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Os organizadores do XII ENAMA expressam sua ao Departamento de Matemática da UnB e a todos os convidados, autores e participantes que contribuíram para o sucesso de mais uma edição do ENAMA.

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LINEAR DYNAMICS OF CONVOLUTION OPERATORS ON THE SPACE OF ENTIRE FUNCTIONS OF INFINITELY MANY COMPLEX VARIABLES

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Abstract

A classical result of Godefroy and Shapiro states that every nontrivial convolution operator on the space $\mathcal{H}(\mathbb{C}^n)$ of entire functions of several complex variables is hypercyclic. In sharp contrast with this result Fávaro and Mujica show that no translation operator on the space $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ of entire functions of infinitely many complex variables is hypercyclic. In this work we study the linear dynamics of convolution operators on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$. First we show that a convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is neither cyclic nor n-supercyclic for any positive integer n. We study the notion of Li–Yorke chaos in non-metrizable topological vector spaces and we show that every nontrivial convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is Li–Yorke chaotic.

1 Introduction

Let V be a subset of a Hausdorff topological complex vector space E and let $T\colon E \to E$ be a continuous linear operator (from now on we just write operator). The *orbit of V under T*, denoted by $\mathrm{orb}_T(V)$, is the subset of E given by

$$\mathrm{orb}_T(V) = \bigcup_{k=0}^{\infty} T^k(V).$$

If $V = \{x\}$ is a singleton and $\mathrm{orb}_T(V) = \{T^k x : k \in \mathbb{N}_0\}$ is dense in E, where $\mathbb{N}_0 = \{0, 1, 2, 3, \ldots\}$, then T is said to be *hypercyclic*. If the linear space generated by $\mathrm{orb}_T(V)$ is dense in E, then T is said to be *cyclic*. If $V = \mathrm{span}\{x\}$ and $\mathrm{orb}_T(V) = \mathbb{C} \cdot \{T^k x : k \in \mathbb{N}_0\}$ is dense in E, then T is said to be *supercyclic*. Finally, if V is a vector subspace of dimension n and $\mathrm{orb}_T(V)$ is dense in E, then T is said to be *n-supercyclic*.

Hypercyclicity is the most important concept in linear dynamics and it has received considerable attention in the last 25 years. There are several important notions of chaos and some authors have started to study this notions in the context of linear dynamics.

In this work we are interested in the linear dynamics of convolution operators on spaces of entire functions of infinitely many complex variables. Recall that a *convolution operator* on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is a continuous linear mapping

$$L\colon \mathcal{H}(\mathbb{C}^{\mathbb{N}}) \to \mathcal{H}(\mathbb{C}^{\mathbb{N}})$$

such that $L(\tau_\xi f) = \tau_\xi(Lf)$ for every $f \in \mathcal{H}(\mathbb{C}^{\mathbb{N}})$ and $\xi \in \mathbb{C}^{\mathbb{N}}$. Analogously we define convolution operators on $\mathcal{H}(\mathbb{C}^n)$ for each $n \in \mathbb{N}$ (we are considering the compact-open topology on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ and $\mathcal{H}(\mathbb{C}^n)$).

A classical result due to Godefroy and Shapiro [2] states that every nontrivial convolution operator on $\mathcal{H}(\mathbb{C}^n)$ is hypercyclic. Moreover, A. Bonilla and K.-G. Grosse-Erdmann [2] showed that these convolution operators are even frequently hypercyclic, which is a stronger notion than hypercyclicity. In contrast with these results, Fávaro and Mujica [1] proved that no convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ can be hypercyclic. Based on these facts, the following question arises:

Do the convolution operators on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ satisfy some notion of the linear dynamics weaker than hypercyclicity?

In sharp contrast with the aforementioned result of Godefroy and Shapiro we will show that no convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ can be either cyclic or n-supercyclic for any positive integer n. By the other hand we will prove that the convolution operators on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ are Li–Yorke chaotic.

It is important to mention that since $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is a non-metrizable complete locally convex space, the classical notion of Li–Yorke chaos does not make sense in this context. Recently T. Arai [1] introduced the notion of Li-Yorke chaos for an action of a group on an uniform space. Since every topological vector space is an uniform space, we will adopt the Arai's definition of Li-Yorke chaos.

For our purpose it is enough to present the definition of Li–Yorke chaos for an operator T on a Hausdorff topological vector space E as follow: A pair $(x, y) \in E \times E$ is said to be *asymptotic* for T if for any neighborhood of zero U, there exists $k \in \mathbb{N}$ such that $T^n(x-y) \in U$ for every $n \geq k$, that is, if $T^n(x-y) \to 0$. A pair $(x, y) \in E \times E$ is said to be *proximal* for T if for any neighborhood of zero U, there exists $n \in \mathbb{N}$ such that $T^n(x-y) \in U$, that is, if the sequence $\{T^n(x-y)\}$ has a subsequence converging to zero.

A pair $(x, y) \in E \times E$ is said to be a *Li–Yorke pair* for T if it is proximal, but it is not asymptotic. In other words, (x, y) is a Li–Yorke pair for T if and only if the sequence $\{T^n(x-y)\}$ does not converge to zero, but it has a subsequence converging to zero.

A *scrambled set* for T is a subset S of E such that (x, y) is a Li–Yorke pair for T whenever x and y are distinct points in S. Finally, we say that T is *Li–Yorke chaotic* if there exists an uncountable scrambled set for T.

2 Main Results

Theorem 2.1. *(a) No convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is cyclic.*

(b) No convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is n-supercyclic, for any $n \in \mathbb{N}$.

Theorem 2.2. *Every nontrivial convolution operator on $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ is Li–Yorke chaotic.*

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ESTIMATES FOR N-WIDTHS OF SETS OF SMOOTH FUNCTIONS ON THE COMPLEX SPHERE

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1 Introduction

In this work, we investigate n-widths of multiplier operators $\Lambda = \{\lambda_{m,n}\}_{m,n\in\mathbb{N}}$ and $\Lambda_* = \{\lambda^*_{m,n}\}_{m,n\in\mathbb{N}}$, $\Lambda, \Lambda_* : L^p(\Omega_d) \to L^q(\Omega_d)$, $1 \le p, q \le \infty$, on the d-dimensional complex sphere Ω_d, where $\lambda_{m,n} = \lambda(|(m,n)|)$ and $\lambda^*_{m,n} = \lambda(|(m,n)|_*)$ for a real function λ defined on the interval $[0,\infty)$ with $|(m,n)| = \max\{m,n\}$ and $|(m,n)|_* = m+n$. Upper and lower bounds are established for n-widths of general multiplier operators and we apply these results to the specific multiplier operators $\Lambda^{(1)} = \{\lambda^{(1)}_{m,n}\}_{m,n\in\mathbb{N}}$ and $\Lambda^{(1)}_* = \{\lambda^{(1),*}_{m,n}\}_{m,n\in\mathbb{N}}$ associated with the function $\lambda^{(1)}(t) = t^{-\gamma}(\ln t)^{-\xi}$ for $t > 1$ and $\lambda^{(1)}(t) = 0$ for $0 \le t \le 1$, and $\Lambda^{(2)} = \{\lambda^{(2)}_{m,n}\}_{m,n\in\mathbb{N}}$ and $\Lambda^{(2)}_* = \{\lambda^{(2),*}_{m,n}\}_{m,n\in\mathbb{N}}$ associated with the function $\lambda^{(2)}(t) = e^{-\gamma t^r}$ for $t \ge 0$, where $\gamma, r > 0$ and $\xi \ge 0$. We have that $\Lambda^{(1)}U_p$ and $\Lambda^{(1)}_*U_p$ are sets of finitely differentiable functions on Ω_d, in particular, $\Lambda^{(1)}U_p$ and $\Lambda^{(1)}_*U_p$ are Sobolev-type classes if $\xi = 0$, and $\Lambda^{(2)}U_p$ and $\Lambda^{(2)}_*U_p$ are sets of infinitely differentiable $(0 < r < 1)$ or analytic $(r = 1)$ or entire $(r > 1)$ functions on Ω_d, where U_p denotes the closed unit ball of $L^p(\Omega_d)$. In particular, we prove that the estimates for the Kolmogorov n-widths $d_n(\Lambda^{(1)}U_p, L^q(\Omega_d))$, $d_n(\Lambda^{(1)}_*U_p, L^q(\Omega_d))$, $d_n(\Lambda^{(2)}U_p, L^q(\Omega_d))$ and $d_n(\Lambda^{(2)}_*U_p, L^q(\Omega_d))$ are order sharp in various important situations. In this work we continue the development of methods of estimating n-widths of multiplier operators begun in [1, 2].

Consider two Banach spaces X and Y. The norm of X will be denoted by $\|\cdot\|_X$. Let A be a convex, compact, centrally symmetric subset of X. The Kolmogorov n-width of A in X is defined by

$$d_n(A;X) = \inf_{X_n} \sup_{x\in A} \inf_{y\in X_n} \|x-y\|_X,$$

where X_n runs over all subspaces of X of dimension n .

Let $l, N, m, n, M_1, M_2 \in \mathbb{N}$, with $M_1 < M_2$, $\mathcal{H}_l = \bigoplus_{(m,n)\in A_l\setminus A_{l-1}} \mathcal{H}_{m,n}$ and $\mathcal{T}_N = \bigoplus_{l=0}^N \mathcal{H}_l = \bigoplus_{(m,n)\in A_N} \mathcal{H}_{m,n}$ where $A_l = \{(m,n)\in\mathbb{N}^2 : |(m,n)| \le l\}$ and $\mathcal{H}_{m,n}$ is the space of all complex spherical harmonics of degree (m, n).

2 Main Results

Theorem 2.1. *Let $1 \le q \le p \le 2$, $0 < \rho < 1$, $s = \dim \mathcal{T}_N$, $d_l = \dim \mathcal{H}_l$ and $\lambda : [0,\infty) \to \mathbb{R}$ a non-increasing function with $\lambda(t) \neq 0$ for $t \ge 0$ and $\Lambda = \{\lambda_{m,n}\}_{m,n\in\mathbb{N}}$, $\lambda_{m,n} = \lambda(|(m,n)|)$. Then there is an absolute constant $C > 0$ such that*

$$d_{[\rho s-1]}(\Lambda U_p, L^p) \ge C'(1-\rho)^{1/2} s^{1/2} \left(\sum_{l=1}^{N} |\lambda(l)|^{-2} d_l\right)^{-1/2} \kappa_s,$$

where $[\rho s - 1]$ denotes the integer part of the number $\rho s - 1$ and were $\kappa_s = 1$ if $1 \le p \le 2$ and $1 < q \le 2$, if $2 \le p < \infty$ and $2 \le q \le \infty$, if $1 \le p \le 2 \le q \le \infty$, and $\kappa_s = (\ln s)^{-1/2}$ if $1 \le p \le 2$ and $q = 1$ and if $p = \infty$ and $2 \le q \le \infty$.

Theorem 2.2. *Let $\lambda : (0,\infty) \longrightarrow \mathbb{R}$ a non-increasing function and let $\Lambda = \{\lambda_{m,n}\}_{m,n\in\mathbb{N}}$,$\lambda_{m,n} = \lambda(|(m,n)|)$ such that $\lambda_{m,n} \neq 0$ for all $m,n \in \mathbb{N}$. Suppose that $1 \le p \le 2 \le q \le \infty$ and that the multiplier operator Λ is bounded* *from L_1 to L_2. Let $\{N_k\}_{k=0}^{\infty}$ and $\{m_k\}_{k=0}^{M}$ be sequences of natural numbers such that $N_k < N_{k+1}$, $N_0 = 0$ and $\sum_{k=0}^{M} m_k \le \beta$. Then there exist an absolute constant $C > 0$ such that*

$$d_\beta(\Lambda U_p; L^q) \le C \left(\sum_{k=1}^{M} |\lambda(N_k)| \varrho_{m_k} + \sum_{k=M+1}^{\infty} |\lambda(N_k)| \left(\theta_{N_k,N_{k+1}}\right)^{1/p-1/q} \right),$$

where

$$\varrho_{m_k} = \frac{\theta_{N_k,N_{k+1}}^{1/p}}{(m_k)^{1/2}} \begin{cases} q^{1/2}, & 2 \le q < \infty, \\ (\ln \theta_{N_k,N_{k+1}})^{1/2}, & q = \infty, \end{cases} \quad \text{and} \quad \theta_{N_k,N_{k+1}} = \sum_{s=N_k+1}^{N_{k+1}} \dim \mathcal{H}_s,\ k \ge 1.$$

Theorem 2.3. *For $\gamma > (2d-1)/2$, $\xi \ge 0$, $1 \le p \le \infty$, $2 \le q \le \infty$ and for all $k \in \mathbb{N}$*

$$\max\{\, d_k(\Lambda^{(1)} U_p, L^q),\ d_k(\Lambda_*^{(1)} U_p, L^q)\} \ll k^{-\gamma/(2d-1)+(1/p-1/2)_+} (\ln k)^{-\xi} \vartheta_k,$$

where $\vartheta_k = 1$ if $2 \le q < \infty$ and $\vartheta_k = (\ln k)^{1/2}$ if $q = \infty$.

Theorem 2.4. *For $\gamma > (2d-1)/2$, $\xi \ge 0$, κ_k as in Theorem 1 and for all $k \in \mathbb{N}$*

$$\min\{\, d_k(\Lambda^{(1)} U_p, L^q),\ d_k(\Lambda_*^{(1)} U_p, L^q)\} \gg k^{-\gamma/(2d-1)} (\ln k)^{-\xi} \kappa_k.$$

Theorem 2.5. *Let $\gamma > 0$, $0 < r \le 1$, and κ_k as in Theorem 1. Then for all $k \in \mathbb{N}$ we have*

$$d_k(\Lambda^{(2)} U_p, L^q) \gg e^{-\mathcal{R} k^{r/(2d-1)}} \kappa_k \quad \text{and} \quad d_k(\Lambda_*^{(2)} U_p, L^q) \gg e^{-\mathcal{R}_* k^{r/(2d-1)}} \kappa_k,$$

where $\mathcal{R} = \gamma \left(d!(d-1)!/2\right)^{r/(2d-1)}$, $\mathcal{R}_* = \gamma \left((2d-1)!/2\right)^{r/(2d-1)}$.

Theorem 2.6. *Let $\gamma > 0$, $0 < r \le 1$, ϑ_k as in Theorem 2.3 and $\mathcal{R}$ and $\mathcal{R}_*$ as in Theorem 2.5. Then for $1 \le p \le \infty$, $2 \le q \le \infty$, for all $k \in \mathbb{N}$, we have*

$$d_k(\Lambda^{(2)} U_p, L^q) \ll e^{-\mathcal{R} k^{r/(2d-1)}} k^{(1-r/(2d-1))(1/p-1/2)_+} \vartheta_k, \quad d_k(\Lambda_*^{(2)} U_p, L^q) \ll e^{-\mathcal{R}_* k^{r/(2d-1)}} k^{(1-r/(2d-1))(1/p-1/2)_+} \vartheta_k.$$

The results for the multiplier operators Λ_* associated with the norm $|\cdot|_*$ were obtained, from results already demonstrated for the real sphere S^{2d-1}, using properties which relate the real spherical harmonics with the complex spherical harmonics. We proved estimates for Levy means of norms on the $\mathbb{R}^n$ spaces, introduced through the multiplier sequence Λ. These estimates were the main tool to prove Theorems 1 and 1. Using Theorems 1 and 1, and the inequality $2/(d!(d-1)!)N^{2d-1} - C_3 N^{2d-2} \le \dim \mathcal{T}_N \le 2/(d!(d-1)!)N^{2d-1} + C_4 N^{2d-2}$ which we proved, we proved the Theorems 2.3, 2.4, 2.5 and 2.6 for the multiplier operators Λ associated with norm $|\cdot|$.

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ON A CLASSIFICATION OF A FAMILY OF ORTHOGONAL POLYNOMIALS ON THE UNIT CIRCLE SATISFYING A SECOND-ORDER DIFFERENTIAL EQUATION WITH VARYING POLYNOMIAL COEFFICIENTS

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Abstract

Consider the linear second order differential equation

$$A_n(z)y'' + B_n(z)y' + C_n y = 0, \tag{1}$$

where $A_n(z) = a_{2,n}z^2 + a_{1,n}z + a_{0,n}$ with $a_{2,n} \neq 0, a_{1,n}^2 - 4a_{2,n}a_{0,n} \neq 0, \forall n \in \mathbb{N}$ or $a_{2,n} = 0, a_{1,n} \neq 0, \forall n \in \mathbb{N}$, $B_n(z) = b_{1,n} + b_{0,n}z$ are polynomials with complex coefficients and $C_n \in \mathbb{C}$. The classification, up to a complex linear change in the variable z, of those sequences of orthogonal polynomials with respect to a measure supported on the unit circle satisfying (2) is given.

1 Introduction

The Bochner Classification Theorem [2] characterizes, under a complex linear change of the variable z, the sequences $(y_n)_{n=0}^{\infty}$ of orthogonal polynomials with respect to a positive Borel measure having finite moments of all orders that simultaneously solve a second order differential of the form

$$A(z)y'' + B(z)y' + C_n y = 0,$$

where A, B are polynomials of degree 2 and 1 respectively, $C_n \in \mathbb{C}$. Such sequences of polynomials turn out to be the classical families of orthogonal polynomials Laguerre, Jacobi and Hermite.

R. Askey in [1] introduced the two–parameter system $\{R_n, S_n\}_{n\geq 0}$ of polynomials given by

$$\begin{aligned} R_n(z;\alpha,\beta) &= {}_2F_1(-n, \alpha+\beta+1; \beta-\alpha+1-n; z), \\ S_n(z;\alpha,\beta) &= R_n(z;\alpha,-\beta), \end{aligned} \tag{2}$$

and pointed out that this system is biorthogonal with respect to the complex valued weight of beta type $\omega(\theta) = (1-e^{\imath\theta})^{\alpha+\beta}(1-e^{-\imath\theta})^{\alpha-\beta} = (2-2\cos\theta)^{\alpha}(-e^{\imath\theta})^{\beta}, \theta \in [-\pi,\pi], \Re(\alpha) > -\frac{1}{2}$, here ${}_2F_1$ denotes the Gauss hypergeometric function. That is

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} R_n(e^{\imath\theta};\alpha,\beta)S_m(e^{-\imath\theta};\alpha,\beta)\omega(\theta)d\theta = \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+\beta+1)\Gamma(\alpha-\beta+1)}\frac{n!}{(2\alpha+1)_n}\delta_{n,m},$$

where Γ denotes the Euler Gamma function.

From known results on hypergeometric functions, the element R_n in the orthogonal system $(R_n(z;\alpha,\beta))_{n=0}^{\infty}$ satisfies the differential equation

$$z(1-z)y'' + (\beta-\alpha+1-n-(-n+2+\alpha+\beta)z)y' + n(\alpha+\beta+1)y = 0.$$

Hence, it is natural to question if there exists other classes of orthogonal polynomials on the unit circle satisfying a linear second order differential equation similar to the Jacobi, Hermite and Laguerre systems of orthogonal polynomials. The above differential equation satisfied by the sequence $(R_n)_{n=0}^{\infty}$ suggest that we should consider a differential equation with varying coefficients in the index n and the associated sequence of orthogonal polynomials as solution. In the present work, we classify those sequences of orthogonal polynomials solving the differential equation for the cases in which A_n satisfies the conditions mentioned.

2 Main Results

Let us consider the differential equation given by (2). Under a linear complex change in the variable z, see [3, Prop. 1.1], this differential equation can be transformed to

$$y'' + P_n(z)y' + Q_n(z)y = 0, \tag{1}$$

where

$$\begin{aligned} P_n(z) &= \frac{c_n - b_n z}{\theta(z)},\ Q_n(z) = \frac{n(n-1+b_n)}{\theta(z)} \quad \text{if} \ \ \theta(z) = z(1-z) \\ P_n(z) &= \frac{c_n - z}{\theta(z)},\ Q_n(z) = \frac{n}{\theta(z)} \quad \text{if } \theta(z) = z, \end{aligned}$$

being $(b_n)_{n\in\mathbb{N}}$ and $(c_n)_{n\in\mathbb{N}}$ sequences of complex numbers; $b_n \notin \{-2n+2, -2n+3, \ldots, -n, -n+1\}$ for $\theta(z) = z(1-z)$. Our main result read as

Theorem 2.1. *Let $(\phi_n)_{n=0}^{\infty}$ be a sequence of orthonormal polynomials with respect to a positive Borel measure on the unit circle satisfying (1). Then*

$$\phi_n(z) = \gamma_n \begin{cases} {}_2F_1(-n, \gamma+1; -\overline{\gamma}+1-n; z); \Re[\gamma] > -\dfrac{1}{2}, & \text{if} \ \ \theta(z) = z(1-z), \\ z^n, & \text{if} \ \ \theta(z) = z, \end{cases}$$

where γ_n is the normalizing coefficient.

Proof See [3] and [4].

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ON THE BEHAVIOR OF NUMERICAL INTEGRATORS FOR D-DIMENSIONAL STOCHASTIC HARMONIC OSCILLATORS

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Abstract

We study the capability of some numerical integrators -thought as discrete dynamical systems- for reproducing the oscillatory behavior of high-dimensional stochastic harmonic oscillators. The results in this work complement previous ones in the literature concerning the preservation of dynamical properties by numerical discretizations.

1 Introduction

Oscillators driven by random forces arise in a variety of models in applications (see, e.g., [1], [4]). It is well known that, in general, noise modifies the dynamics of deterministic oscillators, so new distinctive dynamical features arise in these random systems. In the case of d-dimensional stochastic harmonic oscillators defined by

$$x(t)'' + \Lambda^2 x(t) = \Pi \mathbf{w}_t', \quad x(t_0) = x_0,$$

or equivalently by the $2d$-dimensional system

$$d\mathbf{x}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\Lambda^2 & \mathbf{0} \end{bmatrix} \mathbf{x}(t)\, dt + \begin{bmatrix} \mathbf{0} \\ \Pi \end{bmatrix} d\mathbf{w}_t, \tag{1}$$

where $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, initial condition $\mathbf{x}(t_0) = (x_0, y_0)^\top$, $x_0, y_0 \in \mathbb{R}^d$ and $d \geqslant 1$, with $\Lambda \in \mathbb{R}^{d\times d}$ a nonsingular symmetric matrix, $\Pi \in \mathbb{R}^{d\times m}$, $\mathbf{I}$ the $d-$dimensional identity matrix, and $\mathbf{w}_t$ an m-dimensional standard Wiener process on the filtered complete probability space $\left(\Omega, \mathfrak{F}, (\mathfrak{F}_t)_{t\geq t_0}, \mathbb{P}\right)$, a number of dynamical properties have been studied. It is known (see e.g., [2]) that the expected value of the energy growths linearly, i.e.,

$$E\left(\|y(t)\|^2 + \|\Lambda x(t)\|^2\right) = E\left(\|y_0\|^2 + \|\Lambda x_0\|^2\right) + trace(\Pi^\intercal \Pi)\,(t - t_0)\,;$$

that the phase flow of (1) preserves symplectic structure i.e.,

$$dx(t) \wedge dy(t) = dx(t_0) \wedge dy(t_0), \text{ for all } t \geq t_0;$$

and that for $d = 1$, $x(t)$ has infinitely many zeros on $[t_0, \infty)$.

Since for stochastic models -in particular those containing stochastic oscillators- closed-form solutions are rarely available, numerical integrators able to mimic these dynamical properties of (1) are required. In this direction, recently, the ability of some commonly used integrators to replicate these properties have been studied (see e.g., [3] for a summary). It has been concluded that general multipurpose integrators fail to achieve this target and only exponential-based integrators preserve the dynamics of the oscillators. However, concerning the oscillatory behavior of the discrete maps defining the numerical methods, only the case $d = 1$ have been, so far, considered.

In this work, the ability of discrete dynamical systems defined by numerical integrators for reproducing the oscillatory property of (1) in the multidimensional case $d > 1$ is analyzed. In this way, we complement a recent study carried out in [3] for simple stochastic harmonic oscillators (i.e., for $d = 1$). For this, firstly an early result derived in [5] concerning the oscillatory behavior of simple stochastic harmonic oscillators is extended to the general class of coupled harmonic oscillators. Then, the main theorem characterizing this property for exponential-based numerical integrators is obtained.

2 The oscillatory behavior of coupled Harmonic Oscillators

We first study the infinitely many oscillations of the paths of multidimensional harmonic oscillators (1). We obtain the following Theorem which extends the result of Theorem 3 in [5] restricted to the simple harmonic oscillators (i.e., those defined by (1) with $d = 1$).

Theorem 2.1. *Consider the coupled harmonic oscillator (1). Then, almost surely, each component of the solution* $\mathbf{x}(t)$ *has infinitely many zeros on* $[t_0\ \infty)$ *for every* $t_0 \geq 0$.

3 The oscillatory behavior of exponential-based integrators

Next Theorem deals with the reproduction of the oscillatory behavior of multidimensional harmonic oscillators by the discrete dynamical system defined by the exponential-based numerical integrator considered in [3], [2].

Theorem 3.1. *Let* $\lambda_1, \ldots, \lambda_d$ *be the eigenvalues of* Λ, *and* $|\lambda|_{\max} = \max\limits_{k} (|\lambda_k|)$. *For the* $d-$*dimensional harmonic oscillator (1), each component of the exponential-based integrator considered switches signs infinitely many times as* $n \to \infty$, *almost surely, for any integration stepsize* $h < \pi / |\lambda|_{\max}$.

Concluding Remark: The results in this work extend and complement previous ones obtained in the literature (see [3]) concerning the capability of discrete dynamical system defined by exponential-based integrators for reproducing essential continuous dynamics of multidimensional harmonic oscillators. Remarkably we conclude that, in contrast with the one dimensional case, to replicate this oscillatory behavior for $d > 1$, it is necessary to restrict the stepsize of the numerical methods.

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CLOUD RECOVERY IN ATMOSPHERIC CLIMATE MODELS

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Abstract

A discrete control scheme is presented to provide the unstable trailing variables of an evolutive system of ordinary differential equations with accurate initial values on the system's attractor. The Influence Sampling (IS) scheme adapts sample values of the trailing variables to input values of the determining variables in the attractor. The optimal IS scheme has affordable cost for large systems. The derivation of the scheme and its use for recovering and increasing the predictability of clouds in an Atmospheric climate model is presented.

1 Introduction

When predicting the state of an evolutive system one aims to estimate its trajectory with great confidence for a given time. A major stumbling block in predictions is dynamical instability. As a matter of fact instability is in the nature of most nonlinear processes toward equilibrium of environmental systems involving water and air and, along with model error, is responsible for the loss of information that data provide. In order to increase the predictability of environmental systems, which nowadays is of great economical and social interest, numerical models should be provided with accurate initial values on the attractor of the dynamical system generated by the evolutive system. We present the Influence Sampling (IS) scheme to this end [1]. This is a sampling scheme for trailing variables of an evolutive system of ordinary differential equations (ode) with a global attractor and determining variables on it. It adapts sample values of the trailing variables to reference values of the determining variables. The reference solution is supposed to lie in the attractor.

The derivation of the IS scheme is based on Dyson's splitted action formula [2], which is shown in [1] to hold for $\mathrm{A} = a \cdot \partial_x$ and $\mathrm{B} = b \cdot \partial_x$, where a and b are twice continuously differentiable vector fields in $\mathbb{R}^n$. Next we describe Dyson's formula and show the output of a cloud recovery using the IS scheme with an Atmospheric climate model.

2 Main Results

Let

1. a and b be twice continuously differentiable vector fields in $\mathbb{R}^n$;
2. $\Phi(t, x)$ be the dynamics of $dX/dt = (a + b)(X)$ with $\Phi(t_0, x) = x$;
3. $\Psi(t, x)$ be the dynamics of $dX/dt = a(X)$ with $\Psi(t_0, x) = x$;
4. $e^{(t-t_0)\mathrm{A}}$ be the solution operator of $\partial_t u(t, x) = \mathrm{A}u(t, x)$, $\mathrm{A} = a \cdot \partial_x$, with $e^{0\mathrm{A}} u_0 = u_0$;
5. $e^{(t-t_0)(\mathrm{A}+\mathrm{B})}$ be the solution operator of $\partial_t u(t, x) = (\mathrm{A} + \mathrm{B})u(t, x)$, $\mathrm{B} = b \cdot \partial_x$, with $e^{0(\mathrm{A}+\mathrm{B})} u_0 = u_0$;
6. Ω be the domain of Ψ.

From the method of characteristics for the linear transport equation, one has

$$(e^{(t-t_0)(\mathrm{A}+\mathrm{B})} u_0)(x) = u_0(\Phi(t, x)) \tag{1}$$

and

$$(e^{(t-t_0)\mathrm{A}}u_0)(x) = u_0(\Psi(t,x)), \tag{2}$$

for any $u_0 \in \mathrm{C}^1(\mathbb{R}^n)$. As such

$$(e^{(t-t_0)(\mathrm{A}+\mathrm{B})}u_0)(x) - (e^{(t-t_0)\mathrm{A}}u_0)(x) = u_0(\Phi(t,x)) - u_0(\Psi(t,x)). \tag{3}$$

Also

$$\begin{aligned}&(\textstyle\int_{t_0}^t e^{(t-s)(\mathrm{A}+\mathrm{B})}\mathrm{B}e^{(s-t_0)\mathrm{A}}u_0\,ds)(x)\\ &\quad= \textstyle\int_{t_0}^t (e^{(t-s)(\mathrm{A}+\mathrm{B})}\mathrm{B}e^{(s-t_0)\mathrm{A}}u_0)(x)\,ds\\ &\quad= \textstyle\int_{t_0}^t (e^{(t-s)(\mathrm{A}+\mathrm{B})}\mathrm{B}u_0(\Psi(s,\,\cdot\,)))(x)\,ds\\ &\quad= \textstyle\int_{t_0}^t e^{(t-s)(\mathrm{A}+\mathrm{B})}b(x)\cdot\partial_x u_0(\Psi(s,x))\,ds.\end{aligned} \tag{4}$$

For a class of twice continuously differentiable vector fields b, one has

$$\textstyle\int_{t_0}^t e^{(t-s)(\mathrm{A}+\mathrm{B})}b(x)\cdot\partial_x u_0(\Psi(s,x))\,ds = \int_{t_0}^t \partial_x u_0(\Psi(s,\Phi(t_0+t-s,x)))\,b(\Phi(t_0+t-s,x))\,ds. \tag{5}$$

Therefore

$$(\textstyle\int_{t_0}^t e^{(t-s)(\mathrm{A}+\mathrm{B})}\mathrm{B}e^{(s-t_0)\mathrm{A}}u_0\,ds)(x) = \int_{t_0}^t \partial_x u_0(\Psi(s,\Phi(t_0+t-s,x)))\,b(\Phi(t_0+t-s,x))\,ds. \tag{6}$$

One can also prove that

$$\partial_s u_0(\Psi(s,\Phi(t_0+t-s,x))) = -\partial_x u_0(\Psi(s,\Phi(t_0+t-s,x)))\,b(\Phi(t_0+t-s,x)). \tag{7}$$

Combining Eqs. 5 and 7, one gets

$$\begin{aligned}(\textstyle\int_{t_0}^t e^{(t-s)(\mathrm{A}+\mathrm{B})}\mathrm{B}e^{(s-t_0)\mathrm{A}}u_0\,ds)(x) &= \textstyle\int_{t_0}^t -\partial_s u_0(\Psi(s,\Phi(t_0+t-s,x)))\,ds\\ &= u_0(\Phi(t,x)) - u_0(\Psi(t,x)).\end{aligned} \tag{8}$$

Combining Eqs. 1 and 8, one obtains

$$e^{(t-t_0)(\mathrm{A}+\mathrm{B})} = e^{(t-t_0)\mathrm{A}} + \textstyle\int_{t_0}^t e^{(t-s)(\mathrm{A}+\mathrm{B})}\mathrm{B}e^{(s-t_0)\mathrm{A}}\,ds. \tag{9}$$

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HÉNON ELLIPTIC EQUATIONS IN $\mathbb{R}^2$ WITH CRITICAL EXPONENTIAL GROWTH: LINKING CASE

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Abstract

We study the Dirichlet problem in the unit ball B_1 of $\mathbb{R}^2$ for the Hénon-type equation of the form

$$\begin{cases} -\Delta u &= \lambda u + |x|^\alpha f(u) \quad \text{in} \quad B_1, \\ \quad u &= 0 \qquad\qquad\qquad \text{on} \quad \partial B_1, \end{cases}$$

where λ is between two diferrent eigenvalues of $(-\Delta, H_0^1(B_1))$ and $f(t)$ is a C^1-function in the critical growth range motivated by the celebrated Trudinger-Moser inequality. By variational methods, we study the solvability of this problem in appropriate Sobolev Spaces.

1 Introduction

In this work, using variational methods, we prove the existence of a non-trivial solution for the following class of Hénon-type equations

$$\begin{cases} -\Delta u &= \lambda u + |x|^\alpha f(u) \quad \text{in} \quad B_1, \\ \quad u &= 0 \qquad\qquad\qquad \text{on} \quad \partial B_1, \end{cases} \tag{1}$$

where $\alpha \geq 0$ and B_1 is a unity ball centred at origin of $\mathbb{R}^2$ and λ is between two diferrent eigenvalues of $(-\Delta, H_0^1(B_1))$.

Here we assume that $f(t)$ with exponential critial growth. More precisely, we say that

(CG) $f(t)$ has critical growth at $+\infty$ if there exists $\beta_0 > 0$ such that

$$\lim_{t\to+\infty} \frac{|f(t)|}{e^{\beta t^2}} = 0, \quad \forall \beta > \beta_0; \quad \lim_{t\to+\infty} \frac{|f(t)|}{e^{\beta t^2}} = +\infty, \quad \forall \beta < \beta_0.$$

1.1 Hypotheses

Before stating our main results, we shall introduce the following assumptions on the nonlinearity $f(t)$:

(H_1) The function $f(t)$ is continuous and $f(0) = 0$.

(H_2) There exist t_0 and $M > 0$ such that

$$0 < F(t) =: \int_0^t f(s)\, ds \leq M|f(t)| \quad \text{for all} \quad |t| > t_0.$$

(H_3) $0 < 2F(t) \leq f(t)t$ for all $t \in \mathbb{R} \setminus \{0\}$.

We denote by $0 < \lambda_1 < \lambda_2 \leq \lambda_3 ...$ the eigenvalues of $(-\Delta, H_0^1(B_1))$. We also consider constants $0 < r < 1/2$ and $0 < d < 1$ such that there exists a ball $B_r(x_0) \subset B_1$ so that $|x| > d$ for all $x \in B_r(x_0)$.

2 Main Results

In the critical case, since the weight $|x|^\alpha$ has an important role on the estimate of the minimax levels, the variational setting and methods used in $H_0^1(B_1)$ and $H^1_{0,\mathrm{rad}}(B_1)$ are different and, therefore, are given in two separate theorems.

Theorem 2.1. *(The critical case, saddle point at 0 with $\alpha > 0$). Assume $(H_1) - (H_3)$, (H_5) and that $f(t)$ has critical growth (CG) at both $+\infty$ and $-\infty$. Furthermore, suppose that $\lambda_k < \lambda < \lambda_{k+1}$, $\alpha > 0$ and*

$$(H_7)\ \lim_{t\to+\infty} \frac{f(t)t}{e^{\beta_0 t^2}} \geq \xi, \text{ with } \xi > \frac{4}{\beta_0 r^{2+\alpha}} \left[\left(\frac{d}{r}\right)^\alpha - \frac{2}{2+\alpha} \right]^{-1},$$

where $0 < r < 1/2$ and $0 < d < 1$ are such that

$$\left(\frac{d}{r}\right)^\alpha > \frac{2}{2+\alpha}.$$

Then problem (3) has a nontrivial solution.

Theorem 2.2. *(The radial critical case, saddle point at 0). Assume $(H_1) - (H_3)$, (H_5) and that $f(t)$ has critical growth (CG) at both $+\infty$ and $-\infty$. Furthermore, suppose that $\lambda_k < \lambda < \lambda_{k+1}$ and for all $\gamma \geq 0$ there exists $c_\gamma \geq 0$ such that (H_8) holds for all $t > c_\gamma$, then problem (3) has a nontrivial radially symmetric solution.*

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O SEGUNDO INVARIANTE DE YAMABE EM VARIEDADES CR

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1 Introdução

Em 1987 foi proposto por D. Jerison e J. Lee [4] o seguinte problema de Yamabe sobre variedades CR.

O Problema de Yamabe sobre Variedades CR. Dada uma variedade pseudohermitiana (M, θ) compacta, estritamente pseudoconvexa, encontrar uma estrutura pseudohermitiana $\widetilde{\theta}$ com mesma orientação de θ tal que sua curvatura escalar pseudohermitiana seja constante.

O problema de Yamabe sobre variedades CR compactas, orientáveis, estritamente pseudoconvexas, foi completamente resolvido por D. Jerison, J. Lee ([3], [4], [6], [5]), N. Gamarra e J. Yacoub ([1], [2]).

Seja (M, θ) uma variedade pseudohermitiana compacta, orientável, estritamente pseudoconvexa, de dimensão $2n+1 \geq 3$. A estrutura pseudohermitiana $\widetilde{\theta} = u^{p-2}\theta$ terá curvatura escalar constante λ se, e somente se, u satisfazer a equação

$$p\,\Delta_b u + Ru = \lambda u^{p-1}, \tag{1}$$

em que $\Delta_b u$ é o sublaplaciano de u e ∇u sua derivada covariante, definida com respeito a estrutura pseudohermitiana θ. Esta é a equação de Euler-Lagrange para o funcional

$$Y(\widetilde{\theta}) = \frac{\int_M \widetilde{R} dV_{\widetilde{\theta}}}{\left(\int_M dV_{\widetilde{\theta}}\right)^{2/p}}, \tag{2}$$

em que $dV_\theta = \theta \wedge d\theta^n$ é o elemento de volume CR. Esse funcional é também denominado **funcional de Yamabe CR**. Uma conseqüência da desigualdade de Hölder é que, para variedades compactas, o funcional Y é limitado inferiormente. Portanto podemos considerar

$$\lambda(M) = \inf\left\{Y(\widetilde{\theta}) : \widetilde{\theta} \text{ é conforme à } \theta\right\}. \tag{3}$$

A constante $\lambda(M)$ é um CR-invariante, isto é, depende exclusivamente da estrutura CR e não da escolha da estrutura pseudohermitiana, chamado **invariante de Yamabe CR**. Podemos sintetizar essa solução de acordo com os resultados obtidos por Jerison, Lee, Gamara e Yacoub.
Dessa meneira, Jerison e Lee solucionaram o problema de Yamabe no contexto CR, para os casos em que a variedade não é localmente CR-equivalente à esfera e sua dimensão é diferente de 3. O casos restantes foram solucionados em 2001 por Gamara e Yacoub.

2 Resultados Principais

Agora sendo (M, θ) uma variedade CR pseudohermitiana compacta, conexa e estritamente pseudoconvexa, definimos o **Segundo Invariante de Yamabe CR** como

$$\mu_2(M, \theta) = \inf_{\widetilde{\theta} \in [\theta]} \lambda_2(\widetilde{\theta}) V_{\widetilde{\theta}}^{\frac{1}{n+1}},$$

em que $\lambda_2(\widetilde{\theta})$ é o segundo autovalor do operador de Yamabe CR

$$L_{\widetilde{\theta}} = \left(2 + \frac{2}{n}\right)\Delta_{\widetilde{\theta}} + \widetilde{R}.$$

Com a finalidade de chegar em resultados similares aos do primeiro invariante de Yamabe CR, provamos os seguintes teoremas.

Teorema 2.1 (**Principal**). *Seja (M,θ) uma variedade CR pseudohertiana compacta, conexa e estritamente pseudoconvexa , com dimensão CR igual a $2n+1$ e $n \geq 2$ com $\mu(M,\theta) = \mu_1(M,\theta) > 0$, então $\mu_2(M,\theta) \leq \mu_2(\mathbb{S}^{2n+1})$. Além do mais, a igualdade ocorre se, e somente se, (M,θ) é localmente CR equivalente a $\mathbb{S}^{2n+1}$.*

Teorema 2.2. *Seja (M,θ) uma variedade CR pseudohertiana compacta, conexa e estritamente pseudoconvexa , com dimensão CR igual a $2n+1$ e $n \geq 2$ com $\mu_1(M,\theta) > 0$. Suponha também que exista $B_0(M,\theta) > 0$ tal que*

$$\mu(\mathbb{S}^{2n+1}) = \inf_{u \in S_1^2(M)\setminus\{0\}} \frac{\int_M (p\|\nabla_H u\|_\theta^2 + B_0(M,\theta)u^2)dV_\theta}{(\int_M u^p dV_\theta)^{\frac{2}{p}}}.$$

Então, se $\mu_2(M,\theta) < \mu(\mathbb{S}^{2n+1})$, existe uma estrutura pseudohermitiana $\widetilde{\theta}$, da mesma classe conforme de θ, que minimiza $\mu_2(M,\theta)$. Com $\mu(\mathbb{S}^{2n+1})$ sendo primeiro invariante de Yamabe CR da esfera com relação a estrutura pseudohermitiana canônica $\hat{\theta}$.

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FRACTIONAL REGULARITY FOR A CLASS OF QUASILINEAR EQUATIONS

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Abstract

In this talk we are going to address some recent results on the fractional regularity of solutions of Quasilinear degenerate equations of Elliptic type. Our goal will be to link the nonlinear character of the differential operator with spaces of fractional order of differentiability, and to review and present some new and old results regarding the regularity of solutions to the sort of equations as well as the related a priori estimates. Special attention will be delivered for the case of a p-Kirchoff equation, cf. [1] and also to the (p, q)-Laplacian, cf. [2].

1 Introduction

In the past years, the phenomenon of fractional regularity has been addressed for a large class of linear and/or quasilinear differential operators, mostly, in terms of certain Besov spaces. As it turns out, for the the so-called p-Laplacian, this regularity is guided in the light of the Nikolskii class, the case where the interpolation parameter is infinite. Despite of its own interest, fractional regularity methods may be used as a tool for the investigation of some Partial Differential Equations which are not usually addressed in this manner. Thus, the purpose of the present paper is to exploit such methods in order to provide some results regarding existence and regularity of solutions to a class nonlocal elliptic equations which are linked to the p-Laplacian. This is done by means of explicit a priori estimates regarding Lebesgue and Nikolskii spaces, which are part of the present contribution. As a consequence, this approach allows a relaxation on some of the standard conditions employed in this class of problems.

Throughout the talk, we present an investigation on the existence and fractional regularity of solutions for

$$\begin{cases} -\left[a\left(\|u\|_{1,p}^{p}\right)\right]^{p-1}\Delta_p u + u &= f(x,u) \quad \text{in} \quad \Omega \\ \dfrac{\partial u}{\partial \eta} &= 0 \quad \text{on} \quad \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N,\ N \geq 2$, is an open bounded domain with smooth boundary $\partial\Omega$, Δ_p is the p-Laplacian operator

$$\Delta_p u = \operatorname{div}\Big(|\nabla u|^{p-2}\nabla u\Big), \qquad \text{with } p > 2$$

and $a(.)$ is the so-called p-Kirchhoff, or Kirchhoff term, which will be assumed to be continuous and bounded by below.

Moreover, we will also addresses the gain of global fractional regularity in Nikolskii spaces for solutions of a class of quasilinear degenerate equations with (p, q)-growth. Indeed, we investigate the effects of the datum on the derivatives of order greater than one of the solutions of the (p, q)-Laplacian operator, under Dirichlet's boundary conditions. As it turns out, even in the absence of the so-called Lavrentiev phenomenon and without variations on the order of ellipticity of the equations, the fractional regularity of these solutions ramifies depending on the interplay between the growth parameters p, q and the data. Indeed, we are going to exploit the absence of this phenomenon in order to prove the validity up to the boundary of some regularity results, which are known to hold locally, and as well provide new fractional regularity for the associated solutions. In turn, there are obtained certain global regularity results by means of the combination between new a priori estimates and approximations of the differential operators, whereas the nonstandard boundary terms are handled by means of a careful choice for the local frame.

Indeed, we will discuss the fractional regularity of solutions to the following class of degenerate elliptic equations

$$\begin{cases} -\alpha\Delta_p u - \beta\Delta_q u &= f \text{ in } \Omega \\ \qquad\qquad\qquad u &= 0 \text{ on } \partial\Omega \end{cases} \tag{$\mathcal{D}$}$$

where $q \geq p > 2$, $\alpha > 0$, $\beta \geq 0$, and $\Omega \subset \mathbb{R}^N$ is an open bounded domain of class $C^{2,1}$. Indeed, our aim is to describe the effects of the parameters α and β, which control the ellipticity of (1), and also the interference of the interplay between p, q, and the order of integrability of f on the spatial derivatives of order greater than one of the solutions to this class of equations, the well-known (p, q)-Laplacian operator.

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ON A SYSTEMS INVOLVING FRACTIONAL KIRCHHOFF-TYPE EQUATIONS AND KRASNOSELSKII'S GENUS

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Abstract

We consider a class of variational systems involving fractional Kirchhoff-type equations of the form

$$\begin{cases} M_1(\|u\|_X^2)(-\Delta)^s u &= F_u(x,u,v) \text{ in } \Omega, \\ M_2(\|v\|_X^2)(-\Delta)^s v &= F_v(x,u,v) \text{ in } \Omega, \\ u = v = 0 & \text{ in } \mathbb{R}^N \backslash \Omega, \end{cases}$$

where $s \in (0,1)$, $N > 2s$, $\Omega \subset \mathbb{R}^N$ a smooth and bounded domain, the functions F_u, F_v, M_1 and M_2 are continuous and $(-\Delta)^s$ is the fractional laplacian operator. In this paper we show that, under appropriate growth conditions on the nonlinearities F_u and F_v and on the non-negative functions M_1 and M_2, the (weak) solutions are precisely the critical points of a related functional defined on a fractional Hilbert space $Y(\Omega) = X(\Omega) \times X(\Omega)$ and the existence infinitely many solutions can be obtained by the use of the Krasnoselskii's genus. Besides, a regularity result can also be obtained by using specific results for systems in conjunction with the growth assumptions of these functions.

1 Introduction

Precisely, we assume that $M_1, M_2 : [0,+\infty) \to [0,+\infty)$ are continuous functions that satisfy growth conditions which will be stated later, the function $F \in C^1(\overline{\Omega} \times \mathbb{R}^2, \mathbb{R})$ is such that $\nabla F = (F_u, F_v)$ denotes the gradient of F in the variables u and v, and $(-\Delta)^s$ is the fractional laplacian operator.

In this work, we assume the following hypotheses on M_1, M_2 and F.

The non-negative functions $M_1, M_2 \in C([0,+\infty))$ are such that, there are positive constants a_i, b_i, α_i, $i = 1,2,3,4$, with $1 \leq \alpha_1 \leq \alpha_2$ and $1 \leq \alpha_3 \leq \alpha_4$ satisfying

$$a_1 + b_1 t^{\alpha_1} \leq M_1(t) \leq a_2 + b_2 t^{\alpha_2} \tag{1}$$

and

$$a_3 + b_3 t^{\alpha_3} \leq M_2(t) \leq a_4 + b_4 t^{\alpha_4} \quad \text{for all} \quad t \geq 0. \tag{2}$$

While $F \in C^1(\overline{\Omega} \times \mathbb{R}^2, \mathbb{R})$ satisfies the following assumptions

(f_1) $\nabla F(x,-z,-t) = -\nabla F(x,z,t)$ for any $(x,z,t) \in \overline{\Omega} \times \mathbb{R}^2$, where $\nabla F = (F_z, F_t)$ is the gradient of F in the variables z and t.

(f_2) There are constants $0 < c_i, d_i$ for $i \in \{1,2,3,4\}$, $1 < \gamma_j \leq \gamma_{j+3}$ and $1 < \eta_j \leq \eta_{j+3}$ for $j \in \{1,2,3\}$ such that F_z and F_t satisfy the growth conditions

$$c_1 z^{\gamma_1 - 1} + c_2 z^{\gamma_2 - 1} t^{\gamma_3} \leq F_z(x,z,t) \leq c_3 z^{\gamma_4 - 1} + c_4 z^{\gamma_5 - 1} t^{\gamma_6},$$

$$d_1 t^{\eta_1 - 1} + d_2 z^{\eta_2} t^{\eta_3 - 1} \quad \leq F_t(x, z, t) \leq \quad d_3 t^{\eta_4 - 1} + d_4 z^{\eta_5} t^{\eta_6 - 1},$$

for all $x \in \overline{\Omega}$ and $z, t \in [0, +\infty)$ with $\gamma_1, \eta_1 < 2$ and

$$\max\{\gamma_4, \eta_4, \gamma_5 + \gamma_6, \eta_5 + \eta_6\} < \min_{i=1,3} \{2(\alpha_i + 1), 2_s^*\}, \tag{3}$$

where $2_s^* := 2N/(N - 2s)$ denotes the fractional critical Sobolev exponent.

2 Main Results

Theorem 2.1. Let $s \in (0, 1)$, $N > 2s$, Ω be an open bounded subset of $\mathbb{R}^N$ with conti- nuous boundary. Let $M_1, M_2 : [0, +\infty) \to [0, +\infty)$ be functions satisfying (1) and (2), $F : \Omega \times \mathbb{R}^2 \to \mathbb{R}$ verifying (f_1) and (f_2). Then, the problem admits infinitely many weak solutions.

Theorem 2.2. (Regularity) If (u, v) is a weak solution to the problem, then $u, v \in C^{1,\alpha}_{loc}(\Omega)$ for $s \in (0, 1/2)$ and $u, v \in C^{2,\alpha}_{loc}(\Omega)$ for $s \in (1/2, 1)$. In particular, (u, v) solves the problem in the classical sense.

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SOME CONTRIBUTIONS OF THE KURZWEIL-HENSTOCK INTEGRATION THEORY

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Abstract

The aim of this talk is to point out the main contributions so far of the non absolute integration thelry in the sense of Jaroslav Kurzweil and Ralph Henstock.

1 Introduction

The main objective of this talk is to share some achievements of the theory of Kurzweil-Henstock integration and consequently of generalized ordinary differential equations (we write generalized EDOs, for short) and some of their applications, specially to other types of classic differential and integral equations whose functions involved may have many discontinuities and be highly oscillating (i.e., of unbounded variation).

A typical example of a function of unbounded variation is $f : [0,1] \to \mathbb{R}$ given by $f(t) = F'(t)$, where $F : (t) = t^2 \sin \frac{1}{t^2}$, for $t \in (0,1]$, and $F(0) = 0$. The function f is neither Riemann nor Lebesgue integrable. It is integrable in the sense of Kurzweil-Henstock however.

It is worth mentioning that generalized ODEs, which are defined in terms of the non-absolute Kurzweil integral, comprise a robust theory in which one can include differential equations such as ordinary and functional differential equations, measure and impulsive differential equations, dynamic equations on time scales and integral equations, among others.

2 Main Results

We pick up a few results from the theory and applications of the Kurzweil-Henstock integration theory. The references below are within the basis on the theory.

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REGULARITY THEORY FOR A NONLINEAR FRACTIONAL DIFFUSION EQUATION

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Abstract

In this paper we study a nonlinear fractional diffusion equation. We analyze the behavior of the resolvent family associated to the problem in the scale of fractional power spaces associated to the Laplace operator. We ensure existence and uniqueness of regular mild solutions to the problem in the L^q setting. Furthermore, we consider global existence or non-continuation by blow-up of such solutions.

1 Introduction

Fractional diffusion equations

$$u_t(t,x) = dg_\alpha * \Delta u(t,x) + r(t,x) \quad t>0,\ x\in\mathbb{R}^n, \tag{1}$$

where $g_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$, $0<\alpha<1$, have attracted much interest mostly due to their applications in the modeling of anomalous diffusion, since this subject involves a large variety of natural sciences such as physics, chemistry, biology, geology and their interfacial disciplines, see e.g. [1, 2, 4, 3] and the references therein.

From the mathematical point of view, the study of these equations was initiated by Schneider and Wyss [5] and has been of interest of many researchers since then. For example, Kemppainen et al. [6] prove optimal estimates for the decay in time of solutions to a class of non-local in time linear subdiffusion equations by using estimates based on the fundamental solution and Young's inequality, see also [7]. In [8], de Andrade and Viana consider the nonlinear fractional diffusion equation

$$u_t(t,x) = \int_0^t dg_\alpha(s)\Delta u(t-s,x) + |u(t,x)|^{\rho-1}u(t,x),\ \text{in } (0,\infty)\times\mathbb{R}^n,$$
$$u(x,0) = u_0(x),\ \text{in } \mathbb{R}^n,$$

and prove a global well-posedness result for initial data $u_0 \in L^p(\mathbb{R}^n)$ in the critical case $p = \frac{\alpha n}{2}(\rho-1)$. They also provide sufficient conditions to obtain self-similar solutions and study spatial decays to the problem.

Stimulated by these works, in this paper we study a following nonlinear fractional diffusion equation

$$\begin{cases} u_t(t,x) &= \displaystyle\int_0^t dg(s)\Delta u(t-s,x) + |u(t,x)|^{\rho-1}u(t,x) \\ & \quad + h(t,x),\ \text{in } (0,\infty)\times\Omega, \\ u(t,x) = 0,\ \text{in } (0,\infty)\times\partial\Omega, & \\ u(0,x) = u_0(x),\ \text{in } \Omega, & \end{cases} \tag{2}$$

where $\rho>1$, $\Omega\subset\mathbb{R}^n$ is a bounded smooth domain, h is a given function and $u_0\in L^q(\Omega)$, $1<q<\infty$. For $\gamma\geq 0$ and $0<\alpha\leq 1$,

$$g(t) = e^{-\gamma t}\frac{t^{\alpha-1}}{\Gamma(\alpha)}, \quad t>0.$$

2 Main Results

Theorem 2.1. *Consider* $\gamma \geq 0$, $\alpha \in (0,1]$, $\max\{1-\frac{n}{2q'},0\} < \beta < 1$ *and* $1 < \rho \leq 1 + \frac{2}{n}(q - \beta q)$*. Let* $u_0 \in X_q^1$ *and suppose* $h : (0,\infty) \to X_q^1$ *a continuous function such that* $\|h(s)\|_{X_q^1} \leq ks^{\varphi}$*, for some* $k > 0$ *and* $\varphi > -1$*. Then, there exist a constant* $\tau_0 > 0$ *and a unique mild solution* $u \in C([0,\tau_0]; X_q^1)$ *of the problem (1). Furthermore,*

$$u \in C((0,\tau_0]; X_q^{1+\theta})$$

and

$$t^{\alpha\theta}\|u(t)\|_{X_q^{1+\theta}} \to 0, \ \ as \ t \to 0^+,$$

for all $0 < \theta < \beta$*.*

Theorem 2.2. *Under the conditions of the Theorem 2.1, let* $\tau_0 > 0$ *and* $u : [0,\tau_0] \to X_q^1$ *the mild solution of problem (1). Then there exist* $T > 0$ *and a unique continuation* u^* *of* u *in* $[0,\tau_0 + T]$*. Furthermore, if* u *is the mild solution of the problem (1) with a maximal time of existence* $\tau_{max} < \infty$ *then*

$$\lim_{t\to\tau_{max}^-} \sup \|u(t)\|_{X_q^1} = \infty.$$

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REGULARIDADE TEMPORAL PARA EQUAÇÕES DE VOLTERRA DE TIPO CONVOLUÇÃO EM TEMPO DISCRETO

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Abstract

Usando técnicas provenientes da Análise de Fourier, obtemos uma caracterização para o problema de ℓ^p-regularidade maximal ($p \in (1, \infty)$) para a equação de Volterra de tipo convolução em tempo discreto via análise espectral.

1 Introdução

Seja X um espaço de Banach complexo e denotemos por $B(X)$ o espaço de Banach de todos os operadores lineares limitados em X. Para $p \in [1, \infty)$, consideremos o espaço de Banach $(\ell^p(X), ||\cdot||_p)$ de todas as sequências $u : \mathbb{Z}^+ \to X$ tais que $||u||_p := [\sum_{n=0}^{\infty} ||u_n||^p]^{\frac{1}{p}} < \infty$ e o espaço de Banach $(\ell^\infty(X), ||\cdot||_\infty)$ de todas as sequências limitadas, munido com a norma do supremo $||\cdot||_\infty$. O conceito de ℓ^p-regularidade maximal de operadores lineares limitados foi introduzido por S. Blunck em [1]: dizemos que $A \in B(X)$ possui ℓ^p-regularidade maximal se $n \mapsto (\Delta u)_n := u_{n+1} - u_n \in \ell^p(\mathbb{Z}^+, X)$ sempre que $f \in \ell^p(\mathbb{Z}^+, X)$. Aqui, $u_\bullet(x, f)$ denota a solução de

$$\begin{cases} u_{n+1} = Au_n + f_n, \ n \in \mathbb{Z}^+ \\ u_0 = x \in X. \end{cases} \tag{1}$$

Usando o Teorema da Aplicação Aberta, é possível mostrar que o problema de ℓ^p-regularidade maximal de $A \in B(X)$ consiste em verificar se o operador $K(f)_n = \sum_{k=0}^{n} A^{n-k}(A - I)f_k$ pertence a $B(\ell^p(X))$. Em [1], foi mostrado que, se A for um operador limitado em potências (isto é, o semigrupo discreto $n \mapsto A^n$ é limitado em $B(X)$), então a analiticidade de A no sentido de Ritt (isto é, a família $n \mapsto n(A - I)A^n$ é limitada em $B(X)$) é uma condição necessária para a regularidade maximal de A. Logo, o grande problema em estudar esse conceito está no fato do núcleo do operador K ser de ordem $O(\frac{1}{n})$ e portanto K ser de tipo singular. Todavia, a analiticidade de A traz uma vantagem muito importante: $\{\lambda \in \mathbb{C}; |\lambda| \geq 1, \lambda \neq 1\} \subset \rho(A)$ e $z \mapsto (z - 1)R(z, A)$ admite uma extensão H^∞ para um setor $\sum(1, \theta) = \{\lambda \in \mathbb{C}; 0 < |arg(\lambda - 1)| < \theta\}$, $\theta \in (\frac{\pi}{2}.\pi)$. S. Blunck mesclou isso com a teoria de multiplicadores de Fourier e obteve o seguinte resultado de caracterização em espaços UMD:

Teorema 1.1. *Sejam X um espaço UMD, $p \in (1, \infty)$ e $A \in B(X)$ limitado em potências e analítico. São equivalentes:*

a) A possui ℓ^p-regularidade maximal;

b) O conjunto $\{(\lambda - 1)R(\lambda, A); |\lambda| \geq 1, \lambda \neq 1\}$ é R-limitado;

c) O conjunto $\{A^n, n(A - I)A^n; n \in \mathbb{Z}^+\}$ é R-limitado.

O nosso objetivo é tentar extender os resultados de S. Blunck para um operador $A \in B(X)$ associados à equação de Volterra

$$\begin{cases} u_{n+1} = \sum_{k=0}^{n} a_{n-k}Au_n + f_n, \ n \in \mathbb{Z}^+ \\ u_0 = x \in X. \end{cases} \tag{2}$$

A noção de ℓ^p-regularidade maximal aqui segue do mesmo princípio: dizemos que A possui ℓ^p-regularidade maximal associdada à sequência complexa $(a_n)_{n\in\mathbb{Z}^+}$ se o operador $f \mapsto (\Delta u)$ estiver bem definido em $\ell^p(\mathbb{Z}^+, X)$. Por simplicidade, diremos apenas que, nesse caso, a equação (2) possui ℓ^p-regularidade maximal. Aqui, nossa hipótese central é (assim como em [1]) a limitação da família de evolução $S(n)_{n\in\mathbb{Z}^+} \subset B(X)$ associada a (2) dada por $S(n)x = u_n(x, 0)$.

2 Resultados Principais

Para o resultado principal, assumiremos as seguintes condições:

(H1) a função $\beta(z) = \dfrac{z}{\hat{a}(z)}$ é uma função inteira e satisfaz

$$|\beta(z) - \beta(\omega)| \leq L(r)|z - \omega|,$$

para todo $|z|, |\omega| \leq r$ e para alguma função $L : \mathbb{R}^+ \to \mathbb{R}^+$. Aqui, $\hat{a}$ denota a Transformada-Z de $(a_n)_{n\in\mathbb{Z}^+}$;

(H2) $\beta(z) \in \rho(A)$, para todo $|z| = 1$, $z \neq 1$.

O nosso primeiro resultado nos dá condições de tipo analíticas necessárias para que a equação (2) possua ℓ^p-regularidade maximal, desde que a família de evolução associada seja limitada.

Teorema 2.1. *Seja $p \in [1, \infty]$ e assuma que $(H1)$ seja satisfeita e que a família de evolução $(S(n))_{n\in\mathbb{Z}^+}$ da equação (2) seja limitada. Se (2) possuir ℓ^p-regularidade maximal, então existirá $C > 0$ tal que*

$$|\beta(z)| \ ||R(\beta(z), A)||_{B(X)} \leq \frac{C}{|z - 1|},$$

para todo $|z| > 1$. Em particular, existirá $\theta \in (\frac{\pi}{2}, \pi)$ de modo que o mapa

$$z \mapsto (z - 1)\beta(z)R(\beta(z), T)$$

admita uma extensão H^∞ para a região $\{z \in \mathbb{C}; |z| > 1\} \bigcup \sum(1, \theta)$.

O nosso resultado principal é uma caracterização da ℓ^p-regularidade maximal via "R-analiticidade":

Teorema 2.2. *Sejam X um espaço UMD e $p \in (1, \infty)$. Assuma que (H1)-(H2) sejam satisfeitas e que a família de evolução $(S(n))_{n\in\mathbb{Z}^+}$ da equação (2) seja limitada. São equivalentes:*

a) A equação (2) possui ℓ^p-regularidade maximal;

b) O conjunto $\{(\lambda - 1)p(z)R(p(z), A); |\lambda| \geq 1, \lambda \neq 1\}$ é R-limitado.

Além disso, qualquer um dos itens acima é suficiente para a R-limitação da família $\{S(n), n(\Delta S)(n); n \in \mathbb{Z}^+\}$.

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ON EVOLUTIONARY VOLTERRA EQUATIONS WITH STATE-DEPENDENT DELAY

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Abstract

In this work we study topological properties of the solution set of abstract Volterra equations with state-dependent delay, particularly, we ensure that such a set is a nonempty, compact and connected set. As application we consider our abstract results in the framework of integro-differential equations coming from viscoelasticity theory.

1 Introduction

In this work we study some topological properties of the solution set for a class of integro-differential equations with state-dependent delay

$$\begin{cases} u'(t) &= \int_0^t a(t-s)Au(s)ds + f\left(t, u_{\rho(t,u_t)}\right), \quad t \in [0,b], \\ u(0) &= \varphi \in \mathfrak{B}, \end{cases} \tag{1}$$

where $A : D(A) \subset X \to X$ is a closed linear operator defined on a Banach space X, the kernel $a \in L^1_{loc}((0,\infty))$ and the history $u_t : (-\infty, 0] \to X$, given by

$$u_t(\theta) = u(t+\theta),$$

belongs to some abstract phase space $\mathfrak{B}$ described axiomatically. Furthermore, $f : [0,b] \times \mathfrak{B} \to X$ and $\rho : [0,b] \times \mathfrak{B} \to (-\infty, b]$ are given functions. From the mathematical point of view, we are motivated by elegance and simplicity that evolutionary integro-differential equations of the type (1) provides to problems in mathematical physics.

As typical application of (1) we consider the problem

$$\begin{cases} u_t(t,x) = \int_0^t da(s)u_{xx}(t-s,x) + h\big(t,x,u(t-\sigma(\|u(t,x_0)\|),x)\big), \ t \geq 0, \ x \in [0,\pi], \\ u(t,0) = u(t,\pi) = 0, \quad t > 0, \\ u(t,x) = \varphi(t,x), \quad t \leq 0, \quad x \in [0,\pi], \end{cases}$$

where $x_0 \in (0,\pi)$ is fixed, $a : [0,\infty) \to (0,\infty)$ is a function of bounded variation on each compact interval $J = [0,T]$, $T > 0$, with $a(0) = 0$, and

$$\sigma : [0,\infty) \to [0,\infty)$$

is a continuous function. This type of equations has been the subject of many research papers in the last years since it has applications in such different fields as the theory of viscoelastic materials, thermodynamics, electrodynamics and population biology.

2 Main Results

The scope of this work is to study the topological structure of the solution set of (1). Particularly, we establish some sufficient conditions for the existence of mild solutions for this problem. To prove our results we always assume that $\rho : I \times \mathfrak{B} \to (-\infty, b]$ is continuous and $\varphi \in \mathfrak{B}$. Furthermore, we will suppose that the linear operator $A : D(A) \subset X \to X$ is the generator of a solution operator $S(t)$ and there exist a constant $M > 0$ such that $\|S(t)\| \leq M$, for all $t \in [0, b]$. If $u \in C([0, b]; X)$ we define $\overline{u} : (-\infty, b] \to X$ as the extension of u to $(-\infty, b]$ such that $\overline{u}_0 = \varphi$.

In the sequel we introduce some conditions.

($\mathbf{H_1}$) The function $f : [0, b] \times \mathfrak{B} \to X$ verifies the following conditions.

(i) The function $f(t, \cdot) : \mathfrak{B} \to X$ is continuous for almost everywhere $t \in [0, b]$, and for every $\psi \in \mathfrak{B}$, the function $f(\cdot, \psi) : [0, b] \to X$ is strongly measurable.

(ii) There are $m \in C([0,\, b],\, [0,\, \infty))$ and a continuous non-decreasing function $\Omega : [0, \infty) \to (0, \infty)$ such that $\|f(t, \psi)\| \leq m(t)\Omega(|\psi|_{\mathfrak{B}})$, for all $(t, \psi) \in [0, b] \times \mathfrak{B}$.

($\mathbf{H_2}$) For all $t \in [0, b]$ and $r > 0$, the set $\{f(s, \psi) : s \in [0, t], \|\psi\|_{\mathfrak{B}} \leq r\}$ is relativelly compact in X.

($\mathbf{H}_\varphi$) The function $t \to \varphi_t$ is well defined and continuous from the set

$$\mathcal{R}(\rho^-) = \{\rho(s, \psi) : (s, \psi) \in [0, b] \times \mathfrak{B},\ \rho(s, \psi) \leq 0\}$$

into $\mathfrak{B}$ and there is a bounded continuous function $J^\varphi : \mathcal{R}(\rho^-) \to (0, \infty)$ such that $\|\varphi_t\|_{\mathfrak{B}} \leq J^\varphi(t)\|\varphi\|_{\mathcal{B}}$ for every $t \in \mathcal{R}(\rho)$.

Theorem 2.1. *Suppose* ($\mathbf{H_1}$) ,($\mathbf{H_2}$) *and* ($\mathbf{H}_\varphi$) *are fulfilled. If*

$$K_b M \liminf_{\xi \to \infty} \frac{\Omega(\xi)}{\xi} \int_0^b m(s)ds < 1,$$

then the set $\mathcal{S}$ formed by all mild solutions of (1) is a nonempty set. Furthermore, if

$$K_b M \limsup_{\xi \to \infty} \frac{\Omega(\xi)}{\xi} \int_0^b m(s)ds < 1,$$

then $\mathcal{S}$ is compact in $C([0, b], X)$.

Proof See [1].

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O MÉTODO ASSIMPTÓTICO DE LINDSTEDT-POINCARÉ PARA SOLUÇÃO DAS EQUAÇÕES PERTURBADAS DE DUFFING E MATHIEU

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Abstract

Neste trabalho, apresentamos o método de perturbação ou método assimptótico do pequeno parâmetro para equações diferenciais ordinárias, baseado nas transformações das variáveis independentes para obter aproximações analíticas das soluções das equações perturbadas de Duffing e Mathieu.

1 Introdução

O método do pequeno parâmetro de Lindstedt foi introduzido para evitar a aparição de termos resonantes (por exemplo, $t \sin t$ ou $t \cos t$) nas soluções perturbadas das equações da forma

$$u'' + \omega_o^2 u = \varepsilon f(u, u'),\ \varepsilon << 1.$$

Na base do método de Lindstedt descansa a seguinte observação: a não linearidade muda a frequência do sistema desde o valor de ω_o, que corresponde ao sistema linear, até $\omega(\varepsilon)$. Para evitar a mudança de frequência, Lindstedt introduz uma nova variável $\tau = \omega t$ e desenvolveu ω e u em potências de ε:

$$\begin{aligned} u &= u_o(\tau) + \varepsilon u_1(\tau) + \varepsilon^2 u_2(\tau) + \dots, \\ \omega &= \omega_o + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \dots, \end{aligned}$$

e escolhendo os $\omega_i,\ i \geq 1$ adequados para evitar os termos resonantes. Poincaré em 1892 demonstrou que esta série trigonométrica obtida é assimptótica.

2 Resultados Principais. Método de Lindstedt-Poincaré

Como já foi apontado acima, procurar a solução em série de potências de ε da equação

$$u'' + \omega_o^2 u = \varepsilon f(u, u') \tag{1}$$

não é muito útil, devido ao aparecimento de termos resonantes. A essencia do método de Lindstedt-Poincaré consiste em evitar a aparição destes termos resonantes introduzindo uma nova variável

$$t = s(1 + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \dots). \tag{2}$$

Assim, (2) obtém a forma

$$(1 + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \dots)^{-2}\frac{d^2u}{ds^2} + \omega_o^2 u = \varepsilon f\left[u, (1 + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \dots)^{-1}\frac{du}{ds}\right]. \tag{3}$$

Procurando a solução de (2), em série de potências

$$u = \sum_{n=0}^{\infty} \varepsilon^2 u_n, \tag{4}$$

e igualando os coeficientes das mesmas potências de ε, obtemos equações para encontrar os u_m. As soluções dos u_m não contém termos resonantes somente para determinados valores de ω_m.

Proposição 2.1. *Os dois primeiros termos da série assimptótica da equação de Duffing*

$$\frac{d^2u}{dt^2} + u + \varepsilon u^3 = 0, \tag{5}$$

são dados por

$$u = a\cos(\omega t + \theta) + \frac{\varepsilon}{32}a^3\cos 3(\omega t + \theta) + O(\varepsilon^3),$$

onde a e θ são constantes de integração, e

$$\omega = \left(1 - \frac{3}{8}a^2\varepsilon + \frac{51}{256}a^4\varepsilon^2 + \dots\right)^{-1} = 1 + \frac{3}{8}a^2\varepsilon - \frac{15}{256}a^4\varepsilon^2 + O(\varepsilon^3).$$

Proposição 2.2. *Os dois primeiros termos da série assimptótica da equação de Mathieu [1]*

$$\frac{d^2u}{dt^2} + (\delta + \varepsilon\cos 2t)u = 0, \tag{1}$$

são dados por

$$u = ae^{(1/4)(\sin 2\sigma)\varepsilon t}\left[\sin(t-\sigma) + \frac{1}{16}\varepsilon\sin(3t-\sigma)\right] + O(\varepsilon^3),$$

onde a é constante de integração, e

$$\delta = 1 + \frac{1}{2}\varepsilon\cos 2\sigma + \frac{1}{32}\varepsilon^2(\cos 4\sigma - 2) + O(\varepsilon^3).$$

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ALMOST AUTOMORPHIC SOLUTIONS OF SECOND ORDER DYNAMIC EQUATIONS ON TIME SCALES

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Abstract

In this work, we present a general formulation of the solution of nonlinear and linear second order dynamic equation on time scales in the integral form. Also, we prove a result which ensures the existence of almost automorphic solution of the linear second order dynamic equation on time scales and a result which ensures the existence and uniqueness of almost automorphic solution of the nonlinear second order dynamic equation on time scales. Finally, we present some examples and applications to illustrate our results.

1 Introduction

In 1961, S. Bochner introduced the concept of continuous almost automorphic with relation to some aspects of differential geometry [2]. This concept generalized the continuous almost periodicity and periodicity, obtaining a large class of functions which was used to describe several important phenomena. After that, D. Araya, R. Castro and C. Lizama [1] introduced in the literature the concept of discrete almost automorphic functions. Subsequently, J. G. Mesquita and C. Lizama in [3] extended this concept to almost automorphic function with domain in an invariant under translations time scales, which includes many types of time scales that are used to describe more precisely population models, for instance.

On the other hand, the literature concerning the almost automorphic solutions of second order dynamic equations on time scales is still very scarce. Taking into account that these equations play an important role for applications, since they can be used to describe many important models, we focus our attention to investigate in this work the nonlinear and linear second order dynamic equations on time scales, respectively, given by:

$$x^{\Delta\Delta}(t) = A(t)x^{\Delta}(t) + B(t)x(t) + f(t, x(t)), \quad t \in \mathbb{T}, \tag{1}$$

and

$$x^{\Delta\Delta}(t) = A(t)x^{\Delta}(t) + B(t)x(t) + f(t), \quad t \in \mathbb{T} \tag{2}$$

We start by proving an integral formulation of the solution of the problems (1) and (2). Then, using this formulation, we prove a result which ensures the existence of almost automorphic solution of the equation (2) and a result concerning the existence and uniqueness of almost automorphic solution of the equation (1). Finally, we present some examples to illustrate our main results.

2 Main Results

Theorem 2.1. *Let $\mathbb{T}$ be an invariant under translations time scale. Suppose the equations $y^{\Delta}(t) = b(t)y(t)$ and $x^{\Delta}(t) = a(t)x(t)$ admit exponential dichotomy with positive constants K, α, and $\widetilde{K}$, $\widetilde{\alpha}$ respectively, and $f \in C_{rd}(\mathbb{T}, \mathbb{R}^n)$ is almost automorphic function on time scales. Assume also that $A, B, a, b \in \mathcal{R}(\mathbb{T}, \mathbb{R}^{n\times n})$ are almost automorphic matrices functions on time scales. Then, the equation (2) has an almost automorphic solution.*

Theorem 2.2. *Let $\mathbb{T}$ be an invariant under translations time scale and $f \in C_{rd}(\mathbb{T}\times\mathbb{R}^n, \mathbb{R}^n)$ be almost automorphic with respect to the first variable. Assume that $a, b \in \mathcal{R}(\mathbb{T}, \mathbb{R}^{n\times n})$ are almost automorphic and nonsingular matrices functions, the sets $\{a^{-1}(t)\}_{t\in\mathbb{T}}$ and $\{b^{-1}(t)\}_{t\in\mathbb{T}}$ are bounded. Suppose also the equation*

$$x^{\Delta}(t) = A(t)x(t)$$

admits exponential dichotomy on $\mathbb{T}$ with positive constants K and α, and suppose that $f : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies the Lipschitz condition:

$$\|f(t,x) - f(t,y)\| \leq L\,\|x - y\| \tag{1}$$

for every $x, y \in \mathbb{R}^n$, $t \in \mathbb{T}$ and $0 < L < \frac{\alpha}{2K(2+\widetilde{\mu}\alpha)}$, where $\widetilde{\mu} = \sup_{t\in\mathbb{T}}\big|\mu(t)\big|$. Then, the equation

$$x^{\Delta\Delta}(t) = A(t)x^{\Delta}(t) + B(t)x(t) + f(t, x(t)) \tag{2}$$

where $A(t) := b(t) - a^{\sigma}(t)$ and $B(t) := a(t)b(t) - a^{\Delta}(t)$ for all $t \in \mathbb{T}$, and $a, b : \mathbb{T} \to \mathbb{R}^{n\times n}$, has a unique solution which is almost automorphic.

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ON THE NAVIER-STOKES EQUATIONS WITH VARIABLE VISCOSITY IN STATIONARY FORM

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Abstract

In tris paper, we study the existence of weak solutions for the Navier-Stokes equations with variable viscosity in stationary form. We consider that viscosity depends on the velocity of the fluid. Uniqueness of solutions is also considered.

1 Introduction

The mathematical model for description of the motion of a viscous incompressible fluid is given by the following system of partial differential equation:

$$\left| \begin{array}{l} \dfrac{\partial u}{\partial t} - \nu \Delta u + \displaystyle\sum_{i=1}^{n} u_i \frac{\partial u}{\partial x_i} = f - grad\, p \\ div\ u = 0 \end{array} \right. \tag{1}$$

Here $u = (u_1, u_2, .., u_n)$ is a vector function with $u_i = u_i(x,t)$, where $x = (x_1, x_2, ..., x_n)$ belongs to $\mathbb{R}^n$ and $t \geq 0$ is a real number. Note that u is the velocity of fluid, f is the density of forces acting on it and $p = p(x,t)$ it's pressure at point (x,t).

By the constant ν we represent the viscosity of the fluid. We suppose $\nu > 0$.

The mathematical analysis of (1) was done, first time, by J. Laray in 1934. After that it was systematically investigated by O. A. Ladyzhenskaya , 1963; J. L. Lions , 1969; Roger Temam , 1979; Luc Tartar, 1999 and many others mathematicians.

The above problem, when ν is of the form $\nu = \nu_0 + \nu_1 \|u(t)\|^2$, $\nu_0 > 0$ and $\nu_1 > 0$ are positive constants, was investigated by J. L. Lions [1] in a bounded cylindrical domain $Q = \Omega \times]0, T[$ of $\mathbb{R}^{n+1}$, more precisely, he investigated the mixed problem

$$\left| \begin{array}{l} \dfrac{\partial u}{\partial t} - \left(\nu_0 + \nu_1 \|u(t)\|^2\right) \Delta u + \displaystyle\sum_{i=1}^{n} u_i \frac{\partial u}{\partial x_i} = f - grad\, p \text{ in } Q \\ div\, u = 0 \text{ in } Q \\ u = 0 \text{ on } \Sigma \ (\Sigma \text{ lateral boundary of } Q) \\ u(x,0) = u_0(x) \text{ in } \Omega. \end{array} \right. \tag{2}$$

He proved the existence of weak solution for $n \leq 4$ and uniqueness for $n \leq 3$. For the case $\nu_1 = 0$, as we know we have up to now, uniqueness for $n < 3$, cf. Lions and G. Prodi. The noncylindrical case of 7 was investigated by Araújo [2].

In this paper we study the stationary case of the problem (7). Let Ω be a bounded open set of $\mathbb{R}^n$ with boundary Γ supposed regular. The stationary problem correspondent of the evolution problem (7) consist of to determine

$u = \{u_1, u_2, ..., u_n\}$ and p satisfying

$$\left| \begin{array}{l} -\left(\nu_0 + \nu_1 \|u\|^2\right) \Delta u + \sum_{i=1}^{n} u_i \frac{\partial u}{\partial x_i} = f - grad\, p \text{ in } \Omega \\ div\, u = 0 \text{ in } \Omega \\ u = 0 \text{ on } \Gamma \ (\Gamma \text{ lateral boundary of } \Omega). \end{array} \right. \tag{3}$$

When $\nu_1 = 0$, the mathematical analysis of (7) was done by Lions [1] and Temam [1]. In this paper we study the existence of solutions in some sense for the problem (7) when $n \leq 4$. Uniqueness of solutions for the case $n \leq 4$ is also analyzed.

2 Main Results

We define the following spaces

$$\mathcal{V} = \{\varphi \in (\mathcal{D}(\Omega))^n; \quad div\, \varphi = 0\} V = \overline{\mathcal{V}}^{(H_0^1(\Omega))^n} \text{ and } H = \overline{\mathcal{V}}^{(L^2(\Omega)^n}.$$

We consider $a(u,v)$ the bilinear form and the trilinear form defined for $u, v, w \in V$, where

$$a(u,u) = \sum_{i,j=1}^{n} \int_\Omega \left(\frac{\partial u_i}{\partial x_j}(x) \right)^2 dx = \|u\|^2 \quad \text{and} \quad b(u,v,w) = \sum_{i,j=1}^{n} \int_\Omega u_j(x) \frac{\partial v_i}{\partial x_j}(x) w_i(x) dx$$

Definition 2.1. *Consider $f \in V'$. Then a function $u \in V$ is called a weak solution of the problem (3) when it satisfies*

$$(\nu_0 + \nu_1 \|u\|^2) a(u,v) + b(u,u,v) = \langle f, v \rangle \ \forall v \in V. \tag{4}$$

By $\langle .,. \rangle$, we indicate the duality pairing between V' and V, V' being the topological dual of the space V.

Theorem 2.1. *(weak solutions) We suppose $n \leq 4$. If $f \in V'$, then there exists a solution u of (1)*

Following the ideas of J. L. Lions [1] and R. Temam [1], we deduce from the equation

$$-\left(\nu_0 + \nu_1 \|u\|^2\right) \Delta u + \sum_{i=1}^{n} u_i \frac{\partial u}{\partial x_i} = f \text{ in } V'$$

given in (1), that there exists $p \in L^2(\Omega)$ such that

$$-\left(\nu_0 + \nu_1 \|u\|^2\right) \Delta u + \sum_{i=1}^{n} u_i \frac{\partial u}{\partial x_i} = f - grad\, p \text{ in } (H^{-1}(\Omega))^n.$$

Theorem 2.2. *If $n \leq 4$ and ν_0 is "sufficiently large" or $\|f\|_{V'}$ "sufficiently small", then there exist a unique solution u of (1).*

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HYPERBOLIC DIFFERENTIAL INCLUSION WITH NONLOCAL BOUNDARY CONDITION AND SOURCE TERM

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Abstract

The aim of this paper is the investigation of a problem generated by a hyperbolic differential inclusion with nonlocal boundary condition and source term. By the use of Galerkin procedure, we prove the existence of global solutions and the exponential decay of the energy.

1 Introduction

Let Ω be a bounded domain of $\mathbb{R}^n$, $n \geq 1$ with a smooth boundary $\Gamma = \Gamma_0 \bigcup \Gamma_1$, $\overline{\Gamma}_0 \bigcup \overline{\Gamma}_1 = \emptyset$, where Γ_0 , Γ_1 have positive measures. In this work, we are concerned with the following problem

$$
\begin{aligned}
u_{tt} - \Delta u + \Xi = 0 \quad & \text{in } \Omega\times]0,\infty[, \\
\Xi(x,t) \in \varphi(u_t(x,t)) \quad & a.e.\ (x,t) \ \in \Omega\times]0,\infty[\\
u(x,t) = \int_\Omega K(x,y)u(y,t)\,dy \qquad & \text{on } \Gamma_0\times]0,\infty[, \\
\frac{\partial u}{\partial \nu} + u_t = |u|^\gamma u \quad & \text{on } \Gamma_1\times]0,\infty[, \\
u(x,0) = u^0(x) \quad u_t(x,0) = u^1(x) \quad & \text{in } \Omega
\end{aligned}
\tag{1}
$$

where ν represents the unit outward normal to Γ, $0 \leq \gamma < \frac{1}{N-2}$if $N \geq 3, \gamma \geq 0$if $N = 1, 2$, K is a given function satisfying some general properties and φ is a discontinuous and nonlinear set-valued mapping by filling in jumps of a locally bounded function b.

The study of the wave equations with boundary conditions of different type have attracted expensive interest in recent years (see [1, 1] among many others). Motivated by their works, we consider more generalized problem (1) with a discontinuous and nonlinear multi-valued term and a nonlinear source term on the boundary. The background of these variational problems is in physics, especially in solid mechanics,we refer to [4] . We note that it is difficult to apply a method based on the second kind integral operator (see [2, 2]) to solve equation (1). So we use the Galerkin method to attack it.

2 Main Results

First ,we define $V = \{u \in \ H^1(\Omega) : u(x) = \int_\Omega K(x,y)u(y)\,dy \quad \text{on } \Gamma_0\}$ and the potential well
$W = \{u \in V : \ I(u) = |\nabla u|_2^2 - |u|_{\gamma+2,\Gamma_1}^{\gamma+2} > 0\} \bigcup \{0\}$. It is easy to see that V is a closed subspace of $H^1(\Omega)$
Now ,we give the following hypotheses

(A_1) $K : \overline{\Omega} \times \overline{\Omega} \to \mathbb{R}$ satisfies that $K(x, .)$, $\frac{\partial K}{\partial x_i} \in L^2(\Omega)$ and

$$
K(x) := \left(\int_\Omega |K(x,y)|^2\,dy\right)^{1/2} < \infty, \quad K_i(x) := \left(\int_\Omega |\frac{\partial K}{\partial x_i}|^2\,dy\right)^{1/2} < \infty, \ \text{with} \quad \sum_{i=1}^n \int_{\Gamma_0} K(x)K_i\,d\Gamma < c_2, \ c_2 > 0.
$$

Also, for any $x \in \Gamma_0$, $K(x) < \infty$, $K_i(x) < \infty$
(A_2) $b : \mathbb{R} \to \mathbb{R}$ is a locally bounded function satisfying $b(s)s \geq \mu_1 s^2$, $|b(s)| \leq \mu_2 |s|$, $\forall s \in \mathbb{R}$, where μ_1 and μ_2 are some positive constants.
The multi-valued function $\varphi : \mathbb{R} \to 2^{\mathbb{R}}$ is obtained by filling in jumps of the function b by means of the functions $\underline{b}_\epsilon, \overline{b}_\epsilon, \underline{b}, \overline{b} : \mathbb{R} \to \mathbb{R}$ as follows:

$$\underline{b}_\epsilon(t) = \text{essinf}_{|s-t|\leq\epsilon} b(s), \quad \overline{b}_\epsilon(t) = \text{esssup}_{|s-t|\leq\epsilon} b(s), \quad \underline{b}(t) = \lim_{\epsilon\to 0^+} \underline{b}_\epsilon(t), \quad \overline{b}(t) = \lim_{\epsilon\to 0^+} \overline{b}_\epsilon(t), \quad \varphi(t) \in [\underline{b}(t), \overline{b}(t)]$$

Theorem 2.1. *Suppose $u^0 \in W \bigcap H^2(\Omega)$,$u^1 \in V$, and*

$$0 < E(0) = \frac{1}{2}|u^1|_2^2 + \frac{1}{2}|\nabla u^0|_2^2 - \frac{1}{\gamma+2}|u^0|_{\gamma+2,\Gamma_1} < \frac{\gamma}{4(\gamma+2)}\left(\frac{\gamma}{2c_*^{\gamma+2}(\gamma+2)}\right)^{\frac{2}{\gamma}}$$

where c_* is an imbedding constant from V to $L^{2(\gamma+1)}(\Gamma_1)$. . Then problem (1) admits a global weak solution u. This solution satisfies

$$E(t) \leq L_0 e^{-\gamma t}, \forall t \geq 0$$

where L_0 and γ are two positive constants.

Proof We apply the Galerkin method and the potential well theory to prove the existence. The decay estimate of solutions is established by means of Lemma of V. Komornik ■

Proof

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EXPONENTIAL DECAY FOR WAVE EQUATION WITH INDEFINITE MEMORY DISSIPATION

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Abstract

In this work we deal with the following wave equation with localized dissipation given by a memory term

$$u_{tt} - u_{xx} + \partial_x \left\{ a(x) \int_0^t g(t-s) u_x(x,s) ds \right\} = 0.$$

We consider that this dissipation is indefinite due to sign changes of the coefficient a or by sign changes of the memory kernel g. The exponential decay of solutions is proved when the average of coefficient a is positive and the memory kernel g is small.

1 Introduction

In this work we consider the following system involving a wave equation with localized memory

$$\begin{cases} u_{tt} - u_{xx} + \partial_x \left\{ a(x) \int_0^t g(t-s) u_x(x,s) ds \right\} = 0 & \text{in } (0,L) \times (0,\infty), \\ u(0,t) = 0, \; u(L,t) = 0 & \text{for } t > 0, \\ u(0) = u_0 \in H_0^1(0,L), \; u_t(0) = u_1 \in L^2(0,L), \end{cases} \tag{1}$$

where $g : [0,\infty) \to \mathbb{R}$ denotes the memory kernel and $a : [0,L] \to \mathbb{R}$ is a coefficient that define the region where there is dissipation. This coefficient may act in only a part of the domain $[0,L]$.

Here we consider the energy functional associated to the problem (1) given by

$$E(t) := \frac{1}{2} \int_0^L \left\{ |u_t|^2 + d(x,t)|u_x|^2 + a(x) \int_0^t g(t-s)|u(t) - u(s)|^2 ds \right\} dx,$$

where $d(x,t) = 1 - a(x) \int_0^t g(s) ds$. By multiplying the equation (1) by u_t, integrating by parts and using a suitable identity, we get that,

$$\frac{d}{dt} E(t) = \frac{1}{2} \int_0^L a(x) \left\{ \int_0^t g'(t-s)|u(t) - u(s)|^2 \, ds - g(t)|u(t)|^2 \right\} dx.$$

Then, we can observe that if the functions g and $-g'$ of kernel memory are positive and the coefficient a is positive at least in a part of interval $[0,L]$ but without changing sign, then the energy functional is decreasing. By other side, if functions g and $-g'$ are positive, but the function a losses the positivity, then the dissipation given by memory term has indefinite sign. In the same way, the dissipation has undefined sign if the coefficient a is positive but the functions g or $-g'$ change sign.

In the context of partial differential equations systems where the dissipative effects are given by memory terms and they change sign there are few studies about the energy decay rate. One of the earliest studies in this direction is due to Muñoz-Rivera and Naso. They considered the following functional equation with memory term

$$u_{tt} + Au - \int_0^t g(t-s) Au(s) ds = 0,$$

where the memory kernel g can change sign. They proved, in [1], the exponential decay of the solutions if $0 < g(0) < \lambda_1$, where λ_1 is the smaller eigenvalue of the self-adjoint, positive definite operator A in a Hilbert space. In this work, the memory dissipation is distributed on whole domain. Its a open study when this dissipation is distributed only in a part of its domain. In the similar context, Muñoz-Rivera and Sare, in [2], proved the exponential decay for a Timoshenko system with dissipation given by the memory with indefinite sign.

2 Main Results

Here it is considered that the coefficient that determines the region where the dissipation is effective is a function $a \in W^{2,\infty}(0, L)$ satisfying

$$\overline{a} := \frac{1}{L}\int_0^L a(x)\, dx > 0. \tag{1}$$

Note that this function may suffer sign changes or even be null in a part of your domain. Besides, it is supposed that there exists a constant $d > 0$ such that

$$\|a'\|_{L^\infty(0,L)} + \|a''\|_{L^\infty(0,L)} \le d. \tag{2}$$

Finally, it is assumed that the memory kernel is a function $g \in C^2([0,\infty))$ satisfying

$$g(0) > 0, \quad |g(t)| \le g_0 e^{-\alpha t}, \quad |g'(t)| + |g''(t)| \le g_0^2 C_0 e^{-\alpha t}, \quad \forall t \ge 0, \tag{3}$$

where α, g_0 and C_0 are positive constants, with C_0 independent of g_0.

It is important to note that g_0 doesn't necessarily mean $g(0)$ and the function g may suffer sign changes.

The main result of this work is given by the following theorem.

Theorem 2.1. *Suppose that $g \in C^2([0,\infty))$ and $a \in W^{2,\infty}(0, L)$ satisfy (1)-(3) and $d \le g_0 C$. If g_0 is small there exist $\gamma > 0$ and $C(\|u_0\|_{H_0^1}, \|u_1\|_{L^2}) > 0$ such that the weak solution $u \in C([0,\infty[, H_0^1) \cap C^1([0,\infty[, L^2)$ of the system (1) satisfies*

$$\|u(t)\|_{H_0^1} + \|u_t(t)\|_{L^2} \le C(\|u_0\|_{H_0^1}, \|u_1\|_{L^2}) e^{-\gamma t}.$$

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ON STABILITY OF GLOBAL SOLUTIONS FOR SECOND-GRADE FLUIDS FLOW

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Abstract

This paper deals with global existence in time of weak solution, uniqueness, the uniform stability of the energy, and the continuous dependence on the data for an initial-boundary value problem of an incompressible non-Newtonian fluid flow of grade two in three space dimensions.

1 Introduction

Suppose Ω a bounded, simply-connected and open set in $\mathbb{R}^3$ having a smooth boundary $\partial\Omega$ (i.e., at least $\mathcal{C}^{3,1}$-class) and lying at one side of $\partial\Omega$. Let $Q = \Omega \times (0,\infty)$ and $\Sigma = \partial\Omega \times (0,\infty)$ its lateral boundary. We consider the initial-boundary value problem of an incompressible non-Newtonian fluid flow of grade two given by

$$\begin{array}{rcll} \partial_t\left(\boldsymbol{u} - \alpha\Delta\boldsymbol{u}\right) - \nu\Delta\boldsymbol{u} + \operatorname{curl}(\boldsymbol{u} - \alpha\Delta\boldsymbol{u}) \times \boldsymbol{u} + \nabla q & = & \boldsymbol{f} & \text{in} \quad Q, \\ \operatorname{div}\boldsymbol{u} & = & 0 & \text{in} \quad Q, \\ \boldsymbol{u} & = & 0 & \text{on} \quad \Sigma, \\ \boldsymbol{u}(0,\cdot) & = & \boldsymbol{u}_0 & \text{in} \quad \Omega, \end{array} \tag{1}$$

where $\nu > 0$ represents the constant of kinematic viscosity, $\alpha > 0$ is a constant related to the non-newtonian behavior of the fluid, $\boldsymbol{f}$ are external forces, $q = p - \alpha(\boldsymbol{u}\cdot\Delta\boldsymbol{u} + \frac{1}{4}|D\boldsymbol{u}|^2_{\mathbb{R}^3}) - \frac{1}{2}\boldsymbol{u}\cdot\boldsymbol{u}$ is the potential function, where $(D\boldsymbol{u})_{ij} = \partial_j\boldsymbol{u}_i + \partial_i\boldsymbol{u}_j$ is the linear strain tensor. The others objects of system (1) are usual.

Our mean contribution here is to prove the stability of the global weak solutions of system (1) employing some ideas of Ponce et al [3]. For this purpose, we consider the open neighborhood

$$\mathcal{O}_\delta\big((\boldsymbol{u}_0,\boldsymbol{f})\big) = \Big\{(\boldsymbol{y},\boldsymbol{z}) \in \boldsymbol{H}^1_\alpha(\Omega) \times L^2\big(0,\infty;\boldsymbol{L}^2(\Omega)\big); \quad \|\boldsymbol{u}_0 - \boldsymbol{y}\|^2_{\boldsymbol{H}^1_\alpha(\Omega)} + \int_0^\infty \|\boldsymbol{f}(t) - \boldsymbol{z}(t)\|^2 dt < \delta\Big\} \tag{2}$$

such that, for all $(\widetilde{\boldsymbol{u}}_0, \widetilde{\boldsymbol{f}}) \in \mathcal{O}_\delta(\{\boldsymbol{u}_0,\boldsymbol{f}\})$ there exists a unique global weak solution $(\widetilde{\boldsymbol{u}}, \widetilde{q})$ of the perturbed problem

$$\begin{array}{rcll} \partial_t\left(\widetilde{\boldsymbol{u}} - \alpha\Delta\widetilde{\boldsymbol{u}}\right) - \nu\Delta\widetilde{\boldsymbol{u}} + \operatorname{curl}(\widetilde{\boldsymbol{u}} - \alpha\Delta\widetilde{\boldsymbol{u}}) \times \widetilde{\boldsymbol{u}} + \nabla\widetilde{q} & = & \widetilde{\boldsymbol{f}} & \text{in} \quad Q, \\ \operatorname{div}\widetilde{\boldsymbol{u}} & = & 0 & \text{in} \quad Q, \\ \widetilde{\boldsymbol{u}} & = & 0 & \text{on} \quad \Sigma, \\ \widetilde{\boldsymbol{u}}(0,\cdot) & = & \widetilde{\boldsymbol{u}}_0 & \text{in} \quad \Omega. \end{array} \tag{3}$$

2 Main results

Theorem 2.1. *Let $\boldsymbol{u}$ be a global weak solution of system (1) in the class*

$$\boldsymbol{u} \in L^\infty(0,\infty;\boldsymbol{V}_2) \cap L^2(0,\infty;\boldsymbol{V}_2), \quad \boldsymbol{u}' \in L^\infty(0,\infty;\boldsymbol{V}), \tag{1}$$

and satisfying a global criterion of regularity of Leray type

$$\|\boldsymbol{u}(t)\|^2_{\boldsymbol{H}^4(\Omega)} \quad \textit{belongs to} \quad L^1(0,\infty). \tag{2}$$

Then

(i) *If* $(\widetilde{\boldsymbol{u}}_0, \widetilde{\boldsymbol{f}}) \in \mathcal{O}_\delta\big((\boldsymbol{u}_0, \boldsymbol{f})\big)$ *then there exist a unique weak global solution* $(\widetilde{\boldsymbol{u}}, \widetilde{q})$ *of problem* (3) *and a positive real constante* $C = C\big(C_\Omega, \alpha\big) > 0$ *such that*

$$\|\boldsymbol{u}(t) - \widetilde{\boldsymbol{u}}(t)\|_{\boldsymbol{V}} \leq C\delta, \tag{3}$$

and consequently, goes to zero as $\delta \to 0$*. The constant* C_Ω *is defined by* $\|\boldsymbol{z}\|^2 \leq C_\Omega \|\nabla \boldsymbol{z}\|^2$ *for all* $\boldsymbol{z} \in \boldsymbol{V}$.

(ii) *In addition to Leray condition* (2)*, if there exists* $C > 0$ *such that*

$$\|\boldsymbol{u}(t)\|^2_{\boldsymbol{H}^4(\Omega)} \leq \frac{\zeta}{2C} \quad \textit{and} \quad \int_0^\infty \exp(\zeta t)\|\boldsymbol{f}(t)\|^2 dt < \infty, \tag{4}$$

for all $t \geq 0$*, then the energy*

$$E_{\widetilde{\boldsymbol{u}}}(t) = \frac{1}{2}\left\{\|\widetilde{\boldsymbol{u}}(t)\|^2 + \alpha\|\nabla \widetilde{\boldsymbol{u}}(t)\|^2\right\} \tag{5}$$

of system (3) *satisfy*

$$E_{\widetilde{\boldsymbol{u}}}(t) \leq C_0 \exp\left(-\frac{\zeta}{2}\, t\right) \quad \textit{for all} \quad t \geq 0, \tag{6}$$

where

$$\zeta = (\nu/2)\min\{1/\alpha,\, 1/C_\Omega\}, \tag{7}$$

and C_0 *is a positive real constant,*

$$\boldsymbol{V} = \{\boldsymbol{v} \in \boldsymbol{H}^1_0(\Omega) : \ \mathrm{div}\boldsymbol{v} = 0 \text{ on } \Gamma\} \quad \textit{and} \quad \boldsymbol{V}_2 = \{\boldsymbol{v} \in \boldsymbol{V} : \ \textit{curl}(\boldsymbol{v} - \alpha\Delta\boldsymbol{v}) \in L^2(\Omega)\}.$$

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EXISTÊNCIA E NÃO EXISTÊNCIA DE SOLUÇÕES GLOBAIS PARA UM SISTEMA ACOPLADO DE VÁRIAS COMPONENTES COM TERMOS NÃO HOMOGÊNEOS

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Abstract

Neste trabalho é considerado o seguinte sistema parabólico fracamente acoplado de m equações

$$u_{it} - \Delta u_i = \quad a_i \; u_{i+1}^{p_i} \quad \text{em } \mathbb{R}^N \times (0,T) \; (i = 1, ..., m),$$

com condições iniciais não negativas, $u_{m+1} = u_1$, e $p_i > 0$. Os termos não homogêneos $a_i \in C^{\alpha_i}(\mathbb{R}^N)$ são funções não negativas tal que $a_i(x) \sim |x|^{d_i}$ para $|x|$ suficientemente grande, onde $d_i \in \mathbb{R}$. No caso $d_i \geq 0$ $(i = 1, ..., m)$ obtemos resultados tipo Fujita que garantem existência global ou explosão em tempo finito das soluções. No caso $d_i \in \mathbb{R}$ $(i = 1, ..., m)$ obtemos resultados de existência global.

1 Introdução

Considere o seguinte problema parabólico

$$\begin{cases} u_{it} - \Delta u_i = & a_i \; u_{i+1}^{p_i} \quad \text{em } \mathbb{R}^N \times (0,T) \quad (i = 1, ..., m), \\ u_i(0) = & u_{i0} \quad\quad \text{em } \mathbb{R}^N, \end{cases} \tag{1}$$

onde u_{i0} são funções contínuas, limitadas, e não negativas, $u_{m+1} = u_1$, $p_i > 0$, e $a_i \in C^{\alpha_i}(\mathbb{R}^N)$ tal que $a_i(x) \sim |x|^{d_i}$ com $d_i \in \mathbb{R}$.

O problema (1) tem solução $(u_1, ..., u_m) \in [C([0, T_{max}), C_b(\mathbb{R}^N))]^m$, definida num intervalo maximal $[0, T_{max})$, satisfazendo

$$u_i(t) = S(t)u_{i0} + \int_0^t S(t-\sigma) a_i u_{i+1}^{p_i}(\sigma) d\sigma \quad (i = 1, ..., m), \tag{2}$$

onde $u_{m+1} = u_1$ para todo $t \in [0, T_{max})$, e $(S(t))_{t \geq 0}$ é o semigrupo do calor. Ademais, temos a seguinte alternativa, ou $T_{max} = +\infty$ (solução global), ou $T_{max} < \infty$ e

$$\limsup_{t \to T_{max}} \left(\sum_{i=1}^{m} \|u_i(t)\|_\infty \right) = +\infty,$$

neste caso, dizemos que qualquer solução não trivial do problema (1) explode em tempo finito.

Como as não linearidades $a_i(x) u_{i+1}^{p_i}$ são localmente Hölder contínuas em x e localmente Lipschitz em u, segue-se por argumentos conhecidos que $(u_1, ..., u_m)$ é uma solução clássica; isto é, $(u_1, ..., u_m) \in \left[C^{2,1}(\mathbb{R}^d) \times (0,T)\right]^m$.

O sistema acoplado (1) é um exemplo simples de um sistema de reação-difusão mostrando um acoplamento não trivial de $u_1, ..., u_m$. Aparece em diferentes modelos matemáticos, físicos, químicos, biológicos, sociológicos, etc. Por exemplo, veja [1] e as referências contidas nele.

O problema (1) foi estudado em [2] no caso de uma equação (m=1), e no caso de duas equações acopladas (m=2) com $a_1 = 1$ foi estudado em [1].

2 Resultados Principais

Para $i = 1, ..., m$, considere os seguintes valores

$$\alpha_1 = (\tfrac{d_1}{2} + 1)(\beta - 1)^{-1}, (m = 1),$$
$$\alpha_i = \frac{[\sum_{j=1}^{m-1}(\prod_{k=0}^{j-1} p_{i+k})(\frac{d_{i+j}}{2}+1)] + (\frac{d_i}{2}+1)}{\beta - 1}, (m > 1),$$

onde $d_{m+i} = d_i$, e $p_{m+i} = p_i$.

O primeiro resultado deste trabalho é

Teorema 2.1. *Suponha que* $p_i \geq 1$, $d_i \in [0, \infty)$, $\beta > 1$, *e* $\nu = \max \alpha_i$.

(i) Se $\nu \geq \frac{N}{2}$, *então qualquer solução não trivial e não negativa do problema (1) explode em tempo finito.*

(ii) Se $\nu < \frac{N}{2}$, *então o problema (1) tem soluções globais.*

(iii) Se $0 < \frac{a}{2} < \nu < \frac{N}{2}$ *e* $u_{j0} \in I_a$ *para todo* $j \in \{1, ..., m\}$, *então o problema (1) explode em tempo finito. Onde*

$$I_a := \left\{ \varphi \in C_b(\mathbb{R}^N) : 0 \leq \varphi \ e \ \liminf_{|x| \to \infty} |x|^a \varphi(x) > 0 \right\}.$$

Para $i = 1, ..., m$, considere os seguintes valores

$$\rho_1 = (q(d_1, N) + 1)\,(\beta - 1)^{-1}, (m = 1),$$
$$\rho_i = \frac{[\sum_{j=1}^{m-1}(\prod_{k=0}^{j-1} p_{i+k})(q(d_{i+j}, N)+1)] + (q(d_i, N)+1)}{\beta - 1}, (m > 1),$$

onde $d_{m+i} = d_i$, $p_{m+i} = p_i$, e

$$q(d, N) = \begin{cases} \frac{d}{2}, & \text{quando } (d, N) \in ((-1, \infty) \times [1, \infty)) \cup ((-2, \infty] \times [2, \infty)), \\ -\frac{1}{2}, & \text{quando } (d, N) \in [-2, -1] \times \{1\}, \\ -1, & \text{quando } (d, N) \in \{-2\} \times [2, +\infty). \end{cases}$$

Nosso segundo resultado é o seguinte.

Teorema 2.2. *Suponha que* $p_i \geq 1$, $\beta > 1$, $\nu = \max \alpha_i$, *e* $\rho = \max \rho_i$.

(i) Se $d_i \in (-1, \infty)$ *para todo* $i \in \{1, ..., m\}$ *e* $\nu < \frac{N}{2}$, *então (1) tem soluções globais.*

(ii) Suponha $d_i \in (-2, \infty)$ *para todo* $i \in \{1, ..., m\}$. *Se* $N = 1$ *e* $\rho < \frac{N}{2}$, *ou se* $N \geq 2$ *e* $\nu < \frac{N}{2}$, *então o problema (1) tem soluções globais.*

(iii) Se $d_i \in [-2, \infty)$ *para todo* $i \in \{1, ..., m\}$ *e* $\rho < \frac{N}{2}$, *então o problema (1) tem soluções globais.*

Prova dos Teoremas 2.1 e 2.2.- Para provar o Teorema 2.1 usamos a técnica iterativa de semigrupos adaptado para o caso não homogêneo. Para provar o Teorema 2.2 adaptamos a técnica de [2] e [1].

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SOLUÇÃO GLOBAL FORTE PARA AS EQUAÇÕES DE FLUIDOS MICROPOLARES INCOMPRESSÍVEIS

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Abstract

Estudamos o PVI para as equações de um fluido micropolar incompreensível viscoso com densidade constante $\rho = 1$ em $\mathbb{R}^3$. Inicialmente, baseado em estimativas de energia, mostramos a existência e unicidade de solução local forte para o problema. Ademais, impondo uma condição de pequenez nos dados iniciais, provamos a unicidade da solução global forte.

1 Introdução

Consideramos o problema de Cauchy

$$\begin{cases} \boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - (\mu+\mu_r)\Delta\boldsymbol{u} + \nabla p - 2\mu_r \operatorname{rot}\boldsymbol{w} = \boldsymbol{0}, \\ \operatorname{div}\boldsymbol{u} = 0, \\ \boldsymbol{w}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{w} - (c_a+c_d)\Delta\boldsymbol{w} - (c_0+c_d-c_a)\nabla(\operatorname{div}\boldsymbol{w}) + 4\mu_r\boldsymbol{w} - 2\mu_r\operatorname{rot}\boldsymbol{u} = \boldsymbol{0}, \\ \boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0(\boldsymbol{x}), \quad \boldsymbol{w}(\boldsymbol{x},0) = \boldsymbol{w}_0(\boldsymbol{x}), \end{cases} \tag{1}$$

em $\mathbb{R}^3 \times (0,T)$, onde $\boldsymbol{u}_0$ e $\boldsymbol{w}_0$ são funções dadas.

No sistema (1), as incógnitas são as funções $\boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^3$, $p(\boldsymbol{x},t) \in \mathbb{R}$ e $\boldsymbol{w}(\boldsymbol{x},t) \in \mathbb{R}^3$, as quais representam, respectivamente, a velocidade linear, a pressão hidrostática e a velocidade angular de rotação das particulas do fluido em um ponto $\boldsymbol{x} \in \mathbb{R}^3$ no tempo $t > 0$. Este sistema descreve o movimento de um fluido micropolar (ou assimétrico) homogêneo, viscoso e incompressível (veja [1] e [2]). As constantes positivas μ, μ_r, c_a, c_d e c_0 estão relacionadas com a viscosidade e satisfazem $c_0 + c_d > c_a$. Vale salientar que o sistema (1) inclui, como caso particular, as clássicas equações de Navier-Stokes ($\boldsymbol{w} = \boldsymbol{0}$ e $\mu_r = 0$).

2 Resultados Principais

Os resultados que provamos são similares ao de ***Xin Zhong*** para as equações de Navier-Stokes com condução do calor (veja [3]) e com amortecimento (veja [4]). Por simplicidade, assumimos $\mu = \mu_r = 1/2$ e $c_a + c_d = c_0 + c_d - c_a = 1$.

Antes de enunciarmos o principal resultado obtido, apresentaremos a definição de solução forte para o PVI (1).

Definição 2.1. *Suponha que* $(\boldsymbol{u}_0, \boldsymbol{w}_0) \in \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3)$ *com* $\operatorname{div}\boldsymbol{u}_0 = 0$. *Por uma* ***solução forte*** *do problema* (1), *entendemos funções*

$$\boldsymbol{u},\, \boldsymbol{w} \in L^\infty\left(0,T;\boldsymbol{H}^1(\mathbb{R}^3)\right) \cap L^2\left(0,T;\boldsymbol{H}^2(\mathbb{R}^3)\right),$$

com $(\boldsymbol{u},\boldsymbol{w})$ *satisfazendo as equações* $(1)_1$, $(1)_2$, $(1)_3$ *q.s. em* $\mathbb{R}^3 \times (0,T)$, *e as condições iniciais* $(1)_4$ *em* $\boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3)$.

Teorema 2.1. *Assuma que as velocidades iniciais* $(\boldsymbol{u}_0, \boldsymbol{w}_0)$ *satisfazem*

$$(\boldsymbol{u}_0, \boldsymbol{w}_0) \in \boldsymbol{H}^1(\mathbb{R}^3) \times \boldsymbol{H}^1(\mathbb{R}^3), \quad \operatorname{div}\boldsymbol{u}_0 = 0.$$

Então, existe uma constante $\varepsilon_0 > 0$*, independente de* $\boldsymbol{u}_0$ *e* $\boldsymbol{w}_0$ *tal que se*

$$\left(\|\boldsymbol{u}_0\|^2 + \|\boldsymbol{w}_0\|^2\right)\left(\|\nabla \boldsymbol{u}_0\|^2 + \|\nabla \boldsymbol{w}_0\|^2\right) \leq \varepsilon_0,$$

o problema de Cauchy (1) *possui uma única solução global forte.*

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HIERARCHICAL CONTROL FOR THE ONE-DIMENSIONAL PLATE EQUATION WITH A MOVING BOUNDARY

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Abstract

In this work we investigate the controllability for the one-dimensional plate equation in intervals with a moving boundary. This equation models the vertical displacement of a point x at time t in a bar with uniform cross section. We assume the ends of the bar with small and uniform variations. More precisely, we have introduced functions $\alpha(t)$ and $\beta(t)$ modeling the motion of these ends.

1 Introduction

As in [1], let $\alpha : [0, \infty) \to \mathbb{R}$ and $\beta : [0, \infty) \to \mathbb{R}$ be two functions satisfying the following conditions:

(H1) $\alpha,\, \beta \in C^3([0, \infty); \mathbb{R})$ with $\alpha',\, \alpha'',\, \beta',\, \beta'' \in L^1(0, \infty)$.

(H2) $\alpha(t) < \beta(t)$ for all $t \geq 0$ and $0 < \gamma_0 = inf_{t\geq 0}\gamma(t)$, where $\gamma(t) = \beta(t) - \alpha(t)$.

Given $T > 0$, we consider the non-cylindrical domain defined by

$$\widehat{Q} = \left\{(x, t) \in \mathbb{R}^2;\ \alpha(t) < x < \beta(t),\ \forall\, t \in (0, T)\right\}.$$

Its lateral boundary is defined by $\widehat{\Sigma} = \widehat{\Sigma}_0 \cup \widehat{\Sigma}_0^*$, where

$$\widehat{\Sigma}_0 = \{(\alpha(t), t);\ \forall t \in (0, T)\} \quad \text{and} \quad \widehat{\Sigma}_0^* = \widehat{\Sigma}\backslash\widehat{\Sigma}_0 = \{(\beta(t), t);\ \forall t \in (0, T)\}.$$

We also represent by Ω_t and Ω_0 the intervals $(\alpha(t), \beta(t))$ and (α_0, β_0), respectively.

Thus we consider the mixed problem

$$\left|\begin{array}{l} u'' + u_{xxxx} = 0 \quad \text{in} \quad \widehat{Q}, \\ u(x,t) = 0 \quad \text{on} \quad \widehat{\Sigma}, \\ u_x(x,t) = \begin{cases} \widetilde{w} & \text{on} \quad \widehat{\Sigma}_0, \\ 0 & \text{on} \quad \widehat{\Sigma}_0^*, \end{cases} \\ u(x,0) = u_0(x), \quad u'(x,0) = u_1(x) \quad \text{in} \quad \Omega_0, \end{array}\right. \tag{1}$$

where u is the state variable, $\widetilde{w}$ is the control variable and $(u_0(x), u_1(x)) \in L^2(\Omega_0) \times H^{-2}(\Omega_0)$. By $u' = u'(x,t)$ we represent the derivative $\dfrac{\partial u}{\partial t}$ and by $u_{xxxx} = u_{xxxx}(x,t)$ the fourth order partial derivative $\frac{\partial^4 u}{\partial x^4}$.

The approach proposed consists in a suitable change of variables transforming the system (1) into an equivalent system written over a fixed domain, i.e.,

$$v'' + L(y,t)v = 0, \quad (y,t) \in Q, \tag{2}$$

for $Q = (0,1) \times (0,T)$, where $L = L(y,t)$ is a variable-coefficient operator.

In contrast to [2], the main difficulty in the present work is that we can not apply Holmgren's Theorem because the variable coefficients are not necessarily analytic.

2 Main Results

Associated with the solution $u = u(x,t)$ of (1), we will consider the (secondary) functional

$$\widetilde{J}_2(\widetilde{w}_1, \widetilde{w}_2) = \frac{1}{2}\int\int_{\widehat{Q}} (u(\widetilde{w}_1, \widetilde{w}_2) - \widetilde{u}_2)^2 \, dxdt + \frac{\widetilde{\sigma}}{2}\int_{\widehat{\Sigma}_2} \widetilde{w}_2^2 \, d\widehat{\Sigma}, \tag{1}$$

and the (main) functional

$$\widetilde{J}(\widetilde{w}_1) = \frac{1}{2}\int_{\widehat{\Sigma}_1} \widetilde{w}_1^2 \, d\widehat{\Sigma}, \tag{2}$$

where $\widetilde{\sigma} > 0$ is a constant and $\widetilde{u}_2$ is a given function in $L^2(\widehat{Q})$.

The control problem that we will consider is as follows: the follower $\widetilde{w}_2$ assumes that the leader $\widetilde{w}_1$ has made a choice. Then, it tries to find an equilibrium of the cost $\widetilde{J}_2$, that is, it looks for a control $\widetilde{w}_2 = \mathfrak{F}(\widetilde{w}_1)$ (depending on $\widetilde{w}_1$), satisfying:

$$\widetilde{J}_2(\widetilde{w}_1, \widetilde{w}_2) = \inf_{\widehat{w}_2 \in L^2(\widehat{\Sigma}_2)} \widetilde{J}_2(\widetilde{w}_1, \widehat{w}_2). \tag{3}$$

This process is called Stackelberg-Nash strategy; see Díaz and Lions [3].

As in [1], we assume that

$$T > T_0, \tag{4}$$

where T_0 is given in [4].

Theorem 2.1. *Assume that $T > T_0$. Let us consider $w_1 \in L^2(\Sigma_1)$ and w_2 a Nash equilibrium in the sense (3). Then $(v(T), v'(T)) = (v(.,T,w_1,w_2), v'(.,T,w_1,w_2))$, where v solves (2), generates a dense subset of $L^2(0,1) \times H^{-2}(0,1)$.*

Proof To prove theorem, we use Inverse Inequality (cf. [4]). ∎

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WAVE MODELS WITH TIME-DEPENDENT POTENTIAL AND SPEED OF PROPAGATION

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Abstract

This is a joint work with Prof. Marcelo R. Ebert accepted for publication in the journal Differential and Integral Equations. We study the long time behavior of energy solutions for a class of wave equation with time-dependent potential and speed of propagation. We introduce a classification of the potential term, which clarifies whether the solution behaves like the solution to the wave equation or Klein-Gordon equation. Moreover, the derived linear estimates are applied to obtain global (in time) small data energy solutions for the Cauchy problem to semilinear Klein-Gordon models with power nonlinearity.

1 Introduction

Let us consider the Cauchy problem for the wave equation with time-dependent potential and speed of propagation

$$\begin{cases} u_{tt} - a(t)^2\Delta u + m(t)^2 u = 0, \quad (t,x) \in (0,\infty)\times\mathbb{R}^n, \\ (u(0,x), u_t(0,x)) = (u_0(x), u_1(x)), \quad x \in \mathbb{R}^n. \end{cases} \tag{1}$$

The Klein-Gordon type energy for the solution to (1) is given by

$$E_{a,m}(t) \doteq \frac{1}{2}\big(\|u_t(t,\cdot)\|_{L^2}^2 + a(t)^2\|\nabla_x u(t,\cdot)\|_{L^2}^2 + m(t)^2\|u(t,\cdot)\|_{L^2}^2\big). \tag{2}$$

One can observe many different effects for the behavior of $E_{a,m}(t)$ as $t\to\infty$ according to the properties of the speed of propagation $a(t)$ and the coefficient $m(t)$ in the potential term.

We first discuss properties of the energy in the case $m(t)\equiv 0$ in (1). If $0 < a_0 \le a(t) \le a_1$ for any $t \ge 0$ with a suitable control of the oscillations it is possible to prove that $E_{a,0}(t)$ has the so-called *generalized energy conservation* property (see [3]). In [2] the authors proved energy estimates considering $a(t) \ge a_0 > 0$ an increasing function also satisfying suitable control on the oscillations.

In the case $a(t)\equiv 1$, $E_{1,m}(t)$ is a conserved quantity for the classical Klein-Gordon equation, whereas it is known that the behavior of the potential energy $\|u(t,\cdot)\|_{L^2}$ changes accordingly to the cases $\lim_{t\to\infty} tm(t) = \infty$ or $\lim_{t\to\infty} tm(t) = 0$. To explain this effect, let us consider the energy

$$E_p(u)(t) \doteq \frac{1}{2}\Big(\|u_t(t,\cdot)\|_{L^2}^2 + \|\nabla_x u(t,\cdot)\|_{L^2}^2 + p(t)^2\|u(t,\cdot)\|_{L^2}^2\Big).$$

In the PhD thesis [1], the author studied decreasing coefficients $m = m(t)$ which satisfy among other things $\lim_{t\to\infty} tm(t) = \infty$. In this case the potentials are called *effective*, i.e., the decay of solutions and its derivatives is related to the decay of solutions of the classical Klein-Gordon equation measured in the L^q norm. Under some additional condition on m, was proved that $E_p(u)(t) \le CE_p(u)(0)$, with $p(t)^2 = m(t)$. In [1], the authors also derived the energy estimate $E_p(u)(t) \le CE_p(u)(0)$, for scale invariant models $m(t) = \frac{\mu}{1+t}, \mu > 0$, but now the constant μ has an influence on the function $p(t)$.

In [5, 4] the authors explained qualitative properties of solutions to (1) in the case $a\equiv 1$ and $\lim_{t\to\infty} tm(t) = 0$. Under a suitable control on the oscillations of m, if $(1+t)m(t)^2 \in L^1(\mathbb{R}^+)$, it was proved a scattering result to free wave equation, whereas the potentials are called *non-effective* if $(1+t)m(t)^2 \notin L^1(\mathbb{R}^+)$ and $\limsup_{t\to\infty}(1+t)\int_t^\infty m(s)^2 ds < \frac{1}{4}$. In the case of *non-effective* potentials, the decay of the solutions and its derivatives is related to the decay of solutions to the free wave equation measured in the L^q norm.

In this work we introduced a classification for the potentials in (1) in terms of the time-dependent speed of propagation $a(t) \notin L^1$. In the case of *effective* and *non-effective* potentials we derive sharp energy estimates. As an application to our derived linear estimates, we proved global existence (in time) of small data energy solutions, in the case of effective potentials, for semilinear models with power nonlinearity associated to (1).

2 Main Results

Let $a \in \mathcal{C}^2[0,\infty)$ be a strictly positive function, such that $a \notin L^1$. We define

$$A(t) \doteq 1 + \int_0^t a(\tau)d\tau, \quad \eta(t) \doteq \frac{a(t)}{A(t)}, \quad m(t) = \mu(t)\eta(t) > 0.$$

Theorem 2.1. *If $a(t)$ and $\mu(t)$ satisfy suitables oscillations conditions, then*

1. *The potential term $m(t)^2u$ generates* scattering *to the corresponding wave model if $\mu^2\eta \in L^1([0,\infty))$.*

2. *The potential term $m(t)^2u$ represents a* non-effective potential *if $\mu^2\eta \notin L^1$ and*

$$\limsup_{t\to\infty} A(t)\left\{\int_t^\infty \frac{\mu(s)^2 a(s)}{A(s)^2}ds + \frac{a'(t)}{2a(t)^2} - \frac{1}{4}\int_t^\infty \frac{[a'(s)]^2}{a(s)^3}ds\right\} < \frac{1}{4}.$$

3. *The potential term $m(t)^2u$ generates an* effective potential *if $\lim_{t\to\infty}\mu(t) = \infty$.*

Theorem 2.2. *Suppose that potential term $m(t)^2u$ generates an effective potential. If $\frac{\eta}{\mu} \in L^1[0,\infty)$ and $1 < p \le \frac{n}{[n-2]_+}$ such that $\int_0^t a(s)^{-\frac{1-k}{2}} m(s)^{-\frac{p+k}{2}} \left(\frac{m(s)}{a(s)}\right)^{\frac{n(p-1)}{4}} ds < \infty, k = 0,1,$ then there exists a constant $\epsilon > 0$ such that for all $(u_0,u_1) \in H^1(\mathbb{R}^n)\cap L^2(\mathbb{R}^n)$ with $||(u_0,u_1)||_{H^1\cap L^2} \le \epsilon$ there exists a uniquely determined energy solution $u \in C([0,\infty),H^1(\mathbb{R}^n))\cap C^1([0,\infty),L^2(\mathbb{R}^n))$ to the semilinear model with power nonlinearity associated to* (1) *.*

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ON A NONLINEAR ELASTICITY SYSTEM

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Abstract

This article refers to the existence of weak solution for a nonlinear elastic system with coefficients depending on the time and damping boundary conditiond.

1 Introduction

Let Ω be an open bounded of $\mathbb{R}^n$ with boundary of class C^2 and $T > 0$ be a real number. Assume that Γ is constituted by two nonempty disjoint closed sets Γ_0 and Γ_1. Denote by $\nu(x)$ the outward unit normal vector at $x \in \Gamma_1$. Consider the mixed problem:

$$\begin{aligned}
&u''(x,t) - \mu b(t)\triangle u(x,t) - (\lambda+\mu)b(t)\text{div}u(x,t) + h(u(x,t) = 0 \ in \ \Omega\times]0,\infty[,\\
&u = 0 \ in \ \Gamma_0\times]0,\infty[,\\
&\mu b(t)\frac{\partial u}{\partial \nu}(x,t) + (\lambda+\mu)b(t)\nu(x)\text{div}u(x,t) + \delta(x)u'(x,t) = 0 \ on \ \Gamma_1\times]0,\infty[,\\
&u(x,0) = u^0(x),\ u'(x,0) = u^1(x) \ in \ \Omega,
\end{aligned} \tag{1}$$

where $u = (u_1, \ldots, u_n)$ is a vectorial function; $b(t)$ a real function; $\lambda \geq 0$ and $\mu > 0$, the Lamé coefficients; $h(x)$, a continuous function defined on $\mathbb{R}^n$; and $\delta(x)$ a function defined on Γ_1.

2 Main Results

Let us represented by $H^1_{\Gamma_0}(\Omega)$ the Hilbert space

$$H^1_{\Gamma_0}(\Omega) = \{v \in H^1(\Omega) : v = 0 \text{ on } \Gamma_0\},$$

equipped with the scalar product

$$((u,v))_{H^1_{\Gamma_0}(\Omega)} = \sum_{i=1}^{n}\int_\Omega \frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}\ dx$$

and norm $\|u\|_{H^1_{\Gamma_0}(\Omega)} = ((u,u))^{1/2}_{H^1_{\Gamma_0}(\Omega)}$. Introduce the Hilbert spaces

$$H = (L^2(\Omega))^n,\ (u,v)_H = \sum_{i=1}^{n}(u_i,v_i),\ \forall u,v \in H$$

and

$$V = (H^1_{\Gamma_0}(\Omega))^n,\ ((u,v))_V = \sum_{i=1}^{n}((u_i,v_i))_{H^1_{\Gamma_0}(\Omega)},\ \forall u,v \in V.$$

Consider the following hypotheses

(**H1**) $b \in W^{1,\infty}_{loc}(0,+\infty),\ b(t) \geq b_0 > 0,\ \forall t \in [0,\infty)$

(**H2**) $W^{1,\infty}(\Gamma_1),\ \delta(x) \geq \delta_0 > 0,\ \forall x \in \Gamma_1$;
(**H3**) $h : \mathbb{R}^n \to \mathbb{R}^n$ where $h(x_1, \dots, x_n) = (h_1(x_1), h_2(x_2) \dots, h_n(x_n))$ with $h_i : \mathbb{R} \to \mathbb{R}$ lipschtizian functions, $h_i(s)s \geq s,\ \forall s \in \mathbb{R},\ i = 1, 2, \dots, n.$

Theorem 2.1. *Assume that hypotheses $(H1) - (H3)$ are satisfied and that $u^0 \in V \cap H$ and $u^1 \in V$. Then there exists a unique function u in the class*

$$u \in L^\infty(0,T; V \cap H),\ u' \in L^\infty(0,T; V),\ u'' \in L^\infty(0,T; H)$$

such that u satisfies

$$u'' - \mu b \triangle u - (\lambda + \mu) b div u + h(u) = 0 \ in\ L^\infty(0,T; H);$$

$$\mu b \frac{\partial u}{\partial \nu} + (\lambda + \mu) b \nu div\, u + \delta u' = 0 \ in\ L^\infty(0,T; (H^{\frac{1}{2}}(\Gamma_1))^n);$$

$$u(0) = u^0,\ u'(0) = u^1 \ in\ \Omega.$$

Proof To prove the theorem above, we use the Galerkin method with a special basis, the compactness method, and the results on trace of nonsmooth functions ■

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AN IMPROVEMENT IN KAHANE–SALEM–ZYGMUND'S MULTILINEAR INEQUALITY AND APPLICATIONS

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Abstract

We improve the supremum norm upper estimate in Kahane–Salem–Zygmund's inequality for multilinear forms: given positive integers $d, n_1, \ldots, n_d \geq 1$ and $(p_1, \ldots, p_d) \in [1, +\infty]^d$, there exists a d-linear complex or real valued map $A : \ell_{p_1}^{n_1} \times \cdots \times \ell_{p_d}^{n_d} \to \mathbb{K}$ with sup norm

$$\|A\| \leq B_d \cdot \left(\sum_{k=1}^{d} n_k \right)^{1-\frac{1}{\gamma}} \cdot \prod_{k=1}^{d} n_k^{\max\left(\frac{1}{\gamma} - \frac{1}{p_k}, 0\right)},$$

where $\gamma := \min\{2, \max\{p_k : p_k \leq 2\}\}$ and $B_d > 0$ is a positive constant. Applications involving the multilinear Hardy–Littlewood inequality are presented.

1 Introduction

Paraphrasing H. Boas [3], the main purpose of Kahane–Salem–Zygmund's inequality is to construct a homogeneous polynomial on ℓ_p^n (or a d-linear form on $(\ell_p^n)^d$) with a relatively small supremum norm but relatively large majorant function. Boas' original goal was to quantify the rate at which the Bohr radius decays as the dimension n increases. The Kahane–Salem–Zygmung inequality is nowadays a fundamental tool in modern analysis with a broad range of applications (see [1, 4, 5]).

The main result we prove is an improved version of the multilinear Kahane–Salem–Zygmund inequality on the space $\ell_{p_1}^{n_1} \times \cdots \times \ell_{p_d}^{n_d}$ and with sup norm refined when dealing with some p_k between 1 and 2. Applications concerning Hardy–Littlewood's inequality are provided.

Recall that ℓ_p^n stands for the n-dimensional scalar fields $\mathbb{K}^n$ of real or complex numbers with the ℓ_p-norm, $p \in [1, +\infty]$. For the sake of clarity we fix some useful notation: for $p_1, \ldots, p_d \in (0, +\infty)$, we define $\mathbf{p} := (p_1, \ldots, p_d)$, $\left|\frac{1}{\mathbf{p}}\right| := \frac{1}{p_1} + \cdots + \frac{1}{p_d}$. The cardinal of a set $\mathcal{I}$ is denoted by $\operatorname{card} \mathcal{I}$. Throughout this, X_p stands for ℓ_p if $1 \leq p < \infty$ and $X_\infty := c_0$. The symbol $e_j^{n_j}$ stands for $(e_j, \overset{n_j times}{\ldots}, e_j)$, with $e_j \in X_p$ the j-th canonical vector. Also we use the usual multi-index notation $\mathbf{j} := (j_1, \ldots, j_d) \in \mathbb{N}^d$ and q' denotes the conjugate of $q \in [1, +\infty]$, i.e., $\frac{1}{q} + \frac{1}{q'} = 1$.

2 Main Results

The main result is presented in next (see [1, Theorem 3.1]). We borrow ideas from [2, 3].

Theorem 2.1. *Let $d, n_1, \ldots, n_d \geq 1$ be positive integers and $p_1, \ldots, p_d \in [1, +\infty]$. Then there exist signs $\varepsilon_{\mathbf{j}} = \pm 1$ and a d-linear map $A : \ell_{p_1}^{n_1} \times \cdots \times \ell_{p_d}^{n_d} \to \mathbb{K}$ of the form $A\left(z^1, \ldots, z^d\right) = \sum_{j_1=1}^{n_1} \cdots \sum_{j_d=1}^{n_d} \varepsilon_{\mathbf{j}} z_{j_1}^1 \cdots z_{j_d}^d$, such that*

$$\|A\| \leq (C_d)^{2\left(1-\frac{1}{\gamma}\right)} \cdot \left(\sum_{k=1}^{d} n_k \right)^{1-\frac{1}{\gamma}} \cdot \prod_{k=1}^{d} n_k^{\max\left(\frac{1}{\gamma} - \frac{1}{p_k}, 0\right)},$$

with $\gamma := \min\{2, \max\{p_k : p_k \leq 2\}\}$ and $C_d := 2(d!)^{1-\max\left(\frac{1}{2}, \frac{1}{p}\right)} \sqrt{16 \log(1 + 4d)}$.

Kahane–Salem–Zygmund's inequality is a great tool to gain efficient exponents in Hardy–Littlewood's inequality. The next results are applications in this vein. The fact that the multilinear form provided in Theorem 2.1 is defined on $\ell_{p_1}^{n_1} \times \cdots \times \ell_{p_d}^{n_d}$ for arbitrary finite dimensions $n_1, \cdots, n_d$ has a crucial role.

Theorem 2.2. *Let $1 \leq k \leq d$ and $m_1, \ldots, m_k$ be positive integers such that $m_1 + \cdots + m_k = d$. Also let $\mathbf{p} := \left(\mathbf{p}^1, \ldots, \mathbf{p}^k\right) \in [1, +\infty]^d$, $\quad \mathbf{p}^j := \left(p_1^j, \ldots, p_{m_j}^j\right) \in (1, +\infty]^{m_j}$, with $j = 1, \ldots, k$ and $\rho := (\rho_1, \ldots, \rho_k) \in (0, +\infty)^k$. If there is a constant $C_{k,\boldsymbol{\rho},\mathbf{p}}^{\mathbb{K}} \geq 1$ such that*

$$\left(\sum_{j_1=1}^{+\infty}\left(\cdots\left(\sum_{j_k=1}^{+\infty}\left|T\left(e_{j_1}^{m_1}, \ldots, e_{j_k}^{m_k}\right)\right|^{\rho_k}\right)^{\frac{\rho_{k-1}}{\rho_k}} \cdots\right)^{\frac{\rho_1}{\rho_2}}\right)^{\frac{1}{\rho_1}} \leq C_{k,\boldsymbol{\rho},\mathbf{p}}^{\mathbb{K}}\|T\|, \tag{1}$$

for all d-linear forms $T : X_{p_1^1} \times \cdots \times X_{p_{m_1}^1} \times \cdots \times X_{p_1^k} \times \cdots X_{p_{m_k}^k} \to \mathbb{K}$. Then

$$\sum_{j \in \mathcal{I}} \frac{1}{\rho_j} \leq \frac{card\mathcal{I} + 1}{2} - \sum_{j \in \mathcal{I}}\left|\frac{1}{\mathbf{p}^j}\right|,$$

for all $\mathcal{I} \subset \{1, \ldots, k\}$.

Theorem 2.3. *Let $d \geq 2$ be an integer, $\mathbf{p} := (p_1, \ldots, p_d) \in [1, +\infty]^d$ be such that $\left|\frac{1}{\mathbf{p}}\right| \leq \frac{1}{2}$ and also let $s_k, \rho_k \in (0, \infty)$, for $k = 1, \ldots, d$. Suppose there exists $D_{d,\boldsymbol{\rho},\mathbf{p}}^{\mathbb{K}} \geq 1$ such that*

$$\left(\sum_{j_1=1}^{n_1}\left(\cdots\left(\sum_{j_d=1}^{n_d}\left|T\left(e_{j_1}, \ldots, e_{j_d}\right)\right|^{\rho_d}\right)^{\frac{\rho_{d-1}}{\rho_d}} \cdots\right)^{\frac{\rho_1}{\rho_2}}\right)^{\frac{1}{\rho_1}} \leq D_{d,\boldsymbol{\rho},\mathbf{p}}^{\mathbb{K}} n_1^{s_1} \cdots n_d^{s_d}\|T\|,$$

for all bounded d-linear operators $T : \ell_{p_1}^{n_1} \times \cdots \times \ell_{p_d}^{n_d} \to \mathbb{K}$ and any positive integers $n_1, \ldots, n_d$. Then for all $\mathcal{I} \subset \{1, \ldots, d\}$,

$$\sum_{j \in \mathcal{I}} s_j \geq \max\left\{0, \sum_{j \in \mathcal{I}} \frac{1}{\rho_j} - \frac{card\mathcal{I} + 1}{2} + \sum_{j \in \mathcal{I}} \frac{1}{p_j}\right\}.$$

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CONFORMAL MEASURES ON GENERALIZED RENAULT-DEACONU GROUPOIDS

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Abstract

Countable Markov shifts, which we denote by Σ_A for a 0-1 infinite matrix A, are central objects in symbolic dynamics and ergodic theory. The corresponding operator algebras have been introduced by M. Laca and R. Exel as a generalization of the Cuntz-Krieger algebras for the case of an infinite and countable alphabet. By a result of J. Renault, this generalization may be realized as the C*-algebra of the Renault-Deaconu groupoid for a partially defined shift map σ defined on a locally compact set X_A which is a spectrum of a certain C*-algebra. This set X_A contains Σ_A as a dense subset. We introduced the notion of conformal measures in X_A and, inspired by the thermodynamic formalism for renewal shifts on classical countable Markov shifts, we show that there exists a potential f depending on the first coordinate which presents phase transition, in other words, we have existence and absence of conformal measures μ_β for βf for different values of β. These conformal measures when do exist for some β, satisfy $\mu_\beta(\Sigma_A) = 0$. As a consequence, we have shown the existence of conformal probability measures which are not detected by the classical thermodynamic formalism when the matrix A is not row-finite.

1 Introduction

It is well known that Cuntz-Krieger algebras [2] are the corresponding C*-algebras to Markov shifts when the alphabet is finite and, when the alphabet is infinite but countable, that is, countable Markov Shifts, the algebra associated was introduced by R. Exel and M. Laca in [1]. These both algebras we denote by $\mathcal{O}_A$.

There are some clear connections between the world of the Markov shifts and the operator algebras at the level of the thermodynamic formalism. For example, depending on the potential, there exist a bijection between the conformal measures, in Σ_A and the KMS states in the correspondent algebra $\mathcal{O}_A$. This bijection can be established in both compact and non-compact cases when the potential has suitable properties [4]. But this bijection is, in some sense, one exception, since concrete results between countable Markov shifts [5] and the algebras defined by Exel and Laca are rare. Both theories are growing essentially independently, and the goal of this first paper is to start the measure-theoretical study on the Exel-Laca algebras and then to develop the thermodynamic formalism which naturally emerges from this algebraic setting.

The paper [1] has a significant influence on the community of C*-algebras. However, results exploring the fact that this algebra comes from a matrix A which give us the non-compact shift space Σ_A where the alphabet is $\mathbb{N}$, are very few. O. Sarig and many others developed in the last two decades a good literature extending the thermodynamic formalism from finite alphabet to the case when the alphabet is the set of natural numbers $\mathbb{N}$, see [2]. They explore the similarities and show some fundamental differences with respect to the compact case.

Exel and Laca [1] considered a commutative sub-C*-algebra $\mathcal{D}_A \subseteq \mathcal{O}_A$ and his spectrum X_A, which is a locally compact space where we can identify $\Sigma_A \subseteq X_A$. The set X_A is our primary object. We have that Σ_A and its complement are Borel and dense subsets of X_A. Then, any (conformal or not) probability measure obtained by the thermodynamic formalism on Σ_A generates a probability measure on X_A. Besides, since X_A is locally compact, we can use the true duality between functions and measures via the Riesz representation theorem and not the weak notion of dual operators used on countable Markov shifts [2]. Depending on properties of the matrix A both spaces X_A and Σ_A coincide, for row-finite matrices, for example. In this case Σ_A is locally compact. This fact indicates that X_A can be realized as a locally compact representant of the symbolic space Σ_A. So, it is natural to study the thermodynamic formalism on the space X_A, which contains the standard thermodynamic formalism of Σ_A. After this, the natural question is:

Does exist some conformal probability measure μ which *lives* on $Y_A = X_A \backslash \Sigma_A$, in other words, a conformal probability measure such that $\mu(\Sigma_A) = 0$?

The existence of such measure leads us to conclude that there exist thermodynamic quantities associated to the dynamic structure given by the matrix A, which are not detected by the theory developed on the space Σ_A. Now, with the advantage that we work in a locally compact space and with dual operators in a more strict sense of Analysis than the approach used by Sarig on Countable Markov shifts.

On this paper we gave the first step showing that this direction can be fruitful and we consider a particular Renewal shift [8] and its associated space X_A. We show that we can see even phase transitions on the set of probability measures which vanishes on Σ_A.

2 Main Results

Theorem: Let $f \equiv 1$ be the constant potential. Then, for $\beta_c = \log 2$, we prove the following:

For $\beta > \beta_c$ we have a unique e^{β}-conformal probability measure that vanishes on Σ_A.

For $\beta \leq \beta_c$ there is no e^{β}- conformal probability measure that vanishes on Σ_A.

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ON THE DUAL OF A SEQUENCE CLASS

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Abstract

In a work of 2017 we introduce an abstract environment, based on the concept of sequence classes, that characterizes operator ideals determined by transformations of vector-valued sequences. In this paper we advance in this subject defining a dual of a sequence class, providing a necessary environment and proving some distinguish related results.

1 Introduction

Classes of operators that improve convergence of vector-valued series, as the class of the absolutely summing operators (see [2]), are broadly studied in the last decades. These classes can be characterized by the transformation of vector-valued sequences belonging known sequence spaces and can be studied from the point of view of the Theory of Operator Ideals [2]. A usual approach, proving all the desired properties for the studied classes using the definitions of the underlying sequence spaces, would lead to long and boring proofs.

In the work [1] of 2017 we synthesize the study of these Banach operator ideals and multi-ideals by introducing an abstract framework that generalizes ideals characterized by means of transformation of vector-valued sequences and accommodates the already studied ideals as particular instances. This environment is based in the new concept of sequence classes.

In the current paper our goal is to enrich this abstract approach providing a new sequence class related object that somehow characterizes its dual.

The letters E, F shall denote Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. We use $x \cdot e_j$ to denote the sequence $(0, \ldots, 0, x, 0, 0, \ldots)$, with x in the j-th coordinate. The symbol $E \overset{1}{\hookrightarrow} F$ means that E is a linear subspace of F and $\|x\|_F \leq \|x\|_E$, for every $x \in E$. The theory, definitions and results of sequence classes will be used indistinctly and can be found in paper [1].

2 Main Results

We start presenting a distinguished property that a sequence class can enjoy.

Definition 2.1. A sequence class X is *spherically closed* if, for all $(x_j)_{j=1}^\infty \in X(E)$, we have $(\lambda_j x_j)_{j=1}^\infty \in X(E)$, whenever $(\lambda_j)_{j=1}^\infty \in \mathbb{K}^\mathbb{N}$ with $|\lambda_j| = 1$, for all j, and $\|(\lambda_j x_j)_{j=1}^\infty\|_{X(E)} = \|(x_j)_{j=1}^\infty\|_{X(E)}$.

For a spherically closed sequence class X, the next equivalence of convergence is valid and we will use later to define our dual and its norm.

Lemma 2.1. *Let X be a spherically closed sequence class and $(x_j)_{j=1}^\infty \in E^\mathbb{N}$. Then the following sentences are equivalent:*

(a) *The series $\sum_{j=1}^\infty \varphi_j(x_j)$ converges for all $(\varphi_j)_{j=1}^\infty \in X(E')$.*

(b) *The series $\sum_{j=1}^\infty |\varphi_j(x_j)|$ converges for all $(\varphi_j)_{j=1}^\infty \in X(E')$.*

More than that,

$$\sup_{(\varphi_j)_{j=1}^{\infty}\in B_{X(E')}}\left|\sum_{j=1}^{\infty}\varphi_j(x_j)\right| = \sup_{(\varphi_j)_{j=1}^{\infty}\in B_{X(E')}}\sum_{j=1}^{\infty}|\varphi_j(x_j)|.$$

Let us define a dual of a given sequence class X.

Definition 2.2. A *dual* of a sequence class X is a rule that assigns to each space $E \in BAN$ the E-valued sequence space

$$X^{\mathrm{d}}(E) = \left\{ (x_j)_{j=1}^{\infty} \text{ in } E : \sum_{j=1}^{\infty}\varphi_j(x_j) \text{ converges, } \forall\ (\varphi_j)_{j=1}^{\infty} \text{ in } X(E') \right\}.$$

It is immediate to verify that the above definition, in fact, characterizes a linear sequence space with the coordinatewise operations and that $c_{00}(E) \subseteq X^{\mathrm{d}}(E)$, for all Banach space E.

Here and henceforth, we assume that the sequence class X has the property: for every Banach space E and every $x \in E$, we have $\|x \cdot e_j\|_{X(E)} = \|x\|_E$, for all $j \in \mathbb{N}$. With this, a complete norm for $X^{\mathrm{d}}(E)$ is given by the next

Proposition 2.1. *If X is a spherically closed sequence class, then the expression*

$$\|(x_j)_{j=1}^{\infty}\|_{X^{\mathrm{d}}(E)} := \sup_{(\varphi_j)_{j=1}^{\infty}\in B_{X(E')}}\sum_{j=1}^{\infty}|\varphi_j(x_j)|$$

defines a complete norm on $X^{\mathrm{d}}(E)$ and $X^{\mathrm{d}}(E) \overset{1}{\hookrightarrow} \ell_{\infty}(E)$, for all Banach space E.

With the preceding definitions and results we can assert that X^{d} is a sequence class and the following proposition states more properties enjoyed by the sequence class X^{d}.

Proposition 2.2. *Let X be a spherically closed sequence class. Then X^{d} is finitely determined and spherically closed sequence class. Moreover, if X is linearly stable, then so is X^{d}.*

One of our main results, that justify the used terminology, is presented in the next theorem.

Theorem 2.1. *Let X be a finitely determined, linearly stable and spherically closed sequence class. Then there is an isometric isomorphism between $X^{\mathrm{d}}(E')$ and $(X(E))'$, for all Banach space E.*

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APPROXIMATION PROPERTY AND ERGODICITY OF BANACH SPACES

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Abstract

We obtain a criterion for ergodicity of Banach spaces based on a construction of spaces without approximation property. We prove that a non ergodic Banach space must be near Hilbert. This reinforces the conjecture that ℓ_2 is the only non ergodic Banach space. As an application of our criterion, we prove that there is no separable Banach space which is complementably universal for the class of all subspaces of ℓ_p, for $1 \leq p < 2$. This solves a question left open by W. B. Johnson and A. Szankowski in 1976.

1 Introduction

The solution of Gowers [3] and Komorowski–Tomczak-Jaegermann [4] to the homogeneous Banach space problem, provides that every Banach space having only one equivalence class for the relation of isomorphism between its infinite dimensional subspaces must be isomorphic to ℓ_2. G. Godefroy formulated the question about the number of non isomorphic subspaces of a Banach space X not isomorphic to ℓ_2. This question was studied, in the context of descriptive set theory, by V. Ferenczi and C. Rosendal [2] who introduced the notion of *ergodic Banach space* to study the classification of the relative complexity of the isomorphism relation between the subspaces of a separable Banach space.

The central concept to study the complexity of analytic and Borel equivalence relations on Borel standard spaces is *Borel reducibility*.

Definition 1.1. *Let R and S be two Borel equivalence relations on Borel standard spaces X and Y, respectively. One says that R is Borel reducible to S, (denoted by $R \leq_B S$) if there exists a Borel function $\phi : X \to Y$ such that*

$$xRy \iff \phi(x)S\phi(y),$$

for all $x, y \in X$.

The simplest example of a non-smooth equivalence relation (i.e., that is not reducible to $\mathrm{id}(\mathbb{R})$) is the relation of eventual agreement E_0 on $2^{\mathbb{N}}$: for $x, y \in 2^{\mathbb{N}}$,

$$xE_0y \iff (\exists N \in \mathbb{N})(x(n) = y(n),\, n \geq N).$$

Definition 1.2 (Ferenczi-Rosendal)**.** *A separable Banach space X is ergodic if*

$$(2^{\mathbb{N}}, E_0) \leq_B (\mathcal{SB}(X), \simeq).$$

It follows that an ergodic Banach space has at least $2^{\mathbb{N}}$ non-isomorphic subspaces and the equivalence relation of isomorphism between its subspaces is non-smooth. It was conjectured in [2] that every separable Banach space not isomorphic to ℓ_2 must be ergodic.

2 Main Results

A Banach space X has the approximation property (AP) if the identity operator on X can be approximated uniformly on compact subsets of X by linear operators of finite rank. In 1973, Enflo [1] presented the first example of Banach space without the AP and therefore without a Schauder basis. The criterion we introduce to study ergodicity in Banach spaces is based on a criterion introduced by Enflo to prove that a space fails the AP.

We first introduce some notation. For every $n \in \mathbb{N}$, denote by $I_n = \{2^n, 2^n+1, \dots, 2^{n+1}-1\}$. Given a Banach space X and sequences of vectors $(z_{n,\epsilon})_{n\in\mathbb{N}}$ in X, $(z^*_{n,\epsilon})_{n\in\mathbb{N}}$ in X^*, $(\epsilon = 0,1)$, we denote by $Z = \overline{span}\{z_{j,\epsilon} : j \in \mathbb{N},\ \epsilon = 0,1\}$ and we shall consider for every $t \in 2^{\mathbb{N}}$ the closed subspace

$$X_t = \overline{span}\left\{z_{j,t(n)} \,:\, j \in I_n,\ n = 1,2,3,\dots\right\}.$$

If $T : X_t \to Z$ is a bounded and linear operator the n-trace of T is defined as

$$\beta^n_t(T) = 2^{-n} \sum_{j\in I_n} z^*_{j,t(n)} T(z_{j,t(n)}).$$

Definition 2.1. *A Banach space X satisfies the* Cantorized-Enflo criterion *if there exist bounded sequences of vectors $(z_{n,\epsilon})_{n\in\mathbb{N}}$ in X, $(z^*_{n,\epsilon})_{n\in\mathbb{N}}$ in X^* $(\epsilon = 0,1)$ and a sequence of real scalars $(\alpha_n)_n$ such that*

1. *$z^*_{i,\epsilon}(z_{j,\tau}) = \delta_{ij}\delta_{\epsilon\tau}$ for all $i,j \in \mathbb{N}$ and $\epsilon,\tau = 0,1$.*
2. *For every $t,s \in 2^{\mathbb{N}}$ and every operator $T : X_t \to X_s$*

$$\left|\beta^n_t(T) - \beta^{n-1}_t(T)\right| \le \alpha_n \|T\|$$

3. *$\sum_n \alpha_n < \infty$.*

Theorem 2.1. *Every separable Banach space satisfying the Cantorized-Enflo criterion is ergodic.*

Recall that a Banach space is called *near Hilbert* if it has type $2-\epsilon$ and cotype $2+\epsilon$ for every $\epsilon > 0$.

Theorem 2.2. *Every separable Banach space non near Hilbert satisfies the Cantorized-Enflo criterion and therefore is ergodic. Furthermore, the reduction uses subspaces without AP.*

Theorem 2.3. *There is no separable Banach space which is complementably universal for the class of all subspaces of X when X is non near Hilbert.*

These results are part of the work *Non-ergodic Banach spaces are near Hilbert*, to appear in *Trans. of the Amer. Math. Soc.* https://doi.org/10.1090/tran/7319.

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O DUAL TOPOLÓGICO DO ESPAÇO DOS POLINÔMIOS HIPER-(R, P, Q)-NUCLEARES

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Abstract

Neste trabalho caracterizamos os funcionais lineares contínuos no espaço dos polinômios homogêneos hiper-(r, p, q)-nucleares, via transformada de Borel, como operadores lineares quasi-dominados.

1 Introdução

Neste trabalho E e F denotam espaços de Banach e E' o dual topológico de E. $\mathcal{P}(^nE; F)$ denota o espaço vetorial dos polinômios n-homogêneos contínuos de E em F. Quando $F = \mathbb{K}$ denotamos simplesmente $\mathcal{P}(^nE; \mathbb{K}) = \mathcal{P}(^nE)$. Um polinômio n-homogêneo $P \in \mathcal{P}(^nE; F)$ é dito de posto finito se podemos escrever

$$P = \sum_{j=1}^{k} P_j \otimes y_j,$$

onde $P_j \otimes y_j(x) = P_j(x)y_j$, $k \in \mathbb{N}$, $P_j \in \mathcal{P}(^nE)$ e $y_j \in F$ para todos $j = 1, \ldots, k$, e $x \in E$. Denotamos por $\mathcal{P}_{\mathcal{F}}(^nE; F)$ a classe dos polinômios n-homogêneos de posto finito.

Definição 1.1. Seja uma subclasse $\mathcal{P}_\theta$ da classe $\mathcal{P}$ dos polinômios homogêneos contínuos tal que, para todo $n \in \mathbb{N}$ e quaisquer espaços de Banach E e F a componente $\mathcal{P}_\theta(^nE; F) = \mathcal{P}(^nE; F) \cap \mathcal{P}_\theta$, satisfaz as seguintes condições:

(1) $\mathcal{P}_\theta(^nE; F)$ é um subespaço vetorial de $\mathcal{P}(^nE; F)$ e $\mathcal{P}_{\mathcal{F}}(^nE; F) \subseteq \mathcal{P}_\theta(^nE; F)$.

(2) Se existem $0 < s \leq 1$ e uma função $\|\cdot\|_\theta \colon \mathcal{P}_\theta \longrightarrow [0, \infty)$ tais que:

(i) A função $\|\cdot\|_\theta$ restrita a $\mathcal{P}_\theta(^nE; F)$ é uma s-norma para quaisquer espaços de Banach E e F e todo $n \in \mathbb{N}$.

(ii) Para cada $n \in \mathbb{N}$ e espaços de Banach E e F, existe uma constante $C > 0$ tal que $\|P \otimes y\|_\theta \leq C \cdot \|P\| \cdot \|y\|$, para todos $P \in \mathcal{P}(^nE)$ e $y \in F$;

(iii) A inclusão $\iota_\theta \colon (\mathcal{P}_\theta, \|\cdot\|_\theta) \longrightarrow (\mathcal{P}, \|\cdot\|)$ é contínua.

Nesse caso dizemos que a classe $(\mathcal{P}_\theta, \|\cdot\|_\theta)$ é uma classe s-normada de polinômios. Mais ainda, se todas as componentes $\mathcal{P}_\theta(^nE; F)$ são espaços completos relativamente a $\|\cdot\|_\theta$, então dizemos que $(\mathcal{P}_\theta, \|\cdot\|_\theta)$ é um classe s-Banach de polinômios (Banach, quando $s = 1$).

2 Resultados Principais

Começamos com o seguinte resultado:

Proposição 2.1. *Seja $(\mathcal{P}_\theta, \|\cdot\|_\theta)$ uma classe s-normada de polinômios. A transformada de Borel*

$$\beta_\theta \colon (\mathcal{P}_\theta(^nE; F), \|\cdot\|_\theta)' \longrightarrow (\mathcal{L}(\mathcal{P}(^nE), F'), \|\cdot\|)\ ,\ \beta_\theta(\psi)(P)(y) = \psi(P \otimes y),$$

está bem definida e é um opeador linear contínuo.

Sobre a injetividade da transformada de Borel temos o seguinte resultado:

Proposição 2.2. *Seja* $(\mathcal{P}_\theta, \|\cdot\|_\theta)$ *uma classe s-Banach de polinômios.*
(i) Se $\overline{\mathcal{P}_\mathcal{F}}^{\|\cdot\|_\theta} = \mathcal{P}_\theta$, *isto é,* $\mathcal{P}_\mathcal{F}$ *é denso em* $\mathcal{P}_\theta$ *na* θ*-norma, então* β_θ *é injetiva.*
(ii) Se $(\mathcal{P}_\theta, \|\cdot\|_\theta)$ *é uma classe Banach de polinômios, então* β_θ *é injetiva se, e somente se,* $\overline{\mathcal{P}_\mathcal{F}}^{\|\cdot\|_\theta} = \mathcal{P}_\theta$.

Estudaremos a seguinte classe s-Banach de polinômios:

Definição 2.1. *Sejam* $r \in (0,\infty)$ *e* $p,q \in [1,\infty]$ *tais que* $\frac{1}{r} \geq \frac{1}{p} + \frac{1}{q} - 1$. *Um polinômio* $P \in \mathcal{P}(^nE;F)$ *é hiper-*(r,p,q)*-nuclear se existem escalares* $(\lambda_j)_{j=1}^\infty \in \ell_r$, *polinômios* $(P_j)_{j=1}^\infty \in \ell_{p'}^w(\mathcal{P}(^nE))$ *e* $(y_j)_{j=1}^\infty \in \ell_{q'}(F)$ *tais que*

$$P(x) = \sum_{j=1}^{\infty} \lambda_j P_j \otimes y_j(x) = \sum_{j=1}^{\infty} \lambda_j P_j(x) y_j, \tag{1}$$

para todo $x \in E$. *Denotamos o espaço vetorial de tais polinômios por* $\mathcal{P}_{\mathcal{HN}_{(r,p,q)}}(^nE;F)$. *Chamando*

$$\|P\|_{\mathcal{HN}_{(r,p,q)}} = \inf\{\|(\lambda_j)_{j=1}^\infty\|_r \cdot \|(P_j)_{j=1}^\infty\|_{w,p'} \cdot \|(y_j)_{j=1}^\infty\|_{w,q'}\},$$

onde o ínfimo é tomado sobre todas as representações de P *como em (1), temos uma norma em* $\mathcal{P}_{\mathcal{HN}_{(r,p,q)}}(^nE;F)$.

Note que quando $n = 1$ a definição acima recupera o conceito clássico [2, 18.1.1]. Neste caso escrevemos $\mathcal{N}_{(r,p,q)}(E;F)$. No caso $q = 1$ recuperamos os polinômios hiper-(p,q)-nucleares definidos em [2] e denotados por $\mathcal{P}_{\mathcal{HN}_{(p,q)}}(^nE;F)$. Se $r = p = q = 1$, dizemos que o polinômio é hiper nuclear e escrevemos $\mathcal{P}_{\mathcal{HN}}(^nE;F)$.

Proposição 2.3. *Se* $\frac{1}{s} = \frac{1}{r} + \frac{1}{p'} + \frac{1}{q'}$, *então* $\mathcal{P}_{\mathcal{HN}_{(r,p,q)}}(^nE;F)$ *é uma classe s-Banach de polinômios.*

Prova-se que se a transformada de Borel é um isomorfismo sobre sua imagem em $\mathcal{L}(\mathcal{P}(^nE),F')$, então $\mathcal{P}_{\mathcal{HN}}(^nE;F) = \mathcal{P}_\theta(^nE;F)$. Então precisamos descobrir a imagem de $\mathcal{P}_{\mathcal{HN}_{(r,p,q)}}(^nE;F)'$ pela transformada de Borel.

Definição 2.2. Sejam $l,t,s \in (0,\infty]$ tais que $\frac{1}{t} + \frac{1}{s} \geq \frac{1}{l}$. Dizemos que $u \in \mathcal{L}(E;F')$ é um operador *quasi-*(l,s,t)*-dominado* se existe uma constante $C \geq 0$ tal que

$$\|((u(x_j))(y_j))_{j=1}^m\|_l = \|(J_F(y_j)(u(x_j)))_{j=1}^m\|_l \leq C \cdot \|(x_j)_{j=1}^m\|_{w,t} \cdot \|(y_j)_{j=1}^m\|_{w,s}, \tag{2}$$

para todos $x_1,\ldots,x_m \in E$ e $y_1,\ldots,y_m \in F$, $m \in \mathbb{N}$. Neste caso escrevemos $u \in q\mathcal{D}_{(l,t,s)}(E;F')$. Definimos a função $\|\cdot\|_{q\mathcal{D}_{(l,t,s)}} : q\mathcal{D}_{(l,t,s)} \longrightarrow [0,\infty)$, dada por

$$\|u\|_{q\mathcal{D}_{(l,t,s)}} = \inf\{C;\ C \text{ satisfazendo } (2)\},$$

a qual torna $q\mathcal{D}_{(r,s)}(E;F')$ um espaço quasi-Banach, se $0 < l < 1$), e Banach, se $l \geq 1$.

Teorema 2.1. *Se* $\mathcal{P}(^nE)$ *ou* F *tem a propriedade da aproximação* λ*-limitada, então a transformada de Borel*

$$\beta_{\mathcal{P}_{\mathcal{HN}_{(r,p,q)}}} : \left[\mathcal{P}_{\mathcal{HN}_{(r,p,q)}}(^nE;F), \|\cdot\|_{\mathcal{HN}_{(r,p,q)}}\right]' \longrightarrow \left[q\mathcal{D}_{(r',p',q')}(\mathcal{P}(^nE);F'), \|\cdot\|_{q\mathcal{D}_{(r',p',q')}}\right]$$

é um isomorfismo isométrico.

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SPACEABILITY AND RESIDUALITY ON A SUBSET OF BLOCH FUNCTIONS

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Abstract

We show that the set of Bloch functions on the unit disc which are not bounded analytic functions is spaceable, maximal lineable and residual, but is not algebrable.

1 Introduction

In the last two decades there has been a crescent interest in the search of nice algebraic-topological structures within sets (mainly sets of functions or sequences) that do not enjoy themselves such structures. Here, we study algebraic and topological structures in a certain subset of Bloch space. Now we fix the notation.

Let D denote the unit disk in the complex plane, $\mathcal{H}(D)$ be the space of all analytic functions on D and $\mathcal{H}^\infty(D)$ be the subspace of $\mathcal{H}(D)$ of all bounded functions. We define the set

$$\mathcal{B} = \left\{ f \in \mathcal{H}(D) : \sup_{|z|<1} (1-|z|^2)|f^{'}(z)| < \infty. \right\}$$

and the norm $||f||_{\mathcal{B}} = |f(0)| + \sup_{|z|<1}(1-|z|^2)f^{'}(z)|$. The Bloch space is the set $\mathcal{B}$ with the norm $\|\cdot\|_{\mathcal{B}}$. It is well-known that $\mathcal{B}$ is a Banach space under the above norm. Every function f in $\mathcal{B}$ is called Bloch function. A Bloch function f is an analytic function on D whose derivate grows so faster than a constant times the reciprocal of the distance from z to ∂D. We suggest [1] to see the basic idea of Bloch functions. During the period from 1925 through 1968 Bloch$^{'}$s result motivated works of various nature.

We call by $\mathcal{F} = \mathcal{B} \setminus \mathcal{H}^\infty(D)$. In our work we are interested to see, in a linear/algebraic sense, if these diferences are big or not. In this direction, our aim in this note is to establish some structure in the set $\mathcal{F}$. Indeed, we show that $\mathcal{F}$ is spaceable, maximal lineable and residual, but is not algebrable. Research on the theme of describing spaceability, algebrability and residuality has been carried on in recent years. We refer to [2, 1] for a background about these concepts and a good history of the publication on the theme.

2 Main Results

The Bloch space is the largest possible space of holomorphic functions whose (semi-)norm is invariant under the action of the automorphism group. The definition of Bloch space can be generalized to higher dimensions in several possible ways. However, the definition in higher dimension is not invariant under the action of the automorphism group. Here, we are interested only in the classical Bloch space. In classical geometric function theory of the open unit disk D in the complex plane $\mathbb{C}$, the Bloch space is a central object of study and several outstanding problems remain unresolved. The examples of Bloch functions are the set of polynomials and also the bounded analytic functions.

If Y is a topological vector space, a subset A of Y is called: **lineable** if $A \cup \{0\}$ contains an infinite dimensional vector space; **spaceable** if $A \cup \{0\}$ contains a closed infinite dimensional vector space; **maximal lineable** if $A \cup \{0\}$ contains a vector subspace S of Y with $dim(S) = dim(Y)$. If Y is a function algebra, $A \subset Y$ is said to be:

algebrable if there is an algebra $\mathcal{B} \subset A \cup \{0\}$, such that $\mathcal{B}$ has an infinite minimal system of generators. If Y is a Fréchet space, a set $A \subset Y$ is called **residual in** Y if $Y \setminus A = \cup_{n=1}^{\infty} F_n$, with $\overset{\circ}{\overline{F_n}} = \emptyset$.

The function $g : \mathbb{C} \to \mathbb{C}$ defined by $g(z) = log(1 - z)$, for all $z \in \mathbb{C}$ is a Bloch function, but is not bounded in D. Thus the set $\mathcal{F} = \mathcal{B} \setminus \mathcal{H}^{\infty}(D)$ is not empty and $\mathcal{F}$ is not a vector space. Then it seems natural to study some algebraic structure inside $\mathcal{F}$. Now, we get the following result:

Proposition 2.1. *1. For each $\alpha \in \mathbb{C}$ with $|\alpha| \leq 1$ and $\alpha \neq 0$ the function $g_\alpha = g(\alpha z)$ belongs to $\mathcal{F}$, $\{g_\alpha\}$ is a linearly independent set in $\mathcal{B}$ and $[g_\alpha : \alpha > 0] \subset \mathcal{F} \cup \{0\}$.*

Corollary 2.1. *$\mathcal{F}$ is lineable.*

We remark that as a consequence of Propostion 2.1 and the $dim\mathcal{B} = c$ we have that $\mathcal{F}$ is maximal lineable.

Proposition 2.2. *$\mathcal{F}$ is spaceable.*

Proof. As a consequence of a result of Kiltson and Timoney in [4], it is possible to show that $\mathcal{F}$ is spaceable.

Naturally, if $\mathcal{F}$ is spaceable then it implies $\mathcal{F}$ is lineable, but here we use a different technique to do this, then we decide to include both results.

Proposition 2.3. *$\mathcal{B}$ is residual.*

Proof. Let $S_n = \{f \in \mathcal{B} : \exists z \in D|f(z)| > n\}$. Then S_n is an open set and dense in $\mathcal{B}$. It is possible to show that $\mathcal{F} = \cap_{n=1}^{\infty} S_n$.

Proposition 2.4. *$\mathcal{F}$ is not algebrable*

Proof. The space $\mathcal{B}$ is not an algebra under the usual multiplication. For instance the square of the Bloch function $log(1 + z)$ does not belong to $\mathcal{B}$. Now, by the following result: For a holomorphic function $f \in \mathcal{B}$ the following conditions are equivalent: (i) $f\mathcal{B} \subset \mathcal{B}$ (ii) $f \in \mathcal{H}^{\infty}(D)$ and the function $(1 - |z|^2)|f(z)| \log \frac{1}{1-|z|^2}$ is bounded in D. That means, every algebra which contain $\mathcal{B}$ is contained $\mathcal{H}^{\infty}(D)$. So $\mathcal{F}$ is not algebrable. In fact, the algebra $\mathcal{H}^{\infty}(D)$ is the largest algebra contained in $\mathcal{B}$.

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THE POSITIVE SCHUR PROPERTY ON THE SPACE OF REGULAR MULTILINEAR OPERATORS

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Abstract

In this paper we give conditions under which the space of multilinear regular operators from the product of Banach lattices to a Dedekind complete Banach lattice has the positive Schur property. We also give equivalent conditions for the dual of the Banach lattice positive projective tensor product to have the PSP.

1 Introduction

A Banach lattice E has the *positive Schur property* (PSP in short) if every weakly null sequence formed by positive elements of E is norm null.

Given Banach lattices E and F with F Dedekind complete, it is known that the space of regular linear operators from E to F has the positive Schur property if and only if F and the norm dual of E have the PSP. It is of interest to know whether the space of bilinear regular operators has the positive Schur property under similar conditions. The aim of this work is to give a (positive) solution to this question. Then the bilinear result is generalized for the multilinear case using induction in our proof. We also show that given Banach lattices E and F, a necessary and sufficient condition for the dual of their Fremlin projective tensor product to have the PSP is the possession of the PSP by the duals of E and F, as well as the wot-PSP property of the closed sublattice of regular linear operators from the double dual of F to the dual of E, consisting of *weak**- to -*weak**- continuous positive operators.

2 Main Results

Theorem 2.1. *Given Banach lattices $E_1, \ldots, E_n, F$, by $\mathcal{L}^r(E_1, \ldots, E_m; F)$ we denote the space of regular m-linear operators from $E_1 \times \cdots \times E_m$ to F.*

We start with the bilinear case.

Theorem 2.2. *Let E_1, E_2, F be Banach lattices such that E_2^* and F are Dedekind complete. Then, $\mathcal{L}^r(E_1, E_2; F)$ has the positive Schur property (PSP) if and only if E_1^*, E_2^*, F have the PSP.*

Definition Let E and F be Banach spaces.
(a) A sequence of operators $(T_n)_{n=1}^{\infty} \subseteq \mathcal{L}(E; F)$ converges to zero in the weak operator topology (wot), if for every $x \in E$ and $y^* \in F^*$ we have

$$\langle y^*, T_n x \rangle \to 0, \ as\ n \to \infty.$$

(b) $\mathcal{L}^r(E; F)$ has the wot-positive Schur property if for every sequence of positive operators $(T_n)_{n=1}^{\infty} \subseteq \mathcal{L}^+(E; F)$ with $T_n \to 0$ in the weak operator topology, it follows that $\|T_n\| \overset{n}{\to} 0$.

Theorem 2.3. *Let E and F be Banach lattices. Let $\mathcal{L}^r_{w^*}(F^{**}; E^*)$ denote the closed sublattice of $\mathcal{L}^r(F^{**}; E^*)$, consisting of w^*-to-w^*-continuous positive operators. Then the following are equivalent.*

1. E^* *and* F^* *have the PSP.*

2. $\mathcal{L}^r_{w^*}(F^{**}; E^*)$ *has the wot-PSP.*

3. $(E\hat{\otimes}_{|\pi|}F)^*$ *has the PSP.*

Theorem 2.4. *For the Banach lattices* $E_1, E_2, \ldots, E_m$*, the following are equivalent.*

1. $E_1^*, E_2^*, \cdots, E_m^*$ *have the PSP.*

2. $(E_1\hat{\otimes}_{|\pi|}\cdots\hat{\otimes}_{|\pi|}E_m)^*$ *has the PSP.*

Next we have the multilinear case of Theorem 2.1.

Theorem 2.5. *Let* $E_1, \ldots, E_m, F$ *be Banach lattices such that* $E_2^*, \ldots, E_m^*, F$ *are Dedekind complete. Then* $\mathcal{L}^r(E_1, \ldots, E_m; F)$ *has the PSP if and only if* $E_1^*, \ldots, E_m^*$ *and* F *have the PSP.*

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OPERADORES MULTILINEARES SOMANTES POR BLOCOS ARBITRÁRIOS: OS CASOS ISOTRÓPICOS E ANISOTRÓPICOS

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Abstract

Definimos neste trabalho uma classe geral de operadores multilineares que recupera como caso particulares muitas classes de operadores absolutamente somantes estudados na literatura, incluindo os casos da diagonal e da matriz toda e também os casos isotrópicos e anisotrópicos.

1 Introdução

A teoria dos operadores multilineares absolutamente somantes tem se desenvolvido fortemente nos últimos 30 anos e várias abordagens foram consideradas, cada uma com vantagens. No início considerava-se apenas a soma na diagonal (operadores absolutamente somantes), depois passou-se a estudar a soma na matriz toda (operadores múltiplo somantes), e mais recentemente têm sido estudados alguns casos de somas em determinados blocos da matriz. Ao mesmo tempo, pode-se considerar os casos isotrópico (com a soma sendo feita de uma só vez) e anisotrópico (com a soma iterada ou encaixada).

O objetivo deste trabalho é introduzir um conceito que unifica todos esses casos estudados separadamente. Cada um dos casos estudados até agora será caso particular do conceito aqui introduzido.

Usaremos a noção de classes de sequências vetoriais, introduzido em [1]. Assim, dados uma classe de sequências X e um espaço de Banach E, $X(E)$ será um espaço de sequências a valores em E, de acordo com [1].

2 Resultados Principais

Neste resumo apresentaremos apenas o caso bilinear da construção. Os casos n-lineares, para $n \geq 2$, são análogos.

As letras E, E_1, E_2 e F denotarão espaços de Banach. Dados um subconjunto não vazio B de $\mathbb{N}^2$, denotaremos por $B^{i_1} = \{i_2 \in \mathbb{N} : (i_1, i_2) \in B\}$. É claro que eventualmente podemos ter $B^{i_1} = \emptyset$.

Proposição 2.1. *Sejam X_1, X_2, Y_1 e Y_2 classes de sequências e $B \subseteq \mathbb{N}^2$ não vazio. São equivalentes para um dado operador bilinear $T \in \mathcal{L}(E_1, E_2; F)$:*

(i) $\left((T(x_{i_1}, y_{i_2}))_{i_2 \in B^{i_1}}\right)_{i_1=1}^{\infty} \in Y_1(Y_2(F))$ *sempre que* $(x_i)_{i=1}^{\infty} \in X_1(E_1)$, $(y_i)_{i=1}^{\infty} \in X_2(E_2)$.

(ii) O operador induzido $\widehat{T}_B : X_1(E_1) \times X_2(E_2) \longrightarrow Y_1(Y_2(F))$ *definido por*

$$\widehat{T}_B\left((x_i)_{i=1}^{\infty}, (y_i)_{i=1}^{\infty}\right) = \left((T(x_{i_1}, y_{i_2}))_{i_2 \in B^{i_1}}\right)_{i_1=1}^{\infty},$$

está bem definido, é bilinear e contínuo.

Definição 2.1. Nas condições da Proposição 2.1, um operador bilinear $T \in \mathcal{L}(E_1, E_2; F)$ é dito *absolutamente* $(B; X_1, X_2; Y_1, Y_2)$*-somante* se valem as equivalências da Proposição 2.1. Em tal caso, escrevemos

$$T \in \mathcal{L}_{B;X_1,X_2;Y_1,Y_2}(E_1, E_2; F) \text{ e } \|T\|_{B;X_1,\cdots,X_2;Y_1,\cdots,Y_n} := \|\widehat{T}_B\|.$$

A classe de todos os operadores bilineares contínuos que são absolutamente $(B; X_1, X_2; Y_1, Y_2)$-somante é denotada por $\mathcal{L}_{B;X_1,X_2;Y_1,Y_2}$

Definição 2.2. Dizemos que a quádrupla ordenada (X_1, X_2, Y_1, Y_2) de classes de sequências é B-compatível, $B \subseteq \mathbb{N}^2$, se vale $((\lambda^1_{i_1}\lambda^2_{i_2})_{i_2\in B^{i_1}})_{i_1=1}^{\infty} \in Y_1(Y_2(\mathbb{K})$ sempre que $(\lambda^k_i)_{i=1}^{\infty} \in X_j(\mathbb{K})$, $k = 1, 2$.

Teorema 2.1. *Sejam $B \subseteq \mathbb{N}^2$ não vazio e (X_1, X_2, Y_1, Y_2) uma quádrupla ordenada B-compatível de classes de sequências linearmente estáveis. Então $(\mathcal{L}_{B;X_1,X_2;Y_1,Y_2}, \|\cdot\|_{B;X_1,X_2;Y_1,Y_2})$ é um ideal de Banach de operadores multilineares.*

Além de condição suficiente, a B-compatibilidade também é uma condição necessária para não trivializar o ideal: prova-se que se a quádrupla (X_1, X_2, Y_1, Y_2) não for B-compatível, então $\mathcal{L}_{B;X_1,X_2;Y_1,Y_2} = \{0\}$ para quaisquer E_1, E_2 e F.

Exemplo 2.1 (O caso isotrópico)**.** Sejam $1 \le p_1, p_2, q < \infty$, $X_1 = \ell^w_{p_1}(\cdot)$, $X_2 = \ell^w_{p_2}(\cdot)$, $Y_1 = Y_2 = \ell_q(\cdot)$. Tomando o bloco $B = \{(i, i) : i \in \mathbb{N}\}$, recuperamos os operadores absolutamente $(q; p_1, p_2)$-somantes de [1]. E tomando o bloco $B = \mathbb{N}^2$, recuperamos os operadores múltiplo $(q; p_1, p_2)$-somantes (veja, por exemplo, [2, 2]). E para um bloco arbitrário B, recupera-se a classe estuda em [2].

Exemplo 2.2 (O caso anisotrópico)**.** *Sejam $1 \le p_1, p_2, q_1, q_2 < \infty$, $X_1 = \ell^w_{p_1}(\cdot)$, $X_2 = \ell^w_{p_2}(\cdot)$, $Y_1 = \ell_{q_1}(\cdot)$, $Y_1 = \ell_{q_2}(\cdot)$, $Y_1 = \ell_{q_2}(\cdot)$ e o bloco $B = \mathbb{N}^2$. Um operador $T \in \mathcal{L}(E_1, E_2; F)$ é $(B; X_1, X_2; Y_1, Y_2)$-somante se, e somente se, para quaisquer sequências $(x_i)_{i=1}^{\infty} \in X_1(E_1)$ e $(y_i)_{i=1}^{\infty} \in X_2(E_2)$ tem-se*

$$\left((T(x_{i_1}, y_{i_2}))_{i_2\in\mathbb{N}}\right)_{i_1=1}^{\infty} \in \ell_{q_1}(\ell_{q_2}(F)), \text{ ou seja, } \left(\sum_{i_1=1}^{\infty}\left(\sum_{i_2=1}^{\infty}\|T(x_{i_1}, y_{i_2})\|_F^{q_2}\right)^{\frac{q_1}{q_2}}\right)^{\frac{1}{q_1}} < \infty.$$

Para uma escolha adequada de bloco B, recupera-se também o caso anisotrópico dos operadores $\mathcal{I}$-parcialmente somantes de [2].

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GENERALIZAÇÃO DAS APLICAÇÕES MULTILINEARES MÚLTIPLO SOMANTES EM ESPAÇOS DE BANACH

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Abstract

Neste trabalho introduzimos o conceito de aplicações múltiplo $(\gamma, \gamma_1, \ldots, \gamma_n)$-somantes, seu ideal de polinômios homogêneos gerado e apresentaremos diversas propriedades, dentre elas, que esta classe é um ideal de Banach de aplicações multilineares, culminando na coerência e compatibilidade.

1 Introdução

O conceito de operadores múltiplo somantes tem sido trabalhado em diversos artigos, por exemplo [1, 2, 3]. Nestes trabalhos os operadores múltiplo somantes tem sido abordados utilizando espaços de sequências já bem estudados na literatura, como por exemplo $\ell_p(E)$ e $\ell_p^w(E)$. Em [4] G. Botelho e J. Campos introduziram o conceito de classes de sequências finitamente determinadas e linearmente estáveis, conceito esse fundamental para a abordagem abstrata que nos propomos a fazer para os operadores múltiplo somantes. Vale ressaltar que já existe uma outra abordagem abstrata feita por D. Serrano em [2], para os operadores absolutamente somantes.

Definição 1.1. *Uma n-sequência em um espaço de Banach E é uma aplicação $f : \mathbb{N}^n \to E$, dada por*

$$f(j_1, \ldots, j_n) = x_{j_1,\ldots,j_n}.$$

Por simplicidade iremos representar a n-sequência por $(x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{\infty}$. O espaço de todas as n-sequências é denotado por $E^{\mathbb{N}^n}$. Este é um espaço vetorial quando consideramos as operações coordenadas naturais.

Definição 1.2. *Uma classe de n-sequências a valores vetoriais $\gamma(\cdot, \mathbb{N}^n)$ é uma regra que associa a cada espaço de Banach E um espaço de Banach $\gamma(E; \mathbb{N}^n)$ de n-sequências a valores em E, isto é, $\gamma(E; \mathbb{N}^n)$ é um subespaço vetorial normado do espaço de todas as n-sequências a valores em E com as operações coordenadas, tal que:*

$$c_{00}(E; \mathbb{N}^n) \subset \gamma(E; \mathbb{N}^n) \overset{1}{\hookrightarrow} \ell_\infty(E; \mathbb{N}^n) \quad e \quad \|e_{k_1,\ldots,k_n}\|_{\gamma_s(\mathbb{K}; \mathbb{N}^n)} = 1,$$

onde $e_{k_1,\ldots,k_n} = (x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{\infty}$ é uma n-sequência definida por:

$$x_{j_1,\ldots,j_n} = \begin{cases} 1, & se \quad j_1 = k_1, \ldots, j_n = k_n \\ 0, & \quad caso\ contrário \end{cases}$$

e o símbolo $E \overset{1}{\hookrightarrow} F$ significa que E é um subespaço vetorial de F e $\|x\|_F \le \|x\|_E$.

Uma classe de n-sequências $\gamma(\cdot; \mathbb{N}^n)$ é dita *finitamente determinada* se para qualquer n-sequência $(x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{\infty}$ a valores em E

$$(x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{\infty} \in \gamma(E; \mathbb{N}^n) \iff \sup_{m_1,\ldots,m_n \in \mathbb{N}} \left\| (x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{m_1,\ldots,m_n} \right\|_{\gamma(E;\mathbb{N}^n)} < \infty,$$

e neste caso $\left\|(x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{\infty}\right\|_{\gamma(E;\mathbb{N}^n)} = \sup_{m_1,\ldots,m_n \in \mathbb{N}} \left\| (x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{m_1,\ldots,m_n} \right\|_{\gamma(E;\mathbb{N}^n)}$, onde $(x_{j_1,\ldots,j_n})_{j_1,\ldots,j_m=1}^{m_1,\ldots,m_n}$ é uma n-sequência $(y_{j_1,\ldots,j_n})_{j_1,\ldots,j_m=1}^{\infty}$, tal que, para $1 \le j_i \le m_i$, $i = 1, \ldots, n$ ela assume os valores $x_{j_1,\ldots,j_n}$ e assume valor zero nos outros casos.

Definição 1.3. *Dizemos que uma aplicação n-linear é múltiplo $(\gamma, \gamma_1, \ldots, \gamma_n)$-somante se*

$$\left(T\left(x_{j_1}^{(1)}, \ldots, x_{j_n}^{(n)}\right)\right)_{j_1,\ldots,j_n=1}^{\infty} \in \gamma\left(F; \mathbb{N}^n\right)$$

sempre que $\left(x_j^{(i)}\right)_{j=1}^{\infty} \in \gamma_i(E_i), i = 1, \ldots, n$.

A classe das aplicações n-lineares que são múltiplo $(\gamma, \gamma_1, \ldots, \gamma_n)$-somantes será denotada por $\mathcal{L}^m_{\gamma,\gamma_1,\ldots,\gamma_n}$.

Definição 1.4. *Seja $\gamma(\cdot, \mathbb{N}^n)$ uma classe de n-sequências. Dizemos que $\gamma(\cdot, \mathbb{N}^n)$ é linearmente estável se*

$$(u(x_{j_1,\ldots,j_n}))_{j_1,\ldots,j_n=1}^{\infty} \in \gamma\left(F; \mathbb{N}^n\right)$$

sempre que $(x_{j_1,\ldots,j_n})_{j_1,\ldots,j_n=1}^{\infty} \in \gamma\left(E; \mathbb{N}^n\right)$ *e* $\|\hat{u} : \gamma\left(E; \mathbb{N}^n\right) \to \gamma\left(F; \mathbb{N}^n\right)\| = \|u\|$, *para todo* $u \in \mathcal{L}(E; F)$.

Dadas classes de sequências $\gamma_1, \ldots, \gamma_n$ e uma classe de n-sequências $\gamma(\cdot; \mathbb{N}^n)$, dizemos que $\gamma_1(\mathbb{K}) \cdots \gamma_n(\mathbb{K}) \overset{mult,1}{\hookrightarrow} \gamma\left(\mathbb{K}; \mathbb{N}^n\right)$ quando $\left(\lambda_{j_1}^{(1)} \cdots \lambda_{j_n}^{(n)}\right)_{j_1,\ldots,j_n=1}^{\infty} \in \gamma\left(\mathbb{K}; \mathbb{N}^n\right)$ e $\left\|\left(\lambda_{j_1}^{(1)} \cdots \lambda_{j_n}^{(n)}\right)_{j_1,\ldots,j_n=1}^{\infty}\right\|_{\gamma(\mathbb{K};\mathbb{N}^n)} \leq \prod_{i=1}^{\infty} \left\|\left(\lambda_j^{(i)}\right)_{j=1}^{\infty}\right\|_{\gamma_i(\mathbb{K})}$, sempre que $\left(\lambda_j^{(i)}\right)_{j=1}^{\infty} \in \gamma_{s_i}(\mathbb{K}), i = 1, \ldots, n$.

2 Resultados Principais

Supondo que $\gamma_1, \ldots, \gamma_n$ são classes de sequências e $\gamma(\cdot, \mathbb{N}^n)$ é uma classe de n-sequências finitamente determinadas e linearmente estáveis. Podemos mostrar os seguintes resultados:

Teorema 2.1. *(a) Supondo que $\gamma_1(\mathbb{K}) \cdots \gamma_n(\mathbb{K}) \overset{mult,1}{\hookrightarrow} \gamma\left(\mathbb{K}; \mathbb{N}^n\right)$. Então $\left(\mathcal{L}^m_{\gamma,\gamma_1,\ldots,\gamma_n}, \|\cdot\|_{\mathcal{L}^m_{\gamma,\gamma_1,\ldots,\gamma_n}}\right)$ é um Multi-ideal de Banach.*

(b) A sequência de pares $\left(\left(\mathcal{L}^{m,n}_{\gamma,\gamma_i,\ldots,\gamma_i}, \|\cdot\|_{\mathcal{L}^{m,n}_{\gamma,\gamma_i,\ldots,\gamma_i}}\right); \left(\mathcal{P}_{\mathcal{L}^{m,n}_{\gamma,\gamma_i}}, \|\cdot\|_{\mathcal{P}_{\mathcal{L}^{m,n}_{\gamma,\gamma_i}}}\right)\right)_{n=1}^{\infty}$ é coerente e compatível com $\mathcal{L}_{\gamma,\gamma_i}$.

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VERSÕES NÃO-LINEARES DO TEOREMA DE BANACH-STONE

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Abstract

Nosso estudo tem como ponto de partida o Teorema clássico de Banach-Stone e propõe-se a estudar generalizações deste para a classe de funções não-lineares das quasi-isometrias. Mais precisamente, apresentamos duas versões não-lineares do Teorema de Banach-Stone que generalizam o Teorema de Amir-Cambern e o Teorema de Cambern para espaços de Hilbert de dimensão finita.

1 Introdução

Dados K um espaço de Hausdorff localmente compacto e X um espaço de Banach, denotamos por $C_0(K, X)$ o espaço de Banach das funções contínuas de K a valores em X que se anulam no infinito, munido da norma do supremo. No caso em que $X = \mathbb{R}$, denotaremos este espaço por $C_0(K)$.

O Teorema clássico de Banach-Stone [2] estabelece que se existe uma isometria linear de $C_0(K)$ sobre $C_0(S)$ então K e S são homeomorfos. Amir [1] e Cambern [3] generalizaram este resultado, de modo independente, provando que se existe um isomorfismo linear T de $C_0(K)$ sobre $C_0(S)$ satisfazendo $\|T\|\|T^{-1}\| < 2$, então K e S são homeomorfos.

Posteriormente, Cambern [4] obteve uma versão do Teorema de Banach-Stone para o caso em que $X = H$, um espaço de Hilbert de dimensão finita, provando que se existe um isomorfismo de $C_0(K, H)$ sobre $C_0(S, H)$ satisfazendo $\|T\|\|T^{-1}\| < \sqrt{2}$, então K e S são homeomorfos.

Em 1989, Jarosz deu início aos estudos de generalizações do Teorema de Banach-Stone para classes de funções não-lineares, provando uma versão deste para funções bi-Lipschitz [9]. Tais estudos culminaram nos resultados de Górak para a classe das quasi-isometrias, que destacamos a seguir.

Dizemos que uma função entre espaços de Banach $T : E \to F$ é uma (M, L)-quasi-isometria se satisfaz

$$\frac{1}{M}\|u - v\| - L \leq \|Tu - Tv\| \leq M\|u - v\| + L, \quad \forall u, v \in E,$$

e se a imagem de T é ξ-densa em F, para algum $\xi > 0$, isto é,

$$\forall w \in F, \ \exists u \in E : \|w - Tu\| \leq \xi.$$

Górak provou em [2] que se existe uma (M, L)-quasi-isometria $T : C_0(K) \to C_0(S)$ satisfazendo $M < \sqrt{16/15}$, então K e S são homeomorfos. Além disso, outro resultado também devido a Górak [8] estabelece que, no caso em que K e S são compactos, é suficiente que T satisfaça $M < \sqrt{6/5}$.

As técnicas aplicadas por Górak em [2] e [8] foram objeto de estudo em nosso trabalho de mestrado, realizado no Instituto de Matemática e Estatística da Universidade de São Paulo, sob orientação do Professor Eloi Medina Galego. No doutorado demos continuidade ao trabalho, tendo como objetivo aumentar as constantes $\sqrt{16/15}$ e $\sqrt{6/5}$ nos resultados de Górak. Como fruto desta pesquisa, desenvolvemos uma nova técnica para a demonstração de teoremas do tipo Banach-Stone que nos possibilitou obter uma versão ótima dos resultados de Górak. Além disso, obtivemos versões não-lineares para o caso em que X é um espaço vetorial de dimensão maior que 1 que alcançam os resultados lineares mais gerais atuais.

Em nossa apresentação, será feita uma rápida exposição da técnica aplicada na demonstração dos Teoremas 1 e 1 abaixo, apontando as principais ideias empregadas, e posteriormente discutiremos alguns problemas em aberto relacionados.

2 Resultados Principais

Como generalização do Teorema de Amir-Cambern, provamos em [5] o seguinte:

Teorema 2.1. *Sejam K e S espaços de Hausdorff localmente compactos. Suponha que existe uma (M, L)-quasi-isometria de $T : C_0(K) \to C_0(S)$ com $M < \sqrt{2}$. Então K e S são homeomorfos.*

Em [6], foi obtida a seguinte generalização do Teorema de Cambern:

Teorema 2.2. *Sejam K e S espaços de Hausdorff localmente compactos e H um espaço de Hilbert de dimensão finita. Suponha que existe uma (M, L)-quasi-isometria $T : C_0(K, H) \to C_0(S, H)$ com $M < \sqrt[4]{2}$. Então K e S são homeomorfos.*

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O IDEAL DE COMPOSIÇÃO COMO UM IDEAL BILATERAL

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Abstract

O objetivo desse trabalho é estudar os ideais de composição de polinômios homogêneos $\mathcal{I}\circ\mathcal{P}$ sob a perspectiva do conceito de ideal bilateral, conceito este mais restritivo que o já bem estudado conceito de ideal de polinômios derivado da noção de multi-ideais introduzido por Pietsch em [6]. O objetivo principal é investigar quais propriedades o ideal de operadores $\mathcal{I}$ deve possuir para que o ideal de composição seja um ideal bilateral. Uma vez determinada tal propriedade, vários exemplos são apresentados.

1 Introdução

Começamos com a definição formal de ideais bilaterais de polinômios homogêneos, que foi introduzida em [2].

Definição 1.1. Sejam $0 < p \leq 1$, $(\mathcal{Q}, \|\cdot\|_{\mathcal{Q}})$ uma classe de polinômios homogêneos entre espaços de Banach e $(C_n, K_n)_{n=1}^{\infty}$ uma sequência de pares de números reais positivos com $C_n, K_n \geq 1$ para todo $n \in \mathbb{N}$ e $C_1 = K_1 = 1$. Para todo $n \in \mathbb{N}$ e quaisquer espaços de Banach E e F, suponha que:
(i) A componente

$$\mathcal{Q}(^nE;F) := \mathcal{P}(^nE;F) \cap \mathcal{Q}$$

é um subespaço de $\mathcal{P}(^nE;F)$ contendo os polinômios n-homogêneos de tipo finito.
(ii) A restrição de $\|\cdot\|_{\mathcal{Q}}$ a $\mathcal{Q}(^nE;F)$ é uma p-norma.
(iii) $\|\widehat{I_n}\colon \mathbb{K} \longrightarrow \mathbb{K}\ ,\ \widehat{I_n}(\lambda) = \lambda^n\|_{\mathcal{Q}} = 1$ para todo n.

Dizemos que $(\mathcal{Q}, \|\cdot\|_{\mathcal{Q}})$ é um $(C_n, K_n)_{n=1}^{\infty}$*-ideal bilateral p-normado de polinômios polynomial* se a seguinte condição está satisfeita:

Propriedade de ideal bilateral: Para $n, m, r \in \mathbb{N}$ e espaços de Banach E, F, G e H, se $P \in \mathcal{Q}(^nE;F)$, $Q \in \mathcal{P}(^mG;E)$ e $R \in \mathcal{P}(^rF;H)$, então $R \circ P \circ Q \in \mathcal{Q}(^{rmn}G;H)$ e

$$\|R \circ P \circ Q\|_{\mathcal{Q}} \leq K_r \cdot C_m^{rn} \cdot \|R\| \cdot \|P\|_{\mathcal{Q}}^r \cdot \|Q\|^{rn}.$$

Quando $C_n = K_n = 1$, para todo $n \in \mathbb{N}$ dizemos que $(\mathcal{Q}, \|\cdot\|_{\mathcal{Q}})$ é um *ideal bilateral de polinômios.* A noção de $(C_n, K_n)_{n=1}^{\infty}$-ideal bilateral de Banach (p-Banach) é definida da maneira óbvia.

Os ideais de composição, definidos a seguir, além de serem o objeto de estudo deste trabalho, fornecem vários exemplos de ideais bilaterais.

Definição 1.2. Seja $(\mathcal{I}, \|\cdot\|_{\mathcal{I}})$ um ideal de operadores p-normado. Um polinômio $P \in \mathcal{P}(^nE;F)$ pertence a $\mathcal{I} \circ \mathcal{P}$ se existem um espaço de Banach G, um polinômio $Q \in \mathcal{P}(^nE;G)$ e um operador $u \in \mathcal{I}(G;F)$ tais que $P = u \circ Q$. Definimos ainda $\|\cdot\|_{\mathcal{I}\circ\mathcal{P}}\colon \mathcal{I}\circ\mathcal{P} \longrightarrow [0,\infty)$ por

$$\|P\|_{\mathcal{I}\circ\mathcal{P}} = \inf\{\|u\|_{\mathcal{I}} \cdot \|Q\| : P = u \circ Q, u \in \mathcal{I}\}.$$

Por último iremos necessitar da seguinte definição:

Definição 1.3. Um ideal de operadores p-normado $(\mathcal{I}, \|\cdot\|_{\mathcal{I}})$ é chamado *simetricamente tensor-estável* se existe uma sequência $(C_n)_{n=1}^{\infty}$ de números reais positivos tal que, para quaisquer $n \in \mathbb{N}$ e $u \in \mathcal{I}(E;F)$, vale

$$\otimes^{n,s} u \in \mathcal{I}\left(\widehat{\otimes}_{\pi_s}^{n,s} E; \widehat{\otimes}_{\pi_s}^{n,s} F\right) \text{ e } \|\otimes^{n,s} u\|_{\mathcal{I}} \leq C_n \|u\|_{\mathcal{I}}^n.$$

2 Resultados Principais

Antes de começarmos recordamos que um polinômio n-homogêneo $P \in \mathcal{P}(^nE;F)$ pertence a $\mathcal{P}\circ\mathcal{I}(^nE;F)$ se existem um espaço de Banach G, um operator linear $u \in \mathcal{I}(E;G)$ e um polinômio n-homogêneo $Q \in \mathcal{P}(^nG;F)$ tais que $P = Q \circ u$, além disso

$$\|P\|_{\mathcal{P}\circ\mathcal{I}} = \inf\{\|Q\| \cdot \|u\|_{\mathcal{I}}^n : P = Q \circ u \text{ com } u \in \mathcal{I}\}.$$

Teorema 2.1. *Seja* $(\mathcal{I}, \|\cdot\|_{\mathcal{I}})$ *um ideal de operadores p-normado (p-Banach). São equivalentes:*
(a) $(\mathcal{I}\circ\mathcal{P}, \|\cdot\|_{\mathcal{I}\circ\mathcal{P}})$ *é um* $(1, C_n)_{n=1}^{\infty}$*-ideal bilateral p-normado (p-Banach).*
(b) $\mathcal{P}\circ\mathcal{I} \subseteq \mathcal{I}\circ\mathcal{P}$ *e* $\|P\|_{\mathcal{I}\circ\mathcal{P}} \leq C_n\|P\|_{\mathcal{P}\circ\mathcal{I}}$ *para todo* $P \in \mathcal{Q}(^nE;F)$*.*
(c) $(\mathcal{I}, \|\cdot\|_{\mathcal{I}})$ *é simetricamente tensor-estável com constantes* $(C_n)_{n=1}^{\infty}$*.*

Exemplo 2.1. Os seguintes ideais de operadores $(\mathcal{I}, \|\cdot\|_{\mathcal{I}})$ são simetricamente tensor-estáveis, logo $(\mathcal{I}\circ\mathcal{P}, \|\cdot\|_{\mathcal{I}\circ\mathcal{P}})$ é um ideal bilateral de Banach segundo o Teorema 2.1:
(a) O dual Π_p^{dual} do ideal Π_p dos operadores absolutamente p-somantes, que coincide com a envoltória convexa $\mathcal{K}_p^{\max}$ do ideal dos operadores p-compactos $\mathcal{K}_p$ [7, Theorems 12, 24, 25].
(b) O ideal fechado $\mathcal{S}$ dos operadores separáveis[1, Example 3.5(a)].
(c) O ideal $\overline{\mathcal{F}}^{\|\cdot\|}$ dos operadores aproximáveis por tipo finito e o ideal $\mathcal{N}$ dos operadores nucleares [4, 34.1].
(d) O ideal $\mathcal{J}$ dos operadores integrais [5, Theorem 2].
(e) Os ideais $\mathcal{L}_{1,q}$, $q > 1$, dos operadores $(1, q)$-factoráveis e $\mathcal{K}_{1,p}$, $p > 1$, dos operadores $(1, p)$-compactos [3, Theorem 2.1].

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GENERALIZED ADJOINTS OF LINEAR OPERATORS AND HOMOGENEOUS POLYNOMIALS

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Abstract

In this work we introduce a generalization of the concepts of adjoints of a linear operator and of a homogeneous polynomial between Banach spaces. An illustrative example of how these generalized notions reproduce the properties of the classical concepts is provided.

1 Introduction

E and F are (real or complex) Banach spaces, $\mathcal{L}(E;F)$ is the space of bounded linear operators from E to F and $\mathcal{P}(^mE;F)$ is the space of continuous m-homogeneous polynomials from E to F, $m \in \mathbb{N}$. If F is the scalar field, we simply write E^* and $\mathcal{P}(^mE)$, respectively.

We first remember the usual concepts of adjoints of linear operators (which is folklore) and homoegeneous polynomials (which was introduced by Aron and Schottenloher [1] – see also [1]):

Definition 1.1. (a) The adjoint of an operator $u \in \mathcal{L}(E;F)$ is the operator

$$u^* \colon F^* \longrightarrow E^* \ , \ u^*(\varphi)(x) = \varphi(u(x)).$$

(b) The adjoint of an m-homogeneous polynomial $P \in \mathcal{P}(^mE;F)$ is the linear operator

$$P^* \colon F^* \longrightarrow \mathcal{P}(^mE) \ , \ P^*(\varphi)(x) = \varphi(P(x)).$$

It is clear that $\|u^*\| = \|u\|$ and $\|P^*\| = \|P\|$.

The aim of this work is to generalize these classical notions, in the sense of obtaining a new concept which: (i) recover the classical concepts as particular instances, (ii) behave, in some sense, as the original notions, (iii) has nice applications.

2 Main Results

The new concept we introduce is the following:

Definition 2.1. Let m, n, k be given natural numbers. Given a continuous m-homogeneous polynomial $P \in \mathcal{P}(^mE;F)$, define

$$\Delta_k^n P \colon \mathcal{P}(^kF) \longrightarrow \mathcal{P}(^{mnk}E) \ , \ \Delta_k^n P(q)(x) = q(P(x))^n.$$

Proposition 2.1. *$\Delta_k^n P$ is a well defined continuous n-homogeneous polynomial, that is,*

$$\Delta_k^n P \in \mathcal{P}(^n\mathcal{P}(^kF) \ , \ \mathcal{P}(^{mnk}E)),$$

and $\|\Delta_k^n P\| = \|P\|^{kn}$.

Let us see that, in fact, this concept recovers the classical notions as particular cases:
- For $u \in \mathcal{L}(E;F)$, $u^* = \Delta_1^1 u$.
- For $P \in \mathcal{P}(^m E;F)$, $P^* = \Delta_1^1 P$.

Next we give an illustrative example of how the generalized notion reproduces the behavior of the original adjoints.

Definition 2.2. Given $m, n \in \mathbb{N}$ and a Banach space E, define

$$K_E^{m,n} \colon E \longrightarrow \mathcal{P}\left(^m\mathcal{P}(^n E)\right)\ ,\ K_E^{m,n}(x)(q) = q(x)^m.$$

It is not difficult to check that $K_E^{m,n}$ is a well defined continuous mn-homogeneous polynomial and $\|K_E^{m,n}(x)\| = \|x\|^{mn}$ for every $x \in E$. Moreover, letting $m = n = 1$ we have that $K_E^{1,1}$ is the canonical embedding $J_E \colon E \longrightarrow E^{**}$.

Proposition 2.2. *Given m, n, k, r, $s \in \mathbb{N}$ and $P \in \mathcal{P}(^m E;F)$, the following diagram is commutative:*

$$\begin{array}{ccc} E & \xrightarrow{\quad P \quad} & F \\ {\scriptstyle K_E^{r,mnk}}\downarrow & & \downarrow{\scriptstyle K_F^{nrs,k}} \\ \mathcal{P}\left(^r\mathcal{P}(^{mnk}E)\right) & \xrightarrow{\Delta_r^s(\Delta_k^n P)} & \mathcal{P}(^{nrs}\mathcal{P}(^k F)) \end{array}$$

Letting $m = n = k = r = s = 1$, the diagram above recovers the classical commutative diagram for a linear operator $u \in \mathcal{L}(E;F)$ (see [2]):

$$\begin{array}{ccc} E & \xrightarrow{\ u\ } & F \\ {\scriptstyle J_E}\downarrow & & \downarrow{\scriptstyle J_F} \\ E^{**} & \xrightarrow{u^{**}} & F^{**} \end{array}$$

Further properties and applications of these generalized adjoints shall be given in a forthcoming work.

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ESPAÇABILIDADE DO CONJUNTO DE FUNÇÕES INTEIRAS EM ÁLGEBRAS DE BANACH QUE NÃO SÃO LORCH-ANALÍTICAS

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Abstract

Mostramos que o conjunto das funções inteiras em uma álgebra de Banach e que não são Lorch-analíticas é espaçável. É também obtido o mesmo resultado para as funções inteiras de tipo limitado que não são Lorch-analíticas.

1 Introdução

Nas últimas duas décadas, tem havido um interesse crescente na busca de boas estruturas algébricas e topológicas dentro de conjuntos (principalmente conjuntos de funções ou sequências) que não possuem tais estruturas. Nesta nota, estudamos tais estruturas em certos conjuntos de funções analíticas. Um dos primeiros autores a estudar o assunto é Gurariy em [2], que mostrou que existe um espaço vetorial de dimensão infinita contido no conjunto das funções *nowhere differentiable* em $[0, 1]$. A referência [1] apresenta uma vasta gama de resultados sobre o tema.

O espaço de todas as funções analíticas de E em E, munido da topologia compacto-aberta será indicado por $\mathcal{H}(E, E)$. Denotamos o conjunto de todas as funções (L)-analíticas de E em E por $\mathcal{H}_L(E, E)$. A classe das aplicações (L)-analíticas (cf. Definição 2.1) foi introduzida por E. R. Lorch em [4]. Chamamos de $\mathcal{G}(E; E) = \mathcal{H}(E, E) \setminus \mathcal{H}_L(E, E)$. Em [3] foi provado que, para $E = \mathbb{C}^2$, $\mathcal{G}(\mathbb{C}^2, \mathbb{C}^2)$ é espaçável e fortemente $\mathfrak{c}$-algebrável. Neste trabalho investigamos o conjunto $\mathcal{G}$ para uma álgebra de Banach E qualquer, e mostramos que $\mathcal{G}(E, E)$ é espaçável.

O subespaço de $\mathcal{H}(E, E)$ formado das funções inteiras de tipo limitado, munido da topologia da convergência uniforme sobre os limitados, será denotado por $\mathcal{H}_b(E, E)$. Também vale que $\mathcal{H}_L(E, E) \subset \mathcal{H}_b(E, E)$. Por outro lado, quando $\dim E = \infty$, os espaços $\mathcal{H}(E, E)$ e $\mathcal{H}_b(E, E)$ são diferentes, e neste caso, também estudamos a diferença $\mathcal{G}_b(E; E) = \mathcal{H}_b(E, E) \setminus \mathcal{H}_L(E, E)$ e mostramos que $\mathcal{G}_b(E, E)$ é espaçável.

2 Resultados Principais

Começamos com a definição de função (L)-analítica em uma álgebra de Banach com identidade.

Definição 2.1. *[4] Seja E uma álgebra com Banach comutativa sobre $\mathbb{C}$ com identidade. Uma aplicação $f : E \longrightarrow E$ é* ***(L)-analítica em*** *$\omega \in E$ se existir $\zeta \in E$ tal que*

$$lim_{h \to 0} \frac{\|f(\omega + h) - f(\omega) - \zeta \cdot h\|}{\|h\|} = 0.$$

Dizemos que f é *(L)-analítica em E* se f é *(L)-analítica* em cada ponto de E. Denotamos o conjunto de todas as funções (L)-analíticas de E em E por $\mathcal{H}_L(E, E)$.

É claro que uma função (L)-analítica é diferenciável no sentido de Fréchet e, portanto, holomorfa em E. Entretanto, nem toda aplicação holomorfa em uma álgebra de Banach comutativa com identidade é analítica no sentido de Lorch. De fato, temos a seguinte caracterização de funções (L)-analíticas.

Observação 1. *[5][Remark 2.3] Uma aplicação holomorfa $f : E \longrightarrow E$ é (L)-analítica em E se, e somente se, existe uma única sequência $(a_n)_{n\in\mathbb{N}} \subset E$ tal que $\lim_{n\to\infty} \|a_n\|^{\frac{1}{n}} = 0$ e $f(z) = \sum_{n=0}^{\infty} a_n z^n$, para todo $z \in E$.*

Dado $\omega \in \mathbb{C}$, $\omega \neq 0$, seja $a_n = \omega^n \cdot e \in E$, onde e é a unidade de E. Então $f(z) = \sum_{n=0}^{\infty} a_n z^n$, para todo $z \in E$, é tal que $f \in \mathcal{H}(E, E) \setminus \mathcal{H}_L(E, E)$. Vamos denotar $\mathcal{G}(E, E) = \mathcal{H}(E, E) \setminus \mathcal{H}_L(E, E)$.

Sejam Y um espaço vetorial topológico e $A \subset Y$. Dizemos que A é *lineável (espaçável)* se existe um espaço vetorial (fechado) de dimensão infinita $\mathcal{B} \subset A \cup \{0\}$.

Para mostrar que um conjunto é lineável, em vários casos é importante ter uma *função mãe*, isto é, uma função que pertence ao conjunto de interesse. A partir desta "função mãe", pode ser possível construir um espaço vetorial contido no conjunto em questão. Neste caso, a função f construída acima fará o papel desta "função mãe", como mostra o próximo resultado.

Teorema 2.1. *Para cada $\alpha \in \mathbb{R}$, seja $f_\alpha(z) = f(\alpha z)$, para todo $z \in E$. Seja $S = \{f_\alpha : \alpha \in \mathbb{R}\}$. Então o conjunto S é l.i., $[S] \subset \mathcal{G}(E, E)$ e $\overline{[S]} \subset \mathcal{G}(E, E)$.*

Em [5], Proposição 2.2, é mostrado que $\mathcal{H}_L(E, E) \subset \mathcal{H}_b(E, E)$. No entanto, se tomarmos um funcional linear contínuo $\varphi \in E'$ tal que φ não é multiplicativo, então a função $g : E \longrightarrow E$ definida por $g(z) = \sum_{n=0}^{\infty} b_n \varphi(z)^n$, onde (b_n) é uma sequência em E tal que $\lim \|b_n\|^{\frac{1}{n}} = 0$, é tal que $g \in \mathcal{H}_b(E, E) \setminus \mathcal{H}_L(E, E)$. Com esta "função mãe" g podemos obter o seguinte resultado.

Teorema 2.2. *Para cada $\beta \in \mathbb{C}$, seja $g_\beta(z) = g(\beta z)$, para todo $z \in E$. Seja $T = \{g_\beta : \beta \in \mathbb{C}, |\beta| = 1\}$. Então o conjunto T é l.i. e $[T] \subset \mathcal{G}_b(E, E)$.*

A demonstração do Teorema 2.1 é bastante técnica e usa fortemente a Observação 1. Para demonstrar o Teorema 2.2, fazemos uso do Teorema de Gleason-Kahane-Zelasko, bem como a Observação 1.

Corolário 2.1. *Os conjuntos $\mathcal{G}(E, E)$ e $\mathcal{G}_b(E, E)$ são espaçáveis.*

O fato de $\mathcal{G}(E, E)$ ser espaçável é consequência direta do Teorema 2.1, já a espaçabilidade de $\mathcal{G}_b(E, E)$ é obtida a partir dos Teoremas 2.2 e [1, Theorem 7.4.1], uma vez que $H_b(E, E)$ é uma álgebra de Fréchet.

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STABILIZATION FOR AN EQUATION WITH OPERATOR Δ^{2P} WITH NON LINEAR TERM

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Abstract

Our main objective is to study the exact controllability of problem,

$$(*)\quad \begin{cases} \mathcal{L}\, w + w^3 \;=\; 0 & \text{in} \quad \text{Q} \\ \dfrac{\partial^j \, w}{\partial \eta^j} \;=\; 0 & \text{on} \quad \Sigma \quad \text{where} \quad j = 1, 2, \cdots, 2(p-1), \\ w(0) = w_0 \qquad w'(0) = w_1 & \text{in} \quad \Omega, \end{cases}$$

where

$$\mathcal{L}\, w = w'' + b(t)\Delta^{2p} w + \sum_{i,j=1}^{n} \frac{\partial}{\partial y_i}\left(a_{ij}(y,t) \frac{\partial w}{\partial y_j}\right) + \sum_{i=1}^{n} b_i(y,t) \frac{\partial w'}{\partial y_i} + \sum_{i=1}^{n} d_i(y,t) \frac{\partial w}{\partial y_i}.$$

1 Introduction

Let Ω be a bounded domain of $\mathbb{R}^n$ with regular boundary of type C^{4p}, where $p \geq 1$ so that Ω contains the origin of $\mathbb{R}^n$. We consider the continuous function $k : [0, \infty[\to \mathbb{R}$ checking appropriate hypotheses.
Define the subset Ω_t of $\mathbb{R}^n$, as follows

$$\Omega_t = \{\, x \in \mathbb{R}^n : \; x = k(t)y, y \in \Omega \,\} \quad \text{for all} \quad 0 \leq t \leq T$$

with boundary denoted by Γ_t.

We denote by $\widehat{Q}$ the non-cylindrical domain a set of $\mathbb{R}^{n+1}$ defined by

$$\widehat{Q} = \bigcup_{0<t<T} \Omega_t \times \{t\} \quad \text{with boundary} \quad \widehat{\Sigma} = \bigcup_{0<t\;<T} \Gamma_t \times \{t\}.$$

Consider the non homogeneous problem

$$\begin{cases} u''(t) + \Delta^{2p}\, u(t) \; + u(t)^3 \;=\; 0 & \text{in} \quad \widehat{Q} \\ u \;=\; 0, \quad \dfrac{\partial u}{\partial \nu} \;=\; v & \text{on} \quad \widehat{\Sigma} \\ u(0) = u_0 \quad u'(0) = u_1 & \text{on} \quad \Omega_0 \end{cases} \tag{1}$$

Therefore, to solve the problem of exact controllability of the problem (*) will, through the transformation, solve the problem of exact controllability of problem (1.1). We will initially approach the study of exact controllability on the boundary of the problem

$$\begin{cases} \mathcal{L}\,w \;+ w^3 \,=\, 0 & \text{in} \quad \text{Q} \\ \dfrac{\partial^j\, w}{\partial \eta^j} \;=\, 0 & \text{on} \quad \Sigma \quad \text{where} \quad j = 1, 2, \cdots, 2(p-1), \\ \dfrac{\partial^{2p-1}\, w}{\partial \eta^{2p-1}} \;=\, g & \text{on} \quad \Sigma, \\ w(0) = w_0 \qquad w'(0) = w_1 & \text{in} \quad \Omega, \end{cases} \tag{2}$$

where

$$\mathcal{L}\,w = w'' + b(t)\Delta^{2p} w + \sum_{i,j=1}^{n} \frac{\partial}{\partial y_i}\left(a_{ij}(y,t)\frac{\partial w}{\partial y_j}\right) + \sum_{i=1}^{n} b_i(y,t)\frac{\partial w'}{\partial y_i} + \sum_{i=1}^{n} d_i(y,t)\frac{\partial w}{\partial y_i}.$$

2 Main Results

Theorem 2.1. *For $T > T_0$, and for each $\{w_0, w_1\} \in L^2(\Omega) \times H^{-2p}(\Omega) + L^{3/4}(\Omega)$, exist a control function at the boundary $g \in L^2(\Sigma)$, such that the ultraweak solution w of (*) satisfies the final condition*

$$w(T) \,=\, w'(T) \,=\, 0, \qquad \textit{in} \quad \Omega$$

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SOBRE A DINÂMICA DE SOLUÇÕES DO SISTEMA ACOPLADO DE EQUAÇÕES DE SCHRODINGER NO TORO UNIDIMENSIONAL

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Abstract

A proposta deste trabalho é o estudo do problema de Cauchy para um sistema acoplado de equações tipo Schrödinger no toro.

Resultados de boa colocação local deste sistema, para o caso contínuo, foram obtidos em [2]. Neste trabalho obtemos resultados de boa colocação em diferentes regiões do plano que dependem do valor da constante $\sigma > 0$. Discutimos como diferentes valores desta constante mudam a dinâmica do sistema.

1 Introdução

Este trabalho é dedicado ao estudo do Problema de Cauchy para um sistema que modela problemas da óptica não-linear. De maneira mais precisa estudaremos o seguinte modelo matemático

$$\begin{cases} i\partial_t u(x,t) + p\partial_x^2 u(x,t) - \theta u(x,t) + \bar{u}(x,t)v(x,t) = 0, & x \in [0,L],\ t \geq 0, \\ i\sigma\partial_t v(x,t) + q\partial_x^2 v(x,t) - \alpha v(x,t) + \frac{1}{2}u^2(x,t) = 0, & p,q = \pm 1,\ \sigma > 0 \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad (u_0.v_0) \in H^\kappa([0,L]) \times H^s([0,L]). \end{cases} \tag{1}$$

Observamos que o modelo estabelece o acoplamento não-linear de duas equações dispersivas de tipo Schrödinger através de termos quadráticos

$$N_1(u,v) = \bar{u}v \quad \text{e} \quad N_2(u) = \frac{1}{2}u^2. \tag{2}$$

Fisicamente, de acordo com o trabalho [1], as funções complexas u e v representam pacotes de amplitudes do primeiro e segundo harmônico, respectivamente, de uma onda óptica. Os valores de p e q podem ser 1 ou -1, dependendo dos sinais fornecidos entre as relações de dispersão/difração e a constante positiva σ mede os índices de grandeza de dispersão/difração. O interesse em propriedades não-lineares de materiais ópticos têm atraído a atenção de físicos e matemáticos nos últimos anos. Diversas pesquisas sugerem que explorando a reação não-linear da matéria, a capacidade *bit-rate* de fibras ópticas pode ser aumentada substancialmente e consequentemente uma melhoria na velocidade e economia de transmissão e manipulação de dados. Particularmente, em materiais não centrossimétricos (aqueles que não possuem simetria de inversão ao nível molecular) os efeitos não-lineares de ordem mais baixa originam a susceptibilidade de segunda ordem, o que significa que a resposta não-linear para o campo elétrico é de ordem quadrática ver, por exemplo, os artigos [2] e [4].

2 Resultados Principais

Provaremos resultados de boa colocação local para dados $(u_0, v_0) \in H^\kappa([0,L]) \times H^s([0,L])$ com índices $(\kappa, s) \in \mathcal{W}_\sigma$, onde a região plana $\mathcal{W}_\sigma$.

Este trabalho encontra-se em fase de revisão da região do plano $\mathcal{W}_\sigma$ no qual o teorema abaixo é válido.

Teorema 2.1. *Sejam $\sigma > 0$ e $(u_0, v_0) \in H^\kappa \times H^s$ com $(\kappa, s) \in \mathcal{W}_\sigma$, definida em (**??**). O problema de Cauchy (1) é localmente bem posto em $H^\kappa \times H^s$ no seguinte sentido: para cada $\rho > 0$, existem $T = T(\rho) > 0$ e $b > 1/2$ tais que para todo dado inicial com $\|u_0\|_{H^\kappa} + \|v_0\|_{H^s} < \rho$, existe uma única solução (u, v) para (1) satisfazendo as seguintes condições:*

$$\psi_T(t)u \in X^{\kappa,b} \quad e \quad \psi_T(t)v \in X^{s,b}_\sigma, \tag{1}$$

$$u \in C\big([0,T]; H^\kappa\big) \quad e \quad v \in C\big([0,T]; H^s\big). \tag{2}$$

Além disso, a aplicação dado-solução é localmente Lipschitziana.

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BOA COLOCAÇÃO PARA A EQUAÇÃO DE ONDAS LONGAS INTERMEDIÁRIAS REGULARIZADA (RILW)

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Abstract

Apresentam-se neste trabalho os resultados obtidos em [3], que abordam os problemas de boa colocação local e global para a equação de ondas longas intermediárias regularizada (rILW)

$$\eta_t + \eta_x - \frac{3}{2}\alpha\eta\eta_x - \sqrt{\beta}\,\frac{\rho_2}{\rho_1}\mathcal{T}(\eta_{xt}) = 0, \tag{1}$$

nos espaços de Sobolev H^s, $s > \frac{1}{2}$.

A equação (1) é um modelo não linear para a evolução de ondas na interface entre dois fluidos com densidades diferentes, onde $\eta(x,t)$ representa o reescalamento do deslocamento da interface. Ambos os fluidos são considerados invíscidos, imiscíveis, incompressíveis e irrotacionais. A espessura imperturbada da camada inferior (h_2) é comparável ao comprimento de onda característico da interface perturbada (L) e é muito maior que a espessura imperturbada da camada superior. Essa configuração corresponde ao regime de águas rasas para a camada superior e ao regime intermediário para a camada inferior. A versão não-regularizada de (1), conhecida equação de ondas longas intermediárias (ILW), foi primeiramente estudada por Joseph [2] em 1977.

O operador $\mathcal{T}$, conhecido como *transformada de Hilbert na faixa de espessura* $h = \frac{h_2}{L} > 0$, é definido no domínio da frequência por

$$\widehat{\mathcal{T}f}(k) = i\coth(hk)\widehat{f}(k), \qquad k \in \mathbb{R} \text{ (or } \mathbb{Z}),\ k \neq 0,$$

onde $\widehat{}$ indica a transformada de Fourier. As constantes positivas α e β que aparecem em (1) são chamadas parâmetro não linear e parâmetro dispersivo, respectivamente.

Seguem abaixo os resultados obtidos para a boa colocação local e global, que são válidos tanto no domínio periódico quanto no não-periódico:

Teorema 0.2. *Sejam $s > \frac{1}{2}$ e $\phi \in H^s$, então existe $T = T(s, \|\phi\|_s) > 0$ tal que o problema de Cauchy não linear*

$$\begin{cases} \eta \in C([-T,T], H^s) \\ \eta_t + \eta_x - \frac{3}{2}\alpha\eta\eta_x - \sqrt{\beta}\,\frac{\rho_2}{\rho_1}\mathcal{T}(\eta_{xt}) = 0 \ \in \ H^s \\ \eta(0) = \phi \ \in H^s, \end{cases}$$

é localmente bem-posto.

A existência de solução local é demonstrada a partir do teorema do ponto fixo de Banach e de propriedades do operador $\mathcal{T}$ e do espaço de Sobolev considerado. A desigualdade de Gronwall garante a unicidade de solução e um argumento envolvendo o intervalo maximal de existência leva à continuidade da solução com relação aos dados inciais.

Teorema 0.3. *Sejam $s > \frac{1}{2}$ e $\phi \in H^s$, então o problema de Cauchy não linear*

$$\begin{cases} \eta \in C\left(\mathbb{R}, H^s\right) \\ \eta_t + \eta_x - \dfrac{3}{2}\alpha\eta\eta_x - \sqrt{\beta}\,\dfrac{\rho_2}{\rho_1}\mathcal{T}(\eta_{xt}) = 0 \ \in \ H^s \\ \eta(0) = \phi \ \in H^s, \end{cases}$$

é globalmente bem-posto.

A boa colocação global é obtida combinando o princípio da extensão com uma estimativa global a priori para as soluções locais na norma H^s. A desigualdade do tipo Brezis-Gallouet

$$\|\eta\|_\infty \leq C\left(1 + \sqrt{\log\left(1 + \|\eta\|_s\right)}\right)\|\eta\|_{\frac{1}{2}},$$

proposta por Angulo, Scialom e Banquet em [1], é essencial para a obtenção do resultado global.

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DECAY RATES FOR A POROUS-ELASTIC SYSTEM

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Abstract

In this work we analyze the porous elastic system, where we use two dissipative mechanisms, one of the viscoelastic type and the other porous, which act in the same equation and are not strong enough to make the solutions decay in an exponential way, independently of any relationship between the coefficients of wave propagation speed, that is , we show that the resolvent operator is not limited uniformly along the imaginary axis. However, it decays polynomially with optimal rate.

1 Introduction

In this work we present a porous elastic system with two dissipative mechanisms is considered. Thus, the system of equations considered here is

$$\begin{cases} \rho u_{tt} - \mu u_{xx} - b\phi_x = 0 \ \text{ in } (0,L)\times(0,\infty), \\ J\phi_{tt} - \delta\phi_{xx} + bu_x + \xi\phi + \tau\phi_t - \gamma\phi_{xxt} = 0 \ \text{ in } (0,L)\times(0,\infty). \end{cases} \tag{1}$$

We added to system (1) the initial conditions given by

$$\begin{aligned} u(x,0) &= u_0(x),\ u_t(x,0) = u_1(x), \\ \phi_x(0,t) &= \phi_0(x),\ \phi_t(x,0) = \phi_1(x),\ \forall x \in (0,L), \end{aligned} \tag{2}$$

and Dirichlet-Neumann boundary conditions

$$u(0,t) = u(L,t) = \phi_x(0,t) = \phi_x(L,t) = 0,\ \forall t > 0. \tag{3}$$

Two dissipative mechanisms are present in the system (1), one of the viscoelastic type (Kelvin-Voigt) and the other porous, with τ and γ nonnegative, which act in the same equation, volume fraction.
The constitutive coefficients, in one-dimensional case (see [1, 4]), satisfy

$$\xi > 0,\ \delta > 0,\ \mu > 0,\ \rho > 0,\ J > 0, \text{ and } \mu\xi \geq b^2. \tag{4}$$

Let us consider the Hilbert space and inner product given by

$$\mathcal{H} = H_0^1(0,L)\times L^2(0,L)\times H_*^1(0,L)\times L_*^2(0,L), \tag{5}$$

$$\langle U,V\rangle_{\mathcal{H}} := \int_0^L \left(\rho\varphi\overline{\Phi} + \mu u_x\overline{v}_x + J\psi\overline{\Psi} + \delta\phi_x\overline{\zeta}_x + \xi\phi\overline{\zeta} + b(u_x\overline{\zeta} + \overline{v}_x\phi)\right)\,dx, \tag{6}$$

with $U = (u,\varphi,\phi,\psi)' \in \mathcal{D}(\mathcal{A})$, where the operator $\mathcal{A}$ is given by

$$\mathcal{A}U = \mathcal{A}\{u,\varphi,\phi,\psi\} = \{\varphi, \frac{\mu}{\rho}u_{xx} + \frac{b}{\rho}\phi_x, \psi, \frac{\delta}{J}\phi_{xx} - \frac{b}{J}u_x - \frac{\xi}{J}\phi - \frac{\tau}{J}\phi_t + \frac{\gamma}{J}\phi_{xxt}\},$$

2 Main Results

Theorem 2.1. *The operator $\mathcal{A}$ generates a C_0semigroup $S(t)$ of contraction on $\mathcal{H}$. Thus, for any initial data $U_0 \in \mathcal{H}$, the system (1)-(3) has a unique mild solution $U \in C^0([0,\infty[;\ \mathcal{H})$. Moreover, if $U_0 \in \mathcal{D}(\mathcal{A})$, then U is the classical solution of (1)-(3), that is $U \in C^0([0,\infty[;\ \mathcal{D}(\mathcal{A})\,) \cap C^1([0,\infty[;\ \mathcal{H})$.*

Theorem 2.2. *Let $S(t) = e^{\mathcal{A}}t$ the C_0-semigroup of contraction on Hilbert space $\mathcal{H}$ associated with the system (1)-(3). Then $S(t)$ is not exponentially stable, independently of any relationship between the coefficients μ, ρ, δ e J.*

Proof: To do this, we use the following well-known result due to *Gearhart-Herbst-Prüss-Huang* for dissipative sistems, from semigroup theory (see [2, 2]), where we will argue by contradiction, that is, we will show that there exists a sequence of imaginary number λ_n with $\lim_{n\to\infty} |\lambda_n| = \infty$ and $U_n = (u, \varphi, \phi, \psi)' \subset \mathcal{D}(\mathcal{A})$ for $F_n = (f^1, f^2, f^3, f^4)' \in \mathcal{H}$, with $\|F_n\|_{\mathcal{H}} \leq 1$, that is F_n is bounded in $\mathcal{H}$, such that $\lambda_n U_n - \mathcal{A}U_n = F_n$.

Lemma 2.1. *For the system (1)-(3), we have $i\mathbb{R} \subset \rho(\mathcal{A})$.*

Theorem 2.3. *The semigroup $S(t) = e^{\mathcal{A}t}$ associated with the system (1)-(3), satisfies*

$$\|S(t)U_0\|_{\mathcal{H}} \leq \frac{M}{\sqrt{t}} \|U_0\|_{\mathcal{D}(\mathcal{A})}, \forall U_0 \in \mathcal{D}(\mathcal{A}).$$

Moreover, this rate is optimal.

Proof: Let $U = (u, \varphi, \phi, \psi)^{'}$ the system solution (1)-(3), we can get the inequalities

$$\left(1 - \frac{1}{|\lambda|}\right) \int_0^L |\phi_x|^2\, dx \leq \frac{C}{|\lambda|} \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}}, \tag{1}$$

$$\delta \int_0^L |\phi_x|^2\, dx + b \int_0^L u_x \overline{\phi}\, dx + \xi \int_0^L |\phi|^2\, dx \leq \frac{C}{|\lambda|} \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + C\|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}}, \tag{2}$$

$$\frac{\mu}{2} \int_0^L |u_x|^2\, dx \leq \varepsilon\rho \int_0^L |\varphi|^2\, dx + C|\lambda|^2 \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + C_\varepsilon |\lambda|^2 \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + \frac{C}{|\lambda|} \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + C\|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}}, \tag{3}$$

$$\rho \int_0^L |\varphi|^2\, dx \leq C|\lambda|^2 \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + C_\varepsilon |\lambda|^2 \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + \frac{C}{|\lambda|} \|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}} + C\|U\|_{\mathcal{H}} \|F\|_{\mathcal{H}}, \tag{4}$$

to any $\varepsilon > 0$ there is a constant $C_\varepsilon > 0$, and for $|\lambda|$ larg enough and C a positive constant that does not depend λ. Then using (1)-(4), lemma 2.1 and the Theorem due to A. Borichev and Y. Tomilov (see [1], **Theorem 2.4**), one has the conclusion of Theorem.

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EXISTENCE AND DECAY OF SOLUTIONS TO A GENERALIZED FRACTIONAL SEMILINEAR TYPE PLATE EQUATION

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Abstract

In this work we study the existence and uniqueness of solutions for a linear and semilinear type plate equation with fractional damping term and under effects of a generalized rotational inertia term in the case of plate equation.

1 Introduction

We consider in this work the following Cauchy problem for the plate/Boussinesq type equation with a fractional damping and a generalized fractional rotational inertia term in $\mathbb{R}^n$:

$$\begin{cases} \partial_t^2 u + (-\Delta)^{\delta}\partial_t^2 u + \Delta^2 u + a(-\Delta)^{\alpha} u + (-\Delta)^{\theta}\partial_t u = \beta(-\Delta)^{\gamma}\, u^p, \\ u(0,\, x) = u_0(x), \quad \partial_t u(0,\, x) = u_1(x) \end{cases} \tag{1}$$

with $u = u(t, x)$, $(t, x) \in\,]0,\ \infty[\times\mathbb{R}^n$, $a > 0$, $\beta \in \mathbb{R}$, $p > 1$ integers and u_0, u_1 are the initial date. The Laplacian power δ, θ and γ are such that $0 \leq \delta \leq 2$, $0 \leq \theta \leq (2+\delta)/2$ and $0 \leq \gamma \leq (2+\delta)/2$, but in some cases we need to restrict more the Laplacian power.

2 Linear Problem

Through the Semigroup Theory we will show the existence and uniqueness of solutions to the following Cauchy problem associated with an equation of plates with structural rotational inertia and fractional dissipation in $\mathbb{R}^n$ with $n \geq 1$,

$$\begin{cases} \partial_t^2 u + (-\Delta)^{\delta}\partial_t^2 u + \Delta^2 u + a(-\Delta)^{\alpha} u + (-\Delta)^{\theta}\partial_t u = 0 \\ u(0,\, x) = u_0(x), \quad \partial_t u(0,\, x) = u_1(x), \end{cases} \tag{1}$$

where $u = u(t, x)$, with $(t, x) \in\,]0,\ \infty[\times\mathbb{R}^n$ and $a > 0$ is a constant. The potential of Laplacian δ, α, θ and are such that $0 \leq \delta \leq 2$, $0 \leq \alpha \leq 2$ and $0 \leq \theta \leq (2+\delta)/2$.

To show the existence and uniqueness of the solution, let's divide the problem into cases and for each case consider spaces for the energy that are acquired.

Using the energy space $X = H^2(\mathbb{R}^n) \times H^{\delta}(\mathbb{R}^n)$ we can rewrite the problem (1) in matrix form

$$\begin{cases} \dfrac{dU}{dt} = BU + J(U) \qquad \text{for} \quad t > 0 \\ U(0) = U_0, \end{cases}$$

where $U = (u,\, \partial_t u)$, $U(0) = (u_0,\, u_1)$ and the operators B and J adequate for each case.

Using the Lumer Phillips Theorem we proof that B is the infinitesimal generator of contraction semigroup of class C_0 in X and that J is a bounded operator in X, that is, exist only one solution for Cauchy Problem (1).

Theorem 2.1. *Let $n \geq 1$, $0 \leq \delta \leq 2$ and $0 \leq \theta \leq (2+\delta)/2$. If $u_0 \in H^{4-\delta}(\mathbb{R}^n)$ e $u_1 \in H^2(\mathbb{R}^n)$ then the Cauchy Problem (1) have only one solution u in following class*

$$u \in C^2\big([0,\, \infty[;\, H^{\delta}(\mathbb{R}^n)\big) \cap C^1\big([0,\, \infty[;\; H^2(\mathbb{R}^n)\big) \cap C\left([0,\, \infty[;\, H^{4-\delta}(\mathbb{R}^n)\right).$$

2.1 Semilinear Problem

We consider the following Cauchy problem for the semilinear equation in $\mathbb{R}^n$ of the Plates-Boussineq type with a fractional damping and, in the case of plates, a generalized rotational inertia type term

$$\begin{cases} \partial_t^2 u + (-\Delta)^{\delta} \partial_t^2 u + \Delta^2\, u + a(-\Delta)^{\alpha} u + (-\Delta)^{\theta} \partial_t u = \beta(-\Delta)^{\gamma}\, u^p, \\ u(0,\, x) = u_0(x), \quad \partial_t u(0,\, x) = u_1(x), \end{cases} \tag{2}$$

where $u = u(t,\, x)$, with $(t,\, x) \in\,]0,\, \infty[\times \mathbb{R}^n$, $a > 0$, $\beta \neq 0$ and $p > 1$ integer. The fractional powers of the Laplacian operator are considered as follows $0 \leq \delta \leq 2$, $0 \leq \alpha \leq 2$, $0 \leq \theta \leq (2+\delta)/2$ and $1/2 \leq \gamma \leq (2+\delta)/2$.

We reduce the order of the Cauchy Problem (2) and rewrite it in the following matrix form

$$\begin{cases} \dfrac{dU}{dt} = BU + F(U) \qquad \text{if} \quad t > 0 \\ U(0) = U_0 \end{cases}$$

where $U = (u,\, \partial_t u)$, $U_0 = (u_0,\, u_1)$ and the operator B is define in the Section 2 of according to each both cases mentioned above and is the infinitesimal generator of contraction of semigroup of class C_0 in X. The operator F is the operator which contains the non-linear term.

Theorem 2.2. *Let $0 \leq \theta < \delta$, $0 \leq \delta \leq 2$, $0 \leq \gamma \leq (2+\delta)/2$, $p > 1$ integer and $0 < n < 8 - 4\delta$. Then, for initial data $(u_0,\, u_1) \in H^{4-\delta}(\mathbb{R}^n) \times H^2(\mathbb{R}^n)$ there exist only one solution for semilinear Cauchy problem (2) defined in a maximal interval $[0,\, T_m[$ in class*

$$u \in C^2\big([0,\, T_m[;\; H^{\delta}(\mathbb{R}^n)\big) \cap C^1\big([0,\, T_m[;\; H^2(\mathbb{R}^n)\big) \cap C\big([0,\, T_m[;\; H^{4-\delta}(\mathbb{R}^n)\big),$$

with one and only one of the following conditions true

$$\textbf{i)} \quad T_m = \infty \qquad\qquad \textbf{ii)} \quad T_m < \infty \quad \text{and} \quad \lim_{t \to T_m} \|U\|_X + \|B_1 U\|_X = \infty.$$

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ON A NONLINEAR ELASTICITY SYSTEM WITH NEGATIVE ENERGY

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Abstract

The objective of this paper is to show the existence of global solutions of a nonlinear elasticity system with nonlinear boundary conditions which has negative energy..

1 Introduction

Let Ω be a bounded open set of $\mathbb{R}^n$ with boundary Γ of class C^2. We consider $\{\Gamma_0, \Gamma_1\}$ a partition of Γ, with Γ_0 and Γ_1 having positive Lebesgue measure and $\overline{\Gamma_0} \cap \overline{\Gamma_1} = \emptyset$. By $\nu = \nu(x)$ is denoted the unit outward normal vector at $x \in \Gamma$.

The purpose of this work is to show the existence of global solutions of the following mixed problem:

$$\left| \begin{array}{l} u''(x,\,t) - \mu b(t)\Delta u(x,\,t) - (\lambda+\mu)b(t)\nabla div(u(x,\,t)) + |u(x,\,t)|^{\rho} = 0 \text{ in } \Omega \times (0,\infty); \\ u = 0 \text{ on } \Gamma_0 \times (0,\,\infty); \\ \mu b(t)\dfrac{\partial u}{\partial \nu}(x,\,t) + (\lambda+\mu)\, b(t) div(u(x,\,t))\,\nu(x) + \delta(x) h(u'(x,\,t)) = 0 \text{ on } \Gamma_1 \times (0,\,\infty)\,; \\ u(x,\,0) = u^0(x),\; u'(x,\,0) = u^1(x) \text{ in } \Omega, \end{array} \right. \tag{1}$$

where $u(x,\,t) = (u_1(x,\,t),\ldots,u_n(x,\,t))$, $\Delta u = (\Delta u_1,\ldots,\Delta u_n)$, $\nabla div(u) = \left(\frac{\partial}{\partial x_1}(div(u)),\ldots,\frac{\partial}{\partial x_n}(div(u))\right)$, $div(u) = \displaystyle\sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i}$, $|u|^{\rho} := (|u_1|^{\rho},\ldots,|u_n|^{\rho})$, $\rho > 1$, $h(x) := (h_1(x_1),...,h_n(x_n))$, $x \in \mathbb{R}^n$, and $b(t), \delta(x)$ are functions defined on $[0,\infty)$ and Γ_1, respectively. Here $\lambda \geq 0$ and $\mu > 0$ are the Lamé's constants of the material.

2 Main Results

Consider the Hilbert space $V = (H^1_{\Gamma_0}(\Omega))^n$ where $H^1_{\Gamma_0}(\Omega) := \{v \in H^1(\Omega) : v = 0 \text{ on } \Gamma_0\}$ is equipped with the gradient norm. The space V is provided with the scalar product $((u,v))_V = \mu((u,v))_{(H^1_{\Gamma_0}(\Omega))^n} + (\lambda + \mu)(div\, u, div\, v)_{L^2(\Omega)}$. Note that $((u,v))_{(H^1_{\Gamma_0}(\Omega))^n}$ and $((u,v))_V$ are equivalent in V. Consider $H = (L^2(\Omega))^n$, provided with the scalar product $(u,v)_H = \sum_{i=1}^n (u_i,v_i)_{L^2(\Omega)}$ and the Hilbert space $W = \{u \in V; Au \in H\}$, where $Au = -\mu\triangle u - (\lambda+\mu)\nabla div\, u$. This space is equipped with the scalar product $(u,v)_W = ((u,v))_V + (Au,Av)_H$.

Introduce the following hypotheses:

- **(H.1)**. $b \in W^{1,\infty}_{loc}(0,\,\infty)$, $b(t) \geq b_0 > 0$, $b' \in L^1(0,\infty)$.
- **(H.2)**. $\delta \in W^{1,\infty}(\Gamma_1)$, $\delta(x) \geq \delta_0 > 0$.
- **(H.3)**. The real number ρ has the following restrictions $\rho > 1$ if $n = 1, 2$; $\frac{n+1}{n} \leq \rho \leq \frac{n}{n-2}$ if $n \geq 3$.
- **(H.4)**. Consider $\lambda^* = \left(\dfrac{\mu b_0}{3nk_0^{\rho+1}}\right)^{\frac{1}{\rho-1}}$ and $N^* = \dfrac{\mu b_0(\lambda^*)^2}{4exp\left(\int_0^\infty \frac{3}{b_0}|b'(s)|ds\right)}$.

where the constant k_0 verifies

$$||v||_{L^{\rho+1}(\Omega)} \leq k_0||v||_{H^1_{\Gamma_0}(\Omega)}\ ,\ \forall v \in H^1_{\Gamma_0}(\Omega).$$

• **(H.5)**. Each component $h_i(s)$, $s \in \mathbb{R}$, $i = 1, ..., n$, of the vectorial function h, satisfies

$$\left| \begin{array}{ll} h_i & \text{is a Lipschitz continuous function with } h_0(0) = 0; \\ h_i & \text{is a strongly monotonous function, that is,} \\ & (h_i(r) - h_i(s))(r - s) \geq d_0(r - s)^2, \ \forall r, s \in \mathbb{R} \ (d_0 \text{ positive constant}). \end{array} \right.$$

Theorem 2.1. *If $u \in W$, then $\gamma_1 u \in (H^{-\frac{1}{2}}(\Gamma_1))^n$ where $\gamma_1 u = \mu \dfrac{\partial u}{\partial \nu} + (\lambda + \mu)(div\, u)\nu$.*

Proof The proof is obtained by constructing a one order trace for functions $u \in W$.

■

Let A be the self-adjoint operator of H defined by the triplet $\{V, H, ((u, v))_V\}$. Then $D(A) = \{u \in W; \gamma_1 = 0 \text{ on } \Gamma_1\}$ and $Au = -\mu \triangle u - (\lambda + \mu)\nabla div\ u$.

Theorem 2.2. *Consider $u^0 \in D(A)$ and $u^1 \in (H_0^1(\Omega))^n$ such that*
(i) $\|u^0\|_V < \lambda^*$
(ii) $N = \dfrac{1}{2}|u^1|_H^2 + \dfrac{1}{4}\left[\mu b_0 \|u^0\|_V\right] + \dfrac{1}{2}[(\lambda + \mu) b_0 |div\, u^0|_H^2] + \dfrac{n}{\rho + 1} k_0^{\rho+1} \|u^0\|_V^{\rho+1} < N^*.$
Assume $(H1)$-$(H5)$. Then there exists a function u in the class

$$u \in L^\infty(0, T; V) \cap L^2_{loc}(0, \infty; W), \ u' \in L^\infty(0, T; H) \cap L^\infty_{loc}(0, T; V), \ u'' \in L^\infty(0, \infty; H) \text{ and } div u \in L^\infty(0, \infty; L^2(\Omega))$$

such that u satisfies

$$\left| \begin{array}{l} u'' - \mu b \Delta u - (\lambda + \mu) b \nabla div\, u + |u|^\rho = 0 \ in\, L^2_{loc}(0, \infty; H) + L^2_{loc}(0, \infty; W), \\ \mu b \dfrac{\partial u}{\partial \nu} + (\lambda + \mu) b\, (div\, u)\, \nu + \delta(\cdot) h(u') = 0 \ in\, L^\infty_{loc}(0, \infty; (L^2(\Gamma_1))^n), \\ u(0) = u^0, \ u'(0) = u^{\cdot} \end{array} \right.$$

Proof The theorem is deriving by using the Galerkin method with a special basis, a modification of the Tartar approach, compactness argument and Theorem 2.1.

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SMALL VIBRATIONS OF A BAR

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Abstract

This paper is concerned with the existence of global solutions of a mathematical model that describes the small vibrations of the cross sections of a bar which is clamped in one end and the other end is glued a spring

1 Introduction

Consider a bar of length L which is clamped in one end and the other end is glued a spring. On the action of a force F on the set, the cross sections of the bar begin to vibrate longitudinally. The linear model of this physical phenomenon was given by Timoshenho [2] . In this paper we introduce a nonlinear model of the same phenomenon, which is obtained by using a nonlinear Hooke's law. This mathematical problem has the form:

$$\left| \begin{array}{l} \rho A u''(x,t) - \frac{\partial}{\partial x}\sigma(u_x(x,t)) = 0,\ 0<x<L, t>0; \\ u(0,t)=0\,,\ \sigma(u_x(L,t)) + ku(L,t) = 0\,,\ t>0; \\ u(x,0) = u^0(x)\,,\ u'(x,0) = u^1(x)\,,\ 0<x<L. \end{array} \right. \tag{1}$$

where $u(x,t)$ denotes the displacement of the cross section x of the bar at time t. Here ρ and A represent the constant density and the area of the uniform cross section, respectively, of the bar and $k>0$ denotes the stifness constant of the spring.

The n-dimensional formulation of problem (1) with an internal damping is the following:

$$\left| \begin{array}{l} u''(x,t) - \displaystyle\sum_{i=1}^{n} \frac{\partial}{\partial x_i}\left[\sigma_i(\frac{\partial u}{\partial x_i}) + \frac{\partial u'}{\partial x_i}\right] = 0 \text{ in } \Omega\times(0,\infty); \\ u = 0 \text{ on } \Gamma_0\times(0,\infty); \\ \displaystyle\sum_{i=1}^{n}\left[\sigma_i(\frac{\partial u}{\partial x_i}) + \frac{\partial u'}{\partial x_i}\right]\nu_i + ku = 0 \text{ on } \Gamma_1\times(0,\infty); \\ u(0) = u^0\ ,\ u'(0) = u^1 \text{ in } \Omega. \end{array} \right. \tag{2}$$

where Ω is an open bounded set of $\mathbb{R}^n$ whose boundary Γ is constituted of two disjoint nonempty parts Γ_0 and Γ_1, and $\nu(x) = (\nu_1(x),...,\nu_n(x))$ is the exterior unit normal at $x\in\Gamma_1$.

We introduce the Hilbert space

$$H^1_{\Gamma_0}(\Omega) = \{u\in H^1(\Omega); u = 0 \,\text{on}\, \Gamma_0\}$$

equipped with the gradient norm

2 Main Results

Theorem 2.1. *Assume that the real functions $\sigma_i (i = 1, ..., n)$ satisfy*

$$\sigma_i \text{ is globally Lipschitz}, \, \sigma_i \text{ is increasing and} \, \sigma_i(0) = 0.$$

Consider

$$u^0 \, , \, u^1 \in H^1_0(\Omega) \cap H^2(\Omega) \text{ with } \frac{\partial u^0}{\partial \nu} = \frac{\partial u^1}{\partial \nu} = 0 \, on \, \Gamma_1.$$

Then there exits a unique function u in the class

$$\begin{array}{l} u \in L^\infty_{loc}(0, \infty; H^1_{\Gamma_0}(\Omega)) \cap L^\infty(0, \infty; L^2(\Gamma_1)); \\ u' \in L^\infty(0, \infty; L^2(\Omega)) \cap L^2(0, \infty; H^1_{\Gamma_0}(\Omega)); \\ u'' \in L^\infty(0, \infty; L^2(\Omega)) \cap L^2(0, \infty; H^1_{\Gamma_0}(\Omega)); \end{array} \quad (1)$$

such that u is the solution of problem (2)

Let $E(t)$ be the energy of Problem (2), that is,

$$E(t) = \frac{1}{2}|u'(t)|^2_{L^2(\Omega)} + \sum_{i=1}^{n} \int_\Omega \hat{\sigma}_i(\frac{\partial u}{\partial x_i}) \, dx + |u|^2_{L^2(\Gamma_1)} \, , \; t \geq 0,$$

where $\hat{\sigma}_i = \int_0^s \sigma_i(\tau) \, d\tau \; i = 1, ...n.$

Theorem 2.2. *Assume that there exists a constant $b > 0$ such that*

$$s^2 \leq b\hat{\sigma}_i(s) \, , \, \forall s \in \mathbb{R} \, , \, i = 1, ..., n.$$

Let u be the solution obtained in Theorem 2.1. Then

$$E(t) \leq 3E(0) exp(-\frac{2}{3}\eta t) \;, \; \forall t \geq 0$$

for some positive constant η.

In the proof of Theorem 2.1 we apply the Galerkin method with a special basis, the theory of monotone operators(cf. J.L.Lions [1] and Medeiros-Pereira [2]) and results of the trace of non smooth functions. The decay of solutions is derived by using a Liapunov functional.

In [1] can be seen a problem related to (2) .

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A POHOZAEV IDENTITY FOR A CLASS OF ELLIPTIC HAMILTONIAN SYSTEMS AND THE LANE-ENDEM CONJECTURE

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Abstract

In this talk we discuss about nonexistence of solution for a class of Hamiltonian elliptic systems involving Schrödinger equations. In this direction we prove a general nonexistence result that provide, as a particular case, a partial answer for the Lane-Emden conjecture.

1 Introduction

The purpose of this work is to study nonexistence of solution result for the following class of hamiltonian elliptic systems

$$\begin{cases} -\Delta v + V(x)v = f(x,u) & \text{in } \mathbb{R}^N, \\ -\Delta u + V(x)u = b(x)g(v) & \text{in } \mathbb{R}^N. \end{cases} \tag{S}$$

To illustrate this difficult when dealing with hamiltonian systems, let us consider $f \sim |u|^{p-2}u$ and $g \sim |v|^{p-2}v$. It was discussed in [2, 3] that the correct notion of subcriticallity with respect to (S) occurs when

$$\frac{1}{p} + \frac{1}{q} > 1 - \frac{2}{N},\ p,q > 1, \tag{$\mathcal{H}_{sub}$}$$

while the true notion of criticality is given for (p,q) such that

$$\frac{1}{p} + \frac{1}{q} = 1 - \frac{2}{N},\ p,q > 1,\ N \geq 3, \tag{$\mathcal{H}_{crit}$}$$

i.e., (p,q) lies on the so called Sobolev *critical hyperbola*.

2 Main Results

Our main result is the key to prove general nonexistence results for System (S) and it asserts that some solutions of (S) must satisfy a certain integral identity. The proof follows by a "local-to-global" cutoff argument.

Theorem 2.1. *Let* $V(x) \in C^1(\mathbb{R}^N \setminus \mathcal{O})$, *where* $\mathcal{O}$ *is a finite set,* $0 < \beta \leq b(x) \in L^\infty(\mathbb{R}^N) \cap C^1(\mathbb{R}^N)$, $u \in W^{2,q/(q-1)}_{loc}(\mathbb{R}^N)$ *and* $v \in W^{2,p/(p-1)}_{loc}(\mathbb{R}^N)$ *be a pair of strong solution for* (S). *Suppose that*

$$F(x,u),\ b(x)G(v),\ \sum_{i=1}^N x_i F_{x_i}(x,u),\ \langle \nabla b(x), x\rangle G(v),\ V(x)uv,\ \langle \nabla V(x), x\rangle uv,\ uf(x,u),\ vb(x)g(v) \in L^1(\mathbb{R}^N)$$

$$\text{and} \qquad 2\int_{\mathbb{R}^N} \langle \nabla u, \nabla v\rangle\, dx = \int_{\mathbb{R}^N} v(b(x)g(v) - V(x)u) + u(f(x,u) - V(x)v)dx. \tag{1}$$

Then the following Pohozaev type identity holds

$$N \int_{\mathbb{R}^N} F(x,u) + b(x)G(v)dx + \sum_{i=1}^{N} \int_{\mathbb{R}^N} x_i F_{x_i}(x,u)dx + \int_{\mathbb{R}^N} \langle x, \nabla b(x) \rangle G(v)dx = \frac{N-2}{2} \int_{\mathbb{R}^N} uf(x,u) + vb(x)g(v)dx + \int_{\mathbb{R}^N} [2V(x) + \langle x, \nabla V(x) \rangle] uvdx. \quad (2)$$

As a consequence of Theorem 2.1, next we have a partial answer for the Lane-Endem conjecture for weak solutions on second order Sobolev spaces (see [4, 5]), which involves the region above the critical hyperbola

$$\frac{1}{p} + \frac{1}{q} < 1 - \frac{2}{N}, \ p, q > 1, \ N \geq 3. \quad (\mathcal{H}_{sup})$$

Corollary 2.1. *i) Assume* $(\mathcal{H}_{crit})$ *or* $(\mathcal{H}_{sup})$ *and let* $u \in W^{2,m}_{loc}(\mathbb{R}^N) \cap W^{1,mN/(N-m)}(\mathbb{R}^N), m = q/(q-1)$, *and* $v \in W^{2,l}_{loc}(\mathbb{R}^N) \cap W^{1,lN/(N-l)}(\mathbb{R}^N), l = p/(p-1)$, *be a pair of weak solution for the System*

$$\begin{cases} -\Delta v + v = |u|^{p-2}u & in \quad \mathbb{R}^N, \\ -\Delta u + u = |v|^{q-2}v & in \quad \mathbb{R}^N. \end{cases}$$

Then $u = v = 0$.

ii) Suppose that $(\mathcal{H}_{sub})$ *holds. Let* $u \in \mathcal{D}^{2,m}(\mathbb{R}^N) \cap \mathcal{D}^{1,mN/(N-m)}(\mathbb{R}^N)$ *and* $v \in \mathcal{D}^{2,l}(\mathbb{R}^N) \cap \mathcal{D}^{1,lN/(N-l)}(\mathbb{R}^N)$ *be a pair of strong solution for*

$$\begin{cases} -\Delta v = |u|^{p-2}u & in \quad \mathbb{R}^N, \\ -\Delta u = |v|^{q-2}v & in \quad \mathbb{R}^N, \ N \geq 3. \end{cases}$$

Then $u = v = 0$.

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MULTIPLICITY OF SOLUTIONS FOR (Φ_1, Φ_2)-LAPLACIAN SYSTEMS INCLUDING SINGULAR NONLINEARITIES

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Abstract

It is established existence of bound and ground state solutions for quasilinear elliptic systems driven by (Φ_1, Φ_2)-Laplacian operator. The main contribution here is obtaining multiplicity of non-negative solutions in the context of quasilinear elliptic systems involving both singular and nonsingular nonlinearities in the presence of a convex superlinear subcritical coupled term.

1 Introduction

In this work we consider both the singular-cooperative and the nonsingular-mixed quasilinear elliptic system driven by the (Φ_1, Φ_2)-Laplacian operator

$$\begin{cases} -\Delta_{\Phi_1} u &= \lambda a(x)|u|^{q-2}u + \frac{\alpha}{\alpha+\beta} b(x)|u|^{\alpha-2}u|v|^{\beta} \text{ in } \Omega, \\ -\Delta_{\Phi_2} v &= \mu c(x)|v|^{q-2}v + \frac{\beta}{\alpha+\beta} b(x)|u|^{\alpha}|v|^{\beta-2}v \text{ in } \Omega, \\ \quad u &= v = 0 \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain with $N \geq 2$ and $\Delta_{\Phi_i} u = \operatorname{div}(\phi_i(|\nabla u|)\nabla u)$ with, $\Phi_i(t) := \int_0^{|t|} s\phi_i(s)ds, t \in \mathbb{R}, i = 1, 2$. We begin by considering the continuous potentials $a, b, c : \Omega \to \mathbb{R}$ on $L^\infty(\Omega)$ and taking C^2-functions $\phi_i : (0, \infty) \to (0, \infty)$ satisfying:

(ϕ_1) $\lim_{t\to 0} t\phi_i(t) = 0, \lim_{t\to\infty} t\phi_i(t) = \infty$;

(ϕ_2) $t \mapsto t\phi_i(t)$ is strictly increasing;

(ϕ_3) $-1 < \ell_i - 2 := \inf_{t>0} \dfrac{(t\phi_i(t))''t}{(t\phi_i(t))'} \leq \sup_{t>0} \dfrac{(t\phi_i(t))''t}{(t\phi_i(t))'} =: m_i - 2 < N - 2, i = 1, 2.$

About the powers, let us assume

(H) $0 < q < \dfrac{(\alpha+\beta-1)\min\{\ell_i\} - \max\{m_i(m_i-1)\}}{\alpha+\beta-\min\{\ell_i\}} \leq \ell_i \leq m_i < \alpha+\beta < \min\{\ell_i^*\}, i = 1, 2.$

Our main interest is to ensure the existence of ground state (minimum energy) and bound state (finite energy) solutions to the problem (1) both to the singular and nonsingular cases.

For nonsingular pertubations, particular forms of the System (1) have been much considered in recently years. These variety of works deal since particular forms of the (Φ_1, Φ_2)-operator, passing to cooperative and non-cooperatives structures, going to consider subcritical, critical and supercitical behavior of the coupled term. More details about nonhomogeneous differential operators with different types of nonlinearity Φ can be found in [1, 2, 3, 6] and references therein. About singular elliptic systems, the are few results dealing System (1) in the context of the (Φ_1, Φ_2)-Laplacian operator. The main difficulty in approaching singular elliptic problems by variational methods comes from the fact that the its energy functional is not in the C^1-class anymore. It is important to emphasize that the scalar case have been widely explored in last years. We quote, for instance, [4, 5] and references therein.

2 Main Results

Below, let us state our main results beginning with the non-singular case. To do this, let us assume:

(A) b is a continuous function satisfying $||b||_\infty = 1$ and $b^+ \neq 0$,

(B) a, c are also continuous functions that satisfy $||a||_\infty = ||c||_\infty = 1$, $a^+ \neq 0$ and $c^+ \neq 0$.

Theorem 2.1 (Nonsingular Case)**.** *Assume that* $(\phi_1) - (\phi_3)$*,* (A)*,* (B) *and* (H) *hold. If* $q > 1$*, then there exists a* $\lambda_\star > 0$ *such that System* (1) *admits at least two nonnegative solutions, for each* $\lambda, \mu \geq 0$ *given satisfying* $0 < \lambda + \mu \leq \lambda_\star$*, being one solution a ground state* $\bar{z}_{\lambda,\mu}$ *and the other one a bound state* $\tilde{z}_{\lambda,\mu}$*, Besides this,* $\bar{z}_{\lambda,\mu}, \tilde{z}_{\lambda,\mu} \in W \setminus \{0, z_1, z_2\}$*,* $J(\bar{z}_{\lambda,\mu}) < 0 < J(\tilde{z}_{\lambda,\mu})$ *and* $\lim_{\lambda,\mu \to 0^+} \|\bar{z}_{\lambda,\mu}\| = 0$.

For the singular case, let us consider the assumption.

(C) a, c and b are nennegative continuous functions satisfying $||a||_\infty = ||b||_\infty = ||c||_\infty = 1$.

Theorem 2.2 (Singular Case)**.** *Assume that* $(\phi_1) - (\phi_3)$*,* (C) *and* (H) *hold. If* $0 < q < 1$*, then there exists a* $\lambda^\star > 0$ *such that System* (1) *admits at least two positive solutions, for each* $\lambda, \mu \geq 0$ *given satisfying* $0 < \lambda + \mu < \lambda^\star$*, being one solution a ground state* $\bar{z}_{\lambda,\mu}$ *and the other one a bound state* $\tilde{z}_{\lambda,\mu}$*. Moreover,* $J(\bar{z}_{\lambda,\mu}) < 0 < J(\tilde{z}_{\lambda,\mu})$ *and* $\lim_{\lambda,\mu \to 0^+} \|\bar{z}_{\lambda,\mu}\| = 0$.

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INTERIOR REGULARITY RESULTS FOR ZERO-TH ORDER OPERATORS APPROACHING THE FRACTIONAL LAPLACIAN

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Abstract

In this work we are interested in interior regularity results for the solution $u_\epsilon \in C(\bar{\Omega})$ of the Dirichlet problem

$$\begin{cases} -I_\epsilon(u) = & f_\epsilon \quad \text{in } \Omega \\ \quad u = & 0 \quad \text{in } \Omega^c \end{cases},$$

where Ω is a bounded, open set and $f_\epsilon \in C(\bar{\Omega})$ for all $\epsilon \in (0,1)$. For some $\sigma \in (0,2)$ fixed, the operator I_ϵ is explicitly given by

$$I_\epsilon(u,x) = \int_{R^N} \frac{[u(x+z)-u(x)]dz}{\epsilon^{N+\sigma}+|z|^{N+\sigma}},$$

which is an approximation of the well-known fractional Laplacian of order σ, as ϵ tends to zero. The purpose of this article is to understand how the interior regularity of u_ϵ evolves as ϵ approaches zero. We establish that u_ϵ has a modulus of continuity which depends on the modulus of f_ϵ, which becomes the expected Hölder profile for fractional problems, as $\epsilon \to 0$. This analysis includes the case when f_ϵ deteriorates its modulus of continuity as $\epsilon \to 0$.

1 Introduction

Let $\Omega \subset R^N$ be a bounded open domain and $\epsilon \in (0,1)$. In this work we are interested in understanding interior regularity of solutions u_ϵ to the Dirichlet problem

$$\begin{cases} -I_\epsilon(u) = & f_\epsilon \quad \text{in } \Omega \\ \quad u = & 0 \quad \text{in } \Omega^c, \end{cases} \tag{1}$$

where I_ϵ is the non-local operator

$$I_\epsilon(u,x) = \int_{R^N} [u(x+z)-u(x)]K_\epsilon(z)dz, \tag{2}$$

with kernel $K_\epsilon : R^N \to R$ explicitly given by

$$K_\epsilon(z) = \frac{1}{\epsilon^{N+\sigma}+|z|^{N+\sigma}}, \tag{3}$$

for some $\sigma \in (0,2)$ fixed. Here we also assume $f_\epsilon \in C(\bar{\Omega})$ for each $\epsilon \in (0,1)$ and the family $\{f_\epsilon\}$ is uniformly bounded, that is, there exists $\Lambda > 0$ such that

$$||f_\epsilon||_{L^\infty(\bar{\Omega})} \leq \Lambda, \quad \text{for all} \quad \epsilon \in (0,1). \tag{4}$$

The characteristic feature of the nonlocal operators like I_ϵ is the integrability of the kernel K_ϵ defining it. In the literature, this fact leads to say that I_ϵ is a *zero-th order nonlocal operator*.

On the other hand, of main importance in this paper is the role of the fractional Laplacian of order σ, defined as

$$(-\Delta)^{\sigma/2}u(x) = -C_{N,\sigma}\ \text{P.V.}\int_{R^N}[u(x+z)-u(x)]|z|^{-(N+\sigma)}dz.$$

Notice that in this case, up to the normalizing constant $C_{N,\sigma}$, the kernel defining $(-\Delta)^{\sigma/2}$ can be formally identified with the limit case of K_ϵ, when $\epsilon = 0$ in (3), that is $K_0(z) = |z|^{-(N+\sigma)}$ for $z \neq 0$, which is a non-integrable around the origin. This non-integrability of the kernel determines a deep qualitative contrast between *zero-th order problems* like (1) and *fractional nonlocal problems* with I_ϵ replaced by the fractional Laplacian in (1).

It is the purpose of this work to contribute in the analysis of regularity of the solution u_ϵ of (1) in the passage to the limit as $\epsilon \to 0$. As I_ϵ is a zero-th order operator, it does not have a regularizing effect, and thus u_ϵ is merely continuous when f_ϵ is continuous. However, when $\epsilon = 0$ the solution u_0 is Hölder continuous, even of class $C^{1,\alpha}$ when $\sigma > 1$. The question is: How does the regularity of u_ϵ improves as ϵ approaches zero? In view of the discussion above, it is natural to ask if the modulus of continuity of the solution to (1) actually *improves* as $\epsilon \to 0$, at least locally in Ω, reaching the known Hölder regularity results for fractional problems described above. Furthermore, of particular interest is the case in which the family $\{f_\epsilon\}$ is not equicontinuous in $\bar{\Omega}$ and therefore its modulus of continuity may worsen in the passage to the limit.

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CRITICAL QUASILINEAR ELLIPTIC PROBLEMS USING CONCAVE-CONVEX NONLINEARITIES

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Abstract

It is established existence, multiplicity and asymptotic behavior of nonnegative solutions for a quasilinear elliptic problems driven by the Φ-Laplacian operator. One of these solutions is obtained as ground state solution by applying the well known Nehari method. The nonlinear term is a concave-convex function which presents a critical behavior at infinity. The concentration compactness principle is used in order to recover the compactness required in variational methods.

1 Introduction

In this work we deal with existence, multiplicity and asymptotic behaviour of nonnegative solutions of the problem

$$-\Delta_\Phi u = \lambda a(x)|u|^{q-2}u + b(x)|u|^{\ell^*-2}u \ \text{ in } \ \Omega, \ u = 0 \,\text{on}\, \partial\Omega, \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $\lambda > 0$ is a parameter, $\ell^* := N\ell/(N-\ell)$ with $1 < \ell < N$ and $a, b : \Omega \to \mathbb{R}$ are two indefinite functions in sign. The operator Δ_Φ is named Φ-Laplacian which is given by

$$\Delta_\Phi u = \text{div}(\phi(|\nabla u|)\nabla u)$$

where $\phi : (0,\infty) \to (0,\infty)$ is a C^2-function satisfying

(ϕ_1) $\lim\limits_{s\to 0} s\phi(s) = 0, \lim\limits_{s\to\infty} s\phi(s) = \infty$;

(ϕ_2) $s \mapsto s\phi(s)$ is strictly increasing.

We extend $s \mapsto s\phi(s)$ to $\mathbb{R}$ as an odd function. The function Φ is given by

$$\Phi(t) = \int_0^t s\phi(s)ds, \ t \ge 0.$$

As a consequence the function Φ satisfies $\Phi(t) = \Phi(-t)$ for each $t \in \mathbb{R}$. Without any loss of generality we assume $\Phi(1) = 1$. For further results on Orlicz and Orlicz-Sobolev framework we refer the reader to [1]. At the same time, the Orlicz-Sobolev space $W^{1,\Phi}(\Omega)$ is a generalization of the classical Sobolev space $W^{1,p}(\Omega)$. Hence, several properties of the Sobolev spaces have been extended to Orlicz-Sobolev spaces. The interest regarding Orlicz-Sobolev spaces is motivated by their applicability in many fields of mathematics, such as partial differential equations, calculus of variations, non-linear potential theory, differential geometry, geometric function theory, the theory of quasiconformal mappings, probability theory, non-Newtonian fluids, image processing, among others. The class of problems introduced in (1) is related with several fields of physics based on the nature of the nonhomogeneous nonlinearity Φ. For instance we cite the following examples:

(i) Nonlinear elasticity: $\Phi(t) = (1 + t^2)^\gamma - 1$, $1 < \gamma < N/(N-2)$;

(ii) Plasticity: $\Phi(t) = t^\alpha(\log(1+t))^\beta$, $\alpha \geq 1$, $\beta > 0$;

(iii) Non-Newtonian fluid: $\Phi(t) = \frac{1}{p}|t|^p$, for $p > 1$;

(iv) Plasma physics: $\Phi(t) = \frac{1}{p}|t|^p + \frac{1}{q}|t|^q$, where $1 < p < q < N$ with $q \in (p, p^*)$;

(v) Generalized Newtonian fluids: $\Phi(t) = \int_0^t s^{1-\alpha}[\sinh^{-1}(s)]^\beta \, \mathrm{d}s$, $0 \leq \alpha \leq 1$, $\beta > 0$.

Recall that when $\phi := 2$, $a = b := 1$ we obtain $\ell = 2$. Then problem (1) reads as

$$-\Delta u = \lambda|u|^{q-2}u + |u|^{2^*-2}u \ \ in \ \ \Omega, \ \ u = 0 \ \ on \ \ \partial\Omega. \tag{2}$$

2 Main Results

Our main results are stated below.

Theorem 2.1. *Suppose $(\phi_1) - (\phi_3)$ and (H). Then there exists $\Lambda_1 > 0$ such that for each $\lambda \in (0, \Lambda_1)$, problem* (1) *admits at least one nonnegative ground state solution $u = u_\lambda$ satisfying $J_\lambda(u) < 0$ and $\lim_{\lambda \to 0^+} \|u_\lambda\| = 0$.*

Furthermore, we can state our second result as follows

Theorem 2.2. *Suppose $(\phi_1) - (\phi_3)$ and (H). Then there exists $\Lambda_2 > 0$ in such way that for each $\lambda \in (0, \Lambda_2)$, problem* (1) *admits at least one nonnegative weak solution $v = v_\lambda$ satisfying $J_\lambda(v) > 0$.*

Putting all results together we obtain the following multiplicity result.

Theorem 2.3. *Suppose $(\phi_1) - (\phi_3)$ and (H). Set $\Lambda = \min\{\Lambda_1, \Lambda_2\}$. Then for each $\lambda \in (0, \Lambda)$, problem* (1) *admits at least two nonnegative weak solutions $u = u_\lambda, v = v_\lambda \in W_0^{1,\Phi}(\Omega)$ satisfying $J_\lambda(u) < 0 < J_\lambda(v)$. Furthermore, the function u is a ground state solution for each $\lambda \in (0, \Lambda)$.*

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$P(X)$-KIRCHHOFF TYPE TRANSMISSION PROBLEM OF A GENERALIZED BIOTHERMAL MODEL FOR THE HUMAN FOOT

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Abstract

In this research, by means of the pseudomonotone operator theory we show the existence of weak solutions for a transmission problem given by a system of two nonlinear elliptic equations of $p(x)$-Kirchhoff type which is the generalization of a bio-thermal model for the human foot.

1 Introduction

We are concerned with the existence of solutions to the following system of nonlinear elliptic system

$$\begin{aligned}
-M_1\Big(\int_{\Omega_1}\frac{1}{p(x)}|\nabla u|^{p(x)}\,dx\Big)\,\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u) &= |u|^{\alpha(x)-2}u \quad in \quad \Omega_1\\
-M_2\Big(\int_{\Omega_2}\frac{1}{p(x)}|\nabla v|^{p(x)}\,dx\Big)\,\mathrm{div}(|\nabla v|^{p(x)-2}\nabla v) &= |v|^{\beta(x)-2}v \quad in \quad \Omega_2\\
\frac{\partial u}{\partial \nu} &= 0 \qquad on \quad \Gamma_1\\
u=v \quad , \quad M_1\Big(\int_{\Omega_1}\frac{1}{p(x)}|\nabla u|^{p(x)}\,dx\Big)\frac{\partial u}{\partial \nu} &= M_2\Big(\int_{\Omega_2}\frac{1}{p(x)}|\nabla v|^{p(x)}\,dx\Big)\frac{\partial v}{\partial \nu} \quad on \quad \Gamma_2\\
M_2\Big(\int_{\Omega_2}\frac{1}{p(x)}|\nabla v|^{p(x)}\,dx\Big)\frac{\partial v}{\partial \nu} &= |v|^{\gamma(x)-2}v \quad on \quad \Gamma_3
\end{aligned} \tag{1}$$

where Ω is a bounded smooth domain in $\mathbb{R}^n, n=2,3$ (the domain occupied by the bare human foot) , such that $\overline{\Omega}=\overline{\Omega}_1\cup\overline{\Omega}_2$, $\Omega_1\cap\Omega_2=\phi$ (Ω_1,Ω_2 defining the internal tissue and the skin, respectively); $\Gamma=\partial\Omega$ is assumed to be splitted in three disjoint parts: Γ_1 the part of foot joined to the rest of the leg, Γ_2 represents the common boundary between Ω_1 and Ω_2; Γ_3 is the part of the skin in contact with the environment where the heat losses are produced and u,v is the temperature in Ω_1,Ω_2 respectively . System (1) is a generalized model to describe the bioheat transfer of the bare foot (see [2]).

Transmission problems arise in several applications of physics and biology (see, for instance, [4]). Our work is motivated by the ones of Copetti M.I.M. et al [2] and Ayoujil A. and Moussaoui M. [1].

2 Main Results

We shall deal with the Lebesgue-Sobolev Spaces with variable exponent $L^{p(x)}(\Omega)$, $L^{p(x)}(\Omega_i)$, $L^{p(x)}(\Gamma_i)$ and $W_0^{1,p(x)}(\Omega_i)$ $i=1,2$, (see [3])

Now ,we give the following hypotheses

(M_0) $M_i : [0, +\infty[\longrightarrow [m_{0i}, +\infty[$, are non decreasing locally Lipschitz continuous functions,

(H_0) $p, \alpha, \beta, \in C_+(\overline{\Omega}) = \{h : h \in C(\overline{\Omega}) : h(x) > 1, \forall x \in \overline{\Omega}\}, \gamma \in C_+(\Gamma)$. For any $h \in C_+(\overline{\Omega})$ we define

$$h^+ = \sup_{x\in\Omega} h(x) \text{ and } h^- = \inf_{x\in\Omega} h(x)$$

and take $\alpha^- \leq \alpha^+ \leq p^- < p^+ < \beta^- \leq \beta^+ < \min\{N, \frac{Np^-}{N-p^-}\}, \quad \gamma^- \leq \gamma^+ < p^-$

We have the following result.

Theorem 2.1. *Assume that* (M_0) *and* (H_0) *hold. Then, problem* (1) *has a weak solution*

$$\{u, v\} \in E = \{\{u, v\} \in W^{1,p(x)}(\Omega_1) \times W^{1,p(x)}(\Omega_2) : u = v \text{ on } \Gamma_2\}$$

Proof: We establish the existence using Brezis' theorem for pseudomonotone operators. □

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SISTEMAS ELÍPTICOS SUPERLINEARES COM RESSONÂNCIA

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Abstract

Os resultados apresentados aqui assim como suas demonstrações são frutos do meu trabalho de doutorado realizado sob a orientação do professor Francisco Odair Vieira de Paiva na Universidade Federal de São Carlos. A motivação para nosso trabalho encontra-se em [3], Cuesta-Figueiredo-Srikanth, tais autores estudaram uma classe de problemas superlineares com ressonância. A estratégia usada consiste em obter estimativas a priori para possíveis soluções dos problemas e a partir daí utilizar a teoria do grau topológico para garantir a existência de soluções.

1 Introdução

Em [3], M. Cuesta, De Figueiredo e Srikanth estudaram a resolubilidade da seguinte classe de sistemas hamiltonianos:

$$\begin{cases} -\Delta u = \lambda_1 u + u_+^p + f(x) & x \in \Omega \\ -\Delta v = \lambda_1 v + v_+^q + g(x) & x \in \Omega \\ u = 0 & x \in \partial\Omega, \end{cases} \tag{1}$$

em que $\Omega \in \mathbb{R}^N$ é domínio limitado suave com $N \geq 3$ e λ_1 denota o primeiro autovalor de $(-\Delta, H_0^1(\Omega))$. Os autores provaram a existência de solução para (1) supondo que $f, g \in L^r(\Omega)$, $r > N$, satisfazendo

$$\int_\Omega f\phi_1 < 0 \quad \text{e} \quad \int_\Omega g\phi_1 < 0, \tag{2}$$

em que ϕ_1 denota a autofunção associada ao primeiro autovalor e $p, q > 1$ satisfazem

$$\frac{1}{p+1} + \frac{N-1}{N+1}\frac{1}{q+1} > \frac{N-1}{N+1} \quad \text{e} \quad \frac{1}{q+1} + \frac{N-1}{N+1}\frac{1}{p+1} > \frac{N-1}{N+1}. \tag{3}$$

As hipérboles acima foram introduzidas por Clément-de Figueiredo-Mitidieri, em [2], para obter estimativas a priori para sistemas elípticos superlineares via técnica de Brezis-Turner [1]. Note que se $p = q$ então (3) se reduz a condição de Brézis-Turner, $p < \frac{N+1}{N-1}$.

As não linearidades consideradas aqui podem ser caracterizadas como assimétricas: superlineares em $+\infty$ e assintoticamente linear em $-\infty$. Além disso, nossos problemas são ressonantes no primeiro autovalor em $-\infty$. Problemas deste tipo foram primeiramente considerados por Ward, em [6], no caso de fronteira do tipo de Neumann e posteriormente por Kannan-Ortega, em [5], para fronteira do tipo de Dirichlet.

Em nosso trabalho estudamos a resolubilidade do seguinte sistema gradiente:

$$\begin{cases} -\Delta u = au + bv + u_+^p + f(x) & x \in \Omega \\ -\Delta v = bu + cv + v_+^q + g(x) & x \in \Omega \\ u = v = 0 & x \in \partial\Omega, \end{cases} \tag{4}$$

em que $\Omega \in \mathbb{R}^N$ é um domínio limitado suave, com $N \geq 3$ e $1 < p, q < \frac{N+1}{N-1}$. Os parâmetros $a, b, c \in \mathbb{R}$ são tais que $\max\{a, c\} > 0$ e $b > 0$. Denotamos $w_+ = \max\{w, 0\}$. E as funções f, g são tais que,

$$f, g \in L^r(\Omega) \text{ para } r > N. \tag{5}$$

Para tratar esse problema utilizamos métodos topológicos. A estratégia é encontrar estimativas a priori para possíveis soluções de sistema (4) e utilizar a Teoria do Grau Topológico para garantir existência de soluções.

2 Resultados Principais

Teorema 2.1. *Considere* $1 < p, q < \frac{N+1}{N-1}$ *as funções* $f, g \in L^r(\Omega)$, $r > N$, *satisfazendo a condição*

$$\int_\Omega f\phi_1 + \frac{\lambda_1 - a}{b} \int_\Omega g\phi_1 < 0 \tag{1}$$

e λ_1 *um autovalor da matriz* $A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \in M_{2\times2}(\mathbb{R})$. *Existe pelo menos uma solução* $U = (u, v)$ *em* $\left(W^{2,r}(\Omega) \cap H_0^1(\Omega)\right)^2$ *do sistema (4).*

Prova: Para demonstrar esse resultado de existência utilizamos uma estimativa a piori para possíveis soluções de (4) e teoria do grau topológico. Para maiores detalhes veja [4].

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A WEIGHTED TRUDINGER-MOSER INEQUALITY AND ITS APPLICATIONS TO QUASILINEAR ELLIPTIC PROBLEMS WITH CRITICAL GROWTH IN THE WHOLE EUCLIDEAN SPACE

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Abstract

We establish a version of the Trudinger-Moser inequality involving unbounded or decaying radial weights in weighted Sobolev spaces. In the light of this inequality and using a minimax procedure we also study existence of solutions for a class of quasilinear elliptic problems involving exponential critical growth.

1 Introduction

We recall that if Ω is a bounded domain in $\mathbb{R}^n$ $(n \geq 2)$, the classical Trudinger-Moser inequality (cf. [2, 3]) asserts that $e^{\alpha|u|^{n'}} \in L^1(\Omega)$, for all $u \in W_0^{1,n}(\Omega)$ and $\alpha > 0$ and there exists a constant $C(n) > 0$ such that

$$\sup_{\|u\|_n \leq 1} \int_\Omega e^{\alpha|u|^{n'}} \, \mathrm{d}x \leq C(n)|\Omega|, \quad \text{if } \alpha \leq \alpha_n, \tag{1}$$

where $n' = \frac{n}{n-1}$, $\alpha_n = n\omega_{n-1}^{\frac{1}{n-1}}$, $\|u\|_n := \left(\int_\Omega |\nabla u|^n \, \mathrm{d}x\right)^{\frac{1}{n}}$ and ω_{n-1} is the surface area of the unit sphere in $\mathbb{R}^n$. Moreover, the inequality (1) is sharp in the sense that if $\alpha > \alpha_n$ the correspondent supremum is $+\infty$ and clearly as the Lebesgue's measure $|\Omega| \to +\infty$ no uniform bound can be retained in (1). Recently, Adimurthi and K. Sandeep in [1] extended the Trudinger-Moser inequality (1) for singular weights. More precisely, they have proved that if Ω is a smooth bounded domain in $\mathbb{R}^n$ containing the origin, $u \in W_0^{1,n}(\Omega)$ and $\beta \in [0, n)$, then there exists a constant $C(n, \beta) > 0$ such that

$$\sup_{\|\nabla u\|_n \leq 1} \int_\Omega \frac{e^{\alpha|u|^{n'}}}{|x|^\beta} \, \mathrm{d}x < C(n, \beta)|\Omega| \Leftrightarrow 0 < \alpha \leq \alpha_n(1 - \frac{\beta}{n}). \tag{2}$$

Throughout this work, we consider some weight functions $V(|x|)$ and $Q(|x|)$ satisfying the following assumptions:

(V) $V \in C(0, \infty)$, $V(r) > 0$ and there exist $a, a_0, a_1 > -n$ such that

$$\liminf_{r \to +\infty} \frac{V(r)}{r^a} > 0, \; \liminf_{r \to 0^+} \frac{V(r)}{r^{a_0}} > 0 \text{ and } \limsup_{r \to 0^+} \frac{V(r)}{r^{a_1}} < \infty;$$

(Q) $Q \in C(0, \infty)$, $Q(r) > 0$ and there exist $b < a$, $b_0 > -n$ such that $\limsup\limits_{r \to 0^+} \dfrac{Q(r)}{r^{b_0}} < \infty$ and $\limsup\limits_{r \to +\infty} \dfrac{Q(r)}{r^b} < \infty$.

In order to state our results, we need to introduce some notations. If $1 \leq p < \infty$ we define the weighted Lebesgue spaces $L^p(\mathbb{R}^n; Q) := \left\{u : \mathbb{R}^n \to \mathbb{R} : u \text{ is measurable and } \int_{\mathbb{R}^n} Q(|x|)|u|^p \, \mathrm{d}x < \infty\right\}$ endowed with the norm $\|u\|_{L^p(\mathbb{R}^n;Q)} = \left(\int_{\mathbb{R}^n} Q(|x|)|u|^p \, \mathrm{d}x\right)^{\frac{1}{p}}$. Let $C_0^\infty(\mathbb{R}^n)$ be the set of smooth functions with compact support. We define the energy space $W^{1,n}_{\mathrm{rad}}(\mathbb{R}^n; V)$ as the subspace of radially symmetric functions in the completion of $C_0^\infty(\mathbb{R}^n)$ with respect to the norm $\|u\| = \left[\int_{\mathbb{R}^n} (|\nabla u|^n + V(|x|)|u|^n) \, \mathrm{d}x\right]^{\frac{1}{n}}$. We use the notation $E = W^{1,n}_{\mathrm{rad}}(\mathbb{R}^n; V)$.

2 Main Results

With the aid of inequalities (1), (2), we establish in this work a Trudinger-Moser inequality in the functional space E. More precisely, one has:

Theorem 2.1. *Assume that $(V)-(Q)$ hold. Then, for any $u \in E$ and $\alpha > 0$, we have that $\Phi_\alpha(u) \in L^1(\mathbb{R}^n; Q)$. Furthermore, if $\alpha < \lambda := \min\{\alpha_n, \alpha_n(1 + \frac{b_0}{n})\}$, there holds $\sup_{u\in E:\ \|u\|\le 1} \int_{\mathbb{R}^n} Q(|x|)\Phi_\alpha(u)\,\mathrm{d}x < \infty$. Moreover, if the function Q is nonincreasing in $|x|$, then $\sup_{u\in E:\ \|u\|\le 1} \int_{\mathbb{R}^n} Q(|x|)\Phi_\lambda(u)\,\mathrm{d}x < \infty$. Furthermore, if we also assume that $-n < b_0 \le 0$, and $\liminf_{r\to 0^+} \frac{Q(r)}{r^{b_0}} > 0$, then the value λ is optimal, that is, $\sup_{u\in E:\ \|u\|\le 1} \int_{\mathbb{R}^n} Q(|x|)\Phi_\alpha(u)\,\mathrm{d}x = +\infty$ for all $\alpha > \lambda$.*

As an application of the previous theorem and using a minimax procedure, we will study the existence of a nontrivial solution for the following quasilinear elliptic problem:

$$\begin{cases} -\operatorname{div}(|\nabla u|^{n-2}\nabla u) + V(|x|)|u|^{n-2}u = Q(|x|)f(u), & \text{in } \mathbb{R}^n, \\ u(x) \to 0, \quad \text{as } |x| \to \infty, \end{cases} \tag{1}$$

$n \ge 2$, when the nonlinear term $f(s)$ is allowed to enjoy an exponential critical growth suggested by the classical Trudinger-Moser inequality (1). In order to perform the minimax approach to the problem (1), we also need to make some suitable assumptions on the behaviour of $f(s)$. More precisely, we shall assume the following conditions:

(f_1) $f : [0, +\infty) \to \mathbb{R}$ is continuous and $f(s)/|s|^{n-1} \to 0$ as $s \to 0^+$;

(f_2) there exists $\theta > n$ such that $0 < \theta F(s) := \theta \int_0^s f(t)\,\mathrm{d}t \le sf(s), \quad \forall s > 0$;

(f_3) there exist $\theta_0 > n$ and $\mu > 0$ such that $F(s) \ge \frac{\mu}{\theta_0} s^{\theta_0}, \quad \forall s \ge 0$.

Next, we state our existence result.

Theorem 2.2. *Suppose that $(V)-(Q)$ hold. If f has exponential critical growth and $(f_1)-(f_3)$ are satisfied, then there exists $\mu_0 > 0$ such that problem (1) has a nontrivial nonnegative weak solution u in E for all $\mu > \mu_0$.*

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A CLASS OF NONLOCAL FRACTIONAL $P-$ KIRCHHOFF PROBLEM WITH A REACTION TERM

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Abstract

The object of this work is to study the existence of solutions for a class of nonlocal p-Kirchhoff problem involving a nonlinear integro-differential operator which are possibly degenerate and covers the case of fractional p-Laplacian operator, with a reaction term. We establish our results by using the Degree theory of (S_+) type mappings.

1 Introduction

This paper is devoted to the study of the following nonlocal fractional $p-$ Kirchhoff problem

$$
\begin{gathered}
-M\Big(\int_{\mathbb{R}^N x\mathbb{R}^N} \mathcal{P}\Phi(u(x)-u(y))K(x,y)\,dxdy\Big)\mathcal{L}_\Phi u = f(x,u)|u|_{\alpha_1}^{\alpha_2} - \operatorname{div}(g(x,u)\nabla r), \;\; in\ \Omega \\
u = 0, \;\; on\ \mathbb{R}^n \setminus \Omega
\end{gathered} \tag{1}
$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain , $-\mathcal{L}_\Phi$ is a nonlocal integrodifferential operator, Φ is a real valued continuous fuction over $\mathbb{R}$, $\mathcal{P}\Phi(t) = \int_0^{|t|}\Phi(\tau)\,d\tau$, the kernel $K : \mathbb{R}^N \times \mathbb{R}^N \to \mathbb{R}$ is a measurable function,$1 \le \alpha_1, 0 < \alpha_2$, $M, f, g,$ and r are functions that satisfy conditions which will be stated later. In [2, 1, 2] the authors consider the problem (1), with the special case $\Phi(t) = |t|^{p-2}t$, $K(x,y) = |x-y|^{-(N+sp)}$, , $\alpha_2 = 0$ and $g = 0$, they showed existence of solutions via the mountain pass theorem and its variants. Because of the presence of the terms $f(x,u)|u|_{\alpha_1}^{\alpha_2}$ and $\operatorname{div}(g(x,u)\nabla r)$ (reaction term), the problem (1) has no variational structure, so the most usual variational techniques can not be applied directly. Motivated by the above references and [1] we deal with the existence of solutions for nonlocal problem (1).

2 Main Results

We denote: $Q = \mathbb{R}^{2n} \setminus (\mathcal{C}\Omega \times \mathcal{C}\Omega)$ and $\mathcal{C}\Omega := \mathbb{R}^n \setminus \Omega$,

$$
W = \big\{u : \mathbb{R}^N \to \mathbb{R} : u|_\Omega \in L^p(\Omega), \; \int\int_Q \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}}\,dx\,dy < \infty\big\},
$$

where $u|_\Omega$ represents the restriction to Ω of function $u(x)$. Also, we define the following linear subspace of W,

$$
W_0 = \big\{u \in W : u = 0 \text{ a.e. in } \mathbb{R}^N \setminus \Omega\big\}.
$$

The linear space W is endowed with the norm

$$
\|u\|_W := \|u\|_{L^p(\Omega)} + \Big(\int\int_Q \frac{|u(x)-u(y)|^p}{|x-y|^{N+ps}}\,dx\,dy\Big)^{1/2}.
$$

It is easily seen that $\|\cdot\|_W$ is a norm on W and $C_0^\infty(\Omega) \subseteq W_0$.
Assume that the following assumptions hold:

(M_0) $M:[0,+\infty[\to]m_0,+\infty[$ is a continuous and nondecreasing function, with $m_0>0$.

(Φ_1) $\Phi:\mathbb{R}\to\mathbb{R}$ is a continuous function, satisfying $\Phi(0)=0$ and

$$C_0^{-1}|t|^p \le \Phi(t)t \le C_0|t|^p \quad \text{for all } t\in\mathbb{R}$$

(F_1) $f:\Omega\times\mathbb{R}\to\mathbb{R}$ is a Carathéodory function and there exist positive constants c_1 and c_2 such that

$$|f(x,s)|\le c_1+c_2|s|^{\gamma-1}, \quad \forall x\in\Omega, \forall s\in\mathbb{R},$$

for some $1<\gamma,\alpha_1<p^*$
(G_1) $|g(x,t)|\le c_3|u|^{\eta}$, $\forall(x,t)\in\Omega\times\mathbb{R}$, where $1<\eta$ with $\eta p'<p$
(R_1) $r:\Omega\to\mathbb{R}$ is some measurable function satisfying that $|\nabla r|\in L^{\frac{\mu}{p'}}(\Omega)\bigcap L^{\infty}(\Omega)$ and there exists $\lambda>1$ such that $p\le\lambda\le p^*$, $\frac{1}{\mu}+\frac{\eta p'}{\lambda}=1$
(K_1) The kernel $K:\mathbb{R}^N\times\mathbb{R}^N\to\mathbb{R}$ is a measurable function such that

$$C_0^{-1}|x-y|^{-(N+sp)}\le K(x,y)\le C_0|x-y|^{-(N+sp)} \quad \forall x,y\in\mathbb{R}^N,\ x\ne y$$

where $C_0\ge 1, s\in(0,1), p>2-\frac{s}{N}$; $-\mathcal{L}_\Phi$ is a nonlocal operator defined as

$$\langle -\mathcal{L}_\Phi u,\varphi\rangle=\int_{\mathbb{R}^{2N}}\Phi(u(x)-u(y))(\varphi(x)-\varphi(y))K(x,y)\,dxdy, \text{for all} \varphi\in C_0^\infty(\mathbb{R}^N)$$

(H_1) $\gamma+\alpha_2<p,\ \alpha_2+1<p$

Theorem 2.1. *Assume that hypotheses (M_0), (F_1), (G_1) and (H_1) hold. Then (1) has a weak solution in W_0.*

Proof We apply the degree theory of (S_+) type mappings . ■

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SOLUTIONS FOR THE SCHRÖDINGER-BOPP-PODOLSKY SYSTEM IN THE RADIAL CASE

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Abstract

We consider the following system in $\mathbb{R}^3$ involving a Schrödinger equation,

$$\begin{cases} -\Delta u + u + q\phi u = |u|^{p-2}u, \\ -\Delta\phi + a\Delta^2\phi = u^2, \end{cases}$$

where $a, q > 0, p \in (2, 6)$. We show existence and nonexistence of solutions, depending on the parameter involved, the radial case.

1 Introduction

The classical electromagnetic theory of Maxwell is based on the fact that the Gauss law for the electrostatic potential ϕ generated by a charge distribution whose density is ρ satisfies the Poisson equation

$$-\Delta\phi = \rho \qquad \text{in } \mathbb{R}^3.$$

If $\rho = 4\pi\delta_{x_0}$, with $x_0 \in \mathbb{R}^3$, the fundamental solution is $\mathcal{G}(x - x_0)$, where $\mathcal{G}(x) = \frac{1}{|x|}$, and the electrostatic energy is

$$\mathcal{E}_{\mathrm{M}}(\mathcal{G}) = \frac{1}{2}\int |\nabla\mathcal{G}|^2 = +\infty$$

which leads to the *infinity problem* of the Maxwell theory of the electromagnetism. To solve this problems many attempts have been done in the past century. A remarkable one is that developed by Bopp and Podolski where the equation of the electrostatic field ϕ is

$$-\Delta\phi + a\Delta^2\phi = \rho \qquad \text{in } \mathbb{R}^3, \quad a > 0.$$

If $\rho = 4\pi\delta_{x_0}$, the solution is $\mathcal{K}(x - x_0)$, where

$$\mathcal{K}(x) := \frac{1 - e^{-|x|/a}}{|x|},$$

which is easily seen to have no singularity in 0 and finite energy:

$$\mathcal{E}_{\mathrm{BP}}(\mathcal{K}) = \frac{1}{2}\int |\nabla\mathcal{K}|^2 + \frac{1}{2}\int |\Delta\mathcal{K}|^2 < +\infty.$$

Here we are interested in a system of two elliptic equation deriving from the coupling of the Schrödinger equation and the equation of the generalized electrodynamics developed by Bopp and Podolsky. The search of standing waves solutions lead to the system in $\mathbb{R}^3$

$$\begin{cases} -\Delta u + u + q\phi u = |u|^{p-2}u, \\ -\Delta\phi + a\Delta^2\phi = u^2. \end{cases} \tag{1}$$

The parameter $a > 0$ is responsible for the Bopp-Podolsky term and the parameter $q > 0$ has the meaning of the electric charge. We focus here on the radial case.

By using variational methods we are reduced to find critical points in $H^1_r(\mathbb{R}^3)$ of the following functional

$$\mathcal{J}_q(u) = \frac{1}{2}\int |\nabla u|^2 + \frac{1}{2}\int u^2 + \frac{1}{4}\int \phi_u u^2 - \frac{1}{p}\int |u|^p.$$

Here ϕ_u is the unique solution of the second equation in the system for fixed u.

2 Main results

In a first paper with P. d'Avenia, see [1], by using Mountain Pass type theorems and truncation arguments, we deduced the following result:

Theorem 2.1. *Problem* (1) *has a nontrivial solution in the following cases:*

- $p \in (3, 6)$ *and* $q > 0$*,*
- $p \in (2, 3]$ *and* $q > 0$ *sufficiently small.*

Indeed the result is true also in the non-radial setting, by using a Splitting Lemma instead of using the standard compactness obtained by the Strauss Lemma.

The case $p \in (2, 3]$ is more subtle due to the growth condition of the nonlocal term in the energy functional, and indeed the geometry of the fiber maps is more complicate. This case was studied in a paper with K. Silva, see [2], where we obtained the following result:

Theorem 2.2. *Let* $p \in (2, 3]$*. There exist* $\varepsilon > 0, q^* > 0, q^*_0 > 0$ *satisfying* $q^*_0 + \varepsilon < q^*$ *such that, problem* (1)

- *has two solutions for* $q \in (0, q^*_0 + \varepsilon)$*,*
- *no solutions for* $q > q^*$*.*

Again we observe that the statement about non existence is true in the general setting: it is not limited to the radial framework.

By using suitable estimates we are also able to pass to the limit when $a \to 0$ in the solutions obtained in the previous theorems. The conclusion is that they converge in a suitable sense to solutions of the Schrödinger-Poisson system: the one obtained by setting $a = 0$ in (1).

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ASYMPTOTIC BEHAVIOR AS $P \to \infty$ OF LEAST ENERGY SOLUTIONS OF A $(P, Q(P))$-LAPLACIAN PROBLEM

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Abstract

We study the asymptotic behavior, as $p \to \infty$, of the least energy solutions of the problem

$$\begin{cases} -\left(\Delta_p + \Delta_{q(p)}\right) u = \lambda_p \left|u(x_u)\right|^{p-2} u(x_u)\delta_{x_u} & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where x_u is the (unique) maximum point of $|u|$, δ_{x_u} is the Dirac delta distribution supported at x_u,

$$\lim_{p\to\infty} \frac{q(p)}{p} = Q \in \begin{cases} (0,1) & \text{if} \quad N < q(p) < p \\ (1,\infty) & \text{if} \quad N < p < q(p) \end{cases}$$

and $\lambda_p > 0$ is such that

$$\min\left\{\|\nabla u\|_\infty / \|u\|_\infty : 0 \not\equiv u \in W^{1,\infty}(\Omega) \cap C_0(\overline{\Omega})\right\} \le \lim_{p\to\infty} (\lambda_p)^{\frac{1}{p}} < \infty.$$

1 Introduction

This work is divided in two parts. In the first one, we study the existence of nonnegative least energy solutions for the Dirichlet problem

$$\begin{cases} -(\Delta_p + \Delta_q)u = \lambda \|u\|_r^{p-r} |u|^{r-2} u & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, $N \ge 2$,

$$(\Delta_p + \Delta_q)u := \operatorname{div}\left[\left(|\nabla u|^{p-2} + |\nabla u|^{q-2}\right)\nabla u\right]$$

is the (p,q)-Laplacian operator, $\lambda > 0$ and $1 \le r < \infty$. ($\|\cdot\|_s$ stands for the standard norm of the Lebesgue space $L^s(\Omega)$, with $1 \le s \le \infty$).

We show the limit problem of (1) as $r \to \infty$ is the following

$$\begin{cases} -(\Delta_p + \Delta_q)u = \lambda \left|u(x_u)\right|^{p-2} u(x_u)\delta_{x_u} & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{2}$$

where x_u is the (unique) maximum point of $|u|$ and δ_{x_u} is the Dirac delta distribution supported at x_u.

More precisely, we prove that if

$$\lambda > \lambda_\infty(p) := \min\left\{\|\nabla u\|_p^p / \|u\|_\infty^p : u \in W_0^{1,p}(\Omega) \setminus \{0\}\right\},$$

and u_n denotes a nonnegative least energy solution of (1) for $r = r_n \to \infty$, then there exists a subsequence of $\{u_n\}$ converging strongly in $X_{p,q} := W_0^{1,\max\{p,q\}}(\Omega)$ to a nonnegative least energy solution of (2).

Least energy solutions for (2) are defined as the minimizers of the energy functional

$$J_\lambda(u) = \frac{1}{p}\|\nabla u\|_p^p + \frac{1}{q}\|\nabla u\|_q^q - \frac{\lambda}{p}\|u\|_\infty^p,$$

either on $W_0^{1,q}(\Omega)$, if $N < p < q < \infty$, or on the "Nehari set"

$$\mathcal{N}_{\lambda,\infty} := \left\{u \in W_0^{1,p}(\Omega) : \|\nabla u\|_p^p + \|\nabla u\|_q^q = \lambda\|u\|_\infty^p\right\},$$

if $N < q < p < \infty$.

Although not differentiable, the functional $u \mapsto \|u\|_\infty^p$ has right Gateaux derivative at any $u \in C(\overline{\Omega})$. Using this fact we show that the least energy solutions of (2) are weak solutions of this problem. It is simple to verify that (2) cannot have weak solutions when $\lambda \leq \lambda_\infty(p)$.

In the second part of this work, we consider $q = q(p)$, with

$$\lim_{p\to\infty}\frac{q(p)}{p} =: Q \in \begin{cases} (0,1) & \text{if} \quad N < q(p) < p \\ (1,\infty) & \text{if} \quad N < p < q(p), \end{cases}$$

and fix

$$\Lambda \geq \Lambda_\infty := \min\left\{\|\nabla u\|_\infty / \|u\|_\infty : 0 \not\equiv u \in W^{1,\infty}(\Omega) \cap C_0(\overline{\Omega})\right\}. \tag{3}$$

Then, taking $\lambda_p > 0$ satisfying

$$\lim_{p\to\infty}(\lambda_p)^{\frac{1}{p}} = \Lambda \geq \Lambda_\infty$$

we study the asymptotic behavior, as $p \to \infty$, of the least energy solutions u_p of (2) with $\lambda = \lambda_p$ and $q = q(p)$.

After deriving suitable estimates for u_p in $W_0^{1,m}(\Omega)$, for each $m > N$, we use the compactness of the embedding $W_0^{1,m}(\Omega) \hookrightarrow C(\overline{\Omega})$ to prove that any sequence $\{u_{p_n}\}$, with $p_n \to \infty$, admits a subsequence converging uniformly in $\overline{\Omega}$ to a function $u_\Lambda \in W^{1,\infty}(\Omega) \cap C_0(\overline{\Omega})$, which is strictly positive in Ω and attains its (unique) maximum point at $x_\Lambda \in \Omega$. Moreover, we prove that u_Λ is ∞-harmonic in the punctured domain $\Omega \setminus \{x_\Lambda\}$, meaning that it satisfies, in the viscosity sense,

$$\Delta_\infty u_\Lambda = 0 \quad \text{in} \quad \Omega \setminus \{x_\Lambda\},$$

where $\Delta_\infty u := \frac{1}{2}\nabla u \cdot \nabla |\nabla u|^2$ denotes the ∞-Laplacian.

In addition, we show that if either $\Lambda = \Lambda_\infty$ or $\Lambda > \Lambda_\infty$ and $Q \in (0,1)$, then u_Λ realizes the minimum in (3) and satisfies

$$\|u_\Lambda\|_\infty = (\Lambda_\infty)^{-1}(\Lambda_\infty/\Lambda)^{\frac{1}{1-Q}} \quad \text{and} \quad \|\nabla u_\Lambda\|_\infty = (\Lambda_\infty/\Lambda)^{\frac{1}{1-Q}}.$$

Hence, taking into account that $\Lambda_\infty = (\|\rho\|_\infty)^{-1}$, where $\rho : \overline{\Omega} \to [0,\infty)$ denotes the distance function to the boundary $\partial\Omega$, we conclude that

$$\rho(x_\Lambda) = \|\rho\|_\infty \quad \text{and} \quad 0 \leq u_\Lambda(x) \leq (\Lambda_\infty/\Lambda)^{\frac{1}{1-Q}}\rho(x), \quad \forall x \in \overline{\Omega}.$$

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GROUND STATE OF A MAGNETIC NONLINEAR CHOQUARD EQUATION

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Abstract

We consider the stationary magnetic nonlinear Choquard equation

$$-(\nabla + iA(x))^2 u + V(x)u = \left(\frac{1}{|x|^\alpha} * F(|u|)\right)\frac{f(|u|)}{|u|}u, \tag{1}$$

where $A\colon \mathbb{R}^N \to \mathbb{R}^N$ is a vector potential, V is a scalar potential, $f\colon \mathbb{R} \to \mathbb{R}$ and F is the primitive of f. Under mild hypotheses, we prove the existence of a ground state solution for this problem. We also prove a simple multiplicity result by applying Ljusternik-Schnirelmann methods.

1 Introduction

In problem (P), $\nabla + iA(x)$ is the covariant derivative with respect to the C^1 vector potential $A\colon \mathbb{R}^N \to \mathbb{R}^N$. (After stating our hypotheses, the form of equation (P) will be changed to (2)). The constant α belongs to the interval $(0, N)$ and

$$\lim_{|x|\to\infty} A(x) = A_\infty \in \mathbb{R}^N.$$

The scalar potential $V\colon \mathbb{R}^N \to \mathbb{R}$ is a continuous, bounded function satisfying

($V1$) $\inf_{\mathbb{R}^N} V > 0$;

($V2$) $V_\infty = \lim_{|y|\to\infty} V(y)$;

($V3$) $V(x) \le V_\infty$ for all $x \in \mathbb{R}^N$.

We also suppose that

(AV) $|A(y)|^2 + V(y) < |A_\infty|^2 + V_\infty$.

The function F is the primitive of the nonlinearity $f\colon \mathbb{R} \to \mathbb{R}$, which is non-negative in $(0,\infty)$ and satisfies, for any $r \in \left(\frac{2N-\alpha}{N}, \frac{2N-\alpha}{N-2}\right)$,

($f1$) $\displaystyle\lim_{t\to 0} \frac{f(t)}{t} = 0$,

($f2$) $\displaystyle\lim_{t\to\infty} \frac{f(t)}{t^{r-1}} = 0$,

($f3$) $\dfrac{f(t)}{t}$ is increasing if $t > 0$ and decreasing if $t < 0$.

We denote

$$\tilde{f}(t) = \begin{cases} \dfrac{f(t)}{t}, & \text{if } \ t \neq 0, \\ 0, & \text{if } \ t = 0. \end{cases}$$

Our hypotheses imply that $\tilde{f}$ is continuous. Therefore, problem (P) can be written in the form

$$-(\nabla + iA(x))^2 u + V(x)u = \left(\frac{1}{|x|^\alpha} * F(|u|)\right)\tilde{f}(|u|)u. \tag{2}$$

The composition of f and F with $|u|$ gives a variational structure to the problem, allowing the application of the Mountain Pass Theorem. So, the right-hand side of problem (2) generalizes the term

$$\left(\frac{1}{|x|^\alpha} * |u|^p\right)|u|^{p-2}u, \tag{3}$$

which was studied by Cingolani, Clapp and Secchi in [2].

(The main part of the interesting paper by Cingolani, Clapp and Secchi [2] is devoted to the existence of multiple solutions of equation (2) - with (3) as the right-hand side - under the action of a closed subgroup G of the orthogonal group $O(N)$ of linear isometries of $\mathbb{R}^N$ if $A(gx) = gA(x)$ and $V(gx) = V(x)$ for all $g \in G$ and $x \in \mathbb{R}^N$. The authors look for solutions satisfying

$$u(gx) = \tau(g)u(x), \quad \text{for all } \ g \in G \ \text{ and } \ x \in \mathbb{R}^N,$$

where $\tau\colon G \to S^1$ is a given continuous group homomorphism into the unit complex numbers S^1. We also address the multiplicity of solutions in a particular case of that treated in [2].)

2 Main Result

Our aim in this paper is to prove the existence of a ground state solution for problem (2). We state our main result:

Theorem 2.1. *Suppose that $\alpha \in (0, N)$ and that conditions $(V1)$-$(V3)$, (AV) and $(f1)$-$(f3)$ are valid. Then, problem (2) has a ground state solution.*

This is accomplished by showing that the mountain pass geometry is satisfied and then considering the asymptotic form of problem (2) and applying Struwe's splitting lemma.

The simple proof of our multiplicity result is stablished in a particular case of that treated in [2], admitting a decomposition of the subgroup G. It only collects classical theorems, see [1, Theorem 10.10].

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NONLOCAL ELLIPTIC SYSTEM ARISING FROM THE GROWTH OF CANCER STEM CELLS

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Abstract

This talk is based on [2] where we show the existence of coexistence states for a nonlocal elliptic system arising from the growth of cancer stem cells. For this, we use the bifurcation method and the theory of the fixed point index in cones. Moreover, in some cases we study the behavior of the coexistence region, depending on the parameters of the problem.

1 Introduction

In this work, we will study the following system:

$$\begin{cases} -D_1\Delta u = \delta\gamma F(u+v)\mathcal{K}(u) & \text{in} \quad \Omega, \\ -D_2\Delta v + \alpha v = (1-\delta)\gamma F(u+v)\mathcal{K}(u) + \rho F(u+v)\mathcal{K}(v) & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded and regular domain of $\mathbb{R}^N$, $D_1, D_2, \gamma, \alpha, \rho > 0$, $\delta \in [0,1]$ and $F \in C^1(\mathbb{R}_+)$ is a decreasing function with $F(0) = 1$ and $F(t) = 0$, for $t \geq 1$. The function $\mathcal{K}(u) : L^\infty(\Omega) \longrightarrow L^\infty(\Omega)$ is given by

$$\mathcal{K}(u)(x) = \int_\Omega K(x,y)u(y)dy,$$

where $K \in C(\overline{\Omega} \times \overline{\Omega})$ is a nonnegative and non-identically zero function.

The system (1) is the stationary counterpart, with homogeneous Dirichlet boundary conditions, of a model of the dynamic of cancer stem cells ($CSCs$) and non-stem tumor cells (TCs) in a certain tissue Ω, proposed in [4] to investigate the "tumor growth paradox", that means: *"an increasing rate of spontaneous cell death in* (TCs) *shortens the waiting time for proliferation and migration of* ($CSCs$)*, and thus facilitates tumor progression"*.

We would like to note that when one group of cell vanishes, the other one verifies an equation of the type:

$$\begin{cases} -d\Delta u + \beta u = \sigma F(u) \displaystyle\int_\Omega K(x,y)u(y)dy & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{2}$$

with $\beta \geq 0$ and $\sigma > 0$. The problem (2) is a nonlocal logistic equation and has been analyzed in [3] when $\beta = 0$ and $F(u) = (A(x) - u^p)^+$, where $p \geq 1$ and $A \in C(\overline{\Omega})$, with $A^+ \neq 0$. To study the coexistence states of (1), we generalize the results of [3] for F as above.

2 Main Results

In what follows, we give a brief summary of the main results obtained. For equation (2), we use the sub-super solution method given in [3] to prove the following result:

(*a*) There exists a real number $\sigma_1 > 0$ such that (2) has a unique positive solution in $C_0^1(\overline{\Omega})$, denoted by $\theta_\sigma[d;\beta;K]$, if and only if $\sigma > \sigma_1$. Moreover,

$$\theta_\sigma[d;\beta;K] \leq 1 \quad \text{in} \quad \Omega.$$

Note that for (1) the trivial solution always exists for any values of the parameters. The existence of semi-trivial solutions of (1) is given by the above result (*a*). For the coexistence states, observe first that when $\delta = 0$ the system (1) is reduced to an equation of the type (2). Therefore, in this case it does not have coexistence states. Because of this, we study the existence of coexistence states only in two cases: $\delta \neq 1$ and $\delta = 1$.

For the case $\delta \neq 1$ we use bifurcation arguments, more precisely the results presented in [5], to find an unbounded continuum of coexistence states of (1) emanating from a specific point. Hence, we have the existence of one curve in the plane $(\gamma - \rho)$, denoted by $\gamma = \mathcal{F}_\delta(\rho)$, and we obtain the following result:

(*b*) Assume that $\delta \in (0,1)$ and $\rho > 0$. If $\gamma > \mathcal{F}_\delta(\rho)$, then there exists at least one coexistence state of (1).

For $\delta = 1$, we use the theory presented in [1] of fixed point index with respect to the positive cone and we obtain the existence of two curves, denoted by $\gamma = \mathcal{F}_1(\rho)$ and $\rho = \mathcal{G}(\gamma)$, and we show the following result:

(*c*) There exist real numbers $\sigma_{1,1}, \sigma_{1,2} > 0$ such that if $\delta = 1$, $\gamma > \sigma_{1,1}$ and $\rho > \sigma_{1,2}$, then there exists at least one coexistence state of (1) when

$$(\gamma - \mathcal{F}_1(\rho)) \cdot (\rho - \mathcal{G}(\gamma)) > 0.$$

Depending on the relative position of these two curves, we can conclude:

(*d*) Assume that $\delta = 1$, $\gamma > \sigma_{1,1}$ and $\rho > \sigma_{1,2}$. If $\gamma > \mathcal{F}_1(\rho)$ and $\rho > \mathcal{G}(\gamma)$, then there exists at least one coexistence state of (1). Moreover, the sum of the indices of all coexistence states of (1) is 1.

(*e*) Assume that $\delta = 1$, $\gamma > \sigma_{1,1}$ and $\rho > \sigma_{1,2}$. If $\gamma < \mathcal{F}_1(\rho)$ and $\rho < \mathcal{G}(\gamma)$, then there exists at least one coexistence state of (1). Moreover, the sum of the indices of all coexistence states of (1) is -1.

We use the above results to understand the behavior of $(CSCs)$ and to study the "tumor growth paradox". More details and the proofs of all presented results can be found in the paper [2].

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EXISTENCE OF POSITIVE SOLUTIONS FOR A CLASS OF SEMIPOSITONE QUASILINEAR PROBLEMS THROUGH ORLICZ-SOBOLEV SPACE

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Abstract

In this paper we show the existence of weak solution for a class of semipositone problem of the type

$$\begin{cases} -\Delta_\Phi u = f(u) - a & \text{in } \Omega, \\ u(x) > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a smooth bounded domain, $f : [0, +\infty) \to \mathbb{R}$ is a continuous function with subcritical growth, $a > 0$, $\Delta_\Phi u$ stands for the Φ-Laplacian operator. By using variational methods, we prove the existence of solution for a small enough.

1 Introduction

In this paper we study the existence of positive weak solutions for the semipositone problem

$$\begin{cases} -\Delta_\Phi u = f(u) - a & \text{in } \Omega, \\ u(x) > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a smooth bounded domain with smooth boundary denoted by $\partial\Omega$, $f : [0, +\infty) \to \mathbb{R}$ is a continuous function with subcritical growth, $a > 0$, and $\Delta_\Phi u = div(\phi(|\nabla u|)\nabla u)$ stands for the Φ-Laplacian operator, where $\phi : (0, \infty) \to (0, \infty)$ is an appropriate C^1-function such that

$$\Phi(t) := \int_0^{|t|} \phi(s) s ds, \ t \in \mathbb{R}$$

is an N-function. In what follows, ϕ satisfies the following conditions

(ϕ_1) $\phi : (0, \infty) \to (0, \infty)$ is a C^1-function;

(ϕ_2) $\phi(t)$, $(\phi(t)t)' > 0$, $t > 0$;

(ϕ_3) there exist $l, m \in (1, N)$ with $m \in [l, l^*)$ and $l^* = \frac{lN}{N-l}$, such that

$$l \leq \frac{\Phi'(t)t}{\Phi(t)} \leq m \quad \forall t > 0;$$

(ϕ_4) there exist $\bar{l}, \bar{m} > 0$ such that

$$\bar{l} \leq \frac{\Phi''(t)t}{\Phi'(t)} \leq \bar{m} \quad \forall t > 0.$$

Related to the function f, we assume that $f:[0,+\infty)\to\mathbb{R}$ is a continuous function and the following conditions:

$$0 = f(0) = \min_{t\in[0,+\infty)} f(t), \tag{f_1}$$

$$\lim_{t\to 0^+} \frac{f(t)}{\phi(t)t} = 0. \tag{f_2}$$

There is $q \in (m, l^*)$ such that

$$\limsup_{|t|\to+\infty} \frac{|f(t)|}{|t|^{q-1}} < +\infty. \tag{f_3}$$

There are $\theta > m$ and $t_0 > 0$ such that

$$\theta F(t) \leq f(t)t, \quad \forall t \geq t_0, \tag{f_4}$$

where $F(t) = \int_0^t f(\tau)\,d\tau$.

In the sequel, we say that $u \in W_0^{1,\Phi}(\Omega)$ is a *weak solution* for (P) if u is a continuous positive function that verifies

$$\int_\Omega \phi(|\nabla u|)\nabla u \nabla\varphi dx = \int_\Omega (f(u)-a)\varphi dx, \quad \forall\varphi \in W_0^{1,\Phi}(\Omega).$$

Hereafter, $W_0^{1,\Phi}(\Omega)$ denotes the completion of $C_0^\infty(\Omega)$ in the norm $\|\ \|_{1,\Phi}$.

2 Main Result

Our main result is the following.

Teorema 2.1. *Assume $(\phi_1)-(\phi_4)$ and $(f_1)-(f_4)$. Then, there exists $a^* > 0$ such that if $a \in (0, a^*)$, problem (P) has a positive weak solution $u_a \in C^{1,\gamma}(\Omega)$ for some $\gamma \in (0,1)$.*

In the proof of Theorem 2.1 we have used variational and regularity results found in Liberman [2, 3]. By using mountain pass theorem we have found a solution u_a for all $a > 0$. By taking the limit of a goes to 0, we were able to show, via regularity results found in [2] and [3], that u_a is positive for a small enough. We believe that this is the first paper involving the Δ_Φ Laplacian and semipositone problem. Finally, we would like point out that a version of Theorem 2.1 can be done for $N = 1$, by supposing $l, m > 1$ and $q \in (m, +\infty)$ in (f_3), because the embedding $W_0^{1,\Phi}(\Omega) \hookrightarrow C(\overline{\Omega})$ is compact, for more details about this embedding see [1] and [4].

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TWO SOLUTIONS FOR A FOURTH ORDER NONLOCAL PROBLEM WITH INDEFINITE POTENTIALS

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Abstract

We study the nonlocal equation

$$\Delta^2 u - m\left(\int_\Omega |\nabla u|^2 dx\right)\Delta u = \lambda a(x)|u|^{q-2}u + b(x)|u|^{p-2}u, \text{ in } \Omega,$$

subject to the boundary condition $u = \Delta u = 0$ on $\partial\Omega$. For m continuous and positive we obtain a nonnegative solution if $1 < q < 2 < p \leq 2N/(N-4)$ and $\lambda > 0$ small. If the affine case $m(t) = \alpha + \beta t$, we obtain a second solution if $4 < p < 2N/(N-4)$ and $N \in \{5, 6, 7\}$. In the proofs we apply variational methods.

1 Introduction

Consider the semilinear problem

$$-\Delta u = \lambda a(x)|u|^{q-2}u + b(x)|u|^{p-2}u \text{ in } \Omega, \;\; u \in W_0^{1,2}(\Omega),$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $N \geq 3$, $\lambda > 0$ is a parameter, $1 < q < 2 < p \leq 2N/(N-2)$ and a, b are potentials defined in Ω. In a celebrated paper, Ambrosetti, Brezis and Cerami [1] supposed that $a \equiv 1$, $b \equiv 1$ and obtained two positive solutions if $\lambda > 0$ is small. In [2], de Figueiredo, Gossez and Ubilla generalized this result by considering nonconstant sign changing potentials. In this setting the Maximun Principle can fail and therefore the solutions are only nonnegative.

We consider here a nonlocal fourth-order version of the above problem, namely

$$(P_\lambda) \qquad \begin{cases} \Delta^2 u - m\left(\int_\Omega |\nabla u|^2 dx\right)\Delta u = \lambda a(x)|u|^{q-2}u + b(x)|u|^{p-2}u, \text{ in } \Omega \\ u = \Delta u = 0 \text{ on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $N \geq 5$ and $\Delta^2 u = \Delta(\Delta u)$ is the biharmonic operator. The equation in (P_λ) is related with the so called Berger plate model

$$u_{tt} + \Delta^2 u + \left(Q + \int_\Omega |\nabla u|^2 dx\right)\Delta u = f(x, u, u_t),$$

and it is a simplification of the von Karman plate equation that describes large deflection of plate. The parameter Q describes in-plane forces applied to the plate and the function f represents transverse loads which may depend on the displacement u and the velocity u_t. The equation is also related with some models which describe the bending equilibrium states of a beam subjected to a force $f(x, u)$ and other elastic force (see [**?**]), namely

$$u_{tt} + \frac{EI}{\rho}u_{xxxx} - \left(\frac{H}{\rho} + \frac{EA}{2\rho L}\int_0^L |u_x|^2 dx\right)u_{xx} = f(x, u).$$

More recent references with important details about the physical motivation

In [3], the authors supposed that m is increasing, $a \equiv 1$, $b \equiv 1$ and obtained infinitely many solutions, for $1 < q < 2$, $p = 2^* := 2N/(N-4)$ and $\lambda > 0$ small. This result was partially extended in [6] where they assumed that $b \equiv 1$, the (nonautonomous) concave term were of type $\lambda h(x,u)$, with $h(x,u) \geq 0$ if $u \geq 0$, and a technical assumption on the growth of the function m. Other results for positive potentials in unbounded domains can be found in [5, 4] and references there in.

Here we are going to consider sign-changing potentials under mild regularity conditions. More specifically, we suppose that

(m_1) $m \in C([0,+\infty))$ is positive;

(a_1) $a \in L^{\sigma_q}(\Omega)$, for some $\sigma_q > 2^*/(2^*-q)$;

(a_2) if we set

$$\Omega_a^+ := \{x \in \Omega : a(x) > 0\},$$

then there exist $x_0 \in \Omega_a^+$ and $\delta > 0$ such that $B_\delta(x_0) \subset \Omega_a^+$;

(b_1) $b \in L^\infty(\Omega)$.

2 Main Results

Theorem 2.1. *Suppose that $1 < q < 2 < p \leq 2^*$. If (m_1), $(a_1) - (a_2)$ and (b_1) hold, then there exists $\lambda^* > 0$ such that, for each $\lambda \in (0,\lambda^*)$, the problem (P_λ) has a nonnegative nonzero solution.*

Theorem 2.2. *Suppose that $N \in \{5,6,7\}$, $1 < q < 2$ and $4 < p \leq 2^*$. If (m_2), (a_1), (ab_1) and $(b_1) - (b_2)$ hold, then there exists $\lambda^* > 0$ such that the problem (P_λ) has at least two nonzero solution for each $\lambda \in (0,\lambda^*)$.*

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MULTIPLICIDADE DE SOLUÇÕES PARA EQUAÇÕES DE SCHRÖDINGER QUASILINEAR ENVOLVENDO EXPOENTE CRÍTICO DE SOBOLEV

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Abstract

Neste trabalho estabelecemos, usando método variacional, a existência e multiplicidade de soluções para equações quasilineares de Schrödinger envolvendo expoente crítico de Sobolev. Usando uma mudança de variável dada em [1], obtemos um problema semilinear cujo funcional energia associado é simétrico e satisfaz a condição de Cerami para um nível abaixo de uma constante obtida.

1 Introdução

Neste trabalho estudamos a existência e multiplicidade de soluções para equações de Schrödinger quasilineares. Trataremos de estudar a seguinte equação quasilinear de Schrödinger

$$\begin{aligned} &-\Delta u + V(x)u - \Delta(u^2)u = \lambda q(x)u + K(x)|u|^{p-2}u + \theta\Gamma(x)|u|^{2\cdot 2^*-2}u, \ x \in \mathbb{R}^N, \\ &u \in H^1(\mathbb{R}^N), \end{aligned} \tag{1}$$

onde $N \geq 3$, $4 < p < 2 \cdot 2^*$ e λ, θ são parâmetros positivos.

Nosso objetivo é obter existência e multiplicidade de soluções para a equação (1). Neste caso, pedimos que o potencial V e a não linearidade cumpra algumas condições que destacamos a seguir.

(V_1) $V \in C(\mathbb{R}^N, \mathbb{R}) \cap L^\infty(\mathbb{R}^N)$ com $\inf_{x\in\mathbb{R}^N} \geq V_0 > 0$.

(q_1) $q \in L^\alpha(\mathbb{R}^N) \cap L^\beta(\mathbb{R}^N)$ para algum $\alpha > N/2$ e $\beta \in (2N/(N+2), 2]$, com $q(x) \geq 0$ q. t. p. $x \in \mathbb{R}^N$.

(K_1) $K(x) \in L^\gamma(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ com $\gamma = (2 \cdot 2^*/p)'$ e $K(x) \geq 0$ q. t. p. $x \in \mathbb{R}^N$.

(Γ_1) $\Gamma_1 \in L^\infty(\mathbb{R}^N)$, $\Gamma(x) \geq 0$ q. t. p. $x \in \mathbb{R}^N$ e $\lim_{|x|\to\infty} \Gamma(x) = 0$.

A grande dificuldade em obter soluções não triviais para o problema (1) está associada a perda de compacidade inerente ao tratar problemas em domínios não limitados. Várias condições sobre o potencial V tem sido tratado de forma que a compacidade seja recuperada para algum subespaço fechado de $E = H^1(\mathbb{R}^N)$ (veja [2] para mais detalhes). A condição (V_1) garante que a norma

$$\|u\|^2 = \left(\int_{\mathbb{R}^N} \left(|\nabla u|^2 + V(x)u^2 \right) dx \right)^{1/2}$$

seja equivalente a norma usual de E. Esta norma induz a mesma topologia em E. Isso faz que não seja possível mostrar que as imersões de E nos espaços $L^s(\mathbb{R}^N)$ sejam compactas para $2 \leq s < 2^*$. Para contornar essa dificuldade, pedimos alguma condição de compacidade na não linearidade da equação (1). As condições (q_1) e (K_1) permitem que a imersão de E nos espaços de Lebesgue com peso, $L^2\left(\mathbb{R}^N, q(x)\right)$ e $L^p\left(\mathbb{R}^N, K(x)\right)$, sejam compactas. A condição (Γ_1) permite juntamente com o princípio de compacidade de Lions mostrar que o funcional energia satisfaz a condição de Cerami para níveis abaixo de um nível crítico.

O parâmetro λ interage com autovalores altos do seguinte problema linear com peso $q(x)$,

$$\begin{aligned} &-\Delta u + V(x)u = \lambda q(x)u, \ x \in \mathbb{R}^N, \\ &u \in H^1(\mathbb{R}^N). \end{aligned} \tag{2}$$

Ou seja, existe $j \in \mathbb{N}$ com $j \geq 1$ tal que $\lambda_j < \lambda < \lambda_{j+1}$.

2 Resultado Principal

Nosso principal resultado é o seguinte:

Teorema 2.1. *Suponha que* (V_1), (q_1), (K_1), *e* (Γ_1) *sejam satisfeitas. Suponha também que existe* $j \in \mathbb{N}$ *com* $j \geq 1$. *Então, dado* $k \in \mathbb{N}$ *existe* θ_k *positivo tal que o problema* (1) *admite pelo menos* k *pares de soluções não triviais para todo* $\theta \in (0, \theta_k)$.

Para obter esse resultado utilizamos a seguinte versão do Teorema do passo da montanha simétrico (ver [1]).

Teorema 2.2. *Seja* $E = E_1 \bigoplus E_2$, *onde* E *é um espaço de Banach real e* E_1 *é um subespaço de dimensão finita. Suponha que* $J \in C^1(E, \mathbb{R})$ *é um funcional par satisfazendo* $J(0) = 0$ *e*

(J_1) *existe uma constante* $\rho > 0$ *tal que* $J(v) \geq 0$ *para todo* $v \in \partial B_\rho \cap E_2$.

(J_2) *existe um subespaço* W *de* E *com* $\dim E_1 < \dim W < \infty$ *e existe* $M > 0$ *tal que* $\max_{v \in W} J(v) < M$.

(J_3) *considerando* $M > 0$ *dado por* (J_2), J *satisfaz a condição de Cerami* $(Ce)_c$ *para* $0 \leq c \leq M$.

Então J *possui pelo menos* $\dim W - \dim E_1$ *pares de pontos críticos não triviais.*

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GEOMETRIC ESTIMATES FOR QUASI-LINEAR ELLIPTIC MODELS WITH FREE BOUNDARIES AND APPLICATIONS

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Abstract

We study geometric regularity estimates and the limiting behavior as $p \to \infty$ of nonnegative solutions for elliptic equations of $p-$Laplacian type $(1 < p < \infty)$ with a strong absorption:

$$-\Delta_p u(x) + \lambda_0(x) u^q_{\{u>0\}}(x) = 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^N,$$

where $\lambda_0 > 0$ is a bounded function, Ω is a bounded domain and $0 \leq q < p-1$. When p is fixed, such a model is mathematically interesting since it permits the formation of *dead core zones*, this is, *a priori* unknown regions where non-negative solutions vanish identically. First, we turn our attention to establishing sharp C^{τ}_{loc} regularity estimates for $p-$dead core solutions along free boundary points, where $\tau = \frac{p}{p-1-q} \gg 1$. Afterwards, assuming that $\ell := \lim\limits_{p \to \infty} \dfrac{q(p)}{p} \in [0, 1)$ exists, we establish existence for limit solutions as $p \to \infty$, as well as we characterize the corresponding limit operator governing the limit problem. We also establish sharp C^{γ}_{loc} regularity estimates for limit solutions along free boundary points, where the sharp regularity exponent is given explicitly by $\gamma = \frac{1}{1-\ell}$.

1 Introduction

Quasi-linear elliptic problems whose nonlinear nature give rise to free boundaries come from a varied phenomena as reaction-diffusion and absorption processes in pure and applied mathematics. An enlightening example is the following model (with sign constrain) of a certain (stationary) isothermal catalytical reaction process

$$\begin{cases} -\Delta_p u + \lambda_0(x) u^q_{\{u>0\}}(x) &= 0 \quad \text{in} \quad \Omega \\ u(x) &= g(x) \quad \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $1 < p < \infty$, $\Omega \subset \mathbb{R}^N$ is a regular and bounded region and λ_0 is a positive bounded function. In such a context, u represents the density of a chemical reagent (or gas), λ_0 is the *Thiele Modulus*, which controls the ratio of reaction rate to diffusion-convection rate, and g is a continuous non-negative boundary datum. In this scenery (1) has an absence of Strong Minimum Principle, i.e., non-negative solutions may vanish completely within an *a priori* unknown region of positive measure $\Omega' \subset \Omega$ known as *Dead Core* set (cf. Díaz's Monograph [3, Chapter 1] for a complete survey about this subject). Such a feature allow us to treat (1) as a free boundary problem.

Although several qualitative properties have been established in [3] (and references therein) in the last decades, quantitative geometric estimates have been few developed up to date (cf. [6]). Therefore, this has been our main impetus in studying diffusion problems governed by quasi-linear elliptic equations like (1) via a systematic and modern geometric approach (see also [2]). In addiction, our estimates are striking, because they supply an unexpected gain of smoothness (along free boundary points) when compared with ones currently available (cf. [1]).

2 Main Results

First, we prove which is the growth rate of solutions leaving their free boundaries.

Theorem 2.1 ([3, Theorem 1.1] and [4, Theorem 4.1])**.** *Let u be a nonnegative, bounded weak solution to* (1)*, $\Omega' \Subset \Omega$ and let $x_0 \in \overline{\{u>0\}} \cap \Omega'$. Then there exists a universal constant $\mathfrak{C}_0 > 0$ such that for all $0 < r < \min\{1, dist(\Omega', \partial\Omega)\}$ there holds*

$$\sup_{\partial B_r(x_0)} u(x) \geq \mathfrak{C}_0 \left(N, p, q, \inf_{\Omega} \lambda_0(x)\right) r^{\frac{p}{p-1-q}}.$$

Next, we prove the following sharp and improved regularity estimate at free boundary points:

Theorem 2.2 ([3, Theorem 1.2] and [4, Theorem 1.2])**.** *Let u be a nonnegative, bounded weak solution to* (1)*, $\Omega' \Subset \Omega$ and $x_0 \in \partial\{u>0\} \cap \Omega'$. Then, there exists a universal constant $\mathfrak{C}_1 = \mathfrak{C}_1 \left(N, p, q, \inf_{\Omega} \lambda_0(x)\right) > 0$ such that*

$$\sup_{B_r(x_0)} u(x) \leq \mathfrak{C}_1 r^{\frac{p}{p-1-q}} \quad \forall \ 0 < r \ll \min\{1, dist(\Omega', \partial\Omega)\}.$$

Finally, we prove the convergence of the family of $p-$dead core solutions, as well as we deduce the corresponding limit operator (in non-divergence form) driving the limit equation.

Theorem 2.3 ([3, Theorems 1.3, 1.4 and 1.5])**.** *Let $(u_p)_{p\geq 2}$ be the family of solutions to* (1) *with $g \in Lip(\partial\Omega)$. Assume that $\ell := \lim_{p\to\infty} \dfrac{q(p)}{p} \in [0,1)$ exists. Then, up to a subsequence, $u_p \to u_\infty$ uniformly in $\overline{\Omega}$. Furthermore, such a limit fulfills*

$$\begin{cases} \max\left\{-\Delta_\infty u_\infty(x),\ u_\infty^{\ell}(x) - |\nabla u_\infty(x)|\right\} &= \quad 0 \quad in \quad \{u_\infty > 0\} \cap \Omega, \\ u_\infty(x) &= \quad g(x) \quad on \quad \partial\Omega, \end{cases}$$

in the viscosity sense. Finally, for every $x_0 \in \partial\{u_\infty > 0\} \cap \Omega'$

$$(1-\ell)^{\frac{1}{1-\ell}} r^{\frac{1}{1-\ell}} \leq \sup_{B_r(x_0)} u_\infty(x) \leq 2 \cdot 2^{\frac{1}{1-\ell}} (1-\ell)^{\frac{1}{1-\ell}} r^{\frac{1}{1-\ell}}.$$

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ON THE EXISTENCE OF GROUND STATES OF LINEARLY COUPLED SYSTEMS

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Abstract

We give a survey on recent results related to the existence of ground states for several classes of linearly coupled systems involving Schrödinger equations

$$\begin{cases} Lu + V_1(x)u = f_1(x,u) + \lambda(x)v, & x \in \mathbb{R}^N, \\ Lv + V_2(x)v = f_2(x,v) + \lambda(x)u, & x \in \mathbb{R}^N, \end{cases}$$

where L denotes a local or nonlocal operator. We discuss the difficulties imposed by these classes of systems and the methods applied to get a ground state solution.

1 Introduction

Our purpose is to give a survey on recent results related to the existence of ground states for linearly coupled systems involving Schrödinger equations

$$\begin{cases} Lu + V_1(x)u = f_1(x,u) + \lambda(x)v, & x \in \mathbb{R}^N, \\ Lv + V_2(x)v = f_2(x,v) + \lambda(x)u, & x \in \mathbb{R}^N, \end{cases} \tag{S}$$

where L denotes a local or nonlocal operator. This System suggests many particular classes of systems which may be motivated both from a pure mathematical point of view and their concrete applications. A first particular case of (S) may be considered by the following class of linearly coupled systems

$$\begin{cases} Lu + V_1(x)u = |u|^{p-2}u + \lambda(x)v, & x \in \mathbb{R}^N, \\ Lv + V_2(x)v = |v|^{q-2}v + \lambda(x)u, & x \in \mathbb{R}^N, \end{cases} \tag{1}$$

where $N \geq 3$, $2 < p, q \leq 2^*$ and $0 \leq \lambda(x) < \sqrt{V_1(x)V_2(x)}$, for all $x \in \mathbb{R}^N$. In the celebrated work [1], H. Brezis and E.H. Lieb (1984) proved the existence of ground states for the following class of systems

$$-\Delta u_i(x) = g^i(u(x)), \quad i = 1, 2, ..., n,$$

where $g^i(u) = \partial G(u)/\partial u_i$, for some $G \in C^1(\mathbb{R}^d)$, $d \geq 2$. As consequence of the above work, we have the existence of ground states for System (1) when $L = -\Delta$, $V_1(x) = \mu$, $V_2(x) = \nu$, $\lambda(x) = \lambda$ and $2 < p, q < 2^*$, precisely

$$\begin{cases} -\Delta u + \mu u = |u|^{p-2}u + \lambda v, & x \in \mathbb{R}^N, \\ -\Delta v + \nu v = |v|^{q-2}v + \lambda u, & x \in \mathbb{R}^N. \end{cases} \tag{2}$$

The critical case of System (2) was studied in [2], where the authors proved that the existence or nonexistence of ground states is related with the intervals which the parameters μ, ν and λ belong. For works considering System (1) with a more general operator L and functions $V_1(x), V_2(x), \lambda(x)$, we refer to [4, 5].

Recently, several other classes of linearly coupled systems were studied. These classes of systems imposed some difficulties, for instance: lack of compactness, the presence of linear coupling functions $\lambda(x)v$ and $\lambda(x)u$ in the right-hand side, the type of operator L if it is local or nonlocal, the behavior of the nonlinear terms, etc. Arguing as in System (1), our purpose is to travel on some recent works, by discussing the difficulties and the method which was used to overcome such difficulties. Naturally, new questions arise which motivate new works regarding the existence of ground states for linearly coupled systems.

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ON THE EXTREMAL PARAMETERS OF A SUBCRITICAL KIRCHHOFF TYPE EQUATION AND ITS APPLICATIONS

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Abstract

We study a superlinear and subcritical Kirchhoff type equation which is variational and depends upon a real parameter λ. The nonlocal term forces some of the fiber maps associated with the energy functional to have two critical points. This suggest multiplicity of solutions and indeed we show the existence of a local minimum and a mountain pass type solution. We characterize the first parameter λ_0^* for which the local minimum has non-negative energy. Moreover we characterize the extremal parameter λ^* for which if $\lambda > \lambda^*$, then the only solution to the Kirchhoff equation is the zero function. In fact, λ^* can be characterized in terms of the best constant of Sobolev embeddings. We also study the asymptotic behavior of the solutions when $\lambda \downarrow 0$.

1 Introduction

In this work we study the following Kirchhoff type equation

$$\begin{cases} -\left(a+\lambda\int|\nabla u|^2\right)\Delta u = |u|^{\gamma-2}u & \text{in } \Omega, \\ u=0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $a>0$, $\lambda>0$ is a parameter, Δ is the Laplacian operator and $\Omega\subset\mathbb{R}^3$ is a bounded regular domain.

Kirchhoff type equations have been extensively studied in the literature. It was proposed by Kirchhoff in [1] as an model to study some physical problems related to elastic string vibrations and since then it has been studied by many author, see for example the works of Lions [2], Alves at al. [3] and the references therein. Our main interest here is to analyze, through the fibering method of Pohozaev, how the Nehari set change with respect to the parameter λ and then apply this analysis to study bifurcation properties of the problem (1) (see for example Chen at al. [4]). In fact, there exists a extremal parameter λ^* (see Il'yasov [5]) which can be characterized variationally by

$$\lambda^* = C_{a,\gamma}\sup\left\{\left(\frac{\left(\int|u|^\gamma\right)^{\frac{1}{\gamma}}}{\left(\int|\nabla u|^2\right)^{\frac{1}{2}}}\right)^{\frac{2\gamma}{\gamma-2}} : u\in H_0^1(\Omega)\setminus\{0\}\right\},$$

where $C_{a,\gamma}$ is some positive constant and if $\lambda>\lambda^*$ then the Nehari set is empty while if $\lambda\in(0,\lambda^*)$ then the Nehari set is not empty. Another interesting paramenter is $\lambda_0^*<\lambda^*$ which is characterized by the property that if $\lambda\in(0,\lambda_0^*)$, then $\inf_{u\in H_0^1(\Omega)}\Phi_\lambda(u)<0$ while if $\lambda\geq\lambda_0^*$ the infimum is zero. When $\lambda\in(0,\lambda_0^*]$ one can easily provide a Mountain Pass Geometry and a global minimizer for the functional Φ_λ, however, when $\lambda>\lambda_0^*$ we need to provide some estimates on the Nehari sets in order to solve some technical issues to obtain again a Mountain Pass Geometry and a local minimizer for the functional Φ_λ.

2 Main Results

Let $H_0^1(\Omega)$ denote the standard Sobolev space and $\Phi_\lambda : H_0^1(\Omega) \to \mathbb{R}$ the energy functional associated with (1), that is

$$\Phi_\lambda(u) = \frac{a}{2}\int |\nabla u|^2 + \frac{\lambda}{4}\left(\int |\nabla u|^2\right)^2 - \frac{1}{\gamma}\int |u|^\gamma. \tag{1}$$

We observe that Φ_λ is a C^1 functional and the critical points of Φ_λ are solutions for the equation (1). The first result deal with the existence of two positive solutions for the problem (1).

Theorem 2.1. *Suppose $\gamma \in (2,4)$. Then there exist parameters $0 < \lambda_0^* < \lambda^*$ and $\varepsilon > 0$ such that for each $\lambda \in (0, \lambda_0^* + \varepsilon)$ the problem (1) has two positive solutions u_λ, w_λ satisfying:*

1) The function u_λ is a global minimizer for Φ_λ when $\lambda \in (0, \lambda_0^*]$ while u_λ is a local minimizer for Φ_λ when $\lambda \in (\lambda_0^*, \lambda_0^* + \varepsilon]$. The function w_λ is a mountain pass critical point for Φ_λ.

2) If $\lambda \in (0, \lambda_0^*)$ then $\Phi_\lambda(u_\lambda) < 0$ while $\Phi_{\lambda_0^*}(u_{\lambda_0^*}) = 0$ and if $\lambda \in (\lambda_0^*, \lambda_0^* + \varepsilon)$ then $\Phi_\lambda(u_\lambda) > 0$.

3) $\Phi_\lambda(w_\lambda) > 0$ and $\Phi_\lambda(w_\lambda) > \Phi_\lambda(u_\lambda)$ for each $\lambda \in (0, \lambda_0^* + \varepsilon)$.

4) If $\lambda > \lambda^*$ then the only solution $u \in H_0^1(\Omega)$ to the problem (1) is the zero function $u = 0$.

The second result concerns the asymptotic behavior of the solutions when $\lambda \downarrow 0$.

Theorem 2.2. *There holds*

i) $\Phi_\lambda(u_\lambda) \to -\infty$ and $\|u_\lambda\| \to \infty$ as $\lambda \downarrow 0$.

ii) $w_\lambda \to w_0$ in $H_0^1(\Omega)$ where $w_0 \in H_0^1(\Omega)$ is a mountain pass critical point associated to the equation $-a\Delta w = |w|^{p-2}w$.

Proof of Theorems 2.1 and 2.2: See [6].

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A BRÉZIS-OSWALD PROBLEM TO Φ-LAPLACIAN OPERATOR WITH A GRADIENT TERM

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Abstract

It is establish existence of solution to the quasilinear elliptic problem

$$\begin{cases} -\Delta_\Phi u = \lambda f(x,u) + \mu|\nabla u|^\sigma \text{ in } \Omega, \\ u > 0 \text{ in } \Omega, \ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where f has a sublinear growth, $\sigma > 0$ is an appropriate power, $\lambda > 0$, and $\mu \geq 0$ are real parameters. Our results are an improvement of the classical Brézis-Oswald result to Orlicz-Sobolev setting by including singular nonlinearity as well as a gradient term.

1 Introduction

In this work deals with existence of solution to elliptic quasilinear problem in the form

$$(P)_\mu : \begin{cases} -\Delta_\Phi u = \lambda f(x,u) + \mu|\nabla u|^\sigma \text{ in } \Omega, \\ u > 0 \text{ in } \Omega, \ u = 0 \text{ on } \partial\Omega, \end{cases}$$

where $\lambda > 0$, $\mu \geq 0$ are real parameters, $\Omega \subset \mathbb{R}^N$ with $N \geq 2$ is a smooth bounded domain, Φ is the even function defined by $\Phi(t) = \int_0^t s\phi(s)ds, \ t \in \mathbb{R}$, where $\phi : (0,\infty) \to (0,\infty)$ is a C^1-function satisfying

(ϕ_1) (i) $t\phi(t) \to 0$ as $t \to 0$, (ii) $t\phi(t) \to \infty$ as $t \to \infty$,

(ϕ_2) $t \mapsto t\phi(t)$ *is odd and strictly increasing from* $\mathbb{R}$ onto $\mathbb{R}$,

(ϕ_3) there exist $\ell, m \in (1,N)$ such that $\ell - 1 \leq \frac{(t\phi(t))'}{\phi(t)} \leq m - 1 < \ell^* - 1, \ t > 0$.

Furthermore, the function $f : \Omega \times (0,\infty) \to \mathbb{R}$ is such that:

(H_0) there exists a small $t_0 > 0$ such that $f(x,t) \geq 0$ for all $(x,t) \in \Omega \times (0,t_0)$;

(H_1) $t \mapsto f(x,t)$, $t > 0$ is a continuous function a.e. $x \in \Omega$ and for each $t > 0$ the function $x \mapsto f(x,t)$ belongs to $L^\infty(\Omega)$;

(H_2) $t \mapsto \dfrac{f(x,t)}{t^{\ell-1}}$ is strictly decreasing on $(0,\infty)$ for a.e. $x \in \Omega$;

(H_3) there exist constant $C > 0$ and $t_C \geq 0$ such that $|f(x,t)| \leq C(1 + t^{\ell-1})$ for all $t > t_C$ and a.e. $x \in \Omega$.

Notice that under the above hypotheses, we can consider $f(x,t)$ behaving as a singular nonlinearity at $t = 0$ as well, that is, $f(x,t) \to \infty$ as $t \to 0$ a.e $x \in \Omega$. For instance, the autonomous nonlinearities $f(t) = t^{-\alpha} + t^\beta$, $t > 0$ and $f(t) = t^{-\alpha} - t^\gamma$ for $t > 0$ with $\alpha > 0$, $-\infty < \beta \leq \ell - 1$ and $\gamma \geq \ell - 1$ satisfy (H_0)-(H_3). In both cases we emphasize that t_C must be taken positive in (H_3). Moreover, when $\phi(t) = p|t|^{p-2}$, $t > 0$, $\mu = 0$ and $f(x,t)$ is continuous on $[0,\infty)$ a.e. $x \in \Omega$ (i.e. we can take $t_C = 0$ in (H_3)), the hypotheses (H_1)-(H_3) hold together with a relationship between $\lambda(a_0)$ and $\lambda(a_\infty)$, Problem $(P)_\mu$ was considered by Brézis & Oswald [1] for $p = 2$ and by Díaz & Sáa [3] for $1 < p < \infty$ and under the more general hypothesis $(\phi_1) - (\phi_3)$ it was studied by Carvalho et al [2].

2 Main Result

Definition 2.1. *Let $u \in W^{1,\Phi}_{loc}(\Omega)$ be a fixed function. Recall that $u \leq 0$ on $\partial\Omega$ when $(u-\epsilon)^+ \in W^{1\Phi}_0(\Omega)$ for every $\epsilon > 0$. Moreover, we say that $u = 0$ on $\partial\Omega$ when u is non-negative and $u \leq 0$ on $\partial\Omega$.*

Definition 2.2. *We mean that $u \in W^{1,\Phi}_{loc}(\Omega)$ is a subsolution (supersolution) of $(P)_\mu$ if for every $U \subset\subset \Omega$ given, we have that $essinf_U u > 0$, $\lambda f(.,u) + \mu|\nabla u|^\sigma \in L^1_{loc}(\Omega)$ and*

$$\int_\Omega \phi(|\nabla u|)\nabla u \nabla \varphi dx \overset{(\geq)}{\leq} \lambda \int_\Omega f(x,u)\varphi dx + \mu \int_\Omega |\nabla u|^\sigma \varphi dx$$

hold for every $\varphi \in W^{1,\Phi}_0(U)$. The function u is said solution for $(P)_\mu$ if u is simultaneously a subsolution and a supersolution for $(P)_\mu$ and $u = 0$ on $\partial\Omega$ in the sense of Definition 2.1.

Now we shall consider the following auxiliary functions

$$a_0(x) := \lim_{t \downarrow 0+} \frac{f(x,t)}{t^{\ell-1}},\ a_\infty(x) := \lim_{t \uparrow \infty} \frac{f(x,t)}{t^{\ell-1}}, \tag{1}$$

and

$$\lambda(a) := \inf_{v \in W^{1,\Phi},\ \|v\|_\Phi = 1} \left\{ \int_\Omega \Phi(|\nabla v|)dx - \frac{1}{\ell}\int_{[v\neq 0]} a(x)|v|^\ell dx \right\},$$

for a function $a : \Omega \to \mathbb{R} \cup \{-\infty, \infty\}$ given. According to hypotheses (H_2) and (H_3) we can infer that

$$-\infty < a_0(x) \leq \infty \text{ and } -\infty \leq a_\infty(x) < \infty \Rightarrow -\infty \leq \lambda(a_0) < \infty \text{ and } -\infty < \lambda(a_\infty) \leq \infty.$$

Theorem 2.1. *Assume that conditions $(\phi_1)-(\phi_3)$, $(H_0)-(H_3)$, $0 < \sigma \leq \ell - 1$ and $-\infty \leq \lambda(a_\infty) < 0 < \lambda(a_0) \leq \infty$ hold. Let $\underline{u} \in W^{1,\Phi}_{loc}(\Omega) \cap C^1(\overline{\Omega})$ be a subsolion of $(P)_\mu$ in the sence of Definition 2.2, then there are $0 < \lambda_*, \mu_* \leq \infty$ such that for all $0 < \lambda < \lambda_*$ and $0 < \mu < \mu_*$ given, the problem $(P)_\mu$ has a minimal solution $u \in W^{1,\Phi}_{loc}(\Omega)$, i. e., there exist $u_* \in \mathcal{S}_{loc}(\underline{u})$ such that $u_* \leq u$, for all $u \in \mathcal{S}_{loc}(\underline{u})$, where*

$$\mathcal{S}_{loc}(\underline{u}) := \{u \in W^{1,\Phi}_{loc}(\Omega) : u \text{ is a solution of } (P)_\mu \text{ in the sence of Definition 2.2 and } u \geq \underline{u}\}.$$

Beside this, if $t_C = 0$ then the problem $(P)_\mu$ has a weak minimal solution $u_* \in \mathcal{S}(\underline{u})$ where

$$\mathcal{S}(\underline{u}) := \{u \in W^{1,\Phi}_0(\Omega) : u \text{ is a weak solution of } (P)_\mu \text{ and } u \geq \underline{u}\}.$$

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EXISTENCE AND NONEXISTENCE OF GROUND STATE SOLUTIONS FOR QUASILINEAR SCHRÖDINGER ELLIPTIC SYSTEMS

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Abstract

In this work we are concerned with the existence and nonexistence of ground state solutions for the following class of quasilinear Schrödinger coupled systems

$$\begin{cases} -\Delta u + a(x)u - \Delta(u^2)u = g(u) + \theta\lambda(x)uv^2, & x \in \mathbb{R}^N, \\ -\Delta v + b(x)v - \Delta(v^2)v = h(v) + \theta\lambda(x)vu^2, & x \in \mathbb{R}^N, \end{cases}$$

where $N \geq 3$, $\theta \geq 0$, $a, b, \lambda : \mathbb{R}^N \to \mathbb{R}$ are periodic or asymptotically periodic functions. The nonlinear terms g, h are superlinear at infinity and at the origin. By using a change of variable, we turn the quasilinear system into a nonlinear system where we can establish a variational approach with a fine analysis on the Nehari method. For the nonexistence result we compare the potentials $a(x), b(x)$ with periodic potentials proving nonexistence of ground state solutions.

1 Main Results

We study the existence of ground state solutions for the following class of coupled systems

$$\begin{cases} -\Delta u + a_p(x)u - \Delta(u^2)u = g(u) + \theta\lambda_p(x)uv^2, & x \in \mathbb{R}^N, \\ -\Delta v + b_p(x)v - \Delta(v^2)v = h(v) + \theta\lambda_p(x)vu^2, & x \in \mathbb{R}^N. \end{cases} \tag{$S_{p,\theta}$}$$

under the following hypotheses:
(a_1) $a_p, b_p, \lambda_p \in C(\mathbb{R}^N, \mathbb{R})$ are 1-periodic functions for each $x_1, x_2, ..., x_N$;
(a_2) There exist $a_0, b_0 > 0$ such that $a_p(x) \geq a_0 > 0$ and $b_p(x) \geq b_0 > 0$, for all $x \in \mathbb{R}^N$;
(a_3) $\lambda(x) \geq 0$ for all $x \in \mathbb{R}^N$ and $\lambda_p(x) > 0$ in a subset of finite measure.
On the nonlinear terms $g, h \in C^1(\mathbb{R}, \mathbb{R})$ we shall assume the following assumptions:
(g_1) $\max\{|g(t)|, |h(t)|\} \leq C\left(1 + |t|^{q-1}\right\}$ for all $t \in \mathbb{R}$ and $q \in (4, 2 \cdot 2^*)$
(g_2) There holds $\lim_{t\to 0} \dfrac{g(t)}{t} = 0, \quad \lim_{t\to 0} \dfrac{h(t)}{t} = 0$;
(g_3) There holds $\lim_{|t|\to+\infty} \dfrac{g(t)}{t^3} = +\infty, \quad \lim_{|t|\to+\infty} \dfrac{h(t)}{t^3} = +\infty$;
(g_4) The functions $t \longmapsto \dfrac{g(t)}{t^3}, \quad t \longmapsto \dfrac{h(t)}{t^3}$ are strictly increasing for on $|t| > 0$;
(g_5) $0 \leq G(t) := \int_0^t g(\tau)\,\mathrm{d}\tau \leq G(|t|)$ and $0 \leq H(t) := \int_0^t h(\tau)\,\mathrm{d}\tau \leq H(|t|)$, for all $t \in \mathbb{R}$.

Our first result can be stated in the following form:

Theorem 1.1 (Periodic case)**.** *Suppose that $(a_1) - (a_3)$ and $(g_1) - (g_5)$ hold. Then, there exists $\theta_0 > 0$ such that System ($S_{p,\theta}$) has at least one positive ground state solution, for all $\theta \geq \theta_0$.*

We are also concerned with existence of positive ground state solutions for the quasilinear coupled systems

$$\begin{cases} -\Delta u + a(x)u - \Delta(u^2)u = g(u) + \theta\lambda(x)uv^2, & x \in \mathbb{R}^N, \\ -\Delta v + b(x)v - \Delta(v^2)v = h(v) + \theta\lambda(x)vu^2, & x \in \mathbb{R}^N, \end{cases} \tag{S_θ}$$

where the functions a, b, λ are asymptotically periodic at infinity. More precisely, we assume that
(a_4) $a, b, \lambda \in C(\mathbb{R}^N, \mathbb{R})$, $0 < a_0 \leq a(x) \leq a_p(x)$, $0 < b_0 \leq b(x) \leq b_p(x)$, $\lambda_p(x) \leq \lambda(x)$ for all $x \in \mathbb{R}^N$ and

$$|a(x) - a_p(x)| \to 0, \quad |b(x) - b_p(x)| \to 0, \quad |\lambda(x) - \lambda_p(x)| \to 0, \quad \text{as } |x| \to +\infty.$$

Furthermore, we assume also that $a \not\equiv a_p$ and $b \not\equiv b_p$ hold true in a subset of finite Lebesgue measure.
In this case, the key point is to compare the energy levels for the original functional and the problem at infinity. Our second main result can be written in the following form:

Theorem 1.2 (Asymptotically periodic case). *Suppose that $(a_1) - (a_4)$ and $(g_1) - (g_5)$ hold. Then, there exists $\theta_0 > 0$ such that System (S_θ) has at least one positive ground state solution, for all $\theta \geq \theta_0$.*

We point out that in the proofs of Theorems 1.1 and 1.2 we get ground state solution for any $\theta \geq 0$. However, the solution could be semitrivial. For this reason, we control the range of $\theta > 0$ in order to get a positive ground state solution.
Now, we consider a non existence result under following assumption:
(a_5) $a, b, \lambda \in C(\mathbb{R}^N, \mathbb{R})$, $0 < a_0 \leq a_p(x) \leq a(x)$, $0 < b_0 \leq b_p(x) \leq b(x)$, $0 \leq \lambda(x) \leq \lambda_p(x)$ for all $x \in \mathbb{R}^N$ and

$$|a(x) - a_p(x)| \to 0, \quad |b(x) - b_p(x)| \to 0, \quad |\lambda(x) - \lambda_p(x)| \to 0, \quad \text{as } |x| \to +\infty,$$

where $a \not\equiv a_p$ and $b \not\equiv b_p$ in a subset of finite measure.

Theorem 1.3 (Nonexistence result). *Suppose that $(a_2), (a_3), (a_5)$ and $(g_1) - (g_4)$ hold. Then, System (S_θ) has no positive ground state solution for any $\theta \geq 0$. If in addition (g_5) holds, then System (S_θ) has no ground state solution for any $\theta \geq 0$.*

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UMA ABORDAGEM VIA ANÁLISE DE FOURIER PARA EQUAÇÕES ELÍPTICAS COM POTENCIAIS SINGULARES E NÃO LINEARIDADES ENVOLVENDO DERIVADAS

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Abstract

Neste trabalho estudamos uma família de problemas elípticos não homogêneos considerando um operador elíptico linear geral com potenciais críticos e não linearidades que dependem de operadores multiplicadores, que podem ser derivadas (mesmo fracionárias), e operadores integrais singulares. O operador elíptico geral pode conter derivadas de ordem superior e de tipo fracionário, como no caso de operadores poli-harmônicos e do Laplaciano fracionário, respectivamente. Apresentamos resultados sobre existência e propriedades qualitativas em um espaço cuja norma é baseada na transformada de Fourier. Nossa abordagem é do tipo não variacional e utiliza um argumento de contração em um espaço crítico para o estudo de EDPs. Os resultados apresentados foram publicados em [4].

1 Introdução

Estudamos a seguinte equação não homogênea que envolve não linearidades que dependem de operadores multiplicadores de Fourier (e.g. derivadas)

$$\mathcal{L}u + \left[\sum_{j=1}^{k} \prod_{i=1}^{l} \left[M_{\alpha_{ij}}(u) \right]^{p_{ij}} \right]^{q} + V(x)u + f(x) = 0 \quad \text{em } \mathbb{R}^n, \tag{1}$$

onde $V(x)$ é um potencial, $k, l, q, p_{ij} \in \mathbb{N}$ são tais que $q\sum_{i=1}^{l} p_{ij} > 1$ e α_{ij} são multi-índices ou números reias não negativos. Os operadores $\mathcal{L}$ e M_α são definidos via transformada de Fourier por $\widehat{\mathcal{L}u} = \sigma(\xi)\widehat{u}$ e $\widehat{M_\alpha u} = m_\alpha(\xi)\widehat{u}$, onde $\widehat{u} = \int_{\mathbb{R}^n} e^{-2\pi i x\cdot\xi} u(x)dx$. Por motivos puramente técnicos impomos

$$q\sum_{i=1}^{l} |\alpha_{ij}| p_{ij} < m < n, \text{ para todo } j = 1, \dots, k. \tag{2}$$

Mais ainda, para certas constantes $\mathcal{M}, \mathcal{N} > 0$, os símbolos σ e m_α satisfazem

$$\frac{1}{|\sigma(\xi)|} \leq \frac{\mathcal{M}}{|\xi|^m} \quad \text{and} \quad |m_\alpha(\xi)| \leq \mathcal{N}|\xi|^{|\alpha|}, \text{ q.t.p. em } \mathbb{R}^n. \tag{3}$$

Uma análise de *scaling* requer a definição das seguintes quantidades

$$\widetilde{a} = \frac{m - q\sum_{i=1}^{l} |\alpha_{ij}| p_{ij}}{q\sum_{i=1}^{l} p_{ij} - 1} \quad \text{e} \quad \widetilde{c} = \widetilde{a} + m = \frac{q\left[\sum_{i=1}^{l} (m - |\alpha_{ij}|) p_{ij}\right]}{q\sum_{i=1}^{l} p_{ij} - 1}, \tag{4}$$

onde impomos que $\widetilde{a}$ (e, assim, $\widetilde{c}$) é invariante por j.

Mediante a aplicação formal da transformada de Fourier em $\mathbb{R}^n$, obtemos a seguinte formulação funcional para o problema (1)

$$u = N(u) + T_V(u) + F(f), \tag{5}$$

onde os operadores $N(u)$, $T_V(u)$ e $F(f)$ são definidos de forma conveniente em variáveis de Fourier.

Como usamos uma abordagem baseada na transformada de Fourier, consideramos o espaço de Banach

$$\mathcal{PM}^a(\mathbb{R}^n) = \{u \in \mathcal{S}'(\mathbb{R}^n) \ : \ \widehat{u} \in L^1_{loc}(\mathbb{R}^n) \ \text{ e } \ \operatorname{ess\,sup}_{\xi\in\mathbb{R}^n} |\xi|^a |\widehat{u}(\xi)| < +\infty\},$$

com $0 \leq a < n$ e norma dada por $\|u\|_{\mathcal{PM}^a} = \operatorname{ess\,sup}_{\xi\in\mathbb{R}^n} |\xi|^a |\widehat{u}(\xi)|$. A análise de EDPs neste tipo de espaços começou no contexto da mecânica dos fluidos e equações parabólicas semilineares (veja, e.g., [1, 2, 3]) e, mais recentemente, foi usada no estudo de problemas elípticos (veja [5]).

2 Resultados Principais

Teorema 2.1. *(i) (Existência e Unicidade) Sejam $0 < m < n$ satisfazendo (2), $k, l, q, p_{ij} \in \mathbb{N}$, $q\sum_{i=1}^{l} p_{ij} > 1$, α_{ij} multi-índices, $n > \widetilde{c}$ e $a = n - \widetilde{a}$, onde $\widetilde{a}$ e $\widetilde{c}$ são como em (4). Sejam também $V \in \mathcal{PM}^{n-m}$, $f \in \mathcal{PM}^{a-m}$,*

$$\tau_a = L_a \|V\|_{PM^{n-m}} \quad e \quad \epsilon_a = \min\left\{\frac{1-\tau_a}{2}, \frac{(1-\tau_a)^{\omega/(\omega-1)}}{2^{\omega/(\omega-1)} K^{1/(\omega-1)}}\right\},$$

onde $K = kK_a$, $\omega = \min_{1\leq j\leq k}\{q\sum_{i=1}^{l} p_{ij}\}$, para certas constantes positivas K_a e L_a. Se V e f satisfazem $\tau_a < 1$ e $\|f\|_{PM^{a-m}} < \epsilon/\mathcal{M}$ com $0 < \epsilon < \epsilon_a$ e $\mathcal{M}$ como em (3), então a equação funcional (5) possui uma única solução $u \in \mathcal{PM}^a$ tal que $\|u\|_{PM^a} \leq 2\epsilon/(1-\tau_a)$. Mais ainda, u é uma função e $u \in L^\infty(\mathbb{R}^n) + L^2(\mathbb{R}^n)$.

(ii) (Simetria radial) Seja u a solução de (5) dada em (i). Suponha que V, $\sigma(\xi)$ e $m_{\alpha_{ij}}$ sejam radiais, para todo i, j. Então, u é radial se, e somente se, f é radial.

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ON THE HÉNON-TYPE EQUATIONS IN HYPERBOLIC SPACE

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Abstract

This paper is devoted to study a semilinear elliptic system of Hénon-type in the hyperbolic space $\mathbb{B}^N$. We prove a compactness result and together with the Clark's theorem we establish the existence of infinitely many solutions.

1 Introduction

This article concerns the existence of infinitely many solutions for the following semilinear elliptic system of Hénon type in hyperbolic space

$$\begin{cases} -\Delta_{\mathbb{B}^N} u = K(d(x))Q_u(u,v) \\ -\Delta_{\mathbb{B}^N} v = K(d(x))Q_v(u,v) \\ u,v \in H^1_r(\mathbb{B}^N),\ N \geq 3 \end{cases} \tag{$\mathcal{H}$}$$

where $\mathbb{B}^N$ is the Poincaré ball model for the hyperbolic space, $H^1_r(\mathbb{B}^N)$ denotes the sobolev space of radial $H^1(\mathbb{B}^N)$ function, $r = d(x) = d_{\mathbb{B}^N}(0,x)$, $\Delta_{\mathbb{B}^N}$ is the Laplace-Beltrami type operator on $\mathbb{B}^N$.

We assume the following hypothesis on K and Q

(K_1) $K \geq 0$ is a continuous function with $K(0) = 0$ and $K \neq 0$ in $\mathbb{B}^N \backslash \{0\}$.

(K_2) $K = O(r^\beta)$ as $r \to 0$ and $K = O(r^\beta)$ as $r \to \infty$, for some $\beta > 0$.

(Q_1) $Q \in C^1(\mathbb{R}\times\mathbb{R},\mathbb{R})$ is such that $Q(-s,t) = Q(s,-t) = Q(s,t)$, $Q(\lambda s, \lambda t) = \lambda^p Q(s,t)$ (Q is p - homogeneous), $\forall \lambda \in \mathbb{R}$ and $p \in (2,\delta)$, where

$$\delta = \begin{cases} \dfrac{2N+2\beta}{N-2} & \text{if } N-2>0 \\ \infty & \text{if otherwise} \end{cases}$$

(Q_2) There exist $C, K_1, K_2 > 0$ such that $Q(s,t) \leq C(s^p + t^p)$, $Q_s(s,t) \leq K_1 s^{p-1}$ and $Q_t(s,t) \leq K_2 t^{p-1}$, $\forall s,t \geq 0$.

(Q_3) There exists $C_1 > 0$ such that $C_1(|s|^p + |t|^p) \leq Q(s,t))$ with $p \in (2,\delta)$.

In the past few years the prototype problem

$$-\Delta_{\mathbb{B}^N} u = d(x)^\alpha |u|^{p-2}u, \quad u \in H^1_r(\mathbb{B}^N)$$

has been attracted attention. Unlike the corresponding problem in the Euclidean space $\mathbb{R}^N$, He in [1] proved the existence of a positive solution to the above problem over the range $p \in (2, \frac{2N+2\alpha}{N-2})$ in the hyperbolic space. More precisely, she explored the Strauss radial estimate for hyperbolic space together with the Mountain Pass Theorem. In a subsequent paper [2], she proved the existence of at least one non-trivial positive solution for the critical Hénon equation

$$-\Delta_{\mathbb{B}^N} u = d(x)^\alpha |u|^{2^*-2}u + \lambda u, \quad u \geq 0, \quad u \in H^1_0(\Omega'),$$

provided that $\alpha \to 0^+$ and for a suitable value of λ, where Ω' is a bounded domain in hyperbolic space $\mathbb{B}^N$.

We would like to mention the paper of Carrião, Faria and Miyagaki [4] where they extended He's result by considering a general nonlinearity

$$\begin{cases} -\Delta^{\alpha}_{\mathbb{B}^N} u = K(d(x))f(u) \\ u \in H^1_r(\mathbb{B}^N). \end{cases} \tag{1}$$

The authors were able to prove the existence of at least one positive solution through a compact Sobolev embedding with the Mountain Pass Theorem.

In this paper we investigate the existence of infinitely many solutions by considering a gradient system that generalizes problem (1). In order to obtain our result, we applied the Clark's theorem and get inspiration on the nonlinearities condition employed by Morais Filho and Souto [3] in a p-laplacian system defined on a bounded domain in $\mathbb{R}^N$.

As regarding the difficulties, many technical difficulties arise when working on $\mathbb{B}^N$, which is a non compact manifold. This means that the the embedding $H^1(\mathbb{B}^N) \hookrightarrow L^p(\mathbb{B}^N)$ is not compact for $2 \leq p \leq \frac{2N}{N-2}$ and the functional related to the system (2) cannot satisfy the $(PS)_c$ condition for all $c > 0$.

We also point out that since the weight function $d(x)$ depends on the Riemannian distance r from a pole o, we have some difficulties in proving that

$$\int_{\mathbb{B}^N} d(x)^{\beta}\Big(|u(x)|^p + |v(x)|^p\Big)\, dV_{\mathbb{B}^N} < \infty, \quad \forall (u,v) \in H^1(\mathbb{B}^N) \times H^1(\mathbb{B}^N)$$

leading to a great effort in proving that the Euler-Lagrange functional associated is well defined.

To overcome these difficulties we restrict ourselves to the radial functions.

Our result is

2 Main Results

Theorem 2.1. *Under hypotheses (K_1)-(K_2) and (Q_1)-(Q_3), the problem (2) has infinitely many solutions.*

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A BREZIS-NIRENBERG PROBLEM ON THE HYPERBOLIC SPACE $\mathbb{H}^N$

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Abstract

A nonhomogeneous Brezis-Nirenberg problem on the hyperbolic space $\mathbb{H}^n$ is considered. By the use of the stereographic projection the problem becomes a singular problem on the boundary of the open ball $B_1(0) \subset \mathbb{R}^n$. Thanks to the Hardy inequality, in a version due to the Brezis-Marcus, this difficulty involving singularity can be overcame. The mountain pass theorem due to Ambrosetti-Rabinowitz combined with Brezis-Nirenberg arguments is used to obtain a nontrivial solution.

1 Introduction

The main purpose of this talk is to present a study of the following nonhomogeneous Brezis-Nirenberg problem on the hyperbolic space $\mathbb{H}^n$, for $n \geq 3$:

$$-\Delta_{\mathbb{H}^n} u = \lambda u^q + u^{2^*-1} \quad \text{in } \mathbb{H}^n, \tag{1}$$

where $\lambda > 0$ is a real parameter, $\Delta_{\mathbb{H}^n}$ denotes the Laplace-Beltrami operator on $\mathbb{H}^n$, and $1 < q < 2^* - 1$, where $2^* := \frac{2n}{n-2}$. $\mathbb{H}^n$ is the hyperbolic space defined as

$$\mathbb{H}^n = \left\{ x \in \mathbb{R}^{n+1}; x_1^2 + x_2^2 + ... + x_n^2 - x_{n+1}^2 = -1 \text{ and } x_{n+1} > 0 \right\}.$$

We make use of the stereographic projection $E : \mathbb{H}^n \to \mathbb{R}^n$, where each point $P' \in \mathbb{H}^n$ is projected to $P \in \mathbb{R}^n$, where P is the intersection of the straight line connecting P' and the point $(0, ..., 0, -1)$. More exactly, we have the explicity projection $G : \mathbb{R}^n \to \mathbb{H}^n$ and $G^{-1} : \mathbb{H}^n \to \mathbb{R}^n$ given by

$$G(x) = (x.p(x), (1 + |x|^2)p/2) \;\text{ and } G^{-1}(y) = \frac{1}{y_{n+1}} y,\; x, y \in \mathbb{R}^n,$$

where $p(x) = \frac{2}{1-|x|^2}$.

This projection takes $\mathbb{H}^n$ onto the open ball $B_1(0) \subset \mathbb{R}^n$, and we denote by $D \subset B_1(0)$ the stereographic projetion of $D' \subset \mathbb{H}^n$. See [1, 2].

We will consider the metric

$$ds = p(x)|dx|, \text{ where } p(x) = \frac{2}{1 - |x|^2}.$$

Also, if u is a solution of (3), then if we define $v := p^{\frac{n-2}{2}} u$, then v satisfies the following problem

$$\begin{cases} -\Delta v + \frac{n(n-2)}{4} p^2 v = \lambda p^\alpha v^q + v^{2^*-1}, & \text{in } B_1(0) \\ v = 0, & \text{on } \partial B_1(0), \end{cases} \tag{2}$$

where $\alpha = n - (q + 1)\frac{n-2}{2}$.

2 Main Results

Theorem 2.1. *Problem (3) has a nontrivial solution $u \in H^1(\mathbb{H}^n)$, provided that the following conditions hold:*

i) $q > 1$, $n \geq 4$, *for any* $\lambda > 0$.

ii) $3 < q < 5$, $n = 3$, *for any* $\lambda > 0$.

iii) $1 < q \leq 3$, $n = 3$, *for any* λ *sufficiently large.*

Proof We consider a nonhomogeneous Brezis-Nirenberg problem on the hyperbolic space $\mathbb{H}^n$. Since we are using the stereographic projection, the original problem in $\mathbb{H}^n$ becomes a singular problem on the boundary of the open ball $B_1(0) \subset \mathbb{R}^n$. Thanks to the Hardy inequality, in a version of the Brezis-Marcus, this difficulty involving singularity was overcame. The criticality is handled by adapting some of the arguments made in Brezis-Nirenberg [1], as well as, in [2]. Then the mountain pass theorem due to Ambrosetti-Rabinowitz is used to obtain a nontrivial solution.

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EXISTENCE OF POSITIVE SOLUTION FOR A SYSTEM OF ELLIPTIC EQUATIONS VIA BIFURCATION THEORY

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Abstract

In this work we study the existence of solution for the following class of elliptic systems

$$\begin{cases} -\Delta u = \left(a - \int_\Omega K(x,y) f(u,v)dy\right) u + bv, & \text{in } \Omega \\ -\Delta v = \left(d - \int_\Omega \Gamma(x,y) g(u,v)dy\right) v + cu, & \text{in } \Omega \\ u = v = 0, & \text{on } \partial\Omega \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 1$, and $K, \Gamma : \Omega \times \Omega \to \mathbb{R}$ are nonnegative functions satisfying some hypotheses and $a, b, c, d \in \mathbb{R}$. The functions f and g satisfy some conditions which permit to use Bifurcation Theory to prove the existence of solution for (P).

1 Introduction

The study of the system (P) comes from a problem that models the behavior of a species inhabiting a smooth bounded domain $\Omega \subset \mathbb{R}^N$, which recently a special attention has been given for the problem

$$\begin{cases} -\Delta u = \left(\lambda - \int_\Omega K(x,y) u^p(y)dy\right) u, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases} \tag{1}$$

by supposing different conditions for K, see for example, Allegretto and Nistri [1], Alves, Delgado, Souto and Suarez [2], Chen and Shi [3] and other references.

In [2], Alves, Delgado, Souto and Suarez have considered the existence and nonexistence of solution for Problem (1). In that paper, the authors have introduced a class of functions $\mathcal{K}$ which is formed by functions $K : \Omega \times \Omega \to \mathbb{R}$ such that:

(i) $K \in L^\infty(\Omega \times \Omega)$ and $K(x,y) \geq 0$ for all $x, y \in \Omega$.

(ii) If w is measurable and $\int_{\Omega\times\Omega} K(x,y)|w(y)|^p |w(x)|^2 dxdy = 0$, then $w = 0$ a.e. in Ω.

Using Bifurcation Theory and supposing that K belongs to the class $\mathcal{K}$, the following result has been proved

Theorem 1.1. *The problem* (3) *has a positive solution if, and only if, $\lambda > \lambda_1$, where λ_1 is the first eigenvalue of the problem*

$$\begin{cases} -\Delta u = \lambda u, & \textit{in } \Omega \\ u = 0, & \textit{on } \partial\Omega. \end{cases}$$

Motivated by [2], a problem can be posed: to model the behavior of two species inhabiting a smooth bounded domain $\Omega \subset \mathbb{R}^N$, similarly to the case of single species in [2]. Inspired by Souto [4], we propose the following system to model this problem

$$\begin{cases} -\Delta u = \left(a - \int_\Omega K(x,y) f(u,v)dy\right) u + bv, & \text{in } \Omega \\ -\Delta v = \left(d - \int_\Omega \Gamma(x,y) g(u,v)dy\right) v + cu, & \text{in } \Omega \\ u = v = 0, & \text{on } \partial\Omega. \end{cases} \tag{P}$$

It is interesting to note that in a situation where $a, b, c, d > 0$, we are with a cooperative system, i.e., the two species involved mutually cooperate to their growth. If $b \cdot c < 0$, we say that we are in a structure involving predator and prey. In the case $b, c < 0$, there is a competition between the two species.

This paper, as well as [2], the functions $K : \Omega \times \Omega \to \mathbb{R}$ and $\Gamma : \Omega \times \Omega \to \mathbb{R}$ belong to class $\mathcal{K}$.

Functions f and g are assumed to be:

(f_0) $f, g : [0, \infty) \times [0, \infty) \to \mathbb{R}^+$ are continuous functions.

(f_1) There exists $\epsilon > 0$ such that $f(t, s) \geq \epsilon|t|^p$ and $g(t, s) \geq \epsilon|s|^p$, for all $t, s \in [0, \infty)$ and $p > 0$.

(f_2) $f(\xi t, \xi s) = \xi^p f(t, s)$ and $g(\xi t, \xi s) = \xi^p g(t, s)$, for all $t, s \in [0, \infty)$ and $\xi > 0$, where $p > 0$.

The functions $f(t, s) = |t|^p + |s|^{p-\mu}|t|^\mu$ and $g(t, s) = c_1|t|^p + c_2|s|^p$ are examples that verify $(f_0) - (f_2)$.

2 Main Results

Theorem 2.1. *Assume that $K, \Gamma \in \mathcal{K}$ and $(f_0) - (f_2)$ hold. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix with $a, b, c, d > 0$ and $\lambda > 0$ its biggest eigenvalue. The system*

$$\begin{cases} -\Delta u = \left(a - \int_\Omega K(x, y) f(u, v) dy\right) u + bv, & in \quad \Omega \\ -\Delta v = \left(d - \int_\Omega \Gamma(x, y) g(u, v) dy\right) v + cu, & in \quad \Omega \\ u, v > 0, \quad in \quad \Omega \\ u = v = 0, \quad on \quad \partial\Omega. \end{cases} \tag{P_1}$$

has a solution if, and only if, $\lambda > \lambda_1$, where λ_1 is the first eigenvalue of the problem $(-\Delta, H_0^1(\Omega))$.

In the case $f = g$ and $K = \Gamma$, we have:

Theorem 2.2. *Assume that $K \in \mathcal{K}$ and $(f_0) - (f_2)$ hold. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a matrix such that: there is a positive and largest eigenvalue of A that is the unique positive eigenvalue λ with an eigenvector $z > 0$ and $dimN(\lambda I - A) = 1$. Then, the system*

$$\begin{cases} -\Delta u = \left(a - \int_\Omega K(x, y) f(u, v) dy\right) u + bv, & in \quad \Omega \\ -\Delta v = \left(d - \int_\Omega K(x, y) f(u, v) dy\right) v + cu, & in \quad \Omega \\ u, v > 0, \quad in \quad \Omega \\ u = v = 0, \quad on \quad \partial\Omega. \end{cases} \tag{P_2}$$

has solution for all $\lambda > \lambda_1$, where λ_1 is the first eigenvalue of $(-\Delta, H_0^1(\Omega))$.

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EXISTÊNCIA DE SOLUÇÕES POSITIVAS PARA UMA CLASSE DE PROBLEMAS ELíPTICOS QUASILINEARES E SINGULARES COM CRESCIMENTO EXPONENCIAL

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Abstract

Neste artigo usamos método de Galerkin para investigar a existência de soluções positivas para uma classe de problemas elípticos quasilineares e singulares dados por

$$\begin{cases} -\mathrm{div}(a_0(|\nabla u|^{p_0})|\nabla u|^{p_0-2}\nabla u) = \dfrac{\lambda_0}{u^{\beta_0}} + f_0(u),\ u>0, \text{em } \Omega, \\ u = 0 \text{ sobre } \ \partial\Omega \end{cases} \tag{1}$$

e a versão para sistemas dada por

$$\begin{cases} -\mathrm{div}(a_1(|\nabla u|^{p_1})\ |\nabla u|^{p_1-2}\ \nabla u) = \dfrac{\lambda_1}{v^{\beta_1}} + f_1(u) \text{ em } \Omega, \\ \\ -\mathrm{div}(a_2(|\nabla v|^{p_2})\ |\nabla v|^{p_2-2}\ \nabla v) = \dfrac{\lambda_2}{u^{\beta_2}} + f_2(v) \text{ em } \Omega, \\ u, v > 0 \text{ em } \Omega, \\ u = v = 0 \text{ sobre } \partial\Omega, \end{cases} \tag{2}$$

onde $\Omega \subset \mathbb{R}^N$ é um domínio limitado suave com $N \geq 3$ e para $i = 0, 1, 2$ temos que $2 \leq p_i < N$, $0 < \beta_i < p_i - 1$, $\lambda_i > 0$, $a_i : \mathbb{R}^+ \to \mathbb{R}^+$ são funções de classe C^1 e $f_i : \mathbb{R} \to \mathbb{R}$ são funções contínuas com crescimento exponencial. As hipóteses sobre as funções a_i permitem considerar uma vasta classe de operadores quasilineares.

1 Introdução

Em um celebrado artigo de 1976 [1], Stuart considerou o problema $L(u) = f(x, u)$ em Ω e $u = \phi(x)$ sobre $\partial\Omega$, onde Ω é um domínio limitado em $\mathbb{R}^N$, $N \geq 2$, L um operador elíptico linear de segunda ordem e $f(x, p) \to \infty$ quando $p \to 0$. Problemas desse tipo são chamados singulares e surgem na teoria da condução de calor em materiais eletricamente condutores.

Mais recentemente, em alguns artigos foram estudados os casos singulares com não-linearidade e crescimento exponencial. No entanto, aqui estudamos um problema singular e um sistema singular com um operador mais geral, o que traz algumas dificuldades técnicas.

As hipóteses sobre as C^1-funcões $a_i : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ e sobre as funções contínuas $f_i : \mathbb{R} \longrightarrow \mathbb{R}$ são as seguintes:

(a_1) Existem constantes k_1, k_2, k_3, $k_4 \geq 0$ tal que

$$k_1 t^{p_i} + k_2 t^N \leq a_i(t^{p_i})t^{p_i} \leq k_3 t^{p_i} + k_4 t^N, \quad \text{para todo } t > 0.$$

(a_2) As funções $t \longmapsto a_i(t^{p_i})t^{p_i-2}$ são crescentes, para todo $t > 0$.

(f_1) Existe $\alpha_0 > 0$ tal que as condições de crescimento exponencial no infinito são dadas por:

$$\lim_{t\to\infty} \frac{f_i(t)}{\exp\left(\alpha|t|^{\frac{N}{N-1}}\right)} = 0 \text{ para } \alpha > \alpha_0 \text{ e } \lim_{t\to\infty} \frac{f_i(t)}{\exp\left(\alpha|t|^{\frac{N}{N-1}}\right)} = \infty, \text{ para } 0 < \alpha < \alpha_0.$$

(f_2) A condição de crescimento na origem: $\lim_{t\to 0^+} \frac{f_i(t)}{t^{p_i-1}} = 0$.

(f_3) Existe $\gamma > N$ tal que $f_i(t) \geq t^{\gamma_i - 1}$, para todo $t \geq 0$.

2 Resultados Principais

Teorema 2.1. *Suponha que as condições* $(a_1) - (a_2)$ *e* $(f_1) - (f_3)$ *são válidas. Então, existem* $\lambda^* > 0$ *tal que o problema (1) possui uma solução fraca positiva, para cada* $\lambda_0 \in (0, \lambda^*)$.

Proof. Para cada $\varepsilon > 0$, consideramos o seguinte problema auxiliar

$$\begin{cases} -\text{div}(a_0(|\nabla u|^{p_0})|\nabla u|^{p_0-2}\nabla u) = \dfrac{\lambda_0}{(|u|+\varepsilon)^{\beta_0}} + f_0(u) \text{ em } \Omega, \\ u > 0 \text{ em } \Omega, \\ u = 0 \text{ sobre } \partial\Omega, \end{cases} \tag{3}$$

onde as funções a_0 e f_0 satisfazem as hipóteses do Teorema 2.1.

A fim de provar o Teorema (2.1), inicialmente mostramos a existência de uma solução para o problema (3). Para isto, aplicamos o método de Galerkin em conjunto com o teorema do ponto fixo e usamos alguns resultados importantes de Análise Funcional para obter uma solução fraca para o problema auxiliar.

Assim, considerando u_n uma solução do problem (3), é necessário usar a única solução positiva do problema

$$-\text{ div}\left(a_0(|\nabla v|^{p_0})|\nabla v|^{p_0-2}\nabla v\right) = \theta > 0 \text{ in } \Omega, \; v = 0 \text{ on } \partial\Omega \tag{4}$$

combinado com (f_3) e o príncipio de comparação fraca, veja [1], para concluir que $u_n(x) \geq v(x) > 0$ em Ω, para todo $n \in \mathbb{N}$. E ainda, de (4) e (a_1) podemos argumentar como em [2] para obter que $v \in C^1(\overline{\Omega})$ e daí, para cada $x \in \Omega$, $u_n(x) \geq v(x) > Kd(x) > 0$, onde $d(x) = dist(x, \partial\Omega)$ e K é uma constante positiva que não dependente de x.

Finalmente, desde que $\phi \in C_0^\infty(\overline{\Omega})$ usamos novamente alguns resultados importantes de Análise Funcional e a desigualdade de Hardy-Sobolev para provar que $u \in W_0^{1,N}(\Omega)$ é uma solução fraca do problema (1). □

O segundo resultado, cuja demosntração segue passos semelhantes da demosntação do Teorema (2.1), é o seguinte:

Teorema 2.2. *Suponha que, para* $i = 1, 2$, a_i *satisfazem* $(a_1) - (a_2)$ *e as funções* f_i *satisfazem* $(f_1) - (f_3)$. *Então, existe* $\lambda^* > 0$ *tal que o problema (2) possui uma solução fraca positiva, para cada* $\lambda_i \in (0, \lambda^*)$.

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EXISTENCE OF SOLUTIONS FOR KIRCHHOFF TYPE INVOLVING THE NONLOCAL FRACTIONAL $P-$LAPLACIAN

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Abstract

This work deals wroth the existence of solutions for a class of nonlocal fractional $p-$Kirchhoff problem. Using a topological approach based on the Leray-Schauder alternative principle we establish the existence theorem under certain conditions.

1 Introduction

In this article, we study the following problem

$$\left| \begin{array}{l} M\Big(\|u\|^p_{W_0}\Big)\Big(-\Delta\Big)^s_p u \in f(x,u), \;\; in\ \Omega \\ \qquad\qquad u = 0, \qquad on\ \partial\Omega \end{array} \right. \tag{1}$$

where Ω is a bounded smooth domain in $\mathbb{R}^n$, $\|u\|_{W_0}$ is the Gagliardo $p-$seminorm of u, $1 < p < \frac{n}{s}$, with $0 < s < 1$, the main Kirchhoff function.
$M : \mathbb{R}^+_0 \longrightarrow \mathbb{R}$ is a continuous and nondecreasing functions,$(-\Delta)^s_p$ is the fractional $p-$Laplacian and
$F : \Omega \times \mathbb{R} \longrightarrow 2^{\mathbb{R}} \setminus \phi$ is multifunction.
In recent years, differential inclusion problem involving $p-$Laplacian has been studied, see for example [1], [2] among many others. Motivated for their works we shall study the existence of weak solutions of problem (1). Main difficulties raise when dealing with this problem because of the presence of the Kirchhoff function and of the nonlocal nature of $p-$fractional Laplacian. Our approach, which is topological, is based on the nonlinear alternative of Leray-Schauder.

2 Notations and Main Results

We denote $Q = \mathbb{R}^{2n} \setminus (\mathcal{C}\Omega \times \mathcal{C}\Omega)$ and $\mathcal{C}\Omega := \mathbb{R}^n \setminus \Omega$.
We define W, the usual fractional Sobolev space

$$W = \Big\{ u : \mathbb{R}^n \to \mathbb{R} : u|_\Omega \in L^p(\Omega), \int\int_Q \frac{|u(x)-u(y)|^p}{|x-y|^{n+ps}}\, dx\, dy < \infty \Big\}$$

where $u|_\Omega$ represents the restriction to Ω of function u. Also, we define the following linear subspace of W,

$$W_0 = \Big\{ u \in W : u = 0 \text{ a.e. in } \mathbb{R}^n \setminus \Omega \Big\}$$

The linear space W is endowed with the norm

$$\|u\|_W := \|u\|_{L^p(\Omega)} + \Big(\int\int_Q \frac{|u(x)-u(y)|^p}{|x-y|^{n+ps}}\, dx\, dy \Big)^{1/p}$$

becomes a uniformly convex Banach Space.

We require the following assumptions.

M) the function $M : \mathbb{R}_0^+ \longrightarrow \mathbb{R}$ is a continuous and nondecreasing function and there is a constant $m_0 > 0$ such that

$$M(t) \geq m_0 \text{for all} t \geq 0.$$

F) $F : \Omega \times \mathbb{R} \longrightarrow \mathcal{P}_{hc}$ is a multifunction satisfying

i) $(x, t) \to F(x, t)$ is graph measurable.

ii) for almost all $x \in \Omega$, $t \to F(x, t)$ has a closed graph.

iii) There exist $1 < \alpha < p$ and $c > 0$ such that

$$|w| \leq c\Big(1 + |t|^{\alpha - 1}\Big), \forall w \in F(x, t)$$

Theorem 2.1. *If hypotheses $(M), (F)$ hold, then problem (1) has at least one weak solution in W_0.*

Proof: We apply the Leray-Schauder fixed point theorem. □

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SUPORTE DAS SOLUÇÕES DA EQUAÇÃO LINEAR DE KLEIN-GORDON E CONTROLE EXATO NA FRONTEIRA EM DOMINIOS NÃO CILINDRICOS

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Abstract

Este trabalho mostra que é possível extender cada par de funções de $H^1(B(\overline{x},r)) \times L^2(B(\overline{x},r))$ para $H^1(\mathbb{R}^N) \times L^2(\mathbb{R}^N)$, $N \geq 2$, tal que a solução do problema de Cauchy, para equação linear de Klein-Gordon, com os dados iniciais estendidos se anulam numa região cônica de $\mathbb{R}^N \times [0,\infty)$. O passo seguinte consiste em aplicar este resultado no estudo problemas de controle exato na fronteira determinado dominios não cilindricos.

1 Introdução

Considere $B(\overline{x},r) \subset \mathbb{R}^N$, $N \geq 2$, como sendo a bola aberta de centro $\overline{x}$ e raio $r > 0$ e (f,g) um par de funções suportados em $B(\overline{x},r)$. Se extendermos, suavemente, as funções (f,g) para todo $\mathbb{R}^N$ de modo que as funções estendidas $(\widetilde{f}_\delta, \widetilde{g}_\delta)$ estejam suportadas em $B(\overline{x}, r+\delta)$, é bem conhecido na literatura que a solução do problema de Cauchy para equação de onda $\frac{\partial^2 u}{\partial t^2} - \triangle u = 0$, com os dados iniciais $(\widetilde{f}_\delta, \widetilde{g}_\delta)$, em dimensão impar $N \geq 3$, se anula no cone infinito

$$C(r+\delta) = \bigcup_{t \geq r+\delta} B(\overline{x}, t-r-\delta), \tag{1}$$

isto é,

$$u(x,t) = 0 = u_t(x,t), \qquad x \in B(\overline{x}, t-r-\delta), \qquad \forall t \geq r+\delta, \tag{2}$$

sendo δ um número real positivo arbitrário. Isto é valido pelo fato de que o operador de onda, em dimensão ímpar, $N \geq 3$, satisfaz o principio de Huygens. No entanto, J. Lagnese provou em [3], considerando $B(\overline{x},r) = B(0,1)$, que é possivel ainda realizar tal extensão, satisfazendo a condição (1), mesmo em dimensões pares $N \geq 2$, dimensões onde o principio de Huygens não se applica ao operador de onda. Neste presente trabalho estaremos mostrando que a propriedade de extensão, juntamente com a condição (1), é satisfeita quando consideramos o operador de Klein-Gordon $\frac{\partial^2 u}{\partial t^2} - \triangle u + c^2 u$, em dimensão $N \geq 2$, o qual não satisfaz o principio de Huygens. O passo seguinte é aplicar tais resultados para obter controle exato na fronteira para equação de Klein-Gordon em domínios não cilindricos. Mais especificamente temos os seguintes resultados.

Teorema 1.1. *Sejam $(f,g) \in H^1(B(\overline{x},r)) \times L^2(B(\overline{x},r))$, e $\delta > 0$ um número real fixo. Para todo $T \geq r+\delta$, existe uma extensão $(\widetilde{f}_\delta, \widetilde{g}_\delta) \in H^1(\mathbb{R}^N) \times L^2(\mathbb{R}^N)$ de (f,g) tal que a solução do problema de Cauchy*

$$\frac{\partial^2 u}{\partial t^2} - \triangle u + c^2 u = 0 \qquad em \qquad \mathbb{R}^N \times \mathbb{R} \tag{3}$$

$$u(.,0) = \widetilde{f}_\delta, \qquad u_t(.,0) = \widetilde{g}_\delta \qquad em \qquad \mathbb{R}^N, \tag{4}$$

se anula no cone finito $\bigcup_{r+\delta \leq t \leq T} B(\overline{x}, t-r-\delta) \times \{t\}$, *isto é*

$$u(.,T) = u(.,T) = 0 \qquad em \qquad B(\overline{x}, T-r-\delta). \tag{5}$$

Além disso, a aplicação $(f,g) \longrightarrow (\widetilde{f}_\delta, \widetilde{g}_\delta)$ é linear e limitadada de $H^1(B(\overline{x},r)) \times L^2(B(\overline{x},r))$ em $H^1(\mathbb{R}^N) \times L^2(\mathbb{R}^N)$.

Seja Q um conjunto aberto em $\mathbb{R}^N \times [0, +\infty)$ tal que a intersecção de Q com o hiperplano $\{(x,t) \in \mathbb{R}^{N+1},\ t \geq 0\}$ seja um conjunto aberto e limitado Ω_t em $\mathbb{R}^N$ de tal forma que $\Omega_0 = B(\overline{x}, r)$. Representamos a fronteira de Ω_t por $\partial\Omega_t$ e $\Gamma = \bigcup_{t\geq 0} \partial\Omega_t \times \{t\}$ é a fronteira lateral de Q. Agora, para $T \geq 0$, colocamos

$$Q_T = \bigcup_{0\leq t\leq T} \Omega_t \times \{t\}, \qquad \Gamma_T = \bigcup_{0\leq t\leq T} \partial\Omega_t \times \{t\}.$$

A fim de garantir a boa colocação do problema de valor inicial e fronteira a ser considerado vamos requerer que Q_T, para $T \geq 0$, esteja contido numa time-like região. O próximo teorema mostra como o Teorema 1.1 pode ser aplicado com o proposito de estudar problemas de controle exato na fronteira, em determinados dominios não cilindricos, para a equação linear de Klein-Gordon.

Teorema 1.2. *Sejam $(f, g) \in H^1(B(\overline{x}, r)) \times L^2(B(\overline{x}, r))$ e $T > r$ tais que $\overline{\Omega}_T \subset B(\overline{x}, r)$. Então, existe uma função controle $h \in L^2(\Gamma_T)$ tal que a solução*

$$\frac{\partial^2 u}{\partial t^2} - \triangle u + c^2 u = 0 \qquad em \qquad Q_T, \tag{6}$$

$$u(.,0) = f, \qquad u_t(.,0) = g \qquad em \qquad \Omega_0, \tag{7}$$

$$\nu_t u_t - \triangle u \cdot \nu_x = h(.,t) \qquad em \qquad \Gamma_T, \tag{8}$$

satisfaz a condição final

$$u(.,T) = u(.,T) = 0 \qquad em \qquad \Omega_T. \tag{9}$$

As demonstrações dos teoremas acima seguem as ideias apresentadas em [3]. As ferramentas essencias para a prova destes teoremas são extensão analitica, semenhante a apresentada em [4] e Teoremas de traço apresentados em [1] para obter a função controle desejada.

Aqui (ν_x, ν_t) denota o vetor normal unitário a superfície Γ_T no ponto (x,t). A expressão $\nu_t u_t - \triangle u \cdot \nu_x$ denota a derivada conormal de u sobre Γ_T no ponto (x,t). Se Q_T fosse um dominio cilindrico teríamos $\nu_t \equiv 0$, assim, a função controle h deveria ser obtida pela derivada normal de u. Em (8) temos uma condição de fronteira que determinada pela derivada conormal, tal condição é muito importante em fisica matemática quando lidamos com problemas de difração envolvendo operadores de onda a qual necessita ser confinada numa região limitada do espaço. Esta limitação faz com que alguns sinais emitidos pela perturbação inicial adiquira uma velocidade normal a supercie de limitação da onda, aparecendo a condição de fronteira expressa em (8), para mais detalhes veja [2].

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ON A COUPLED SYSTEM OF WAVE EQUATIONS TYPE P-LAPLACIAN

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Abstract

In this work we study the existence and asymptotic behaviour for a nonlinear coupled system of wave equation type p-Laplacian.

1 Introduction

Let $T > 0$ be a real number, $\Omega \subset \mathbb{R}^n$ be a bounded open set with sufficiently smooth boundary Γ. We denote by $Q = \Omega \times [0, T]$ the cylinder with lateral boundary $\Sigma = \Gamma \times [0, T]$. Here we consider $2 \leq p < \infty$ and q such that $\dfrac{1}{p} + \dfrac{1}{q} = 1$. We denote the p-Laplacian operator by $\Delta_p u = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$, which can be extended to a monotone, bounded, hemicontinuous and coercive operator between the spaces $W_0^{1,p}(\Omega)$ and its dual by

$$-\Delta_p \colon W_0^{1,p}(\Omega) \to W^{-1,q}(\Omega), \qquad \langle -\Delta_p u, v\rangle_p = \int_\Omega |\nabla u|^{p-2}\nabla u \cdot \nabla v \, dx.$$

The existence of a global solution for wave equation of p-Laplacian type

$$u'' - \Delta_p u = 0 \tag{1}$$

without any additional dissipation term is an open problem. For $n = 1$, M. Derher [1] proved the local in time existence of solution and showed by a generic counter-example that the global in time solution can not be expected. Adding a strong damping $-\Delta u'$ in (1) the well-posedness and asymptotic behavior was studied by Greenberg [2]. Weak solutions and blow-up for wave equations of p-Laplacian type with supercritical sources was considered in [3]. Ma and Soriano [4] gave the weak solution for the problem with a dissipative source term $g(u)$ where $g(u)u \geq 0$ has growth bound. Nevertheless, if the strong damping is replaced by a weaker damping u', then global existence and uniqueness are only know for $n = 1, 2$, see the works of Chueshov and Lasiecka [5] and Zhijian [6]. In the manuscript [7] Gao and Ma analyzed existence of solution with the damping $(-\Delta)^\alpha u'$ with $0 < \alpha \leq 1$ and extended the result of [4] for $g(u)$ without the sign condition $g(u)u \geq 0$. In this work we have interested in to prove existence and energy decay to the following nonlinear coupled system

$$\begin{cases} u'' - \Delta_p u + |u|^{r-1}u|v|^{r+1} - \Delta u' = 0, & \text{in } Q, \\ v'' - \Delta_p v + |v|^{r-1}v|u|^{r+1} - \Delta v' = 0, & \text{in } Q, \\ (u(x,0), v(x,0)) = (u_0(x), v_0(x)), & \text{in } \Omega, \\ (u'(x,0), v'(x,0)) = (u_1(x), v_1(x)), & \text{in } \Omega, \\ u(x,t) = v(x,t) = 0, & \text{in } \Sigma. \end{cases} \tag{2}$$

2 Main Results

Theorem 2.1. *Let us assume* $0 < r < \dfrac{np}{n-p}$ *if* $n > p$ *and* $0 < r < +\infty$ *if* $n \leq p$, *with* $r+1 < p$. *Then given* $(u_0, v_0) \in [W_0^{1,p}(\Omega) \cap L^{r+1}(\Omega)]^2$, $(u_1, v_1) \in [L^2(\Omega)]^2$, $p \geq 2$, *there exist functions* $u, v \colon \Omega \times (0,T) \to \mathbb{R}$ *such that,*

$$(u,v) \in [L^\infty(0,T; W_0^{1,p}(\Omega)) \cap L^\infty(0,T; L^{r+1}(\Omega))]^2, \tag{1}$$

$$(u',v') \in [L^\infty(0,T; L^2(\Omega)) \cap L^2(0,T; H_0^1(\Omega))]^2, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(u',w) + \langle -\Delta_p u, w\rangle + (|u|^{r-1}u|v|^{r+1}, w) + (\nabla u', \nabla w) = 0, \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(v',z) + \langle -\Delta_p v, z\rangle + (|v|^{r-1}v|u|^{r+1}, z) + (\nabla v', \nabla z) = 0, \tag{4}$$

$$\forall\ w, z \in W_0^{1,p}(\Omega) \ \ in \ \ D'(0,T),$$

$$(u(0), v(0)) = (u_0, v_0),\ \ (u'(0), v'(0)) = (u_1, v_1). \tag{5}$$

Proof Faedo-Galerkin procedure. □

Theorem 2.2. *Under the hypotheses of Theorem 2.1, the solution of system (2) satisfies:*

(a) $E(t) \leq C(E(0))e^{-\gamma t}$, *if* $p = 2$,

(b) $E(t) \leq C(E(0))(1+t)^{-\frac{p}{p-2}}$, *if* $p > 2$,

$\forall\ t \geq 0$, *where* $C(E(0))$ *and* γ *are positive constants.*

Proof Nakao's method. □

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UNIFORM ENERGY ESTIMATES FOR A SEMILINEAR TRUNCATED VERSION OF THE TIMOSHENKO WITH MEMORY

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Abstract

In this paper, we show that a mathematical model for viscoelastic beams[based on important physical and historical observations made by Elishakoff(Advances mathematical modeling and experimental methods for materials and structures, solid mechanics and its applications, Springer, Berlin, pp 249-254, 2010), Elishakoff et al.(ASME Am Soc Mech Eng Appl Mech Rev 67(6):1-11 2015) and Elishakoff et al. (Int J Solids Struct 109:143-151, 2017)] has a uniforme estimate for energy for any coefficient values of system.

1 Introduction

We consider the dynamics of the one-dimensional Timoshenko system for beams involving a memory term. For a beam with length L, these system is given by

$$\rho_1\varphi_{tt} - \kappa\left(\varphi_x + \psi\right)_x = f(\varphi), \text{ in } Q, \tag{1}$$

$$-\rho_2\varphi_{ttx} - b\psi_{xx} + \kappa\left(\varphi_x + \psi\right) - \int_0^{\infty}\beta(s)\psi_{xx}(t-s)ds = g(\psi), \text{ in } Q. \tag{2}$$

in a rectangular domain $Q = (0, L) \times (0, T)$ and $\Gamma = \{0; L\}$ represent the domain border and $T > 0$ is a given control time. To facilitate our analysis we consider the following initial conditions:

$$\varphi(x, 0) = \varphi^0(x), \quad \varphi_t(x, 0) = \varphi^1(x), \quad \psi(x, 0) = \psi^0(x) \text{ in } (0, L) \tag{3}$$

and boundary conditions:

$$\varphi(0, t) = \varphi(L, t) = \psi_x(0, t) = \psi_x(L, t) = 0 \text{ in } (0, T) \tag{4}$$

with positive constants $\rho_1, \rho_2, \kappa, b$.
We can define the energy functional as following

$$E(t) = \frac{1}{2}\int_0^L \left(\rho_1|\varphi_t|^2 + \frac{\rho_2\rho_1}{\kappa}|\varphi_{tt}|^2 + \rho_2|\varphi_{xt}|^2 + b|\psi_x|^2 + \kappa|\varphi_x + \psi|^2 + \int_0^{\infty}\mu(s)|\eta_x^t|^2 ds\right)dx. \tag{5}$$

We consider the truncated version of the Timoshenko beam model with weakly dissipative term given by memory and nonlinear sources f and g. This system which has only one spectrum of frequency(physical spectrum), we show that has a uniform estimate of the energy $(1) - (2)$ for any values of the coefficient of the system, regardless of any relationship between wave propagation velocities. That is, we prove that the energy associated with the solution of $(1) - (2)$ has an estimate which does not depend on any stability number.

2 Main Results

Theorem 2.1. *The energy $E(t)$ of the system (1)-(4) have a uniform estimate as time t tends to infinity. That is, there exist positive constants, ω and Λ independent of the initial data, such that*

$$E(t) \leq \mathcal{J}(\mathcal{L}(0))e^{-\omega t} + \Lambda C, \quad \forall t \geq 0. \tag{1}$$

Remark 2.1. *The proof is based energy method. Note that the condition:*

$$\frac{\kappa}{\rho_1} = \frac{b}{\rho_2}$$

it expresses that are equal the propagation speeds of the two deformation waves, associated to φ and ψ. In this paper that condition not is necessary for ensure the estimate of energy (this condition is necessary to holds stability the solution of system of Timoshenko, cf. [1]*).*

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CONTINUITY OF THE FLOWS AND ROBUSTNESS FOR EVOLUTION EQUATIONS WITH NON GLOBALLY LIPSCHITZ FORCING TERM

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Abstract

This talk is about a study of the sensitivity with respect to exponent and diffusion parameter for the problem

$$\begin{cases} \frac{\partial u_\lambda}{\partial t} - \operatorname{div}(D_\lambda(x)|\nabla u_\lambda|^{p_\lambda(x)-2}\nabla u_\lambda) + f(x,u_\lambda) = g(x), \quad t>0, \\ u_\lambda(0) = u_{0\lambda}, \end{cases} \tag{1}$$

under homogeneous Dirichlet boundary conditions, where $\lambda \in [0,\lambda_0]$, $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ is a smooth bounded domain, $u_{0\lambda} \in H := L^2(\Omega)$, $g \in L^2(\Omega)$, $p_\lambda(\cdot) \to p(\cdot)$, $D_\lambda(\cdot) \to D(\cdot)$ in $L^\infty(\Omega)$ as $\lambda \to 0$, and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a non globally Lispchitz Carathéodory mapping.

1 Introduction

In this talk we will present a study of a problem of the form

$$\begin{cases} \frac{\partial u_\lambda}{\partial t} - \operatorname{div}(D_\lambda(x)|\nabla u_\lambda|^{p_\lambda(x)-2}\nabla u_\lambda) + f(x,u_\lambda) = g(x), \quad t>0, \\ u_\lambda(0) = u_{0\lambda}, \end{cases} \tag{2}$$

under homogeneous Dirichlet boundary conditions, where $\lambda \in [0,\lambda_0]$, $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ is a smooth bounded domain, $u_{0\lambda} \in H := L^2(\Omega)$, $g \in L^2(\Omega)$, $D(\cdot), D_\lambda(\cdot) \in C^1(\bar{\Omega})$, for every $\lambda \in [0,\lambda_0]$, $0 < \beta \leq D(\cdot), D_\lambda(\cdot) \leq M < +\infty$, a.e. in Ω and for every $\lambda \in [0,\lambda_0]$, $p_\lambda(\cdot) \in C^1(\bar{\Omega})$ for every $\lambda \in [0,\lambda_0]$. Also, $2 < p_\lambda^- := \operatorname{ess\,inf} p_\lambda(x) \leq p_\lambda(x) \leq p_\lambda^+ := \operatorname{ess\,sup} p_\lambda(x) \leq a$, for every $\lambda \in [0,\lambda_0]$, where $a > 2$ is positive constant, $p_\lambda(\cdot) \to p(\cdot) \geq p^- > 2$ and $D_\lambda(\cdot) \to D(\cdot)$ in $L^\infty(\Omega)$ as $\lambda \to 0$.

As in [1] and [5] we will assume that $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a non globally Lispchitz Carathéodory mapping satisfying the following conditions: there exist positive constants ℓ, k, c_1 and $c_2 \geq 1$ such that

$$(f(x,s_1) - f(x,s_2))(s_1 - s_2) \geq -\ell|s_1 - s_2|^2, \ \forall\, x \in \Omega \text{ and } s_1, s_2 \in \mathbb{R}, \tag{3}$$

$$c_2|s|^{q(x)} - k \leq f(x,s)s \leq c_1|s|^{q(x)} + k, \ \forall\, x \in \Omega \ \text{ and } s \in \mathbb{R}, \tag{4}$$

where $q \in C(\overline{\Omega})$ with $2 < q^- := \inf_{x\in\Omega} q(x) \leq q^+ := \sup_{x\in\Omega} q(x)$. For example, if $\alpha_1 > 1$ and $r > 2$, we observe that the function $f : \Omega \times \mathbb{R} \to \mathbb{R}$ given by $f(x,u) = \alpha_1|u|^{r-2}u - u$ is not globally Lipschitz and satisfies the condition (3) with $\ell = 1$ and the condition (4) with $c_2 = 1$, $c_1 = \alpha_1$ and $q(x) = r$ for all $x \in I$ and for every $\lambda \in [0,\infty)$.

2 Main Results

Assuming that $p_\lambda(\cdot), D_\lambda(\cdot), D(\cdot) \in C^1(\bar{\Omega})$, $p_\lambda(\cdot) \to p(\cdot)$ and $D_\lambda(\cdot) \to D(\cdot)$ both in $L^\infty(\Omega)$ as $\lambda \to 0$, where $p^- > 2$, we will prove continuity of the flows and joint upper semicontinuity of the family of global attractors $\{\mathcal{A}_\lambda\}_{\lambda\in\mathbb{N}}$ as

$\lambda \to 0$ for the problem (2) with respect to the couple of parameters $(D_\lambda(\cdot), p_\lambda(\cdot))$. More specifically, we will prove the joint continuity of the solution with respect to (t,x), that the semigroup $S_\lambda(t)$ associated with problem (2) is compact and that, given $T>0$, the solutions u_λ of (2) go to the solution u of

$$\begin{cases} \frac{\partial u}{\partial t}(t) - \operatorname{div}\left(D(x)|\nabla u|^{p(x)-2}\nabla u\right) + f(x,u) = g(x), \quad t>0 \\ u(0) = u_0 \in H, \end{cases} \tag{1}$$

in $C([0,T];H)$ when $p_\lambda(\cdot) \to p(\cdot)$, $D_\lambda(\cdot) \to D(\cdot)$ both in $L^\infty(\Omega)$ and $u_{0\lambda} \to u_0$ in $H := L^2(\Omega)$ as $\lambda \to 0$, where $p^- > 2$ and $p_\lambda(\cdot), D_\lambda(\cdot), D(\cdot) \in C^1(\overline{\Omega})$. After that, we will obtain the upper semicontinuity on λ in H of the family of global attractors $\{\mathcal{A}_\lambda \subset H; \lambda \in [0,\lambda_0]\}$ of (2) at p.

Theorem 2.1. *i) If $u_{0\lambda}, v_{0\lambda} \in L^2(\Omega)$, $u_\lambda(\cdot) := S_\lambda(\cdot)u_{0\lambda}$ and $v_\lambda(\cdot) := S_\lambda(\cdot)v_{0\lambda}$, then*

$$\|u_\lambda(t) - v_\lambda(t)\|_H \leq \|u_{0\lambda} - v_{0\lambda}\|_H e^{2\ell T}, \quad \textit{for every} \quad t \in [0,T].$$

ii) The map $S_\lambda : \mathbb{R}^+ \times L^2(\Omega) \to L^2(\Omega)$ is continuous.

Theorem 2.2. *Let $\{S_\lambda(t)\}$ be the semigroup associated with problem (2) on $L^2(\Omega)$. Then $S_\lambda(t) : L^2(\Omega) \to L^2(\Omega)$ is of class $\mathcal{K}$.*

If we additionally suppose that f satisfies $\|f(\cdot,u(\cdot)) - f(\cdot,v(\cdot))\|_H \leq L(B)\|u-v\|_H$. for all $u, v \in B$, where B is a bounded set of H, then we have that

Theorem 2.3. *Let u_λ be a solution of (2) with $u_\lambda(0) = u_{0\lambda}$. Suppose that there exists $C>0$, independent of λ, such that $\|u_{0\lambda}\|_{X_\lambda} \leq C$ for every $\lambda \in [0,\lambda_0]$ and $u_{0\lambda} \to u_0$ in H as $\lambda \to 0$. Then, for each $T>0$, $u_\lambda \to u$ in $C([0,T];H)$ as $\lambda \to 0$, where u is a solution of (1) with $u(0) = u_0 \in H$.*

Using uniform estimates of the solutions and the continuity of the flows we get

Theorem 2.4. *The family of global attractors $\{\mathcal{A}_\lambda;\ \lambda \in [0,\lambda_0]\}$ associated with problem (2) is upper semicontinuous on λ at infinity, in the topology of H.*

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REMARKS ABOUT A GENERALIZED PSEUDO-RELATIVISTIC HARTREE EQUATION

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Abstract

Com hipóteses apropriadas sobre a não linearidade f, provamos a existência de solução u de energia mínima (ground state) para a equação

$$\left(-\Delta+m^2\right)^\sigma u+Vu=(W*F(u))\,f(u) \quad \text{em} \quad \mathbb{R}^N,$$

sendo $0<\sigma<1$, V um potencial limitado, não necessariamente contínuo, e F a primitiva de f. Também mostramos resultados sobre a regularidade de qualquer solução deste problema.

1 Introdução

O estudo da equação generalizada de Hartree pseudo-relativista

$$\sqrt{-\Delta+m^2}u+Vu=(W*F(u))\,f(u) \quad \text{em} \quad \mathbb{R}^N, \tag{1}$$

sendo $F(t)=\int_0^t f(s)ds$, foi realizado em [1] com hipóteses adequadas no caso $N\geq 2$. Neste caso, o estudo de (1) baseia-se no trabalho de Coti Zelati e Nolasco [2] e foi generalizado por Cingolani e Secchi [1].

Neste trabalho consideramos a mesma equação (1), substituindo o operador $\sqrt{-\Delta+m^2}$ por $(-\Delta+m^2)^\sigma$, em que $0<\sigma<1$. Ou seja, consideramos a equação

$$(-\Delta+m^2)^\sigma u+Vu=(W*F(u))\,f(u) \quad \text{em} \quad \mathbb{R}^N, \tag{2}$$

supondo que o potencial $V:\mathbb{R}^N\to\mathbb{R}$ satisfaça as seguintes condições:

$(V1)$ $V(y)+V_0\geq 0$ para todo $y\in\mathbb{R}^N$ e alguma constante $V_0<\min\{1,m^2\}\mathcal{K}(\Phi_\sigma)$, em que $\mathcal{K}(\Phi_\sigma)>0$.

$(V2)$ $V_\infty=\lim\limits_{|y|\to\infty}V(y)>0$;

$(V3)$ $V(y)\leq V_\infty$ para todo $y\in\mathbb{R}^N$, $V(y)\neq V_\infty$.

Assumimos que a função W é radial e satisfaz

(W_h) $0\leq W=W_1+W_2\in L^r(\mathbb{R}^N)+L^\infty(\mathbb{R}^N)$, com $r>\frac{N}{N(2-\theta)+2\sigma\theta}$.

A não-linearidade f é uma função C^1 que satisfaz

$(f1)$ $\lim\limits_{t\to 0}\dfrac{f(t)}{t}=0$;

$(f2)$ $\lim\limits_{t\to\infty}\dfrac{f(t)}{t^{\theta-1}}=0$ para algum θ tal que $\max\{2,N/(N-2\sigma)\}<\theta<2^*_\sigma=\frac{2N}{N-2\sigma}$;

$(f3)$ $\dfrac{f(t)}{t}$ é não decrescente para $t>0$.

2 Resultados Principais

Citamos um resultado geral sobre o problema de extensão, apenas mudando a notação:

Teorema 2.1. *Seja $h \in Dom(L^\sigma)$ e Ω um conjunto aberto de $\mathbb{R}^N$. Uma solução do problema de extensão*

$$\begin{cases} -L_y u + \frac{1-2\sigma}{x} u_x + u_{xx} = 0 & em \quad (0,\infty) \times \Omega \\ u(0,y) = h(y) & em \quad \{x=0\} \times \Omega \end{cases}$$

é dada por

$$u(x,y) = \frac{1}{\Gamma(\sigma)} \int_0^\infty e^{-tL}(L^\sigma h)(y) e^{-\frac{x^2}{4t}} \frac{dt}{t^{1-\sigma}}$$

e satisfaz

$$\lim_{x\to 0} \frac{x^{1-2\sigma}}{2\sigma} u_x(x,y) = \frac{\Gamma(-\sigma)}{4^\sigma \Gamma(\sigma)} (L^\sigma h)(y).$$

No nosso caso, o problema de extensão produz, para $(x,y) \in (0,\infty) \times \mathbb{R}^N = \mathbb{R}^{N+1}_+$,

$$\begin{cases} \Delta_y u + \dfrac{1-2\sigma}{x} u_x + u_{xx} - m^2 u = 0 & \text{in } \mathbb{R}^{N+1}_+ \\ \\ \lim\limits_{x\to 0^+} \left(-x^{1-2\sigma} \dfrac{\partial u}{\partial x} \right) = -V(y)u + [W * F(u)]\, f(u), & \text{em } \{0\} \times \mathbb{R}^N \simeq \mathbb{R}^N, \end{cases} \tag{P}$$

Teorema 2.2. *Suponha que as condições $(f1)$-$(f3)$, $(V1)$-$(V3)$ e (W_h) sejam satisfeitas. Então, o problema (P) possui uma solução* positiva *de energia mínima (ground state) $u \in H^1(\mathbb{R}^{N+1}_+, x^{1-2\sigma})$.*

Teorema 2.3. *Toda solução v do problema (P) satisfaz*

$$v \in L^\infty(\mathbb{R}^{N+1}_+) \cap C^\alpha(\mathbb{R}^{N+1}_+).$$

Teorema 2.4. *Suponha que $v \in H^1(\mathbb{R}^{N+1}_+, x^{1-2\sigma})$ seja um ponto crítico do funcional energia associado a (P). Então $v \in C^\alpha(\mathbb{R}^{N+1}_+) \cap L^\infty(\mathbb{R}^{N+1}_+)$ satisfaz*

$$\sup_{y\in\mathbb{R}^N} |v(x,y)| \le C|h|_2 x^{(2\sigma-1)/2} e^{-mx}$$

e portanto

$$|v(x,y)|e^{\lambda x} \to 0$$

quando $x \to \infty$, para todo $\lambda < m$.

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EXISTÊNCIA DE SOLUCÕES POSITIVAS PARA UMA CLASSE DE PROBLEMAS ELÍPTICOS QUASILINEARES E SINGULARES COM CRESCIMENTO EXPONENCIAL

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Abstract

Neste artigo usamos método de Galerkin para investigar a existência de soluções positivas para uma classe de problemas elípticos quasilineares e singulares dados por

$$\begin{cases} -\text{div}(a_0(|\nabla u|^{p_0})|\nabla u|^{p_0-2}\nabla u) = \dfrac{\lambda_0}{u^{\beta_0}} + f_0(u),\ u > 0, \text{em } \Omega, \\ u = 0 \text{ sobre } \partial\Omega \end{cases} \tag{1}$$

e a versão para sistemas dada por

$$\begin{cases} -\text{div}(a_1(|\nabla u|^{p_1})\ |\nabla u|^{p_1-2}\ \nabla u) = \dfrac{\lambda_1}{v^{\beta_1}} + f_1(u) \text{ em } \Omega, \\ \\ -\text{div}(a_2(|\nabla v|^{p_2})\ |\nabla v|^{p_2-2}\ \nabla v) = \dfrac{\lambda_2}{u^{\beta_2}} + f_2(v) \text{ em } \Omega, \\ u, v > 0 \text{ em } \Omega, \\ u = v = 0 \text{ sobre } \partial\Omega, \end{cases} \tag{2}$$

onde $\Omega \subset \mathbb{R}^N$ é um domínio limitado suave com $N \geq 3$ e para $i = 0, 1, 2$ temos que $2 \leq p_i < N$, $0 < \beta_i < p_i - 1$, $\lambda_i > 0$, $a_i : \mathbb{R}^+ \to \mathbb{R}^+$ são funções de classe C^1 e $f_i : \mathbb{R} \to \mathbb{R}$ são funções contínuas com crescimento exponencial. As hipóteses sobre as funções a_i permitem considerar uma vasta classe de operadores quasilineares.

1 Introdução

Em um celebrado artigo de 1976 [1], Stuart considerou o problema $L(u) = f(x, u)$ em Ω e $u = \phi(x)$ sobre $\partial\Omega$, onde Ω é um domínio limitado em $\mathbb{R}^N$, $N \geq 2$, L um operador elíptico linear de segunda ordem e $f(x, p) \to \infty$ quando $p \to 0$. Problemas desse tipo são chamados singulares e surgem na teoria da condução de calor em materiais eletricamente condutores.

Mais recentemente, em alguns artigos foram estudados os casos singulares com não-linearidade e crescimento exponencial. No entanto, aqui estudamos um problema singular e um sistema singular com um operador mais geral, o que traz algumas dificuldades técnicas.

As hipóteses sobre as C^1-funcões $a_i : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ e sobre as funções contínuas $f_i : \mathbb{R} \longrightarrow \mathbb{R}$ são as seguintes:

(a_1) Existem constantes k_1, k_2, k_3, $k_4 \geq 0$ tal que

$$k_1 t^{p_i} + k_2 t^N \leq a_i(t^{p_i})t^{p_i} \leq k_3 t^{p_i} + k_4 t^N, \quad \text{para todo } t > 0.$$

(a_2) As funções $t \longmapsto a_i(t^{p_i})t^{p_i-2}$ são crescentes, para todo $t > 0$.

(f_1) Existe $\alpha_0 > 0$ tal que as condições de crescimento exponencial no infinito são dadas por:

$$\lim_{t\to\infty} \frac{f_i(t)}{\exp\left(\alpha|t|^{\frac{N}{N-1}}\right)} = 0 \quad \text{para } \alpha > \alpha_0 \quad \text{e} \quad \lim_{t\to\infty} \frac{f_i(t)}{\exp\left(\alpha|t|^{\frac{N}{N-1}}\right)} = \infty, \quad \text{para } 0 < \alpha < \alpha_0.$$

(f_2) A condição de crescimento na origem: $\lim_{t\to 0^+} \frac{f_i(t)}{t^{p_i-1}} = 0$.

(f_3) Existe $\gamma > N$ tal que $f_i(t) \geq t^{\gamma_i - 1}$, para todo $t \geq 0$.

2 Resultados Principais

Teorema 2.1. *Suponha que as condições $(a_1) - (a_2)$ e $(f_1) - (f_3)$ são válidas. Então, existem $\lambda^* > 0$ tal que o problema (1) possui uma solução fraca positiva, para cada $\lambda_0 \in (0, \lambda^*)$.*

Proof. Para cada $\varepsilon > 0$, consideramos o seguinte problema auxiliar

$$\begin{cases} -\text{div}(a_0(|\nabla u|^{p_0})|\nabla u|^{p_0-2}\nabla u) = \dfrac{\lambda_0}{(|u|+\varepsilon)^{\beta_0}} + f_0(u) \text{ em } \Omega, \\ u > 0 \text{ em } \Omega, \\ u = 0 \text{ sobre } \partial\Omega, \end{cases} \tag{3}$$

onde as funções a_0 e f_0 satisfazem as hipóteses do Teorema 2.1.

A fim de provar o Teorema (2.1), inicialmente mostramos a existência de uma solução para o problema (3). Para isto, aplicamos o método de Galerkin em conjunto com o teorema do ponto fixo e usamos alguns resultados importantes de Análise Funcional para obter uma solução fraca para o problema auxiliar.

Assim, considerando u_n uma solução do problem (3), é necessário usar a única solução positiva do problema

$$-\text{div}\left(a_0(|\nabla v|^{p_0})|\nabla v|^{p_0-2}\nabla v\right) = \theta > 0 \ \text{ in } \Omega, \ v = 0 \text{ on } \partial\Omega \tag{4}$$

combinado com (f_3) e o príncipio de comparação fraca, veja [1], para concluir que $u_n(x) \geq v(x) > 0$ em Ω, para todo $n \in \mathbb{N}$. E ainda, de (4) e (a_1) podemos argumentar como em [2] para obter que $v \in C^1(\overline{\Omega})$ e daí, para cada $x \in \Omega$, $u_n(x) \geq v(x) > Kd(x) > 0$, onde $d(x) = dist(x, \partial\Omega)$ e K é uma constante positiva que não dependente de x.

Finalmente, desde que $\phi \in C_0^\infty(\overline{\Omega})$ usamos novamente alguns resultados importantes de Análise Funcional e a desigualdade de Hardy-Sobolev para provar que $u \in W_0^{1,N}(\Omega)$ é uma solução fraca do problema (1). □

O segundo resultado, cuja demosntração segue passos semelhantes da demosntação do Teorema (2.1), é o seguinte:

Teorema 2.2. *Suponha que, para $i = 1, 2$, a_i satisfazem $(a_1) - (a_2)$ e as funções f_i satisfazem $(f_1) - (f_3)$. Então, existe $\lambda^* > 0$ tal que o problema (2) possui uma solução fraca positiva, para cada $\lambda_i \in (0, \lambda^*)$.*

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EXISTENCE OF SOLUTIONS FOR KIRCHHOFF TYPE INVOLVING THE NONLOCAL FRACTIONAL $P-$LAPLACIAN

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Abstract

This work deals wroth the existence of solutions for a class of nonlocal fractional $p-$Kirchhoff problem. Using a topological approach based on the Leray-Schauder alternative principle we establish the existence theorem under certain conditions.

1 Introduction

In this article, we study the following problem

$$\left| \begin{array}{l} M\Big(\|u\|^p_{W_0}\Big)\Big(-\Delta\Big)^s_p u \in f(x,u), \;\; in\ \Omega \\ \qquad\qquad u = 0, \qquad on\ \partial\Omega \end{array} \right. \tag{1}$$

where Ω is a bounded smooth domain in $\mathbb{R}^n$, $\|u\|_{W_0}$ is the Gagliardo $p-$seminorm of u, $1 < p < \frac{n}{s}$, with $0 < s < 1$, the main Kirchhoff function.
$M : \mathbb{R}^+_0 \longrightarrow \mathbb{R}$ is a continuous and nondecreasing functions,$(-\Delta)^s_p$ is the fractional $p-$Laplacian and
$F : \Omega \times \mathbb{R} \longrightarrow 2^{\mathbb{R}} \setminus \phi$ is multifunction.
In recent years, differential inclusion problem involving $p-$Laplacian has been studied, see for example [1], [2] among many others. Motivated for their works we shall study the existence of weak solutions of problem (1). Main difficulties raise when dealing with this problem because of the presence of the Kirchhoff function and of the nonlocal nature of $p-$fractional Laplacian. Our approach, which is topological, is based on the nonlinear alternative of Leray-Schauder.

2 Notations and Main Results

We denote $Q = \mathbb{R}^{2n} \setminus (\mathcal{C}\Omega \times \mathcal{C}\Omega)$ and $\mathcal{C}\Omega := \mathbb{R}^n \setminus \Omega$.
We define W, the usual fractional Sobolev space

$$W = \Big\{u : \mathbb{R}^n \to \mathbb{R} : u|_\Omega \in L^p(\Omega),\ \int\int_Q \frac{|u(x)-u(y)|^p}{|x-y|^{n+ps}}\,dx\,dy < \infty\Big\}$$

where $u|_\Omega$ represents the restriction to Ω of function u. Also, we define the following linear subspace of W,

$$W_0 = \Big\{u \in W : u = 0 \text{ a.e. in } \mathbb{R}^n \setminus \Omega\Big\}$$

The linear space W is endowed with the norm

$$\|u\|_W := \|u\|_{L^p(\Omega)} + \Big(\int\int_Q \frac{|u(x)-u(y)|^p}{|x-y|^{n+ps}}\,dx\,dy\Big)^{1/p}$$

becomes a uniformly convex Banach Space.

We require the following assumptions.

M) the function $M : \mathbb{R}_0^+ \longrightarrow \mathbb{R}$ is a continuous and nondecreasing function and there is a constant $m_0 > 0$ such that

$$M(t) \geq m_0 \text{for all} t \geq 0.$$

F) $F : \Omega \times \mathbb{R} \longrightarrow \mathcal{P}_{hc}$ is a multifunction satisfying

i) $(x, t) \to F(x, t)$ is graph measurable.

ii) for almost all $x \in \Omega$, $t \to F(x, t)$ has a closed graph.

iii) There exist $1 < \alpha < p$ and $c > 0$ such that

$$|w| \leq c\Big(1 + |t|^{\alpha-1}\Big), \forall w \in F(x, t)$$

Theorem 2.1. *If hypotheses $(M), (F)$ hold, then problem (1) has at least one weak solution in W_0.*

Proof: We apply the Leray-Schauder fixed point theorem. □

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EXISTENCE OF SOLUTIONS FOR A $P(X)$ KIRCHHOFF TYPE DIFFERENTIAL INCLUSION PROBLEM WITH DEPENDENCE ON GRADIENT

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Abstract

For under certain conditions,we show the existence of weak solutions for a class of $p(x)$ Kirchhoff type differential inclusion problem with dependence on gradient and Dirichlet boundary data. We establish our result by using the degree theory for operators of generalized (S_+) type and working on the variable exponent Lebesgue-Sobolev spaces.

1 Introduction

In this work we consider the problem

$$\begin{gathered} -M\Big(\int_\Omega A(x,\nabla u)dx\Big)div\Big(a(x,\nabla u)\Big)+u\in f(x,u,\nabla u),\;\; in\ \Omega \\ u=0,\;\; on\ \Gamma \end{gathered} \tag{1}$$

where is Ω a bounded domain with smooth boundary Γ in $\mathbb{R}^n$, $(n\geq 3)$ and
(A_1) $M:[0,+\infty[\longrightarrow[m_0,+\infty[$ is a continuous and nondecreasing functions, $m_0>0$.
(A_2) $a(x,\xi):\overline{\Omega}\times\mathbb{R}^n\longrightarrow\mathbb{R}^n$ is the continuous derivate with respect to ξ of the continuous mapping $A:\overline{\Omega}\times\mathbb{R}^n\longrightarrow\mathbb{R}, A=A(x,\xi)$, i,e. $a(x,\xi)=\nabla_\xi A(x,\xi)$; there exists two positive constants $c_1\leq c_2$ such that $c_1|\xi|^{p(x)}\leq a(x,\xi)\xi$, for all $x\in\Omega$, $\xi\in\mathbb{R}^n$ and $a(x,\xi)\leq c_2|\xi|^{p(x)-1}$. Also $A(x,0)=0$, for all $x\in\Omega$ and $A(x,.)$ is strictly convex in $\mathbb{R}^n$, p is a continuous function on $\overline{\Omega}$.
(A_3) $f:\overline{\Omega}\times\mathbb{R}\times\mathbb{R}^n\longrightarrow P_{fc}(\mathbb{R})$ is a multifunction such that
(i)$(x,u,s)\to f(x,u,s)$ is graph measurable;
(ii)for almost all $x\in\Omega$, $(u,s)\to f(x,u,s)$ is closed graph;
(iii)for almost all $x\in\Omega$, and all $(u,s,v)\in Grf(x,.,.)$ we have

$$|v|\leq\gamma_1(x,|u|)+\gamma_2(x,|u|)|s| \qquad \text{with}$$

$$\sup\{\gamma_1(x,t);0\leq t\leq k\}\leq\eta_{1,k}(x) \quad \text{for almost all } x\in\Omega$$

$$\sup\{\gamma_2(x,t);0\leq t\leq k\}\leq\eta_{2,k}(x) \quad \text{for almost all } x\in\Omega$$

and $\eta_{1,k},\eta_{2,k}\in L^\infty(\Omega)$

In recent years, the studies of the Kirchhoff type problem with variable exponent of elliptic inclusion has attracted more and more interest and many results have been obtained on this kind of problems , see [1, 2, 1]. More recently I-S. Kim [2] showed, via topological degree the existence of weak solutions of (1), but with $M=1$, $p(x)=p$ and $f=f(x,u)$. We note that, the presence of the gradient in the multifunction f, precludes the use of variational methods in the analysis of (1).

2 Main Results

Our main result is as follows.

Theorem 2.1. *Under the assumptions* $(A_1)-(A_3)$*, there exists at least one solutions* $u \in W_0^{1,p(x)}(\Omega)$ *of the problem* (1).

Proof We transform the corresponding integral equation to problem (1) in the form of abstract Hammerstein inclusion, then we apply the Berkovits-Tienari degree theory for bounded weakly upper semicontinuous operators of generalized (S_+) type.(See [2], Theorem 2.10) . □

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MULTIPLE SOLUTIONS OF SYSTEMS INVOLVING FRACTIONAL KIRCHHOFF-TYPE EQUATIONS WITH CRITICAL GROWTH

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Abstract

In this work, we investigate existence and multiplicity of solutions of a system involving fractional Kirchhoff-type and critical growth. For this problem we prove the existence of infinitely many solutions, via a suitable truncation argument and exploring the genus theory introduced by Krasnoselskii .

1 Introduction

Precisely, we are concerned with the existence of multiple solutions for a system of a fractional Kirchhoff-type of the following form

$$(P_{\lambda,\gamma})\quad \begin{cases} M_1(||u||_X^2)(-\Delta)^s u = \lambda f(x,v(x)) \left[\displaystyle\int_\Omega F(x,v(x))dx\right]^{r_1} + u^{2_s^*-2}u \text{ in } \Omega, \\ M_2(||v||_X^2)(-\Delta)^s v = \gamma g(x,u(x)) \left[\displaystyle\int_\Omega G(x,u(x))dx\right]^{r_2} + v^{2_s^*-2}v \text{ in } \Omega, \\ u = v = 0 \text{ in } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where $s \in (0,1)$, $n > 2s$, $\Omega \subset \mathbb{R}^n$ is a bounded and open set, $||.||_X$ denotes the norm in the fractional Hilbert Sobolev space $X(\Omega)$, $2_s^* = 2N/(N-2s)$ is the fractional critical Sobolev exponent, r_1 and r_2 are positive constants, λ and γ are real parameters, $F(x,v(x)) = \int_0^{v(x)} f(\tau)d\tau$ and $G(x,u(x)) = \int_0^{u(x)} g(\tau)d\tau$ and $(-\Delta)^s : \mathcal{S}(\mathbb{R}^n) \to L^2(\mathbb{R}^n)$ is the fractional laplacian operator, given by

$$(-\Delta)^s u(x) := \lim_{\epsilon\to 0^+} C(n,s) \int_{\mathbb{R}^n\setminus B(0;\epsilon)} \frac{u(x)-u(y)}{|x-y|^{n+2s}}dy \quad x \in \mathbb{R}^n.$$

The set $\mathcal{S}(\mathbb{R}^n)$ is the set of all tempered distributions and $C(n,s)$ is the following positive constant

$$C(n,s) := \left(\int_{\mathbb{R}^n} \frac{1-\cos(\zeta_1)}{|\zeta|^{n+2s}}d\zeta\right)^{-1},$$

with $\zeta = (\zeta_1, \zeta')$, $\zeta' \in \mathbb{R}^{n-1}$.

The functions M_1 and M_2 has form

$$M_1(t) = m_0 + m_1 t \text{ and } M_2(t) = m_0' + m_1' t. \tag{1}$$

Also, for the problem $(P_{\lambda,\gamma})$ we assume that $f : \Omega \times \mathbb{R} \to \mathbb{R}$ and $g : \Omega \times \mathbb{R} \to \mathbb{R}$ are continuous functions, satisfying

$$f(x,-t) = -f(x,t) \text{ and } g(x,-t) = -g(x,t) \text{ for any } \overline{\Omega} \times \mathbb{R}, \tag{2}$$

and

$$a_1 t^{q_1-1} \le f(x,t) \le a_2 t^{q_1-1} \text{ and } b_1 t^{q_2-1} \le g(x,t) \le b_2 t^{q_2-1}, \tag{3}$$

with $a_i, b_i > 0$ and $1 < q_i < \dfrac{2}{r_i+1}$, for $i = 1,2$.

2 Main Result

Our main result is the following theorem:

Theorem 2.1. *Let $s \in (0,1)$, $n > 2s$, Ω be an open bounded subset of $\mathbb{R}^n$ and $r_1, r_2 \geq 0$. Let M_1 and M_2 with the form (1). Let $f : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ and $g : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$ verifying (2) and (3). Then, there exists $\bar{\lambda}$, $\bar{\gamma} > 0$ such that for any $(\lambda, \gamma) \in (0, \bar{\lambda}) \times (0, \bar{\gamma})$ the problem $(P_{\lambda,\gamma})$ has infinitely many weak solutions.*

The proof of the Theorem 2.1 consists in use truncation arguments to show that exist a functional associated to the problem $(P)_{\lambda,\gamma}$ which satisfies a local Palais-Smale condition and that under some conditions the critical points of this functional are solutions for the problem $(P)_{\lambda,\gamma}$. After that, using well know techniques involving Krasnoselskii's genus, one can show that the functional has infinitely many critical points.

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HIGHER-ORDER STATIONARY DISPERSIVE EQUATIONS ON BOUNDED INTERVALS: A RELATION BETWEEN THE ORDER OF AN EQUATION AND THE GROWTH OF ITS CONVECTIVE TERM

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Abstract

A boundary value problem for a stationary nonlinear dispersive equation of order $2l+1 \quad l \in \mathbb{N}$ with a convective term in the form $u^k u_x \quad k \in \mathbb{N}$ was considered on an interval $(0, L)$. The existence, uniqueness and continuous dependence of a regular solution as well as a relation between l and critical values of k have been established.

1 Introduction

This work concerns a boundary value problem for nonlinear stationary dispersive equations posed on $(0, L)$:

$$au + \sum_{j=1}^{l} (-1)^{j+1} D_x^{2j+1} u + u^k D_x u = f(x), \quad l, k \in \mathbb{N}, \tag{1}$$

subject to the following boundary conditions:

$$D_x^i u(0) = D_x^i u(L) = D_x^l u(L) = 0, \; i = 0, \dots, l-1, \tag{2}$$

where a is a positive constant and $f(x) \in L^2(0, L)$ is a given function. This class of stationary equations appears naturally while one wants to solve a corresponding evolution equation

$$u_t + \sum_{j=1}^{l} (-1)^{j+1} D_x^{2j+1} u + u^k D_x u = 0, \quad l, k \in \mathbb{N} \tag{3}$$

making use of an implicit semi-discretization scheme. For the $k = 1$, the problem (1)-(2) has been studied in [1]. Initial value problems for $l = 1$ has been studied in [3] while an initial-boundary value problem in the case $l = 1$ has been studied in [2]. Dispersive equations of higher orders have been developed for unbounded regions of wave propagations, however, if one is interested in implementing numerical schemes to calculate solutions in these regions, there arises the issue of cutting off a spatial domain approximating unbounded domains by bounded ones. In this case, some boundary conditions are needed to specify a solution. Obviously, boundary conditions for (1) are the same as for (3). Because of this, study of boundary value problems for (1) helps to understand solvability of initial- boundary value problems for (3).

Here, we propose (1) as a stationary analog of (3) because it includes classical models such as the Korteweg-de Vries, $l = 1$, and Kawahara equations, $l = 2$. The goal of our work is to formulate a correct boundary value problem for (1) and to prove the existence, uniqueness and continuous dependence on perturbations of $f(x)$ for regular solutions as well as to study relations between l and the critical values of k.

2 Main Results

Definition 2.1. *For a fixed $l \in \mathbb{N}$, equation (1) is a regular one for $k < 4l$ and is critical when $k = 4l$.*

Theorem 2.1. *Let $f \in L^2(0,L)$, then in the regular case, $k < 4l$, problem (1)-(2) admits at least one regular solution $u \in H^{2l+1}(0,L)$ such that*

$$\|u\|_{H^{2l+1}} \leq \mathcal{C}((1+x), f^2)_{L^2}^{\frac{1}{2}} \tag{1}$$

with the constant $\mathcal{C}$ depending only on L, l, k, a and $((1+x), f^2)_{L^2}$.
In the critical case, $k = 4l$, let f be such that

$$\|f\|_{L^2} < \frac{[(2l+1)(4l+2)]^{\frac{1}{4l}} a}{2^{\frac{2l+1}{4l}}}. \tag{2}$$

Then problem (1)-(2) admits at least one regular solution $u \in H^{2l+1}(0,L)$ such that

$$\|u\|_{H^{2l+1}} \leq \mathcal{C}'((1+x), f^2)_{L^2}^{\frac{1}{2}} \tag{3}$$

with the constant $\mathcal{C}'$ depending only on L, l, a and $((1+x), f^2)_{L^2}$.

Theorem 2.2. *Let $((1+x), f^2)_{L^2}$ be sufficiently small. Then the solution from Theorem 2.1 is unique and depends continuously on perturbations of f.*

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DECAIMENTO DE ONDAS ACOPLADAS COM RETARDO

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Abstract

Neste trabalho, estudamos a existência, unicidade, decaimento ótimo e o não decaimento para um sistema de equações de ondas acopladas com retardo. Utilizando teoremas da teoria de semigrupos de operadores, estabelecemos os resultados de existência, unicidade e a taxa ótima de decaimento. Além disso, para o resultado de instabilidade do sistema, encontramos uma sequência de retardos e soluções correspondentes a esses retardos aos quais não decai para 0 quando $t \to +\infty$.

1 Introdução

O estudo de equações envolvendo retardos pode ser encontrado por exemplo em [1] e [2], porém até então, os problemas tratados é com uma equação. Nesse trabalho, tratamos de um sistema com duas equações, a saber:

$$\rho_1 u_{tt}(t) - \beta_1 \Delta u(t) + \alpha v(t) + \mu_1 u_t(t) + \mu_2 u_t(t-\tau) = 0 \quad \text{in} \quad \Omega \times \mathbb{R}^+, \tag{1}$$

$$\rho_2 v_{tt}(t) - \beta_2 \Delta v(t) + \alpha u(t) = 0 \quad \text{in} \quad \Omega \times \mathbb{R}^+, \tag{2}$$

satisfazendo condições de fronteira

$$u = 0, \quad v = 0 \quad \text{on} \quad \partial\Omega \times \mathbb{R}^+,$$

e dados iniciais

$$u(0) = u_0,\ v(0) = v_0,\ u_t(0) = u_1,\ v_t(0) = v_1, \quad u_t(-s) = \phi(s),\ s \in [0, \tau].$$

Aqui, os coeficientes de densidade e elasticidade $\rho_1,\ \rho_2,\ \beta_1,\ \beta_2$ são positivos, e o coeficiente de acoplamento é tal que $0 < |\alpha| < \gamma_1\sqrt{\beta_1\beta_2}$, em que γ_1 é o primeiro autovalor do operador $-\Delta : H^2(\Omega) \cap H_0^1(\Omega) \subset L^2(\Omega) \subset L^2(\Omega)$. Além disso, consideramos a constante das diferenças entre propagação das velocidades de ambas as ondas, nesse caso denotada por $\chi_0 := \frac{\beta_1}{\rho_1} - \frac{\beta_2}{\rho_2}$. É sobre a relação entre os coeficientes de amortecimento μ_1 e μ_2 que os resultados de decaimento e não decaimento são obtidos, isso estará mais claro nos resultados principais.

Note que, se denotarmos $z(t, s) = u_t(t - s),\ s \in [0, \tau]$, segue que $z_t = -\partial_s z$ e $z_{tt} = \partial_s^2 z$. Se considerarmos $U(t) = (u(t), v(t), u_t(t), v_t(t), z(t, \cdot))$, o sistema anterior pode ser reescrito na forma

$$\frac{d}{dt}U(t) = \mathbb{B}U(t), \quad U(0) = U_0, \tag{3}$$

em que $U_0 = (u_0, v_0, u_1, v_1, \phi)$ e o operador $\mathbb{B}$ é dado por

$$\mathbb{B}U = \left(\dot{u},\ \dot{v},\ \rho_1^{-1}\left\{\beta_1\Delta u - \alpha v - \mu_1\dot{u} - \mu_2 z(\tau)\right\},\ \rho_2^{-1}\left\{\beta_2\Delta v - \alpha u\right\},\ -\partial_s z\right),$$

com $U = (u, v, \dot{u}, \dot{v}, z)$. Esse operador é definido no espaço de Sobolev

$$\mathbb{X} = H_0^1(\Omega) \times H_0^1(\Omega) \times L^2(\Omega) \times L^2(\Omega) \times L^2(0, \tau; L^2(\Omega)),$$

munido do produto interno

$$\begin{aligned}\langle U_1, U_2\rangle &= \rho_1\langle \dot u_1, \dot u_2\rangle + \rho_2\langle \dot u_1, \dot u_2\rangle + \beta_1\langle \nabla u_1, \nabla u_2\rangle + \beta_2\langle \nabla v_1, \nabla v_2\rangle \\ &\quad +\alpha\langle u_1, v_2\rangle + \alpha\langle v_1, u_2\rangle + \mu_1\int_0^\tau \langle z_1(s), z_2(s)\rangle\, ds.\end{aligned}$$

Aqui $\langle\cdot,\cdot\rangle$ irá denotar o produto interno em $L^2(\Omega)$. O domínio do operador $\mathbb{B}$ é definido por

$$D(\mathbb{B}) = \Big\{U \in \mathbb{X} : \dot u, \dot v \in H_0^1(\Omega), u, v \in H^2(\Omega)\cap H_0^1(\Omega), \\ \partial_s z \in L^2(0,\tau; L^2(\Omega)),\ z(0) = \dot u\Big\}.$$

2 Resultados Principais

Em seguida, enunciaremos os resultados obtidos. Para os teoremas 2.1, 2.2 e 2.3, estamos considerando $\mu_1 > \mu_2$.

Teorema 2.1. *Para o dado inicial $U_0 \in D(\mathbb{B})$ existe única solução para o problema (3) satisfazendo*

$$U \in C([0,+\infty[; D(\mathbb{B})) \cap C^1([0,+\infty[; \mathbb{X}).$$

Teorema 2.2. *Nós temos os seguintes resultados sobre o comportamento assintótico da solução:*

1. *Se $\chi_0 = 0$, o semigrupo é polinomialmente estável com taxa de decaimento $t^{-1/2}$, isto é, existe uma constante $C > 0$, tal que*

$$\|e^{tA}U_0\| \le \frac{C}{t^{1/2}}\|U_0\|_{D(\mathbb{B})}, t > 0.$$

2. *Se $\chi_0 \neq 0$, o semigrupo é polinomialmente estável com taxa de decaimento $t^{-1/4}$, isto é, existe uma constante $C > 0$, tal que*

$$\|e^{tA}U_0\| \le \frac{C}{t^{1/4}}\|U_0\|_{D(\mathbb{B})}, t > 0.$$

Teorema 2.3. *As taxas de decaimento obtidas no Teorema 2.2 são ótimas, ou seja,*

1. *Se $\chi_0 = 0$, o semigrupo não decai com taxa t^{-k}, para $k > 1/2$.*
2. *Se $\chi_0 \neq 0$, o semigrupo não decai com taxa t^{-k}, para $k > 1/4$.*

Teorema 2.4. *Se $\mu_1 \le \mu_2$, existe uma sequência de retardos (arbitrariamente pequena ou grande), e soluções do problema (1)-(2) correspondentes a esses retardos, que não decai para 0 quando $t \to +\infty$.*

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DECAY OF SOLUTIONS FOR THE 2D NAVIER-STOKES EQUATIONS POSED ON RECTANGLES AND ON A HALF-STRIP

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Abstract

In this work we consider the initial-boundary value problem for the 2D Navier-Stokes equations. The existence and uniqueness of global regular solutions, as well as exponential decay of solutions have been established.

1 Introduction

Consider the intial-boundary value problem for the two-dimensional Navier-Stokes equations.

$$u_t + (u \cdot \nabla)u = \nu\Delta u - \nabla p, \text{ in } \Omega \times (0,t), \tag{1}$$

$$\nabla u = 0 \text{ in } \Omega, \quad u|_{\partial\Omega} = 0, \qquad u(x,y,0) = u_0(x,y), \tag{2}$$

whether $\Omega \subset \mathbb{R}^2$ is either a rectangle or a half-strip.

The problem of decay of the energy for generalized solutions had been stated by J. Leray [4] and has been studied in [1, 1, 5]. In all of these papers, the decay rate of $\|u\|_{L^2(\Omega)}$ was controlled by the first eigenvalue of the operator $A = P\Delta u$, where P is the projection operator on solenoidal subspace of $L^2(\Omega)$.

It is well-known, [5], that solutions of the 2D Navier-Stokes equations posed on smooth bounded domains with the Dirichlet boundary conditions are globally regular. On the other hand, the question of regularity is not obvious for the case of bounded Lipschitz domains.
It has proved proved in [5] that there exists a unique global generalized solution

$$u, u_t \in L^\infty(0,\infty; L^2(\Omega)) \cap L^2(0,\infty; H^1(\Omega)),$$

but it was not clear whether

$$u \in L^\infty(0,\infty; H^2(\Omega))$$

at least for bounded Lipschitz domains.

In this work, we have established this fact for bounded rectangles making use of ideas of Koshelev [2]. The following inequality holds for bounded rectangles

$$\|u\|^2_{H^2(\Omega)}(t) + \|u_t\|^2_{L^2(\Omega)}(t) \le C\|u_0\|^2_{H^2(\Omega)} \exp\Big(-\frac{\pi^2}{(L^2+B^2)}\nu t\Big)$$

and

$$\|u\|^2_{H^1_0(\Omega)}(t) + \|u_t\|^2_{L^2(\Omega)}(t) \le C\|u_0\|^2_{H^2(\Omega)} \exp\Big(-\frac{\pi^2}{B^2}\nu t\Big)$$

for a half-strip. Moreover, having a s grneralized solution of the problem on the half-strip, we can obtain for a smooth subdomain $\Omega_0 \subset \Omega$ decay for solutions in Ω_0 as the following inequality:

$$\|u\|^2_{H^2(\Omega_0)}(t) \le C\|u_0\|^2_{H^2(\Omega)} \exp\Big(-\frac{\pi^2}{B^2}\nu t\Big).$$

2 Main Results

Theorem 2.1. *Consider a rectangular domain* $\Omega = \{(x, y) \in \mathbb{R}^2; 0 < x < L,\ \ 0 < y < B\}$. *Given* $u_0 \in H^2(\Omega) \cap V$, *the problem (1)-(2) has a unique solution* (u, p) *such that*

$$\begin{aligned} &u \in L^\infty(0, \infty; H_0^1 \cap H^2(\Omega)), u_t \in L^\infty(0, \infty; L^2(\Omega)), \\ &\nabla p \in L^\infty(0, \infty; L^2(\Omega)). \end{aligned} \tag{1}$$

Moreover,

$$\|u_t\|(t) + \|u\|_{H^2(\Omega)}(t) + \|\nabla p\|(t) \leq C_1 e^{-\frac{1}{2}\chi t}, \tag{2}$$

where $\chi = \nu\big(\frac{\pi^2}{L^2} + \frac{\pi^2}{B^2}\big)$ *and* C_1 *depends of* ν, $\|u_0\|_{H^2(\Omega)}$ *and* V *is a closure of* $C_0^\infty(\Omega)$, *div* $u = 0$ *in* $H_0^1(\Omega)$.

Theorem 2.2. *Consider the half-trip* $\Omega = \{(x, y) \in \mathbb{R}^2; 0 < x,\ 0 < y < B\}$. *Given* $u_0 \in H^2(\Omega) \cap V$, *the problem (1)-(2) with the condition*

$$\lim_{x \to \infty} |u(x, y, t)| = 0,$$

has a unique solution (u, p) *such that*

$$\begin{aligned} &u \in L^\infty(0, \infty; H_0^1(\Omega)), u_t \in L^\infty(0, \infty; L^2(\Omega)) \cap L^2(0, \infty; H_0^1(\Omega)), \\ &\nabla p \in L^\infty(0, \infty; L^2(\Omega)). \end{aligned} \tag{3}$$

Moreover,

$$\|u_t\|(t) + \|u\|_{H_0^1(\Omega)}(t) + \|\nabla p\|_{L^{4/3}(\Omega)}(t) \leq C_2 e^{-\frac{1}{2}\chi t}, \tag{4}$$

where $\chi = \nu \frac{\pi^2}{B^2}$ *and* C_2 *depends on* ν, $\|u_0\|_{H^2(\Omega)}$.

Theorem 2.3. *Let* u *be the solution of Theorem 2.2 and* Ω_0 *be a subdomain of* Ω *with boundary of class* C^2, *then, the following estimate takes a place:*

$$\|u\|(t)_{H^2(\Omega_0)} \leq C_3 e^{-\frac{1}{2}\chi t}, \tag{5}$$

where $\chi = \nu \frac{\pi^2}{B^2}$ *and* C_3 *depends on* ν *and* $\|u_0\|_{H^2(\Omega)}$.

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EXISTENCIA DE CONTROLES INSENSIBILIZANTES PARA UM SISTEMA DE GINZBURG-LANDAU

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Abstract

Neste trabalho, investigamos a existência de controles que insensibilizam um funcional energia associado às soluções de um sistema de Ginzburg-Landau com não linearidade cúbica.

1 Introdução

Sejam $\Omega \subset \mathbb{R}^N$ um domínio limitado com fronteira $\partial\Omega$ suficientemente regular, $T > 0$, ω (*domínio de controle*) e $\mathcal{O}$ (*domínio de observação*) subconjuntos abertos e não vazios de Ω. Usaremos as notações $Q_T = \Omega \times (0,T)$, $q_T = \omega \times (0,T)$ e $\Sigma_T = \partial\Omega \times (0,T)$. Considere o sistema de controle dado pela equação não linear de Ginzburg-Landau,

$$\begin{cases} y_t - (1+ia)\Delta y + Ry - (1+ib)|y|^2 y = v1_\omega + f & \text{em} \quad Q, \\ y = 0 & \text{sobre} \quad \Sigma, \\ y(0) = y_0 + \tau\widehat{y_0} & \text{em} \quad \Omega. \end{cases} \tag{1}$$

Aqui, v é a função controle, y é o estado, $a, b, R \in \mathbb{R}$, y_0 e f são funções conhecidas. O dado inicial do sistema acima é parcialmente conhecido, sabemos apenas que se trata de uma perturbação do dado y_0 da forma $y_0 + \tau\hat{y}_0$ onde $\widehat{y_0} \in L^2(\Omega)$ é um dado desconhecido com $||y_0||_{L^2(\Omega)} = 1$ e $\tau \in \mathbb{R}$ é um número real desconhecido suficientemente pequeno. Definimos o funcional $J : \mathbb{R} \times L^2(q_T) \to \mathbb{R}$, chamado sentinela, por:

$$J(\tau, v) := \frac{1}{2}\int_0^T\int_{\mathcal{O}} |\nabla y(x,t;\tau,v)|^2 dxdt, \tag{2}$$

onde $y = y(x,t;\tau,u)$ é a solução de (1) associado ao parâmetro τ e controle v. Uma vez que o dado é desconhecido, faz sentido se perguntar sobre a existência de controles que tornam a variação de uma energia local insensível a pequenas variações de τ. Sendo assim definimos

Definição 1.1. *Dados $y_0 \in L^2(\Omega)$ e $f \in L^2(Q_T)$, dizemos que o controle v insensibiliza o funcional J se*

$$\left.\frac{\partial}{\partial\tau}J(\tau,v)\right|_{\tau=0} = 0, \quad \forall\ \widehat{y}_0 \in L^2(\Omega) \quad \textit{com } \|\widehat{y}_0\|_{L^2(\Omega)} = 1, \tag{3}$$

i.e., J não detecta pequenas perturbações do dado inicial $y(0)$.

Pela escolha do funcional J dado em (2), a existência de controles insensibilizantes, segundo a Definição 1.1, é equivalente à existência de controles que conduzem a zero um determinado sistema de otimalidade. Mais precisamente temos,

Proposição 1.1. *O controle $v \in L^2(q_T)$ insensibiliza o funcional J no sentido da Definição 1.1 se, e somente se, v controla nulamente e parcialmente a solução (w,z) de*

$$\begin{cases} w_t - (1+ia)\Delta w + Rw - (1+ib)|w|^2 w = v1_\omega + f, & em \quad Q_T, \\ -z_t - (1-ia)\Delta z + Rz - (1-ib)\overline{w}^2\overline{z} - 2(1-ib)|w|^2 z = \nabla\cdot(\nabla w 1_{\mathcal{O}}), & em \quad Q_T, \\ w = z = 0, & sobre \quad \Sigma_T \\ w|_{t=0} = y_0, z|_{t=T} = 0, & em \quad \Omega. \end{cases} \tag{4}$$

Ou seja, v é tal que $z(0) = 0$ em Ω.

2 Resultados Principais

O resultado principal deste trabalho é

Teorema 2.1. *Suponha que $\omega \cap \mathcal{O} \neq \emptyset$ e $y_0 \equiv 0$. Então existe constante $C > 0$ tal que para todo $f \in e^{-C/t}L^2(Q_T)$ podemos encontrar um controle $v \in L^2(q_T)$ que insensibiliza o funcional J no sentido da Definição 1.1.*

Pela Proposição 1.1, o Teorema 2.1 é equivalente ao seguinte resultado de controlabilidade nula parcial,

Teorema 2.2. *Suponha que $\omega \cap \mathcal{O} \neq \emptyset$ e $y_0 \equiv 0$. Então existe uma constante $C > 0$ que depende de $a, b, R, \omega, \Omega, \mathcal{O}$ e T, tal que para todo $f \in e^{-C/t}L^2(Q_T)$, podemos encontrar um controle $v \in L^2(Q_T)$ tal que a solução (w, z) de (4) satisfaz $z|_{t=0} = 0$ em Ω.*

Para demonstrar o Teorema 2.2 vamos estudar um sistema linearizado associado ao sistema (4),

$$\begin{cases} w_t - (1 + ia)\Delta w + Rw = v1_\omega + f^0, & \text{em } Q_T, \\ -z_t - (1 - ia)\Delta z + Rz = \nabla \cdot (\nabla w 1_{\mathcal{O}}) + f^1, & \text{em } Q_T, \\ w = z = 0, & \text{sobre } \Sigma_T, \\ w|_{t=0} = y^0, z|_{t=T} = 0, & \text{em } \Omega. \end{cases} \tag{5}$$

Aqui, f^0, f^1 são funções dadas. Provamos o seguinte resultado de controlabilidade nula parcial para esse sistema,

Teorema 2.3. *Suponha que $\omega \cap \mathcal{O} \neq \emptyset$ e $y_0 \equiv 0$. Então existe uma constante $C > 0$ dependendo de $a, R, \omega, \Omega, \mathcal{O}$ e T, tal que para todo f^0 e f^1 em espaços ponderados adequados, $e^{-C/t}L^2(Q_T)$, podemos encontrar um controle $v \in L^2(Q_T)$ tal que a solução (w, z) de (5) satisfaz $z|_{t=0} = 0$ em Ω.*

Por fim, o Teorema 2.1 é obtido como consequência de um argumento de função inversa inspirado nas ideias vistas em [1]: Por meio de uma desigualdade de Carleman com lado direito em $H^{-1}(\Omega)$, deduzimos uma desigualdade do tipo Carleman para as soluções do sistema adjunto de (5). Por meio desta, provamos um resultado de regularidade cuja consequência é o Teorema 2.3. O caso não linear é obtido por meio de um argumento de função inversa onde definimos uma aplicação conveniente que parte do espaço onde o resultado linear foi obtido (espaço com pesos). Mostramos que tal aplicação é de classe C^1 com derivada sobrejetiva, logo admite uma inversa local.

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SISTEMA DE EQUAÇÕES DE ONDAS ACOPLADAS COM DAMPING FRACIONÁRIO E TERMOS DE FONTE

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Abstract

O Objetivo deste trabalho é estudar um problema de ondas acopladas envolvendo damping fracionário e termos de fonte mostrando que o mesmo está bem posto, ou seja, que sob hipóteses adequadas em relação aos dampings e os termos de fonte, podemos garantir a existência e unicidade de solução local fraca além de garantir que esta solução depende continuamente dos dados iniciais. A abordagem deste problema dá-se através de operadores monótonos e acretivos.

1 Introdução

Seja $\Omega = (0, L)$ com fronteira $\partial\Omega = \{0, L\}$. Estudaremos existência e unicidade de soluções fracas para o sistema não linear de ondas acopladas

$$\left\{ \begin{array}{rcl} u_{tt} - \Delta u + (-\Delta)^{\alpha_1} u_t + g_1(u_t) & = & f_1(u, v), \quad \text{em} \quad \Omega \times (0, T) \\ v_{tt} - \Delta v + (-\Delta)^{\alpha_2} v_t + g_2(v_t) & = & f_2(u, v), \quad \text{em} \quad \Omega \times (0, T) \end{array} \right. \tag{1}$$

onde $\alpha_1, \alpha_2 \in (0, 1)$, sujeito às seguintes condições iniciais e de fronteira

$$\left\{ \begin{array}{l} u(0) = u_0 \in H_0^1(\Omega), u_t(0) = u_1 \in L^2(\Omega) \\ v(0) = v_0 \in H_0^1(\Omega), v_t(0) = v_1 \in L^2(\Omega) \\ u = v = 0 \quad \text{em} \quad \partial\Omega \times (0, T). \end{array} \right. \tag{2}$$

As funções $g_1, g_2 : \mathbb{R} \to \mathbb{R}$ são funções globalmente lipschitz, monótonas crescentes com $g_1(0) = g_2(0) = 0$. Além disso, existem constantes positivas a e b tais que, para todo $|s| \geq 1$,

$$\begin{array}{rcl} a|s|^{m+1} \leq & g_1(s)s & \leq b|s|^{m+1}, \quad \text{com} \quad m \geq 1 \\ a|s|^{r+1} \leq & g_2(s)s & \leq b|s|^{r+1}, \quad \text{com} \quad r \geq 1 \end{array}$$

e $f_k(z) \in C^1(\mathbb{R})$ e existe uma constante positiva C tal que

$$|\nabla f_k(z)| \leq C(|u|^{p-1} + |v|^{p-1} + 1), \quad k = 1, 2 \quad \text{com} \quad p \geq 1.$$

Definição 1.1. ***(Solução fraca)*** *Dizemos que uma função vetorial $z = (u, v)$ é solução fraca para o problema (1.1)-(1.2) em $[0, T]$ se:*

- $z \in C(0, T; (H_0^2(\Omega))^2)$, $z(0) \in H_0^1(\Omega)$ *e* $z_t(0) \in L^2(\Omega)$;
- $z_t \in C(0, T; (L^2(\Omega))^2) \cap [(L^{m+1}(\Omega \times (0, T)) \times L^{r+1}(\Omega \times (0, T))) \cap L^2(0, T; D((-\Delta)^\alpha))]$ *sendo* $D((-\Delta)^\alpha) = D((-\Delta)^{\alpha_1}) \times D((-\Delta)^{\alpha_2})$;

- $z = (u, v)$ *satisfaz a identidade*

$$
\begin{aligned}
(z_t(t), \theta(t))_2 \quad - \quad & (z_t(0), \theta(0))_2 + \int_0^t (z_t(\tau), \theta_t(\tau))_2 d\tau - \int_0^t (z(\tau), \theta_{xx}(\tau))_2 d\tau \\
+ \quad & \int_0^t ((-\Delta)^\alpha z_t(\tau), \theta(\tau))_2 d\tau + \int_0^t (G(z_t(\tau)), \theta(\tau))_2 d\tau = \int_0^t (F(z(\tau)), \theta(\tau))_2 d\tau
\end{aligned}
\tag{3}
$$

para todo $t \in [0, T]$ *e toda função teste* θ *que pertence ao conjunto*

$$\Theta = \{\theta = (\theta^1, \theta^2); \theta \in C(0, T; (H_0^2(\Omega))^2) \quad e \quad \theta_t \in L^1(0, T; (L^2(\Omega))^2)\}$$

sendo

$$(-\Delta)^\alpha z_t = ((-\Delta)^{\alpha_1} u_t, (-\Delta)^{\alpha_2} v_t), \quad G(z_t) = (g_1(u_t), g_2(v_t)) \quad e \quad F(z) = (f_1(z), f_2(z)).$$

2 Resultados Principais

Teorema 2.1. ***(Solução local fraca)*** *Existe uma solução local fraca* $z = (u, v)$ *para o problema* (1)-(2) *definida em* $[0, T_0]$ *para algum* $T_0 > 0$ *dependendo da energia inicial* $E(0)$ *onde*

$$E(t) = \frac{1}{2}(||z_x(t)||_2^2 + ||z_t(t)||_2^2), \forall t \in [0, T_0]. \tag{4}$$

Além disso, vale a seguinte identidade de energia para todo $t \in [0, T_0]$*:*

$$E(t) + \int_0^t ((-\Delta)^\alpha z_t(\tau), z_t(\tau))_2 d\tau + \int_0^t (G(z_t(\tau)), z_t(\tau))_2 d\tau = E(0) + \int_0^t (F(z(\tau)), z_t(\tau))_2 d\tau. \tag{5}$$

Teorema 2.2. ***(Unicidade e dependência contínua)*** *A solução fraca dada pelo teorema* (2.1) *depende continuamente dos dados iniciais no espaço* $(H_0^1(\Omega) \times L^2(\Omega))^2$ *e esta solução é única.*

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BIFURCAÇÃO DE HOPF PARA MODELO DE POPULAÇÃO DE PEIXES COM RETARDAMENTO

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Abstract

A dispersão de uma espécie é um fenômeno bem conhecido na natureza e de grande relevância, com impacto na dinâmica da população, em sua genética e na distribuição da espécie. Neste trabalho apresentamos um modelo com retardamento em que consideramos dois ambientes, com uma população de peixes dispersando-se entre essas duas áreas e apresentamos condições que garantam a estabilidade assintótica e a existência de Bifurcação de Hopf.

1 Introdução

Consideramos o seguinte sistema:

$$\begin{aligned} \dot{x}_1(t) &= -dx_1(t) + ax_2(t) + \beta x_1(t-\tau)e^{-x_1(t-\tau)}, \\ \dot{x}_2(t) &= -dx_2(t) + ax_1(t) + \beta x_2(t-\tau)e^{-x_2(t-\tau)}, \end{aligned} \tag{1}$$

onde $d > 0$ é a taxa de mortalidade, $a > 0$ é a taxa de dispersão, $\tau \geq 0$ é o tempo de maturidade, isto é, o tempo necessário para que os recém-nascidos se tornem maduros para a reprodução.

Iremos então impor condições a este modelo para que tenhamos bifurcação de Hopf. Considerando as seguintes hipóteses em nossa análise. Seja a EDFR da forma

$$\dot{x}(t) = F(\alpha, x_t), \tag{2}$$

então $F(\alpha, \phi)$ tem primeira e segunda derivadas em α, ϕ para $\alpha \in \mathbb{R}$, $\phi \in C = C([-r, 0], \mathbb{R}^n)$, e $F(\alpha, 0) = 0$ para todo α. Definindo $L : \mathbb{R} \times C \to \mathbb{R}^n$ por

$$L(\alpha)\psi = D_\phi F(\alpha, 0)\psi,$$

onde $D_\phi F(\alpha, 0)$ é a derivada de $F(\alpha, 0)$ com respeito a ϕ em $\phi = 0$. E definindo também

$$f(\alpha, \phi) = F(\alpha, \phi) - L(\alpha)\phi,$$

teremos ainda as seguintes hipóteses:

1. A EDFR$(L(0))$ linear tem uma raiz característica imaginária pura simples $\lambda_0 = iv_0 \neq 0$ e todas as raízes características $\lambda_j \neq \lambda_0, \bar{\lambda_0}$, satisfazem $\lambda_j \neq m\lambda_0$ para qualquer inteiro m.

2. $Re\lambda'(0) \neq 0$.

Temos então o seguinte teorema,

Teorema 1.1. *Suponha que as hipóteses anteriores são satisfeitas. Então existem contantes $a_0 > 0$, $\alpha_0 > 0$, $\delta_0 > 0$, funções $\alpha(a) \in \mathbb{R}$, $\omega(a) \in \mathbb{R}$, e uma função $\omega(a)$-periódica $x^*(a)$, com todas as funções sendo continuamente diferenciáveis em a para $|a| < a_0$, tal que $x^*(a)$ é uma solução da equação (2) com*

$$x_0^*(a)^{P_\alpha} = \Phi_{\alpha(a)} y^*(a), \quad x_0^*(a)^{Q_\alpha} = z_0^*(a),$$

onde $y^*(a) = (a, 0)^T + o(|a|)$, $z_0^*(a) = o(|a|)$ quando $|a| \to 0$. Além disso, para $|\alpha| < \alpha_0$, $|\omega - (2\pi/\nu_0)| < \delta_0$, toda solução ω-periódica da equação (2) com $|x_t| < \delta_0$ deve ser deste tipo exceto por uma translação na fase.

Prova: Referência [2].

2 Resultados Principais

Supondo que $\beta > d - a$, então o sistema (1) apresenta uma única solução de equilíbrio positivo dada por

$$x^* = (x_1^*, x_2^*) = (\ln \frac{\beta}{d-a}, \ln \frac{\beta}{d-a}).$$

Substituindo $y_1(t) = x_1(t) - x_1^*$ e $y_2(t) = x_2(t) - x_2^*$ no sistema (1), introduzindo a função $h(\alpha) = \alpha e^{-\alpha}$ para $\alpha \in \mathbb{R}$ e linearizando o sistema em torno da origem, obtemos

$$\begin{aligned} \dot{y}_1(t) &= -dy_1(t) + ay_2(t) + \beta h'(x_1^*) y_1(t-\tau), \\ \dot{y}_2(t) &= -dy_2(t) + ay_1(t) + \beta h'(x_2^*) y_2(t-\tau). \end{aligned} \tag{1}$$

Denotamos por $c = \beta h'(x_1^*)$, obtendo a equação característica associada ao sistema (1) dada por

$$(\lambda + d - ce^{-\lambda\tau})^2 = a^2. \tag{2}$$

Fizemos o estudo da existência de raízes desse problema, substituindo $\lambda = u + iv$ em (2) e separando a parte real e a parte imaginária.

Como as soluções de (2) são invariantes por conjugação complexa, estudamos apenas o caso $v \geq 0$. Verificamos a existência de raiz imaginária pura e mostramos então que $Re\frac{d\lambda}{d\tau}\,|_{\lambda=vi} \neq 0$. O que nos levou a concluir o resultado esperado.

E como consequência pudemos estabelecer o seguinte resultado.

Teorema 2.1. *Suponha que no sistema (1) tenhamos $\dfrac{\beta}{d-a} > e^2$, então o equilíbrio de (1) é assintoticamente estável.*

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FLUIDOS MICROPOLARES COM CONVECÇÃO TÉRMICA

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Abstract

Vamos considerar o problema que descreve o movimento micropolar, viscoso, incompressível com convecção termal em um domínio limitado $\Omega \subset \mathbb{R}^3$. Utilizaremos um método iterativo para analizar existência, unicidade e determinar o raio de convergência em várias normas.

1 Introdução

O objetivo do presente trabalho é o estudo de existência e unicidade de soluções fortes das equações que descrevem o movimento de um fluido micropolar viscoso incompressível com convecção termal (problema (3)), ocupando um domínio limitado $\Omega \subset \mathbb{R}^3$ (região de escoamento) com fronteira C^2 compacta, durante um intervalo de tempo $(0,T)$ $(0 < T \leq \infty)$. Quando f e g não dependem de θ em (3) e não temos a equação de balanço de energia, o fluido é denominado fluido micropolar. Se além disso tivermos a velocidade microrotacional $\omega = 0$ obtemos as equações clássicas de Navier Stokes. Neste trabalho usando o método iterativo proposto por Zarubin [2], isto é, consideramos uma solução aproximada para o problema (3), linearizamos o problema, obtemos uma sequência de sistemas lineares que geram uma sequência de soluções aproximadas (a existência pode ser obtida como em Kagei Skowron [1]), logo provamos estimativas uniformes no tempo para as sequências de soluções aproximadas, a seguir provamos que a sequência é de Cauchy em determinados espaços de Banach, portanto a sequência converge fortemente a um elemento do mesmo espaço e então com estas convergências provamos que o elemento limite é a única solução do problema não linear. É importante ressaltar que a grande vantagem neste método iterativo é o não uso de teoremas de compacidade, fato fundamental no método de Galerkin.

O sistema que iremos estudar na região Q_T é o seguinte:

$$\left\{\begin{array}{rl} u_t - (\mu+\mu_r)\Delta u + (u\cdot\nabla)u + \nabla p & = 2\mu_r \text{rot } w + f(\theta), \\ w_t - (c_a+c_d)\Delta w + (u\cdot\nabla)w + 4\mu_r w & = -(c_0+c_d-c_a)\nabla\text{div } w + 2\mu_r \text{rot } u + g(\theta), \\ \theta_t + u\cdot\nabla\theta - \kappa\Delta\theta & = \Phi(u,w) + h, \\ \text{div } u & = 0, \end{array}\right. \tag{1}$$

junto com as seguintes condições iniciais e de fronteira

$$\left\{\begin{array}{l} u = 0, \quad w = 0, \quad \theta = 0 \text{ on } S_T, \\ u(0) = u_0, \; w(0) = w_0, \; \theta(0) = \theta_0, \text{ in } \Omega, \end{array}\right.$$

onde $Q_T \equiv \Omega \times (0,T)$ e $S_T \equiv \partial\Omega \times (0,T)$. As funções vetoriais $u = (u_1, u_2, u_3)$, $w = (w_1, w_2, w_3)$ e as funções escalares p e θ denotam respectivamente a velocidade, velocidade angular e a rotação de partículas, pressão do fluido e a temperatura do fluido. As funções vetoriais f, g e h denotam respectivamente as fontes externas de momento linear, angular e a entrada de calor. As constantes positivas μ, μ_r, c_0, c_a e c_d são coeficientes dos tipo viscosidade satisfazendo a seguinte desigualdade $c_0 + c_d > c_a$ e a constante positiva κ é a condutividade de calor. A função Φ

é dada por $\Phi = \sum_{i=1}^{5} \Phi_i$, donde

$$\begin{aligned}
\Phi_1(u) &= \tfrac{1}{2}\mu \textstyle\sum_{i,j=1}^{3} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2, \\
\Phi_2(u,w) &= 4\mu_r \left(\tfrac{1}{2}\text{rot } u - w \right)^2, \\
\Phi_3(w) &= c_0(\text{div } w)^2, \\
\Phi_4(w) &= (c_a + c_d) \textstyle\sum_{i,j=1}^{3} \left(\frac{\partial w_i}{\partial x_j} \right)^2, \\
\Phi_5(w) &= (c_d - c_a) \textstyle\sum_{i,j=1}^{3} \frac{\partial w_i}{\partial x_j} \frac{\partial w_j}{\partial x_i}.
\end{aligned}$$

Suponha que as funções f, g e h verificam

$$|f(s) - f(t)| \leq M_f |t - s|, \quad |g(s) - g(t)| \leq M_g |t - s| \tag{2}$$

para $s, t \in \mathbb{R}$ e constantes $M_f, M_g > 0$, $f(0) = g(0) = 0$ e $h \in L^2(0,T; L^2(\Omega))$.

2 Resultados Principais

Teorema 2.1. *Seja f, g e h em $L^2(0,T;L^2(\Omega))$, $u^1 = w^1 = \theta^1 = 0$. Para cada n existe uma única solução (u^n, w^n, θ^n) do problema linearizado, definida no intervalo $[0, T_1]$, com $0 < T_1 \leq T$, tal que*

$$\begin{aligned}
u^n &\in L^\infty(0,T_1;V) \cap L^2(0,T_1;D(A)), \\
w^n &\in L^\infty(0,T_1;H_0^1(\Omega)) \cap L^2(0,T_1;D(B)), \\
\theta^n &\in L^\infty(0,T_1;L^2(\Omega)) \cap L^2(0,T_1;H_0^1(\Omega)), \\
u_t^n &\in L^2(0,T_1;H),\ w_t^n \in L^2(0,T_1;L^2(\Omega)), \theta_t^n \in L^2(0,T_1;H^{-1}(\Omega)).
\end{aligned}$$

Além disso, existe uma constante $M_0 > 0$, independente de n, tal que

$$\begin{aligned}
\int_0^t \|\nabla u^n(\tau)\|^2 d\tau + \int_0^t \|\nabla w^n(\tau)\|^2 d\tau + \int_0^t \|\nabla \theta^n(\tau)\|^2 d\tau &\leq M_0, \\
\sup_{t \in (0,T_1)} \{\|\nabla w^n(t)\|^2 + \|\nabla w^n(t)\|^2 + \|\theta^n(t)\|^2\} &\leq M_0, \\
\int_0^t \|Au^n(\tau)\|^2 d\tau + \int_0^t \|Bw^n(\tau)\|^2 d\tau &\leq M_0, \\
\int_0^t \|u_\tau^n(\tau)\|^2 d\tau + \int_0^t \|w_\tau^n(\tau)\|^2 d\tau + \int_0^t \|\theta_\tau^n(\tau)\|_{H^{-1}}^2 d\tau &\leq M_0,
\end{aligned}$$

para cada $t \in (0, T_1)$.

O valor de T_1 depende apenas de $\mu, \mu_r, c_a, c_d, c_0, \kappa, M_f, M_g, \Omega$ e h. além disso, se μ_r, M_f, M_g e h são suficientemente pequenas, então é possível escolher $T_1 = T$.

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CONTROLE MULTIOBJETIVO DAS EQUAÇÕES DOS FLUIDOS MICROPOLARES I: PARETO OTIMALIDADE

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Abstract

Neste trabalho estudamos um problema de controle multiobjetivo, onde a dinâmica é dada pelas equações dos fluidos micropolares.

1 Introdução

Sejam Ω um domínio limitado em $\mathbb{R}^2$, com fronteira $\partial\Omega$ suficientemente regular, ω_1, ω_2 duas regiões abertas de Ω com $\omega_1 \cap \omega_2 = \emptyset$ e $\omega_{1,d}, \omega_{2,d}$ subconjuntos abertos de Ω. Consider o seguinte sistema

$$\boldsymbol{u}_t - \nu_1 \Delta \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \nabla p = 2\nu_r \mathrm{rot} w + \mathbf{f} + \boldsymbol{v}^{(1)}\chi_{\omega_1} + \boldsymbol{v}^{(2)}\chi_{\omega_2} \quad \text{em } Q, \tag{1}$$

$$\mathrm{div}\boldsymbol{u} = 0 \quad \text{em } Q, \tag{2}$$

$$w_t - \nu_2\Delta w + \boldsymbol{u}\cdot\nabla w + 4\nu_r w = 2\nu_r \mathrm{rot}\boldsymbol{u} + g + z^{(1)}\chi_{\omega_1} + z^{(2)}\chi_{\omega_2} \quad \text{em } Q, \tag{3}$$

$$\boldsymbol{u} = \mathbf{0}, \qquad w = 0 \quad \text{sobre } \Sigma, \tag{4}$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0, \qquad w(0) = w_0 \quad \text{em } \Omega. \tag{5}$$

onde $Q = \Omega \times (0,T)$ e $\Sigma = \partial\Omega \times (0,T)$.

Sejam $W_1 = \{\boldsymbol{v} \in L^2(0,T;\mathbf{V}),\ \boldsymbol{v}_t \in L^2(0,T;\mathbf{V}')\}$, $W_2 = \{v \in L^2(0,T;H_0^1(\Omega)),\ v_t \in L^2(0,T;H^{-1}(\Omega))\}$, e $\mathcal{U}_1 = L^2(0,T;\mathbf{H}(\omega_1))$, $\mathcal{U}_2 = L^2(0,T;\mathbf{H}(\omega_2))$, $\hat{\mathcal{U}}_1 = L^2(0,T;L^2(\omega_1))$ e $\hat{\mathcal{U}}_2 = L^2(0,T;L^2(\omega_2))$ os espaços que contém aos controles distribuídos nas regiõnes ω_1 e ω_2. Por outro lado, denotamos por $W = W_1 \times W_2$, $\mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2$ e $\hat{\mathcal{U}} = \hat{\mathcal{U}}_1 \times \hat{\mathcal{U}}_2$, e definimos os funcionais objetivos $J_r : W \times \mathcal{U} \times \hat{\mathcal{U}} \to \mathbb{R}$:

$$\begin{aligned} J_r((\boldsymbol{u},w),\boldsymbol{v},\boldsymbol{h}) &= \frac{\alpha_r}{2}\int_0^T\int_{\omega_{r,d}} |\boldsymbol{u}-\boldsymbol{u}_{r,d}|^2 dxdt + \frac{\beta_r}{2}\int_0^T\int_{\omega_{r,d}} |w - w_{r,d}|^2 dxdt \\ &\quad + \frac{\mu_r}{2}\int_0^T\int_{\omega_r} |\boldsymbol{v}^{(r)}|^2 dxdt + + \frac{\eta_r}{2}\int_0^T\int_{\omega_r} |z^{(r)}|^2 dxdt, \end{aligned} \tag{6}$$

onde $\alpha_r, \beta_r \geq 0$, $\mu_r, \eta_r > 0$, $r = 1,2$, são constantes, $\boldsymbol{u}_{r,d}$ e $w_{r,d}$ são funcões dadas em $L^2(0,T;\mathbf{H}(\omega_{r,d}))$ e $L^2(0,T;L^2(\omega_{r,d}))$, $r = 1,2$, respetivamente. As funcões $\boldsymbol{v}^{(r)}$ e $z^{(r)}$ são os controles considerados nos subconjuntos K_r e $\hat{K}_r$, respetivamente, e são convexos, fechados com interiores não vazios de $\mathcal{U}_r$ e $\hat{\mathcal{U}}_r$, $r = 1,2$, e os estados $\boldsymbol{u}$ e w são dados como solucões do Sistema (1)-(5). Denotaremos quando for necessário $\boldsymbol{v} = (\boldsymbol{v}^{(1)}, \boldsymbol{v}^{(2)})$ e $\boldsymbol{z} = (z^{(1)}, z^{(2)})$.

2 Ótimo de Pareto

Denotando por $\boldsymbol{h} = (\boldsymbol{h}^{(1)}, \boldsymbol{h}^{(2)})$ e $\boldsymbol{k} = (k^{(1)}, k^{(2)})$, o problema de controle considerado pode ser representado pelo seguiente problema de otimização vetorial:

$$\begin{array}{cc} \min & (J_1, J_2)((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k}) \\ ((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k}) \in \mathcal{Q} & \\ \boldsymbol{h}^{(1)} \in K_1,\ \boldsymbol{h}^{(2)} \in K_2 & \\ k^{(1)} \in \hat{K}_1,\ k^{(2)} \in \hat{K}_2 & \end{array} \tag{7}$$

onde $\mathcal{Q} = \{((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k}) \in W \times \mathcal{U} \times \hat{\mathcal{U}};$ e $((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k})$ satisfazem (1)-(5)$\}$.

Definição 2.1. *$(\boldsymbol{v}, \boldsymbol{z})$ é dito ótimo de Pareto para o problema (7) se existir $(\boldsymbol{u}, w) \in W$ tal que*

$$((\boldsymbol{u}, w), \boldsymbol{v}, \boldsymbol{z}) \in \mathcal{Q}, \boldsymbol{v}^{(1)} \in K_1, \boldsymbol{v}^{(2)} \in K_2, z^{(1)} \in \hat{K}_1, z^{(2)} \in \hat{K}_2$$

e não existe $(\boldsymbol{h}, \boldsymbol{k}) \neq (\boldsymbol{v}, \boldsymbol{z})$ e um estado $(\boldsymbol{y}_1, y_2)$ correspondente a $(\boldsymbol{h}, \boldsymbol{k})$ com $((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k})$ satisfazendo as mesmas restrições anteriores, tal que $J_r((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k}) \leq J_r((\boldsymbol{u}, w), \boldsymbol{v}, \boldsymbol{z}),\ r = 1, 2,$ com pelo menos uma desigualdade estrita.

Teorema 2.1. *Seja $\Omega \in \mathbb{R}^2$, e sejam $\mathbf{f} \in L^2(0, T; \mathbf{H})$, $g \in L^2(0, T; L^2(\Omega))$, $\boldsymbol{u}_0 \in \mathbf{H}$, $w_0 \in L^2(\Omega)$, $\boldsymbol{v}^{(r)} \in L^2(0, T; \mathbf{H}(\omega_r))$ e $z^{(r)} \in L^2(0, T; L^2(\omega_r))$, $r = 1, 2$, e além disso, sejam $\boldsymbol{u}_{r,d} \in L^2(0, T; \mathbf{H}(\omega_{r,d}))$ e $w_{r,d} \in L^2(0, T; L^2(\omega_{r,d}))$, $r = 1, 2$. Se $\inf J_r((\boldsymbol{y}_1, y_2), \boldsymbol{h}, \boldsymbol{k}) < J_r((\boldsymbol{u}, w), \boldsymbol{v}, \boldsymbol{z})$, $r = 1, 2$, então as condições necessárias para que o ponto $(\boldsymbol{v}, \boldsymbol{z})$ seja um ótimo de Pareto do Problema (7) é que existam escalares $\lambda_1 \geq 0$, $\lambda_2 \geq 0$, não nulos simultaneamente, e um par $(\boldsymbol{\varphi}, \psi)$ tais que o sistema abaixo seja verificado*

$$\begin{array}{rcl} \boldsymbol{u}_t + \nu_1 A\boldsymbol{u} + B(\boldsymbol{u}) & = & 2\nu_r \mathrm{rot} w + \mathbf{f} + \boldsymbol{v}^{(1)}\chi_{\omega_1} + \boldsymbol{v}^{(2)}\chi_{\omega_2} \quad \text{em } Q, \\ w_t + \nu_2 L w + R(\boldsymbol{u}, w) + 4\nu_r w & = & 2\nu_r \mathrm{rot}\boldsymbol{u} + g + z^{(1)}\chi_{\omega_1} + z^{(2)}\chi_{\omega_2} \quad \text{em } Q, \\ \boldsymbol{u}(0) = \boldsymbol{u}_0, & & w(0) = w_0 \quad \text{em } \Omega, \end{array}$$

$$\begin{array}{rcl} -\boldsymbol{\varphi}_t + \nu_1 A\boldsymbol{\varphi} + [B'(\boldsymbol{u})]^*\boldsymbol{\varphi} - 2\nu_r \mathrm{rot}\psi & = & -\lambda_1\alpha_1(\boldsymbol{u} - \boldsymbol{u}_{1,d})\chi_{\omega_{1,d}} \\ & & -\lambda_2\alpha_2(\boldsymbol{u} - \boldsymbol{u}_{2,d})\chi_{\omega_{2,d}} \quad \text{em } Q, \\ -\psi_t + \nu_2 L\psi + [R'(\boldsymbol{u}, w)]^*(\boldsymbol{\varphi}, \psi) + 4\nu_r\psi - 2\nu_r \mathrm{rot}\boldsymbol{\varphi} & = & -\lambda_1\beta_1(w - w_{1,d})\chi_{\omega_{1,d}} \\ & & -\lambda_2\beta_2(w - w_{2,d})\chi_{\omega_{2,d}} \quad \text{em } Q, \\ \boldsymbol{\varphi}(T) = 0, & & \psi(T) = 0 \quad \text{em } \Omega, \end{array}$$

e que o seguiente princípio de mínimo seja válido

$$\begin{array}{rcl} (\boldsymbol{\varphi}\chi_{\omega_r} - \lambda_r\mu_r\boldsymbol{v}^{(r)}, \boldsymbol{h}^{(r)} - \boldsymbol{v}^{(r)})_{L^2(\omega_r \times (0,T)} & \leq & 0, \forall \boldsymbol{h}^{(r)} \in K_r, r = 1, 2, \\ (\psi\chi_{\omega_r} - \lambda_r\eta_r z^{(r)}, k^{(r)} - z^{(r)})_{L^2(\omega_r \times (0,T)} & \leq & 0, \forall k^{(r)} \in \hat{K}_r, r = 1, 2. \end{array}$$

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SOBRE O PROBLEMA DA EXTENSÃO DE OPERADORES MULTILINEARES

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Abstract

Neste trabalho mostramos que, dentre as técnicas clássicas de extensão de operadores lineares contínuos, não valem para o caso multilinear a extensão de Hahn-Banach e a extensão quando o contradomínio é um espaço injetivo, e vale a extensão para o fecho e a extensão quando o subespaço é complementado.

1 Introdução

Sejam $u\colon G \longrightarrow F$ um operador linear contínuo entre espaços de Banach e E um espaço que contém G como um subespaço. São bem conhecidos os seguintes casos em que é possível estender u para um operador linear contínuo em E: (i) Quando F é o corpo dos escalares $\mathbb{K} = \mathbb{R}$ ou $\mathbb{C}$ (Teorema de Hahn-Banach, veja [1, Corolário 3.1.]), (ii) Quando G é subespaço complementado de E [1, Proposição 3.2.5], (iii) Quando G é denso em E (veja [3]), (iv) Quando F é um espaço de Banach injetivo (veja [2, Ex. 1.7]). O objetivo deste trabalho é mostrar que, com respeito à extensão de operadores multilineares contínuos, valem as extensões análogas aos casos (ii) e (iii), e não valem aquelas análogas aos casos (i) e (iv).

Dados $n \in \mathbb{N}$ e espaços de Banach $E_1, \ldots, E_n$, por $\mathcal{L}(E_1, \ldots, E_n; F)$ denotamos o espaço dos operadores n-lineares contínuos de $E_1 \times \cdots \times E_n$ em F.

2 Resultados Principais

Começamos com a extensão de operadores em subespaços complementados.

Definição 2.1. (a) Seja E um espaço de Banach. Um operador linear contínuo $P : E \longrightarrow E$ é uma *projeção* se $P^2 := P \circ P = P$.
(b) Um subespaço F do espaço de Banach E é *complementado* se existe uma projeção $P : E \longrightarrow E$ cuja imagem coincide com F, ou, equivalentemente, se existe um subespaço fechado G de E tal que $E = F \oplus G$. Dizemos que F é $\lambda-$*complementado*, $\lambda \geq 1$, se $\|P\| \leq \lambda$.

Teorema 2.1. *Sejam $E_1, \ldots, E_n$ espaços de Banach, G_j subespaço λ_j-complementado de E_j, $j = 1, \ldots, n$, e F espaço normado. Se $A \in \mathcal{L}(G_1, \ldots, G_n; F)$ então existe $\widetilde{A} \in \mathcal{L}(E_1, \ldots, E_n; F)$ extensão de A e $\|A\| \leq \|\widetilde{A}\| \leq \|A\| \cdot \lambda_1 \cdots \lambda_n$.*

Demonstração. Para cada $j = 1, \ldots, n$, tome $P_j\colon E_j \longrightarrow E_j$ projeção sobre G_j e chame $Q_j\colon E_j \longrightarrow G_j$, $Q_j(x) = P_j(x)$. Basta definir $\widetilde{A}\colon E_1 \times \cdots \times E_n \longrightarrow F$, $\widetilde{A}(x_1, \ldots, x_n) = A(Q_1(x_1), \ldots, Q_n(x_n))$. $\square$

Lema 2.1. *Seja G um subespaço fechado não-complementado do espaço de Banach E. Então não existe operador linear e contínuo $u : E \longrightarrow G$ tal que $u(x) = x$ para todo $x \in G$.*

Demonstração. Suponha que exista $u\colon E \longrightarrow G$ linear e contínuo tal que $u(x) = x$ para todo $x \in G$. Então $i \circ u\colon E \longrightarrow E$, onde $i\colon G \longrightarrow E$ é a inclusão, é uma projeção e $Im(i \circ u) = G$. Isso é um absurdo pois G não é complementado em E. $\square$

Teorema 2.2. [4, Proposition 1.4] *Sejam* $E_1, \ldots, E_n$ *espaços normados e* F *um espaço de Banach. Então*

$$\psi : \mathcal{L}(E_1, \ldots, E_n; F) \longrightarrow \mathcal{L}(E_j, ; \mathcal{L}(E_1, \ldots, E_{j-1}, E_{j+1}, \ldots, E_n; F)),$$

dado por $\psi(A)(x_j)(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n) = A(x_1, \ldots, x_j, \ldots, x_n)$, *é um isomorfismo isométrico.*

A seguir provamos a não existência de um teorema de Hahn-Banach multilinear:

Proposição 2.1. *Sejam* G, F *espaços de Banach,* $G^{'}$ *subespaço fechado não complementado de* F *e* $A : G \times G^{'} \longrightarrow \mathbb{K},\ A(x, \varphi) = \varphi(x)$. *Então* $A \in \mathcal{L}(G, G^{'}; \mathbb{K})$ *e não existe* $\widetilde{A} \in \mathcal{L}(G, F; \mathbb{K})$ *que estende* A.

Demonstração. Suponha que exista $\widetilde{A} \in \mathcal{L}(G, F; \mathbb{K})$ que estenda A e considere o isomorfismo $\psi : \mathcal{L}(G, F; \mathbb{K}) \longrightarrow \mathcal{L}(F; G^{'})$ do Teorema (2.2). Daí $\psi(\widetilde{A}) \in \mathcal{L}(F; G^{'})$ e $\psi(\widetilde{A})(\varphi) = \varphi$ para todo φ, o que contradiz o Lema (2.1). □

A seguir apresentamos um caso particular em que vale a extensão de funcionais bilineares:

Proposição 2.2. *Sejam* G *um subespaço do espaço normado* E *e* $i\colon G \longrightarrow E$ *a inclusão. Então todo* $A \in \mathcal{L}(G, G^{'}; \mathbb{K})$ *é extensível para um* $\widetilde{A} \in \mathcal{L}(E, G^{'}; \mathbb{K})$ *se, e somente se, existe* $u \in \mathcal{L}(E; G^{''})$ *tal que* $u \circ i = J_G$.

Apesar de c_0 ser subespaço não-complementado de ℓ_∞, temos o:

Corolário 2.1. *Para todo funcional bilinear* $A \in \mathcal{L}(c_0, \ell_1; \mathbb{K})$ *existe* $\widetilde{A} \in \mathcal{L}(\ell_\infty, \ell_1; \mathbb{K})$ *extensão de* A.

Um espaço de Banach F é chamado *injetivo* se para cada espaço de Banach E, cada subespaco $G \subseteq E$ e cada $u \in \mathcal{L}(G; F)$ existe $\widetilde{u} \in \mathcal{L}(E; F)$ extensão de u.

Do Teorema de Hahn-Banach sabemos que $\mathbb{K}$ é um espaço injetivo, portanto a Proposição 2.1 nos diz que nem sempre operadores multilineares tomando valores em espaços injetivos podem ser estendidos.

Por fim temos a extensão ao fecho:

Teorema 2.3. *Sejam* $E_1, \ldots, E_n$ *espaços normados,* F *espaço de Banach,* G_1 *subespaço de* $E_1, \ldots, G_n$ *subespaço de* E_n. *Para todo* $A \in \mathcal{L}(G_1, \ldots, G_n; F)$ *existe um único* $\widetilde{A} \in \mathcal{L}(\overline{G_1}, \ldots, \overline{G_n}; F)$ *que estende* A *e* $\|\widetilde{A}\| = \|A\|$.

É importante notar que a demonstração do caso linear deste resultado, que se baseia no fato de operadores lineares serem uniformemente contínuos, não se adapta ao caso multilinear, pois operadores multilineares não são uniformemente contínuos. Apresentaremos duas demonstrações para este resultado, a primeira utilizando o fato de operadores multilineares serem uniformemente contínuos sobre conjuntos limitados, e a segunda usando o caso linear e procedendo por indução sobre o grau de multilinearidade.

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MULTIPLIERS TECHNIQUES FOR RELATING APPROXIMATION TOOLS IN COMPACT TWO-POINT HOMOGENEOUS SPACES

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Abstract

We prove a characterization of the Peetre type K-functional on a compact two-point homogeneous space, in terms the rate of approximation of a family of averages (multipliers) operator defined to this purpose.

1 Introduction

Our basic framework is a compact two-point homogeneous spaces $\mathbb{M}$. This spaces is both a Riemannian m-manifold and a compact symmetric space of rank 1 for which there is a well-developed harmonic analysis structure on them. They are also completely characterized (see[3]) as: the unit spheres $\mathbb{S}^m$, $m = 1, 2, \ldots$; the real projective spaces $\mathbb{P}^m(\mathbb{R})$, $m = 2, 3, \ldots$; the complex projective spaces $\mathbb{P}^m(\mathbb{C})$, $m = 4, 6, \ldots$; the quaternion projective spaces $\mathbb{P}^m(\mathbb{H})$, $m = 8, 12, \ldots$; 16-dimensional Cayley's elliptic plane $\mathbb{P}^{16}$.

We write $\mathcal{B}$ for *Laplace-Beltrami operator* on $\mathbb{M}$, it is well-known that its differential form depends on a pair of index (α, β) varying according to the space. It has a discrete spectrum which can be arranged in an increasing order and it is given by $\{k(k+\alpha+\beta+1) : k = 0, 1, \ldots\}$. For each k the eigenspace $\mathcal{H}_k^m$ attached to $k(k+\alpha+\beta+1)$ has finite dimension denoted here by $d_k^m := \dim \mathcal{H}_k^m$ and they are mutually orthogonal. If we write $\{Y_{k,j} : j = 1, 2, \ldots, d_k^m\}$ for an orthonormal basis of $\mathcal{H}_k^m$, then $\{Y_{k,j} : k = 0, 1, \ldots, j = 1, 2, \ldots d_k^m\}$ is an orthonormal basis of $L^2(\mathbb{M})$. This permits us to consider naturally Fourier expansions on $L^2(\mathbb{M})$. Here, clearly, $\| \cdot \|_p$ stands for the canonical p-norm in $L^p(\mathbb{M})$, $1 \leq p < \infty$, the equivalence class of p-integrable and real or complex valuable functions from $\mathbb{M}$. In particular, for $p = 2$ we have a Hilbert space such that its inner product generates $\| \cdot \|_2$. All these facts and additional ones can be found in [2], for example.

We write $S_t(\cdot)$ for usual *shifting operator* on $L^2(\mathbb{M})$, which is defined by the average of a function in a "ring" of $\mathbb{M}$, namely for each $x \in \mathbb{M}$ the set is $\sigma_t^x := \{y \in \mathbb{M} : d(x, y) = t\}$, $0 < t < \pi$, with the induced measure. Then the addition formula ([2]) implies the following Fourier expansion of the shifting operator on $L^2(\mathbb{M})$:

$$S_t(\,\cdot\,) \sim \sum_{k=0}^{\infty} Q_k^{(\alpha,\beta)}(\cos t)\, \mathcal{Y}_k(\,\cdot\,), \tag{1}$$

where $Q_k^{(\alpha,\beta)}$ denotes the normalized Jacobi polynomial, it means $Q_k^{(\alpha,\beta)}(1) = 1$, and $\mathcal{Y}_k$ is the projection of $L^2(\mathbb{M})$ onto $\mathcal{H}_k^m$, $k = 0, 1, \ldots$.

We write $\mathcal{B}^r(\,\cdot\,)$ to denote the *fractional derivative of order* r which is defined on $\mathbb{M}$ in the distributional sense and given by

$$\mathcal{B}^r(\,\cdot\,) \sim \sum_{k=0}^{\infty} (k(k+\alpha+\beta+1))^{r/2}\, \mathcal{Y}_k(\,\cdot\,)$$

we are allowed to consider the Sobolev class $W_p^r(\mathbb{M}) := \{f \in L^p(\mathbb{M}) : \mathcal{B}^r(f) \in L^p(\mathbb{M})\}$, endowed which the usual norm $\| \cdot \|_{W_p^r} := \| \cdot \|_p + \|\mathcal{B}^r(\cdot)\|_p$.

Consider $r > 0$ and $t > 0$ real numbers and $f \in L^p(\mathbb{M})$. We introduce the Peetre-type K*-functional of fractional order* r:

$$K_r(f,t)_p := \inf_{g \in W_p^r(\mathbb{M})} \left\{ \|f-g\|_p + t^r \|g\|_{W_p^r} \right\}. \tag{2}$$

The r*-th moduli of smoothness*:

$$\omega_r(f,t)_p := \sup \left\{ \|(I-S_s)^{r/2}(f)\|_p : s \in (0,t] \right\}. \tag{3}$$

And the *generalized shifting operator*:

$$S_{r,t}(f) := \frac{-2}{\binom{2r}{r}} \sum_{j=1}^{\infty} (-1)^j \binom{2r}{r-j} S_{jt}(f), \tag{4}$$

The interrelation of approximation tools above are explored and this the content of next section.

2 Main Results

Platonov ([2, Theorem 1.2]) showed that the K-functional and the moduli of smoothness are related in a asymptotic sense. It reads as follows.

Theorem 2.1. *For* $1 < p < \infty$ *and* $r \geq 1$ *a natural number, it holds*[1]

$$K_{2r}(f,t)_p \asymp \omega_{2r}(f,t)_p \quad f \in L^p(\mathbb{M}),\, t > 0.$$

Our main interest on these tools is their relation with the decay of Fourier coefficients of functions in terms of the rate of approximation of generalized shifting operator. The latter is usually directly related to generalized Hölder conditions (see [1]) and it has shown to be an efficient tool to get good estimates for both Fourier coefficients of functions satisfying a generalized Hölder condition and eigenvalues sequences of positive integral integral operators with Hölderian kernels. The relation we have stablished is the following.

Theorem 2.2. *For* $1 < p < \infty$ *and* $r \geq 1$ *a natural number, it holds*

$$K_{2r}(f,t)_p \asymp \|S_{r,t}(f) - f\|_p, \quad f \in L^p(\mathbb{M}),\, t > 0.$$

The technic employed to prove Theorem 2.2 is to get sharp estimates for the multiplier sequence attached to the generalized shifting operator in order to apply the Marcinkiewicz Multiplier's theorem, from what the asymptotic relation above follows.

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[1]Notation $A(t) \asymp B(t)$ stands for $B(t) \lesssim A(t)$ and $A(t) \lesssim B(t)$, while $A(t) \lesssim B(t)$ means that $A(t) \leq c\,B(t)$, for some constant $c \geq 0$ not depending upon t.

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MOUNTAIN PASS ALGORITHM VIA POHOZAEV MANIFOLD

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Abstract

A new numerical algorithm for solving an asymptotically semilinear elliptic problem is presented. The ground state solution of the problem, which in general is obtained as a min-max of the associated functional, is obtained as a minimum of the functional constrained to the Pohozaev manifold instead. Examples are given of the use of this method for finding numerical solutions depending on various parameters.

1 Introduction

The celebrated *Mountain Pass Theorem* of Ambrosetti and Rabinowitz [1] has been widely used in the past forty years for finding weak solutions of a semilinear elliptic problem as critical points of an associated functional.

Solutions are found on the mini-max levels of the functional. A numerical approach of this theorem was first introduced by Choi and McKenna [2] in . Their work showed that, when carefully implemented, the algorithm is globally convergent and leads to a solution with the required mountain pass property.

Later, Chen, Ni and Zhou [1] in observed that this algorithm may converge to a solution with morse index greater or equal to two, and not to the ground state mountain pass level and, to circumvent this fault, they created a new algorithm based on the fact that the minimum of the associated functional constrained to the Nehari manifold is equal to the min-max level obtained by the Mountain Pass Theorem. This equivalence follows when the nonlinear terms in the equation are superquadratic. For the asymptotically linear problem, this is not true in general. However, more recently, the ground state level was shown to be equal to the minimum of the functional restricted to the Pohozaev manifold (see Jeanjean and Tanaka [2]).

Our new algorithm is based in this analytical result. We obtain numerical positive solutions for an asymptotically linear problem using the well known important fact proved by Pohozaev that any weak solution of an elliptic equation of type

$$\begin{cases} - \quad \Delta u = g(u) \ \text{ in } \mathbb{R}^N, \\ \quad u \quad \in \quad H^1(\mathbb{R}^N), \end{cases} \tag{1}$$

must satisfy the Pohozaev identity, where $G(s) = \int_0^s g(t)dt$.

2 Main Results

We consider the semilinear elliptic problem

$$\begin{cases} -\Delta u \quad + \quad \lambda u = f(u) \ \text{ in } \mathbb{R}^N \\ \qquad\qquad u \ \in \qquad H^1(\mathbb{R}^N) \end{cases} \tag{2}$$

where $N \geq 2$ and λ is a positive constant. The associate functional to this problem is defined in $H^1(\mathbb{R}^N)$ by

$$I(u) = \frac{1}{2}\int_{\mathbb{R}^N}(|\nabla u|^2 + \lambda u^2)dx - \int_{\mathbb{R}^N} F(u)dx, \tag{3}$$

with $F(s) = \int_0^s f(t)dt$. Moreover, the functional is well defined and $I \in C^1(H^1(\mathbb{R})^N, \mathbb{R})$, with

$$I'(u)\varphi = \int_{\mathbb{R}^N}(\nabla u\nabla\varphi + \lambda u\varphi)dx - \int_{\mathbb{R}^N} f(u)dx) \quad \forall\ \varphi \in H^1(\mathbb{R}^N) \tag{4}$$

Weak solutions u of problem (2) are precisely the critical points of I, i.e. $I'(u) = 0$.

Among some other assumptions, we assume that f satisfies the following: there is a positive constant a such that $\frac{f(s)}{s} \to a$, as $|s| \to +\infty$, $a < \lambda$. This assumption implies that the problem is asymptotically linear at infinity and that the well known Ambrosetti and Rabinowitz condition [1] $0 < \mu F(s) \leq sf(s)$, for some $\mu > 2$, is not satisfied. We recall that any solution of (2) satisfies Pohozaev identity, given by

$$(N-2)\int_{\mathbb{R}^N}|\nabla u|^2dx = 2N\int_{\mathbb{R}^N} G(u)dx, \tag{5}$$

where $G(u) = -\frac{\lambda}{2}u^2 + F(u)$.

We recall that the Pohozaev manifold is defined by $\mathbb{P} = \{u \in H^1(\mathbb{R}^N\backslash\{0\} : J(u) = 0\}$.

In this work, we present several lemmas which describe the analytical tools necessary to support the construction of the proposed algorithm. Of those, two of them are a core part to make such a construction: that under some suitable conditions, there exists a unique real number $t > 0$ such that $u(\frac{x}{t}) \in \mathbb{P}$ and $I(u(\frac{x}{t}))$ is the maximum for the function $t \mapsto I(u(\frac{\cdot}{t}))$, $t > 0$, and that a function $u \in H^1(\mathbb{R}^N)$ is a critical point of I if and only if u is a critical point of I restricted to the Pohozaev manifold $\mathbb{P}$.

We present the algorithm below:

2.1 Mountain Pass algorithm using Pohozaev manifold (MPAP)

Step 1. Take an initial guess $w_0 \in H^1(\mathbb{R}^N)$ such that $w_0 \neq 0$ and $\int G(w_0) > 0$;
Step 2 Find t_* such that $I(w_o(\frac{\cdot}{t_*})) = maxI(w_0(\frac{x}{t}))$, $t > 0$, and set $w_1 = w_0(\frac{\cdot}{t_*})$;
Step 3 Find the steepest descent direction $\hat{v} \in H^1(\mathbb{R}^N)$ such that $[I(w_1 + \epsilon\hat{v}) - I(w_1)]/\epsilon$ is as negative as possible as $\epsilon \to 0$, obtaining $\hat{v} = -I'(w_1).If||\hat{v}|| < \tau$, where τ is the estimator for convergence, then output and stop. Else, go to the next step;
Step 4 Let $\hat{\alpha}$ be such that $I(w_1 + \alpha\hat{v})$ attains its minimum at $\alpha = \hat{\alpha}$, $\forall\ \alpha > 0$; redefine $w_0 := w_1 + \hat{\alpha}\hat{v}$. Then, go to step 2.

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MULTIPLICIDADE DE SOLUÇÕES PARA UM PROBLEMA ENVOLVENDO O OPERADOR $(P,Q)-$ LAPLACIANO

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Abstract

Neste trabalho, considerando $\Omega \subset \mathbb{R}^N$ $(N \geqslant 2)$ um domínio limitado suave, através do emprego de métodos variacionais, como o Teorema do Passo da Montanha, pretendemos garantir a existência e multiplicidade de soluções para o problema superlinear envolvendo uma perturbação

$$-\Delta_p u - \Delta_q u = \lambda u^\alpha + (a(x) + \varepsilon)u^r$$

onde $1 < q \leqslant p < \alpha + 1 < r + 1 < p^*$, $\lambda > 0$, $\varepsilon > 0$ e a função $a(x)$ é contínua não-negativa que se anula em um subdomínio de Ω.

1 Introdução

Problemas envolvendo o operador diferencial $\Delta_p + \Delta_q$, chamado de $(p, q)-$Laplaciano, tem sua origem numa reação geral de difusão e sua aplicabilidade é bastante presente em áreas da Física, Química, Biologia e nas ciências relacionadas como Biofísica e Física Plasmática. Devido a esta importância, muitos trabalhos com este operador foram desenvolvidos, por exemplo [4] em 2012 e [2] em 2014, os quais estabeleceram resultados de existência e multiplicidade de soluções para problemas envolvendo não-linearidades críticas. Além destes, podemos citar [3] em 2013, a qual considerou uma não-linearidade que muda de sinal e também obteve resultados de multiplicidade de soluções.

Em nosso trabalho, consideramos o seguinte problema

$$\begin{cases} -\Delta_p u - \Delta_q u = \lambda u^\alpha - (a(x) + \varepsilon)u^r & \text{em } \Omega \\ u = 0 & \text{sobre } \partial\Omega \end{cases} \tag{1}$$

onde $\Omega \subset \mathbb{R}^N$ $(N \geqslant 2)$ é um domínio limitado de fronteira suave, $\lambda > 0$ e $\Delta_s u = div(|\nabla u|^{s-2}\nabla u)$. Ainda, $1 < q \leqslant p < \alpha + 1 < r + 1 < p^*$ sendo p^* o expoente crítico de Sobolev. A função não-negativa $a(x)$ pertence a $C^\theta(\overline{\Omega})$ $(0 < \theta < 1)$ e, além disso,

$$\Omega_0 = \{x \in \Omega : a(x) = 0\}$$

é um domínio não vazio com fronteira suave, $\overline{\Omega}_0 \subset \Omega$, de tal modo que para cada $x \in \Omega \setminus \overline{\Omega}_0$ próximo de $\partial\Omega_0$,

$$a(x) = b(x)[d(x, \partial\Omega_0)]^\gamma. \tag{2}$$

A função contínua positiva $b(x)$ está definida numa pequena vizinhança de $\partial\Omega_0$ e $0 < \gamma \neq \frac{p(r-\alpha)}{\alpha-p+1}$.

Baseados no trabalho desenvolvido por Dong em [1], no qual o autor estudou o problema (1) considerando o caso em que $p = q$, obtemos, sob as mesmas hipóteses, um resultado de existência e multiplicidade de soluções similar ao encontrado por ele.

2 Resultados Principais

Sob as hipóteses apresentadas anteriormente demonstramos o seguinte resultado:

Teorema 2.1. *Para todo $\varepsilon > 0$ existe $\lambda_\varepsilon > 0$ tal que, para cada $\lambda > \lambda_\varepsilon$, o problema (1) possui ao menos duas soluções positivas em $C_0^{1,\sigma}(\overline{\Omega})$ $(0 < \sigma < 1)$. Além disso, o problema (1) com $\lambda = \lambda_\varepsilon$ possui ao menos uma solução positiva em $C_0^{1,\sigma}(\overline{\Omega})$ e não admite solução positiva limitada quando $0 < \lambda < \lambda_\varepsilon$.*

Prova: Para garantirmos a existência das soluções utilizamos alguns problemas auxiliares, um truncamento da não-linearidade e empregamos Métodos Variacionais, dentre eles o Teorema do Passo da Montanha.

■

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HOMOGENIZATION OF P-LAPLACIAN IN THIN DOMAINS: THE UNFOLDING APPROACH

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Abstract

In this work we apply the unfolding operator method to analyze the asymptotic behavior of the solutions of the p-Laplacian equation with Neumann boundary condition set in bounded thin domains of the type $R^\varepsilon = \{(x,y) \in \mathbb{R}^2 : x \in (0,1) \text{ and } 0 < y < \varepsilon g(x/\varepsilon)\}$. We take a L-periodic function $g : \mathbb{R} \mapsto \mathbb{R}$ in $L^\infty(\mathbb{R})$. The thin domain situation is established passing to the limit in the positive parameter ε with $\varepsilon \to 0$.

1 Introduction

Let $R^\varepsilon \subset \mathbb{R}^2$ be the following family of thin domains

$$R^\varepsilon = \left\{(x,y) \in \mathbb{R}^2 : x \in (0,1) \text{ and } 0 < y < \varepsilon g\left(\frac{x}{\varepsilon}\right)\right\} \tag{1}$$

where $g : \mathbb{R} \to \mathbb{R}$ is a strictly positive function, periodic of period L, lower semicontinuous which satisfies

$$0 < g_0 \leq g(x) \leq g_1, \quad \forall x \in (0,L) \tag{2}$$

with $g_0 = \min_{x\in\mathbb{R}} g(x)$ and $g_1 = \max_{x\in\mathbb{R}} g(x)$.

In this work, we are interested in analyzing the asymptotic behavior of the family of solutions set by the following nonlinear elliptic problem

$$\begin{cases} -\Delta_p u_\varepsilon + |u_\varepsilon|^{p-2} u_\varepsilon = f^\varepsilon \text{ in } R^\varepsilon \\ |\nabla u_\varepsilon|^{p-2} \nabla u_\varepsilon \eta_\varepsilon = 0 \text{ on } \partial R^\varepsilon \end{cases} \tag{3}$$

where η_ε is the unit outward normal vector to the boundary ∂R^ε, $1 < p < \infty$ with $p^{-1} + p'^{-1} = 1$ and Δ_p denotes the p-laplacian operator.

Further, the existence and uniqueness of the solutions is guaranteed by Minty-Browder's Theorem for each fixed $\varepsilon > 0$. Hence, we are interested here in analyzing the behavior of the solutions u_ε as $\varepsilon \to 0$, that is, as the domain R^ε gets thinner and thinner although with a high oscillating boundary.

Notice that parameter $\varepsilon > 0$ introduced in (1) models the thin domain situation since $R^\varepsilon \subset (0,1) \times (0, \varepsilon g_1)$. Moreover, we see that R^ε has tickness order ε, and then, it is expected that the sequence of solutions u_ε will converge to a function depending just on the first variable $x \in (0,1)$ as $\varepsilon \to 0$.

We combine techniques as unfolding operator methods for thin domains developed in [1], as well as, that ones presented in [2, 3] in order to analyze monotone operators in perforated domains. We can also obtain a corrector result for the case studied here.

2 Main Results

Before we state the main result, we give the following definition:

Definition 2.1. *Let φ be Lebesgue-measurable in R^ε. The unfolding operator T_ε is 'roughly' defined as*

$$T_\varepsilon \varphi(x, y_1, y_2) = \varphi\left(\varepsilon^\alpha \left[\frac{x}{\varepsilon^\alpha}\right]_L L + \varepsilon^\alpha y_1, \varepsilon y_2\right),$$

where for each $\varepsilon > 0$ and any $x \in (0,1)$, there exists an integer denoted by $\left[\frac{x}{\varepsilon^\alpha}\right]_L$ such that

$$x = \varepsilon^\alpha \left[\frac{x}{\varepsilon^\alpha}\right]_L L + \varepsilon^\alpha \left\{\frac{x}{\varepsilon^\alpha}\right\}_L, \ \left\{\frac{x}{\varepsilon^\alpha}\right\}_L \in [0, L).$$

Theorem 2.1. *Let u_ε be the solution of problem (3) with f^ε satisfying*

$$|||f^\varepsilon|||_{L^{p'}(R^\varepsilon)} \leq c$$

for $c > 0$ independent of $\varepsilon > 0$. Suppose also that

$$T_\varepsilon f^\varepsilon \rightharpoonup \hat{f} \text{ weakly in } L^{p'}\left((0,1) \times Y^*\right). \tag{1}$$

Then, there exists $(u, u_1) \in W^{1,p}(0,1) \times L^p((0,1); W^{1,p}_\#(Y^*))$ such that

$$\begin{cases} T_\varepsilon u_\varepsilon \rightharpoonup u \text{ weakly in } L^p((0,1); W^{1,p}(Y^*)), \\ T_\varepsilon\left(\nabla u_\varepsilon\right) \rightharpoonup (u', 0) + \nabla_y u_1(x, y_1, y_2) \text{ weakly in } L^p\left((0,1); W^{1,p}(Y^*)\right)^2, \\ T_\varepsilon(|\nabla u_\varepsilon|^{p-2} \nabla u_\varepsilon) \rightharpoonup B(u') \text{ weakly in } L^p\left((0,1) \times Y^*\right)^2 \end{cases}$$

where $\nabla_y \cdot = (\partial_{y_1} \cdot, \partial_{y_2} \cdot)$ and u is the solution of the problem

$$\begin{cases} -(B(u'))' + |u|^{p-2} u = \bar{f} \text{ in } (0,1), \\ \quad B(u'(0)) = B(u'(1)) = 0, \end{cases} \tag{2}$$

where

$$\bar{f} = \frac{1}{|Y^*|} \int_{Y^*} \hat{f}\, dy_1 dy_2 \quad \text{and} \quad B(\xi) = \frac{1}{|Y^*|} \int_{Y^*} |\nabla v|^{p-2} \partial_{y_1} v\, dy_1 dy_2,$$

and v is the solution of the auxiliary problem

$$\begin{aligned} &\int_{Y^*} |\nabla_y v|^{p-2} \nabla_y v \nabla_y \varphi dy_1 dy_2 = 0, \quad \forall \varphi \in W^{1,p}_\#(Y^*), \\ &(v - \xi y_1) \in W^{1,p}_\#(Y^*) \quad \text{with} \quad \int_{Y^*} (v - \xi y_1) dy_1 dy_2 = 0, \end{aligned} \tag{3}$$

given for each $\xi \in \mathbb{R}$.

Proof: For a proof see [4]. □

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EXISTÊNCIA E MULTIPLICIDADE DE SOLUÇÕES PARA UM PROBLEMA DE ROBIN

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Abstract

Neste trabalho é exibido um teorema de multiplicidade de soluções, produzindo ao menos, três soluções não-triviais. Seguimos as idéias do artigo “Multiplicity theorems for nonlinear nonhomogeneous Robin problems” de Nikolaos S. Papageorgiou e Vicentiu D. Rădulescu [1]. Os autores estudam problemas com fronteira de Robin não-linear, dirigido por um operador diferencial não-homogêneo com uma reação Carathéodory. O problema é, basicamente, ressonante e a reação não possui restrições de grau de crescimento global. Um detalhe importante é a informação precisa com respeito ao sinal das soluções, que dependem das diferentes condições sobre o grau de crescimento da reação.

1 Introdução

Buscamos soluções para o seguinte problema:

$$\begin{cases} -\text{div}\, a(Du(z)) = f(z,u(z)) & \text{em } \Omega \\ \\ \dfrac{\partial u}{\partial n_a} + \beta(z)|u(z)|^{p-2}u(z) = 0 & \text{sobre } \partial\Omega \end{cases} \tag{1}$$

onde $\Omega \subset \mathbb{R}^N$ é um domínio limitado, $\partial\Omega$ é de classe C^2, a aplicação $a : \mathbb{R}^N \longrightarrow \mathbb{R}^N$ é estritamente monótona, contínua e satisfaz algumas condições de crescimento e de regularidade. Além disso, $\dfrac{\partial u}{\partial n_a} := (a(Du(z)), n)_{\mathbb{R}^N}$, Sendo $(\cdot,\cdot)_{\mathbb{R}^N}$ o produto interno usual para todo $u \in W^{1,p}(\Omega)$ e f é uma função Carathéodory.

2 Resultados Principais

A seguir exibiremos uma série de hipóteses que serão necessárias para a prova do teorema.

Seja $\eta \in C^1(0\infty)$, com $\eta(t) > 0$ para todo $t > 0$, tal que, existem constantes positivas $\bar{c}, c_0, c_1$ e c_2 valendo $0 < \bar{c} \leq \frac{t\eta'(t)}{\eta(t)} \leq c_0$ e $c_1 t^{p-1} \leq \eta(t) \leq c_2(1+t^{p-1})$ para todo $t > 0$. Definimos a aplicação $a : \mathbb{R}^N \longrightarrow \mathbb{R}^N$ por: $a(y) = a_0(|y|)(y)$, para todo $y \in \mathbb{R}^N$, com $a_0(t) > 0$ para todo $t > 0$.
Chamaremos de $H(a)$ as hipóteses abaixo:

(i) $a_0 \in C^1(0,\infty)$, $t \mapsto ta_0(t)$ é estritamente crescente , $\lim_{t\to 0^+} ta_0(t) = 0$ e $\lim_{t\to 0} \dfrac{ta_0'(t)}{a_0(t)} > -1$.

(ii) Existe $c_3 > 0$ tal que $|\nabla a(y)| \leq c_3 \frac{\eta(|y|)}{|y|}$, $\forall y \in \mathbb{R}^N \backslash \{0\}$.

(iii) $(\nabla a(y)\xi, \xi)_{\mathbb{R}^N} \geq \frac{\eta(|y|)}{|y|}|\xi|^2$, $\forall y \in \mathbb{R}^N\backslash\{0\}$, $\forall \xi \in \mathbb{R}^N$.

(iv) Se $G_0(t) = \int_0^t sa_0(s)ds$ para todo $t > 0$, então existe $q \in (1,p]$ tal que a função $t \mapsto G_0(t^{1/q})$ é estritamente convexa em $(0,\infty)$ e $\lim_{t\to 0^+} \dfrac{qG_0(t)}{t^q} = \tilde{c} > 0$.

Denotamos por $H(\beta)$ a condição:

$\beta \in C^{1,\alpha}(\partial\Omega)$, com $\alpha \in (0,1)$, $\beta(z) \geq 0$ para todo $z \in \partial\Omega$.

Precisamos também de informações a respeito do espectro de um problema que é um caso particular do estudado em [2]:

$$\begin{cases} -\Delta_q u(z) = \hat{\lambda}|u(z)|^{q-2}u(z) & \text{em } \Omega \\ \\ \dfrac{\partial u}{\partial n_q} + \beta(z)|u(z)|^{q-2}u(z) = 0 & \text{sobre } \partial\Omega \end{cases} \tag{1}$$

onde, $\dfrac{\partial u}{\partial n_a} := |D(u)|^{q-2}(a(Du(z)), n_q)_{\mathbb{R}^N}$, para todo $u \in W^{1,p}(\Omega)$ e $q \in (1, \infty)$.
O bloco de condições abaixo chamaremos de (H_1):

(i) $|f(z,x)| \leq \tilde{a}(z)(1 + |x|^{p-1})$ para quase todo $z \in \Omega$, para todo $x \in \mathbb{R}$ com $\tilde{a} \in L^{\infty}(\Omega)_+$.

(ii) Para $\hat{\beta} = \frac{p-1}{c_1}\beta \in L^{\infty}(\Omega)_+$ com $c_1 > 0$ vale:

$$\limsup_{x\to\pm\infty} f(z,x)|x|^{q-2}x \leq \frac{c_1}{p-1}\hat{\lambda_1}(q, \tilde{\beta})$$

uniformemente para quase todo $z \in \Omega$.

(iii) Se $F(z,x) = \int_0^t f(z,s)ds$, então $\lim_{x\to\pm\infty}[f(z,x)x - pF(z,x)] = +\infty$ uniformemente para quase todo $z \in \Omega$.

(iv) Existe $\eta_0 \in L^{\infty}(\Omega)_+$ tal que $\tilde{c}\hat{\lambda_1}(q, \tilde{\beta}) \leq \eta_0(z)$ para quase todo $z \in \Omega$, $\eta_0 \neq \tilde{c}\hat{\lambda_1}(q, \tilde{\beta})$ e $\eta_0(z) \leq \liminf_{x\to\pm\infty} f(z,x)|x|^{q-2}x$ uniformemente para quase todo $z \in \Omega$ com $\tilde{\beta} = \frac{1}{\tilde{c}}\beta$, onde $\tilde{c} > 0$ e $q \in (1, p]$ são como nas hipóteses $H(a)$. Ainda, $\hat{\lambda_1}(q, \beta)$ é o principal autovalor do problema (1).

Seja $f : \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ é uma função Carathéodory, $f(z, 0) = 0$ para quase todo $z \in \Omega$.
Chamaremos de (H_2) as condições (i), (ii) e (iii) da lista de hipóteses em H_1, junto com a condição abaixo:

(iv) $\tilde{c}\hat{\lambda_2}(q, \tilde{\beta}) < \liminf_{t\to 0^+} f(z,x)|x|^{q-2}x$ uniformemente para quase todo $z \in \Omega$.

Teorema 2.1. *Supondo válidas as hipóteses $H(a)$, (H_2) e $H(\beta)$, então o problema (2) possui ao menos três soluções não-triviais com:*

$$u_0 \in \text{int}C_+, \quad v_0 \in -\text{int}C_+, \quad e \quad y_0 \in [v_0, u_0] \cap C^1(\overline{\Omega}) \ \textit{nodal}.$$

Para um futuro próximo, pretendemos buscar resultados similares para problemas do tipo:

$$\begin{cases} -\text{div } a(Du(z)) + c(z, u(z)) = f(z, u(z)) & \text{em } \Omega \\ \\ \dfrac{\partial u}{\partial n_a} + \beta(z)|u(z)|^{p-2}u(z) = f(z, u(z)) & \text{sobre } \partial\Omega \end{cases}$$

buscando as hipóteses necessárias sobre as funções a, f, c, β e g, e assim, estendendo o problema (2) para uma classe mais ampla de soluções.

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EXISTENCE OF SOLUTION OF A RADIAL NONLINEAR SCHRÖDINGER EQUATION WITH SIGN-CHANGING POTENTIAL VIA SPECTRAL PROPERTIES

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Abstract

Considering the radial Schrödinger equation

$$-\Delta u + V(x)u = g(x,u) \ \text{ in } \ \mathbb{R}^N, \ N \geq 3 \tag{1}$$

we aim to find a radial nontrivial solution, where V changes sign ensuring problem (1) is indefinite and g is an asymptotically linear nonlinearity. We work with variational methods associating problem (1) to an indefinite functional in order to apply our Abstract Linking Theorem for Cerami sequences in [3] to get a non-trivial critical point for the functional. Our goal is to make use of spectral properties of operator $A := \Delta + V(x)$ restricted to $H^1_r(\mathbb{R}^N)$, the space of radially symmetric functions in $H^1(\mathbb{R}^N)$, for obtaining a linking geometry structure to the problem and by means of special properties of radial functions get the necessary compactness.

1 Introduction

We work with problem (1) with the following hypotheses:

$(V_1)_r$ $V \in L^\infty(\mathbb{R}^N)$ is a radial sign-changing function, $V(x) = V(|x|) = V(r),\ r \geq 0$;

$(V_2)_r$ Setting $\bar{V}(r) = V(r) + \dfrac{(N-1)(N-3)}{4r^2}$ and $\bar{A} := -\dfrac{d^2}{dr^2} + \bar{V}(r)$, an operator of $L^2(0,\infty)$, $0 \notin \sigma_{ess}(\bar{A})$ and

$$\sup\left[\sigma(\bar{A}) \cap (-\infty, 0)\right] = \sigma^- < 0 < \sigma^+ = \inf\left[\sigma(\bar{A}) \cap (0, +\infty)\right].$$

(g_1) $g(x,s) \in C(\mathbb{R}^N \times \mathbb{R}, \mathbb{R})$ is a radial function such that $\displaystyle\lim_{|s|\to 0} \frac{g(x,s)}{s} = 0$, uniformly in x and for all $t \in \mathbb{R}$, $G(x,t) = \displaystyle\int_0^t g(x,s)ds \geq 0$;

(g_2) $\displaystyle\lim_{|s|\to+\infty} \frac{g(x,s)}{s} = h(x)$, uniformly in x, where $h \in L^\infty(\mathbb{R}^N)$;

(g_3) $a_0 = \displaystyle\inf_{x\in\mathbb{R}^N} h(x) > \sigma^+ = \inf\left[\sigma(A) \cap (0, +\infty)\right];$

(g_4) Setting $\mathcal{O} := A - \mathcal{H}$, where $\mathcal{H}$ is the operator multiplication by $h(x)$ in $L^2(\mathbb{R}^N)$ and denoting by $\sigma_p(\mathcal{O})$ the pointing spectrum of $\mathcal{O}$, $0 \notin \sigma_p(\mathcal{O})$.

Inspired by [4] we seek to extract from $(V_1)_r - (V_2)_r$ useful information of operator $\bar{A}$ in order to study the spectrum of operator A restricted to $H^1_r(\mathbb{R}^N)$ and obtain the components to establish a suitable linking geometry. Moreover, following ideas in [1, 2] we are able to treat the problem in $H^1_r(\mathbb{R}^N)$, taking advantage of its properties to get the necessary compactness to the associated functional. Under this setting, we are able to complement and generalize this problem to sign-changing potentials and a broad class of non linearities. As we work with asymptotically linear nonlinearities at infinity, our version of linking theorem for Cerami sequences is applied (cf. [3]).

2 Main Results

Theorem 2.1. ***(Linking Theorem for Cerami Sequences)*** *Let E be a real Hilbert space, with inner product $\big(\cdot,\cdot\big)$, E_1 a closed subspace of E and $E_2 = E_1^{\perp}$. Let $I \in C^1(E,\mathbb{R})$ satisfying:*

(I_1) $I(u) = \dfrac{1}{2}\big(Lu, u\big) + B(u)$, *for all* $u \in E$, *where* $u = u_1 + u_2 \in E_1 \oplus E_2$, $Lu = L_1u_1 + L_2u_2$ *and* $L_i : E_i \to E_i$, $i = 1, 2$ *is a bounded linear self adjoint mapping.*

(I_2) *B is weakly continuous and uniformly differentiable on bounded subsets of E.*

(I_3) *There exist Hilbert manifolds $S, Q \subset E$, such that Q is bounded and has boundary ∂Q, constants $\alpha > \omega$ and $v \in E_2$ such that*
(i) $S \subset v + E_1$ and $I \geq \alpha$ on S;
(ii) $I \leq \omega$ on ∂Q;
(iii) S and ∂Q link.

(I_4) *If for a sequence (u_n), $I(u_n)$ is bounded and $(1 + ||u_n||)\,||I'(u_n)|| \to 0$, as $n \to +\infty$, then (u_n) is bounded.*

Then I possesses a critical value $c \geq \alpha$.

For the proof of this technical result see [3].

Theorem 2.2. *Suppose $(V_1)_r - (V_2)_r$ and $(g_1) - (g_4)$ hold. Then problem (P_r) in (1) possess a radial, nontrivial, weak solution in $H^1(\mathbb{R}^N)$.*

Proof Provided that I satisfies all assumptions $(I_1) - (I_4)$ in Theorem 2.1, applying it provides a critical point $u \in E$ of I, with $I(u) = c \geq \alpha > 0$, hence u is a non-trivial critical point of $I : E \to \mathbb{R}$. It implies that $I'(u)v = 0$, for all $v \in H^1_{rad}(\mathbb{R}^N)$. Nevertheless, the Principle of Symmetric Criticality implies that $I'(u)v = 0$ for all $v \in H^1(\mathbb{R}^N)$, namely, u is a critical point of I as a functional defined on the whole $H^1(\mathbb{R}^N)$. Since $I \in C^1(H^1(\mathbb{R}^N),\mathbb{R})$, it yields that u is a weak solution of (P_r). In addition, since $u \in E$, it is a radial weak solution. □

References

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