Wiener's Tauberian Theorems and Zeros of Entire Functions and Dirichlet Series

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Wiener's Tauberian theorems provide necessary and sufficient condition in order that the translates of a fixed function from either $L^1(\mathbb{R})$ or $L^2(\mathbb{R})$ are dense in the same space in terms of the quantity of real zeros of the Fourier transform of the function. We shall report both the mathematical and historical aspects around these fundamental results which lie on the border of Harmonic Analysis and Approximation Theory. For instance, the density criteria of Nyman-Beurling and Báez-Duarte for location of zeros of Dirichlet L-functions will be discussed. In particular, these criteria provide necessary and sufficient conditions for the truth of the Riemann hypothesis. Some new density criteria analogous to the latter ones will be announced. The main new results are parts of two joint works, one of them with Yuan Xu, and the other with Willian D. Oliveira.