Topics in Inverse Spectral Problems

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We consider a family of inverse spectral problems for self-adjoint elliptci operators. Assuming that $\Omega \subset \mathbb{R}^N$ is a bounded smooth operator, let $a, \rho \in L^\infty(\Omega)$ be such that for some $\varepsilon_0 > 0$ we have

$$a(x) \geq \varepsilon_0, \qquad \rho(x) \geq \varepsilon_0 \qquad \text{a.e. in } \Omega.$$

Then for any potential $q \in L^\infty(\Omega)$ there exists a family of eigenvalues and eigenfunctions $(\lambda_k, \varphi_k)_{k\geq 1}$ for the eigenvalue problem

$$-\mathrm{div}(a\nabla\varphi_k) + q\varphi_k = \lambda_k \rho\varphi_k \qquad \text{in } \Omega, \qquad \int_\Omega \rho\varphi_k\varphi_j \, dx = \delta_{kj},$$

where δ_{kj} denotes the Kronecker symbol.

Then, denoting by $\psi_k := a\partial\varphi_k/\partial\mathbf{n}$ the normal derivative of the eigenfunction φ_k, we consider the so called *Boundary Spectral Data* associated to the coefficients a, ρ, q as being

$$\mathrm{BSD}(a, \rho, q) := \{(\lambda_k, \psi_k) \; ; \; k \geq 1\} .$$

We shall investigate the inverse problem consisting in the following question: if for two sets of coefficients a_j, ρ_j, q_j, for $j = 0, 1$, one has

$$\mathrm{BSD}(a_0, \rho_0, q_0) = \mathrm{BSD}(a_1, \rho_1, q_1)$$

what can be said about the coefficients (a_0, ρ_0, q_0) and (a_1, ρ_1, q_1)?