laplacian.

It is well-known that, for every $T > 0$, each initial condition $u_0 \in L^2(\Omega)$, and any $k \in \mathbb{R}$, there exists a control $f_k \in L^2(\omega \times (0,T))$ such that the associated solution u of system (1) satisfies $u(T) = 0$ in Ω. Moreover, there exists a constant $C_k(T) > 0$, such that

$$\|f_k\|_{L^2(\omega\times(0,T))} \leq C_k(T) \|u_0\|_{L^2(\Omega)}. \tag{2}$$

When $k \to -\infty$, Lions and Zuazua, in [1], showed that $C_k(T) \to +\infty$ due to the instability of system (1). They conjectured the opposite behavior as $k \to +\infty$, suggesting that controllability becomes easier, i.e., $C_k(T) \to 0$.

This work provides a positive and quantitative answer to that conjecture by proving that the cost decays exponentially as $k \to +\infty$. Furthermore, we extend the result to certain semilinear problems, including nonlinearities in both the state and its gradient.

2 Main Results

We begin by proving that the cost of null controllability for the linear equation decays exponentially with respect to the parameter k.

Theorem 2.1. *Let $0 < T_0 \leq 1$ and let $u_0 \in L^2(\Omega)$. Then, for every $T \in (0, T_0)$, there exists a control function $f_k \in L^2(\omega \times (0,T))$ such that the solution to (1) satisfies $u(T) = 0$ in Ω and, for any $k > 0$, the control cost satisfies*

$$\|f_k\|_{L^2(\omega\times(0,T))} \leq C_1 e^{-C_2 kT} e^{\frac{C_3}{T}} \|u_0\|_{L^2(\Omega)},$$

where the constants C_1, C_2, C_3 are positive and do not depend on k.

This result answers the Lions-Zuazua conjecture by proving that the control cost decays exponentially, via a spectral observability inequality inspired by Seidman in [2], and Lebeau-Robbiano in [3]. We also extend the approach to nonlinear systems, showing that the exponential decay in the linear case allows controllability through the source term method developed in [4] and an appropriate inverse mapping theorem.

Theorem 2.2. *Let $1 \leq N \leq 4$, and let $0 < T_0 \leq 1$, $T \in (0, T_0)$. For each $k > 0$, consider the system*

$$\left|\begin{array}{lll} u_t - \Delta u + k(-\Delta)^\theta u + |u|^{l-1}u = f1_\omega & in & Q, \\ u = 0 & on & \Sigma, \\ u(0) = u_0 & in & \Omega, \end{array}\right. \tag{1}$$

for certain values of l that depend on the dimension N. Then, there exists $\gamma > 0$, not depending on k, such that, for any $u_0 \in L^2(\Omega)$ satisfying $\|u_0\|_{L^2(\Omega)} \leq \gamma$, there exists a control $f_k \in L^2(\omega \times (0,T))$, and constants $C_1, C_2 > 0$, independent of k, such that the corresponding solution to (1) satisfies $u(T) = 0$ in Ω and the control has the estimate

$$\|f_k\|^2_{L^2(\omega\times(0,T))} \leq C_1 e^{-C_2 kT}.$$

Theorem 2.3. *Let $1 \leq N \leq 3$, and let $0 < T_0 \leq 1$, $T \in (0, T_0)$ and let $\vec{A} \in \mathbb{R}^N$ be a constant vector. For each $k > 0$, consider the following system*

$$\left|\begin{array}{lll} u_t - \Delta u + k(-\Delta)^\theta u + u(\vec{A}\cdot\nabla u) = f1_\omega & in & Q, \\ u = 0 & on & \Sigma, \\ u(0) = u_0 & in & \Omega. \end{array}\right. \tag{2}$$

Then, there exists $\gamma > 0$, not depending on k, such that, for any $u_0 \in L^2(\Omega)$ satisfying $\|u_0\|_{L^2(\Omega)} \leq \gamma$, there exists a control $f_k \in L^2(\omega \times (0,T))$, and constants $C_1, C_2 > 0$, independent of k, such that the corresponding solution to (2) satisfies $u(T) = 0$ in Ω and the control has the estimate

$$\|f_k\|^2_{L^2(\omega\times(0,T))} \leq C_1 e^{-C_2 kT}.$$

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GLOBAL EXISTENCE FOR A COUPLED PARABOLIC SYSTEMS WITH GENERAL SOURCES

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Abstract

We obtain optimal conditions for the existence of global solutions for the parabolic coupled system $u_t - \Delta u = h(t)f(v)$ and $v_t - \Delta v = l(t)g(u)$ in $\Omega \times (0,T)$ with the Dirichlet boundary conditions. Here, $\Omega \subset \mathbb{R}^N$ is either a bounded or unbounded domain, the initial data belong to $[C_0(\Omega)]^2$, and the functions $f, g, h, l \in C[0,\infty)$.

1 Introduction

Let $\Omega \subset \mathbb{R}^N$ be either a smooth bounded or unbounded domain. We are interested in the following coupled parabolic system

$$\begin{cases} u_t - \Delta u & = h(t)f(v) \quad \text{in } \Omega \times (0,T), \\ v_t - \Delta v & = l(t)g(u) \quad \text{in } \Omega \times (0,T), \\ u = 0, & v = 0 \quad \text{on } \partial\Omega \times (0,T), \\ u(0) = u_0, & v(0) = v_0 \quad \text{in } \Omega, \end{cases} \tag{1}$$

where $u_0, v_0 \in C_0(\Omega)$ are nonnegative functions; $h, l : [0,\infty) \to [0,\infty)$ are continuous; the functions $f, g : [0,\infty) \to [0,\infty)$ are locally Lipschitz continuous. When $u_0 = v_0$, $f(t) = g(t) = t^p$ and $h(t) = l(t)$, problem (1) is the well-known semilinear parabolic problem

$$\begin{cases} u_t - \Delta u & = & h(t)u^p & \text{in } \Omega \times (0,T), \\ u & = & 0 & \text{on } \partial\Omega \times (0,T), \\ u(0) & = & u_0 & \text{in } \Omega. \end{cases} \tag{2}$$

In [?], P. Meier proved that the problem (2) admits a global solution, if there exists $w_0 \in L^\infty(\Omega)$, with $w_0 \neq 0$ and $w_0 \geq 0$ such that the following condition $\int_0^\infty h(\sigma)\|S(\sigma)w_0\|_\infty^{p-1} d\sigma < \infty$ is verified. Here, $\{S(t)\}_{t\geq 0}$ denotes the Dirichlet heat semigroup in Ω. He also analyzed the nonexistence of global solutions. This result was extended for a general coupled system with polynomial sources in [1], [2], and [3] using an iterative semigroup method. In a similar context, in 2016, Ishige, Kawakami and SierÅ¼ega (see [5]) developed a supersolution approach to establish the global existence of solutions for parabolic coupled systems involving polynomial sources. Recently, in 2025, Meier-type results

ave been successfully extended to the Hardy-Henón parabolic equation (see [4]).

The main challenges in our study arose from the general nature of the functions f and g in problem (1), as well as from the generality of the domain Ω. Furthermore, the interaction between u and v complicates any direct extension of the earlier arguments.

2 Main result

Set

$$F(t) = \int_0^t f(s)ds \text{ and } G(t) = \int_0^t g(s)ds.$$

We consider the following assumptions:

(H_1) $\mathcal{F}_0 = \limsup_{s\to 0^+} \frac{f(F^{-1}(G(s)))}{s} < \infty$ and $\mathcal{G}_0 = \limsup_{s\to 0^+} \frac{g(s)}{F^{-1}(G(s))} < \infty$.

(H_2) There exists $m_0 > 0$ such that the functions f and g are nondecreasing in $[0, m_0]$ and $f(s), g(s) > 0$ for all $0 < s < m_0$.

(H_3) The function $F^{-1}(G(s))$ is concave on $[0, m_0]$.

Theorem 2.1. *Suppose that* $hl : [0,\infty) \to [0,\infty)$ *are continuous,* $f, g : [0,\infty) \to [0,\infty)$ *are locally Lipschitz continuous with* $f(0) = g(0) = 0$*, and the conditions* (H_1)*-*(H_3) *hold. If there exists a nontrivial* $w_0 \in C_0(\Omega)$ *with* $w_0 \geq 0$ *and* $\max_{\{\sigma\geq 0\}} \left\{\|w_0\|_\infty, \|F^{-1}(G(S(\sigma)w_0))\|_\infty\right\} \leq m_0$*, such that*

$$\int_0^\infty h(\sigma)\mathcal{F}(\|S(\sigma)w_0\|_\infty)d\sigma < 1 \text{ and } \int_0^\infty l(\sigma)\mathcal{G}(\|S(\sigma)w_0\|_\infty)d\sigma < 1, \text{ with}$$

$$\mathcal{F}(s) := \sup_{0<t\leq s} \frac{f(F^{-1}(G(t)))}{t} \quad \text{and} \quad \mathcal{G}(s) := \sup_{0<t\leq s} \frac{g(t)}{F^{-1}(G(t))} \quad \text{for } s > 0$$

($\mathcal{F}(0) := \mathcal{F}_0$ *and* $\mathcal{G}(0) := \mathcal{G}_0$*), then there exists* $(u, v) \in [L^\infty((0,\infty), C_0(\Omega))]^2$ *which is a nontrivial global solution to* (1).

Remark. Problem (1) with $h(t) = l(t)$, $f(t) = t^p$ and $g(t) = t^q$ was treated in [1, Theorem 2]. In this case, Theorem [**?**] establishes the global existence of a solution when $\int_0^\infty h(\sigma)\|S(\sigma)w_0\|_\infty^{\frac{pq-1}{p+1}} d\sigma < \infty$, where $p \geq q \geq 1$ and $pq > 1$.

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LAMÉ SYSTEM WITH NONLINEAR VECTOR FIELD UNDER CRITICAL EXPONENT

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Abstract

In this work, we investigate the existence of a compact global attractor that has finite fractal dimension for the Lamé system. The decomposition of the nonlinearity into a coupled and a decoupled component is a well-established approach in the mathematical literature. Our objective is to propose an alternative method capable of handling the critical case without requiring this structural assumption on the nonlinearity, that is, we consider the general case for the nonlinear vector fields under critical exponents.

1 The problem

Let us consider the following problem

$$\begin{cases} \mathbf{u}_{tt} - \Delta_e \mathbf{u} + \mathbf{f}(\mathbf{u}) + \alpha \mathbf{u}_t = \mathbf{h} \quad \text{in} \quad \Omega \times \mathbb{R}^+, \\ \mathbf{u} = 0 \quad \text{on} \quad \partial\Omega \times \mathbb{R}^+, \\ \mathbf{u}(0) = \mathbf{u}_0, \mathbf{u}_t(0) = \mathbf{u}_1 \quad \text{in} \quad \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded domain with smooth boundary $\partial\Omega$, $\mathbf{u} = (u_1, u_2, u_3)$ is the unknown vector solution depending on (x, t), with $x = (x_1, x_2, x_3) \in \Omega$ and $t \geq 0$, $\mathbf{f}(\mathbf{u})$ is a vector-valued nonlinear perturbation, $\mathbf{h} = \mathbf{h}(x)$ is an external force in $L^2(\Omega)$, $\alpha > 0$ is the damping coefficient, and Δ_e stands for the elastic LamÃ© operator given by

$$\Delta_e \mathbf{u} = \mu \Delta \mathbf{u} + (\lambda + \mu) \nabla \mathrm{div} \mathbf{u}, \ \mu > 0, \ \lambda + \mu \geq 0. \tag{2}$$

2 Main Results

Assumption 1. Let us assume $\mathbf{h} \in \mathbf{L}^2(\Omega)$ and $\alpha > 0$.

Concerning the nonlinear operator $\mathbf{f}(\mathbf{u}) = (\mathbf{f_1}(\mathbf{u}), \mathbf{f_2}(\mathbf{u}), \mathbf{f_3}(\mathbf{u}))$, we assume the following growth condition in critical scenarios.

Assumption 2. Let us assume that there exist $\mathbf{g} = (g_1, g_2, g_3) \in \mathbf{C}^1(\mathbb{R}^3)$ and functions $G \in C^2(\mathbb{R}^3)$ and $\gamma_i \in C^2(\mathbb{R})$, $i = 1, 2, 3$, such that the vector field $\mathbf{f} = (f_1, f_2, f_3)$ satisfies:

$$f_i(u_1, u_2, u_3) = g_i(u_1, u_2, u_3) + \gamma_i(u_i), \quad i = 1, 2, 3,$$

$$f_i(0) = g_i(0) = \gamma_i(0) = 0, \quad i = 1, 2, 3,$$

$$\mathbf{g} = (g_1, g_2, g_3) = \nabla G.$$

In addition, there exist constants $M, m_f \geq 0$ such that

$$f(u) \cdot u - G(u) - \sum_{i=1}^{3} \int_0^{u_i} \gamma_i(s)\, ds \geq -M|u|^2 - m_f, \quad \forall u \in \mathbb{R}^3, \tag{1}$$

$$G(u) + \sum_{i=1}^{3} \int_0^{u_i} \gamma_i(s)\, ds \geq -M|u|^2 - m_f, \quad \forall u \in \mathbb{R}^3, \tag{2}$$

with $0 \leq M < \frac{\mu\lambda_1}{2}$, where $\lambda_1 > 0$ denotes the first eigenvalue of the Laplacian operator $-\Delta$. With respect to functions g_i and γ_i, $i = 1, 2, 3$, we assume:

- g fulfills the growth restriction: there exist $M_g > 0$ such that, for $i = 1, 2, 3$,

$$|\nabla g_i(u)| \leq M_g \left(1 + |u_1|^{p-1} + |u_2|^{p-1} + |u_3|^{p-1}\right), \quad \forall u = (u_1, u_2, u_3) \in \mathbb{R}^3, \tag{3}$$

  for $1 \leq p \leq 3$.

- For each $i = 1, 2, 3$, h_i fulfills the critical growth restriction: there exists a constant $c_h > 0$ such that

$$|\gamma_i'(x)| \leq c_\gamma(1 + |x|^2), \quad \forall x \in \mathbb{R}, \quad i = 1, 2, 3. \tag{4}$$

By denoting $\mathbf{v} = \mathbf{u}_t$ and $\mathbf{U}(t) = (\mathbf{u}(t), \mathbf{v}(t))$, $t \geq 0$, we rewrite problem (1) in the following abstract Cauchy problem

$$\begin{cases} \mathbf{U}_t = \Delta_{\mathcal{E}}\mathbf{U} + \mathbf{F}(\mathbf{U}), \quad t > 0, \\ \mathbf{U}(0) = (\mathbf{u}_0, \mathbf{u}_1) := \mathbf{U}_0, \end{cases} \tag{5}$$

with linear $\Delta_{\mathcal{E}} : D(\Delta_{\mathcal{E}}) \subset \mathcal{H} \to \mathcal{H}$ and nonlinear $\mathbf{F} : \mathcal{H} \to \mathcal{H}$ operators given by

$$\Delta_{\mathcal{E}}\mathbf{U} = (\mathbf{v}, \Delta_e\mathbf{u} - \alpha\mathbf{v}) \quad \text{and} \quad \mathbf{F}(\mathbf{U}) = (0, \mathbf{H} - \mathbf{f}(\mathbf{u})),$$

with $\mathcal{H} = [H_0^1(\Omega)]^3 \times [L^2(\Omega)]^3$ and $D(\Delta_{\mathcal{E}}) = [(H^2(\Omega) \cap H_0^1(\Omega))]^3 \times [H_0^1(\Omega)]^3$. Under Assumptions 1 and 2, one can show that problem (5) is well-posed on $\mathcal{H}$. Then, we can generate the dynamical system $(\mathcal{H}, S(t))$ corresponding to problem (1) via mild solutions $\mathbf{U}(t) = (\mathbf{u}(t), \mathbf{u}_t(t))$, $t \geq 0$, of (5), namely,

$$S(t)(\mathbf{u}_0, \mathbf{u}_1) = (\mathbf{u}(t), \mathbf{u}_t(t)), \quad (\mathbf{u}_0, \mathbf{u}_1) \in \mathcal{H}. \tag{6}$$

In [1, 2] the authors consider $p < 3$ (subcritical growth condition) in (3) or $g_i = 0$ for $i = 1, 2, 3$ in (3). Here, our main goal is to explore the critical case $p = 3$ in (3), that is, critical growth condition for the vector field $\mathbf{g} = (g_1, g_2, g_3)$. More precisely, we have:

Theorem 2.1. *Under the Assumptions 1 and 2 with $p = 3$, the dynamical system $(\mathcal{H}, S(t))$ given by (6) possesses a unique compact global attractor $\mathcal{A}$ with finite fractal dimension in $\mathcal{H}$ ($\dim_{\mathcal{H}}^f \mathcal{A} < \infty$). Moreover, the attractor $\mathcal{A}$ is bounded in $D(\Delta_{\mathcal{E}})$.*

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A STUDY OF THE GLOBAL WELL-POSEDNESS FOR THE INHOMOGENEOUS NAVIER-STOKES EQUATIONS WITH VARIABLE VISCOSITY IN BESOV SPACES

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Abstract

In this work, we consider the incompressible inhomogeneous Navier-Stokes equations with density-dependent viscosity. Our main objective is to analyze the local and global well-posedness in time, assuming initial conditions belonging to homogeneous Besov spaces. In order to ensure the global existence of strong solutions, a smallness condition on the initial velocity is required. In this work, we establish regularity propagation estimates from initial data $u_0 \in \dot{B}^{\frac{3}{p_2}+s}_{p_2,r}$ to the solution u for $t \in (0,1)$, under the assumptions $q \in [1,2]$ and $p \in \left(\frac{6}{5}, 6\right)$, with

$$\frac{1}{2} - \frac{1}{q} \leq \frac{1}{p_1} \leq \frac{1}{2} + \frac{1}{q}.$$

The approach based on the auxiliary space $\dot{B}^{\frac{3}{p_2}+s}_{p_2,r}$ constitutes a key step toward handling a new class of initial conditions and extending the admissible range of the parameter p.

1 Introduction

The inhomogeneous incompressible Navier-Stokes equations with density-dependent viscosity are described by the system:

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0, & (t,x) \in \mathbb{R}^+ \times \mathbb{R}^3 \\ \partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) - \operatorname{div}(2\mu(\rho)\mathbb{D}u) + \nabla \pi = 0, & \\ \operatorname{div} u = 0, & \\ (\rho, u)|_{t=0} = (\rho_0, u_0), & \end{cases} \tag{1}$$

where ρ denotes the fluid density, u the velocity field, $\mathbb{D}u = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$ represents the symmetric part of the velocity gradient, π is the pressure, and $\mu(\rho)$ is a smooth positive function defined on $[0, \infty)$.

Our approach will be based on the framework of homogeneous Besov spaces $\dot{B}^s_{p,q}$. Given $s \in \mathbb{R}$, $p, q \in [1, \infty]$, and $f \in \mathcal{S}'_h(\mathbb{R}^3)$, we define the norm:

$$\|f\|_{\dot{B}^s_{p,q}} := \left\| \left(2^{js} \|\dot{\Delta}_j f\|_{L^p} \right)_{j \in \mathbb{Z}} \right\|_{l^q},$$

where $\dot{\Delta}_j f$ denotes the homogeneous dyadic localization of f at frequency level $j \in \mathbb{Z}$. Accordingly, we define the space:

$$\dot{B}^s_{p,q} := \left\{ f \in \mathcal{S}'_h(\mathbb{R}^3) \mid \|f\|_{\dot{B}^s_{p,q}} < +\infty \right\}.$$

2 Main Results

We establish the inequality

$$\begin{aligned}
&\|u\|_{\tilde{L}^r_t(\dot{B}^{\frac{3}{p_2}+s}_{p_2,r})} + \|u\|_{\tilde{L}^1_t(\dot{B}^{\frac{3}{p_2}+s+2}_{p_2,r})} + \|\nabla\pi\|_{\tilde{L}^\infty_t(\dot{B}^{\frac{3}{p_2}+s}_{p_2,r})} \\
&\qquad \lesssim \Big(\|u_0\|_{\dot{B}^{\frac{3}{p_2}+s}_{p_2,r}} + t^{\frac{1}{4}} 2^{k(s+\frac{3}{2})}\big(1+\|u\|_{L^\infty_t(\dot{B}^{-1+\frac{3}{p_1}}_{p_1,1})}\big) \\
&\qquad\qquad \times \|u\|^{\frac{1}{4}}_{L^\infty_t(\dot{B}^{-1+\frac{3}{p_1}}_{p_1,1})} \|u\|^{\frac{1}{4}}_{L^1_t(\dot{B}^{1+\frac{3}{p_1}}_{p_1,1})}\Big) \\
&\qquad\qquad \times \exp\Big\{C\|u\|_{\tilde{L}^1_t(\dot{B}^{\frac{3}{p_1}+1}_{p_1,r})} + Ct2^{2k}\|(b,\lambda)\|^2_{\tilde{L}^1_t(\dot{B}^{\frac{3}{q}}_{q,r})} \\
&\qquad\qquad\qquad + C\|(a,b,\lambda)\|^2_{\tilde{L}^r_t(\dot{B}^{\frac{3}{q}}_{q,r})}\Big\},
\end{aligned}$$

where $1 \le q \le p_1 \le p_2 \le \infty$, and

$$\frac{6}{5} < p_1 < 6, \qquad \frac{1}{2} - \frac{1}{q} \le \frac{1}{p_1} \le \frac{1}{2} + \frac{1}{q}, \qquad -1 > s > \max\left\{-\frac{3}{p_1} - \frac{3}{p_2} + 1, -2\right\}.$$

Taking into account the local well-posedness result established in [2], our present goal is to obtain the global extension of the solution to system (1) by deriving regularity propagation estimates in the auxiliary norm $\dot{B}^{\frac{3}{p_2}+s}_{p_2,r}$, under the smallness condition

$$\|u_0\|_{\dot{B}^{-1+\frac{3}{p_1}}_{p_1,1}} \le \varepsilon,$$

where ε depends on $\|\rho_0 - 1\|_{B^{\frac{3}{q}}_{q,1}}$, $\|\nabla\mu(\rho_0)\|_{B^{\frac{3}{q}-\frac{3}{r}}_{q,1}}$, and $\|u_0\|_{\dot{B}^{\frac{3}{p_2}+s}_{p_2,r}}$.

This work is based on [1]. In that article, published in 2024, the authors established both local and global well-posedness results for this system in the case $q \in [1,2]$ and $p \in \left(1, \frac{9}{2}\right]$, under the index condition

$$\max\left\{\frac{1}{p}, 1 - \frac{2}{p}\right\} \le \frac{1}{q} \le \frac{1}{p} + \frac{1}{3},$$

with initial data $(\rho_0 - 1, \nabla\mu(\rho_0), u_0) \in B^{\frac{3}{q}}_{q,1}(\mathbb{R}^3) \times L^r(\mathbb{R}^3) \times \dot{B}^{-1+\frac{3}{p}}_{p,1} \cap \dot{H}^{-2\delta}(\mathbb{R}^3)$.

A key aspect of their analysis was the smallness assumption on the initial velocity in the auxiliary space $\dot{H}^{-2\delta}$. However, the use of the space $\dot{H}^{-2\delta}$ imposes certain limitations, which we aim to overcome by employing the homogeneous Besov space $\dot{B}^{\frac{3}{p_2}+s}_{p_2,r}$ as an alternative auxiliary space.

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EXISTENCE OF GLOBAL SOLUTION AND GLOBAL ATTRACTOR FOR A SUSPENDED BRIDGE SYSTEM OF KIRCHHOFF TYPE WITH FRACTIONAL DERIVATIVE DAMPING

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Abstract

This paper investigates an abstract Kirchhoff-type suspension bridge model with fractional internal damping. Using the theory of semigroups of bounded linear operators, we establish the existence and uniqueness of global strong solutions for the model. Through the theory of continuous (nonlinear) operator semigroups, we prove that the associated semigroup is gradient and asymptotically compact, which enables us to demonstrate the existence of a global attractor characterized by the system's stationary solutions. Furthermore, we show that the semigroup is asymptotically quasistable, yielding two important consequences: (1) the attractor has finite fractal dimension and (2) enhanced solution regularity.

1 Introduction

In addition to the classical Timoshenko beam model, one of the most significant contributions to beam theory was introduced by Kirchhoff in 1876. This model describes the nonlinear transverse vibrations of a stretched string and is characterized by the following integro-differential equation:

$$w_{tt}(x,t) - m\left(\int_\Omega |\nabla w(x,t)|^2 dx\right)\Delta w(x,t) = f(x,t);\ x\in\Omega \ \text{ and } \ t>0. \tag{1}$$

Nonlinear models based on Kirchhoff's theory have been widely studied. In this regard, [2] studied the existence and uniqueness of global solutions, as well as exponential stability, for a coupled system of Kirchhoff beams with weak damping and a logarithmic source. A central aspect in the study of nonlinear systems involves the analysis of global attractors. In this direction, [1] investigated the existence of attractors for a nonlinear thermoelastic system with rotational inertia and time delay. De Jesus and collaborators in [3], studied the well-posedness and asymptotic behavior of a Timoshenko beam model with internal damping of the Caputo fractional derivative type.

Motivated by the works of [1, 3, 2], we propose to investigate the existence of solutions and global attractors for a nonlinear suspension bridge system where the deck is modeled by Kirchhoff beam theory with fractional damping. More precisely, we consider the following system:

$$u_{tt} - \Delta u + |u|^{\rho_1} - \tau(w-u) + c_1\partial_t^{\alpha,\eta}u = f, \quad \text{on } \ \Omega\times(0,+\infty), \tag{2}$$

$$w_{tt} + \Delta^2 w - m\left(\int_\Omega |\nabla w|^2 dx\right)\Delta w + |w|^{\rho_2}w + \tau(w-u) + c_2\partial_t^{\beta,\zeta}w = g, \quad \text{on } \ \Omega\times(0,+\infty), \tag{3}$$

$$u = 0 \text{ and } w = \frac{\partial w}{\partial \nu} = 0, \quad \text{on } \ \partial\Omega\times(0,+\infty), \tag{4}$$

$$u(x,0)=u_0(x),\ u_t(x,0)=u_1(x),\ w(x,0)=w_0(x) \text{ and } \ w_t(x,0)=w_1(x), \quad \text{on } \Omega, \tag{5}$$

where $\partial_t^{\omega,\delta}$ denotes the *exponentially modified Caputo fractional derivative operator of order* ω *and weight* δ.

2 Main Results

Theorem 2.1. *If $u_0 \in H_0^1(\Omega)$, $w_0 \in H_0^2(\Omega)$ and $u_1, w_1 \in L^2(\Omega)$, then the initial-boundary value problem (2)-(5) admits a unique mild solution (u, w) with the following regularity:*

$$\begin{cases} u \in C^0\left([0,+\infty); H_0^1(\Omega)\right) \cap C^1\left([0,+\infty); L^2(\Omega)\right) \\ w \in C^0\left([0,+\infty); H_0^2(\Omega)\right) \cap C^1\left([0,+\infty); L^2(\Omega)\right). \end{cases} \tag{1}$$

Moreover, if $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$, $w_0 \in H_0^4(\Omega) \cap H_0^2(\Omega)$, $u_1 \in H_0^1(\Omega)$ and $w_1 \in H_0^2(\Omega)$, then the problem (2)-(5) admits a unique strong solution (u, w) satisfying:

$$\begin{cases} u \in L^\infty_{loc}\left(0,+\infty;\ H^2(\Omega) \cap H_0^1(\Omega)\right) \\ w \in L^\infty_{loc}\left(0,+\infty;\ H^4(\Omega) \cap H_0^2(\Omega)\right) \\ u_t \in L^\infty_{loc}\left(0,+\infty;\ H_0^1(\Omega)\right) \\ w_t \in L^\infty_{loc}\left(0,+\infty;\ H_0^2(\Omega)\right) \\ u_{tt}, w_{tt} \in L^\infty_{loc}\left(0,+\infty;\ L^2(\Omega)\right) \end{cases}$$

Theorem 2.2. *The semigroup $\{T(t)\}_{t\geq 0}$ of continuous operators associated with problem (2)-(5) possesses a global attractor with finite-fractal dimension in $\mathcal{H}$ and characterized by the unstable manifold $\mathcal{M}^u(\mathcal{N})$ of the set of stationary solutions.*

Corollary 2.1. *The solution (u, w) to Problem (2)-(5) has the following additional regularity properties: $u_t \in L^\infty\left(0,+\infty;\ H_0^1(\Omega)\right)$, $w_t \in L^\infty\left(0,+\infty;\ H_0^2(\Omega)\right)$ and $u_{tt}, w_{tt} \in L^\infty\left(0,+\infty;\ L^2(\Omega)\right)$.*

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BLOW-UP AND GLOBAL SOLUTIONS FOR A SINGULAR PARABOLIC EQUATION ON THE HEISENBERG GROUP

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Abstract

We study global solutions and blow-up for the parabolic problem $u_t - \mathcal{L}u = |\cdot|_{\mathbb{H}}^{\gamma} u^p$ in $\mathbb{H}^N \times (0,T)$, with $u(0) = u_0$ in $\mathbb{H}^N$, where $\mathcal{L}$ is a sub-Laplacian, and $\mathbb{H}^N$ denotes the Heisenberg group.

1 Introduction

The Heisenberg group $\mathbb{H}^N$ is the prototypical example of a 2-step stratified nilpotent Lie group. As a smooth manifold, $\mathbb{H}^N \cong \mathbb{R}^{2N+1}$, with group law $\circ$ given by

$$(x,y,t) \circ (\tilde{x}, \tilde{y}, \tilde{t}) = (x + \tilde{x},\ y + \tilde{y},\ t + \tilde{t} + 2\,(x \cdot \tilde{y} - \tilde{x} \cdot y)), \tag{1}$$

where $\cdot$ denotes the standard scalar product in $\mathbb{R}^N$. The identity element in $\mathbb{H}^N$ is 0 and the inverse is $z^{-1} = -z$ for any $z \in \mathbb{H}^N$. The Hausdorff dimension of $\mathbb{H}^N$ is $Q = 2N + 2$, which is strictly greater than the topological dimension $2N + 1$.

The associated Lie algebra $\mathfrak{h}^N$ of $\mathbb{H}^N$ admits a stratification

$$\mathfrak{h}^N = V_1 \oplus V_2, \quad \text{with } \dim V_1 = 2N,\ \dim V_2 = 1,$$

where $V_2 = [V_1, V_1]$ and $[V_1, V_2] = \{0\}$, with the notation $[V_i, V_j] := \operatorname{span}\{[X,Y] \mid X \in V_i,\ Y \in V_j\}$ denoting the subspace generated by all Lie brackets between elements of V_i and V_j. A basis for V_1 is given by the vector fields $X_j = \partial_{x_j} - 2y_j\partial_t$, $Y_j = \partial_{y_j} + 2x_j\partial_t$ $(1 \le j \le N)$, with the only nontrivial commutators being $[X_j, Y_j] = 4\partial_t$. The vector fields X_j, Y_j span the horizontal distribution $\mathcal{H} \subset T\mathbb{H}^N$, which defines the sub-Riemannian structure in $\mathbb{H}^N$.

The group $\mathbb{H}^N$ admits a family of anisotropic dilations δ_λ given by

$$\delta_\lambda(x,y,t) = (\lambda x, \lambda y, \lambda^2 t), \quad \lambda > 0.$$

The function $|\eta|_{\mathbb{H}} = \left[\left(|x|^2 + |y|^2\right)^2 + |\tau|^2\right]^{1/4}$ is a symmetric homogeneous norm on the group $\mathbb{H}^N$, in the sense that it is homogeneous with respect to the group dilations $|\delta_\lambda z|_{\mathbb{H}} = \lambda |z|_{\mathbb{H}}$ for all $z \in \mathbb{H}^N$ and $\lambda > 0$. Such a function is known as the Korányi norm, and it is equivalent to the Carnot-Carathéodory distance d_{cc}, which is the natural sub-Riemannian distance on $\mathbb{H}^N$ ([1, Proposition 5.1.4 and Theorem 5.2.8]).

The natural differential operator associated with the sub-Riemannian geometry is the sub-Laplacian, defined by

$$\mathcal{L} := \sum_{j=1}^{N} \left(X_j^2 + Y_j^2\right).$$

It generates a strongly continuous, symmetric Markovian semigroup $(e^{-t\mathcal{L}})_{t \ge 0}$ on $L^2(\mathbb{H}^N)$, which admits an integral kernel $h_t(x,y)$. This kernel is smooth on $(0,\infty) \times \mathbb{H}^N \times \mathbb{H}^N$, strictly positive, and satisfies the following Gaussian-type estimate

$$h_t(x,y) \sim \frac{1}{t^{Q/2}} \exp\left(-c\,\frac{d_{cc}(x,y)^2}{t}\right), \quad t > 0,$$

for some constant $c > 0$.

In this work, we study global existence and blow-up for nonnegative solutions of the parabolic problem

$$\begin{cases} u_t - \mathcal{L}u & = |\cdot|_{\mathbb{H}}^{\gamma} u^p \quad \text{in } \mathbb{H}^N \times (0,T), \\ u(0) & = u_0 \quad \text{in } \mathbb{H}^N, \end{cases} \tag{2}$$

where $T > 0$, $p > 1$ and $\gamma > -2$.

In the Euclidean case $\mathbb{R}^N$, equation (2) has already been extensively studied. In particular, several authors (see [3], for example) have shown that the Fujita critical exponent for equation (2) is $p_c = 1 + (2+\gamma)/N$. The exponent p_c is called the Fujita critical exponent: if $1 < p \leq p_c$, then all nonnegative solutions blow up in finite time, while for $p > p_c$, global nonnegative solutions exist. When $\gamma = 0$, problem (2) was studied in the setting of general stratified nilpotent Lie groups in [2], where it was shown that the Fujita critical exponent depends on the Hausdorff dimension. Specifically, it was proved that $p_c = 1 + 2/Q$.

2 Main Results

The notion of solution considered in this work is that of a mild solution; that is, we study equation (2) via the associated integral equation:

$$u(t) = e^{-t\mathcal{L}} u_0 + \int_0^t e^{-(t-s)\mathcal{L}} |\cdot|_{\mathbb{H}}^{\gamma} [u(s)]^p ds.$$

When $\gamma < 0$ the solution space is $L^\infty(\mathbb{H}^N)$, while for $\gamma \geq 0$, we need to consider the weighted space $L^\infty_{\gamma/(p-1)}(\mathbb{H}^N) = \left\{\varphi \in L^\infty(\mathbb{H}^N);\ \|(1 + |\cdot|_{\mathbb{H}})^{\gamma/(p-1)} \cdot \varphi\|_{L^\infty(\mathbb{H}^N)} < +\infty\right\}$. Our main results are the following:

Theorem 2.1. *Suppose $p > 1$ and $0 \leq \gamma < Q(p-1)$. Then the Fujita critical exponent for problem (2) is $p_c = 1 + (2+\gamma)/Q$.*

Theorem 2.2. *Assume $p > 1$ and $-2 < \gamma < 0$. If $1 < p \leq 1 + (2+\gamma)/Q$, then all nontrivial nonnegative solutions of problem (2) blow up in finite time. If $1 + (2+\gamma)/(Q+\gamma) < p$, then there are nontrivial and nonnegative initial data in $L^\infty(\mathbb{H}^N)$ such that the corresponding solution to problem (2) is global.*

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GLOBAL WELL-POSEDNESS FOR A QUASILINEAR COMBUSTION MODEL

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Abstract

We investigate a nonlinear reaction-diffusion-convection system modeling combustion fronts in multilayer porous media. The model accounts for both temperature and fuel concentration dynamics, treating them as coupled unknowns. By integrating the fuel concentration equations, we derive a quasilinear, non-autonomous evolution problem expressed purely in terms of temperature. A fourth-order parabolic regularization allows us to prove the global existence of solutions in Sobolev spaces. The analysis relies on Banach's fixed-point theorem and precise Sobolev estimates, which provide uniform bounds and ensure continuous dependence on the initial data and model parameters.

1 Introduction

Combustion fronts in porous media have been studied by many authors in recent years. It is important in various industrial applications, including heat exchangers, waste-to-energy systems, and enhanced oil recovery. Da Mota and Schecter [5] derived a two-layer model describing lateral combustion front propagation in porous media with distinct physical properties. This study was followed by [7, 6] and later extended in [3, 2]. More recent analytical results can be found in [4, 1].

In the present work, we generalize the previous problems by studying the full quasilinear structure of the multilayer combustion system, treating both the temperature and fuel concentration as unknowns. Following the integration of the ODEs for fuel concentrations, the resulting PDE system for the temperature vector $u = (u_1, \ldots, u_n)$ takes the form:

$$\begin{cases} \partial_t u + L(x,t,u)u = f(x,t,u), & x \in \mathbb{R},\ t > 0, \\ u(x,0) = \phi(x), \end{cases} \tag{1}$$

where $L(x,t,u)$ is a nonlinear differential operator reflecting diffusion, convection, and reactive terms, and $f(x,t,u)$ includes source terms arising from the fuel dynamics. This formulation leads to a non-autonomous quasilinear evolution equation outside the scope of classical semigroup theory.

We state our two main results, which summarize the key mathematical results of this work.

Proposition 1.1 (Global solution)**.** *Suppose $s \geq 2$, and let R be a constant defined in (??). If $\phi = (\phi_1, \ldots, \phi_n) \in H^s(\mathbb{R})^n$, then for all $T > 0$, there exists a unique solution $u = (u_1, \ldots, u_n) \in \mathcal{C}\big([0,T], H^s(\mathbb{R})^n\big)$ to the initial-value problem (1), in the sense of the H^{s-2}-norm.*

The proof of Proposition 1.1 relies on a sequence of estimates, established in the H^s, L^2, and L^∞ norms.

Proposition 1.2 (Continuous dependence)**.** *Under the same assumptions as in Proposition 1.1, the solution $u \in \mathcal{C}\big([0,T], H^s(\mathbb{R})^n\big)$ depends continuously on the initial data ϕ and on the model parameters, for every $T > 0$.*

Proposition 1.2 establishes the continuous dependence of solutions on both the initial data and model parameters, based on uniform estimates and convergence arguments.

2 Conclusions

We have established a well-posedness theory for a class of quasilinear reaction-diffusion-convection systems modeling combustion in multilayer porous media, where both temperature and fuel concentrations evolve as coupled unknowns. By integrating the fuel equations and introducing a fourth-order parabolic regularization, we proved the lack of a linear structure and proved the global existence, uniqueness, and continuous dependence of solutions in Sobolev spaces. This analysis significantly extends previous models by eliminating the assumption of prescribed fuel profiles and incorporating spatially heterogeneous parameters.

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LONG-WAVE APPROXIMATION IN THE CAUCHY PROBLEM FOR THE WAVE PROPAGATION IN THE CONTINUOUS MODEL FOR THE 1D DIATOMIC CRYSTAL

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Abstract

We discuss the Cauchy problem for a continuous model for the nearest-neighbor interaction in the one-dimensional diatomic crystal lattice. We construct the analytic asymptotic solution for it based on the pseudo-differential operators technique developed by V.P. Maslov. These asymptotic formulas allow us to describe effects previously not mentioned in the literature.

A crucial aspect of our analysis is the proper formulation of initial conditions for the continuous model. While discrete initial perturbations are specified simultaneously for both atom types, the continuous model requires separate initial functions for each type of atoms.

1 Introduction

The one-dimensional diatomic crystal model consists of two types of atoms with different masses m_1 and m_2 (where $m_1 > m_2$). The atoms arranged in an alternating pattern such that each atom is surrounded by atoms of the opposite type. In equilibrium, the atoms located in the lattice points $x_n = nh$, $n \in \mathbb{Z}$, where h is the nearest-neighbor distance (lattice step).

Under the nearest-neighbor interaction the oscillations of the atoms described by the equations

$$\ddot{u}_{2k}(t) = \gamma_1(v_{2k-1}(t) - 2u_{2k}(t) + v_{2k+1}(t)), \quad \ddot{v}_{2k+1}(t) = \gamma_2(u_{2k}(t) - 2v_{2k+1}(t) + u_{2k+2}(t)). \tag{1}$$

Here $\gamma_{1,\,2}$ describe to the parameters of the lattice, we assume $\gamma_1 < \gamma_2$. Functions $u_{2k}(t)$ describe the displacement of heavy atoms (mass m_1) and $v_{2k+1}(t)$ correspond to the light atoms (mass m_2).

A key characteristic of the diatomic model is the existence of two distinct frequency branches for propagating waves. These branches are separated by a frequency gap and define the acoustic and optical wave modes.

2 Main Results

Let μ be the characteristic size of the initial perturbation. We assume that there is a function $W(\xi) \in S(\mathbb{R})$, such that at the initial time moment, the atoms are subject to the localized perturbation described by the values

$$W_n = W\left(\frac{x_n}{\mu}\right) \equiv W\left(n\delta\right), \quad \delta = \frac{h}{\mu}. \tag{1}$$

This initial condition generates the propagating waves, we assume that μ and h are small with respect to the distance of propagation. On the other hand, the ratio δ can be small or quite large. This depends on the size of the initial perturbation. If $h \ll \mu$, then also $\delta \ll 1$ and we say that this case corresponds to the sufficiently long waves with respect to the lattice step.

Let $p \in B_1 \equiv [-\pi/2\delta,\, \pi/2\delta]$, then we define the following functions

$$\tilde{W}_1(p) = \sum_{k\in\mathbb{Z}} W_{2k} e^{-ip2k\delta}, \quad \tilde{W}_2(p) = \sum_{k\in\mathbb{Z}} W_{2k+1} e^{-ip(2k+1)\delta}. \tag{2}$$

The Kotel'nikov-Whittaker-Shannon interpolation formulas give the continuous functions

$$W_1\left(\frac{x}{\mu}\right) = \frac{\delta}{\pi}\int\limits_{B_1} \tilde{W}_1(p)e^{\frac{i}{\mu}xp}dp, \quad W_2\left(\frac{x}{\mu}\right) = \frac{\delta}{\pi}\int\limits_{B_1} \tilde{W}_2(p)e^{\frac{i}{\mu}xp}dp. \tag{3}$$

Let smooth functions $u(x, t)$ and $v(x, t)$ coincide with the displacement $u(x_{2k}, t) = u_{2k}(t)$ and $v(x_{2k+1}, t) = v_{2k+1}(t)$. We introduce [1, 2] the shift operators $T_h^{\pm} f(x) = f(x \pm h) = e^{\pm h\frac{\partial}{\partial x}} f(x)$.

Let us denote $U(x, t) = \big(u(x, t), v(x, t)\big)^T$ the vector-function of the two unknown functions, then the continuous Cauchy problem for the equations (1) has the form

$$-h^2\frac{\partial^2}{\partial t^2}U(x, t) = 2\Gamma L(hD)U(x, t), \quad U(x, 0) = (W_1(x/\mu), W_2(x/\mu)), \tag{4}$$

where

$$L(hD) = \begin{pmatrix} 1 & -\cos(hD) \\ -\cos(hD) & 1 \end{pmatrix}, \Gamma = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}, D = -i\frac{d}{dx}, \gamma_1 < \gamma_2.$$

In the long-wave approximation the atoms of both types oscillate almost similar and we can describe the solution of the system (4) with the help of the scalar equation.

Theorem 2.1. *Let $h^2 = O(\mu^3)$ and $\hat{W}(p)$ be the Fourier transform of $W(\xi)$, then we have the following relation for the solution of the system (4)*

$$U(x, t) = U_{as}(x, t)\begin{pmatrix} 1 \\ 1 \end{pmatrix}(1 + O(\mu)), \quad U_{as}(x, t) = \frac{1}{\sqrt{2\pi}}Re\int\limits_{\mathbb{R}} \hat{W}(p)e^{\frac{i}{\mu}px}exp\left[it\left(\frac{1}{\mu}c|p| - q\frac{h^2}{3\mu^3}p^2|p|\right)\right]dp.$$

Here $c = \sqrt{2\gamma_1\gamma_2/(\gamma_1+\gamma_2)}$ and $q = c(\gamma_1^2 - \gamma_1\gamma_2 + \gamma_2^2)/(2(\gamma_1+\gamma_2)^2)$.

Corollary 2.1. *Let $h^2 = O(\mu^{3+\alpha})$ where $\alpha > 0$ is some parameter, then*

$$U(x, t) = U_{as}(x, t)\begin{pmatrix} 1 \\ 1 \end{pmatrix}(1 + O(\mu^\alpha)), \qquad U_{as}(x, t) = \frac{1}{2}\left(W\left(\frac{x+ct}{\mu}\right) + W\left(\frac{x-ct}{\mu}\right)\right).$$

Important consequence, not mentioned in the literature before, of this analysis is that the optical mode does not contribute significantly to the wave propagation in the long-wave approximation. The amplitude of the corresponding wave is very small and the waves propagate only with respect to the acoustical mode.

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GLOBAL SOLVABILITY TO A MASS-CONSERVED ALLEN-CAHN PHASE-FIELD MODEL FOR TWO-PHASE FERROFLUID FLOWS

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Abstract

In this work, we study the global solvability of a simplified version of the system introduced by Zang G.-D., He X., and Yang X. [1], which describes the behavior of two-phase ferrofluid flows using a phase-field approach. The model combines the mass-conserved Allen-Cahn equation, the Navier-Stokes equations, and the magnetization equation. We prove the existence of global-in-time weak solutions by using a regularization and a semi-Galerkin method.

1 Introduction

Ferrofluids are materials that combine the characteristics of both liquids and magnetic substances. We study the flow of an incompressible, viscous, Newtonian ferrofluid within a smooth bounded domain $\Omega \subset \mathbb{R}^3$. The model proposed by Zhang, He, and Yang [1] is considered, assuming a linear magnetization law $M \approx \chi H$. Under this assumption, the terms $\mu \operatorname{curl}(M \times H)$ and $\beta M \times (M \times H)$ are neglected. Assuming that convection and reaction dominate in the domain, the term $\frac{1}{2}\operatorname{curl}(U) \times M$ in the magnetization equation is also neglected. Additionally, the effective magnetic field H is assumed to be given [2]. With these simplifications, the system reduces to the following form:

$$\begin{aligned}
&\phi_t + U \cdot \nabla\phi - \Delta\phi - f(\phi) + \tfrac{1}{|\Omega|}\int_\Omega f(\phi)dx = 0,\\
&\operatorname{div} U = 0,\\
&U_t + (U\cdot\nabla)U - \nabla\cdot(\nu(\phi)D(U)) + \nabla P + \Delta\phi\nabla\phi = (M\cdot\nabla)H,\\
&M_t + (U\cdot\nabla)M + \tfrac{1}{\tau}M = \tfrac{1}{\tau}\chi(\phi)H,
\end{aligned} \tag{1}$$

in which the unknowns are: the vector field U, representing the fluid velocity field; the pressure P; the magnetization field M; and the phase field variable ϕ that distinguishes the ferrofluid from the ambient fluid. Furthermore, $f(s) = \sum_{i=0}^n a_i s^i$, $n > 1$ is an odd number, $a_n < 0$, and τ is the relaxation time constant. Here, the functions $\nu(\phi)$ and $\chi(\phi)$ represent the viscosity and magnetic susceptibility, respectively, as functions of the phase-field variable ϕ. Both functions are Lipschitz continuous with respect to ϕ and satisfying

$$0 < \min\{\nu_w, \nu_f\} \le \nu(\phi) \le \max\{\nu_w, \nu_f\}, \quad 0 \le \chi(\phi) \le \chi_0$$

where $\chi_0 > 0$ is the magnetic susceptibility of the ferrofluid phase and the parameters ν_w and ν_f denote the viscosity of the non-magnetic and ferrofluid phases, respectively. The symmetric part of the velocity gradient is defined as $D(U) = \frac{1}{2}(\nabla U + (\nabla U)^T)$.

System (1) is equipped with initial and boundary conditions

$$\begin{aligned}
&\phi(x,0) = \phi_0(x), \quad U(x,0) = U_0(x), \quad M(x,0) = M_0(x), \text{ em } \Omega\\
&\tfrac{\partial\phi}{\partial\mathbf{n}} = 0, \quad U = 0, \text{ em } \Gamma_T
\end{aligned} \tag{2}$$

where $\Gamma_T = \partial\Omega \times (0,T)$ and $\mathbf{n}$ is the outward unit normal vector on the boundary of Ω.

2 Main Result

By using a regularization and a semi-Galerkin method we have the following result.

Theorem 2.1. *Suppose that* $\Omega \subset \mathbb{R}^3$ *is a bounded domain with smooth boundary. Let* $U_0 \in \mathcal{H} = \{u \in \mathbb{L}^2 : \operatorname{div} u = 0,\ u \cdot \boldsymbol{n}|_{\partial\Omega} = 0\}$, $M_0 \in \mathbb{L}^2$, *and* $\phi_0 \in H^1 \cap L^\infty$, *with* $s_1 \le \phi_0 \le s_2$ *in* Ω. *Assume* $H \in H^1(0,T;\mathbb{L}^2)$ *and* $\nabla H \in L^\infty(0,T;\mathbb{L}^\infty)$. *Then, for any* $T > 0$, *there exist functions* U, M, *and* ϕ *such that*

$$U \in L^\infty(0,T;\mathcal{H}) \cap L^2(0,T;V), \quad M \in L^\infty(0,T;\mathbb{L}^2), \quad \phi \in L^\infty(0,T;H^1 \cap L^\infty) \cap L^2(0,T;H^2),$$

with $s_1 \le \phi \le s_2$ *almost everywhere in* $\Omega_T = \Omega \times (0,T)$, *and satisfying the following conditions:*

i) For all $v \in V$, *we have*

$$\begin{aligned} \langle U_t, v\rangle + ((U\cdot\nabla)U, v) + \int_\Omega \nu(\phi) D(U) : D(v)dx + (\Delta\phi\nabla\phi, v) &= ((M\cdot\nabla)H, v) \quad \text{in } \mathcal{D}'((0,T)) \\ U(0) &= U_0; \end{aligned} \tag{1}$$

ii) Equation $(1)_4$ *holds in the weak sense, that is, for every test function* $\psi \in C_c^\infty(\Omega \times [0,T))$

$$\textstyle\int_{\Omega_T} M\psi_t\, dxdt + \int_{\Omega_T} (U\cdot\nabla)\,\psi M\, dxdt + \displaystyle\int_\Omega M_0 \psi(x,0)\, dxdt = \frac{1}{\tau}\int_{\Omega_T} (M - \chi(\phi) H)\,\psi\, dxdt; \tag{2}$$

iii)

$$\begin{aligned} \phi_t + U\cdot\nabla\phi - \Delta\phi - f(\phi) + \tfrac{1}{|\Omega|}\int_\Omega f(\phi), dx &= 0 \quad \text{a.e. in} \quad \Omega_T \\ \phi(0) &= \phi_0 \quad \text{a.e. in} \quad \Omega \\ \tfrac{\partial\phi}{\partial\mathbf{n}} &= 0 \quad \text{a.e. in} \quad \Gamma_T = \partial\Omega \times (0,T). \end{aligned} \tag{3}$$

In equation (1), $D(U) : D(v) := \operatorname{tr}(D(U)^T D(v))$. *The triple* (U, M, ϕ) *is called a weak solution to the problem* (1)-(2).

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GLOBAL WELL-POSEDNESS FOR THE NONLINEAR SCHRÖDINGER EQUATION

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Abstract

In this work, we study the initial-value problem associated with the nonlinear Schrödinger equation. We discuss the global well-posedness of the problem, that is, existence, uniqueness, persistence, and continuous dependence upon the data. To this end, we use analytical tools such as Strichartz estimates, the Banach fixed point theorem, and conservation of mass and energy.

1 Introduction

Consider the initial-value problem (IVP) associated with the nonlinear Schrödinger equation

$$\begin{cases} iu_t + \Delta u + \lambda |u|^{\alpha-1}u = 0, \\ u(x,0) = u_0(x), \end{cases} \tag{1}$$

where $u : \mathbb{R}^n \times \mathbb{R} \to \mathbb{C}$ is the unknown function, $t \in \mathbb{R}$, $x \in \mathbb{R}^n$, Δ is the Laplacian with respect to the variable x, and λ and α are real constants with $\alpha > 1$. In the case where $\lambda > 0$, we say that equation (1) is focusing, and in the case where $\lambda < 0$, we say it is defocusing.

Initially, it is natural to ask whether this IVP admits a solution. Here, the concept of well-posedness we adopt includes existence and uniqueness, persistence, and continuous dependence. By means of contraction arguments and using Strichartz estimates, we show that (1) is locally well-posed in the Sobolev spaces $L^2(\mathbb{R}^n)$ and $H^1(\mathbb{R}^n)$ (see, for instance, [1], [4], or [2]).

Having established the local well-posedness of the problem, it is natural to ask whether such solutions can be extended globally in time, that is, whether the existence time T can be taken arbitrarily large. To answer this question, we employ different techniques depending on the Sobolev space considered and the nonlinearity α. However, it is important to emphasize that the Schrödinger conserves the mass and the energy, given respectively by

$$M(u) := \|u\|_{L^2}^2 \text{ and } E(u) := \|\nabla u\|_{L^2}^2 - \frac{2\lambda}{\alpha+1}\int_{\mathbb{R}^n} |u|^{\alpha+1}\, dx.$$

The conservation of these quantities is of great importance in obtaining global results.

2 Main Results

Based on assumptions regarding the dimension n, the nonlinearity α, the sign of λ, and the norm of the initial data, it is possible to ensure that the solutions of (1) are defined globally in time, that is, on any interval $[-T, T]$ with $T > 0$, as shown by the theorems below.

Theorem 2.1 (Global solution in L^2, subcritical case)**.** *Let $1 < \alpha < 1 + \frac{4}{n}$. For any $u_0 \in L^2(\mathbb{R}^n)$, problem (1) admits a unique solution $u = u(x,t)$, and it extends globally with $u \in C((-\infty,\infty); L^2(\mathbb{R}^n)) \cap L^q_{loc}((-\infty,\infty); L^p(\mathbb{R}^n))$, where (q,p) is any admissible pair.*

Theorem 2.2. *Consider problem (1) with $u_0 \in H^1(\mathbb{R}^n)$. Under any of the following set of hypotheses, (1) has a unique local solution $u \in C([-T,T];H^1(\mathbb{R}^n)) \cap L^q((-T,T);H^{1,p}(\mathbb{R}^n))$ and this solution can be extended globally in time, where (q,p) is any admissible pair.*

1. $\lambda < 0$, $1 < \alpha < \frac{n+2}{n-2}$, *if* $n \geq 3$ *and* $1 < \alpha < \infty$, *if* $n = 1,2$,
2. $\lambda > 0$ *and* $1 < \alpha < 1 + \frac{4}{n}$,
3. $\lambda > 0$, $\alpha = 1 + \frac{4}{n}$ *and* $\|u_0\|_{L^2} < \|\varphi\|_{L^2}$,
4. $\lambda > 0$, $1 + \frac{4}{n} < \alpha < \frac{n+2}{n-2}$, $E(u_0)^{s_c} M(u_0)^{1-s_c} < E(\varphi)^{s_c} M(\varphi)^{1-s_c}$ *and* $\|\nabla u_0\|_{L^2}^{s_c} \|u_0\|_{L^2}^{1-s_c} < \|\nabla \varphi\|_{L^2}^{s_c} \|\varphi\|_{L^2}^{1-s_c}$.

Here, φ is the ground state solution and it is the unique positive and radial solution to the elliptic problem $-(1-s_c)\varphi + \Delta\varphi + |\varphi|^{\alpha-1}\varphi = 0$, where $s_c := \frac{n}{2} - \frac{2}{\alpha-1}$.

The hypotheses established in the global existence theorems in H^1 are ideal, that is, if the established conditions are not satisfied, then there exists an initial data $u_0 \in H^1$ such that the corresponding solution blows-up in finite time.

Theorem 2.3. *Let u be a solution in $C([0,T];H^1(\mathbb{R}^n)) \cap L^2(\mathbb{R}^n, |x|^2, dx)$ of (1) with $\lambda = 1$. If the initial condition u_0 and the nonlinearity α satisfy $E(u_0) < 0$ and $1 + \frac{4}{n} < \alpha < \frac{n+2}{n-2}$, then there exist $0 < T_{min}, T_{max} < \infty$ such that*

$$\lim_{t \to T_{max}^-} \|\nabla u(t)\|_{L^2} = \infty \ e \lim_{t \to -T_{min}^+} \|\nabla u(t)\|_{L^2} = \infty.$$

Proposition 2.1. *Let $u(x,t) = e^{it}\varphi(x)$ be a solution of (1) with $\alpha = 1 + \frac{4}{n}$ and $\lambda = 1$. Then,*

$$z(x,t) = \frac{e^{i|x|^2/4t}}{|t|^{n/2}} e^{-i/t} \varphi(x/t) = \frac{e^{i(|x|^2-4)/4t}}{|t|^{n/2}} \varphi(x/t)$$

is also a solution in $C(\mathbb{R}n\{0\}; H^1(\mathbb{R}^n) \cap L^2(|x|^2\,dx))$, $\|z(t)\|_{L^2} = \|\varphi\|_{L^2}$ and z blows-up in finite time.

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MATHEMATICAL AND NUMERICAL INVESTIGATION OF THE EXPONENTIAL STABILITY OF PIEZOELECTRIC BEAMS UNDER MAGNETIC AND MICROTEMPERATURE EFFECTS

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Abstract

In this work, we study the exponential stability of a piezoelectric beam subjected to magnetic effects and dissipation via microtemperatures. The model is formulated as a system of coupled partial differential equations, including a thermal equation to capture the dissipative effects at a microscopic scale. First, we establish the well-posedness of the problem within the framework of semigroup theory. Then, we use the energy method to derive sufficient conditions for the exponential stability of the system. Additionally, we conduct numerical experiments using finite differences to validate the theoretical results. Our findings indicate that the inclusion of dissipation due to microtemperatures plays a significant role in stabilizing the system, reducing undesired oscillations, and improving the energy decay rate.

1 Introduction

The study of piezoelectric materials dates back to the late 19th century when brothers Pierre and Jacques Curie discovered in 1880 that certain materials, such as quartz, Rochelle salt, and barium titanate, generate electric charge when subjected to mechanical pressure. This phenomenon, known as the direct piezoelectric effect, marked the beginning of a new era in materials science. Shortly thereafter, in 1881, Gabriel Lippmann demonstrated the inverse phenomenon, namely that these same materials undergo mechanical deformations when exposed to an electric field, which became known as the inverse piezoelectric effect [1]. These pioneering discoveries paved the way for the development of a class of materials that are now widely utilized in various technological applications.

Morris and Özer in [3] proposed a variational approach for a piezoelectric beam model with a magnetic effect, based on Euler-Bernoulli and Rayleigh beam theory for small displacements (the same equations for the model are obtained when using the Mindlin-Timoshenko small displacement assumptions [4]). They considered an elastic beam covered by a piezoelectric material on its upper and lower surfaces, isolated at the edges and connected to an external electrical circuit to feed charge to the electrodes. They obtained the system:

$$
\begin{aligned}
\rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} &= 0, \\
\mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} &= 0.
\end{aligned} \tag{1}
$$

In this work, we analyze a piezoelectric beam model that incorporates microtemperature effects, introducing an additional dissipation mechanism that influences the system's stability. The model is governed by the following system of partial differential equations:

$$\rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} + k_3\omega_x = 0, \quad \text{in} \quad (0,l)\times(0,\infty), \tag{2}$$

$$\mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, \quad \text{in} \quad (0,l)\times(0,\infty), \tag{3}$$

$$\delta\omega_t - k_1\omega_{xx} + k_2\omega + k_3 v_{xt} = 0, \quad \text{in} \quad (0,l)\times(0,\infty). \tag{4}$$

We consider the following initial and boundary conditions

$$v(0,t) = \alpha v_x(l,t) - \gamma\beta p_x(l,t) = p(0,t) = p_x(l,t) - \gamma v_x(l,t) = \omega_x(0,t) = \omega(l,t) = 0, \qquad t > 0, \tag{5}$$

$$v(x,0) = v_0(x), v_t(x,0) = v_1(x), p(x,0) = p_0(x), p_t(x,0) = p_1(x), \omega(x,0) = \omega_0(x) \quad x \in \ (0,l). \tag{6}$$

2 Main Results

Theorem 2.1. *The initial-boundary value problem associated with system* (2)–(5) *is well-posed in the Hilbert space* $\mathcal{H}$*. That is, for any initial data* $U_0 \in \mathcal{H}$*, there exists a unique solution* $U(t) \in C([0,\infty);\mathcal{H})$*, continuously depending on the initial data. Moreover, if* $U_0 \in D(\mathcal{A})$*, then* $U(t) \in C^1([0,\infty);\mathcal{H}) \cap C([0,\infty);D(\mathcal{A}))$*.*

Theorem 2.2. *The energy* $E(t)$ *of the system* (2)–(5) *decays exponentially as* $t \to \infty$*. That is, there exist two positive constants* M *and* σ*, independent of the initial data, such that*

$$E(t) \le ME(0)e^{-\sigma t}, \quad \textit{for all} \quad t \ge 0. \tag{1}$$

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THE EFFECTS OF NON-INTEGRABLE DATA ON THE CRITICAL EXPONENT FOR A σ-EVOLUTION EQUATION WITH STRUCTURAL DAMPING AND NONLINEAR MEMORY

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Abstract

In this work, we investigate how assuming non-integrable regularity for the initial data influences the critical exponent of a structurally damped σ-evolution equation with nonlinear memory.

The main novelty lies in identifying how the interplay between the slower decay caused by the nonlinear memory term and the lower regularity of the initial data taken in L^m with $m > 1$ instead of L^1 leads to a new critical exponent. Notably, this exponent differs from the one obtained in the L^m framework for the corresponding equation with a standard power-type nonlinearity of the form $|u|^p$.

We establish the sharpness of this new critical exponent by applying the test function method.

1 Introduction

In this work, we are interested in the following $\sigma-$evolution equation with structural damping

$$\begin{cases} u_{tt} + (-\Delta)^\sigma u + (-\Delta)^\theta u_t = F(t,u), & t \geq 0,\ x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $\sigma > 0$, $2\theta = \sigma$ and $F(t,u) = \displaystyle\int_0^t (t-s)^{-\gamma} |u(s,x)|^p ds$, $\gamma \in (0,1)$, represents a memory term, since it is a fractional Riemann-Liouville integral of a power nonlinearity $|u|^p$.

In the case of a power nonlinearity $F = |u|^p$ it has been proved [1] that the critical exponent, assuming small data in L^m, is $p_c = 1 + \frac{2m\sigma}{n-2\sigma}$. When we refer to p_c as a critical exponent, we mean that global solutions exist for small initial data when the power is supercritical, i.e., $p > p_c$. In contrast, global solutions do not exist for subcritical powers, $1 < p < p_c$, even with small initial data in the same norm, given appropriate sign assumptions on the data. The critical power $p = p_c$ may either lie within the existence or nonexistence range of global solutions, depending on the specific problem considered.

In [2] the author proved that the critical exponent for (1) in the special case $\theta = \frac{1}{2}$ and $\sigma = 1$, assuming data in L^1 is $p_c = \max\{p_\gamma(n), \gamma^{-1}\}$, where $p_\gamma = 1 + \frac{3-\gamma}{n+\gamma-2}$.

In this talk, we will analyze the influence of the L^m smallness of the initial data. We will conclude that a new critical exponent emerges from the interplay between the loss of decay rate due to the nonlinear memory term and the loss of decay resulting from relaxing the L^1 condition on the initial data.

2 Main Results

If the loss of decay due to the assumption of L^m on the initial data becomes irrelevant with respect to the loss os decay rate related to the presence of the nonlinear memory term, i.e., if

$$\gamma > 1 - \frac{n}{\sigma}\left(1 - \frac{1}{m}\right), \tag{1}$$

the expected critical exponent is

$$p_c = \max\{p^1_{m,\gamma}(n,\sigma), p^2_{m,\gamma}(n,\sigma)\}, \tag{2}$$

where

$$p^1_{m,\gamma}(n,\sigma) = 1 + \frac{\gamma-3}{\left[1-\frac{n}{\sigma m}\right]_+} = 1 + \frac{\sigma m(3-\gamma)}{[n-\sigma m]_+} \tag{3}$$

$$p^2_{m,\gamma}(n,\sigma) = 1 + \frac{1-\gamma}{\left[1-\frac{n}{\sigma}\left(1-\frac{1}{m}\right)\right]_+} = 1 + \frac{\sigma m(1-\gamma)}{[\sigma m - n(m-1)]_+}. \tag{4}$$

On one hand, we proved that the exponent $p^1_{m,\gamma}(n,\sigma)$ is indeed critical. On the other hand, we only proved the existence of global small data energy solutions when $p > p^2_{m,\gamma}(n,\sigma)$. The main results are:

Theorem 2.1. *Let $n \le 2\sigma$ and $m \in (1,\infty)$ or $n > 2\sigma$ and $m \in (1, n/(n-\sigma))$. Let us assume (1) and $p > p_c$, or $p > n/(n-2\sigma)$ if $n > 2\sigma$ and $m = \frac{2n}{2\sigma(\gamma-2)+n(3-\gamma)}$. Then there existe $\varepsilon > 0$ such that for any initial data $(u_0,u_1) \in \mathcal{A} = (L^m \cap L^\infty) \cap (L^m \cap L^\infty)$, with $\|(u_0,u_1)\|_{\mathcal{A}} := \|u_0\|_{L^m} + \|u_1\|_{L^m} \le \varepsilon$, there is a unique global energy solution $u \in \mathcal{C}([0,\infty), H^1) \cap \mathcal{C}^1([0,\infty), L^2)$ to (1). Moreover, it satisfies*

$$\|u(t,\cdot)\|_{L^q} \lesssim (1+t)^{1-\frac{n}{\sigma}\left(\frac{1}{m}-\frac{1}{q}\right)} \|u\|_{L^m \cap L^\infty}, \tag{5}$$

for any $q \in [m,\infty]$ if $n < 2\sigma$ and for any $q \in [m, n/(n-2\sigma)]$ if $n \ge 2\sigma$, where $n/(n-2\sigma) = \infty$ if $n = 2\sigma$. If $n \ge 2\sigma$ and $q = \bar{q} = n/(n-2\sigma)$, its satisfies

$$\|u(t,\cdot)\|_{L^{\bar{q}}} \lesssim (1+t)^{1-\frac{n}{\sigma}\left(1-\frac{1}{m}\right)} \log(e+t) \|u\|_{L^m \cap L^\infty}. \tag{6}$$

If $n > 2\sigma$ and $q \in (n/(n-2\sigma), \infty]$, its satisfies the following decay estimate

$$\|u(t,\cdot)\|_{L^q} \lesssim (1+t)^{1-\frac{n}{\sigma}\left(1-\frac{1}{m}\right)} \|u\|_{L^m \cap L^\infty}. \tag{7}$$

Theorem 2.2. *Let $n \ge 1$, $\gamma \in (0,1)$, $m \in (1,\infty)$. Assume that $(u_0,u_1) \in L^1_{loc}$ with $u_0 \ge 0$, $(u_0(x)+u_1(x)) \ge \varepsilon |x|^{-\frac{n}{m}} \log|x|$, for $|x| >> 1$ and that $u \in L^p([0,\infty) \times \mathbb{R}^n)$ is a global solutioin to (1). Then $p \ge p^1_{m,\gamma}(n,\sigma)$.*

The proofs of these theorems follow the techniques used in [3]

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EXISTENCE AND CONTINUOUS DEPENDENCE OF THE LOCAL SOLUTION OF NON HOMOGENEOUS THIRD ORDER EQUATION

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Abstract

In this work, we prove that initial value problem associated to the nonhomogeneous third order equation on periodic Sobolev spaces has a local solution in $[0,T]$ with $T>0$, and the solution has continuous dependence with respect to the initial data and the nonhomogeneous part of the problem. We do this in an intuitive manner using Fourier theory and introducing a C_o-semigroup inspired by the work of Iorio [4] and Ayala [1]. Also, we prove the uniqueness solution of the homogeneous and nonhomogeneous equation, using its dissipative property, inspired by the work of Iorio [4], Liu [5] and Ayala [2].

1 Introduction

First, we want to comment that from Theorem 3.1 in [3], we have that the homogeneous problem is globally well posed and, in addition to the inequality (3.2) in [3], we have the continuous dependence of the solution of homogeneous problem. In this work, in Theorem 2.1 we will prove the existence and uniqueness of the local solution for the non-homogeneous problem and from inequality (4) we will get the continuous dependence of the solution with respect to the initial data and respect to the non-homogeneous part. Thus, in both homogeneous and non-homogeneous cases, the estimatives are made from the explicit form of the solution, that is, by applying the Fourier transform to the respective equation. Another result in this work is about the dissipative property of the homogeneous problem and some estimates of it, using differential calculus in H^s_{per}. This is included in Theorem 2.2 which we will develop. So, using Theorem 2.2, we deduce the results of continuous dependence and uniqueness of solution for both homogeneous and non-homogeneous problems, respectively. Finally, we give some conclusions and generalizations.

2 Main Results

We prove that the non-homogeneous problem (P^F) is locally well posed.

Theorem 2.1. *Let $\varphi \in H^s_{per}$, $s \in R$, $a>0$, $F \in C([0,T], H^s_{per})$, where $T>0$, and $\{S(t)\}_{t\geq 0}$ the semigroup of class C_o of contraction in H^s_{per} associated to homogeneous case $(F=0)$, introduced in Theorem 3.2 from [3], then*

1. *The function*

$$u^F(t) := S(t)\varphi + \underbrace{\int_0^t S(t-\tau)F(\tau)\,d\tau}_{u_p(t):=}\,, \quad t \in [0,T] \tag{1}$$

belong to $C([0,T], H^s_{per}) \cap C^1([0,T], H^{s-3}_{per})$ and

2. *u is the unique solution of*

$$(P^F) \quad \left| \begin{array}{l} u_t + \partial_x^3 u + au = F\,, \quad F(t) \in H^{s-3}_{per} \\ u(0) = \varphi \end{array} \right. \tag{2}$$

with the derivative given by

$$\lim_{h\to 0}\left\|\frac{u(t+h)-u(t)}{h}+\partial_x^3 u+au-F(t)\right\|_{s-3}=0 \tag{3}$$

3. *Let* $\varphi_j \in H^s_{per}$, $F_j \in C([0,T],H^s_{per})$ *for* $j=1,2$. *Then the map* $\varphi \longrightarrow u$ *is continuous in the sense: Let* u_j *be the corresponding solutions to initial data* φ_j *with non homogeneity* F_j, *respectively, then*

$$\|u_1-u_2\|_{\infty,s} \le \|\varphi_1-\varphi_2\|_s+T\|F_1-F_2\|_{\infty,s} \tag{4}$$

$$\begin{aligned}\|\partial_t u_1-\partial_t u_2\|_{s-3} &\le (1+a)\|u_1-u_2\|_{\infty,s}+\|F_1-F_2\|_{\infty,s-3}\\ &\le (1+a)\|\varphi_1-\varphi_2\|_s+[(1+a)T+1]\|F_1-F_2\|_{\infty,s}\end{aligned} \tag{5}$$

where we have used the notation:

$$\|g\|_{\infty,r}=\sup_{t\in[0.T]}\|g(t)\|_r\ , \quad g\in C([0,T],H^r_{per}) \tag{6}$$

Now, we study the dissipative property of the homogeneous problem Let $a>0$, $s\in R$ and the homogeneous problem:

$$(P)\quad \left|\begin{array}{l} w\in C([0,\infty),H^s_{per})\cap C^1((0,\infty),H^{s-3}_{per})\\ w_t+\partial_x^3 w+aw=0\,,\\ w(0)=\psi\,,\quad \psi\in H^s_{per}\end{array}\right.$$

Theorem 2.2. *Let* w *the solution of* (P) *with initial data* $\psi\in H^s_{per}$, *then the following statements hold:*

1. $\partial_t\|w(t)\|^2_{s-3}=-2a\|w(t)\|^2_{s-3}\le 0\,.$

2. $\|w(t)\|_{s-3}=e^{-at}\|\psi\|_{s-3}\le\|\psi\|_{s-3}\le\|\psi\|_s$, *for* $t\ge 0$.

3. $\lim_{t\to+\infty}\|w(t)\|_{s-3}=0\,.$

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A SUMMABILITY PRINCIPLE AND APPLICATIONS

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Abstract

This paper investigates summability principles for multilinear summing operators. The main result presents a novel inclusion theorem for a class of summing operators, which generalizes several classical results. As applications, we derive improved estimates for Hardy–Littlewood inequalities on multilinear forms and prove a Grothendieck-type coincidence result in anisotropic settings.

1 Introduction

The main contribution of this work is an anisotropic inclusion theorem for the class of Λ-summing operators, which extends two of the main important multilinear summing classes: *absolutely* and *multiple* summing operators. For instance, a particular case of the main result we prove is an inclusion of the type

$$\Pi^{\Lambda}_{(r;p)}\left(E_1,\ldots,E_m;F\right)\subset\Pi^{\Lambda}_{(s;q)}\left(E_1,\ldots,E_m;F\right)$$

where Λ can be both classes *absolutely* or *multiple* summing operators, r,s,p,q are suitable parameters and, as usual, $E_1,\ldots,E_m,F$ stand for Banach spaces. The main result (Theorem 2.1) yields several applications, including connections with the Bohnenblust–Hille and Hardy–Littlewood inequalities, as well as a Grothendieck-type theorem.

Given $\mathbf{r},\mathbf{p}\in[1,+\infty)^m$ and $\Lambda\subset\mathbb{N}^m$ a non-empty set of indices, an m-linear operator $T:E_1\times\cdots\times E_m\to F$ is Λ-$(\mathbf{r};\mathbf{s})$-summing if there is a constant $C>0$ such that

$$\left\|(Tx_{\mathbf{i}})_{\mathbf{i}\in\Lambda}\right\|_{\ell_{\mathbf{r}}(F)}\le C\prod_{j=1}^{m}\|(x_i^j)_{i\in\mathbb{N}}\|_{w,p_k},\tag{1}$$

for all $(x_i^j)_{i\in\mathbb{N}}\in\ell^w_{p_j}(E_j)$, $j=1,\ldots,m$. The $\ell_{\mathbf{r}}$-norm in the left can be seen as $\left\|(Tx_{\mathbf{i}})_{\mathbf{i}\in\Lambda}\right\|_{\ell_{\mathbf{r}}(E)}=\left\|(Tx_{\mathbf{i}}\cdot 1_\Lambda(\mathbf{i}))_{\mathbf{i}\in\mathbb{N}^m}\right\|_{\ell_{\mathbf{r}}(E)}$, where 1_Λ is the characteristic function of Λ. The class of all operators that satisfies (1) is denoted by $\Pi^{\Lambda}_{(\mathbf{r};\mathbf{s})}\left(E_1,\ldots,E_m;F\right)$, which is a Banach space endowed with the usual norm $\pi^{\Lambda}_{(\mathbf{r};\mathbf{s})}(\cdot)$. By taking $\Lambda=\mathrm{Diag}\left(\mathbb{N}^m\right):=\{(n,\cdots,n)\in\mathbb{N}^m:n\in\mathbb{N}\}$ and $\Lambda=\mathbb{N}^m$, the Λ-summing class Π^Λ recovers both absolutely summing class Π^{as}, and multiple summing class Π^{ms}, respectively.

The study of Λ-summing operators, where Λ is an arbitrary non-empty set of indices, can be a challenging problem. A primary difficulty lies in computing the norm on the left-hand side of (1). One approach to address this is to introduce a well-behaved *block structure* on Λ, as outlined below.

For $i_j\in\mathbb{N}$, $j\in\{1,\ldots,m\}$, we define $i_j\cdot e_j:=(0,\ldots,0,i_j,0,\ldots,0)\in\mathbb{N}^m$, with i_j in the j-th coordinate. For $1\le d\le m$ and $\mathcal{I}:=\{I_1,\ldots,I_d\}$ a partition of non-empty disjoints subsets of $\{1,\ldots,m\}$ such that $\cup_{i=1}^d I_i=\{1,\ldots,m\}$, the set of index $\Lambda=\mathcal{B}_{\mathcal{I}}:=\left\{\sum_{n=1}^d\sum_{j\in I_n}i_n\cdot e_j\ :\ i_1,\ldots,i_d\in\mathbb{N}\right\}\subseteq\mathbb{N}^m$, is called a *block of $\mathcal{I}$-type*. We focus our attention this block structure. Given Banach spaces $E_1,\ldots,E_m$ and $x_j\in E_j$, for some $j\in\{1,\ldots,m\}$, we define $x_j\cdot e_j$ as the element of $E_1\times\cdots\times E_m$ with x_j in the j-th coordinate and 0 elsewhere. With this we have

$$Tx_{\mathbf{i}}=T\left(\sum_{n=1}^{d}\sum_{j\in I_n}x^j_{i_n}\cdot e_j\right),\qquad\text{for }\mathbf{i}\in\mathcal{B}_{\mathcal{I}}.$$

2 Main Results

The main results are listed next. Before proceeding, we introduce some notation. For $A \subset \{1,\ldots,m\}$ and $p_1,\ldots,p_m \in [1,\infty]$, we define $\left|\frac{1}{\mathbf{p}}\right|_{j\in A} := \sum_{j\in A}\frac{1}{p_j}$. We write $|1/\mathbf{p}|$ as a shorthand for $|1/\mathbf{p}|_{j\geq 1}$.

Theorem 2.1. *Let $1 \leq d \leq m$ be positive integers and $\mathcal{I} = \{I_1,\ldots,I_d\}$ be a partition of $\{1,\ldots,m\}$ and and $\mathcal{B}_{\mathcal{I}}$ a block of $\mathcal{I}$-type. Let also $r \geq 1$, $\mathbf{p},\mathbf{q} \in [1,\infty)^m$ and $r \leq s_d \leq \cdots \leq s_2 \leq s_1$ such that*

$$\frac{1}{s_k} - \left|\frac{1}{\mathbf{q}}\right|_{j\in\bigcup_{i=k}^d I_i} = \frac{1}{r} - \left|\frac{1}{\mathbf{p}}\right|_{j\in\bigcup_{i=k}^d I_i},\ k = 1,\ldots,d.$$

If one of the following conditions holds,

(A) $q_j \geq p_j,\ j = 1,\ldots,m,$ *and* $\frac{1}{r} - \left|\frac{1}{\mathbf{p}}\right| + \left|\frac{1}{\mathbf{q}}\right| > 0;$

(B) $q_1 > p_1,\ q_j \geq p_j,\ j = 2,\ldots,m,$ *and* $\frac{1}{r} - \left|\frac{1}{\mathbf{p}}\right| + \left|\frac{1}{\mathbf{q}}\right| = 0.$

then

$$\Pi^{\mathcal{B}_{\mathcal{I}}}_{(r;\mathbf{p})}(E_1,\ldots,E_m;F) \subset \Pi^{\mathcal{B}_{\mathcal{I}}}_{(\mathbf{s};\mathbf{q})}(E_1,\ldots,E_m;F)$$

for any Banach spaces $E_1,\ldots,E_m,F$. Moreover, the inclusion operator has norm 1.

Proposition 2.1 (Grothendieck-type coincidence)**.** *Let m be a positive integer, $p \leq 2$ or $r > p > 2$ and $\mathbf{s},\mathbf{q} \in [1,\infty)^m$. If $1/r - m/p + |1/\mathbf{q}| > 0$, and $q_k \geq p$ and $\frac{1}{s_k} - \left|\frac{1}{\mathbf{q}}\right|_{\geq k} = \frac{1}{r} - \frac{m-k+1}{\mathbf{p}}$ for $k = 1,\ldots,m$, then*

$$\Pi^{\mathrm{ms}}_{(\mathbf{s};\mathbf{q})}({}^m\ell_1;\ell_2) = \mathcal{L}({}^m\ell_1;\ell_2).$$

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TRANSITION PROBABILITIES AND GENERALIZED DIFFERENTIAL GEOMETRY

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Abstract

Given a classical oriented Riemannian manifold M, we embed M discretely into a generalized manifold M^* in such a way that M and its differential structure are the shadow of the differential structure of M^*. We proof a Fixed Point Theorem and extend an existing embedding theorem by proving that there exists an algebra embedding $\kappa : \hat{\mathcal{G}}(M) \longrightarrow \mathcal{C}^\infty(M^*, \widetilde{\mathbb{R}}_f)$, thus relating the generalized construction on classical manifolds to the classical construction on generalized manifolds. The existence of transition probabilities, as proposed by Colombeau-Gsponer, is indicated in the finite dimensional case and for infinite dimensional Hilbert modules.

1 Introduction

The theory of generalized functions goes back to Schwartz. More recently, J.F. Colombeau and E. Rosinger has undertaken the challenge of developing a nonlinear theory of generalized functions, thus extending Schwartz premier work. Colombeau's proposal has been extensively used. In spite of these important contributions and development, somehow the underlying algebraic structure remained $\mathbb{R}$ or $\mathbb{C}$. One of the reasons to sticks to the classical underlying structure is the concern that introducing another underlying structure might lead to controversies either about the existence and rigor or about how much of nonstandard tools one needs to know to understand these structures. However, this should not really be a concern. In [1], the Fermat reals ${}^\bullet\mathbb{R}$ were used as the algebraic underlying structure of a generalized differential calculus. The totally ordered topological ring ${}^\bullet\mathbb{R}$ is basically the union of hallows of real numbers, each hallow consisting of unique real number ${}^\circ x$ and elements $y = {}^\circ x + dt_a$, with $dt_a, a \in [1, \infty[$ being nilpotent elements. In particular, the group of invertible elements $Inv({}^\bullet\mathbb{R})$ is open but not dense and zero and nonzero infinitesimals are precisely the noninvertible elements.

In applications and in certain areas, one must deal with infinitesimals and infinities which, in certain situations, are cancelled out by each other and thus suggesting that they are invertible elements. Can an environment be constructed in which infinitesimals and infinities coexists and some of which are invertible elements or at least invertible in some sense? Non-Archimedean rings with these properties originated with J. Tate. Here we focus on $\widetilde{\mathbb{R}}$ which was constructed in Colombeau's approach to generalized functions. Originally, it was just a ring where generalized functions took values, but, over time, it turned out to have a very rich topological and algebraic structure making it suitable to be the underlying algebraic milieu of a new Differential Calculus, a Generalized Differential Calculus. Let's sum up some of its features. Infinitesimals and infinities live like ebony and ivory in $\widetilde{\mathbb{R}}$ and when rendezvous occurs an interleaving of real numbers may be the result. Moreover, $Inv(\widetilde{\mathbb{R}})$ is open and dense, $\mathcal{B}(\widetilde{\mathbb{R}})$, its Boolean algebra of idempotent elements, consists of the characteristic functions of subsets of the real interval $I =]0, 1]$ and if $x \notin Inv(\widetilde{\mathbb{R}})$ then there exist $e, f \in \mathcal{B}(\widetilde{\mathbb{R}})$ such that $e \cdot x = 0$ and $f \cdot x \in Inv(f \cdot \widetilde{\mathbb{R}})$.

Piecing the puzzle, result in an ultra-metric milieu $\widetilde{\mathbb{R}}^n$ in which $\mathbb{R}^n$ is the shadow, or support, of points of $\widetilde{\mathbb{R}}^n$. In $\widetilde{\mathbb{R}}^n$, $\mathbb{R}^n - \{\vec{0}\}$ is a grid of equidistant points sitting between infinitesimals, the elements of $B_1(\vec{0}) - \{\vec{0}\}$, and infinities and hence, algebraically, it is the result of the rendezvous of such elements which go undetected in physical reality. If $\Omega \subset \mathbb{R}^n$ is open, then there exists a discrete embedding of $\mathcal{D}'(\Omega)$ into $\mathcal{C}^\infty(\widetilde{\Omega}_c, \widetilde{\mathbb{R}})$, where $\widetilde{\Omega}_c$ is a subset of $\widetilde{\mathbb{R}}^n$ consisting of those elements of $\overline{B}_1(\vec{0})$ whose support is contained in Ω and their norm is less than some real number. In particular, Dirac's infinity δ, becomes a $\mathcal{C}^\infty-$function on $\widetilde{\mathbb{R}}_c$ and $x\delta$ becomes nonzero and, when evaluated at certain infinitesimals, produces real values. Generalized Space-Time is constructed and applications to physical reality are suggested. In the finite dimensional case, the existence of transition probabilities, of important in quantum mechanics, is proved under some conditions, some of which also hold for infinite Hilbert modules. Concrete examples are considered.

2 Main Results

Theorem 2.1. *Let $\Omega \subset \mathbb{R}^n$, $A = [(A_\varphi)_\varphi] \subset B_r(0) \cap \mathcal{G}_f(\Omega)$, $r < 2$, be an internal set, and $T : A \to A$ be a mapping with representative $(T_\varphi : A_\varphi \to A_\varphi)_{\varphi \in A_0(n)}$. If there exists $k = [(k_\varphi)_\varphi] \in \tilde{\mathbb{N}}$ such that each $T_\varphi^{k_\varphi}$ is a λ-contraction, then T^k is well-defined, continuous, and has a unique fixed point $f_0 \in A$.*

Theorem 2.2. *Let M be an $n-$dimensional orientable Riemannian manifold. There exists an $n-$dimensional $\mathcal{G}_f-$manifold M^*, in which M is discretely embedded, and an algebra monomorphism $\kappa : \hat{\mathcal{G}}(M) \longrightarrow \mathcal{C}^\infty(M^*, \widetilde{\mathbb{R}}_f)$ which commutes with derivation. Moreover, equations whose data have singularities or nonlinearities defined on M naturally extend to equations on M^* and, on M^*, these data become $\mathcal{C}^\infty-$functions.*

Theorem 2.3. *Let $A \in M_n(\overline{\mathbb{R}})$ be self-adjoint, $U \in M_n(\overline{\mathbb{R}})$ a unitary matrix such that $UAU^* = Diag(\lambda_1, \cdots \lambda_n)$, $u, v \in \mathbb{R}^n$ ortogonal unit vectors, such that Uu does not belong to the canonical basis of $\mathbb{R}^n$ and $\Delta(Spec(A)) = \{\lambda_k - \lambda_j : \lambda_k \neq \lambda_j \in Spec(A)\} \subset \overline{\mathbb{R}}$. Then $\nu(A, u, v) = |\langle u \mid exp(iA)v\rangle|^2$ is given by $\nu(A, u, v) = \|a\|^2 + 2 \cdot \sum_{k<j} a_k a_j \cos(\lambda_k - \lambda_j) = -4 \cdot \sum_{k<j} a_k a_j \sin^2\left(\frac{\lambda_k - \lambda_j}{2}\right)$ and satisfies the perfect destructive interference conditions: (i). $a = (a_1, \cdots, a_n)$, (ii). $\sum_i a_i = 0$, (iii). $\|a\| < 1$. Moreover, if $\Delta(Spec(A))$ consists of pure infinities and $a \in \overline{\mathbb{R}}_{as}$, with $a \approx (b_1, \cdots, b_n) \in \mathbb{R}^n$, then the transition probability exists and is given by $\mathcal{P}_0(A, u, v) = -2 \cdot \sum_{k<j} b_k b_j$.*

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STABILITY RESULTS OF THE GENERALIZED APPROXIMATE HYPERPLANE SERIES PROPERTY

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Abstract

We study the generalized approximate hyperplane series property (generalized AHSP) for pairs (X, Y) of Banach spaces. This concept was introduced to characterize when the pair $(\ell_1(X), Y)$ has the Bishop-Phelps-Bollobás property.

1 Introduction

Throughout the paper, X is a Banach space over a real or complex field $\mathbb{K}$ and B_X (resp. S_X) is the closed unit ball (resp. unit sphere) of X. For Banach spaces X, Y over the same scalar field $\mathbb{K}$, $\mathcal{L}(X, Y)$ is the Banach space of all bounded linear operators from X into Y and $X^\star = \mathcal{L}(X, \mathbb{K})$ stands for the dual space of X. We say that an operator $T \in \mathcal{L}(X, Y)$ *attains its norm* if there exists a point $x_0 \in S_X$ such that $\|Tx_0\| = \|T\| = \sup\{\|Tx\| : x \in B_X\}$.

E. Bishop and R. Phelps [2] proved that every continuous linear functional $x^\star$ on a Banach space X can be approximated, uniformly on the closed unit ball, by a continuous linear functional $y^\star$ that attains its norm. This result is called the Bishop-Phelps Theorem. Later, B. Bollobás in [3] showed that this approximation can be done in such a way that the point at which $x^\star$ almost attains its norm is close in norm to a point at which $y^\star$ attains its norm. This is a version of the Bishop-Phelps Theorem, known as the Bishop-Phelps-Bollobás Theorem.

The Bishop-Phelps-Bollobás property was introduced in 2008 [1] as an extension of the Bishop-Phelps-Bollobás Theorem to the vector-valued case.

Definition 1.1. *[1, Definition 1.1] Let X and Y be real or complex Banach spaces. We say that the pair (X, Y) has the Bishop-Phelps-Bollobás property for operators (shortly, BPBp) if given $\varepsilon > 0$, there are $\eta(\varepsilon) > 0$ and $\beta(\varepsilon) > 0$ with $\lim_{t\to 0} \beta(t) = 0$ such that for all $T \in S_{\mathcal{L}(X,Y)}$, if $x_0 \in S_X$ is such that $\|Tx_0\| > 1 - \eta(\varepsilon)$, then there exist a point $u_0 \in S_X$ and an operator $S \in S_{\mathcal{L}(X,Y)}$ that satisfy the following conditions:*

$$\|Su_0\| = 1, \quad \|u_0 - x_0\| < \beta(\varepsilon) \quad \text{and} \quad \|S - T\| < \varepsilon.$$

Kim et al. introduced in [5] the notion of the generalized approximate hyperplane series property (generalized AHSP, for short) for a pair (X, Y) of Banach spaces. This property provides a necessary and sufficient condition for the pair $(\ell_1(X), Y)$ to satisfy the BPBp, where $\ell_1(X)$ denotes the Banach space of all sequences (x_k) in X such that the series $\sum_{k=1}^{\infty} \|x_k\|$ converges.

Definition 1.2. *A pair of Banach spaces (X, Y) is said to have the generalized AHSP if, for every $\varepsilon > 0$, there exists $0 < \eta(\varepsilon) < \varepsilon$ such that given sequences $(T_k) \subset S_{\mathcal{L}(X,Y)}$ and $(x_k) \subset S_X$, and a convex series $\sum_{k=1}^{\infty} \alpha_k$ such that*

$$\left\| \sum_{k=1}^{\infty} \alpha_k T_k x_k \right\| > 1 - \eta(\varepsilon),$$

there exist a subset $A \subset \mathbb{N}$, $y^* \in S_{Y^*}$, and sequences $(S_k) \subset S_{\mathcal{L}(X,Y)}$, $(z_k) \subset S_X$ satisfying the following:

(1) $\sum_{k\in A} \alpha_k > 1-\varepsilon$,

(2) $\|z_k - x_k\| < \varepsilon$ *and* $\|S_k - T_k\| < \varepsilon$ *for all* $k \in A$,

(3) $y^*(S_k z_k) = 1$ *for every* $k \in A$.

We refer the reader to [5] for examples of pairs of Banach spaces that satisfy the generalized AHSP.

2 Main Results

Let L be a locally compact Hausdorff space. We denote by $\mathcal{C}_0(L, X)$ the Banach space of all continuous functions from L into X that vanish at infinity, endowed with the supremum norm. Given a compact Hausdorff space K, let $\mathcal{C}(K)$ denote the Banach space of complex-valued continuous functions on K. A *uniform algebra* $\mathcal{A}$ is a closed subalgebra of $\mathcal{C}(K)$ that separates the points of K. We denote by $\mathcal{A}^X$ the subspace of $\mathcal{C}(K, X)$ defined by $\mathcal{A}^X = \{f \in \mathcal{C}(K,X) : x^* \circ f \in \mathcal{A} \,\text{for all}\, x^* \in X^*\}$, where $\mathcal{A}$ is a uniform algebra. We prove the following results.

Theorem 2.1. *[4, Theorem 2.4] Let X be a finite-dimensional space, let Y be a Banach space, and let L be a locally compact Hausdorff space. If the pair (X, Y) has the generalized AHSP, then the pair $(X, \mathcal{C}_0(L, Y))$ has the generalized AHSP. Equivalently, if the pair $(\ell_1(X), Y)$ has the BPBp, then so does the pair $(\ell_1(X), \mathcal{C}_0(L, Y))$.*

Theorem 2.2. *[4, Theorem 2.7] Let X be a finite-dimensional complex space, let Y be a complex Banach space, and let $\mathcal{A}$ be a uniform algebra. If the pair (X, Y) has the generalized AHSP, then the pair $(X, \mathcal{A}^Y)$ has the generalized AHSP. Equivalently, if the pair $(\ell_1(X), Y)$ has the BPBp, then so does the pair $(\ell_1(X), \mathcal{A}^Y)$.*

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TENSORS AND NUCLEAR OPERATORS THAT ATTAIN THEIR NORMS

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Abstract

Following [1], we study the norm-attainment for the projective tensor product space $X\widehat{\otimes}_\pi Y$ and the space of nuclear operators $\mathcal{N}(X,Y)$, where X and Y are Banach spaces. The analysis addresses two fundamental questions: under what conditions does every element $z \in X\widehat{\otimes}_\pi Y$ attains its projective norm, and failing that, when the set $NA_\pi(X\widehat{\otimes}_\pi Y)$ dense in $X\widehat{\otimes}_\pi Y$. Similarly, we studied the norm-attainment of nuclear operators $T : X \to Y$ with its respective nuclear norm.

1 Introduction

Let X and Y be two Banach spaces. The projective tensor product $X\widehat{\otimes}_\pi Y$ is the completion of the algebraic tensor product $X \otimes Y$ endowed with the projective norm:

$$\|z\|_\pi = \inf\left\{\sum_{n=1}^{\infty}\|x_n\|\|y_n\| : \sum_{n=1}^{\infty}\|x_n\|\|y_n\| < \infty, z = \sum_{n=1}^{\infty} x_n \otimes y_n\right\}.$$

The topological dual $(X\widehat{\otimes}_\pi Y)^*$ of the projective tensor product is identified with $\mathcal{L}(X, Y^*)$, where the action of $G : X \longrightarrow Y^*$ as a linear functional on $X\widehat{\otimes}_\pi Y$ is given by

$$G\left(\sum_{n=1}^{\infty} x_n \otimes y_n\right) = \sum_{n=1}^{\infty} G(x_n)(y_n).$$

A bounded linear operator $T : X \to Y$ is said to be *nuclear* if it is in range of the canonical mapping $J : X^*\widehat{\otimes}_\pi Y \longrightarrow \mathcal{L}(X,Y)$ defined by $J\left(\sum_{n=1}^{\infty}\varphi_n \otimes y_n\right)(x) = \sum_{n=1}^{\infty}\varphi_n(x)\, y_n$. The set of all nuclear operators $\mathcal{N}(X,Y)$ is a Banach space endowed with the *nuclear norm*

$$\|T\|_{\mathcal{N}} = \inf\left\{\sum_{n=1}^{\infty}\|\varphi_n\|\|y_n\| : T(x) = \sum_{n=1}^{\infty}\varphi_n(x) y_n\right\}.$$

The following definitions regarding the norm-attainment of tensor products and nuclear operators were introduced by S. Dantas, M. Jung, O. Roldán and A. Rueda Zoca in [1].

Definition 1.1. *Let X, Y be Banach spaces. we say that*

1. $z \in X\widehat{\otimes}_\pi Y$ attains its projective norm *if there is a bounded sequence $(x_n, y_n) \subset X \times Y$ with $\sum_{n=1}^{\infty}\|x_n\|\|y_n\| < \infty$ such that $z = \sum_{n=1}^{\infty} x_n \otimes y_n$ and that $\|z\|_\pi = \sum_{n=1}^{\infty}\|x_n\|\|y_n\|$. The set of all such vectors is denoted by $NA_\pi(X\widehat{\otimes}_\pi Y)$.*

2. $T \in \mathcal{N}(X,Y)$ attains its nuclear norm *if there is a bounded sequence $(\varphi_n, y_n) \subset X^* \times Y$ with $\sum_{n=1}^{\infty}\|\varphi_n\|\|y_n\| < \infty$ such that $T = \sum_{n=1}^{\infty}\varphi_n \otimes y_n$ and that $\|T\|_{\mathcal{N}} = \sum_{n=1}^{\infty}\|\varphi_n\|\|y_n\|$. The set of all such operators is denoted by $NA_{\mathcal{N}}(X,Y)$.*

2 Main Results

Theorem 2.1. *[1, Theorem 3.1] Let X, Y be Banach spaces. Let $z \in X\widehat{\otimes}_\pi Y$ with $z = \sum_{n=1}^{\infty} \lambda_n x_n \otimes y_n$, where $(\lambda_n)_n \subset \mathbb{R}^+$ and $\|x_n\| = \|y_n\| = 1$ for every $n \in \mathbb{N}$. Then the following are equivalent:*

1. $\|z\|_\pi = \sum_{n=1}^{\infty} \lambda_n$.
2. *There exists some $G \in \mathcal{L}(X, Y^*)$ with $\|G\| = 1$ such that $G(x_n)(y_n) = 1$ for every $n \in \mathbb{N}$.*
3. *Every norm one operator $G \in \mathcal{L}(X, Y^*)$ such that $G(z) = \|z\|_\pi$ satisfies that $G(x_n)(y_n) = 1$ for every $n \in \mathbb{N}$.*

Theorem 2.2. *[1, Theorem 3.2] Let X, Y be Banach spaces. Let $T \in \mathcal{N}(X, Y)$ with $T = \sum_{n=1}^{\infty} \lambda_n \varphi_n \otimes y_n$, where $(\lambda_n)_n \subset \mathbb{R}^+$ and $\|\varphi_n\| = \|y_n\| = 1$ for every $n \in \mathbb{N}$. Then the following are equivalent:*

1. $\|T\|_{\mathcal{N}} = \sum_{n=1}^{\infty} \lambda_n$
2. *There exists some $G \in (\ker J)^\perp$ with $\|G\| = 1$ such that $G(\varphi_n)(y_n) = 1$ for every $n \in \mathbb{N}$.*
3. *Every norm one operator $G \in \ker(J)^\perp$ such that $G(T) = \|T\|_{\mathcal{N}}$ satisfies that $G(x_n)(y_n) = 1$ for every $n \in \mathbb{N}$.*

Next, we list some examples from [1]:
(a) If X and Y are finite-dimensional spaces, then $NA_\pi(X\widehat{\otimes}_\pi Y) = X\widehat{\otimes}_\pi Y$.
(b) If Y is a Banach space then $NA_{\mathcal{N}}(c_0, Y) = \mathcal{L}(c_0, Y)$, and $NA_\pi(\ell_1\widehat{\otimes}_\pi Y) = \ell_1\widehat{\otimes}_\pi Y$.
(c) If H is a complex Hilbert space, then $NA_{\mathcal{N}}(H, H) = \mathcal{L}(H, H)$.

The following is an interesting result that relates the norm-attainment of tensors and the density of the norm-attainment of operators:

Corollary 2.1. *[1, Corollary 3.11] Let X and Y be Banach spaces such that every tensor in $X\widehat{\otimes}_\pi Y$ attains its projective norm. Then, the set of norm-attaining operators from X in Y^* is dense in $\mathcal{L}(X; Y^*)$.*

The last corollary provides a method for finding tensor products containing non-norm attaining tensors. For instance, if Y^* is a strictly convex Banach space without the Radon-Nikodým property, then $NA_\pi(L_1([0,1])\widehat{\otimes}_\pi Y^*) \neq L_1([0,1])\widehat{\otimes}_\pi Y^*$ (see [1, Example 3.12 (b)]) .

This leads to the question whether the set of norm-attaining tensors is at least dense in the projective tensor product.

Theorem 2.3. *[1, Theorem 4.8] Let X be a Banach space satisfying the metric $\pi-$property. If either Y satisfies the metric $\pi-$property or Y is uniformly convex, then $\overline{NA_\pi(X\widehat{\otimes}_\pi Y)} = X\widehat{\otimes}_\pi Y$.*

An analogous result holds for the density of norm-attaining nuclear operators.

Corollary 2.2. *[1, Corollary 4.9] Let X be a Banach space such that X^* satisfies the metric $\pi-$property. If Y satisfies the metric $\pi-$property or Y is uniformly convex, then $\overline{NA_{\mathcal{N}}(X, Y)} = \mathcal{N}(X, Y)$.*

For instance, every Banach space with a Schauder basis can be renormed to have the metric π-property. Also, L_1-preduals and L_p-spaces $(1 \leq p < \infty)$ have the metric π-property.

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STRICTLY POSITIVE DEFINITE FUNCTIONS ON REAL SPHERES

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Abstract

Our objective is to present the characterizations of positive definite functions of any order [1] on unit spheres $\mathbb{S}^{d-1}$ in the d-dimensional Euclidean space, as well as on the unit sphere $\mathbb{S}^{\infty}$ of the infinite-dimensional Hilbert space l_2, as established by Isaac Jacob Schoenberg in [2]. Furthermore, we will talk about some necessary conditions to strict positive definitiness [4] of order $n \in \mathbb{Z}^+$ and necessary and sufficient conditions when $n = 2$, as established by Valdir Antônio Menegatto in [4].

1 Introduction

Definition 1.1. *A real continuous function f defined on $[-1,1]$ is positive definite (PD) of any order on $\mathbb{S}^d$ when the matrix with ij-entry given by $(f(\langle x_i, x_j\rangle))_{i,j=1}^n$ is nonnegative definite for all $n \in \mathbb{N}$ and distinct points $x_1, \ldots, x_n \in \mathbb{S}^d$. On the other hand, if $(f(\langle x_i, x_j\rangle))_{i,j=1}^n$ is positive definite for a fixed n, f is said to be strictly positive definite (SPD) of order $n \in \mathbb{N}$ in $\mathbb{S}^d$.*

Definition 1.2. *Let $d \geq 3$ and $k \in \mathbb{N}_0$. The normalized Gegenbauer polynomial $G_k^{(d)}$ of degree k with index d is*

$$G_k^{(d)}(t) := \frac{\Gamma(\frac{d-1}{2})}{\sqrt{\pi}\Gamma(\frac{d-2}{2})} \int_{-1}^{1} [t + i(1-t^2)^{\frac{1}{2}} s]^k (1-s^2)^{\frac{d-4}{2}} ds.$$

Theorem 1.1. *([2, Theorem 3.1]) Let $d \geq 3$ and f a real continuous function in $[-1,1]$. Then, f is positive definite on $\mathbb{S}^{d-1}$ if, and only if, f has a representation in a Gegenbauer polynomials series of the form*

$$f(t) = \sum_{k=0}^{\infty} a_k^{(d)}(f) G_k^{(d)}(t), \quad a_k^{(d)} \geq 0, \quad \forall k \geq 0 \tag{1}$$

and

$$\sum_{k=0}^{\infty} a_k^{(d)}(f) < \infty. \tag{2}$$

Lemma 1.1. *([2, Lemma 4.1]) Let $t \in (-1,1)$. For every $\epsilon > 0$, there exists $L = L(t,\epsilon) > 0$ such that $(d-2)/2 > L(t,\epsilon)$ implies*

$$|G_k^{(d)}(t) - t^k| < \epsilon, \quad \textit{for all } k \geq 0. \tag{3}$$

Theorem 1.2. *([2, Theorem 4.2]) A PD function f on $\mathbb{S}^{\infty}$ is necessarily of the form*

$$f(t) = \sum_{k=0}^{\infty} a_k(f) t^k, \ a_k \geq 0, \ \forall k \geq 0, \ \textit{and} \ \sum_{k=0}^{\infty} a_k(f) < \infty. \tag{4}$$

Remark 1.1. *Theorem 1.2 is also sufficient. It follows from the fact that PD functions in $\mathbb{S}^{\infty}$ form a convex cone and the identity function is PD in $\mathbb{S}^{\infty}$.*

2 Main Results

It is not hard to prove that whether or not a function f as in (1) is strictly positive definite of a certain order on $\mathbb{S}^d$ depends only on the set $K_d(f) := \{k : a_k^{(d+1)}(f) > 0\}$ and not on the particular values of the $a_k^{(d+1)}(f)$. In other words, if two positive definite functions f and g on $\mathbb{S}^d$ are such that $K_d(f) = K_d(g)$, then f is strictly positive definite of order n on $\mathbb{S}^d$ if, and only if, g is so. Consequently, we say that a nonempty subset K of $\mathbb{N} := \{0, 1, \ldots\}$ *induces strict positive definiteness (shortly induces SPD) of order n on* $\mathbb{S}^d$, whenever all f as in (1) and satisfying $K_d(f) = K$, are strictly positive definite of order n on $\mathbb{S}^d$.

The first theorem is a necessary condition for SPD on $\mathbb{S}^1$:

Theorem 2.1. *([4, Theorem 2.2]) Let K be a subset of $\mathbb{N}$ and let n be an integer bigger than 1. If K contains an odd multiple of $\lfloor n/2 \rfloor$, then K induces SPD of order n on S^1.*

The condition presented in the next theorem characterizes SPD of order n on $\mathbb{S}^1$. However, that condition alone will allow us to characterize SPD of order n on $\mathbb{S}^d$, $n \leq d$, as Menegatto showed in ([4, Theorem 2.14]), which we will present as the last theorem of this work.

Theorem 2.2. *([4, Theorem 2.5]) A subset K of $\mathbb{N}$ induces SPD of order 2 on $\mathbb{S}^1$ if, and only if, it contains an even integer and a relatively prime subset. It induces SPD of order 2 on $\mathbb{S}^d$, $d \geq 2$ if, and only if, it contains an even integer and an odd integer.*

Theorem 2.3. *([4, Theorem 2.8]) Let K be a subset of $\mathbb{N}$ and let n be an integer bigger than 1. If K induces SPD of order n on $\mathbb{S}^d$, $d = 1, 2, \ldots, \infty$, then the set $\{k \in K : k \geq \lfloor n/2 \rfloor - 1\}$ contains an even integer and an odd integer.*

A characterization of SPD on $\mathbb{S}^\infty$ is now at hand.

Theorem 2.4. *([4, Theorem 2.9]) Let n be an integer bigger than 1. A subset K of $\mathbb{N}$ induces SPD of order n on $\mathbb{S}^\infty$ if, and only if, the set $\{k \in K : k \geq \lfloor n/2 \rfloor - 1\}$ contains an even integer and an odd integer.*

Theorem 2.5. *([4, Theorem 2.14]) Let d be an integer at least 2 and let n be at most d. A subset K of $\mathbb{N}$ induces SPD of order n on $\mathbb{S}^d$ if, and only if, the set $\{k \in K : k \geq \lfloor n/2 \rfloor - 1\}$ contains an even integer and an odd integer.*

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CONVOLUTION ROOTS AND FOURIER-JACOBI APPROXIMATION ON COMPACT TWO-POINT HOMOGENEOUS SPACES

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Abstract

We establish new spectral criteria for the existence of convolution roots of isotropic positive definite kernels on compact two-point homogeneous spaces. Our results generalize classical theorems for spheres, extending them to all such spaces. We also construct a family of weighted Fourier-Jacobi approximation operators with L^2-convergence guarantees. Each operator admits a convolution-type representation, providing practical tools for kernel approximation.

1 Introduction

Compact two-point homogeneous spaces, such as spheres and projective spaces, are Riemannian manifolds characterized by their high degree of symmetry: any two pairs of points at the same distance can be mapped into each other by an isometry. This property supports a rich framework of harmonic analysis, where addition formulas and Jacobi polynomials play central roles.

Isotropic positive definite kernels on these spaces depend only on the geodesic distance and admit spectral representations in terms of nonnegative Fourier-Jacobi expansions. Foundational results in this direction were established by Schoenberg [4] and Gangolli [3].

In this work, we present new results on the existence of convolution roots for such kernels, extending previous analyses on spheres (e.g., [5], [1]) to all compact two-point homogeneous spaces. We also introduce a family of weighted Fourier-Jacobi approximation operators, study their L^2-convergence, and interpret each operator as a convolution with an explicitly constructed kernel.

Complete proofs and further technical details appear in [2].

2 Main Results

Let $L^2(M^d)$ denote the space of square-integrable functions on the compact two-point homogeneous space M^d, with respect to its invariant probability measure σ_d. Given two kernels $K_1, K_2 \in L^2(M^d \times M^d)$, their convolution is defined by

$$(K_1 * K_2)(x, y) = \int_{M^d} K_1(x, \xi)\, K_2(\xi, y)\, d\sigma_d(\xi).$$

Our first main result gives sufficient conditions for the existence of convolution roots.

Theorem 2.1. *Let $K \in L^2(M^d \times M^d)$ be an isotropic kernel with Fourier-Jacobi coefficients satisfying*

$$\langle K, J_n^{(\alpha,\beta)} \rangle \geq 0, \quad \forall\, n \geq 0, \quad \textit{and} \quad \sum_{n=0}^{\infty} \frac{\langle K, J_n^{(\alpha,\beta)} \rangle^2}{h_n^{(\alpha,\beta)}} < \infty.$$

Then there exists an isotropic positive definite kernel L such that

$$K = L * L.$$

This result generalizes classical findings for spheres [1, 5] to the wider class of compact two-point homogeneous spaces.

To establish this result, we first describe an orthonormal basis for the space of radial kernels.

Lemma 2.1. *The family*

$$\left\{\left(r_n\, h_n^{(\alpha,\beta)}\right)^{-1/2} J_n^{(\alpha,\beta)}\right\}_{n=0}^{\infty}$$

forms an orthonormal basis for the subspace of radial functions in $L^2(M^d \times M^d)$.

We then introduce an approximation scheme based on spectral filtering.

Proposition 2.1. *Let* K *be isotropic with Fourier coefficients* $\hat{K}_k$. *For each* $n \geq 0$, *define*

$$T_n(K)(x,y) = \sum_{k=0}^{n} \sum_{j=1}^{\delta(k,d)} w_{k,j}(n)\, \hat{K}_k\, S_{k,j}(x)\, S_{k,j}(y),$$

where $w_{k,j}(n)$ *are spectral weights. If*

$$\lim_{n\to\infty} w_{k,j}(n) = 1, \quad \text{for all } k, j,$$

then $T_n(K)$ *converges to* K *in* $L^2(M^d \times M^d)$.

Corollary 2.1. *If, for each* k, *the weights* $w_{k,j}(n)$ *are independent of* j, *then each* $T_n(K)$ *is isotropic.*

Each operator T_n admits a convolution-type representation, with kernel expressible in terms of Jacobi polynomials and the addition formula for spherical harmonics. Together with the spectral criterion for convolution roots, this constitutes the main contribution of this work.

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STRUCTURAL PROPERTIES OF BANACH SPACES OF $\mathcal{I}$-NULL SEQUENCES

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Abstract

Using a natural way to define convergence of sequences in terms of a fixed ideal $\mathcal{I}$ over the natural numbers, it is possible to define a Banach space of $\mathcal{I}$-null sequences. The classical Banach space c_0 is a particular case of this construction, when one considers the ideal of finite subsets. In this work we discuss some classical results of c_0 that are preserved for the spaces of $\mathcal{I}$-null sequences.

1 Introduction

An ideal on $\mathbb{N}$ is a proper subset $\mathcal{I} \subset \mathcal{P}(\mathbb{N})$ that is hereditary (for subsets) and closed under finite unions. In addition, $\mathcal{I}$ is said admissible if the ideal of finite subsets $\mathrm{Fin} \subseteq \mathcal{I}$. Throughout this work, unless stated otherwise, $\mathcal{I}$ will always denote an admissible ideal on $\mathbb{N}$, X, Y denote Banach spaces.

Definition 1.1. *Let $\mathcal{I}$ an ideal and a sequence $(x_n)_{n\in\mathbb{N}}$ in a Banach space X, we say that $y \in X$ is the $\mathcal{I}$-limit of the sequence, denoted by $y = \mathcal{I} - \lim_n x_n$, if:*

$$\forall \varepsilon > 0 : \quad A(\varepsilon, y) \doteq \{n \in \mathbb{N} : |x_n - y| \geq \varepsilon\} \in \mathcal{I}.$$

The notion of $\mathcal{I}$-convergence defined above was introduced in [4], although other authors have already studied it in different particular cases (we refer [3, 1, 2]). In a natural way, for a fixed Banach space X, one can define the space of $\mathcal{I}$-null sequences in X as follows:

$$c_{0,\mathcal{I}}(X) \doteq \left\{(x_n) \in \ell_\infty(X) : \mathcal{I} - \lim_n x_n = 0\right\},$$

where $\ell_\infty(X)$ denotes the space of bounded sequences in X. In the particular case $X = \mathbb{R}$, we simply denote this space by $c_{0,\mathcal{I}}(\mathbb{R}) = c_{0,\mathcal{I}}$.

Observe that for the ideal Fin of finite substets, $c_{0,\mathrm{Fin}} = c_0$. In this work we will present which properties of c_0 are preserved in the spaces $c_{0,\mathcal{I}}$, and which are not.

2 Main Results

In this section we will analyze some classical properties of the structure of the $c_{0,\mathcal{I}}$ spaces, from the standpoint of the structural properties that are satisfied by c_0.

Definition 2.1. *Let $\mathcal{I}$ an ideal. We say that $\mathcal{I}$ is separably injective (or has the Sobczyk property) if the space $c_{0,\mathcal{I}}$ is separably injective.*

It can be proved that for most of the simple constructions involvind ideals $\mathbb{N}$ (finite intersection of maximal ideals or the direct sum of ideals, each having the Sobczyk property) that $c_{0,\mathcal{I}}$ is separably injective. To prove the general case, we need the following:

Proposition 2.1 ([5, Proposition 3.1]). *If $\mathcal{I}$ is an ideal on $\mathbb{N}$, then $c_{0,\mathcal{I}}$ is Banach isometric to $C_0(U_{\mathcal{I}})$, where $U_{\mathcal{I}}$ is an open set of $\beta\mathbb{N}$.*

Using the above identification and a classical result for $C(K)$ spaces yields:

Theorem 2.1. *If $\mathcal{I}$ is an ideal on $\mathbb{N}$, then $c_{0,\mathcal{I}}$ is separably injective.*

If X is a Banach space, it easy to check that the space of bounded operators $B(X, c_{0,\mathcal{I}})$ is isometric to the space $c^{w^*}_{0,\mathcal{I}}(X^*) = \{(x^*_n) \in \ell_\infty(X^*) : \mathcal{I} - \lim x^*_n(x) = 0 \text{ for all } x \in X\}$ in the same way it is done for c_0. Although, the isometry is essentially the same, it remains open if there exists a classical proof of the Sobczyk theorem for the spaces $c_{0,\mathcal{I}}$ without passing to $\beta\mathbb{N}$ as it was done in Theorem 2.1.

It is easy to check that $c_0 \subseteq c_{0,\mathcal{I}}$ for every ideal $\mathcal{I}$, in respect to complementation, we have the following:

Proposition 2.2. $c_0 \overset{c}{\hookrightarrow} c_{0,\mathcal{I}}(X)$ *if, and only if,* $c_0 \overset{c}{\hookrightarrow} c_{0,\mathcal{I}^\omega}(X)$.

A map $m\colon \mathcal{I} \to \mathbb{R}$ is called a *signed finitely additive measure* if $m(A \cup B) = m(A) + m(B)$ whenever $A \cap B = \emptyset$. Let $ba(\mathcal{I})$ be the space of all signed finitely additive measures with finite total variation.

Theorem 2.2. *Let $\mathcal{I}$ be an ideal on $\mathbb{N}$. There exists an isometry from $c^*_{0,\mathcal{I}}$ onto $ba(\mathcal{I})$. More precisely, for $\Phi \in c^*_{0,\mathcal{I}}$, we let $m_\Phi(A) = \Phi(\chi_A)$ for all $A \in \mathcal{I}$. Then, the map $T : c^*_{0,\mathcal{I}} \to ba(\mathcal{I})$ defined by $T(\varphi) = m_\varphi$ is an onto isometry.*

Moreover, when considering the space $c_{0,\mathcal{I}}(X)$ we have the following theorem:

Theorem 2.3. *Let X be a Banach space and $\mathcal{I}$ an ideal. The operator $S : c_{0,\mathcal{I}}(X)^* \to ba(\mathcal{I}, \ell_\infty(X)^*)$, given by $S(\varphi) = m_\varphi$, where:*

$$m_\varphi(A)\Big((x_n)_n\Big) \doteq \varphi\Big(\chi_A \cdot (x_n)\Big), \qquad \forall A \in \mathcal{I}, (x_n) \in \ell_\infty(X)$$

is an isometry.

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ON NORM-ATTAINING POSITIVE OPERATORS BETWEEN BANACH LATTICES

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Abstract

In this talk, we present recent results considering the norm-attainment of positive operators between Banach lattices. By considering an absolute version of James boundaries, we prove that: If E is a reflexive Banach lattice whose order is given by a basis and F is a Dedekind complete Banach lattice, then every positive operator from E to F is compact if and only if every positive operator from E to F attains its norm. An analogue result considering that E is reflexive and the order in F is continuous and given by a basis was proven.

1 Introduction

A classical problem in Functional Analysis consists in studying the norm attainment of bounded linear operators between Banach spaces. One of the most known results in this direction is the James theorem which states that a Banach space X is reflexive if and only if every bounded linear functional on X attains its norm. Recently, S. Dantas, M. Jung and G. Martínez-Cervantes proved that if X and Y are Banach spaces such that X is reflexive and the pair (X, Y) satisfies the bounded compact approximation property, then every bounded linear operator from X to Y is compact if and only if every bounded linear operator from X to Y attains its norm (see [2, Theorem B]).

In the Banach lattice setting, the study of norm-attaining positive linear functionals has appeared, indirectly, in the last decade. For instance, D. Ji, B. Lee, Q. Bu (2014) and T. Oikhberg, M. A. Tursi (2020) proved, separately, the lattice version of James theorem: An order continuous Banach latice E is reflexive if and only if every positive linear functional on E attains its norm. Moreover, S. Dantas, G. MartÃnez-Cervantes, J. D. Rodríguez-Abellán and A. Rueda-Zoca (2022) studied norm-attaining lattice-homomorphisms. Motivated by these recent results, we obtained a positive version of [2, Theorem B] in [3, Theorem 2.12].

2 Main Results

The main result of this presentation is the following:

Theorem 2.1. *Let E be a reflexive Banach lattice and let F be a Dedekind complete Banach lattice. Consider the following conditions:*
(1) *Every positive operator $T : E \to F$ is compact.*
(2) *Every positive operator $T : E \to F$ attains its norm.*
(3) *$B_{\mathcal{K}^+(E;F)}$ is sequentially closed in the absolutely strong operator topology.*
Then (1)⇒(2)⇒(3). In addition, if the order of E is given by a basis or F has order continuous norm whose order is defined by a basis, then (3)⇒(1).

The proof of implication (1)⇒(2) is a classical exercise in Functional Analysis. To prove (2)⇒(3), we needed to introduce two new concepts in the Banach lattice setting: an absolute version of the James boundaries and an absolute version of the strong operator topology that we define as follows. Let E and F be Banach lattices with F

being Dedekind complete. We define the absolute strong operator topology ($|SOT|$, for short) in $\mathcal{L}^r(E;F)$ by the following basic neighborhoods

$$N(T;A,\varepsilon) = \{S \in \mathcal{L}^r(E;F) : \||T-S|(x)\| < \varepsilon \text{ for every } x \in A\},$$

where $A \subset E^+$ is an arbitrary finite set and $\varepsilon > 0$. Thus, a net $(T_\alpha)_\alpha \subset \mathcal{L}^r(E;F)$ converges to T in the $|SOT|$ if and only if $(|T_\alpha - T|(x))_\alpha$ converges to 0 for every $x \in E^+$. The implication (2)⇒(3) of Theorem 2.1 is essentially in the following lemma.

Lemma 2.1. *Let E and F be Banach lattices with F being Dedekind complete. If there exists a norm-closed convex set $C \subset \mathcal{L}^r(E;F)$ that is not sequentially $|SOT|$-closed, then there exists a non-norm attaining positive operator $T : E \to F$.*

Finally, the implication (3)⇒(1) of Theorem 2.1 is an application of our next lemma.

Lemma 2.2. *If G is a Banach lattice with order continuous norm whose order is defined by a basis, then there exists a sequence $(S_n)_n \subset B_{\mathcal{K}^+(G;G)}$ such that $\lim_{n\to\infty} \||S_n - id_G|(x)\| = 0$ for every $x \in G$.*

Remark 2.1. (1) *It is important to observe that Theorem 2.1 can be applied to $E = \ell_p$ and $F = L_q(\mu)$ whenever $1 \leq q < p \leq 2$, because every positive linear operator $T : E \to F$ is compact ([1, Theorem 4.9]). Note, however, that there exist non-compact linear operators from ℓ_p into $L_q(\mu)$ by [1, Theorem 4.7]. Thus, it follows from [2, Theorem B] that there exists a non-norm attaining operator from ℓ_p into $L_q(\mu)$, while every positive operator from ℓ_p into $L_q[0,1]$ attains its norm.*
(2) *Let $E = L_p(\mu)$ and $F = \ell_q$ with $2 \leq q < p < \infty$. By [1, Theorem 4.7], there exists a non-compact bounded linear operator from $L_p(\mu)$ into ℓ_q, and so by [2, Theorem B] there exists a non-norm attaining operator from $L_p(\mu)$ into ℓ_q. Nevertheless, every positive operator from $L_p(\mu)$ into ℓ_q is norm attaining from Theorem 2.1 and [1, Theorem 4.9].*
(3) *If E is an AM-space with an order unit e, i.e. $B_E = [-e,e]$, then every positive linear operator from E into any Banach lattice F attains its norm in e. However, since E is not reflexive, there exists a (non-positive) continuous linear functional $f \in E^*$ that does not attain its norm. Thus, for every Banach lattice F and every $0 \neq y \in F$, the non-positive linear operator $T(x) = f(x)y$ does not attain its norm.*

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ON INESSENTIAL PERTURBATIONS AND KALTON-PECK SPACE

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Abstract

We consider quantitative notions of strictly singular and inessential operators and study their spectral properties through perturbations with Fredholm operators. We examine the associated notion of singular twisted sums, with special attention paid to the Kalton-Peck space, which is a non-trivial twisted Hilbert space. Additionally, we explore perturbations by complex structures and symplectic structures to generalize some results from [1, 2] concerning the problem of determining whether the Kalton-Peck space is isomorphic to its hyperplanes. This work will be submitted to arXiv soon.

1 Introduction

The ideal $\mathcal{IN}$ of *inessential operators* consists of operators $R \in L(X,Y)$ for which $\mathrm{Id}_X - TR$ is Fredholm for all $T \in L(Y,X)$ (or equivalently, $\mathrm{Id}_Y - RT$ is Fredholm for all $T \in L(Y,X)$). Here, an operator is *Fredholm* if it has closed range, finite dimensional kernel, and finite codimensional image. The index of a Fredholm operator T is defined as $\mathrm{ind}(T) := \dim(\ker T) - \dim(\operatorname{coker} T)$. A key property of $\mathcal{IN}$ is the stability of Fredholm operators under perturbations: if $F \in L(X,Y)$ is Fredholm and $R \in \mathcal{IN}(X,Y)$, then $F + R$ remains Fredholm (with the same index).

A short exact sequence of Banach spaces is a diagram of Banach spaces and operators

$$0 \longrightarrow Y \xrightarrow{\ j\ } X \xrightarrow{\ q\ } Z \longrightarrow 0 \tag{1}$$

such that the image of each arrow coincides with the kernel of the subsequent one. The middle space X is usually called a *twisted sum* of Y and Z. It follows from the open mapping theorem that X contains a copy of Y such that the corresponding quotient is isomorphic to Z.

Definition 1.1. *A homogeneous map $\Omega : Z \to Y$ is quasi-linear if*

$$\|\Omega(z_1 + z_2) - \Omega(z_1) - \Omega(z_2)\| \leq C(\|z_1\| + \|z_2\|)$$

for some constant $C > 0$ and every $z_1, z_2 \in Z$. Denote by $c(\Omega)$ the optimal constant C that satisfies the condition above.

Let us denote by $Y \oplus_\Omega Z$ the product space $Y \times Z$ endowed with the quasi-norm $\|(y,z)\|_\Omega = \|y - \Omega z\|_Y + \|z\|_Z$. In this case, the sequence

$$0 \longrightarrow Y \xrightarrow{\ j\ } Y \oplus_\Omega Z \xrightarrow{\ q\ } Z \longrightarrow 0,$$

where $j(y) = (y, 0)$ and $q(y,z) = z$, is exact making Y isometric to its copy in $Y \oplus_\Omega Z$ such that the respective quotient is also isometric to Z. Kalton and Peck proved that every short exact sequence is equivalent to one of the type above for some quasi-linear map.

The fundamental example in the theory is the *Kalton-Peck space* $Z_2 = \ell_2 \oplus_{\Omega_2} \ell_2$, where the quasi-linear map defined on vectors of finite support by

$$\Omega_2(x)(n) = x(n) \log \frac{\|x\|_2}{|x(n)|}$$

generates a non-trivial twisted sum of Hilbert spaces.

An important open question is whether Z_2 is isomorphic to its hyperplanes. Although Banach's general hyperplane problem was solved by Gowers [3], see also the class of HI spaces defined by Gowers and Maurey [4], the case of Z_2 is still open.

2 Main Results

Definition 2.1. *Let S be an operator between Banach spaces X and Y, and $C > 0$. We say that*

S is C-singular if every infinite dimensional subspace of X contains a normalized vector x such that $\|Sx\| \leq C$. We denote by $\rho(S)$ the infimum of all constants C such that S is C-singular.

S is C-inessential if $\mathrm{Id}_Y - ST$ (equivalently, $\mathrm{Id}_X - TS$) is Fredholm whenever $T \in L(Y, X)$ satisfies $\|T\| < C$.

It follows that if S is $1/C$-singular operator, then S is C-inessential operator.

Lemma 2.1. *Let $T : X \to Y$ be a Fredholm operator. Then there exists a constant $C(T) > 0$ such that if S is C-inessential for some $C > C(T)$, we have $T + S$ is Fredholm, and $i(T + S) = i(T)$.*

Proposition 2.1. *Let Y be an infinite dimensional real Banach space and let $X = \mathbb{R} \oplus Y$. Let $I \in GL(X)$ such that $I^2 + Id$ is C-inessential for some $C > 2$. Then there exists a constant $\alpha = \alpha(I) > 0$ and a C/α-inessential operator S such that either*

$$(I + S)^2 = -Id \text{ or } I + S = \begin{bmatrix} 1 & 0 \\ 0 & J \end{bmatrix}$$

where $J \in \mathcal{L}(Y)$ satisfies $J^2 = -Id$.

Theorem 2.1. *Consider an exact sequence $0 \to X \to Z \to Y \to 0$ with quotient map π. Let j be a complex structure on X which extends to a complex structure on Z of norm at most K. Then any complex structure on H, H a hyperplane of Z, extending j, must have norm at least $2\rho(\pi)^{-1}K^{-1} - K$.*

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P-HARMONIC FUNCTIONS IN THE UPPER HALF SPACE

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Abstract

This paper investigates the existence, nonexistence, and qualitative properties of p-harmonic functions in the upper half-space $\mathbb{R}^N_+$ $(N \geq 3)$ satisfying nonlinear boundary conditions for $1 < p < N$. Moreover, the symmetry of positive solutions is shown by using the method of moving planes.

1 Introduction

This work treats some aspects of the $p-$harmonic equations in the upper half-space under nonlinear boundary conditions. The discussion focuses on the existence, nonexistence and qualitative properties of solutions for the following model of quasilinear elliptic problems with nonlinear boundary conditions:

$$\begin{cases} \Delta_p u \equiv \operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0 & \text{in} \quad \mathbb{R}^N_+, \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} + \lambda |u|^{p-2}u = |u|^{q-2}u & \text{on} \quad \mathbb{R}^{N-1}, \end{cases} \tag{$\mathcal{P}_\lambda$}$$

where $\mathbb{R}^N_+ := \{x = (x', x_N) \in \mathbb{R}^N : x' \in \mathbb{R}^{N-1}, x_N > 0\}$ standards for the upper half-space, λ is a real parameter, ν is the unit outer normal to the boundary $\partial\mathbb{R}^N_+ := \mathbb{R}^{N-1}$, $1 < p < N$ and $p \leq q < \infty$. The nonlinear differential operator $\Delta_p u \equiv \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is known as the $p-$Laplacian.

Problem ($\mathcal{P}_\lambda$) is related to the Euler-Lagrange equation associated with the Sobolev embedding

$$\left(\int_{\mathbb{R}^{N-1}} |u|^q \, \mathrm{d}x x'\right)^{p/q} \leq S_\lambda^{-1}(p,q) \left(\int_{\mathbb{R}^N_+} |\nabla u|^p \, \mathrm{d}x x + \lambda \int_{\mathbb{R}^{N-1}} |u|^p \, \mathrm{d}x x'\right), \quad u \in C_0^\infty(\mathbb{R}^N), \tag{1}$$

where $S_\lambda(p,q)$ is the best constant for (1). J. Escobar in the remarkable paper [2], by exploiting the conformal invariance of (1) when $p = 2$, $\lambda = 0$ and $N \geq 3$, characterize the minimizers of $S_0(2, 2_*)$. He also conjectured if a similar result holds for minimizers of $S_0(p, p_*)$, which was proved by B. Nazaret [3] for the case $N \geq 3$ and $1 < p < N$ based on the mass transportation approach. We also mention that a significant number of authors studied inequality (1) in bounded domains (or compact manifolds) see [1] and their references.

2 Main Results

Theorem 2.1. *Assume $p < q < p_* := p(N-1)/(N-p)$. Then, Problem ($\mathcal{P}_\lambda$) has a positive ground state solution for all $\lambda > 0$.*

Next, we investigate regularity and asymptotic behavior for solutions of Problem ($\mathcal{P}_\lambda$). By using a Moser iteration procedure and a Harnack type inequality, we show that weak solutions of Problem ($\mathcal{P}_\lambda$) decay to zero at infinity. To be precise, we state the following result:

Theorem 2.2. *Let u_λ be a weak solution of Problem ($\mathcal{P}_\lambda$) with $p < q < p_*$. Then, in the trace sense $u_\lambda|_{\mathbb{R}^{N-1}} \in L^\infty(\mathbb{R}^{N-1})$ and $u_\lambda \in L^\infty(\mathbb{R}^N_+)$. Consequently, weak solutions of ($\mathcal{P}_\lambda$) are of class $C^{1,\alpha}_{\text{loc}}(\overline{\mathbb{R}^N_+})$ for some $0 < \alpha < 1$. Furthermore, u_λ has the following decay rate at infinity*

$$u_\lambda(x) = O\left(|x|^{\frac{p-N}{p-1}}\right) \quad \text{as} \quad |x| \to +\infty.$$

On the nonexistence of solutions for Problem ($\mathcal{P}_\lambda$), we mention the result proved by B. Hu [4] for the particular case that $\lambda = 0$ and $p = 2$. Here, we complete his analysis by using a Pohožaev type identity. Indeed, we show the nonexistence results of weak solutions stated as follows.

Theorem 2.3. *Let $u_\lambda \in E \cap C^1(\overline{\mathbb{R}^N_+})$ be a weak solution of Problem ($\mathcal{P}_\lambda$). Then $u_\lambda \equiv 0$ if one of the conditions holds:*

i) If $\lambda = 0$ and $q \in [p, p_*) \cup (p_*, +\infty)$,

ii) If $\lambda > 0$ and $q \in [p_*, +\infty)$,

iii) If $\lambda < 0$ and $q \in [p, p_*]$.

Moreover, when the solution u_λ is nontrivial, it holds $\lambda \leq \|u_\lambda\|^{q-p}_{L^\infty(\mathbb{R}^{N-1})}$.

Next, we state our symmetric result for positive solutions of ($\mathcal{P}_\lambda$), which is the first one when the domain is the half-space and involves nonlinear boundary conditions to the best of our knowledge.

Theorem 2.4. *Let u be a positive weak solution of Problem ($\mathcal{P}_\lambda$) with $p < q < p_*$. Then u is radially symmetric with respect to $(N-1)$ first variables, that is*

$$u(x', x_N) = u(r, x_N),\ \forall\ (x', x_N) \in \mathbb{R}^{N-1} \times \mathbb{R}_+,\ \text{with}\ \ |x'| = r.$$

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