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ASPECTOS GEOMÉTRICOS DE DISCRETIZAÇÃO DE EDPS EM SUPERFíCIES

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Abstract

Neste trabalho investigamos a influência da curvatura na discretização via elementos finitos de EDPs em superfícies [1]. Em particular propomos uma estratégia de discretização que leva em conta como regiões de alta curvatura afetam a solução. Neste sentido propomos uma estratégia de geração de malhas que são menos ou mais refinada em sítios com curvatura menor ou maior, levando em conta a transição de uma região para outra.

O objetivo maior desta apresentação entretanto é tentar estabelecer um diálogo com a comunidade de análise de EDPs em superfícies. Não sabemos responder a algumas questões de geometria, que provavelmente são conhecidas dos analistas. Em particular a influência da curvatura na solução de EDPs e questões relacionadas a espaços de aproximação em superfícies são de interesse.

1 Introdução

Considere o problema de Poisson para o operador de Laplace–Beltrami Δ_Γ. Seja $\Gamma \subset \mathbb{R}^3$ uma superfície bidimensional C^3 compacta orientável. Para $f \in L^2(\Gamma)$ com $\int_\Gamma f = 0$, seja $u \in \mathring{H}^1(\Gamma)$ tal que

$$\int_\Gamma \boldsymbol{\nabla}_\Gamma u \cdot \boldsymbol{\nabla}_\Gamma v \, dx = \int_\Gamma f v \, dx \quad \text{para todo } v \in \mathring{H}^1(\Gamma), \tag{1}$$

onde $\boldsymbol{\nabla}_\Gamma$ denota o gradiente tangencial e $\mathring{H}^1(\Gamma) = \{v \in H^1(\Gamma) : \int_\Gamma v = 0\}$. Suponha que $\Gamma_1 \subsetneq \Gamma$ seja uma região com curvatura alta, e $\Gamma_2 = \Gamma \backslash \overline{\Gamma_1}$.

Seja Γ_h uma "triangularização" de Γ, e o conjunto $\mathcal{T}_h$ dos triângulos T e $h_T = \operatorname{diam}(T)$. Para todo $x \in K \in \mathcal{T}_h$ suponha que haja exista um único ponto $a(x) \in \Gamma$ tal que

$$x = a(x) + d(x)\nu(a(x)), \tag{2}$$

onde $d(x)$ é a distância (com sinal) de x a Γ, e $\nu(a(x))$ é a normal a Γ em $a(x)$.

Considere a seguinte aproximação de (1). Seja o espaço de dimensão finita

$$S_h = \{v_h \in C^0(\Gamma_h) : \int_{\Gamma_h} v = 0,\, v_h|_T \text{ é linear em todo } T \in \mathcal{T}_h\}, \qquad S_h^l = \{a \circ v_h : v_h \in S_h\}.$$

Para $F_h \in L^2(\Gamma_h)$ com $\int_{\Gamma_h} F_h = 0$, seja $U_h \in S_h$ tal que

$$\int_{\Gamma_h} \boldsymbol{\nabla}_\Gamma h U_h \cdot \boldsymbol{\nabla}_\Gamma h v_h \, dx = \int_{\Gamma_h} F_h v_h \, dx \quad \text{para todo } v_h \in S_h. \tag{3}$$

2 Resultados Principais

Nesta seção, C denota uma constante universal, em particular independente da curvatura.

Lema 2.1 (Desigualdade de Poincaré)**.** *Existe constante* c_p *tal que*

$$\|\phi\|_{L^2(\Gamma)} \leq c_p \|\boldsymbol{\nabla}_\Gamma \phi\|_{L^2(\Gamma)}, \tag{1}$$

para todo ϕ *tal que* $\int_\Gamma \phi \, dx = 0$.

Pergunta: como c_p depende da curvatura de Γ?

Teorema 2.1 (Melhor aproximação)**.** *Seja* U_h *solução de* (3) *e* $u_h \circ a = U_h$*. Então existe constante* C *tal que*

$$\|\boldsymbol{\nabla}_\Gamma(u - u_h)\|_{L^2(\Gamma)} \leq C \min_{\phi_h \in S_h^\ell} \|\boldsymbol{\nabla}_\Gamma(u - \phi_h)\|_{L^2(\Gamma)} + C c_p (\boldsymbol{\Psi}_h \|f\|_{L^2(\Gamma)} + \|f - f_h\|_{L^2(\Gamma)})$$

onde $\boldsymbol{\Psi}_h = \max_{T \in \mathcal{T}_h} \kappa_T^2 h_T^2$*, para* $\kappa_T = \max\{|\kappa_1|, |\kappa_2|\}$ *e* κ_1*,* κ_2 *são as curvaturas principais.*

Seja $I_{h,\ell}$ o interpolador de Lagrange em Γ_h sobre S_h e defina a interpolação $I_h u \in S_h^\ell$ em Γ tal que $I_h u \circ a = I_{h,\ell}\eta$. Então vale a seguinte estimativa.

Lema 2.2. *Seja* u *solução de* (1)*. Então*

$$\|\boldsymbol{\nabla}_\Gamma(u - I_h u)\|_{L^2(\Gamma)} \leq C c_p (\boldsymbol{\Lambda}_h + \boldsymbol{\Psi}_h)\|f\|_{L^2(\Gamma)} + \left(\sum_{T \in \mathcal{T}_h} h_T^2 \|\boldsymbol{\nabla}_\Gamma{}^2 u\|_{L^2(T^\ell)}^2 \right)^{1/2},$$

onde $\boldsymbol{\Lambda}_h = \max_{T \in \mathcal{T}_h} h_T^3 \psi_T \kappa_T$ *e* $\psi_T = \max\{\|\boldsymbol{\nabla}_\Gamma \kappa_1\|_{L^\infty(T)}, \|\boldsymbol{\nabla}_\Gamma \kappa_2\|_{L^\infty(T)}\}$.

Pergunta: é possível eliminar dependência nas derivadas da curvatura?

Pergunta: o espaço polinomial S_h é o mais adequado?

Para obter estimativa de convergência usando o Lema 2.2, é necessário um resultado de regularidade *local*.

Lema 2.3 (Regularidade H^2 ponderada)**.** *Seja* $u \in H^2(\Gamma)$ *solução de* (1)*, e o subconjunto* $\Gamma_2 \subseteq \Gamma$*. Considere* $\rho \in W^{1,\infty}(\Gamma)$ *tal que supp*$\rho \subseteq \Gamma_2$*. Então existe* C *tal que*

$$\|\rho \boldsymbol{\nabla}_\Gamma{}^2 u\|_{L^2(\Gamma_2)} \leq C \|\rho\|_{W^{1,\infty}(\Gamma)} \big(1 + c_p(1 + \kappa^{(2)})\big) \|f\|_{L^2(\Gamma_2)}. \tag{2}$$

Pergunta: considere $\rho(x) = dist\{x, \Gamma_1\}$. Qual a "distância" mais adequada? Geodésica? Euclidiana?

Seja $\overline{h}$ um tamanho de malha de referência longe da região de alta curvatura Γ_1. Seja $\rho_T = \max\{1, dist(\Gamma_1, T^\ell)\}$, e defina a domínio $\mathcal{N}_1 = \{x : dist(\Gamma_1, x) \leq d_1\}$, para uma determinada distância d_1. Sejam $\tilde{\mathcal{T}}_h^1 = \{T \in \mathcal{T}_h : T \cap \mathcal{N}_1 \neq \emptyset\}$ e $\tilde{\mathcal{T}}_h^2 = \{T \in \mathcal{T}_h : T \notin \tilde{\mathcal{T}}_h^1\}$.

Teorema 2.2. *Suponha que: 1)* $(\boldsymbol{\Lambda}_h + \boldsymbol{\Psi}_h) \leq \overline{h}\kappa^{(2)}$*, 2)* $h_T \leq h_1$ *para todo* $T \in \tilde{\mathcal{T}}_h^1$ *e 3)* $h_T \leq \rho_T \overline{h}$ *para todo* $T \in \tilde{\mathcal{T}}_h^2$*. Então*

$$\|\boldsymbol{\nabla}_\Gamma(u - u_h)\|_{L^2(\Gamma)} \leq C(1 + c_p(1 + \kappa^{(2)}))\overline{h}\|f\|_{L^2(\Gamma)} + C c_p \|f - f_h\|_{L^2(\Gamma)}.$$

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EXISTENCE AND MULTIPLICITY OF SOLUTIONS FOR NONLOCAL NEUMANN PROBLEM WITH NON-STANDARD GROWTH

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Abstract

In this paper we are concerning with questions of existence of solution for a nonlocal and non-homogeneous Neumann boundary value problems involving the $p(x)$-Laplacian in which the non-linear terms have critical growth. The main tools we will use are the generalized Sobolev spaces and the Mountain Pass Theorem.

1 Introduction

In this paper we are going to study questions of existence of solutions for the nonlocal and non-homogeneous equation, with critical growth and Neumann boundary conditions, given by

$$\begin{array}{rcl} M\left(\displaystyle\int_\Omega \frac{1}{p(x)}(|\nabla u|^{p(x)}+|u|^{p(x)})dx\right)(-div(|\nabla u|^{p(x)-2}\nabla u)+|u|^{p(x)-2}u) & & \\ = & \lambda f(x,u)\left[\displaystyle\int_\Omega F(x,u)dx\right]^r & \text{in } \Omega \\ M\left(\displaystyle\int_\Omega \frac{1}{p(x)}(|\nabla u|^{p(x)}+|u|^{p(x)})dx\right)|\nabla u|^{p(x)-2}\frac{\partial u}{\partial \nu} & & \\ = & \gamma g(x,u)\left[\displaystyle\int_{\partial\Omega} G(x,u)dS\right]^\kappa & \text{on } \partial\Omega, \end{array} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain of $\mathbb{R}^N$, $p \in C(\overline{\Omega})$, $f : \overline{\Omega} \times \mathbb{R} \to \mathbb{R}$, $g : \partial\Omega \times \mathbb{R} \to \mathbb{R}$ and $M : \mathbb{R}^+ \to \mathbb{R}^+$ are continuous functions enjoying some conditions which will be stated later, $F(x,u) = \displaystyle\int_0^u f(x,\xi)d\xi$, $G(x,u) = \displaystyle\int_0^u g(x,y)dy$, $\dfrac{\partial u}{\partial \nu}$ is the outer unit normal derivative, $\lambda, r, \gamma, \kappa$ are real parameters, and $\Delta_{p(x)}$ is the $p(x)$-Laplacian operator, that is,

$$\Delta_{p(x)}u = \sum_{i=1}^N \frac{\partial}{\partial x_i}\left(|\nabla u|^{p(x)-2}\frac{\partial u}{\partial x_i}\right), \qquad 1 < p(x) < N.$$

In this article, we discuss existence of solutions for the nonlocal problem (1) with critical growth on the domain and on its boundary. We study the nonlocal condition for the two following important classes of functions: $M(\tau) = a + b\tau^\eta$ with $a \geq 0$, $b > 0$, $\eta \geq 1$ and $m_0 \leq M(\tau) \leq m_1$, where m_0 and m_1 are positive constants. Note that the original Kirchhoff term is included in our analysis.

We will study the problem with the following critical Sobolev exponents

$$p^*(x) = \frac{Np(x)}{N-p(x)} \quad \text{and} \quad p_*(x) = \frac{(N-1)p(x)}{N-p(x)}, \tag{2}$$

where p_* is critical exponent from the point of view of the trace.

2 Main Results

Theorem 2.1.

(*i*) Assume $\kappa = 0$, $g(x,u) = |u|^{q(x)-2}u$, $q : \partial\Omega \to [1,\infty)$ and $\mathcal{A} := \{x \in \partial\Omega : q(x) = p_*(x)\} \neq \emptyset$ and $M(\tau) = a + b\tau^\eta$, with $a \geq 0, b > 0, \eta \geq 1$. Moreover, assume the existence of functions $p(x), q(x), \beta(x) \in C_+(\overline{\Omega})$, positive constants A_1, A_2 such that $A_1 t^{\beta(x)-1} \leq f(x,t) \leq A_2 t^{\beta(x)-1}$ for all $t \geq 0$ and for all $x \in \overline{\Omega}$, with $f(x,t) = 0$ for all $t < 0$. Furthermore, $(\eta+1)p^+ < \beta^-(r+1) < q^-$ and $\dfrac{(\eta+1)(p^+)^{\eta+1}}{(p^-)^\eta} < \left(\dfrac{A_1}{A_2}\right)^{r+1} \dfrac{(\beta^-)^{r+1}(r+1)}{(\beta^+)^r}$. Then there exists $\lambda_0 > 0$ such that for all $\lambda > \lambda_0$ and for all $\gamma > 0$ there exists a nontrivial solution to (1).

(*ii*) Assume $\kappa = 0$, $g(x,u) = |u|^{q(x)-2}u$, $q : \partial\Omega \to [1,\infty)$ and $\mathcal{A} := \{x \in \partial\Omega : q(x) = p_*(x)\} \neq \emptyset$. Moreover, assume the existence of functions $p(x), q(x), \beta(x) \in C_+(\overline{\Omega})$, positive constants A_1, A_2 such that $A_1 t^{\beta(x)-1} \leq f(x,t) \leq A_2 t^{\beta(x)-1}$ for all $t \geq 0$ and for all $x \in \overline{\Omega}$, with $f(x,t) = 0$ for all $t < 0$. Furthermore, assume there exists $0 < m_0$ and m_1 such that $m_0 \leq M(\tau) \leq m_1$, with $\dfrac{m_1 p^+}{m_0} < \left(\dfrac{A_1}{A_2}\right)^{r+1} \dfrac{(\beta^-)^{r+1}(r+1)}{(\beta^+)^r}$ and $p^+ < \beta^-(r+1) < q^-$. Then there exists $\lambda_1 > 0$ such that for all $\lambda > \lambda_1$ and for all $\gamma > 0$ there exists a nontrivial solution to (1).

(*iii*) Suppose $r = 0$, $f(x,u) = |u|^{q(x)-2}u$, $q : \Omega \to [1,\infty)$ and $\mathcal{A} := \{x \in \Omega : q(x) = p^*(x)\} \neq \emptyset$. Moreover, assume the existence of functions $p(x), q(x), \beta(x) \in C_+(\partial\Omega)$, positive constants A_1, A_2 such that $A_1 t^{\beta(x)-1} \leq g(x,t) \leq A_2 t^{\beta(x)-1}$ for all $t \geq 0$ and for all $x \in \partial\Omega$, with $g(x,t) = 0$ for all $t < 0$. Furthermore, assume there exists $0 < m_0$ and m_1 such that $m_0 \leq M(\tau) \leq m_1$, with $\dfrac{m_1 p^+}{m_0} < \left(\dfrac{A_1}{A_2}\right)^{\kappa+1} \dfrac{(\beta^-)^{\kappa+1}(\kappa+1)}{(\beta^+)^\kappa}$ and $p^+ < \beta^-(\kappa+1) < q^-$. Then there exists $\gamma_1 > 0$ such that for all $\gamma > \gamma_1$ and for all $\lambda > 0$ there exists a nontrivial solution to (1).

(*iv*) Suppose $r = 0$, $f(x,u) = |u|^{q(x)-2}u$, $q : \Omega \to [1,\infty)$, $\mathcal{A} := \{x \in \Omega : q(x) = p^*(x)\} \neq \emptyset$ and $M(\tau) = a + b\tau^\eta$, com $a \geq 0, b > 0, \tau \geq 0$, and $\eta \geq 1$. Moreover, assume the existence of functions $p(x), q(x), \beta(x) \in C_+(\partial\Omega)$, positive constants A_1, A_2 such that $A_1 t^{\beta(x)-1} \leq g(x,t) \leq A_2 t^{\beta(x)-1}$ for all $t \geq 0$ and for all $x \in \partial\Omega$, with $g(x,t) = 0$ for all $t < 0$. Furthermore, assume $(\eta+1)p^+ < \beta^-(\kappa+1) < q^-$ and $\dfrac{(\eta+1)(p^+)^{\eta+1}}{(p^-)^\eta} < \left(\dfrac{A_1}{A_2}\right)^{\kappa+1} \dfrac{(\beta^-)^{\kappa+1}(\kappa+1)}{(\beta^+)^\kappa}$. Then there exists $\gamma_2 > 0$ such that for all $\gamma > \gamma_2$ and for all $\lambda > 0$ there exists a nontrivial solution to (1).

Proof. It was inspired in the references [1, 3, 4, 5].

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QUASILINEAR ELLIPTIC PROBLEMS WITH CONCAVE-CONVEX NONLINEARITIES

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Abstract

It is establish existence and multiplicity of solutions for a quasilinear elliptic problem driven by Φ-Laplacian operator. These solutions are also built as ground state solutions using the Nehari method. The main difficult arises from the fact that Φ-Laplacian operator is not homogeneous and the nonlinear term is indefinite.

1 Introduction

In this work we consider the quasilinear elliptic problem driven by the Φ-Laplacian operator given by

$$\begin{cases} -\Delta_\Phi u &= \lambda a(x)|u|^{q-1}u + b(x)|u|^{p-1}u \text{ in } \Omega, \\ u &= 0, \text{ in } \partial\Omega, \end{cases} \tag{1}$$

where $\lambda > 0$, $\Omega \subset \mathbb{R}^N$ is bounded and smooth domain. Throughout this work we assume that $\phi : (0,\infty) \to (0,\infty)$ is of C^2 class which satisfies the following conditions

(ϕ_1) $\lim_{t\to 0} t\phi(t) = 0, \lim_{t\to\infty} t\phi(t) = \infty$;

(ϕ_2) $t \mapsto t\phi(t)$ is strictly increasing;

(ϕ_3) $-1 < \ell - 2 := \inf_{t>0} \frac{(t\phi(t))''t}{(t\phi(t))'} \le \sup_{t>0} \frac{(t\phi(t))''t}{(t\phi(t))'} =: m - 2 < N - 2.$

On the nonlinear problem (1) we shall assume the following inequalities $1 < q+1 < \ell \le m < p+1 < \ell^*$ and $1 < \ell \le m < N, \ell^* = \ell N/(N-\ell)$. Furthermore, we shall assume the following assumption

(H) $p(m-\ell) < ((p+1)-\ell)(m-(q+1)),\ a, b \in L^\infty(\Omega), a^+, b^+ \not\equiv 0.$

Due to the nature for the nonlinear operator

$$\Delta_\Phi u = \operatorname{div}(\phi(|\nabla u|)\nabla u)$$

we shall work in the framework of Orlicz-Sobolev spaces $W_0^{1,\Phi}(\Omega)$. Throughout this paper we define

$$\Phi(t) = \int_0^t s\phi(s)ds, t \in \mathbb{R}.$$

Recall that hypotheses $(\phi_1) - (\phi_2)$ allows us to use Orlicz and Orlicz-Sobolev spaces. Here we emphasize that hypothesis (ϕ_3) ensures that $W_0^{1,\Phi}(\Omega)$ is a reflexive and separable Banach space.

Quasilinear elliptic problem such as problem (1) have been considered in order to explain many physical problems arising from Nonlinear Elasticity, Plasticity, Generalized Newtonian Fluids, Non-Newtonian Fluids and Plasma Physics. For further applications and more details we infer the reader to [4, 6, 7] and the references therein.

2 Main Results

Using a regularity result for quasilinear elliptic problems we can state the following multiplicity result

Theorem 2.1. *Suppose* $(\phi_1) - (\phi_3)$ *and* (H)*. Then problem* (1) *admits at least two positive ground state solutions* u^+, u^- *which belongs to* $C^{1,\alpha}(\bar{\Omega})$ *whenever* $0 < \lambda < \lambda_1$.

Quasilinear elliptic problems driven by Φ-Laplacian operator have been extensively considered during the last years. We refer the reader to important works [1, 2, 3]. In [2] the authors considered existence of positive solutions for quasilinear elliptic problems where the nonlinear term is superlinear at infinity.

In order to achieve our results we shall consider the Nehari manifold $\mathcal{N}_\lambda$ introduced in [6]. In this work the main difficult is that a and b is not defined in sign, i.e, we consider also the case where a, b are sign changing functions. In order to overcome this difficulty we split the Nehari manifold into two parts $\mathcal{N}_\lambda = \mathcal{N}_\lambda^+ \cup \mathcal{N}_\lambda^-$. In this way, we obtain that problem (1) admits at least two positive solutions thanks to the fact that the fibering maps give us an unique projection in each part ${\mathcal{N}_\lambda}^{\pm}$.

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LOCAL EXACT CONTROLLABILITY TO TRAJECTORIES OF GENE REGULATORY NETWORKS

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Abstract

This work deals with the local exact controllability to trajectories of an initial-boundary value problem for a parabolic system with nonlinear terms. The control is distributed in space and time and it is exerted through one scalar function whose support can be arbitrarily small. We prove that if the initial data are sufficiently small and the linearized system satisfies an appropriate condition, the equations can be driven exactly to a constant trajectory.

1 Introduction

Let Ω be a non-empty open bounded set of $\mathbb{R}^n$ $(n \leq 3)$ with C^2 boundary $\partial\Omega$. For a real number $T > 0$, we consider the cylinder $Q = \Omega \times (0, T)$ of $\mathbb{R}^{n+1}$, with lateral boundary $\Sigma = \partial\Omega \times (0, T)$. The points of Ω are represented by $x = (x_1, \ldots, x_n)$, $x_i \in \mathbb{R}$, $i = 1, \ldots, n$ and those of Q are represented by (x, t), with $x \in \Omega$ and $0 < t < T$. By ω we consider a subset of Ω, that is, $\omega \Subset \Omega$; as usual, 1_ω denotes the characteristic function of ω.

In this work, we consider the simplified biological model, incorporating the genes FlbA, FlbB, FlbC, FlbD, brlA and PkaA and one external signal (light). We will concerned with the local exact controllability to trajectories of nonlinear parabolic system

$$\left| \begin{array}{lll} u_t - D\Delta u = -\alpha u + F(u) + Bv1_\omega & \text{in} & Q, \\ \dfrac{\partial u}{\partial n} = 0 & \text{on} & \Sigma, \\ u(x,0) = u^0(x) & \text{in} & \Omega, \end{array} \right. \tag{1}$$

where $u = [u_1 \cdots u_6]^T$, $u^0 = \left[u_1^0 \cdots u_6^0\right]^T$, $v = [v_1\ v_2\ v_3]$, $D = \text{diag}(d_1 \cdots d_6)$, $\alpha = \text{diag}(\alpha_1 \cdots \alpha_6)$, with $d_i > 0$ and $\alpha_i > 0$, for $i = 1, \ldots, 6$,

$$F(u) = \begin{bmatrix} F_1(u) \\ F_2(u) \\ F_3(u) \\ F_4(u) \\ F_5(u) \\ F_6(u) \end{bmatrix} = \begin{bmatrix} 0 \\ A_{21} + A_{23} \\ 0 \\ A_{42} \\ A_{52} + A_{53} + A_{54} + R_{56} \\ R_{61} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

where functions A_{ij} and R_{ij}, for $i = 1$, ..., 6, are given by

$$\begin{aligned} A_{ij} = A(k_{ij}, \beta_{ij}, u_j) &:= \frac{k_{ij} u_j}{1 + \beta_{ij}(u_j)_+} \qquad \text{(activation functions)}, \\ R_{ij} = R(k_{ij}, \beta_{ij}, u_j) &:= \frac{k_{ij}}{1 + \beta_{ij}(u_j)_+} \qquad \text{(repression functions)}. \end{aligned} \tag{3}$$

The functions $u_i = u_i(x,t)$ indicate the amounts of FlbA, FlbB, FlbC, FlbD, brlA and PkaA. The control function is $v = v(x,t)$ and it is related to light intensity.

Definition 1.1. *It will be said that (1) is locally exactly controllable to the trajectory $\overline{u}$ at time T if there exists $\varepsilon > 0$ such that, for any $u^0 \in (H^1(\Omega))^6$ with*

$$\|u^0 - \overline{u}^0\|_{(H^1(\Omega))^6} \leq \varepsilon,$$

there exist at least a control function $v \in L^2(\omega \times (0,T))$ such that the associated state u satisfy

$$u(T) = \overline{u}(T) \text{ in } \Omega. \tag{4}$$

It will be said that system (1) is exactly controllable to the trajectory $\overline{u}$ at time T if, for any $u^0 \in (H^1(\Omega))^6$ there exist at least a control function $v \in L^2(\omega \times (0,T))$ such that the associated state u satisfy (4).

2 Main Result

Our main result is the following:

Theorem 2.1. *Let be $Q = \Omega \times (0,T)$, where $\Omega \subset \mathbb{R}^n$, with $n \leq 3$, is a non-empty open and bounded C^2-domain and $0 < T < \infty$. If $(\lambda_p d_1 - \alpha_1) \neq (\lambda_p d_3 - \alpha_3)$ for all $p \geq 1$, then system (1) is locally exactly controllable to the constant trajectory at time T, where λ_p denote the eigenvalue of $-\Delta$ with Neumann boundary condition.*

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INJECTIVE IDEALS AND THE DOMINATION PROPERTY

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Abstract

We prove that an operator ideal is injective if and only if it has the domination property. Then we address the question if an analogous characterization holds for multi-ideals.

1 Introduction

An ideal $\mathcal{I}$ of linear operators between Banach spaces (operator ideal), in the sense of [4, 3]:
• Is *injective* if the following holds: if $u \in \mathcal{I}(E,G)$ and $v \in \mathcal{L}(F,G)$ is a metric injection ($\|v(y)\| = \|y\|$ for every $y \in F$) such that $v \circ u \in \mathcal{I}(E,G)$, then $u \in \mathcal{I}(E,F)$.
• Has the *domination property* if the following holds: if $u \in \mathcal{I}(E;F)$ and $v \in \mathcal{L}(E;G)$ are such that $\|v(x)\| \leq C\|u(x)\|$ for every $x \in E$ and some constant C (depending eventually on E, F, G, u, v), then $v \in \mathcal{I}(E;G)$.

As we are about to see, an operator ideal is injective if and only if it has the domination property (cf. Propostion 2.1). In this work we address the question if the same characterization holds for ideals of multilinear operators (multi-ideals). For the definition of multi-ideals, see, e.g. [6]. The definitions of an injective multi-ideal and of a multi-ideal with the domination property are obvious. We prove that some knwon ideals are injective and have the domination property and that the multi-ideals generated by the linearization method are injective and have the domination property. Although these results support the conjecture that the characterization holds in general, we use composition ideals to show that things may not work so well.

2 Main Results

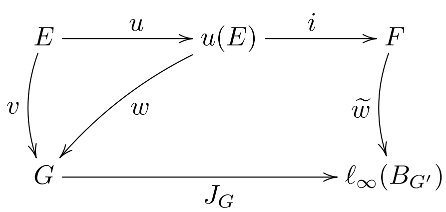
The following proof was taught to the author by Prof. A. Pietsch.

Proposition 2.1. *An operator ideal $\mathcal{I}$ is injective if and only if it has the domination property.*

Proof. Assume that $\mathcal{I}$ is injective and let $u \in \mathcal{I}(E;F)$, $v \in \mathcal{L}(E;G)$ be such that $\|v(x)\| \leq C\|u(x)\|$ for every $x \in E$. This inequality guarantees that the map

$$w\colon u(E) \subseteq F \longrightarrow G\ ,\ w(u(x)) = v(x),$$

is a well defined continuous linear operator. Considering the canonical metric injection $J_G\colon G \longrightarrow \ell_\infty(B_{G^*})$, by the extension property of $\ell_\infty(B_{G^*})$ [3, Proposition C.3.2] there is an extension $\widetilde{w} \in \mathcal{L}(F;\ell_\infty(B_{G^*}))$ of $J_G \circ w$ to the whole of F.



From $\widetilde{w} \circ u = J_G \circ v$ we conclude that $J_G \circ v$ belongs to $\mathcal{I}$, and the injectivity of $\mathcal{I}$ gives $v \in \mathcal{I}(E;G)$. The converse is obvious. $\square$

Obviously the proof above does not work for nonlinear operators. Addressing the multilinear case, it is clear that ideals of absolutely-summing-type multilinear operators (such as dominated operators and strongly summing operators [5]) are injective and have the domination property. They are all defined or characterized by an inequality of the type

$$\left(\sum_j \|A(x_j^1, \ldots, x_j^n)\|^p \right)^p \le C \cdots ,$$

where $A\colon E_1 \times \cdots \times E_n \longrightarrow F$, from which the injectivity and the domination property follow. Let us formalize this: let $\ell_p(\cdot)$ be the correspondence that takes a Banach space E to the space $\ell_p(E)$ of E-valued p-summable sequences. Let $X_1, \ldots, X_n$ be linearly stable sequences classes, according to [2], such that $X_1(\mathbb{K}) \cdots X_n(\mathbb{K}) \overset{1}{\hookrightarrow} \ell_p$.

Proposition 2.2. *The ideal of $(X_1, \ldots, X_n; \ell_p(\cdot))$-summing multilinear operators is injective and has the domination property.*

By $[\mathcal{I}]$ we denote the multi-ideal generated by the linearization method starting with the operator ideal $\mathcal{I}$ (see, e.g. [1]).

Proposition 2.3. *If the operator ideal $\mathcal{I}$ is injective, then the multi-ideal $[\mathcal{I}]$ is injective and has the domination property.*

On the other hand, things do not work so well for composition ideals $\mathcal{I} \circ \mathcal{L}$ (see [3]). Remember that this class includes the multi-ideals of compact and weakly compact operators. By A_L we denote the linearization of the multilinear operator A on the projective tensor product. All we know is the following quite unsatisfactory result:

Proposition 2.4. *Let $\mathcal{I}$ be an injective operator ideal. If $A \in \mathcal{I} \circ \mathcal{L}(E_1, \ldots, E_n; F)$ and $B \in \mathcal{L}(E_1, \ldots, E_n; G)$ are such that $\|B_L(z)\| \le C\|A_L(z)\|$ for every $z \in \widehat{\otimes}_\pi^n E$, then $B \in \mathcal{I} \circ \mathcal{L}(E_1, \ldots, E_n; G)$.*

The general question of whether a multi-ideal is injective if and only if it has the domination property remains open.

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NEW LOWER BOUNDS FOR THE CONSTANTS IN THE REAL POLYNOMIAL HARDY–LITTLEWOOD INEQUALITY

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Abstract

In this work we obtain new lower bounds for the constants of the real scalar-valued Hardy–Littlewood inequality for m-homogeneous polynomials on ℓ_p^2 spaces when $p = 2m$ and for certain values of m.

1 Introduction

Henceforth, for any map $f : \mathbb{R} \to \mathbb{R}$ we define $f(\infty) := \lim_{p\to\infty} f(p)$.

For $\mathbb{K}$ be $\mathbb{R}$ or $\mathbb{C}$ and $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{N}^n$, we define $|\alpha| := \alpha_1 + \cdots + \alpha_n$. By $\mathbf{x}^\alpha$ we shall mean the monomial $x_1^{\alpha_1} \cdots x_n^{\alpha_n}$ for $\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{K}^n$. The polynomial Bohnenblust–Hille inequality (see [3], 1931) asserts that, given $m, n \geq 1$, there is a constant $B_{\mathbb{K},m}^{\mathrm{pol}} \geq 1$ such that

$$\left(\sum_{|\alpha|=m} |a_\alpha|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq B_{\mathbb{K},m}^{\mathrm{pol}} \|P\|$$

for all m-homogeneous polynomials $P : \ell_\infty^n \to \mathbb{K}$ given by

$$P(x_1, ..., x_n) = \sum_{|\alpha|=m} a_\alpha \mathbf{x}^\alpha,$$

and all positive integers n, where $\|P\| := \sup_{z \in B_{\ell_\infty^n}} |P(z)|$. It is well-known that the exponent $\frac{2m}{m+1}$ is sharp.

When one tries to replace ℓ_∞^n by ℓ_p^n the extension of the polynomial Bohnenblust–Hille inequality is called polynomial Hardy–Littlewood inequality and the optimal exponents are $\frac{2mp}{mp+p-2m}$ for $2m \leq p \leq \infty$ and $\frac{p}{p-m}$ for $m < p < 2m$. More precisely, given $m, n \geq 1$, there is a constant $C_{\mathbb{K},m,p}^{\mathrm{pol}} \geq 1$ such that

$$\left(\sum_{|\alpha|=m} |a_\alpha|^{\frac{2mp}{mp+p-2m}} \right)^{\frac{mp+p-2m}{2mp}} \leq C_{\mathbb{K},m,p}^{\mathrm{pol}} \|P\|,$$

for all m-homogeneous polynomials on ℓ_p^n with $2m \leq p \leq \infty$ given by $P(x_1, \ldots, x_n) = \sum_{|\alpha|=m} a_\alpha \mathbf{x}^\alpha$ and all positive integers n. When $m < p < 2m$ the optimal exponent is $\frac{p}{p-m}$.

The search for precise estimates in the polynomial and multilinear Bohnenblust–Hille inequalities is important for many different reasons besides its intrinsic mathematical challenge. The knowledge of precise estimates for the constants of the Bohnenblust–Hille inequalities is a crucial point for applications (see [1] and the references therein). In this work we improve the best known lower bounds for the constants of the polynomial Hardy–Littlewood inequality for the case of real scalars for certain values of m. In some sense, there is a big difference between the Hardy–Littlewood and Bohnenblust–Hille inequalities. While in the Bohnenblust–Hille inequality the domain ℓ_∞^n remains unchanged, in the Hardy–Littlewood inequality the variable p in ℓ_p^n depends on the degree of homogeneity. In this work, we choose the case $p = 2m$, but similar investigation can be done for the other cases.

In order to find good lower bounds for the estimates for the quotient $\frac{|P_m|_2}{\|P_m\|}$, we have made a wide search (numerically) for appropriate coefficients. Of course, the exactness via numerical methods cannot be obtained. So, in the absence of analytical mathematical approaches the aid of our numerical approach can be of help with a very strong confidence. Our numerical method has shown to be quite effective, since when we restrict ourselves to the case of the Bohnenblust-Hille inequality, we perfectly recover the results analytically predicted in [3].

2 Main Results

Theorem 2.1. *The constants of the real polynomial Hardy–Littlewood inequality satisfy:*

$$\begin{aligned}
C_{\mathbb{R},2,4}^{\mathrm{pol}} &\geq 1.414213 \approx \sqrt{2} \\
C_{\mathbb{R},3,6}^{\mathrm{pol}} &\geq 2.236067 \approx \sqrt{5} > (1.30)^3 \\
C_{\mathbb{R},5,10}^{\mathrm{pol}} &\geq 6.191704 > (1.44)^5 \\
C_{\mathbb{R},6,12}^{\mathrm{pol}} &\geq 10.636287 > (1.48)^6 \\
C_{\mathbb{R},7,14}^{\mathrm{pol}} &\geq 18.095148 > (1.51)^7 \\
C_{\mathbb{R},8,16}^{\mathrm{pol}} &\geq 31.727174 > (1.54)^8 \\
C_{\mathbb{R},10,20}^{\mathrm{pol}} &\geq 91.640152 > (1.57)^{10}
\end{aligned}$$

We have numerical evidence that the best constant $C_{\mathbb{R},2,4}$ when restricted to two variables (in this case the optimal constant is denoted by $C_{\mathbb{R},2,4}^{\mathrm{pol}}(2)$) is given by the estimate of the previous theorem. In fact, we corroborate analytically this numerical evidence by the result:

Theorem 2.2. *If* $2 < p \leq 4$*, then*

$$C_{\mathbb{R},2,p}^{\mathrm{pol}}(2) = 2^{\frac{2}{p}}.$$

This work is contained in [4].

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PERSISTENT EIGENVALUES AND EIGENVECTORS OF A PERTURBED FREDHOLM OPERATOR

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Abstract

Let H be a Hilbert space and S its unit sphere. Consider the nonlinear eigenvalue problem $Lx+\varepsilon N(x)=\lambda x$, where $\varepsilon,\lambda\in\mathbb{R}$, $L:H\to H$ is a bounded, self-adjoint, linear operator with nontrivial kernel, and $N:H\to H$ is a nonlinear perturbation. A unit eigenvector x of L is said to be persistent if it is close to solutions $x\in S$ of the above equation for small values of the parameters ε and λ. In this paper we extend the problem to the context of Banach spaces and we give an affirmative answer to a conjecture by R. Chiappinelli, M. Furi and M.P. Pera, proving that, if N is Lipschitz continuous and the eigenvalue $\lambda=0$ has odd multiplicity, then the sphere S contains at least one persistent eigenvector. This is a joint work with A. Calamai, M. Furi and M.P. Pera.

1 Introduction

Let L be a self-adjoint operator defined on a real Hilbert space H. Suppose that $\lambda_0\in\mathbb{R}$ is an isolated simple eigenvalue of L and let $N:H\to H$ be a Lipschitz continuous map. Consider the problem

$$\begin{cases} Lx+\varepsilon N(x)=\lambda x \\ \|x\|=1, \end{cases} \tag{1}$$

where ε and λ are real parameters. When $\varepsilon=0$ and $\lambda=\lambda_0$, denote by x^1 and x^2 the two unit eigenvectors of L corresponding to the eigenvalue λ_0. Considering small values of ε, R. Chiappinelli in [2] obtained a sort of persistence result of these eigenvectors as well as of the eigenvalue λ_0. More precisely, he proved that, defined in a neighborhood V of $0\in\mathbb{R}$, there exist two H-valued Lipschitz curves, $\varepsilon\mapsto x^1_\varepsilon$ and $\varepsilon\mapsto x^2_\varepsilon$, as well as two real Lipschitz functions, $\varepsilon\mapsto\lambda^1_\varepsilon$ and $\varepsilon\mapsto\lambda^2_\varepsilon$, such that for $i=1,2$ and $\varepsilon\in V$ one has

$$Lx^i_\varepsilon+\varepsilon N(x^i_\varepsilon)=\lambda^i_\varepsilon x^i_\varepsilon,\quad \|x^i_\varepsilon\|=1.$$

It is natural to study the case when the multiplicity of the eigenvalue λ_0 is bigger than one. We introduce some terminology. Denote by S the unit sphere in H. By a *solution* of (1) we mean a triple $(x,\varepsilon,\lambda)\in S\times\mathbb{R}\times\mathbb{R}$ verifying the equation $Lx+\varepsilon N(x)=\lambda x$. In the set of all solutions of (1) we distinguish the subset $S\times\{0\}\times\{\lambda_0\}$ of the *trivial* ones, so that the notion of *nontrivial solution* is well defined. We say that an element $\bar{x}\in S\cap\operatorname{Ker}(L-\lambda_0 I)$ is a *bifurcation point* (or a *persistent eigenvector*) for problem (1) if any neighborhood of the associated trivial solution $(\bar{x},0,\lambda_0)$ contains nontrivial solutions. In [3] the persistence problem was investigated in the more general context of real Banach spaces. Namely, it was considered the system

$$\begin{cases} Lx+\varepsilon N(x)=\lambda Cx \\ g(x)=1, \end{cases}$$

where $L,C\colon E\to F$ are bounded linear operators between real Banach spaces, $N\colon E\to F$ is a nonlinear map, and g is a continuous norm in E, not necessarily equivalent to the Banach norm of E. Under a natural transversality condition between L and C (see (3) below), it was shown that, if L is Fredholm of index zero with odd dimensional kernel and N is C^1, then one obtains the persistence of at least one element of $g^{-1}(0) \cap \mathrm{Ker}L$.

In this paper we obtain a further generalization, allowing N to be locally α-Lipschitz, where α stands for the Kuratowski measure of noncompactness. This result was conjectured in [3] and, thus, we give here a positive answer. What is more, instead of merely considering eigenvectors of the type $g^{-1}(0) \cap \mathrm{Ker}L$, we seek for vectors belonging to $\Sigma := \partial\Omega \cap \mathrm{Ker}L$, where Ω is any open set of E containing the origin and such that Σ is nonempty and compact. The results we obtain here are based upon a notion of topological degree, developed in [1], for a class of noncompact perturbations of Fredholm maps of index zero between Banach spaces, whose definition is related to the measure of noncompactness α.

2 Main Results

Let E and F be two real Banach spaces, and let $\Omega \subseteq E$ be an open subset such that $0 \in \Omega$. Consider the system

$$\begin{cases} Lx + \varepsilon N(x) = \lambda C x \\ x \in \partial\Omega, \end{cases} \tag{2}$$

where $L : E \to F$ and $C : E \to F$ are bounded linear operators, $N : \overline{\Omega} \to F$ is continuous and locally α-Lipschitz. We assume that L is Fredholm of index zero with nontrivial kernel and that

$$\mathrm{Im}L \oplus C(\mathrm{Ker}L) = F. \tag{3}$$

Theorem 2.1 (Persistence of the eigenvalues and the eigenvectors)**.** *Suppose that the set $\Sigma = \partial\Omega \cap \mathrm{Ker}L$ of the trivial $\partial\Omega$-eigenvectors is nonempty and compact, and that $\mathrm{Ker}L$ is odd dimensional. Then, given $c > 0$ sufficiently small, there exist $a > 0$ and $b > 0$ such that:*

1. *$\forall \varepsilon \in [-a, a]$, the set $\Gamma_\varepsilon = \{\lambda \in [-b, b] : \; Lx + \varepsilon N(x) = \lambda C x \;$ for some $x \in \partial\Omega$ with $dist(x, \Sigma) < c\}$ of the eigenvalues and the set $\Xi_\varepsilon = \{x \in \partial\Omega \cap V_c : Lx + \varepsilon N(x) = \lambda C x$ for some $\lambda \in [-b, b]\}$ of the eigenvectors are nonempty;*
2. *the multivalued eigenvalue map $\varepsilon \in [-a, a] \multimap \Gamma_\varepsilon$ and the multivalued eigenvector map $\varepsilon \in [-a, a] \multimap \Xi_\varepsilon$ are upper semicontinuous;*
3. *$\Gamma_0 = \{0\}$ and $\Xi_0 = \Sigma$.*

Proof It is based on a relation between the odd dimension of $\mathrm{Ker}L$ and the cited degree introduced in [1]. The persistence phenomenon is ensured by the homotopy invariance of the degree. ■

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UNIFORM STABILITY OF THE ENERGY FOR A KIRCHHOFF-TYPE PROBLEM

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Abstract

This paper deals with the global existence and uniqueness of solutions, and uniform stabilization of the energy of an initial-boundary value problem for a quasilinear equation of Kirchhoff type. The asymptotic stabilization of the energy of problem (1) bellow, is obtained without any dissipative mechanism acting in the displacement variable, u, of equation $(1)_1$.

1 Introduction

In this paper we investigate the initial-boundary value problem

$$\begin{array}{lll} u'' - M\left(\cdot,\cdot,|\nabla u|^2\right)\Delta u = 0 & \text{in} & Q, \\ u = 0 & \text{on} & \Sigma, \\ u(x,0) = u_0(x), \;\; u'(x,0) = u_1(x) & \text{in} & \Omega. \end{array} \tag{1}$$

The main goals in this paper are to establish, under suitable conditions, that system (1) has the same properties obtained in Límaco et al [1], including the uniform stability of the energy without the internal damping. We emphasize that the asymptotic stability of the energy is obtained without any dissipative mechanism acting in the displacement variable, u.

As far as we know, all articles in the literature on problems of Kirchhoff type, like (1) or when $M = M(\lambda)$, the energy at the initial time $t_0 = 0$ is the same at any time $t > 0$. Thus, this paper is the first work bringing an asymptotic analysis of the energy of system (1), under suitable assumption only on the coefficient function, M.

2 Main Results

The existence of global solutions for the mixed problem (1) is established imposing some restrictions on the norm of the initial data u_0 and u_1. Actually, we suppose $u_0 \in H_0^1(\Omega) \cap H^2(\Omega)$, $u_1 \in H_0^1(\Omega)$ and consider the positive real constant

$$K_0 = \frac{1}{2}\left[|\nabla u_1|^2 + C_0 f\left(|\nabla u_0|^2\right)\right]|\Delta u_0|^2, \tag{1}$$

which will have some restrictive conditions to be imposed later. Moreover, we also assume some hypotheses on the function M. Namely, there are positive real constants m_0, C_k for $k = 0, 1, 2$, and a negative real constant ρ, such that

$$\left|\begin{array}{l} M \in C^1\left(\overline{\Omega} \times [0,\infty) \times [0,\infty)\right), \quad 0 < m_0 \le M(x,t,\lambda) \le C_0\, f(\lambda), \\ M(x,t,\lambda) := M_1(x,t) + M_2(x,t,\lambda), \\ \partial_t M_1 \le \rho < 0, \quad |\partial_t M_2|_{\mathbb{R}} \le C_1\, g(\lambda), \quad |\partial_\lambda M|_{\mathbb{R}} \le C_2\, h(\lambda), \\ f,\, g,\, h \in C^1\big([0,\infty);[0,\infty)\big) \text{ are strictly increasing.} \end{array}\right. \tag{2}$$

Remark 2.1. *The hypothesis $\partial_t M_1 \le \rho < 0$ is only necessary to prove the uniform stability of the energy of system* (1)*, i.e., if it is replaced by $\partial_t M_1 \le 0$ then Theorem 2.1 remains true.*

Definition 2.1. *A global strong solution to the initial-boundary value problem* (1) *is a function u defined on $\Omega \times [0,\infty)$ with real values, such that*

$$u \in L^\infty_{loc}\big(0,\infty;\, H^1_0(\Omega) \cap H^2(\Omega)\big),\;\; u' \in L^\infty_{loc}\big(0,\infty; H^1_0(\Omega)\big),\;\; u'' \in L^\infty_{loc}\big(0,\infty; L^2(\Omega)\big), \tag{3}$$

and the function u satisfies the system (1) *almost everywhere.*

Throughout this paper C_Ω represents a positive real constant that depends on the measure of set Ω. Namely, $|z|^2 \le C_\Omega |\nabla z|^2$ and $|\nabla z|^2 \le C_\Omega |\Delta z|^2$, which hold for all $z \in H^1_0(\Omega) \cap H^2(\Omega)$, since Γ is a smooth boundary.

In these conditions we can state the main results of this paper.

Theorem 2.1. *Suppose $u_0 \in H^1_0(\Omega) \cap H^2(\Omega)$, $u_1 \in H^1_0(\Omega)$ and $C_3\Big[g\Big(\dfrac{2C_\Omega K_0}{m_0}\Big) + h\Big(\dfrac{2C_\Omega K_0}{m_0}\Big)K_0\Big] < -\dfrac{\rho}{2}$, where C_3 and K_0 are positive real constants. Then there exists at least a global strong solution of system* (1)*, provided that hypothesis* (2) *holds. Moreover, if ℓ is a real-valued continuous and increasing function defined on $[0,\infty[$, and there exists a positive real constant C_4 such that $|\nabla M(x,t,\lambda)|_{\mathbb{R}} \le C_4 \ell(\lambda)$, then the solution of problem* (1) *is unique.*

The energy defined by the strong solution of system (1) given by

$$E(t) = \frac{1}{2}\Big\{|u'(t)|^2 + \int_\Omega M\big(\cdot, t, |\nabla u(t)|^2\big)|\nabla u(t)|^2_{\mathbb{R}} dx\Big\}$$

is uniformly stable. Namely,

Theorem 2.2. *Let u be a global strong solution of system* (1)*. Then the energy of system* (1) *satisfies*

$$E(t) \le 3C_\Omega \exp\Big\{-\frac{4\tau}{3}t\Big\} \quad \textit{for all} \quad t \ge 0,$$

where

$$\tau = \min\Big\{\delta,\, \frac{\gamma_0}{4C_0C_5}\Big\}, \qquad 0 < \delta < \min\left\{\frac{1}{2C_6},\, \frac{\gamma_0/4}{C_0 f\big((2C_\Omega K_0)/m_0\big)}\right\},$$

and γ_0 is a positive real constant, $C_5 = \sup\Big\{f(\varphi);\ |\varphi|_{L^\infty_{loc}(0,\infty,H^1_0(\Omega))} \le C\Big\}$ and $C_6 = C_\Omega + \dfrac{1}{m_0}$.

Two relevant references for this paper are:

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UNIFORM STABILIZATION OF A WAVE EQUATION WITH LOCALIZED INTERNAL DAMPING AND ACOUSTIC BOUNDARY CONDITIONS WITH VISCOELASTIC DAMPING

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Abstract

In this work we deal with the uniform stabilization for a nonlinear wave equation with acoustic boundary conditions for a non-locally reacting boundary. The main purpose is to study the stability when the internal damping acts over a subset ω of the domain Ω and the boundary damping is of the viscoelastic type.

1 Introduction

The main purpose of this work is to study stability for the initial boundary value problem

$$u'' - \Delta u + a(x)\varphi(u') = 0 \quad \text{in}\, \Omega \times (0,\infty); \tag{1}$$

$$u = 0 \quad \text{on}\, \Gamma_0 \times (0,\infty); \tag{2}$$

$$\frac{\partial u}{\partial \nu} = \delta' \quad \text{on}\, \Gamma_1 \times (0,\infty); \tag{3}$$

$$m\delta'' - c^2\Delta_\Gamma \delta + \int_0^t g(t-s)\Delta_\Gamma \delta(s)ds + r\delta = -\rho_0 u' \,\text{on}\, \Gamma_1 \times (0,\infty); \tag{4}$$

$$u(x,0) = u_0(x),\ u'(x,0) = u_1(x) \quad \text{for}\, x \in \Omega; \tag{5}$$

$$\delta(x,0) = \delta_0(x)\,, \quad \delta'(x,0) = \frac{\partial u_0}{\partial \nu}(x) \ \text{for}\ x \in \Gamma_1, \tag{6}$$

where $\Omega \subset \mathbb{R}^n$ is an open, bounded and connected set with smooth boundary Γ made up of two disjoint and closed parts Γ_0 and Γ_1, both with positive measure; $r \in C^0(\Gamma_1, \mathbb{R})$ is a nonnegative function; $\varphi, g : \mathbb{R} \to \mathbb{R}$, $u_0, u_1 : \Omega \to \mathbb{R}$ and $\delta_0 : \Gamma_1 \to \mathbb{R}$ are given functions; m, c and ρ_0 are positive constants; and $a : \Omega \to \mathbb{R}$ is such that $a(x) \geq a_0 > 0$ over $\omega \subset \Omega$ and ω is a neighborhood of a boundary portion.

Since the pioneer work of Beale and Rosencrans [1], where the acoustic boundary conditions were introduced into the rigorous wave propagation literature, many authors have been studying stability of problems with this boundary condition. In [3], Frota, Medeiros and Vicente introduced a new formulation for the acoustic boundary conditions which includes a more general and realistic model. This model involves the Laplace-Beltrami operator and brings more regularity to the problem which allows us to consider additional nonlinear models involving the function δ, as in [7]. Recently, in [8], we showed that the uniform stabilization holds even when the internal frictional damping acts only on a small region close to a boundary portion. The main tolls in getting such result was a Lemma due to Lasiecka-Tataru [4] and the techniques of Cavalcanti *et al.* [2].

Lastly, the interaction of acoustic boundary conditions and memory terms has been extensively studied, see Liu-Sun [5], Park-Park [6] and references therein, but nobody has taken into account acoustic boundary condition with memory terms involving the function δ which is our goal in this work. In some sense boundary dissipation of viscoelastic type involving the function δ extends the previous results. Moreover, here we also preserve the internal localized damping, as in [8].

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STACKELBERG-NASH STRATEGIES FOR THE 2-D NAVIER-STOKES SYSTEM

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Abstract

This work deals with the application of Stackelberg-Nash strategies to the control of Navier-Stokes system. More precisely, we assume that we can act on the system through a hierarchy of controls. A first control (the leader) is assumed to choose the policy. Then, a Nash equilibrium pair (corresponding to a noncooperative multiple-objective optimization strategy) is found; this governs the action of the other controls (the followers).

1 Introduction and main result

In control theory, an interesting situation arises when several (in general, conflictive or contradictory) objectives are considered. This may happen, for example, if the cost function is the sum of several terms and so it is not clear how to average. It can also be expected to have more than one control acting on the equation. In these cases, we are led to consider multi-objective control problems. Nash equilibria define a noncooperative multiple-objective optimization strategy (see [3]). In this work, we are interested in applying a Stackelberg hierarchical strategy (see [4]) to find the Nash equilibrium associated to solutions of the following controlled version of the Navier-Stokes system:

$$\begin{cases} y_t - \Delta y + (y\cdot\nabla)\,y + \nabla p = f1_{\mathcal{O}} + v^1 1_{\mathcal{O}_1} + v^2 1_{\mathcal{O}_2} & \text{in} \quad \Omega\times(0,T), \\ \nabla\cdot y = 0 & \text{in} \quad \Omega\times(0,T), \\ y = 0 & \text{on} \quad \Gamma\times(0,T), \\ y(0) = y^0 & \text{in} \quad \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded connected open set with regular enough boundary Γ and $T > 0$. The variables $y = y\,(x,t)$ and $p = p(x,t)$ denote, respectively, the velocity of the fluid and the hydrostatic pressure. The subset $\mathcal{O} \subset \Omega$ is the main control domain and $\mathcal{O}_1$, $\mathcal{O}_2$ are the secondary control domains (all of them are supposed to be small); $1_{\mathcal{O}}$, $1_{\mathcal{O}_1}$ and $1_{\mathcal{O}_2}$ are the characteristic functions of $\mathcal{O}$, $\mathcal{O}_1$ and $\mathcal{O}_2$, respectively; the controls are f, v^1 and v^2, where f is the leader and v^1 and v^2 are the followers. For simplicity, we will assume that only three controls are applied (one leader and two followers), but very similar considerations hold for systems with a higher number of controls.

Let $\mathcal{O}_{1,d}, \mathcal{O}_{2,d} \subset \Omega$ be open sets, representing observation domains for the followers. We will consider the functionals

$$J_i(f;y,v^1,v^2) := \frac{\alpha_i}{2}\iint_{\mathcal{O}_{i,d}\times(0,T)} |y - y_{i,d}|^2 \, dx\,dt + \frac{\mu_i}{2}\iint_{\mathcal{O}_i\times(0,T)} |v^i|^2 \, dx\,dt, \quad i = 1,2, \tag{2}$$

where the $\alpha_i > 0$, $\mu_i > 0$ are constants and $y_{i,d} = y_{i,d}(x,t)$ are given functions.

The followers v^1 and v^2 assume that the leader f has made a choice and they try to find a Nash equilibrium for the costs J_i $(i = 1,2)$, which means finding a solution for the following optimal control problems:

$$J_1(f;v^1,v^2) = \min_{\hat{v}^1} J_1(f;\hat{v}^1,v^2), \quad J_2\left(f;v^1,v^2\right) = \min_{\hat{v}^2} J_2(f;v^1,\hat{v}^2). \tag{3}$$

Any pair (v^1, v^2) satisfying (3) is called a Nash equilibrium for J_1 and J_2.

If we were dealing with convex functionals, a common way to solve (3) is to find solutions for the following problems:

$$J_1'(f; v^1, v^2)(\hat{v}^1, 0) = 0, \quad \forall \hat{v}^1 \in L^2(\mathcal{O}_1 \times (0,T)), \quad v^i \in L^2(\mathcal{O}_i \times (0,T)) \tag{4}$$

and

$$J_2'(f; v^1, v^2)(0, \hat{v}^2) = 0, \quad \forall \hat{v}^2 \in L^2(\mathcal{O}_2 \times (0,T)), \quad v^i \in L^2(\mathcal{O}_i \times (0,T)). \tag{5}$$

However, in principle, this is not the case, due to the fact that we are considering the nonlinear Navier-Stokes system. Our strategy is to show that, under suitable conditions, the pair of functions (v^1, v^2) satisfying (4) and (5) is unique and that

$$\langle J_1''(f; v^1, v^2), (\hat{v}^1, 0)\rangle > 0, \quad \forall \hat{v}^1 \in L^2(\mathcal{O}_1 \times (0,T)), \quad v^i \in L^2(\mathcal{O}_i \times (0,T)) \tag{6}$$

and

$$\langle J_2''(f; v^1, v^2), (0, \hat{v}^2)\rangle > 0, \quad \forall \hat{v}^2 \in L^2(\mathcal{O}_2 \times (0,T)), \quad v^i \in L^2(\mathcal{O}_i \times (0,T)). \tag{7}$$

In this way, to find a Nash equilibrium $\left(v^1, v^2\right)$ becomes equivalent to prove that $\left(v^1, v^2\right)$ is unique and satisfies $(4) - (7)$.

Let us consider the vector spaces

$$V = \{u \in \left[H_0^1(\Omega)\right]^2;\ \nabla \cdot u = 0 \quad \text{in} \quad \Omega\}$$

and

$$H = \{u \in \left[L^2(\Omega)\right]^2;\ \nabla \cdot u = 0 \quad \text{in} \quad \Omega, \quad u \cdot \eta = 0 \quad \text{on} \quad \Gamma\},$$

where $\eta = \eta(x)$ denotes the outward unit normal to Ω at the point $x \in \Gamma$.

The following result holds.

Theorem 1.1. *Let us consider $f \in L^2(\mathcal{O} \times (0,T))$, $y^0 \in H$, $y_{i,d} \in L^2(\mathcal{O}_{i,d} \times (0,T))$ $(i = 1,2)$. Given $A > 0$, there is some $\mu_0 = \mu_0(A)$ such that whenever $\|f\|_{L^2(\mathcal{O}\times(0,T))} \leq A$ and $\mu_i \geq \mu_0$ $(i = 1,2)$, there exists a unique Nash equilibrium associated to* (1) *in sense of* (3).

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SOBRE A DINÂMICA DE SOLUÇÕES DE UM SISTEMA ACOPLADO DE EQUAÇÕES DE SCHRODINGER

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Resumo

A proposta deste trabalho é o estudo do problema de Cauchy para um sistema acoplado de equações tipo Schrödinger conhecido em óptica não linear como Equações de geração de segundo harmônico, χ^2-*SHG equations*.

Resultados de boa colocação local deste sistema, para o caso periódico, foram obtidos por Angulo e Linares em [1]. Neste trabalho obtemos resultados de boa colocação em diferentes regiões do plano que dependem do valor da constante $\sigma > 0$. Discutimos como diferentes valores desta constante mudam a dinâmica do sistema. Utilizando o método-I obtemos resultados de boa colocação global nos diferentes casos. Por fim, apresentamos resultados de má colocação obtidos para este sistema.

1 Introdução

Este trabalho é dedicado ao estudo do Problema de Cauchy para um sistema que modela problemas da óptica não-linear. De maneira mais precisa estudaremos o seguinte modelo matemático

$$\begin{cases} i\partial_t u(x,t) + p\partial_x^2 u(x,t) - \theta u(x,t) + \bar{u}(x,t)v(x,t) = 0, & x \in \mathbb{R},\ t \geq 0, \\ i\sigma\partial_t v(x,t) + q\partial_x^2 v(x,t) - \alpha v(x,t) + \frac{1}{2}u^2(x,t) = 0, & p,q = \pm 1,\ \sigma > 0 \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad (u_0.v_0) \in H^\kappa(\mathbb{R}) \times H^s(\mathbb{R}). \end{cases} \tag{1}$$

Observamos que o modelo estabelece o acoplamento não-linear de duas equações dispersivas de tipo Schrödinger através de termos quadráticos

$$N_1(u,v) = \bar{u}v \quad \text{e} \quad N_2(u) = \frac{1}{2}u^2. \tag{2}$$

Fisicamente, de acordo com o trabalho [3], as funções complexas u e v representam pacotes de amplitudes do primeiro e segundo harmônico, respectivamente, de uma onda óptica. Os valores de p e q podem ser 1 ou -1, dependendo dos sinais fornecidos entre as relações de dispersão/difração e a constante positiva σ mede os índices de grandeza de dispersão/difração. O interesse em propriedades não-lineares de materiais ópticos têm atraído a atenção de físicos e matemáticos nos últimos anos. Diversas pesquisas sugerem que explorando a reação não-linear da matéria, a capacidade *bit-rate* de fibras ópticas pode ser aumentada substancialmente e consequentemente uma melhoria na velocidade e economia de transmissão e manipulação de dados. Particularmente, em materiais não centrossimétricos (aqueles que não possuem simetria de inversão ao nível molecular) os efeitos não-lineares de ordem mais baixa originam a susceptibilidade de segunda ordem, o que significa que a resposta não-linear para o campo elétrico é de ordem quadrática ver, por exemplo, os artigos [4] e [5].

2 Resultados Principais

Provaremos resultados de boa colocação local para dados $(u_0, v_0) \in H^\kappa \times H^s$ com índices $(\kappa, s) \in \mathcal{W}_\sigma$, onde a região plana $\mathcal{W}_\sigma$ é definida da seguinte forma:

Definição 2.1. *Dado $\sigma > 0$, dizemos que o par de índices de Sobolev (κ, s) verifica a hipótese H_σ se satisfaz uma das seguintes condições:*

a) $|\kappa| - 1/2 < s < \min\{\kappa + 1/2,\ 2\kappa + 1/2\}$ *para* $0 < \sigma < 2$;

b) $\kappa = s \geq 0$ *para* $\sigma = 2$;

c) $|\kappa| - 1 < s < \min\{\kappa + 1,\ 2\kappa + 1\}$ *para* $\sigma > 2$.

Desse modo, podemos trabalhar com um conjunto $\mathcal{W}_\sigma$ definido como

$$\mathcal{W}_\sigma = \Big\{(\kappa, s) \in \mathbb{R}^2;\ (\kappa, s) \text{ verifica a hipótese } H_\sigma\Big\}. \tag{1}$$

Obtemos,

Teorema 2.1. *Sejam $\sigma > 0$ e $(u_0, v_0) \in H^\kappa \times H^s$ com $(\kappa, s) \in \mathcal{W}_\sigma$, definida em (1). O problema de Cauchy (1) é localmente bem posto em $H^\kappa \times H^s$ no seguinte sentido: para cada $\rho > 0$, existem $T = T(\rho) > 0$ e $b > 1/2$ tais que para todo dado inicial com $\|u_0\|_{H^\kappa} + \|v_0\|_{H^s} < \rho$, existe uma única solução (u, v) para (1) satisfazendo as seguintes condições:*

$$\psi_T(t)u \in X^{\kappa,b} \quad e \quad \psi_T(t)v \in X^{s,b}_\sigma, \tag{2}$$
$$u \in C\big([0,T]; H^\kappa\big) \quad e \quad v \in C\big([0,T]; H^s\big). \tag{3}$$

Além disso, a aplicação dado-solução é localmente Lipschitziana.

Para finalizar provaremos, fazendo uso do I-método,

Teorema 2.2. *Seja $(u_0, v_0) \in H^s(\mathbb{R}) \times H^s(\mathbb{R})$. As soluções locais obtidas no Teorema 2.1 para o Problema de Cauchy* (1) *podem ser estendidas a todo intervalo de tempo positivo $0 \leq t \leq T$, preservando todas as propriedades da teoria local, em cada uma das seguintes situações:*

(a) $-1/2 \leq s \leq 0$ *quando* $\sigma > 2$,

(b) $-1/4 \leq s \leq 0$ *quando* $0 < \sigma < 2$.

No caso $\sigma = 2$. Temos solução global em $L^2 \times L^2$.

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A MATHEMATICAL ANALYSIS OF A MODEL FOR GEOGRAPHIC SPREAD OF DENGUE DISEASE

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Abstract

In this work it is considered a system of nonlinear partial differential equations coupled with nonlinear ordinary differential equations corresponding to a generalization of a mathematical model for geographical spreading of dengue disease proposed by Maidana and Yang in [1]. As in that article, the mosquito population is divided in winged form and aquatic forms and the human population is divided in the susceptible, infected and removed subpopulations. On the other hand, in the mentioned work of Maidana and Yang it is considered just the one dimensional case with constant coefficients, but in the present there are allowed higher spatial dimensions and parameters depending on space and time. This last generalization is done to cope with possible abiotic effects as variations in temperature, humidity, wind velocity, carrier capacities, and so on; thus, the results hold for more realistic situations. Moreover, effects of additional control terms are considered.

In the present work it is proved a result on existence and uniqueness of solutions of the problem. Furthermore, it is obtained an estimate of the solution in terms of certain norms of the parameters of the problem.

1 Introduction

The main objective of this work is to perform a rigorous mathematical analysis of a system of nonlinear differential equations corresponding to mathematical model for geographical spreading of dengue disease.

To describe the model, let be $\Omega \subset \mathbb{R}^n$, $n \leq 3$, be an open and bounded set associated to a geographical region where a certain human population is settled and the *Aedes aegypti* mosquito population is spreading; let also denote $0 < T < \infty$ be a given final time of interest and $Q = \Omega \times (0,T)$ the space-time cylinder. Then, the system of equations we are considering is the following:

$$\begin{cases} M_{S,t} - D\Delta M_S + v.\nabla M_S = \gamma A\left(1 - \dfrac{M}{k_1}\right) - \mu_1 M_S - \beta_1 M_S I - h_1 M_S \mathbf{1}_{\omega_1} & \text{in } Q, \\ M_{I,t} - D\Delta M_I + v.\nabla M_I = \mu_1 M_I + \beta_1 M_S I - h_1 M_I \mathbf{1}_{\omega_1} & \text{in } Q, \\ A_t = r\left(1 - \dfrac{A}{k_2}\right) M - \mu_2 A - \gamma A - h_2 A \mathbf{1}_{\omega_2} & \text{in } Q, \\ H_t = \mu_H N - \mu_H H - \beta_2 H M_I & \text{in } Q, \\ I_t = \beta_2 H M_I - \sigma I - \mu_H I & \text{in } Q, \\ R_t = \sigma I - \mu_H R & \text{in } Q, \end{cases} \tag{1}$$

together with the following given boundary conditions:

$$\frac{\partial M_S}{\partial \eta} = \frac{\partial M_I}{\partial \eta} = 0 \quad \text{on } \partial\Omega \times (0,T), \tag{2}$$

where η is the unitary exterior normal to Ω at the boundary, and the following initial conditions

$$M_S(\cdot,0) = M_{S0}(\cdot),\ M_I(\cdot,0) = M_{I0}(\cdot),\ A(\cdot,0) = A_0(\cdot),\ H(\cdot,0) = H_0(\cdot),\ I(\cdot,0) = I_0(\cdot),\ R(\cdot,0) = R_0(\cdot) \quad \text{on } \Omega. \tag{3}$$

In this system the mosquito population is divided 3 subpopulations and represented in the following way: the spatial densities of winged form (mature female mosquitoes) and aquatic form (comprising eggs, larvae and pupae) are denoted by $M(x,t)$ and $A(x,t)$, respectively. In the winged population there are considered the susceptible and infected forms, that are respectively denoted by $M_S(x,t)$ and $M_I(x,t)$, and $M(x,t) = M_S(x,t) + M_I(x,t)$. Concerning the human population, it is divided in the subpopulations: susceptible, infected and removed (or immune); their respective spatial densities are denoted by $H(x,t)$, $I(x,t)$ and $R(x,t)$.

The parameters in the previous equations represent: D is the diffusion coefficient; $v = (v_1, \ldots, v_n)$ is the advection velocity (related to the wind velocity, for instance); $\gamma > 0$ is the per capita rate of maturation from the aquatic form to the winged form; k_1 is the carrying capacity regarding the winged form of the mosquitoes; k_2 is the carrying capacity regarding the aquatic form of the mosquitoes; $\beta_1 \geq 0$ is the transmission coefficient measuring the rate of effective contact between uninfected mosquitos and infected humans; $\beta_2 \geq 0$ is the transmission coefficient measuring the rate of effective contact between uninfected humans and infected mosquitoes; $r \geq 0$ is the intrinsic oviposition rate; $\mu_1 \geq 0$ is the per capita mortality rate of the winged form of the population; $\mu_2 \geq 0$ is the per capita mortality rate of the aquatic population; $\mu_H \geq 0$ is the per capita mortality rate of humans; $\sigma \geq 0$ is the transfer rate of infected humans to the removed class; these previous parameters are assumed to be at least mensurable and bounded functions. The parameters related to control actions, $h_1 \geq 0$ and $h_2 \geq 0$, act on subsets ω_1 and ω_2 of Ω.

2 Main Results

Theorem 2.1. *Assume that previous hypotheses hold. Suppose that $D \geq d_0 > 0$, $D \in C^1(\overline{Q})$, The functions v_i, $1 \leq i \leq n$, γ, $k_1 \geq k_0 > 0$, $k_2 \geq k_0 > 0$, r, μ_1, μ_2, μ_H, β_1, β_2 and σ are $L^\infty(Q)$; $h_1 \in L^{5/2}(\omega_1 \times (0,T))$; $h_2 \in L^1(\omega_1 \times (0,T))$. For the initial data suppose that M_{I0}, $M_{S0} \in H^1(\Omega)$ are nonnegative functions such that $M_{I0} + M_{S0} \leq \|k_1\|_{L^\infty(Q)}$ and A_0, H_0, $I_0 \in L^\infty(\Omega)$ are nonnegative functions satisfying $A_0(\cdot) \leq \|k_2\|_{L^\infty(Q)}$.*

Then, there exists a unique solution $(M_I, M_S, A, H, I) \in W_2^{2,1}(Q) \times W_2^{2,1}(Q) \times L^\infty(Q) \times L^\infty(Q) \times L^\infty(Q)$ of Problem (1)-(3). Moreover, M_I, M_S, A, H and I are nonnegative functions satisfying $M = M_I + M_S \leq \|k_1\|_{L^\infty(Q)}$, $A \leq \|k_2\|_{L^\infty(Q)}$ and

$$\|M_I\|_{W_2^{2,1}(Q)} + \|M_S\|_{W_2^{2,1}(Q)} + \|H\|_{L^\infty(Q)} + \|I\|_{L^\infty(Q)} \leq C,$$

where C is a constant depending only on Ω, T, $\|\mu_1\|_{L^\infty}$, $\|h_1\|_{L^{s_1}(\omega_1 \times (0,T))}$, $\|D\|_{C^1(\overline{Q})}$, $\|v_i\|_{L^\infty(Q)}$, $\|\gamma\|_{L^\infty(Q)}$, k_0, $\|k_1/k_2\|_{L^\infty(Q)}$, $\|\mu_1\|_{L^\infty(Q)}$, $\|\mu_H\|_{L^\infty(Q)}$, $\|\beta_1\|_{L^\infty(Q)}$, $\|\beta_2\|_{L^\infty(Q)}$, $\|M_{I0}\|_{H^1(\Omega)}$, $\|M_{S0}\|_{H^1(\Omega)}$, $\|H_0\|_{L^\infty(\Omega)}$, $\|I_0\|_{L^\infty(\Omega)}$.

Remark 2.1. *The constraints about positivity and upper limitation imposed on the initial conditions are natural biological requirements.*

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A ESTABILIDADE DO MODELO VLASOV-HMF

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Resumo

Vamos apresentar uma equação cinética que é um modelo derivado do sistema de Vlasov-Poisson chamado de "Hamiltonian Mean Field model"(HMF) ou também Vlasov-HMF. Examinaremos a questão da estabilidade orbital de estados estacionários não homogêneos para soluções do sistema Vlasov-HMF por meio de técnicas recentes desenvolvidas para o sistema de Vlasov-Poisson fundamentadas sobre rearranjos simétricos generalizados. Apresentaremos um resultado que garante a estabilidade orbital de estados estacionários que são funções decrescentes da energia microscópica e satisfazem um simples critério. Este resultado é um trabalho conjunto com Florian Méhats e Mohammed Lemou da Université de Rennes 1.

1 Introdução

O modelo HMF (Hamiltonian Mean Field model) é uma variação do caso unidimensional do sistema de Vlasov-Poisson. O sistema HMF é um modelo cinético que descreve partículas se movendo em um círculo unitário com interações de longo alcance via um potencial atrativo cossenoidal. A função de distribuição de partículas $f(t,\theta,v)$ resolve o seguinte problema de valor inicial

$$\begin{aligned}&\partial_t f + v\partial_\theta f - \partial_\theta \phi_f \partial_v f = 0, \qquad (t,\theta,v)\in\mathbb{R}_+\times\mathbb{T}\times\mathbb{R}, \\ &f(0,\theta,v) = f_{\text{init}}(\theta,v)\geq 0,\end{aligned} \tag{1}$$

onde $\mathbb{T}$ é o toro plano $[0,2\pi]$ e o potencial ϕ_f associado a função de distribuição f é definido por

$$\phi_f(\theta) = -\int_0^{2\pi}\rho_f(\theta')\cos(\theta-\theta')d\theta', \qquad \rho_f(\theta)=\int_{\mathbb{R}} f(\theta,v)dv. \tag{2}$$

Podemos ainda definir ϕ_f em termos da magnetização por partícula que é um vetor bidimensional definido por

$$M_f = \int_0^{2\pi}\rho_f(\theta)u(\theta)d\theta, \qquad \text{with} \quad u(\theta)=(\cos\theta,\sin\theta)^T \tag{3}$$

e nós temos que

$$\phi_f(\theta) = -M_f\cdot u(\theta). \tag{4}$$

Assim como acontece com o sistema de Vlasov-Poisson, o modelo HMF tem a propriendade de preservar várias quantidades como o Hamiltoniano, os funcionais de Casimir e o momento total.

No trabalho desenvolvido até o momento consideramos soluções estacionárias da forma

$$f_0(\theta,v) = F(e_0(\theta,v)), \qquad \text{com} \quad e_0(\theta,v)=\frac{v^2}{2}+\phi_0(\theta), \tag{5}$$

assim como em [1]). Temos que e_0 é a energia microscópica e consideramos o potencial associado à f_0 de acordo com (2) na forma

$$\phi_0(\theta) = -m_0\cos\theta, \qquad \text{with } m_0\geq 0.$$

Existem várias hipóteses técnicas sobre a função F. Vejamos:

Hipótese 1 *A função F é uma função $\mathcal{C}^0$ em $\mathbb{R}$ satisfazendo as seguintes propriedades: Ela é $\mathcal{C}^1$ em $(-\infty, e_*)$, para algum $e_* \in \mathbb{R} \cup \{+\infty\}$, com $F' < 0$ neste intervalo. Também assumimos que $F(e) = 0$ para $e \geq e_*$ quando e_* é finito, e que $\lim_{e \to +\infty} F(e) = 0$ se $e_* = +\infty$. Sua inversa F^{-1} é uma função $\mathcal{C}^1$ definida de $(0, \sup F)$ em $(-\infty, e^*)$. Supomos que a função f_0 dada por (5) pertence ao espaço de energia $L^1((1+|v|^2)d\theta dv)$. Além disso, no caso $e_* < +\infty$ e $m_0 = e_*$, assumimos que $\int_{-m_0}^{m_0} \log(m_0 - e)F'(e)de < +\infty$.*

2 Resultados Principais

Nosso principal resultado em [2] foi prova a da estabilidade orbital sob o seguinte critério:

Critério 1 [Critério de estabilidade não-linear] *Assumiremos que f_0 satisfaz o seguinte critério*

$$\kappa_0 < 1 \quad com \quad \kappa_0 = \int_0^{2\pi} \int_{-\infty}^{+\infty} |F'(e_0(\theta, v))| \left(\frac{\int_{\mathcal{D}} (\cos\theta - \cos\theta')(e_0(\theta, v) - \phi_0(\theta'))^{-1/2} d\theta'}{\int_{\mathcal{D}} (e_0(\theta, v) - \phi_0(\theta'))^{-1/2} d\theta'} \right)^2 d\theta dv, \tag{1}$$

onde $\mathcal{D} = \{\theta' \in \mathbb{T} : \ \phi_0(\theta') < e_0(\theta, v)\}$.

O resultado está no seguinte teorema:

Teorema 2.1. *Seja f_0 um estado estacionário da forma (5) satisfazendo a hipótese 1 e o critério 1. Existe $\delta > 0$ tal que, para todo $f \in L^1\left((1+|v|^2)d\theta dv\right)$ satisfazendo $|M_f - M_{f_0(\cdot - \theta_f)}| < \delta$, nós temos*

$$\|f - f_0(\cdot - \theta_f)\|_{L^1}^2 \leq C \Bigg(\mathcal{H}(f) - \mathcal{H}(f_0) + C(1 + \|f\|_{L^1})\|f^* - f_0^*\|_{L^1}$$
$$+ C \int_0^{+\infty} s^2 \left(f_0^\sharp(s) - f^\sharp(s) \right)_+ ds + C \int_0^{+\infty} \mu_{f_0}(s)^2 \beta_{f^*, f_0^*}(s) ds \Bigg), \tag{2}$$

onde $\beta_{f^*, f_0^*}(s) = |\{(\theta, v) \in \mathbb{T} \times \mathbb{R} : f^*(\theta, v) \leq s < f_0^*(\theta, v)\}|$, para todo $s \geq 0$, e onde C é uma constante positiva dependendo somente de f_0. O parâmetro θ_f é definido por $M_f = |M_f|(\cos\theta_f, \sin\theta_f)^T$. Em particular, se f_0 é um estado estacionário de suporte compacto, então (2) reduz-se a

$$\|f - f_0(\cdot - \theta_f)\|_{L^1}^2 \leq C \left(\mathcal{H}(f) - \mathcal{H}(f_0) + C(1 + \|f\|_{L^1})\|f^* - f_0^*\|_{L^1} \right). \tag{3}$$

Deste teorema deduzimos a estabilidade orbital de f_0. A prova deste teorema pode ser resumida nas seguintes etapas: primeiramente introduzimos um funcional de energia reduzido que depende somente do vetor de magnetização usando os rearranjos generalizados; depois mostramos que se $\kappa_0 < 1$ então a magnetização do estado estacionário m_0 é um mínimo local do funcional de energia reduzido. Deste modo, através do desenvolvimento de Taylor deste funcional na vizinhança de m_0, controlamos o vetor de magnetização pelo hamiltoniano relativo e pelo rearranjo relativo. Por fim conseguimos controlar $f - f_0$ (que aparece na expressão do hamiltoniano relativo) usando uma desigualdade funcional.

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HÖLDER'S INEQUALITY FOR MIXED SUMS AND APPLICATIONS

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Abstract

Hölder's inequality for mixed norm spaces is used to obtain new results, approaches and applications: the Bohnenblust-Hille and Hardy-Littlewood multilinear inequalities and also summability of multilinear operators.

1 Introduction

A. Benedek and R. Panzone introduced the mixed L_p spaces notion on [1]: given σ-finite measurable spaces (X_i, Σ_i, μ_i), $i = 1, \ldots, m$, let $(\mathbf{X}, \boldsymbol{\Sigma}, \boldsymbol{\mu}) := \left(\prod_{i=1}^m X_i, \prod_{i=1}^m \Sigma_i, \prod_{i=1}^m \mu_i\right)$ be the product space and $\mathbf{p} := (p_1, \ldots, p_m) \in [1, \infty]^m$. The space $L_{\mathbf{p}}(\mathbf{X})$ consists in all measurable functions $f : \mathbf{X} \to \mathbb{K}$, for $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, with the following property: for any $(x_1, \ldots, x_{m-1}) \in \prod_{i=1}^{n-1} X_i$, $f(x_1, \ldots, x_{m-1}, \cdot) \in L_{p_m}(X_m)$, i.e., $\|f\|_{p_m} := \|f(x_1, \ldots, x_{m-1}, \cdot)\|_{p_m} < \infty$, also $\|f\|_{p_m}$, results in a measurable function. This process is repeated successively: the resulting p_{m-1}-norm, p_{m-2}-norm,…, p_1-norm (in this order) are finite. There are several classical properties and deep results concerning the $L_{\mathbf{p}}$ spaces, among others, $L_{\mathbf{p}}(\mathbf{X})$ is a Banach space (as expected), the correspondent of the Monotone and Lebesgue's dominated convergence theorems.

We are interested in a particular environment: by considering $X_i = \mathbb{N}$, $i = 1, \ldots, n$ endowed with the counting measure, we recover the mixed norm sequence space $\ell_{\mathbf{p}} := L_{\mathbf{p}}(\mathbb{N}^m)$, which gathers all multi-index scalars valued matrices $\mathbf{a} := (a_{\mathbf{i}})_{\mathbf{i} \in \mathbb{N}^m}$ with finite $\mathbf{p}$-norm. For two multi-indexes scalar matrices $\mathbf{a} := (\mathrm{a}_{\mathbf{i}})_{\mathbf{i} \in \mathbb{N}^m}$ and $\mathbf{b} := (\mathrm{b}_{\mathbf{i}})_{\mathbf{i} \in \mathbb{N}^m}$ we consider the coordinate product, that is, $\mathbf{a} \cdot \mathbf{b} := (\mathrm{a}_{\mathbf{i}} \mathrm{b}_{\mathbf{i}})_{\mathbf{i} \in \mathbb{N}^m}$. The Hölder's inequality for mixed sums we need is stated as follows.

Theorem 1.1 (Hölder's inequality for mixed $\ell_{\mathbf{p}}$ spaces)**.** *Let m, n, N be positive integers, $\mathbf{r}, \mathbf{p}(1), \ldots, \mathbf{p}(N) \in (0, \infty]^m$ be such that*

$$\frac{1}{r_j} = \frac{1}{p_j(1)} + \cdots + \frac{1}{p_j(N)}, \quad \text{for } j = 1, \ldots, m,$$

and also let $\mathbf{a}_k := \left(\mathrm{a}_{\mathbf{i}}^k\right)_{\mathbf{i} \in \mathcal{M}(m,n)}$, $k = 1, \ldots, N$ be scalar matrices. Then,

$$\|\mathbf{a}_1 \cdots \mathbf{a}_N\|_{\mathbf{r}} \leq \|\mathbf{a}_1\|_{\mathbf{p}(1)} \cdots \|\mathbf{a}_N\|_{\mathbf{p}(N)}.$$

Hölder's inequality for mixed norms (although well-known in PDEs) was essentially overlooked in Functional and Complex Analysis and has had a crucial (and in some sense unexpected) influence in very recent advances in [2, 3, 4, 5]. Some of these results are related to the Bohnenblust-Hille and Hardy-Littlewood multilinear inequalities and its consequences and also with the summability of multilinear operators. We present some of these results here.

2 Main Results

For the sake of clarity we fix the following notation: we define $X_p := \ell_p$, for $1 \leq p < \infty$, and $X_\infty := c_0$. For $\mathbf{p} := (p_1, \ldots, p_m) \in [1, \infty]^m$ let $\left|\frac{1}{\mathbf{p}}\right| := \frac{1}{p_1} + \cdots + \frac{1}{p_m}$. For $\rho, r_1, \ldots, r_m \in (0, \infty)$, $M_<^\rho := \{j \in \{1, \ldots, m\} : r_j < \rho\}$, $M_\geq^\rho := \{1, \ldots, m\} \setminus M_<^\rho$. We shall denote by $|M_<^\rho|$ the cardinality of $M_<^\rho$. A particular case of ρ will be useful: $M_<^{\mathrm{HL}} := M_<^{2m/(m+1-2\left|\frac{1}{\mathbf{p}}\right|)}$.

Theorem 2.1 (Generalized Bohnenblust-Hille / Hardy–Littlewood inequality)**.** *Let $m \geq 2$ be a positive integer, $\mathbf{p} := (p_1, \ldots, p_m) \in [1, \infty]^m$ such that $\left|\frac{1}{\mathbf{p}}\right| \leq \frac{1}{2}$ and $\mathbf{s} := (s_1, \ldots, s_m) \in \left[\left(1 - \left|\frac{1}{\mathbf{p}}\right|\right)^{-1}, 2\right]^m$. The following assertions are equivalent:*
(a) There exists $D^{\mathbb{K}}_{m,\mathbf{s},\mathbf{p}} \geq 1$ satisfying,

$$\left(\sum_{i_1=1}^{\infty}\left(\cdots\left(\sum_{i_m=1}^{n}\left|A\left(e_{i_1}, \ldots, e_{i_m}\right)\right|^{s_m}\right)^{\frac{s_{m-1}}{s_m}}\cdots\right)^{\frac{s_1}{s_2}}\right)^{\frac{1}{s_1}} \leq D^{\mathbb{K}}_{m,\mathbf{s},\mathbf{p}}\|A\|.$$

for every continuous m-linear map $A : X_{p_1} \times \cdots \times X_{p_m} \to \mathbb{K}$.
(b) $\dfrac{1}{s_1} + \cdots + \dfrac{1}{s_m} \leq \dfrac{m+1}{2} - \left|\dfrac{1}{\mathbf{p}}\right|$.

The next result deals with the case $\frac{1}{s_1} + \cdots + \frac{1}{s_m} > \frac{m+1}{2} - \left|\frac{1}{\mathbf{p}}\right|$.

Theorem 2.2. *Let $m \geq 2$ be an integer, $\mathbf{p} := (p_1, \ldots, p_m) \in [2, \infty]^m$ and $\mathbf{r} := (r_1, \ldots, r_m) \in (0, \infty)^m$. Then, for all positive integer n and for all bounded m-linear forms $T : \ell^n_{p_1} \times \cdots \times \ell^n_{p_m} \to \mathbb{K}$ there exists a constant $D^{\mathbb{K}}_{m,\mathbf{r},\mathbf{p}} \geq 1$ (not depending on n) such that*

$$\left(\sum_{i_1=1}^{n}\left(\cdots\left(\sum_{i_m=1}^{n}\left|T(e_{i_1}, \ldots, e_{i_m})\right|^{r_m}\right)^{\frac{r_{m-1}}{r_m}}\cdots\right)^{\frac{r_1}{r_2}}\right)^{\frac{1}{r_1}} \leq D^{\mathbb{K}}_{m,\mathbf{r},\mathbf{p}} n^s \|T\|,$$

with the exponent s summarized as follows:

$$s = \begin{cases} \displaystyle\sum_{j \in M^2_<} \frac{1}{r_j} + \left|\frac{1}{\mathbf{p}}\right| - \frac{1}{2}\left(|M^2_<| + 1\right), & \text{if } \mathbf{p} \in [2, 2m]^m \\ \displaystyle\sum_{j \in M^{HL}_<} \frac{1}{r_j} - \frac{m + 1 - 2\left|\frac{1}{\mathbf{p}}\right|}{2m} \cdot |M^{HL}_<|, & \text{if } \mathbf{p} \in [2m, \infty]^m. \end{cases}$$

Moreover, the exponent is optimal in the following cases: (i) $M^2_< = \{1, \ldots, m\}$ when $\mathbf{p} \in [2, 2m]^m$; (ii) $M^{HL}_< = \{1, \ldots, m\}$ or $M^{HL}_< = \emptyset$ when $\mathbf{p} \in [2m, \infty]^m$.

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ON THE OPTIMAL CONSTANTS OF THE BOHNENBLUST–HILLE AND HARDY–LITTLEWOOD INEQUALITIES

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Abstract

For $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$ and $m \geq 2$, among other results, we show that for $p > 2m(m-1)^2$ the optimal constants satisfying the Hardy–Littlewood inequality for m-linear forms in ℓ_p spaces are dominated by the best known constants of the corresponding Bohnenblust–Hille inequality. For instance, we show that if $p > 2m(m-1)^2$, then

$$C_{m,p}^{\mathbb{C}} \leq \prod_{j=2}^{m} \Gamma\left(2 - \frac{1}{j}\right)^{\frac{j}{2-2j}} < m^{\frac{1-\gamma}{2}},$$

where $C_{m,p}^{\mathbb{C}}$ is the optimal constant of the Hardy–Littlewood inequality and γ is the Euler–Mascheroni constant.

1 Introduction

Let $\mathbb{K}$ be $\mathbb{R}$ or $\mathbb{C}$ and $m \geq 2$ be a positive integer. In 1931, F. Bohnenblust and E. Hille [3] proved that there exists a constant $B_m^{\mathbb{K}} \geq 1$ such that for all m–linear forms $T : \ell_\infty^n \times \cdots \times \ell_\infty^n \to \mathbb{K}$, and all positive integers n, we have

$$\left(\sum_{j_1,\ldots,j_m=1}^{n} \left| T(e_{j_1}, \ldots, e_{j_m}) \right|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq B_m^{\mathbb{K}} \left\| T \right\|. \tag{1}$$

In an easier presentation, the best known estimates for the constants in (1), which were recently presented in [3], are

$$\begin{array}{lll} B_m^{\mathbb{C}} & < & m^{\frac{1-\gamma}{2}} < m^{0.21139}, \\ B_m^{\mathbb{R}} & < & 1.3 \cdot m^{\frac{2-\log 2-\gamma}{2}} < 1.3 \cdot m^{0.36482}, \end{array} \tag{2}$$

where γ denotes the Euler–Mascheroni constant. From (2) we can deduce that the growth of the constants $B_m^{\mathbb{K}}$ is sublinear (see [3]). The Hardy–Littlewood inequality [6, 8] is a natural extension of the Bohnenblust–Hille inequality and asserts that for $2m \leq p \leq \infty$ there exists a constant $C_{m,p}^{\mathbb{K}} \geq 1$ such that, for all m–linear forms $T : \ell_p^n \times \cdots \times \ell_p^n \to \mathbb{K}$, and all positive integers n,

$$\left(\sum_{j_1,\ldots,j_m=1}^{n} \left| T(e_{j_1}, \ldots, e_{j_m}) \right|^{\frac{2mp}{mp+p-2m}} \right)^{\frac{mp+p-2m}{2mp}} \leq C_{m,p}^{\mathbb{K}} \left\| T \right\|,$$

and the exponent $\frac{2mp}{mp+p-2m}$ is optimal. This inequality was proved by G. Hardy and J.E. Littlewood [6] for $m = 2$ and later extended to $m > 2$ by Praciano-Pereira [8].

The achievement of the optimal values of $B_m^{\mathbb{K}}$ and/or $C_{m,p}^{\mathbb{K}}$ is a quite challenging problem (and also important for applications) in Mathematical Analysis and seems to be far from a definitive answer. For complex scalars, having good estimates for the polynomial version of the Bohnenblust–Hille inequality is crucial in applications in Complex Analysis and Analytic Number Theory (we mention, for instance, the striking paper [5]); for real scalars, the estimates of $B_m^{\mathbb{R}}$ are important in Quantum Information Theory (see [7]).

The original estimates for $C^{\mathbb{K}}_{m,p}$ were of the form $C^{\mathbb{K}}_{m,p} \leq \left(\sqrt{2}\right)^{m-1}$, and nowadays the best known estimates for the constants $C^{\mathbb{K}}_{m,p}$ can be found in [2]. For $p > 2m(m-1)^2$, these best known estimates are

$$\begin{aligned} C^{\mathbb{C}}_{m,p} &\leq \left(\tfrac{2}{\sqrt{\pi}}\right)^{\frac{2m(m-1)}{p}} \left(m^{\frac{1-\gamma}{2}}\right)^{\frac{p-2m}{p}}, \\ C^{\mathbb{R}}_{m,p} &\leq \left(\sqrt{2}\right)^{\frac{2m(m-1)}{p}} \left(1.3 \cdot m^{\frac{2-\log 2-\gamma}{2}}\right)^{\frac{p-2m}{p}}. \end{aligned} \tag{3}$$

Note that the presence of the parameter p in the formulas of (3), if compared to the original estimates, catches more subtle information: now the estimates become "better" as p grows. As p tends to infinity we note that the above estimates tend to the best known estimates for $B^{\mathbb{K}}_{m}$. In this work, among other results, we show that for $p > 2m(m-1)^2$ the constant $C^{\mathbb{K}}_{m,p}$ has the exactly same upper bounds that we have now for the Bohnenblust–Hille constants (2).

2 Main Results

Our main result shows that for $p > 2m(m-1)^2$ the optimal constants satisfying the Hardy–Littlewood inequality for m-linear forms in ℓ_p spaces are dominated by the best known estimates for the constants of the m-linear Bohnenblust–Hille inequality; this result improves the most recent estimates we have thus far, and may suggest a more subtle connection between the optimal constants of these inequalities.

Theorem 2.1 ([1])**.** *Let $m \geq 2$ be a positive integer and $2m(m-1)^2 < p \leq \infty$. Then,*

$$\begin{aligned} C^{\mathbb{C}}_{m,p} &< m^{\frac{1-\gamma}{2}} < m^{0.21139}, \\ C^{\mathbb{R}}_{m,p} &< 1.3 \cdot m^{\frac{2-\log 2-\gamma}{2}} < 1.3 \cdot m^{0.36482}, \end{aligned} \tag{1}$$

It is not difficult to verify that (1) in fact improves (3). These new estimates are precisely the best known estimates for the constants of the Bohnenblust–Hille inequality. It is important to say that to prove these new estimates we also improve the best known estimates for the generalized Bohnenblust–Hille inequality.

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ORTHOGONAL EXPANSIONS RELATED TO COMPACT GELFAND PAIRS

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Abstract

We generalize a recent result of Berg and Porcu to compact Gelfand pairs.

1 Introduction

Let $\mathbb{S}^d$ denote the unit sphere of $\mathbb{R}^{d+1}$ and let L denote an arbitrary locally compact group with neutral element e_L. The set of continuous positive definite functions $f : L \to \mathbb{C}$ is denoted $\mathcal{P}(L)$. In a recent paper [1] Berg and Porcu consider the product space $\mathbb{S}^d \times L$ and define the class $\mathcal{P}(\mathbb{S}^d, L)$ of continuous functions $f : [-1, 1] \times L \to \mathbb{C}$ such that $f(\xi \cdot \eta, u^{-1}v)$ is a positive definite kernel on $\mathbb{S}^d \times \mathbb{S}^d \times L \times L$. They prove that this class of functions is exactly the functions with a (uniformly convergent) orthogonal expansion

$$f(x, u) = \sum_{n=0}^{\infty} \varphi_{n,d}(u) c_n(d, x), \quad (x, u) \in [-1, 1] \times L,$$

where $(\varphi_{n,d})_{n\geq 0}$ is a sequence in $\mathcal{P}(L)$ such that $\sum_n \varphi_{n,d}(e_L) < \infty$ and

$$c_n(d, x) = C_n^{((d-1)/2)}(x)/C_n^{((d-1)/2)}(1)$$

are the Gegenbauer polynomials normalized to 1 at 1. These functions are the spherical functions for the compact Gelfand pair $(O(d + 1), O(d))$ as defined below and $\mathbb{S}^d$ is isomorphic to the homogeneous space $O(d + 1)/O(d)$.

When L is the trivial group of one element, the theorem above becomes a classical Theorem of Schoenberg, see [3].

The case $L = \mathbb{R}$ is of special importance for probability theory and stochastic processes, because it characterizes completely the class of space-time covariance functions, where the space is the sphere.

We indicate how this result can be generalized by replacing $\mathbb{S}^d$ by the homogeneous space of an arbitrary compact Gelfand pair.

2 Main Results

Let G be a locally compact group. For a compact subgroup K of G we call a function $\varphi : G \to \mathbb{C}$ bi-invariant under K if

$$\varphi(kul) = \varphi(u), \quad u \in G, \ k, l \in K. \tag{1}$$

For a set A of functions on G we denote by $A^\sharp$ the set of bi-invariant functions from A. In particular $C_c^\sharp(G)$ denotes the set of continuous complex-valued functions on G with compact support and bi-invariant under K. It is easy to see that for $f, g \in C_c^\sharp(G)$ the convolution

$$f * g(x) = \int_G f(y)g(y^{-1}x)\, d\omega_G(y), \quad x \in G, \tag{2}$$

where ω_G denotes a left Haar measure on G, is again a bi-invariant function on G, and $C_c^\sharp(G)$ becomes a subalgebra of the group algebra $L^1(G)$.

We say that (G,K) is a **Gelfand pair** if $C_c^\sharp(G)$ is commutative, cf. [4, p. 75].

A spherical function for (G,K) is a continuous function $\varphi : G \to \mathbb{C}$ satisfying

$$\int_K \varphi(xky)\, d\omega_K(k) = \varphi(x)\varphi(y), \quad x,y \in G; \quad \varphi(e_G) = 1, \tag{3}$$

where ω_K is Haar measure on K normalized to $\omega_K(K) = 1$. A spherical function is necessarily bi-invariant.

The dual space of the Gelfand pair (G,K) is the set Z of positive definite spherical functions. It is a locally compact space in the topology inherited from $C(G)$, which carries the topology of uniform convergence on compact subsets of G, cf. [4, p.83].

A Gelfand pair (G,K) is called compact if G is compact. In this case the dual space Z is discrete and an orthogonal system in $L^2(G)$. It is countable if G in addition is metrizable.

For a compact Gelfand pair (G,K) and a locally compact group L we denote by $\mathcal{P}^\sharp(G/K, L)$ the set of functions in $\mathcal{P}(G \times L)$ which are bi-invariant under K in the first variable.

Theorem 2.1. *Let (G,K) be a compact Gelfand pair, L a locally compact group and $f : G \times L \to \mathbb{C}$ a continuous function. Then f belongs to $\mathcal{P}^\sharp(G/K, L)$ if and only if there exists a function $B : Z \to \mathcal{P}(L)$ satisfying $\sum_{\varphi \in Z} B(\varphi)(e_L) < \infty$ such that*

$$f(x,u) = \sum_{\varphi \in Z} B(\varphi)(u)\varphi(x), \quad (x,u) \in G \times L. \tag{4}$$

The expansion (4) is uniformly convergent for $(x,u) \in G \times L$, and we have

$$B(\varphi)(u) = \int_G f(x,u)\overline{\varphi(x)}\, d\omega_G(x), \quad u \in L. \tag{5}$$

Let $O(n)$ denote the compact group of $n \times n$ real orthogonal matrices. Then $(O(d+1), O(d))$ is a compact Gelfand pair, the homogeneous space $O(d+1)/O(d)$ is isomorphic with $\mathbb{S}^d$, and the bi-invariant functions can be identified with functions on $[-1,1]$.

Let $U(q)$ denote the compact group of unitary $q \times q$ complex matrices, $q \geq 2$. Then $(U(q), U(q-1))$ is a compact Gelfand pair, the homogeneous space $U(q)/U(q-1)$ is isomorphic with Ω_{2q}–the complex unit sphere in $\mathbb{C}^q$–and the bi-invariant functions can be identified with functions on the unit disc $\{z \in \mathbb{C} : |z| \leq 1\}$, see [2].

In both cases the spherical functions are well-known.

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OPERATOR IDEALS RELATED TO ABSOLUTELY SUMMING AND COHEN STRONGLY SUMMING OPERATORS

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Abstract

We study the ideals of linear operators between Banach spaces determined by the transformation of vector-valued sequences involving the new sequence space introduced by Karn and Sinha [5] and the classical spaces of absolutely, weakly and Cohen strongly summable sequences.

1 Introduction

For a Banach space E, let $\ell_p(E)$, $\ell_p^w(E)$ and $\ell_p\langle E\rangle$ denote the spaces of absolutely, weakly and Cohen strongly p-summable E-valued sequences, respectively. Karn and Sinha [5] recently introduced a space $\ell_p^{mid}(E)$ of E-valued sequences such that

$$\ell_p\langle E\rangle \subseteq \ell_p(E) \subseteq \ell_p^{mid}(E) \subseteq \ell_p^w(E). \tag{1}$$

In the realm of the theory of operator ideals, it is a natural step to study the classes of operators $T\colon E \longrightarrow F$ that send: (i) sequences in $\ell_p^w(E)$ to sequences in $\ell_p^{mid}(F)$, (ii) sequences in $\ell_p^{mid}(E)$ to sequences in $\ell_p(F)$, (iii) sequences in $\ell_p^{mid}(E)$ to sequences in $\ell_p\langle F\rangle$. This is the basic motivation of the our work [2], which is the subject of this communication.

The letters E, F shall denote Banach spaces over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$. The symbol $E \overset{1}{\hookrightarrow} F$ means that E is a linear subspace of F and $\|x\|_F \leq \|x\|_E$ for every $x \in E$. By $\mathcal{L}(E;F)$ we denote the Banach space of all continuous linear operators $T\colon E \longrightarrow F$ endowed with the usual sup norm. By $\Pi_{p;q}$ we denote the ideal of absolutely $(p;q)$-summing linear operators [1]. If $p = q$ we simply write Π_p. The ideal of Cohen strongly p-summing linear operators [3] shall be denoted by $\mathcal{D}_p$. We use the standard notation of the theory of operator ideals [2].

Due to the nature of this short communication, the proofs of all presented results will be omitted.

2 Main Results

Instead of the original definition of the space $\ell_p^{mid}(E)$ given in [5], here we call *the space of mid p-summable E-valued sequences*, we shall use a characterization proved in [5, Lemma 2.3, Proposition 2.4]:

Definition 2.1. A sequence $(x_j)_{j=1}^\infty$ in a Banach space E is said to be *mid p-summable*, $1 \leq p < \infty$, if $\left((x_n^*(x_j))_{j=1}^\infty\right)_{n=1}^\infty \in \ell_p(\ell_p)$ whenever $(x_n^*)_{n=1}^\infty \in \ell_p^w(E^*)$.

We prove that the expression $\|(x_j)_{j=1}^\infty\|_{mid,p} := \sup\limits_{(x_n^*)_{n=1}^\infty \in B_{\ell_p^w(E^*)}} \left(\sum\limits_{n=1}^\infty \sum\limits_{j=1}^\infty |x_n^*(x_j)|^p\right)^{1/p}$ defines a norm that makes $\ell_p^{mid}(E)$ a Banach space and $\ell_p(E) \overset{1}{\hookrightarrow} \ell_p^{mid}(E) \overset{1}{\hookrightarrow} \ell_p^w(E)$. It is worth noting that the original paper does not provide a norm for this space. However it proves that, if E is a Banach space and $1 \leq p < \infty$, then:

(i) $\ell_p^{mid}(E) = \ell_p^w(E)$ if and only if $\Pi_p(E;\ell_p) = \mathcal{L}(E;\ell_p)$.
(ii) $\ell_p^{mid}(E) = \ell_p(E)$ if and only if E is a subspace of $L_p(\mu)$, for some Borel measure μ.

Taking a closer look at the space $\ell_p^{mid}(E)$, we establish more of its distinguished features:

(iii) $\ell_p^{mid}(E)$ and $\ell_p^u(E)$ (unconditionally p-summable sequences) are incomparable (by inclusion) in general.
(iv) $\ell_p^{mid}(E)$ is finitely determined and linearly stable (cf. [1], Definitions 1.1 and 2.2)

Following the classical line of studying operators that improve the summability of sequences, we investigate the obvious classes of operators, involving mid p-summable sequences, determined by the chain (1). From now on, $1 \le q \le p < \infty$ are real numbers and $T \in \mathcal{L}(E;F)$ is a continuous linear operator.

Definition 2.2. An operator T is said to be:
(i) *Absolutely mid $(p;q)$-summing* if $(T(x_j))_{j=1}^{\infty} \in \ell_p(F)$ whenever $(x_j)_{j=1}^{\infty} \in \ell_q^{mid}(E)$.
(ii) *Weakly mid $(p;q)$-summing* if $(T(x_j))_{j=1}^{\infty} \in \ell_p^{mid}(F)$ whenever $(x_j)_{j=1}^{\infty} \in \ell_q^w(E)$.
(iii) *Cohen mid p-summing* if $(T(x_j))_{j=1}^{\infty} \in \ell_p\langle F\rangle$ whenever $(x_j)_{j=1}^{\infty} \in \ell_p^{mid}(E)$.

The spaces formed by the operators above shall be denoted by $\Pi_{p;q}^{mid}(E;F), W_{p;q}^{mid}(E;F)$ and $\mathcal{D}_p^{mid}(E;F)$, respectively. When $p=q$ we simply write mid p-summing instead mid $(p;p)$-summing and use the symbols Π_p^{mid} and W_p^{mid}.

Observing the theory of operator ideals, we show the following two results:

Theorem 2.1. *The classes $\left(\Pi_{p;q}^{mid}, \|\cdot\|_{\Pi_{p;q}^{mid}}\right)$, $\left(W_{p;q}^{mid}, \|\cdot\|_{W_{p;q}^{mid}}\right)$ and $\left(\mathcal{D}_p^{mid}, \|\cdot\|_{\mathcal{D}_p^{mid}}\right)$ are Banach operator ideals.*

Proposition 2.1. *The operator ideal $\Pi_{p;q}^{mid}$ is injective and the operator ideals $W_{p;q}^{mid}$ and $\mathcal{D}_p^{mid}$ are regular.*

It is clear from the definitions that $\Pi_{p,r}^{mid} \circ W_{r,q}^{mid} \subseteq \Pi_{p,q}$ for $q \le r \le p$. More than that we show that the equality holds if $p=q$, what gives a new factorization theorem for absolutely p-summing operators:

Theorem 2.2. *Every absolutely p-summing linear operator factors through absolutely and weakly mid p-summing linear operators, that is, $\Pi_p = \Pi_p^{mid} \circ W_p^{mid}$.*

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EXISTÊNCIA E APROXIMAÇÃO DE SOLUÇÕES DE INCLUSÇÕES DINÂMICAS EM ESCALAS TEMPORAIS

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Resumo

No presente trabalho, nós extendemos um resultado de existência e aproximação de soluções para inclusões diferenciais a inclusões dinâmicas em escalas temporais.

1 Introdução

Resultados de existência de soluções para inclusões dinâmicas em escalas temporais podem ser encontrados, por exemplo, em [1, 3, 4, 2, 6].

Nós estabelecemos um resultado de existência e aproximação de soluções para inclusões dinâmicas em escalas temporais. Assim, nós obtemos uma generalização do resultado [[5], 3.1.6 Theorem].

2 Escalas Temporais

Abaixo consideramos conceitos e resultados básicos da teoria de escalas temporais.

Uma escala temporal $\mathbb{T} \subset \mathbb{R}$ é um conjunto não-vazio e fechado.

Se $\mathbb{T}$ é uma escala temporal, define-se:

(i) a função $\mu : \mathbb{T} \to [0, +\infty)$ é dada por

$$\mu(t) = \sigma(t) - t \ .$$

(ii) se $\sup \mathbb{T} < +\infty$ tem-se

$$\mathbb{T}^\kappa = \mathbb{T} \setminus (\rho(\sup \mathbb{T}), \sup \mathbb{T}]_\mathbb{T}$$

e se $\sup \mathbb{T} = +\infty$ então $\mathbb{T}^\kappa = \mathbb{T}$.

Seja $\mathbb{T}$ uma escala temporal. Define-se a função $\sigma : \mathbb{T} \to \mathbb{T}$ como

$$\sigma(t) = \inf\{s \in \mathbb{T}T : s > t\}$$

e a função $\rho : \mathbb{T} \to \mathbb{T}$ como

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\}.$$

Estamos supondo que $\inf \emptyset = \sup \mathbb{T}$ e $\sup \emptyset = \inf \mathbb{T}$.

Se $\mathbb{T}$ é uma escala temporal, considere uma função $f : \mathbb{T} \to \mathbb{R}$ e $t \in \mathbb{T}^\kappa$. Se existe $\xi \in \mathbb{R}$ tal que, para todo $\varepsilon > 0$ existe $\delta > 0$ de modo que

$$| \ f(\sigma(t)) - f(s) - \xi(\sigma(t) - s) \ | \leq \ \varepsilon \ | \ \sigma(t) - s \ |$$

para todo $s \in (t - \delta, t + \delta)_\mathbb{T}$, diz-se que ξ é a derivada delta de f em t e denota-se $\xi = f^\Delta(t)$.

3 Resultado Principal

Teorema 3.1. *Seja $F : \mathbb{T} \times \mathbb{R}^n \rightsquigarrow \mathbb{R}^n$ uma multifunção não-vazia e fechada. Suponha que F é $\Delta \times \mathcal{B}$-mensurável e mensuravelmente Lipschitz de posto k.*

Considere constantes $K := \exp\left(\int_{[a,b)_\mathbb{T}} k(\tau)\Delta\tau\right)$ e $\varepsilon > 0$. Se um arco x satisfaz

$$\rho_F(x) = \int_{[a,b)_\mathbb{T}} \rho(s, x(s), x^\Delta(s))\Delta s < \frac{\varepsilon}{K}$$

então existe uma trajetória y de F tal que

$$y(a) = x(a), \qquad \|y - x\|_\infty \leq \int_{[a,b)_\mathbb{T}} \|y^\Delta(s) - x^\Delta(s)\|\Delta s \leq K\rho_F(x)$$

sendo

$$\rho(t, u, v) = \inf\{\|v - w\| : w \in F(t, u)\}.$$

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APLICAÇÕES DE UMA EQUAÇÃO ABSTRATA DEGENERADA

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Abstract

Este artigo está direcionado a mostrar as aplicações do problema abstrato degenerado do tipo

$$Bu''(x,t) + M(t)Au(t) = 0 \quad \text{em} \quad Q, \tag{1}$$

com condições iniciais e de fronteira

$$u(x,0) = u^0(x), \quad u'(x,0) = u^1(x) \quad \text{em} \quad \Omega,$$
$$u(x,t) = 0 \quad \text{sobre} \quad \Gamma \times]0,\infty[,$$

onde u é o deslocamento, $A : V \to V'$ denota o operador definido por $\langle Au, v\rangle = a(u,v),\ \forall u, v \in V$ e $B : H \to H$ é um operador linear, simétrico não negativo, estritamente convexo, M é uma função real derivável, estrictamente positiva. Os espaços de Hilbert $\{H, (.)\}$ e $\{V, ((.))\}$ verificam a imersão densa e compacta $V \hookrightarrow H$.

1 Introdução

Sejam $(H, (u,v))$ e $(V, a(u,v))$ espaços de Hilbert reais com imersão densa e compacta de V em H. Consideramos os operadores lineares $A, B : V \to V'$ com

$$\langle Au, v\rangle = a(u,v),\ \forall u, v \in V$$

$$\langle Bv, v\rangle \geq 0,\ \forall v \in V,\ v \neq 0;$$

$$S : H \to H, \text{un operador lineal, simétrico y estrictamente positivo. } (Su, u) > 0,\ u \neq 0$$

Denotamos $c(u,v) = (Su, v)$, então c é uma forma bilinear, simétrica e positiva.
Contruímos $\widehat{H} = (H, c(u,v))$ e C o operador definido pela terna $\left\{V, \widehat{H}, a(u,v)\right\}$.

2 Resultados Principais

Teorema 2.1. *Assumimos hipóteses adequadas e* $u^0, v^0 \in D(C), u^1, v^1 \in D(C^{1/2}), z^i \in D(C) \cap D(B)$, $i = 1, 2, z(t) = z^0 + tz^1$, *existe uma única* $v = v(x,\ t)$ *tal que verifica*

$$(P) \left| \begin{array}{l} Bv'' + M\,(t)\,Av(t) = M(t)Az(t) \text{ em } L^\infty(0,T,H), \\ v\,(0) = v^0 \ \ , v'(0) = v^1. \end{array} \right.$$

A função $u : [0,T] \to V$, *definida por* $u(t) = v(t) + z(t) \in L^\infty(0,T,H)$ *é única solução do problema degenerado:*

$$\begin{cases} Bu''(t) + M\,(t)\,Au(t) = 0 \\ u(0) = u^0 = z^0 + v^0, \ \ u'(0) = u^1 = z^1 + v^1 \end{cases}$$

Prova: *Existência* - Para mostrar a existência de solução global, usamos propriedades de espaços de Hilbert, teorema espectral, construção de espaços adequados e método de Faedo-Galerkim.

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CARLEMAN ESTIMATES FOR A CLASS OF DEGENERATE PARABOLIC EQUATIONS AND APPLICATIONS

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Abstract

This work deals with a class of degenerate parabolic equations in a square. We consider a differential operator degenerating only in a part of the boundary. Furthermore the differential operator can be, simultaneausly, weakly degenerate in a part of the boundary and strongly degenerate in another part. The goal is to deduce Carleman estimates to obtain controllability results. This goal is reached in any space dimension, but for a space dimension higher than 1, we can prove the results only for a class of control regions.

1 Introduction

In this work we consider the following class of degenerate parabolic equations

$$\begin{cases} u_t - div(A\nabla u) + bu = g1_\omega & \text{in} \quad Q, \\ B.C. & \text{on} \quad \Sigma, \\ u(\cdot,0) = u_0 & \text{in} \quad \Omega, \end{cases} \tag{1}$$

where $\Omega = (0,1)\times(0,1)$, $\Gamma := \partial\Omega$, $T > 0$, $Q = \Omega\times(0,T)$, $\Sigma := \Gamma\times(0,T)$, $\omega \subset \Omega$ is open and 1_ω its the characteristic function, $b \in L^\infty(Q)$, $g \in L^2(Q)$, $u_0 \in L^2(\Omega)$, $A : \overline{\Omega} \mapsto M_{2\times2}(\mathbb{R})$ is given by

$$A(x) = diag(x_1^{\alpha_1}, x_2^{\alpha_2}),$$

$$B.C. := \begin{cases} u = 0 \ \text{ on } \ \Sigma & \text{if} \quad \alpha_1,\ \alpha_2 \in [0,1), \\ u = 0 \ \text{ on } \ \Sigma_{3,4} \ \text{ and } \ (A\nabla u)\nu = 0 \ \text{ on } \ \Sigma_{1,2} & \text{if} \quad \alpha_1,\ \alpha_2 \in [1,+\infty), \\ u = 0 \ \text{ on } \ \Sigma_{1,3,4} \ \text{ and } \ (A\nabla u)\nu = 0 \ \text{ on } \ \Sigma_2 & \text{if} \quad \alpha_1 \in [0,1) \ \text{ and } \ \alpha_2 \in [1,+\infty), \\ u = 0 \ \text{ on } \ \Sigma_{2,3,4} \ \text{ and } \ (A\nabla u)\nu = 0 \ \text{ on } \ \Sigma_1 & \text{if} \quad \alpha_1 \in [1,+\infty) \ \text{ and } \ \alpha_2 \in [0,1), \end{cases}$$

$\alpha = (\alpha_1, \alpha_2) \in [0,+\infty)\times[0,+\infty)$, $\Sigma_{i,j,l} := (\Gamma_i \cup \Gamma_j \cup \Gamma_l)\times(0,T)$, and

$$\Gamma_1 := \{0\}\times[0,1],\ \Gamma_2 := [0,1]\times\{0\},\ \Gamma_3 := \{1\}\times[0,1], \ \text{ and } \ \Gamma_4 := [0,1]\times\{1\}.$$

Developing a trace theory for convenient Hilbert spaces we can prove well posedness results for (1).

The main goal of this work is obtain controllability results for (1). A classical way to do that is to deduce a observabillity estimate for the associated adjoint system:

$$\begin{cases} -v_t - div(A\nabla v) + bv = f & \text{in} \quad Q, \\ B.C. & \text{on} \quad \Sigma, \\ v(\cdot,T) = v_T & \text{in} \quad \Omega. \end{cases} \tag{2}$$

It is well know that observabillity estimates of this kind can be deduced from Carleman estimates.

2 Main Result

Assume that there exists $a_0, b_0, \delta_0 \in (0,1)$ such that

$$\omega \supset \omega_0 := \begin{cases} (0,\delta) \times (0,\delta) & \text{if} \quad \alpha_1, \alpha_2 \in (0,+\infty) \\ (0,\delta) \times (a_0,b_0) & \text{if} \quad \alpha_1 \in (0,+\infty) \quad \text{and} \quad \alpha_2 = 0 \\ (a_0,b_0) \times (0,\delta) & \text{if} \quad \alpha_1 = 0 \quad \text{and} \quad \alpha_2 \in (0,+\infty). \end{cases} \tag{3}$$

Now, we will introduce the weight function $\eta \in C^\infty(\overline{\Omega})$ given by

$$\eta(x) := \begin{cases} -(x_1^2 + x_2^2)/2 & \text{if} \quad \alpha_1, \alpha_2 \in (0,+\infty) \\ \eta_0(x_2) - x_1^2/2 & \text{if} \quad \alpha_1 \in (0,+\infty) \quad \text{and} \quad \alpha_2 = 0 \\ \eta_0(x_1) - x_2^2/2 & \text{if} \quad \alpha_1 = 0 \quad \text{and} \quad \alpha_2 \in (0,+\infty), \end{cases}$$

where $\eta_0 \in C^\infty([0,1])$ is such that $\eta_0(x) = x$ in $[0,a_0]$ and $\eta_0(x) = -x$ in $[b_0,1]$. Furthermore, for $\lambda > \lambda_0$, let us introduce

$$\theta(t) := [t(T-t)]^{-4}, \quad \xi(x,t) := \theta(t) e^{2\lambda(|\eta|_\infty + \eta(x))}, \quad \text{and} \quad \sigma(x,t) := \theta(t) e^{4\lambda|\eta|_\infty} - \xi(x,t).$$

Theorem 2.1. *Assume (3). There exists positive constants C, s_0, λ_0 such that for any $\lambda > \lambda_0$, $s > s_0$ and v solution of (2) one has*

$$\iint_Q e^{-2s\sigma} \left[s^{-1}\xi^{-1} \left(|v_t|^2 + |div(A\nabla v)|^2 \right) + s\lambda^2 \xi |\nabla v A \nabla v| + s^3 \lambda^4 \xi^3 |v|^2 \right] dx\, dt$$
$$\leq C \left[\iint_Q e^{-s\sigma} |f|^2 \, dx\, dt + s^3 \lambda^4 \iint_{\omega_T} e^{-2s\sigma} \xi^3 |v|^2 \, dx\, dt \right].$$

Remark 2.1. In one-dimensional case, we can deduce a similar estimate without assuming (3).
Remark 2.2. If $\alpha_1, \alpha_2 \in (0,2)$ we again can deduce a similar estimate considering the weaker assumption: There exists numbers $a_i, b_i, \delta \in (0,1)$ with $a_i < b_i$, such that

$$\omega_0 := [(a_1,b_1) \times (0,\delta)] \cup [(0,\delta) \times (a_2,b_2)] \subset \omega.$$

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SOME RESULTS OF INTERNAL CONTROLLABILITY FOR THE KORTEWEG-DE VRIES EQUATION IN BOUNDED DOMAIN

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Abstract

This work is devoted to the study of the internal controllability for the Korteweg-de Vries equation posed on a bounded interval. The main part of the work focus on the null controllability property of a linearized equation. Following a classical duality approach [4] the problem is reduced to the study of an observability inequality which is proved by using a Carleman estimate. Then, making use of a cut-off argument and the duality approach, the exact controllability is also investigated. In both cases, we return to the nonlinear system by means of a fixed point argument.

1 Introduction

The Korteweg–de Vries equation (KdV) was first derived in [3] as a model for propagation of surface water waves along a channel. At present it is known that the KdV equation is not only a good model for water waves but also a very useful approximation model in nonlinear studies whenever one wishes to include and balance weak nonlinear and dispersive effects. In particular, the equation is commonly accepted as a mathematical model for the unidirectional propagation of small amplitude long waves in nonlinear dispersive systems. In mathematical studies, considerations have been given principally to pure initial value problems and well-posedness results. It is our purpose here to establish this as a fact, at least in the context of the control problem for the KdV equation posed on a bounded interval $(0, L)$. Therefore, we consider the following equation

$$u_t + u_x + u_{xxx} + uu_x = 0, \qquad \text{where } x \in [0, L] \text{ and } t \geq 0. \tag{1}$$

In order to study the controllability properties, we introduce a dynamical system (control system) on which we can act by means of a control term in order to reach some goal. Here, we consider a control system where the state, at each time, is given by the solution of the KdV equation with a control term distributed in a subset of the domain $(0, L)$. More precisely, we study the following distributed parameter control system

$$\begin{cases} u_t + u_x + uu_x + u_{xxx} = 1_\omega f(t, x) & \text{in } (0, T) \times (0, L), \\ u(t, 0) = u(t, L) = u_x(t, L) = 0 & \text{in } (0, T), \\ u(0, x) = u_0(x) & \text{in } (0, L). \end{cases} \tag{2}$$

where f is acts as control inputs supported in a given open set $\omega \subset (0, L)$.

Our main purpose is to see whether one can force the solutions of (2) to have certain desired properties by choosing an appropriate control input f. Therefore, we study the following fundamental problems that arises in control theory for partial differential equations:

Problem: *Given an initial state $u_0(x)$ and a terminal state $u_1(x)$ in a certain space, can one find an appropriate control input f so that the equation (2) admits a solution u which equals u_0 at time $t = 0$ and equals u_1 at time $t = T$?*

If one can always find a control input f to guide the system described by (2) from any given initial state u_0 to any given terminal state u_1, then the system (2) is said to be **exactly controllable**. If the system can be driven, by means of a control f, from any state to the origin (i. e. $u_1 \equiv 0$), one says that system is **null controllable**.

2 Main Results

Theorem 2.1. *[1] Let $\omega = (l_1, l_2)$ with $0 < l_1 < l_2 < L$, and let $T > 0$. For $\bar{u}_0 \in L^2(0, L)$, let $\bar{u} \in C^0([0, T]; L^2(0, L)) \cap L^2(0, T; H^1(0, L))$ denote the solution of*

$$\begin{cases} \bar{u}_t + \bar{u}_x + \bar{u}\,\bar{u}_x + \bar{u}_{xxx} = 0 & \text{in } (0,T) \times (0,L), \\ \bar{u}(t,0) = \bar{u}(t,L) = \bar{u}_x(t,L) = 0 & \text{in } (0,T), \\ \bar{u}(0,x) = \bar{u}_0(x) & \text{in } (0,L). \end{cases} \tag{3}$$

Then there exists $\delta > 0$ such that for any $u_0 \in L^2(0, L)$ satisfying $\|u_0 - \bar{u}_0\|_{L^2(0,L)} \leq \delta$, there exists $f \in L^2((0, T) \times \omega)$ such that the solution $u \in C^0([0, T]; L^2(0, L)) \cap L^2(0, T, H^1(0, L))$ of (2) satisfies $u(T, \cdot) = \bar{u}(T, \cdot)$ in $(0, L)$.

Proof The idea this proof is based in [2]. We show an observability inequality which is proved by using a new Carleman estimate for the system (2). ■

Theorem 2.2. *[1] Let $T > 0$ and $\omega = (l_1, l_2)$. Then, there exist $\delta > 0$ such that the following holds:*

i. If $= (L - \nu, L)$ where $0 < \nu < L$, for any u_0, $u_1 \in L^2_{\frac{1}{L-x}dx}$ satisfying $\|u_0\|_{L^2_{\frac{1}{L-x}dx}} \leq \delta$ and $\|u_1\|_{L^2_{\frac{1}{L-x}dx}} \leq \delta$, one can find a control input $f \in L^2(0, T; H^{-1}(0, L))$ with supp$(f) \subset (0, T) \times \omega$ such that the solution $u \in C^0([0, L], L^2(0, L)) \cap L^2(0, T, H^1(0, L))$ of (2), satisfies $u(T, .) = u_1$ in $(0, L)$ and $u \in C^0([0, T], L^2_{\frac{1}{L-x}dx})$. Furthermore, $f \in L^2_{(T-t)dt}(0, T, L^2(0, L))$.

ii. If $\omega = (l_1, l_2)$ with $0 < l_1 < l_2 < L$. Pick any number $l_1' \in (l_1, l_2)$. Then, for any $u_0, u_1 \in L^2(0, L)$ satisfying $||u_0||_{L^2(0,L)} \leq \delta$ and $||u_1||_{L^2(0,L)} \leq \delta$, one can find a control $f \in L^2(0, T, H^{-1}(0, L))$ with supp$(f) \subset (0, T) \times \omega$ such that the solution $u \in C^0([0, T], L^2(0, L)) \cap L^2(0, T, H^1(0, L))$ of (2), satisfies

$$u(T,x) = \begin{cases} u_1(x) & \text{if } x \in (0, l_1'); \\ 0 & \text{if } x \in (l_2, L). \end{cases} \tag{4}$$

Proof The idea this proof is use the Theorem 2.1 through with cut-off argument and the duality approach to ensure the exact controllability for the system (2). ■

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ON OPTIMAL DECAY RATES FOR WEAKLY DAMPED IBQ-BEAM TYPE EQUATIONS ON THE 1-D HALF LINE

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Abstract

We consider the mixed problem for weakly damped IBq-Beam equations on the one dimensional half line $(0, +\infty)$. We shall derive fast decay results of the total energy and L^2-norm of solutions based on the idea due to [1]. In order to apply such ideas to the one dimensional exterior mixed problem, one also constructs an important Hardy-Sobolev type inequality, which holds only in the 1-D half line case.

1 Introduction

We consider the mixed problem for weakly damped IBQ-Beam equations on the one dimensional half line $(0, +\infty)$ with Stokes damped term

$$u_{tt}(t,x) - \gamma u_{xxtt}(t,x) + u_{xxxx}(t,x) + u_t(t,x) = 0, \quad (t,x) \in (0,\infty) \times (0,+\infty), \tag{1}$$

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad x \in (0,+\infty), \tag{2}$$

$$u(t,0) = u_{xx}(t,0) = 0, \quad t > 0, \tag{3}$$

where the constant coefficient γ is non negative. For $\gamma > 0$ the equation (1) is related to the IBq equation. When $\gamma = 0$ the equation (1) is the well known Beam equation.

We derive fast decay results of the total energy and L^2-norm of solutions based on the idea due to [1], which is an essential modification of that developed by C. Morawetz. In order to apply that idea due to [1] to the one dimensional exterior mixed problem, one also constructs an important Hardy-Sobolev type inequality, which holds only in the 1-D half line case.

Let $H := L^2(0,\infty)$, and we define the operator $A : D(A) \subset H \to H$ by

$$D(A) := H^2(0,\infty) \cap H_0^1(0,\infty),$$

$$Au := -u_{xx} \quad (u \in D(A)).$$

It is well-known that the operator A is nonnegative and self-adjoint in H, and the fractional Laplacian A^α $(\alpha \geq 0)$ can be well-defined in H, and also becomes nonnegative and self-adjoint. Furthermore, it is also known that

$$D(A^{3/2}) = \{v \in H^3(0,\infty) |\, v(0) = v_{xx}(0) = 0\},$$

and

$$D(A^2) = \{v \in H^4(0,\infty) |\, v(0) = v_{xx}(0) = 0\}.$$

Under these preparations, by using a semi-group theory as Luz-Charão [2] we may obtain a unique existence of the weak solution $u = u(t,x)$ for each initial data $[u_0, u_1] \in D(A^{3/2}) \times D(A)$, to problem (1)-(3) in the class

$$u \in X(0,+\infty) := C([0,+\infty); D(A^{3/2})) \cap C^1([0,+\infty); D(A)) \cap C^2([0,+\infty); D(A^{1/2}))$$

satisfying the variational form:

$$(u_{tt}(t,\cdot),\psi)+\gamma(u_{xtt}(t,\cdot),\psi_x)+(u_{xx}(t,\cdot),\psi_{xx})+(u_t(t,\cdot),\psi)=0,\qquad t>0$$

for all $\psi\in D(A)$.

For more regular initial data $[u_0,u_1]\in D(A^2)\times D(A^{3/2})$ the problem admits also a unique strong solution in the class

$$X_1(0,+\infty):=C([0,+\infty);D(A^2))\cap C^1([0,+\infty);D(A^{3/2}))\cap C^2([0,+\infty);D(A)).$$

The total energy $E(t)$ for the solution $u(t,x)$ of (1) is denoted by

$$E(t)=E(u(t,\cdot)):=\frac{1}{2}(\|u_t(t,\cdot)\|^2+\gamma\|u_{xt}(t,\cdot)\|^2+\|u_{xx}(t,\cdot)\|^2)$$

and the second order energy is defined by

$$E_1(t)=E_1(u(t,\cdot)):=\frac{1}{2}(\|u_{xt}(t,\cdot)\|^2+\gamma\|u_{xxt}(t,\cdot)\|^2+\|u_{xxx}(t,\cdot)\|^2)$$

Our main result in this work read as follows.

Teorema 1.1. *If the initial data $[u_0,u_1]\in D(A^2)\times D(A^{3/2})$, further satisfies*

$$\int_0^{+\infty}(1+x)^{3/2}|u_0(x)+u_1(x)-\gamma(u_1)_{xx}(x)|dx<+\infty.$$

then the unique solution $u(t,x)\in X_1(0,+\infty)$ to the problem (1)–(3) satisfies

$$\|u(t,\cdot)\|^2\le I_\gamma^2(1+t)^{-1},\quad E(t)\le J_\gamma^2(1+t)^{-2}$$

for $t>0$, where I_γ^2 and J_γ^2 are constants depending only on the initial data and the coefficient γ.

Corolário 1.1. *Under the hypothesis as in Theorem 1.1 one has*

$$E_1(t)\le C_1(1+t)^{-1}\ ,\qquad ||u_x(\cdot,t)||^2\le C_2(1+t)^{-1}$$

for $t>0$, where the constants C_1 and C_2 are depending only on the initial data and γ.

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CONTROLLABILITY RESULTS FOR SOME PSEUDO-PARABOLIC EQUATIONS

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Abstract

In this work, we deal with the null controllability problem for some Sobolev type equations. We show that these equations can not be driven to zero if the control region is strictly supported within the domain. Nevertheless, we prove that if we consider the control having a moving support, and under some assumptions on the movement of the control region, it is possible to steer the solution to zero.

1 Introduction

In this work, we will be concerned with the null controllability problem of the following two problems:

$$\left| \begin{array}{lll} y_t - \Delta y_t - \Delta y = v 1_{\omega(t)} & \text{in} & Q := \Omega \times (0,T), \\ y = 0 & \text{on} & \Sigma := \partial\Omega \times (0,T), \\ y(\cdot,0) = y_0 & \text{in} & \Omega \end{array} \right. \tag{1}$$

and

$$\left| \begin{array}{lll} z_t - \Delta z_t + div(A(x,t)z) = u 1_{\omega(t)} & \text{in} & Q, \\ z = 0 & \text{on} & \Sigma, \\ z(\cdot,0) = z_0 & \text{in} & \Omega, \end{array} \right. \tag{2}$$

where $T > 0$, $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a bounded open set and $A = (a_1, \ldots, a_N)$ is a vector field such that $a_i \in W^{1,\infty}(Q)$ and $\nabla \cdot A_t \in L^\infty(Q)$.

Equation (1) is called *Barenblatt-Zheltov-Kochina* equation (see, for instance, [2]). On the other hand, equation (2) is the multidimensional analogous of the Benjamin-Bona-Mahony equation (see [1]). These equations are particular examples of the so called equations of Sobolev type (see [3, 4]) and they appear for instance in the study of problems associated with the flow of certain viscous fluids or in the theory of seepage of homogeneous liquids in fissured rocks.

We prove that none of the equations (1) or (2) can be driven to zero by means of control localized in a small subset of Ω (see Theorems 2.1 and 2.3). This can be seen at the level of the dual observability problem. In fact, the structure of the underlying PDE operators and, in particular, the existence of time-like characteristic hyperplanes, makes impossible the propagation of information in the space-like directions, thus making the observability inequality also impossible.

For this reason, we make the control to move in time in order to cover all the domain, adding in some sense a transport effect into the equations. As we will see, this make both equations (1) and (2) null controllable at any given time $T > 0$.

2 Main Results

We prove the following results for the Barenblatt-Zheltov-Kochina equation.

Theorem 2.1. *Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a bounded open set. Suppose $\omega(t) = \omega_0$ with $\overline{\omega}_0 \neq \Omega$ and let $T > 0$ and $y_0 \in H^2(\Omega) \times H_0^1(\Omega)$. Then, there is no $v \in L^2(Q)$ such that the solution y of (1) satisfies*

$$y(T) = 0 \quad in \quad \Omega.$$

Proof We use a Gaussian beams argument to construct a solution to the adjoint problem of (1) which is localized outside the control region. This allows us to show that the observability inequality cannot hold. ■

Theorem 2.2. *Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a bounded open set. Under appropriate assumptions on the trajectory of the control region $\omega(t)$, for any $y_0 \in H^2(\Omega) \cap H_0^1(\Omega)$ and any $T > 0$, there exists a control $v \in L^2(Q)$ such that the associated solution to (1) satisfies*

$$y(\cdot, T) = 0 \quad in \quad \Omega.$$

Proof By means of some new Carleman estimates for elliptic systems with a moving region, we show an observability inequality for the adjoint system of (1). ■

For the generalized Benjamin-Bona-Mahony equation (2), we prove the following results.

Theorem 2.3. *Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a bounded open set. Assume A regular enough and suppose $\omega(t) = \omega_0$ with $\overline{\omega}_0 \neq \Omega$ and let $T > 0$ and $z_0 \in H_0^1(\Omega)$. Then, there is no $u \in L^2(Q)$ such that the solution z of (2) satisfies*

$$z(T) = 0 \quad in \quad \Omega.$$

Proof Here we cannot use directly a Gaussian beams argument, so we construct an approximate solution to the adjoint equation of (2) and use it to show that the observability inequality is not true. ■

Theorem 2.4. *Let $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ be a bounded open set. Under appropriate assumptions on the trajectory of the control region $\omega(t)$, for any $z_0 \in H^2(\Omega) \cap H_0^1(\Omega)$ and any $T > 0$, there exists a control $u \in L^2(Q)$ such that the associated solution z to (2) satisfies*

$$z(\cdot, T) = 0 \quad in \quad \Omega.$$

Proof Using Carleman inequalities for ODE's and elliptic equations, both with moving regions and same weight functions, we prove that the observability inequality for the adjoint of (2) holds. ■

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FLUIDOS MICROPOLARES NÃO-HOMOGÊNEOS: ESTIMATIVAS DE ERRO PARA AS APROXIMAÇÕES SEMI-GALERKIN

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Abstract

Estabeleceremos, em um domínio limitado Ω de $\mathbb{R}^3$, estimativas de erro para as aproximações semi-Galerkin espectrais das soluções globais fortes das equações dos fluidos micropolares não-homogêneos incompressíveis. Mais precisamente, obteremos, na norma $\boldsymbol{H}^1(\Omega) = \big(H^1(\Omega)\big)^3$, estimativas de erro uniformes no tempo para as aproximações das velocidades linear e angular. Também deduziremos estimativas de erro para as aproximações da densidade em alguns espaços de Lebesgue $L^r(\Omega)$.

1 Introdução

Seja $\Omega \subsetneq \mathbb{R}^3$ um domínio regular limitado de classe $C^{1,1}$, cuja fronteira é denotada por $\partial\Omega$. Consideramos, no espaço-tempo $\Omega \times (0,\infty)$, o seguinte sistema de equações:

$$\begin{cases} \rho\big(\boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\big) + \nabla p = (\mu+\mu_r)\Delta\boldsymbol{u} + 2\mu_r \operatorname{rot}\boldsymbol{w} + \rho\boldsymbol{f}, \\ \operatorname{div}\boldsymbol{u} = 0, \\ \rho\big(\boldsymbol{w}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{w}\big) - (c_0+c_d-c_a)\nabla(\operatorname{div}\boldsymbol{w}) + 4\mu_r\boldsymbol{w} = (c_a+c_d)\Delta\boldsymbol{w} + 2\mu_r\operatorname{rot}\boldsymbol{u} + \rho\boldsymbol{g}, \\ \rho_t + \operatorname{div}(\rho\boldsymbol{u}) = 0. \end{cases} \tag{1}$$

Complementamos o sistema acima com as seguintes condições iniciais e de fronteira:

$$\begin{cases} \rho(\boldsymbol{x},0) = \rho_0(\boldsymbol{x}), \quad \boldsymbol{u}(\boldsymbol{x},0) = \boldsymbol{u}_0(\boldsymbol{x}) \quad \text{e} \quad \boldsymbol{w}(\boldsymbol{x},0) = \boldsymbol{w}_0(\boldsymbol{x}) \quad \text{em } \Omega, \\ \boldsymbol{u}(\boldsymbol{x},t) = \boldsymbol{0}, \quad \boldsymbol{w}(\boldsymbol{x},t) = \boldsymbol{0} \quad \text{sobre } \partial\Omega\times(0,\infty), \end{cases} \tag{2}$$

onde ρ_0, $\boldsymbol{u}_0$ e $\boldsymbol{w}_0$ são funções dadas.

No sistema (1), as incógnitas são as funções $\rho(\boldsymbol{x},t)\in\mathbb{R}$, $\boldsymbol{u}(\boldsymbol{x},t)\in\mathbb{R}^3$, $p(\boldsymbol{x},t)\in\mathbb{R}$ e $\boldsymbol{w}(\boldsymbol{x},t)\in\mathbb{R}^3$, as quais representam, respectivamente, a densidade de massa, a velocidade linear, a pressão e a velocidade angular de rotação das particulas do fluido em um ponto $\boldsymbol{x}\in\Omega$ no tempo $t\in\mathbb{R}^+$. Por outro lado, as funções $\boldsymbol{f}(\boldsymbol{x},t)\in\mathbb{R}^3$ e $\boldsymbol{g}(\boldsymbol{x},t)\in\mathbb{R}^3$ são forças externas dadas. Este sistema descreve o movimento de um fluido micropolar (ou assimétrico) não-homogêneo, viscoso e incompressível (veja [3] e [4]). Fisicamente, a primeira equação no sistema (1) corresponde a lei de conservação do momento linear; a segunda é a condição de incompressibilidade do fluido; a terceira é a lei de conservação do momento angular, e a quarta é a lei de conservação da massa. As constantes positivas μ, μ_r, c_0, c_d e c_a estão relacionadas com a viscosidade e satisfazem $c_0+c_d > c_a$. Vale salientar que o sistema (1) inclui, como caso particular, as clássicas equações de Navier-Stokes com densidade variável ($\boldsymbol{w}=\boldsymbol{g}=\boldsymbol{0}$ e $\mu_r=0$).

2 Resultados Principais

Os resultados que provaremos são similares ao de ***Braz e Silva e Rojas-Medar*** em [2] para as equações de Navier-Stokes com densidade variável e ao de ***Heywood*** em [5] para as equações de Navier-Stokes usuais (i.e., com densidade constante).

Teorema 2.1. *Suponha que* $(\rho, \boldsymbol{u}, \boldsymbol{w})$ *é uma solução* p_0*-condicionalmente assintoticamente estável do problema* (1)-(2)*, para algum* p_0*,* $6 \leq p_0 \leq \infty$ *(num certo sentido a ser precisado). Então, existem constantes* $N \in \mathbb{N}$ *e* $C \geq 0$*, tais que se* $k \geq N$ *tem-se, para todo* $t \geq 0$*,*

$$\left\|\nabla(\boldsymbol{u} - \boldsymbol{u}^k)(\cdot, t)\right\|^2_{\boldsymbol{L}^2(\Omega)} + \left\|\mathcal{L}^{1/2}(\boldsymbol{w} - \boldsymbol{w}^k)(\cdot, t)\right\|^2_{\boldsymbol{L}^2(\Omega)} \leq \frac{C}{\lambda_{k+1}} + \frac{C}{\gamma_{k+1}}.$$

Ademais, se $6 \leq p_0 < \infty$*, então*

$$\left\|(\rho - \rho^k)(\cdot, t)\right\|_{L^r(\Omega)} \leq \frac{Ct}{(\lambda_{k+1})^{1/2}} + \frac{Ct}{(\gamma_{k+1})^{1/2}}, \quad 2 \leq r \leq \frac{6p_0}{6 + p_0},$$

e se $p_0 = \infty$*, então*

$$\left\|(\rho - \rho^k)(\cdot, t)\right\|_{L^r(\Omega)} \leq \frac{Ct}{(\lambda_{k+1})^{1/2}} + \frac{Ct}{(\gamma_{k+1})^{1/2}}, \quad 2 \leq r \leq 6.$$

Aqui, $\left(\rho^k, \boldsymbol{u}^k, \boldsymbol{w}^k\right)$ *são as aproximações semi-Galerkin espectrais das soluções globais fortes do problema* (1)-(2)*, assim como* λ_k *e* γ_k *denotam, respectivamente, os autovalores do* ***operador de Stokes*** $\mathcal{A} := -P\Delta$ *e do* ***operador de Lamé*** $\mathcal{L} := -(c_a + c_d)\Delta - (c_0 + c_d - c_a)\nabla\mathrm{div}$.

Prova: Inicialmente obtemos várias estimativas *a priori* para as soluções do problema (1)-(2), bem como para as "soluções perturbadas" deste problema (ou seja, soluções do sistema (1) começando num tempo $t_0 \geq 0$). Também são obtidas estimativas para as soluções do problema aproximado. Em seguida, escrevemos as expressões das soluções $\boldsymbol{u}$ e $\boldsymbol{w}$ em termos das autofunções do operador de Stokes $\mathcal{A}$ e do operador de Lamé $\mathcal{L}$, respectivamente. Denotando as k-ésimas somas parciais destas séries por $\boldsymbol{\alpha}^k$ e $\boldsymbol{\beta}^k$, respectivamente, obtemos estimativas para os "restos" $\boldsymbol{E}^k := \boldsymbol{u} - \boldsymbol{\alpha}^k$ e $\boldsymbol{\mathcal{E}}^k := \boldsymbol{w} - \boldsymbol{\beta}^k$, bem como para $\boldsymbol{\eta}^k := \boldsymbol{u}^k - \boldsymbol{\alpha}^k$ e $\boldsymbol{r}^k := \boldsymbol{w}^k - \boldsymbol{\beta}^k$. Basicamente, todas estas estimativas são suficientes para provar o Teorema 2.1. $\square$

Observação 1. *O resultado enunciado acima, faz parte de um projeto que está em fase de conclusão (ver [1]).*

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SUFFICIENT CONDITIONS FOR EXISTENCE OF POSITIVE PERIODIC SOLUTION OF A GENERALIZED NONRESIDENT COMPUTER VIRUS MODEL

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Abstract

In this paper, we introduce a nonresident computer virus model and prove the existence of at least one positive periodic solution. The proposed model is based on a biological approach and is obtained by considering that all rates (rates that the computers are disconnected from the Internet, the rate that the computers are cured, etc) are time dependent real functions. Assuming that the initial condition is a positive vector and the coefficients are positive $\omega-$periodic and applying the topological degree arguments we deduce that generalized nonresident computer virus model has at least one positive $\omega-$periodic solution. The proof consists of two big parts. First, an appropriate change of variable which "conserves" the periodicity property and implies the positive behavior. Second, a reformulation of transformed system as an operator equation which is analyzed by applying the continuation theorem of the coincidence degree theory .

1 Introduction

In this paper, we are interested in the deterministic model proposed by for nonresident virus propagation [2, 3, 4]. Let us consider that the varying total numbers of computers in the network are further divided at any time t into three compartments denoted by $S(t), L(t)$ at $A(t)$. Here, $S(t)$ denotes the average numbers of uninfected computers (susceptible computers) at time t, $L(t)$ denotes the average numbers of infected computers (latent computers) in which viruses are not yet loaded in their memory at time t; and $A(t)$ denotes the average numbers of infected computers (infectious computers) in which viruses are located in memory at time t. Thus, generalizing the hypotheses of [2], we obtain the following model:

$$\frac{dS(t)}{dt} = b(t) - \mu_1(t)S(t) - \beta_1(t)S(t)L(t) - \beta_2(t)S(t)A(t) + \gamma_1(t)L(t) + \gamma_2(t)A(t), \tag{1a}$$

$$\frac{dL(t)}{dt} = \beta_1(t)S(t)L(t) + \beta_2(t)S(t)A(t) + \alpha_2(t)A(t) - [\mu_2(t) + \alpha_1(t) + \gamma_1(t)]L(t), \tag{1b}$$

$$\frac{dA(t)}{dt} = \alpha_1(t)L(t) - [\mu_3(t) + \alpha_1(t) + \gamma_2(t)]A(t). \tag{1c}$$

Here we consider all rates are time dependent, i.e. some assumptions are more general in the sense that the parameters $b, \alpha_i, \beta_i, \mu_i$ and γ_i with $i = 1, 2, 3$, are time dependent real functions. Now, in order to understand the dynamics we study the existence of positive periodic solution for (1).

2 Main Results

The main results of the paper are given by the following theorems (see [1] for details):

Theorem 2.1. *Assume that the coefficients of the system* (1) *satisfy the following hypothesis:*

$$\left.\begin{array}{c}\text{The initial condition } (S(0),L(0),A(0))\in\mathbb{R}^3_+ \text{ and the coefficient functions}\\ b,\mu_1,\beta_1,\beta_2,\gamma_1,\gamma_2,\alpha_1 \text{ and } \alpha_2 \text{ are positive, continuous, } \omega\text{-periodic on } [0,\omega] \text{ and}\\ \max_{t\in[0,\omega]}\left[\dfrac{\alpha_1(\alpha_2+\gamma_2)}{(\alpha_1+\mu_2)}\right](t)\leq \min_{t\in[0,\omega]}(\mu_3+\alpha_2+\gamma_2)(t).\end{array}\right\} \tag{1}$$

Then, the system (1) *has at least one positive* ω*-periodic solution.*

Theorem 2.2. *Let us consider* X *and* Y *are the Banach spaces defined by*

$$X=Y=\Big\{\mathbf{x}^T=(x_1,x_2,x_3)^T\in C(\mathbb{R},\mathbb{R}^3)\quad:\quad \mathbf{x}(t+\omega)=\mathbf{x}(t),\qquad \Big\|\mathbf{x}\Big\|=\sum_{i=1}^{3}\max_{t\in[0,\omega]}|x_i(t)|<\infty\Big\}.$$

The operators Q *defined by* $L:X\to Y$ *and* $N:X\to Y$ *defined as follows*

$$Q(\mathbf{x})=\frac{1}{\omega}\int_0^{\omega}\mathbf{x}^T(\tau)d\tau,\quad L(\mathbf{x})=\frac{d\mathbf{x}}{dt},\quad N(\mathbf{x})=\Big(\mathcal{N}_1(\mathbf{x}(t)),\mathcal{N}_2(\mathbf{x}(t)),\mathcal{N}_3(\mathbf{x}(t))\Big)^T,$$

where

$$\begin{aligned}\mathcal{N}_1(\mathbf{x}(t)) &= b(t)\exp(-x_1(t))-\mu_1(t)-\beta_1(t)\exp(x_2(t))-\beta_2(t)\exp(x_3(t))\\ &\quad+\gamma_1(t)\exp(x_2(t)-x_1(t))+\gamma_2(t)\exp(x_3(t)-x_1(t))\\ \mathcal{N}_2(\mathbf{x}(t)) &= \beta_1(t)\exp(x_1(t))+\beta_2(t)\exp(x_1(t)+x_3(t)-x_2(t))+\alpha_2(t)\exp(x_3(t)-x_2(t))-[\mu_2(t)+\alpha_1(t)+\gamma_1(t)]\\ \mathcal{N}_3(\mathbf{x}(t)) &= \alpha_1(t)\exp(x_2(t)-x_3(t))-[\mu_3(t)+\alpha_2(t)+\gamma_2(t)].\end{aligned}$$

respectively. Moreover, assume that the hypothesis (1) *is satisfied. Then, there are the positive constants* $\rho_1,\rho_2,\rho_3,d_1,d_2,d_3,\delta_1,\delta_2$ *and* δ_3*, such that the following assertions are valid*

(a) If $\lambda\in(0,1)$ *and* $(x_1,x_2,x_3)\in\mathrm{Dom}\ L$ *are such that* $L(x_1,x_2,x_3)=\lambda N(x_1,x_2,x_3)$*, the inequalities* $x_i(t)<\ln(\rho_i/w)+di$, $\ln(\delta_i)<x_i(t)$, $\quad i=1,2,3,$ *holds for all* $t\in[0,\omega]$.

(b) If $(x_1,x_2,x_3)\in\mathrm{Ker}\ L$ *are such that* $QN(x_1,x_2,x_3)=0$*, the inequalities* $x_i(t)<\ln(\rho_i/w)$, $\ln(\delta_i)<x_i(t)$, $\quad i=1,2,3,$ *holds for all* $t\in[0,\omega]$.

To prove the Theorem 2.1 we apply the coincidence degree theory. The proof is self-contained and and given by introducing several lemmas which implies that the hypotheses of continuation theorem are valid in this context. Moreover, it is worthwhile to remark that another important result of the paper is the a priory estimates given on Theorem 2.2, which is useful to get the contradiction in one of the steps of the proof of Theorem 2.1.

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A CONNECTION BETWEEN ALMOST PERIODIC FUNCTIONS DEFINED ON TIME SCALES AND $\mathbb{R}$

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Abstract

In this work, we prove a strong connection between almost periodic functions on time scales and almost periodic functions on $\mathbb{R}$. Also, we present an application to difference equations on $\mathbb{T} = h\mathbb{Z}$.

1 Introduction

In the last few decades, the theory of discrete processes has been attracting the attention of several researchers. Basic problems such as stability and other asymptotic behavior, the presence of oscillations in such systems, and many other features related to the solutions of difference equations have been investigated successfully by many scientists. The applications of such topics to various fields have been a concern of researchers, and a rich body of literature has appeared relatively recently.

It is interesting to point out the fact that a certain parallelism takes place between the theory of almost periodic functions in the case of continuous time and that in the case of discrete time. In [1, Theorem 1.27] (see also [3, Proposition 3.35]), C. Corduneanu investigated a surprising and strong connection between almost periodic functions defined on $\mathbb{Z}$ and almost periodic functions defined on $\mathbb{R}$, namely: A necessary and sufficient condition for $f \in AP(\mathbb{Z})$ is the existence of a function $\varphi \in AP(\mathbb{R})$ such that $f(n) = \varphi(n), n \in \mathbb{Z}$.

From this fact, a natural and more general open question which appears is the following: Let $\mathbb{T}$ be an arbitrary time scale, i.e. a closed and nonempty subset of $\mathbb{R}$. For a given almost periodic function $g : \mathbb{T} \to \mathbb{R}$, is there an almost periodic function $F_g : \mathbb{R} \to \mathbb{R}$ such that the restriction of F_g to $\mathbb{T}$ coincides with g? In this work, we will answer this question affirmatively.

In addition, we prove that this fact turns out to be a very useful tool to provide criteria for the almost periodicity of solutions to diamond-α dynamic equations on time scales.

2 Main Results

The next result presents a strong connection between almost periodic functions defined on $\mathbb{T}$ and $\mathbb{R}$.

Theorem 2.1. *If $\mathbb{T}$ is invariant under translations, a necessary and sufficient condition for a continuous function $g : \mathbb{T} \to \mathbb{E}^n$ to be almost periodic on $\mathbb{T}$ is the existence of an almost periodic function $f : \mathbb{R} \to \mathbb{E}^n$ such that $f(t) = g(t)$ for every $t \in \mathbb{T}$.*

As an application of the connection presented in Theorem 2.1, we obtain the following theorem.

Theorem 2.2. *Let A be a nonsingular square matrix of order n. Let $g : h\mathbb{Z} \to \mathbb{E}^n$ be an almost periodic function, then if $x : \mathbb{R} \to \mathbb{E}^n$ is the almost periodic solution of*

$$x'(t) = -Ax(t) + f(t), \quad t \in \mathbb{R}, \tag{1}$$

where $f : \mathbb{R} \to \mathbb{E}^n$ is defined by

$$f(t) = \begin{cases} \left(1 - \dfrac{(t - t_*)}{\mu(t_*)}\right) g(t_*) + \dfrac{t - t_*}{\mu(t_*)} g(\sigma(t_*)), & t \in \mathbb{R} \setminus h\mathbb{Z}, \\ g(t), \quad t \in h\mathbb{Z}, \end{cases}$$

then the restriction of x to the time scale $\mathbb{T} = h\mathbb{Z}$, i.e. $x : h\mathbb{Z} \to \mathbb{E}^n$, is the almost periodic solution of

$$x^{\diamond_\alpha}(t) = -Ax(t) + g(t), \quad t \in h\mathbb{Z}, \tag{2}$$

for $\alpha = \frac{1}{2}$.

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REACTION-DIFFUSION EQUATIONS WITH SPATIALLY VARIABLE EXPONENTS

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Abstract

This talk is based on [5] where we proved continuity of solutions with respect to initial conditions and couple parameters and we proved upper semicontinuity of a family of global attractors for the problem

$$\begin{cases} \dfrac{\partial u_s}{\partial t}(t) - \operatorname{div}(D_s|\nabla u_s|^{p_s(x)-2}\nabla u_s) + |u_s|^{p_s(x)-2}u_s = B(u_s(t)), \ \ t > 0, \\ u_s(0) = u_{0s}, \end{cases}$$

under homogeneous Neumann boundary conditions, $u_{0s} \in H := L^2(\Omega)$, $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ is a smooth bounded domain, $B : H \to H$ is a globally Lipschitz map with Lipschitz constant $L \geq 0$, $D_s \in [1, \infty)$, $p_s(\cdot) \in C(\bar{\Omega})$, $p_s^- :=$ ess inf $p_s \geq p$, $p_s^+ :=$ ess sup $p_s \leq a$, for all $s \in \mathbb{N}$, when $p_s(\cdot) \to p$ in $L^\infty(\Omega)$ and $D_s \to \infty$ as $s \to \infty$, with $a, p > 2$ positive constants.

1 Introduction

In [3, 4] we investigated in which way the parameter $p(x)$ affects the dynamics of problems involving the $p(x)$-Laplacian. In [2] it was made the diffusion parameter D go to infinity and proved that the family of global attractors for the problem

$$\begin{cases} \dfrac{\partial u^D}{\partial t}(t) - D\Delta_{p(x)}(u^D) = B(u^D(t)), \ \ t > 0 \\ u^D(0) = u_0^D \in H = L^2(\Omega) \end{cases}$$

is upper and lower semicontinuous at infinity.

In [5] we studied a problem of the form

$$\begin{cases} \dfrac{\partial u_s}{\partial t}(t) - \operatorname{div}(D_s|\nabla u_s|^{p_s(x)-2}\nabla u_s) + |u_s|^{p_s(x)-2}u_s = B(u_s(t)), \ \ t > 0, \\ u_s(0) = u_{0s}, \end{cases} \tag{1}$$

under homogeneous Neumann boundary conditions, $u_{0s} \in H := L^2(\Omega)$, $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$ is a smooth bounded domain, $B : H \to H$ is a globally Lipschitz map with Lipschitz constant $L \geq 0$, $D_s \in [1, \infty)$, $p_s(\cdot) \in C(\bar{\Omega})$, $p_s^- :=$ ess inf $p_s \geq p$, $p_s^+ :=$ ess sup $p_s \leq a$, for all $s \in \mathbb{N}$. We assumed that $p_s(\cdot) \to p$ in $L^\infty(\Omega)$ and $D_s \to \infty$ as $s \to \infty$, where $a, p > 2$ are positive constants. We proved continuity of the flows and joint upper semicontinuity of the family of global attractors $\{\mathcal{A}_s\}_{s \in \mathbb{N}}$ as s goes to infinity for the problem (1) with respect to the couple of parameters (D_s, p_s), where p_s is the variable exponent and D_s is the diffusion coefficient.

It is also worth remembering that the authors in [1] considered the constant exponent case for which it was easy to see that a solution of the limit problem is also a solution for (1) (but for the variable exponent case this is no longer true) and the lower semicontinuity of the global attractors was an immediate consequence of this.

2 Main Results

Our objective is to prove that the limit problem of problem (1) as D_s increases to infinity and $p_s(\cdot) \to p > 2$ in $L^\infty(\Omega)$ as $s \to \infty$ is described by an ordinary differential equation. Firstly we observe that the gradients of the solutions u_s of problem (1) converge in norm to zero as $s \to \infty$, which allows us to guess the limit problem

$$\begin{cases} \dfrac{du}{dt}(t) + |u(t)|^{p-2}u(t) = \tilde{B}(u(t)), \quad t > 0, \\ u(0) = u_0 \in \mathbb{R}, \end{cases} \tag{1}$$

with $\tilde{B} := B_{|\mathbb{R}}$ if we identify $\mathbb{R}$ with the constant functions which are in H, since Ω is a bounded set.

The proofs of the next two results follow the ideas of [1], but some adjustments are necessary for this variable exponent case.

Theorem 2.1. *Given $T_0 > 0$, if for each s, u_s is a solution of (1) in $(0, \infty)$, then for each $t \geq T_0$, the sequence of real numbers $\{\|\nabla u_s(t)\|_H\}$ has a subsequence $\{\|\nabla u_{s_\ell}(t)\|_H\}$ which converges to zero as $\ell \to \infty$.*

Moreover, by Lemma 3.2 and Theorem 3.1 in [1] problem (1) has a unique global solution and a maximal compact invariant global $B-$attractor $\mathcal{A}^\infty$, given as the union of all bounded complete trajectories in $\mathbb{R}$.

The next result guarantees that (1) is in fact the limit problem for (1), as $s \to \infty$.

Theorem 2.2. *Let u_s be a solution of (1) with $u_s(0) = u_{0s}$ and let u be a solution of (1) with $u(0) = u_0$. If $u_{0s} \to u_0$ in H as $s \to \infty$, then for each $T > 0$, $u_s \to u$ in $C([0,T]; H)$ as $s \to +\infty$.*

Finally we have

Theorem 2.3. *The family of global attractors $\{\mathcal{A}_s;\ s \in \mathbb{N}\}$ associated with problem (1) is upper semicontinuous on s at infinity, in the topology of H.*

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ANÁLISE NUMÉRICA PARA UMA FORMULAÇÃO PRIMAL HÍBRIDA APLICADA AO PROBLEMA DE CONDUÇÃO DE CALOR

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Resumo

Com o intuito de obter uma solução mais estável para o problema de condução de calor em regime transiente, propomos o uso de um método híbrido estabilizado de elementos finitos para a variável espacial com o método de Euler implícito para a variável do tempo. O método híbrido, originalmente introduzido para problemas elípticos, consiste no acoplamento de problemas resolvidos locamente por métodos de Galerkin Descontínuo (GD) para a variável primal, temperatura, com um problema global para o multiplicador de Lagrange, que é identificado como o traço da temperatura, que impõe fracamente a continuidade entre os elementos. A formulação híbrida é estável, robusta e flexível, como os métodos GD, e também possue um custo computacional e complexidade de implementação reduzidos.

A análise para a formulação totalmente discretizada e condições de estabilidade independentes da discretização espacial serão obtidas neste trabalho.

1 Introdução

Métodos híbridos estabilizados vem se destacando nos últimos anos, pois estes possibilitam uma maior flexibilidade na construção de novos métodos de elementos finitos estáveis, precisos e localmente conservativos, bem como na implementação de algoritmos hp-adaptativos mais simples e computacionalmente eficientes. Além disso, estes métodos possuem uma conexão com os métodos de Galerkin Descontínuo (GD) [4] que permitem utilizar em sua análise numérica os mesmos argumentos são aplicados aos métodos GD, como foi feito em [1]. Para o problema elíptico, Arruda et al. [1] propuseram e analisaram uma formulação híbrida estabilizada onde o multiplicador de Lagrange é identificado como o traço da variável primal. Este método, denominado LDGC, consiste no acoplamento de problemas locais, onde a solução da variável primal é dada pelo método GD, com um problema global para os multiplicadores de Lagrange, tendo a continuidade imposta de forma fraca. Com base nestas ideias uma formulação híbrida estabilizada para a variável espacial combinada com um esquema de Euler implícito para a variável temporal foi proposta em [3] para o problema parabólico. Esta formulação evita as oscilações espúrias que surgem nos tempos iniciais de simulação quando, por exemplo, é usado o método de Galerkin usual para discretização espacial.

Então, neste trabalho apresentamos os principais resultados da análise numérica para essa formulação. Desta forma, algumas definições importantes que serão utilizadas nas análises de estabilidade neste trabalho são definidas. Seja a partição de elementos finitos $\mathcal{T}_h = \{\mathcal{K}\} := \{$ união de todos os elementos $\mathcal{K}\}$, $\mathcal{E}_h$ o conjunto de todas as arestas e dos elementos $\mathcal{K}$, $\mathcal{E}_h^0$ o conjunto das arestas interiores e $\mathcal{E}_h^\partial = \mathcal{E}_h \cap \partial\Omega$ o conjunto de arestas da fronteira de Ω. Denotaremos o espaço para a temperatura (variável espacial), como: $V_h = \{v_h \in L^2(\Omega) : v_h|_\mathcal{K} \in S_\mathcal{K}(\mathcal{K}) \quad \forall \mathcal{K} \in \mathcal{T}_h\}$, e para o multiplicador de Lagrange, considerando funções de interpolação contínuas, $M_h = \{\mu_h \in C^0(\mathcal{E}_h) : \mu_h|_e = p_l(e),\ \forall e \in \mathcal{E}_h^0,\ \mu_h|_e = 0,\ \forall e \in \mathcal{E}_h^\partial\}$, ou aproximações descontínuas $M_h = \{\mu_h \in L^2(\mathcal{E}_h) : \mu_h|_e = p_l(e),\ \forall e \in \mathcal{E}_h^0,\ \mu_h|_e = 0,\ \forall e \in \mathcal{E}_h^\partial\}$ onde $p_l(e)$ é o espaço de funções polinomiais de grau igual ou menor do que l em cada aresta e.

2 Resultados Principais

A formulação variacional totalmente discreta para o problema proposto se segue como: Para $n = 0, \cdots, N-1$, $\Delta t = T/N, t^n = n\Delta t$, encontre o par $[u_h^{n+1}, \lambda_h^{n+1}] \in V_h^k \times M_h^l$, para todo $[v_h, \mu_h] \in V_h^k \times M_h^l$, tal que

$$\sum_{\mathcal{K}} \left(\frac{u_h^{n+1} - u_h^n}{\Delta t}, v_h \right)_{\mathcal{K}} + a([u_h^{n+1}, \lambda_h^{n+1}], [v_h, \mu_h]) = F(t^{n+1}; v_h), \tag{1}$$

onde $a\left(\left[u_h^{n+1}, \lambda_h^{n+1}\right], [v_h, \mu_h]\right) = \sum_{\mathcal{K}} \left(\nabla u_h^{n+1}, \nabla v_h\right)_{\mathcal{K}} + \int_{\mathcal{E}_h} \left(\alpha\{\nabla v_h\} \cdot \left[u_h^{n+1}\right] - \left\{\nabla u_h^{n+1}\right\} \cdot [v_h]\right)\, ds +$
$\int_{\mathcal{E}_h^0} \left(\alpha\left[\nabla v_h\right]\left(\left\{u_h^{n+1}\right\} - \lambda_h^{n+1}\right) - \left[\nabla u_h^{n+1}\right]\left(\{v_h\} - \mu_h\right)\right)\, ds + \frac{\beta_0}{2h}\int_{\mathcal{E}_h} [v_h] \cdot \left[u_h^{n+1}\right]\, ds$
$+ \frac{2\beta_0}{h}\int_{\mathcal{E}_h} \left(\left\{u_h^{n+1}\right\} - \lambda_h^{n+1}\right)\left(\{v_h\} - \mu_h\right)\, ds, \quad \beta_0 > 0, \quad \alpha = \{-1, 1, 0\}$ e $F(t^{n+1}; v_h) = \sum_{\mathcal{K}} \int_{\mathcal{K}} f(v_h)$.
Os próximos lemas apresentam resultados de estabilidade para a formulação totalmente discretizada, em termos da norma da energia: $\|\,[v_h, \mu_h]\,\|_E^2 = \int_{\mathcal{K}} |\nabla v_h|^2\, dx + \sum_{e\in\mathcal{E}_h} h^{-1}\int_e |\,[v_h]\,|^2\, ds + \sum_{e\in\mathcal{E}_h} h^{-1}\int_e |\{v_h\} - \mu_h|^2\, ds$ e da norma L^2 definida de uma forma geral como: $\|f\| = \|f\|_0 = (f,f) = \int_\Omega f^2\, dx, \quad \forall f \in L^2(\Omega)$.

Lema 2.1. *Existe uma constante C independente do parâmetro de malha h e Δt, tal que,*

$$\|u_h^m\|^2 + \Delta t \sum_{n=1}^{N_T} \|[u_h^n, \lambda_h^n]\|_E^2 \le C\left(\|\tilde{u}_0\|^2 + \Delta t \sum_{n=1}^{N_T} \|f^n\|^2\right) \qquad \forall m \le N_T, \tag{2}$$

onde N_T é o número total de elementos da malha. Se $\beta_0 = 0$ para alguma face e, então este limite se aplica se $\Delta t < 1$.

Lema 2.2. *Assumindo que a solução exata $u(t,\cdot)$ para o problema satisfaça*

$$u(t,\cdot) \in H^1(\mathcal{T}_h), \quad \frac{\partial^2 u(t,\cdot)}{\partial t^2} \quad \in \quad L^2(\Omega) \quad \forall t \in [0,T], \tag{3}$$

onde $H^1(\mathcal{T}_h) = \left\{v \in L^2(\Omega) : v|_{\mathcal{K}} \in H^1(\mathcal{T}_h), \quad \forall\, \mathcal{K} \in \mathcal{T}_h\right\}$ é o espaço de Sobolev quebrado. Existe uma constante C independente do parâmetro de malha h e Δt tal que

$$\|u_h^m - u^m\| \quad \le \quad Ch\int_0^T \left\|\frac{\partial u^m}{\partial t}\right\| dt + C\Delta t\int_0^T \left\|\frac{\partial^2 u^m}{\partial t^2}\right\| dt \quad \forall m > 0, \tag{4}$$

$$\left(\Delta t \sum_{n=1}^{m} \|u_h^n - u^n\|_E^2\right)^{1/2} \quad \le \quad Ch\int_0^T \left\|\frac{\partial u^m}{\partial t}\right\| dt + C\Delta t\int_0^T \left\|\frac{\partial^2 u^m}{\partial t^2}\right\| dt \quad \forall m > 0. \tag{5}$$

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ESQUEMAS WENO-Z E WENO-Z+ DE TERCEIRA ORDEM PARA LEIS DE CONSERVAÇÃO HIPERBÓLICAS

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Resumo

As versões de terceira ordem do esquema WENO-Z [1] e de sua generalização WENO-Z+ [2] são apresentadas e analisadas. O desempenho de ambos os esquemas em problemas clássicos de leis de conservação é comparado com o de outros esquemas de mesma ordem. Os resultados demonstram a viabilidade do WENO-Z e a robustez e poder de resolução do WENO-Z+ de terceira ordem.

1 O esquema WENO-Z de terceira ordem

O esquema WENO-Z usa uma fórmula aprimorada para seus pesos [1] que reduz em muito a dissipação numérica comparada ao WENO clássico (doravante, WENO-JS) [3], possuindo uma maior resolução por tempo de processamento que este último [4]. Isto decorre do maior peso atribuído aos subestênceis menos suaves [1, 2]. Devido a uma análise superficial, pensava-se que o WENO-Z de terceira ordem não existisse. Todavia, uma análise mais aprofundada [5] demonstrou sua existência.

Considere uma malha uniforme $x_i = x_0 + i\Delta x$ e uma função $f(x)$ suave por partes tal que $f(x_i) = f_i$, $i = 0, \ldots, N$. Para i fixo, os pesos ω_0 e ω_1 do WENO-Z de terceira ordem são dados por

$$\alpha_k = d_k \left[1 + \left(\frac{\tau}{\beta_k + \varepsilon}\right)^p\right], \qquad \omega_k = \frac{\alpha_k}{\alpha_0 + \alpha_1}, \qquad k = 0, 1, \tag{1}$$

onde $\beta_0 = (f_i - f_{i-1})^2$, $\beta_1 = (f_{i+1} - f_i)^2$, $\tau = |\beta_0 - \beta_1|$, e ε e p são parâmetros do esquema. Neste estudo, $\varepsilon = 10^{-40}$ e $p = 1$. Para mais detalhes, veja [1, 2, 5] e suas referências.

Um resultado de Henrick et al. [6] nos diz que um esquema WENO terá ordem 3 se $\alpha_k = d_k(1 + \mathrm{O}(\Delta x^2))$ ($d_0 = 1/3$ e $d_1 = 2/3$ são os chamados pesos ideais). Sabe-se que $\beta_0 = \mathrm{O}(\Delta x^2)$, $\beta_1 = \mathrm{O}(\Delta x^2)$ e $\tau = \mathrm{O}(\Delta x^3)$. Numa análise grosseira, isto daria $\alpha_k = d_k(1 + \mathrm{O}(\Delta x))$, o que não é suficiente para garantir que o WENO-Z possua ordem 3. No entanto, a análise mais fina feita em [5] mostrou que $\alpha_k = d_k(1 + \mathrm{O}(\Delta x^2))$, o que garante que o esquema WENO-Z (Eq. (1)) é de fato de terceira ordem.

2 O esquema WENO-Z+ de terceira ordem

O WENO-Z+ é uma generalização do WENO-Z com um termo adicional que aumenta ainda mais a contribuição dos subestênceis menos suaves [2] (uma versão preliminar deste esquema foi apresentada no VIII ENAMA). Isto faz com que o WENO-Z+ tenha um poder de resolução ainda maior. Mais detalhes podem ser encontrados em [2, 7].

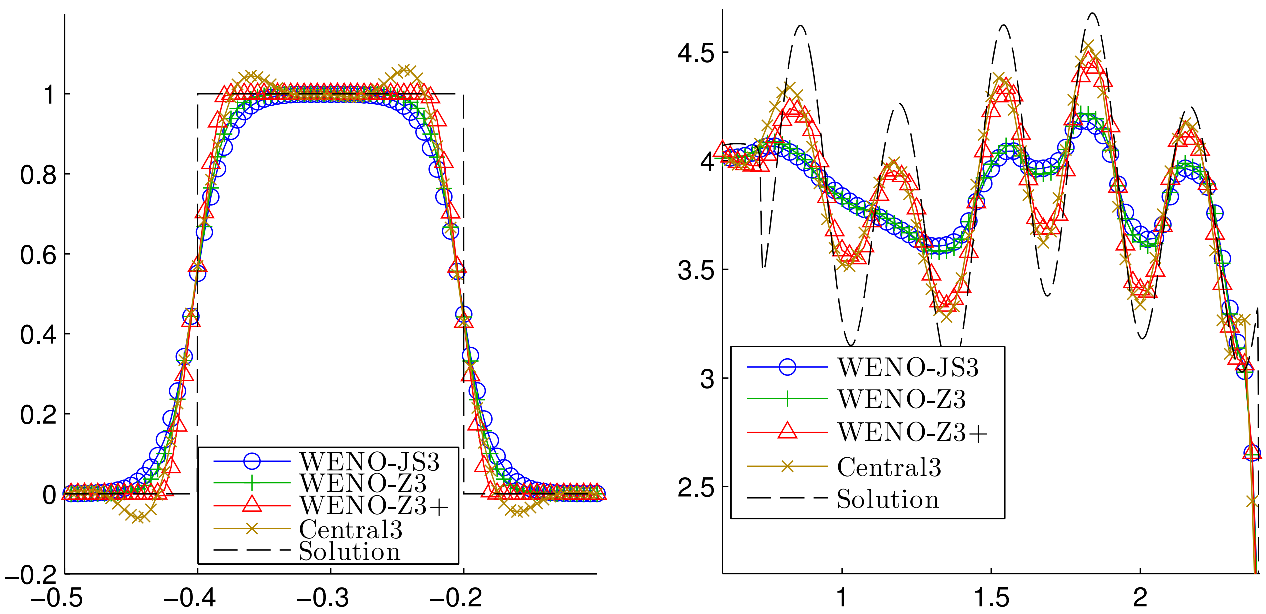
Os pesos ω_0 e ω_1 do WENO-Z+ de terceira ordem são dados por

$$\alpha_k = d_k \left[1 + \left(\frac{\tau + \varepsilon}{\beta_k + \varepsilon}\right)^p + \lambda \left(\frac{\beta_k + \varepsilon}{\tau + \varepsilon}\right)\right], \qquad \omega_k = \frac{\alpha_k}{\alpha_0 + \alpha_1}, \qquad k = 0, 1, \tag{2}$$

onde λ é o parâmetro de dissipação do esquema (neste estudo, $\lambda = 16$).

3 Resultados numéricos

Os resultados mostram que o esquema WENO-Z é apenas um pouco menos dissipativo que o WENO-JS. Já o esquema WENO-Z+ é bem menos dissipativo que estes dois, tendo uma resolução comparável à do Upstream Central mas sem apresentar oscilações espúrias perto de descontinuidades. Isto é evidente na figura abaixo: à esquerda, temos a solução da equação do transporte linear de uma onda quadrada; à direita, a solução do problema de Shu–Osher das equações de Euler (mais informações em [2]). Em ambos os casos, a malha possui $N = 400$ pontos. Outros resultados (não exibidos aqui) confirmam que ambos os esquemas possuem ordem 3.



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SUPERLINEAR PROBLEMS AND NONQUADRITICY CONDITION

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Abstract

In this talk we present some sufficient conditions to obtain compactness properties for the Euler-Lagrange functional of an elliptic equation. As an application we extend some existence and multiplicity results for superlinear problems.

1 Introduction

In this talk we consider the nonlinear elliptic equation

$$\begin{cases} -\Delta u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N, N \geq 3$, is a bounded smooth domain and $f \in C(\Omega \times \mathbb{R}, \mathbb{R})$ satisfies the standard subcritical growth condition

(f_0) there exist $a_1 > 0$ and $p \in (2, 2^*)$ such that

$$|f(x,s)| \leq a_1(1 + |s|^{p-1}), \text{ for any } (x,s) \in \Omega \times \mathbb{R}.$$

Under this condition the weak solutions of the problem are precisely the critical points of the C^1-functional

$$I(u) := \frac{1}{2}\int_\Omega |\nabla u|^2 \mathrm{d}x - \int_\Omega F(x,u)\mathrm{d}x, \ u \in H_0^1(\Omega),$$

where $F(x,s) := \int_0^s f(x,\tau)\mathrm{d}\tau$. Hence, we can use all the machinery of the Critical Point Theory to look for weak solutions. As it is well known, this theory is based on the existence of a linking structure and on deformation lemmas on the associated functional. In general, to be able to derive such deformation results, it is supposed that the functional satisfies some compactness condition. We use here the Cerami condition, which reads as: the functional I satisfies the Cerami condition at level $c \in \mathbb{R}$ ($(\mathrm{Ce})_c$ for short) if any sequence $(u_n) \subset H_0^1(\Omega)$ such that $I(u_n) \to c$ and $\|I'(u_n)\|_{H_0^1(\Omega)'}(1 + \|u_n\|) \to 0$ has a convergent subsequence.

Our main objective is presenting sufficient conditions to assure that the functional satisfies the Cerami condition. More specifically, we shall consider the *nonquadraticity condition at infinity* introduced by Costa and Magalhães [2], whose statement is

(NQ) setting $H(x,s) := f(x,s)s - 2F(x,s)$, we have that

$$\lim_{|s|\to\infty} H(x,s) = +\infty, \text{ uniformly for } x \in \Omega.$$

In the core result of this paper we show that the above condition and (f_0) are suffice to guarantee compactness for the functional I.

2 Main Results

At this stage we can state our main result:

Theorem 2.1. *Suppose that f satisfies (f_0) and (NQ). Then the functional I satisfies the Cerami condition at any level $c \in \mathbb{R}$.*

As an application of this theorem we prove some new results for the problem (P) in the case that f is superlinear at infinity and at the origin. Furthermore, we give an unnified approach for any superlinear elliptic problem using the nonquadraticiy condition. In order to better explain our results we recall that, in their seminal work, Ambrosetti and Rabinowitz [1] introduced the condition

(AR) there exist $\theta > 2$ and $s_0 > 0$ such that

$$0 < \theta F(x,s) \leq sf(x,s), \text{ for any } x \in \Omega, \, |s| > s_0.$$

A straightforward calculations shows that it provides $c_1 > 0$ such that $F(x,s) \geq c_1|s|^\theta$ for $|s|$ large. Thus, the problem is called *superlinear* in the sense that the primitive of f lives above any parabola of the type $c_2 s^2$. Unfortunately, there are several nonlinearities which are superlinear but do not satisfy the above inequality. For example, if we take $f(s) = |s|\ln(1+|s|)$, we can easily check that $\lim_{s\to+\infty} F(s)/s^\theta = 0$ for any $\theta > 2$. So, it is natural to ask if we can replace (AR) condition for a more natural one, namely

(SL) the following limit holds

$$\lim_{|s|\to+\infty} \frac{2F(x,s)}{s^2} = +\infty, \text{ uniformly for } x \in \Omega.$$

One of the main feature of condition (AR) is that it provides the boundedness of Palais-Smale sequences. In the past 40 years many authors tried to obtain solution in situations where (AR) is no longer valid. Instead, they consider the condition (SL) with extra assumptions (see [2, 5, 3, 4] and references therein). In the most of them, there are some kind of monotonicity assumption on the functions $F(x,s)$ or $f(x,s)/s$, or some convexity condition on the function $f(x,s)s - 2F(x,s)$.

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CONTROL PROBLEM OF MICROPOLAR FLOW WITH SLIP BOUNDARY CONDITION

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Abstract

An optimal boundary control problem for the micropolar fluid equations in 3D bounded domains, with mixed boundary conditions, is analyzed. By considering boundary controls for the velocity vector and the angular velocity of rotation of particles, the existence of optimal solutions is proved. By using the Theorem of Lagrange multipliers, an optimality system is derived. A second-order sufficient condition is also give.

1 Introduction

The stationary micropolar fluid model was introduced by Eringen in [1], and is given by the following system of partial differential equations:

$$\begin{cases} -(\nu+\nu_r)\Delta\mathbf{u}+(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p &= 2\nu_r \mathrm{rot}\,\mathbf{w}+\mathbf{f} \ \text{ in } \Omega, \\ \mathrm{div}\mathbf{u} &= 0 \ \text{ in } \Omega, \\ -(c_a+c_d)\Delta\mathbf{w}-(c_0+c_d-c_a)\nabla\mathrm{div}\,\mathbf{w}+(\mathbf{u}\cdot\nabla)\mathbf{w}+4\nu_r\mathbf{w} &= 2\nu_r\mathrm{rot}\,\mathbf{u}+\mathbf{h} \ \text{ in } \Omega, \end{cases} \tag{1}$$

where Ω is a smooth bounded domain of $\mathbb{R}^3$ with boundary Γ, and $\mathbf{u}$, p and $\mathbf{w}$ denote the velocity field, the pressure and the microrotational velocity of the fluid, respectively; the fields $\mathbf{f}$ and $\mathbf{h}$ represent external sources of linear and angular momentum, respectively; $\nu, \nu_r, c_0, c_a, c_d$ are positive constants which characterize isotropic properties of the fluid; in particular, ν denotes the viscosity, and ν_r, c_0, c_a, c_d are new viscosities connected with physical characteristics of the fluid. When the microrotation viscous effects are neglected, that is, ν_r, or $\mathbf{w}=\mathbf{0}$, model (1) reduces to the incompressible Navier-Stokes system. We consider the following boundary conditions:

$$\mathbf{u}=\begin{cases} \mathbf{g}_1 & \text{on } \Gamma_1, \\ \mathbf{u}_0 & \text{on } \Gamma_2^1, \end{cases} \quad \mathbf{w}=\begin{cases} \mathbf{w}_0 & \text{on } \Gamma_3, \\ \mathbf{g}_2 & \text{on } \Gamma_4, \end{cases} \quad \mathbf{u}\cdot\mathbf{n}=0,\ [D(\mathbf{u})\mathbf{n}+\alpha\mathbf{u}]_{\mathrm{tang}}=0 \ \text{ on } \Gamma_2^2. \tag{2}$$

Here, the boundary $\Gamma=\Gamma_1\cup\Gamma_2=\Gamma_3\cup\Gamma_4$, where $\Gamma_1\cap\Gamma_2=\emptyset$, $\Gamma_3\cap\Gamma_4=\emptyset$, and $\Gamma_2=\Gamma_2^1\cup\Gamma_2^2$ with $\Gamma_2^1\cap\Gamma_2^2=\emptyset$, where Γ_2^2 may be empty. The parts Γ_1 and Γ_4 may coincide or differ, but at leats, one of them must has positive measure; moreover, any one of them may coincide with Γ, independently of the other.

The fields $\mathbf{u}_0$, $\mathbf{w}_0$ are given on the respective parts Γ_2^1 and Γ_3, and $\mathbf{g}_1$, $\mathbf{g}_2$ describe the Dirichlet boundary control for $\mathbf{u}$ and $\mathbf{w}$ on the parts Γ_1 and Γ_4, which lie in closed convex sets $\mathcal{U}_1\subset\mathbf{H}^{1/2}(\Gamma_1)$ and $\mathcal{U}_2\subset\mathbf{H}^{1/2}(\Gamma_4)$, respectively. The term $[D(\mathbf{u})\mathbf{n}+\alpha\mathbf{u}]_{\mathrm{tang}}:=D(\mathbf{u})\mathbf{n}+\alpha\mathbf{u}-[(D(\mathbf{u})\mathbf{n}+\alpha\mathbf{u})\cdot\mathbf{n}]\mathbf{n}$ represents the tangential component of the vector $D(\mathbf{u})\mathbf{n}+\alpha\mathbf{u}$, where $D(\mathbf{u}):=\frac{1}{2}(\nabla\mathbf{u}+\nabla^T\mathbf{u})$ is the deformation tensor, $\mathbf{n}$ denotes the exterior normal vector to Γ, and $\alpha\geq 0$ is the friction coefficient which measures the tendency of the fluid to slip on Γ_2^2. The condition $[D(\mathbf{u})\mathbf{n}+\alpha\mathbf{u}]_{\mathrm{tang}}=0$, $\mathbf{u}\cdot\mathbf{n}=0$ on Γ_2^2 is a Navier friction boundary (see [4]). We consider the following minimization problem with constraints being the weak solutions of (1)-(2.2): Find $((\mathbf{u},\mathbf{w}),\mathbf{g}_1,\mathbf{g}_2)\in\mathbb{H}\times\mathcal{U}_1\times\mathcal{U}_2$ such that, for $2\leq p\leq 6$ and $2\leq q\leq 6$, the functional

$$\begin{aligned} J(\mathbf{u},\mathbf{w},\mathbf{g}_1,\mathbf{g}_2) \;&=\; \frac{\beta_1}{2}\|\mathrm{rot}\,\mathbf{u}-\mathbf{u}_d\|+\frac{\beta_2}{p}\|\mathbf{u}-\mathbf{u}_b\|_p^p+\frac{\beta_3}{q}\|\mathbf{w}-\mathbf{w}\|_q^q \\ &\quad+\frac{\beta_4\nu}{2}\|D(\mathbf{u})\|^2+\frac{\beta_5}{2}\|\mathbf{g}_1\|^2_{\mathbf{H}^{1/2}(\Gamma_1)}+\frac{\beta_6}{2}\|\mathbf{g}_2\|^2_{\mathbf{H}^{1/2}(\Gamma_4)}, \end{aligned} \tag{3}$$

is minimized, subject to $(\mathbf{u}, \mathbf{w})$ be weak solution of (1)-(2.2) (see [3]).

2 Main Results

Here are the most important results this presentation, in relation to the functional spaces that appear there, see [2] and [3] for more details.

Theorem 2.1. *Let $(\mathbf{f}, \mathbf{h}) \in \mathbb{X}'$ and assume $\mathbf{g}_1 \in \tilde{\mathbf{H}}_{00}^{1/2}(\Gamma_1)$, $\mathbf{u}_0 \in \tilde{\mathbf{H}}_{00}^{1/2}(\Gamma_2^1)$, $\mathbf{w}_0 \in \mathbf{H}^{1/2}(\Gamma_3)$, $\mathbf{g}_2 \in \mathbf{H}_{00}^{1/2}(\Gamma_4)$. Let $\nu, (c_a + c_d)$ large enough such that $\kappa > C$, where $C > 0$ depends only Ω and $\nu, \nu_r, c_0, c_a, c_d$. Then, there exists a unique weak solution of (1)-(2.2).*

Theorem 2.2. *Under the conditions of Theorem 2.1, the optimal control problem (3) has at least a optimal solution.*

Remark 2.1. *Under certain conditions of smallness on the data and by using the Lagrange method we derive the first-order optimality conditions and we obtain an optimality system for problem (3), we also give a second-order sufficient optimality condition.*

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STATIONARY SCHRÖDINGER EQUATIONS IN $\mathbb{R}^2$ WITH POTENTIALS UNBOUNDED OR VANISHING AT INFINITY AND INVOLVING CONCAVE NONLINEARITIES

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Abstract

We study the existence and multiplicity of solutions for the following class of stationary nonlinear Schrödinger equations:

$$-\Delta u + V(|x|)u = Q(|x|)f(u) + \lambda g(x,u), \quad x \in \mathbb{R}^2,$$

where λ is a nonnegative parameter, V and Q are unbounded or decaying radial potentials, the nonlinearity $f(s)$ may exhibit exponential growth and $g(x,s)$ is a concave term. The approach used here is based on a version of the Trudinger-Moser inequality, mountain-pass theorem and the Ekeland's variational principle in a suitable weighted Sobolev space.

1 Introduction

This paper is concerned with the existence and multiplicity of solutions for nonlinear elliptic equations of the form

$$-\Delta u + V(|x|)u = Q(|x|)f(u) + \lambda g(x,u), \quad x \in \mathbb{R}^2, \tag{1}$$

where the nonlinear term $f(s)$ is allowed to enjoy the exponential growth by mean of the Trudinger-Moser inequality (see [3, 4]), $g(x,s)$ may be a concave term, the radial potentials $V, Q : \mathbb{R}^2 \to \mathbb{R}$ are unbounded, singular at the origin or decaying to zero at infinity and λ is a nonnegative parameter. Explicitly, we make the following assumptions on the potential $V(|x|)$ and the weight function $Q(|x|)$:

$(V0)$ $V \in C(0,\infty)$, $V(r) > 0$ and there exists $a > -2$ such that $\liminf_{r\to+\infty} \dfrac{V(r)}{r^a} > 0$;

$(Q0)$ $Q \in C(0,\infty)$, $Q(r) > 0$ and there exist $b < (a-2)/2$ and $b_0 > -2$ such that

$$\limsup_{r\to 0^+} \frac{Q(r)}{r^{b_0}} < \infty \quad \text{and} \quad \limsup_{r\to+\infty} \frac{Q(r)}{r^{b}} < \infty;$$

$(Q1)$ there exist $q \in (1,2)$ and $1 < m \le 2/(2-q)$ such that $Q(|x|)^{1-m} \in L^1(\mathbb{R}^2)$.

In order to state our main results, we need to introduce some notation. In all the integrals we omit the symbol $\mathrm{d}x$. If $1 \le p < \infty$ we define the Lebesgue space $L^p(\mathbb{R}^2;Q) := \left\{ u : \mathbb{R}^2 \to \mathbb{R} \;\middle|\; u \text{ is measurable and } \int_{\mathbb{R}^2} Q(|x|)|u|^p < \infty \right\}$. Similarly, we can define $L^p(\mathbb{R}^2;V)$. Let $C_0^\infty(\mathbb{R}^2)$ be the set of smooth functions with compact support. Here we follow [1] define the energy space $H^1_{\mathrm{rad}}(\mathbb{R}^2;V)$ as the subspace of radially symmetric functions in the completion of $C_0^\infty(\mathbb{R}^2)$ with respect to the norm $\|u\| := \left[\int_{\mathbb{R}^2} \left(|\nabla u|^2 + V(|x|)|u|^2\right)\right]^{1/2}$. Equivalently, $H^1_{\mathrm{rad}}(\mathbb{R}^2;V)$ can be considered as the Sobolev space modeled in the Lebesgue space $L^2(\mathbb{R}^2;V)$ defined by $H^1_{\mathrm{rad}}(\mathbb{R}^2;V) := \left\{ u \in L^2_{\mathrm{rad}}(\mathbb{R}^2;V) : |\nabla u| \in L^2(\mathbb{R}^2) \right\}$, where the derivative above is understand in the sense of distributions. We use the notation $E = H^1_{\mathrm{rad}}(\mathbb{R}^2;V)$ and $\langle \cdot\,,\cdot \rangle$ for its inner product $\langle u, v \rangle := \int_{\mathbb{R}^2} \left(\nabla u \nabla v + V(|x|)uv\right), \quad u, v \in E.$

2 Main Results

Here, we are also interested in the case where the nonlinear term $f(s)$ has maximal growth on s which allows us to treat problem (1) variationally. Explicitly, in view of the classical Trudinger-Moser inequality, and following [2], we say that a function $f(s)$ has *exponential subcritical growth* at $+\infty$ if for all $\alpha > 0$ we have $\lim_{s\to+\infty} \dfrac{f(s)}{e^{\alpha s^2}} = 0$, and $f(s)$ has α_0-*exponential critical growth* at $+\infty$ for $\alpha_0 > 0$ if $\lim_{s\to+\infty} \dfrac{f(s)}{e^{\alpha s^2}} = \begin{cases} 0, & \forall\alpha > \alpha_0, \\ +\infty, & \forall \alpha < \alpha_0. \end{cases}$ Similarly we define exponential subcritical and critical growth at $-\infty$. Throughout this paper the following hypotheses on $f(s)$ and $g(x,s)$ will be assumed:

(f_0) $f : [0, +\infty) \to \mathbb{R}$ is continuous and $f(s) = o(s)$ as $s \to 0^+$;

(f_1) there exists $\theta > 2$ such that for all $s > 0$, $0 < \theta F(s) := \theta \int_0^s f(t)\, dt \leq s f(s)$;

(f_2) there holds $\lim_{s\to+\infty} \frac{sf(s)}{F(s)} = +\infty$;

(f_3) there exist $\theta_0 > 2$, $\mu_0 > 0$ and $s_0 > 0$ such that for all $0 \leq s \leq s_0$, $F(s) \geq \mu_0 s^{\theta_0}$;

(g_4) the function $g : \mathbb{R}^2 \times [0, +\infty) \to [0, +\infty)$ is continuous and there exists a constant $\beta_0 > 0$ such that for all $(x, s) \in \mathbb{R}^2 \times [0, +\infty)$, $g(x, s) \leq \beta_0 s^{q-1}$, where $q \in (1, 2)$ is given in the hypothesis $(Q1)$.

In the subcritical case, we have the following results:

Theorem 2.1. *Suppose that $f(s)$ has exponential subcritical growth and $(V0)$, $(Q0)$, $(Q1)$, $(f_0) - (f_3)$, (g_4) hold. Then there exists a constant $\lambda_0 > 0$ such that for all $\lambda \in [0, \lambda_0]$ problem* (1) *possesses a positive weak solution $u_\lambda \in E$.*

Theorem 2.2. *In addition to the hypotheses in Theorem 2.1, suppose that the function $g(x, s)$ satisfies the following condition (g_5) there exist constants $\beta_1 > 0$, $q_1 \in (1, 2)$ and $s_1 > 0$ such that for all $(x, s) \in \mathbb{R}^2 \times [0, s_1]$, $g(x, s) \geq \beta_1 s^{q_1-1}$. Then, for all $\lambda > 0$, problem* (1) *possesses a nonnegative solution $v_\lambda \in E$ which is different of u_λ when $\lambda \in (0, \lambda_0]$.*

Denote by $S > 0$ the best constant of the Sobolev embedding $E \hookrightarrow L^{\theta_0}(\mathbb{R}^2; Q)$, where θ_0 is given in (f_3). When $f(s)$ exhibits critical growth we obtain the following results:

Theorem 2.3. *Suppose that $f(s)$ has α_0-exponential critical growth at $+\infty$ and $(V0)$, $(Q0)$, $(Q1)$, (g_4), $(f_0) - (f_3)$ hold, with $\mu_0 > \left[\frac{\alpha_0(\theta_0-2)}{\alpha'}\right]^{(\theta_0-2)/2} S^{\theta_0/2}$, $\quad \alpha' := \min\{4\pi, 4\pi(1 + b_0/2)\}$. Then, for all $\lambda \in [0, \lambda_0]$, problem* (1) *has a nonnegative weak solution $u_\lambda \in E$.*

Theorem 2.4. *Under the hypotheses of Theorem 2.3, if in addition we assume (g_5), then for all $\lambda > 0$, problem* (1) *has a second nonnegative weak solution $v_\lambda \in E$, which is different from u_λ when $\lambda \in [0, \lambda_0]$.*

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EXISTENCE OF SOLUTIONS FOR A NONLOCAL P-LAPLACIAN EQUATION WITH SECOND KIND INTEGRAL BOUNDARY CONDITION

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Abstract

In this research we consider a nonlocal p-Laplacian equation with second kind integral boundary condition.We do not use the theory of integral operator to solve it.By means of Galerkin approximation we establish the existence of weak solutions for the problem.

1 Introduction

The purpose of this work is to investigate the existence of weak solutions to the class of nonlocal boundary value problem of the p-Kirchhoff type

$$\begin{aligned} -\Big[M\left(\int_\Omega |\nabla u|^p\,dx\right)\Big]^{p-1}\Delta_p u = b\left(\int_\Omega F(x,u)\right) f(x,u) \quad \text{in } \Omega, \\ u(x) = \int_\Omega K(x,y)u(y)\,dy \quad \text{on } \Gamma \end{aligned} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ $(N \geq 2)$with a smooth boundary $\partial\Omega$, M, b, K, and f are given fuctions, $1 < p < \infty$ and $F(x,s) = \int_0^s f(x,t)\,dt$.

The study of the Kichhoff type equations with boundary conditions of different type have attracted expensive interest in recent years (see [1][3]among many others).We note that it is difficult to apply a method based on the second kind integral operator (see [4, 2]) to solve equation (1).So we use the Galerkin method to attack it.

2 Main Result

We give the following hypotheses

(A_1) $M : [0,+\infty[\to [m_0,+\infty[$, $m_0 > 0$ is a continuous function .

(A_2) For all $s \in \mathbb{R}$, the function $f : \overline{\Omega}\times\mathbb{R} \to \mathbb{R}$ is a Caratheodory function satisfying:
there exists $C_1 > 0$ such that $|f(x,s)| \leq C_1(1 + |s|^{q-1}),\ p < q \leq p^+,\ s \in \mathbb{R}$

(A_3) b is a continuous function with $|b(s)| \leq C_2|s|^\beta$ for some $C_2 > 0$, with $p + 1 > \beta + q$

(A_4) $K : \overline{\Omega}\times\overline{\Omega} \to \mathbb{R}$ satisfies that $K(x,.), \frac{\partial K}{\partial x_i} \in L^q(\Omega)$ and

$$K(x) := \left(\int_\Omega |K(x,y)|^q\,dy\right)^{1/q} < \infty, \quad K_i(x) := \left(\int_\Omega |\frac{\partial K}{\partial x_i}|^q\,dy\right)^{1/q} < \infty, \text{ with } \sum_{i=1}^n \int_\Gamma K(x)^{q-1}K_i\,d\Gamma < c_p,\ c_p > 0$$

Our main result is the following theorem

Theorem 2.1. *Assume that $(A_1) - (A_4)$ hold.In addition we suppose that , there exist positive constants λ, η such that*

$$f(x,s)s \leq \lambda|s|^p + \eta|s| \quad \forall x \in \Omega, \forall s \in \mathbb{R}$$

then problem (1) has at least one weak solution.

Proof. We construct a special basis of a suitable separable subspace of $W^{1,p}(\Omega)$ and , then, we apply the Galerkin method .□

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ATRATORES PARA EQUAÇÕES DE ONDAS EM DOMÍNIOS DE FRONTEIRA MÓVEL

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Resumo

Este trabalho contém um estudo sobre equações de ondas fracamente dissipativas definidas em domínos de fronteira móvel. Nossa contribuição é dividida em três etapas. 1- Provamos que o problema munido da condição de fronteira de Dirichlet é bem posto no sentido Hadamard para soluções fortes e fracas. 2- Buscamos uma teoria de sistemas dinâmicos não autônomos para estudar o operador solução do problema como um processo.

1 Introdução

Seja $\{\mathcal{O}_t\}_{t\in\mathbb{R}}$ uma família de abertos limitados de $\mathbb{R}^N$ cuja fronteira $\partial\mathcal{O}_t$ pode variar continuamente com relação ao parâmetro temporal t. Então podemos definir a equação de ondas

$$\begin{aligned} u_{tt} - \Delta u + \eta u_t + g(u) = f(x,t), \quad x \in \mathcal{O}_t, \quad t \geq \tau \\ u(x,t) = 0, \quad x \in \partial\mathcal{O}_t, \quad t \geq \tau \\ u(x,\tau) = u_{0\tau}(x), \quad u_t(x,\tau) = u_{1\tau}(x), \quad x \in \mathcal{O}_\tau, \end{aligned} \tag{1}$$

onde $\tau \in \mathbb{R}$ é um tempo inicial. Assim o domínio da variável independente (x,t) é definido por

$$\widehat{\mathcal{D}}_\tau = \bigcup_{t\in(\tau,+\infty)} \mathcal{O}_t \times \{t\},$$

e possui fronteira lateral

$$\widehat{\mathcal{E}}_\tau = \bigcup_{t\in(\tau,+\infty)} \partial\mathcal{O}_t \times \{t\}.$$

Através de uma mudança de variáveis o problema original é transformado em um problema de domínio fixo,

$$\begin{aligned} v_{tt} - A(t)v + a_2 \cdot \nabla v_t + a_3 \cdot \nabla v + \eta v_t + g(v) = f(y,t), \quad (y,t) \in \mathcal{Q}_\tau \\ v(y,t) = 0, \quad (y,t) \in \partial\mathcal{O} \times \{t\} \\ v(y,\tau) = v_{0\tau}(y), \quad v_t(y,\tau) = v_{1\tau}(y), \quad y \in \mathcal{O}. \end{aligned} \tag{2}$$

Seque que a equação (1) a coeficientes constantes no domínio de fronteira móvel é equivalente a uma equação (2) de coeficientes dependentes de t no domínio fixo, que é por definição, não autônomo. Dessa forma o estudo da dinâmica assintótica de equações de onda em domínios de fronteira móvel conduz necessariamente ao estudo de sistemas dinâmicos não autônomos.

Definição 1.1. *Fixado $X_t = H_0^1(\mathcal{O}_t) \times L^2(\mathcal{O}_t)$. Uma família $\widehat{\mathcal{A}} = \{\mathcal{A}(t) : \mathcal{A}(t) \subset X_t,\ \mathcal{A}(t) \neq \emptyset,\ t \in \mathbb{R}\}$ é chamado atrator no sentido $\mathcal{D}$-pullback associado ao processo $\{U(t,\tau) : t \geq \tau\}$ se:*

1. *$\mathcal{A}(t)$ é um subconjunto compacto de X_t para todo $t \in \mathbb{R}$,*

2. $\widehat{\mathcal{A}}$ *atrai subconjuntos de* $\mathcal{D}$ *no sentido pullback, ou seja,*

$$\lim_{\tau \to -\infty} dist_t(U(t,\tau)D(\tau), \mathcal{A}(t)) = 0, \text{ para todo } \widehat{D} \in \mathcal{D} \text{ e para todo } t \in \mathbb{R},$$

3. $\widehat{\mathcal{A}}$ *é invariante, ou seja*

$$U(t,\tau)\mathcal{A}(\tau) = \mathcal{A}(t) \text{ para todo } -\infty < \tau \leq t < +\infty.$$

2 Resultados Principais

Teorema 2.1. *Supondo* $f \in H^1_{loc}(\mathbb{R}, L^2(\mathcal{O}_t))$*, condições dissipativas apropriadas e* $0 < \frac{\sigma(N-1)}{N-2} < \frac{\eta}{5}$*, para algum* $\sigma > 0$*. Então o processo* $\{U(t,\tau) : t \geq \tau\}$ *associado ao problema (1) possui um atrator no sentido* $\mathcal{D}_\sigma -$ *pullback.*

Prova: Para provar que o processo de evolução $\{U(t,\tau) : t \geq \tau\}$ associado ao problema (1) tem um atrator no sentido $\mathcal{D}_\sigma$-pullback é necessário verificar os seguintes lemas:

Lema 2.1. *A família de conjuntos*

$$\widehat{B} := \{B(t) : t \in \mathbb{R}\} = \{\{(u,u') \in X_t : \|(u,u')\|_{X_t} \leq R(t)\} : t \in \mathbb{R}\}$$

é um conjunto absorvente no sentido $\mathcal{D}_\sigma$*-pullback para o processo de evolução* $\{U(t,\tau) : t \geq \tau\}$*, onde*

$$R^2(t) = C_1 e^{-\sigma t} \int_{-\infty}^{t} e^{\sigma\theta} \|f(\theta)\|^2_{H^{-1}(\mathcal{O}_\theta)} d\theta + C_2.$$

Lema 2.2. *O processo de evolução* $\{U(t,\tau) : t \geq \tau\}$ *gerado pelas soluções fracas do problema (1) é assintoticamente compacto no sentido* $\mathcal{D}_\sigma$*-pullback.*

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ORBITAL STABILITY OF PERIODIC TRAVELING WAVES FOR DISPERSIVE MODELS

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Abstract

In this work we study the orbital stability of periodic traveling waves for two classes of nonlinear dispersive models. The first one includes the generalized Korteweg-de Vries (KdV) equation and the second one includes the Benjamin-Bona-Mahony (BBM) equation. By adapting the classical theory developed by Grillakis, Shatah and Strauss, [3], we bring to the light the orbital stability of a series of periodic waves for several models.

1 Introduction

This work brings new contributions on the orbital stability theory of periodic traveling-wave solutions for nonlinear dispersive models which can be written in the forms

$$u_t - \mathcal{M}u_x + (f(u))_x = 0 \tag{1}$$

and

$$u_t + \mathcal{M}u_t + (f(u))_x = 0 \tag{2}$$

where $f:\mathbb{R}\to\mathbb{R}$ is at least a C^1-function, in general representing the nonlinearity, and $\mathcal{M}$ is a differential or pseudo-differential operator. In particular, when $\mathcal{M} = -\partial_x^2$ and f is a function having the form $f(u) = u + u^{p+1}$, where $p \geq 1$ is an integer number, equation (1) is the well known generalized Korteweg-de Vries (KdV) equation

$$u_t + u_{xxx} + u_x + \partial_x(u^{p+1}) = 0, \tag{3}$$

whereas (2) reduces to the generalized regularized long-wave equation or generalized Benjamin-Bona-Mahony (BBM) equation

$$u_t - u_{xxt} + u_x + \partial_x(u^{p+1}) = 0. \tag{4}$$

Traveling-wave solutions for (1) and (2) are those solutions having the form

$$u(x,t) = \phi(x - ct), \tag{5}$$

where $\phi:\mathbb{R}\to\mathbb{R}$ is a suitable function and c is a real constant representing the wave speed. Thus, by replacing this waveform in (1) (similar considerations apply to (2)), we see that ϕ must be a solution of the differential or pseudo-differential equation

$$(\mathcal{M} + c)\phi - f(\phi) = -A, \tag{6}$$

where A appears as an integration constant. In this context, explicit periodic waves can be obtained by solving equation (6).

When the study of orbital stability of such explicit periodic waves was initiated, in almost all cases, the constant A above was set to be zero, see for example [1] and [2]. Our contributions here consist in removing such restriction by making some changes in the theories developed in [3] in order to attending those new periodic waves that appear when A is not zero. It is important to mention that some works in the same direction has been recently appeared, but the waves in question are not explicit, see [4] and [5]. At a first glance, the theory for the non-explicit waves is satisfactory, but some assumptions are not easy to be checked. On the other hand, in many cases, our assumptions can be checked in a very simple way.

2 Main Results

In order to stating our main theorem, we observe that (1) conserves the following functionals:

$$E(u) = \frac{1}{2}\int_0^L (u\mathcal{M}u - 2F(u))dx, \quad \text{with} \quad F(u) = \int_0^u f(s)ds, \tag{7}$$

$$Q(u) = \frac{1}{2}\int_0^L u^2 dx, \qquad V(u) = \int_0^L u\, dx, \qquad F_k := E + cQ + AV. \tag{8}$$

Let $\mathcal{L}_k = F_k''(\phi_k)$, where ϕ_k is the periodic wave we want to prove the orbital stability. Also, we consider the hypotheses:

(H0) There are an interval $J \subset \mathbb{R}$, C^1-functions $k \in J \mapsto c = c(k)$ and $k \in J \mapsto A = A(k)$, and a nontrivial smooth curve of L-periodic solutions for (6), $k \in J \mapsto \phi_k := \phi_{(c(k),A(k))} \in H^{s_2}_{per}([0,L])$.

(H1) The linearized operator $\mathcal{L}_k := \mathcal{L}_{(c(k),A(k))}$ has a unique negative eigenvalue, which is simple.

(H2) Zero is a simple eigenvalue of $\mathcal{L}_k$ with associated eigenfunction ϕ_k'.

(H3) The quantity Φ defined by $\Phi := \left\langle \mathcal{L}_k\left(\frac{\partial \phi_k}{\partial k}\right), \frac{\partial \phi_k}{\partial k}\right\rangle$ is negative.

(H4) It holds $M_k(\phi_k) \neq -\dfrac{\partial c}{\partial k} Q(\phi_k)$.

Our main theorem reads as follows.

Theorem 2.1. *Under assumptions* **(H0)-(H4)**, *for each $k \in J$, the periodic traveling wave ϕ_k is orbitally stable by the flow of* (1) *in the energy space.*

A similiar result holds for equation (2).

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TRANSFORMADA SUMUDO E O MODELO FRACIONÁRIO DE DINÂMICA POPULACIONAL

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Resumo

Apresentam-se aplicações da transfomada sumudu[1] em métodos numéricos, Método de Pertubação Homotópica (MPH) e o Método da Decomposição de Adomian (MDA) em um modelo de dinâmica populacional descrito por uma equação diferencial parcial fracionária não-linear. Mostramos que as soluções numéricas encontradas apresentam métodos eficientes do ponto de vista computacional.

1 Introdução

Considere a seguinte equação diferencial parcial fracionária não-linear para a densidade populacional ρ

$$\frac{d^\alpha \rho}{dt^\alpha} = \frac{\partial^2 \rho^2}{\partial x^2} + \frac{\partial^2 \rho^2}{\partial y^2} + h\rho^a(1 - r\rho^b), \quad t \geq 0, x, \, y \in \mathbb{R}, \tag{1}$$

onde $0 < \alpha \leq 1$ para $t \geq 0$ e $a, \; b, \; h, \; r, \; x, \; y \in \mathbb{R}$ e com condição inicial dada por $\rho(x, y, 0)$. Se $\alpha = 1$ a equação (1) se reduz a um modelo clássico de dinâmica populacional [1, 3].

Para a construção do método para solução da equação(1) precisamos da definição da derivada fracionária de Caputo e da transforamda sumudu, que é uma transformada integral similiar à clássica transformada de Laplace, introduzida no início da década de 90 por Watugala [4] para resolver equações diferenciais envolvendo problemas de controle em engenharia.

Definição 1.1. *A derivada fracionária de Caputo é dada por com $\alpha > 0$ e $m \in \mathbb{N}$*

$$ {}_0^*D_t^\alpha f(t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^t \frac{f^{(m)}(\tau) d\tau}{(t-\tau)^{\alpha+1-m}}, & m-1 < \alpha < m, \\ \\ \dfrac{d^m}{dt^m} f(t), & \alpha = m. \end{cases} \tag{2}$$

com $\alpha > 0$ e $m \in \mathbb{N}$.

Definição 1.2. *Considere o conjunto de funções*

$$A = \{f(t) : \exists M, \tau_1, \tau_2 > 0, |f(t)| < M e^{\frac{t}{\tau_j}}, \; se \;\; t \in (-1)^j \times [0, \infty)\},$$

tal que para $f(t) \in A$ temos que a transformada sumudu de uma função $f(t) \in A$ é dada por

$$S[f(t); u] = G_s(u) = \int_0^\infty f(u \; t) \, e^{-t} \, dt, \quad u \in (-\tau_1, \tau_2). \tag{3}$$

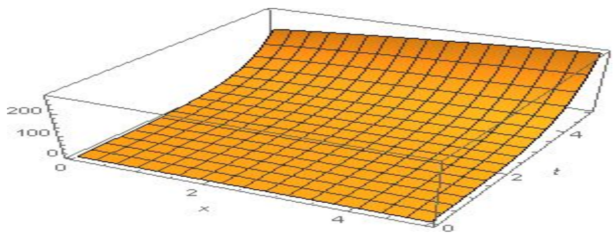
[1]Sumudu é uma palavra no idioma sinhala que significa suave.

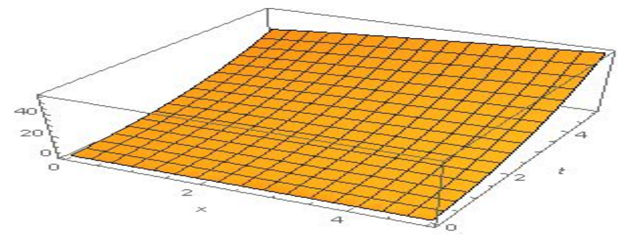
2 Resultados Principais

Utilizamos a transformada sumudu aplicada aos MPH [5] e MDA [2] para resolver equações diferenciais parciais fracionárias relacionadas à dinâmica populacional.

As técnicas propostas pelos método de pertubação homotópica usando a transformada sumudu (MPHS) e método da decomposição de Adomian usando a transformada sumudu (MADS) se mostraram eficazes produzindo os mesmos resultados e de acordo com as soluções exatas para modelos conhecidos. O esforço computacional em ambos os métodos, dentro do modelo analisado, foi otimizado mantendo a precisão dos resultados.

Esses métodos são importantes ferramentas para a análise da dinâmica populacional fracionária e também para modelos que apresentam termos não-lineares, como por exemplo a equação fracionária de Fokker-Planck [6] e também a equação fracionária de Navier-Stokes [7]. Nas figuras, a seguir, vê-se a solução exata e aproximada respectivamente.





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ANÁLISE TEÓRICA E COMPUTACIONAL DE UMA EQUAÇÃO DE SCHRÖDINGER NÃO LINEAR COM FRONTEIRA MÓVEL

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Resumo

Neste trabalho, estudamos os aspectos matemáticos e numéricos de uma equação de Schrödinger não linear em domínio não-cilíndrico. Uma mudança da variável permitirá transformar o problema em estudo em um equivalente, com domínio cilíndrico, permitindo obter a existência e a unicidade da solução utilizando o método de Galerkin e resultados de compacidade. A simulação numérica será realizada para o caso unidimensional. Vamos aplicar o método dos elementos finitos no espaço associado e o método de diferenças finitas na parte temporal, para obter uma solução numérica aproximada. Além disso, faremos uma análise da taxa de convergência dos métodos aplicados. Isso nos permitirá estudar o comportamento da solução aproximada no domínio não cilíndrico.

1 Introdução

Sejam $T > 0$ e Ω um conjunto aberto limitado do $\mathbb{R}^n$, com fronteira regular Γ de classe C^2. Neste trabalho, provamos a existência e unicidade de solução e apresentamos uma simulação numérica, no caso unidimensional, para o seguinte problema de valor inicial com fronteira móvel:

$$\left| \begin{array}{l} u'(x,t) - i\Delta u(x,t) + |u(x,t)|^{\rho}\, u(x,t) = \hat{f} \;\; \text{em} \;\; \hat{Q}; \\ u(x,t) = 0 \;\; \text{em} \;\; \hat{\Sigma}; \qquad\qquad\qquad\qquad (i^2 = -1) \\ u(x,0) = u_0(x) \;\; \text{em} \;\; \Omega_0; \end{array} \right. \tag{1}$$

em que o domínio $\hat{Q} = \{(x,t) \in \mathbb{R}^n \times]0,T[\,; x \in \Omega_t\}$ com $\Omega_t = \{x \in \mathbb{R}^n; x = k(t)y, y \in \Omega\}$, cuja fronteira lateral $\hat{\Sigma} = \bigcup_{0<t<T} (\Gamma_t \times \{t\})$. Sendo Γ_t a fronteira de Ω_t. Além disso, $\rho > 0$.

Usando a mudança de variável: $\tau(x,t) = (x/k(t), t)$, transformamos este problema em um outro equivalente, mas em domínio cilíndrico. Possibilitando o uso de técnicas de compacidade para assim obtermos a existência e unicidade de solução pelo método de Galerkin.

Consideramos as seguintes hipóteses: (H1) $k \in W^{2,\infty}_{loc}([0,\infty[); k(t) \geq k_0 > 0$ para todo $t \geq 0$; (H2) $0 \leq \rho < \infty$ se $n = 1,2$ e $0 \leq \rho \leq \frac{2}{n-2}$ se $n \geq 3$.

2 Resultado Principal

Teorema 2.1. *Sob as hipóteses (H1) e (H2), considerando o dado inicial* $u_0 \in H^1_0(\Omega_0)$ *e* $\hat{f} \in L^2(0,T; H^1_0(\Omega_t))$, *existe uma única função* $u : \hat{Q} \to \mathbb{C}$, *satisfazendo as condições:*

$$\begin{array}{l} u \in L^{\infty}(0,T; H^1_0(\Omega_t)) \cap L^p(0,T; L^p(\Omega_t)), \;\; \textit{com}\; p = \rho + 2; \\ u' \in L^{p'}(0,T; H^{-1}(\Omega_t)), \;\; \textit{com}\; p' = \frac{\rho+2}{\rho+1}; \\ u' - i\Delta u + |u|^{\rho}\, u = \hat{f} \quad \textit{em} \quad L^{\frac{\rho+2}{\rho+1}}(0,T; H^{-1}(\Omega_t)); \\ u(0) = u_0 \;\; \textit{em}\; \Omega_0. \end{array} \tag{2}$$

Demonstramos a existência e a unicidade de solução do problema principal garantindo a existência e a unicidade da solução do problema equivalente em domínio cilíndrico.

3 Simulação Numérica

Para a simulação numérica, consideramos o problema para o caso unidimensional. Aplicamos o método de elementos finitos na parte espacial, resultando em um sistema de equações diferenciais ordinárias (EDO) não-linear. Aplicamos o método de diferenças finitas de ordem dois no sistema de EDO, resolvendo o sistema não-linear resultante pelo método de Newton, para cada passo de tempo.

Consideramos a fronteira dada pelas funções $\alpha(t)$ e $\beta(t)$ definidas como $\alpha(t) = (\cos(2\pi t) - 3)/4$ e $\beta(t) = (16t+1)/(16t+2)$, e como solução exata a função $v(y,t) = \sin(\pi y)(\cos(\pi t) + 2) + i\sin(\pi y)(\cos(\pi t) + 2)$.

A tabela 1 apresenta a taxa de convergência da solução numérica para $\rho = 0, \ldots, 3$. Considerando as discretizações $\Delta y = \Delta t$, os resultados numéricos mostram que a ordem de convergência do erro associado aos métodos aplicados é quadrática no tempo e no espaço.

	p_{v_m}			
$\Delta y = \Delta t$	$\rho = 0$	$\rho = 1$	$\rho = 2$	$\rho = 3$
2^{-6}	2.0002	2.0007	2.0028	2.0085
2^{-7}	2.0001	2.0002	2.0007	2.0022
2^{-8}	2.0492	2.0014	1.9999	2.2190
2^{-9}	2.0205	2.0006	2.0000	2.0191
2^{-10}	2.0058	2.0001	2.0000	2.0043

Tabela 1: Taxa de convergência do erro da solução numérica para $\rho = 0, \ldots, 3$.

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ON THE UNIQUENESS AND CONDITIONAL STABILITY IN SOURCE RECONSTRUCTION FOR MODIFIED HELMHOLTZ EQUATIONS

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Abstract

In this work we present a result that guarantee the stability in the reconstruction of characteristic sources in modified Helmholtz model ($\lambda = \kappa^2 > 0$), under a separation condition over the Neumann data. When we make a priori assumptions about the unknown terms (the source term in our case) and prove a stability result, we say that this is *conditional stability* result.

1 Introduction

The inverse source problem for modified Helmholtz equation can be stated as: given $(g, g_\eta) \in H^{\frac{1}{2}}(\Omega) \times H^{-\frac{1}{2}}(\Omega)$ to find (u, f) in the model problem

$$\begin{cases} (-\Delta + \kappa^2)u = f, & \text{in } \Omega, \\ u = g, & \text{on } \partial\Omega, \\ \frac{\partial u}{\partial \eta} = g_\eta, & \text{on } \partial\Omega. \end{cases} \tag{1}$$

By defining

$$\mathcal{R}[f](v) = \int_\Omega v f dx, \tag{2}$$

and using Green's Identity we obtain the Reciprocity Functional formulation

$$\mathcal{R}[f](v) := \int_{\partial\Omega} u\frac{\partial v}{\partial \eta} - v\frac{\partial u}{\partial \eta} d\sigma = \int_{\partial\Omega} g\frac{\partial v}{\partial \eta} - v g_\eta d\sigma, \tag{3}$$

for all test functions $v \in \mathcal{H}_{-\Delta+\kappa^2}(\Omega) = \{v \in H^1(\Omega); (-\Delta + \kappa^2)v = 0\}$. In this way, the inverse source problem is determine a function f such that

$$\int_\Omega v f dx = \mathcal{R}[f](v) = \int_{\partial\Omega} g\frac{\partial v}{\partial \eta} - v g_\eta d\sigma,$$

for all $v \in \mathcal{H}_{-\Delta+\kappa^2}(\Omega)$.

The simultaneous prescription of Dirichlet and Neumann,i.e., the Cauchy data at the boundary does not give sufficient information for uniquely solve the problem when data are consistent with the direct Dirichlet, Neumann or mixed problems related with these over prescribed data. Also, it can be proved that problems with different Cauchy data consistents with this model are equivalent in the sense that they formulates the same inverse source problem [1]. In this way, without loss of generality, we need investigate only the $g = 0$ case.

When $\kappa = 0$, we have the classical inverse source problem for the Laplace operator and uniqueness for consistent data can be proved for many class of source such as harmonic, characteristic star shaped support. For $\kappa \in \mathcal{R}$ we have the modified Helmholtz model. In this case numerical reconstruction of characteristic supported star shaped source can be done with the Method of Fundamental Solution, but we have difficult in prove the existence of solution in the case o consistent data. Fortunately, with help of the reverse Holder inequality, we can prove stability for this inverse source problem, that is, if two characteristic star shaped source problems with zero Dirichlet data have different Neumann data, then they have different supports.

Note that in a practical problem, we does not have knowledge about the original Neumann data g_η but only artificial Neumann data g_η^{art}, which is a data with measurement or numerical error.

2 Main Results

2.1 The Conditional Stability for Boundary Support Reconstruction

In the next theorem, we present a stability result for boundary support reconstruction in inverse problem under a *separation condition* over the Neumann data.

Definition 2.1. *Let $f_1 = \chi_{\omega_1}$ and $f_2 = \chi_{\omega_2}$ be two sources. Suppose that f_1 and f_2 generate the Neumann data on the boundary g_η^1 and g_η^2, respectively, with null Dirichlet data. We say that the Neumann data are separated, or satisfies a separation condition, if*

$$\|g_\eta^1 - g_\eta^2\|_{H^{-1/2}(\partial\Omega)} > 0. \tag{1}$$

Theorem 2.1 (Conditional Stability in L^1 and L^∞)**.** *Consider two characteristic sources $f_1(x) = \chi_{\omega_1}(x)$ and $f_2(x) = \chi_{\omega_2}(x)$, where ω_1 and ω_2 are open, connected and bounded subsets of Ω. Let $R_1, R_2 : \mathbb{S}^{N-1} \to \mathbb{R}_*^+$ be parametrizations of $\partial\omega_1$ and $\partial\omega_2$, respectively, with $R_1, R_2 \in L^1(\mathbb{S}^{N-1})$. If the Neumann data generated by the sources are separated , then there exists constants $C_1 = C_1(N) > 0$ and $C_\infty = C_\infty(N) > 0$, such that*

$$\|R_2 - R_1\|_{L^1(\mathbb{S}^{N-1})} \leq C_1 \|\mathcal{R}[\chi_{\omega_1}] - \mathcal{R}[\chi_{\omega_2}]\|_{L^1(\mathbb{S}^{N-1})}^{\frac{1}{N}}, \tag{2}$$

$$\|R_2 - R_1\|_{L^1(\mathbb{S}^{N-1})} \leq C_\infty \|\mathcal{R}[\chi_{\omega_1}] - \mathcal{R}[\chi_{\omega_2}]\|_{L^\infty(\mathbb{S}^{N-1})}^{\frac{1}{N}}, \tag{3}$$

where

$$\|R_2 - R_1\|_{L^1(\mathbb{S}^{N-1})} = \int_{\mathbb{S}^{N-1}} |R_1(\theta) - R_2(\theta)| d\theta$$

$$\|\mathcal{R}[\chi_{\omega_1}] - \mathcal{R}[\chi_{\omega_2}]\|_{L^1(\mathbb{S}^{N-1})} = \int_{\mathbb{S}^{N-1}} |\mathcal{R}[\chi_{\omega_1}](\varphi) - \mathcal{R}[\chi_{\omega_2}](\varphi)|\, d\varphi.$$

$$\|\mathcal{R}[\chi_{\omega_1}] - \mathcal{R}[\chi_{\omega_2}]\|_{L^\infty(\mathbb{S}^{N-1})} = \operatorname{ess}\sup_{\varphi \in \mathbb{S}^{N-1}} |\mathcal{R}[\chi_{\omega_1}](\varphi) - \mathcal{R}[\chi_{\omega_2}](\varphi)|.$$

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A UNIFIED APPROACH TO A PRIORI ERROR ESTIMATION OF APPROXIMATIONS BY CLASSICAL DISCRETIZATION METHODS

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Abstract

In the framework of the numerical solution of boundary value problems, three families of discretization methods play a leading role, namely, the finite-difference (FD) method, the finite-element (FE) method and the finite-volume (FV) method. In order to derive a priori error estimates for approximations generated by these methods, specialists in each one of them use a specific technique of analysis, supposedly better suited to the corresponding approach. However, at least for linear problems, the Lax-Richtmyer Equivalence Theorem [1] introduced for studying the FD method, provides a convenient mathematical environment for a unified treatment of the three methods, as shown in [3]. The technique allowing for this consists of studying approximate problems with more general (given) right sides, which mimic in a sense the operators appearing on equations' left side.

1 Introduction

Although the technique of analysis advocated in this work applies to a very wide class of problems (see e.g. [3]), for the sake of clarity and brevity we present it in the framework of the very simple model linear ODE (1), namely,

$$-[pu']' + qu = f \quad \text{in } (0,1) \tag{1}$$

with the boundary conditions $u(0) = 0$ and $u'(1) = 0$, where p, q and f are given real continuous functions in $L^\infty(0,1)$. p is assumed to be strictly positive in $[0,1]$ and such that $p' \in L^\infty(0,1)$, while q is non negative in $[0,1]$. Referring to [3] for Sobolev space notations, these assumptions guarantee existence and uniqueness of $u \in W^{2,\infty}(0,1)$.

Formulation (1) is the usual form adopted to solve the problem by the FD method. On the other hand the FE method is applied to problem's equivalent (weak) variational form, namely,

$$\text{Find } u \in V \text{ such that } \int_0^1 [pu'v' + quv]dx = \int_0^1 fvdx \quad \forall v \in V, \tag{2}$$

where $V = \{v|\ v \in H^1(0,1),\ v(0) = 0\}$. The FV method in turn applies to problem's conservative form, namely,

$$\text{Find } u \in C^1[0,1] \text{ such that } [pu'](a_\omega) - [pu'](b_\omega) + \int_0^1 \chi_\omega qudx = \int_0^1 \chi_\omega fdx \quad \forall \omega \in \Omega \text{ and } u(0) = u'(1) = 0, \tag{3}$$

Ω being the set of all 'control volumes' $\omega = [a_\omega, b_\omega]$ contained in $[0,1]$, χ_ω being the characteristic function of ω.

2 Error estimation for the FD, FE and FV methods

Let n be an integer greater than or equal to 2, and let us be given a FD grid $\mathcal{G} := \{x_0, x_1, \ldots, x_n\}$ with $0 = x_0 < x_1 < \ldots < x_{n-1} < x_n = 1$. Henceforth we set $u_0 = 0$, $u_{n+1} = u_n$, $x_{n+1} = 2x_n - x_{n-1}$, $f_i = f(x_i)$

and $p_{i-1/2} = p([x_i + x_{i-1}]/2)$ for $i = 1, 2, \ldots, n$, and $p_{n+1/2} = p(x_n)$. We also denote by h the maximum of $h_i := x_i - x_{i-1}$ over $i \in \{1, 2, \ldots, n+1\}$, and set $h_{i+1/2} := (h_i + h_{i+1})/2$ for $i = 1, 2, \ldots, n-1$ and $h_{n+1/2} = h_n/2$. The simplest FD method consists of finding approximation u_i of $u(x_i)$ for $i = 1, 2, \ldots, n$ such that,

$$\left[p_{i-1/2} \frac{u_i - u_{i-1}}{h_i} - p_{i+1/2} \frac{u_{i+1} - u_i}{h_{i+1}} \right] / h_{i+1/2} + q(x_i) u_i = f_i \tag{4}$$

Multiplying both sides of (4) by $u_i h_{i+1/2}$, and adding up the resulting relations from $i = 1$ through $i = n$, after straightforward manipulations (cf. [3]) the left side is found to be bounded below by $p_{min} |\vec{u}_h|^2_{1,h}$, where $\vec{u}_h := [u_1, u_2, \ldots, u_n]^{\mathcal{T}}$, $p_{min} := \min_{x \in [0,1]} p(x)$ and $|\cdot|_{1,h}$ denotes the discrete semi-norm of $H^1(0,1)$ given by $|\vec{u}_h|^2_{1,h} = \sum_{i=1}^n (u_i - u_{i-1})^2 / h_i$. The right side in turn equals $\sum_{i=1}^n h_{i+1/2} f_i u_i$. Then further introducing the discrete L^2-norm $\| \vec{f}_h \|_{0,h}$ of $\vec{f}_h = [f_1, f_2, \ldots, f_n]^{\mathcal{T}}$ given by $\| \vec{f}_h \|^2_{0,h} = \sum_{i=1}^n f_i^2 h_{i+1/2}$, and combining the discrete Poincaré inequality $\| \vec{u}_h \|_{0,h} \leq |\vec{u}_h|_{1,h}$ with the Cauchy-Schwarz inequality, we derive the stability result:

$$|\vec{u}_h|_{1,h} \leq [p_{min}]^{-1} \| \vec{f}_h \|_{0,h} \,. \tag{5}$$

Now in a classical manner, we replace the components of $\vec{f}_h$ in (4) by those of the residual vector $\vec{r}_h(u)$ between both sides of (4), when $\vec{u}_h$ is substituted by $\vec{U}_h = [u(x_1), u(x_2), \ldots, u(x_n)]^{\mathcal{T}}$. Assume that $u \in C^2[0,1]$ and $pu' \in C^1[0,1]$. Since $\| \vec{r}_h(u) \|_{0,h}$ is bounded above by h times a constant multiplied by terms involving higher order derivatives of both u and p resulting from Taylor expansions, the stability inequality (5) immediately yields an estimate for the error $|\vec{u}_h - \vec{U}_h|_{1,h}$ in terms of an $O(h)$ for the FD method.

Next we associate with the FD grid $\mathcal{G}$ the FE mesh $\mathcal{T} := \{[x_{i-1}, x_i]\}_{i=1}^n$. In the simplest FE method to solve (1) we search for an approximation u_h of u in the subspace V_h of V consisting of continuous functions whose restriction to every $T \in \mathcal{T}$ is linear. However, although in the problem to solve the right side is given by $\int_0^1 f v dx$, we assume that we are solving the more general problem for another given function $g \in L^2(0,1)$, namely,

$$\text{Find } u_h \in V_h \text{ such that } \int_0^1 [p u_h' v' + q u_h v] dx = \int_0^1 [g v' + f v] dx \quad \forall v \in V_h, \tag{6}$$

Using the Poincaré inequality in $(0,1)$, and denoting by $\| \cdot \|_0$ the norm of $L^2(0,1)$, it is easy to see that the following stability result holds for (6) in the semi-norm $|\cdot|_1$ of $H^1(0,1)$:

$$|u_h|_1 \leq \sqrt{5} [p_{min}]^{-1} [\| g \|_0^2 + \| f \|_0^2]^{1/2}, \tag{7}$$

When we replace in (6) u_h by the function $U_h \in V_h$ interpolating u at the mesh nodes x_i for $i = 0, 1, \ldots, n$, the difference between both sides of this equation for the true problem, i.e. for $g \equiv 0$, is easily seen to be given by $\int_0^1 [p(U_h - u)' v' + q(U_h - u) v] dx$. It immediately follows from (7) that the deviation $|u_h - U_h|_1$ is bounded above by the H^1-norm of the interpolation error function $u - U_h$, multiplied by $\sqrt{5}$ times the maximum between the L^∞-norms of p and q divided by p_{min}. Then the well-known estimates for such an interpolation error in terms of an $O(h)$, together with the triangle inequality, lead to the conclusion that the error $|u_h - u|_1$ is also an $O(h)$.

An analogous trick allows us to derive very easily first order error estimates in the discrete H^1 semi-norm for the popular cell-centered FV method (see e.g. [3]). More precisely we consider a modified right side by the addition of incoming and outgoing fluxes at control volume ends, in the balance equation (3), taking $\omega = T$, $\forall T \in \mathcal{T}$, thereby mimicking $[pu'](x_{i-1})$ and $-[pu'](x_i)$. For more details the authors refer to [3].

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ANÁLISE DE ESTABILIDADE E CONVERGÊNCIA DE UM MÉTODO ESPECTRAL TOTALMENTE DISCRETO PARA SISTEMAS DE BOUSSINESQ

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Resumo

Neste trabalho, apresentamos a análise de estabilidade da família de sistemas de Boussinesq com o objetivo de determinar a influência de seus parâmetros (a, b, c, d) na eficiência e precisão do método espectral de colocação de Fourier aplicado na variável espacial, juntamente com o método de Runge Kutta de quarta ordem aplicado na variável temporal. São identificadas quais regiões de parâmetros são as mais adequadas para a obtenção de uma solução numérica consistente no caso linear, e essa classificação também pode ser observada no caso não linear. Para os casos em que a condição de estabilidade é dada por $\Delta t \leq C\Delta x$, é feito um estudo teórico e numérico da convergência dos métodos numéricos aplicados. Experimentos numéricos são fornecidos com o objetivo de verificar a estabilidade das soluções do problema linear em cada região de parâmetros, e confirmar a ordem de precisão das soluções do problema não linear.

1 Introdução

A família (a, b, c, d) de sistemas de Boussinesq foi obtida e analisada em [1], como um modelo assintótico obtido a partir das equações de Euler para ondas de pequena amplitude e grande comprimento de onda. É dada por:

$$\eta_t + u_x + (u\eta)_x + au_{xxx} - b\eta_{xxt} = 0, \tag{1}$$

$$u_t + \eta_x + uu_x + c\eta_{xxx} - du_{xxt} = 0, \tag{2}$$

onde

$$a = \frac{1}{2}\Big(\theta^2 - \frac{1}{3}\Big)\lambda, \quad b = \frac{1}{2}\Big(\theta^2 - \frac{1}{3}\Big)(1-\lambda), \quad c = \frac{1}{2}\Big(1-\theta^2\Big)\mu, \quad d = \frac{1}{2}\Big(1-\theta^2\Big)(1-\mu)$$

com λ, $\mu \in \mathbb{R}$ e $0 \leq \theta \leq 1$.

O sistema (1)-(2) descreve a propagação não linear de ondas de pequena amplitude em um canal. As variáveis dependentes $\eta = \eta(t, x)$ e $u = u(t, x)$ representam, respectivamente, a altura da superfície livre do fluido em relação a superfície de repouso e a velocidade horizontal do fluido em algum ponto acima do fundo do canal. Até onde sabemos, a maior parte dos resultados numéricos para esses sistemas são concentrados em escolhas específicas dos parâmentros (a, b, c, d), como por exemplo em [2].

Embora esses resultados respondam a uma importante pergunta levantada em [1], sobre a construção de métodos numéricos eficientes e precisos para a obtenção de soluções aproximadas de PVICs relacionados com esses sistemas, seria útil explorar de maneira mais abrangente e consistente a escolha desses parâmetros na construção destes esquemas numéricos.

2 Resultados Principais

Analisamos os sistemas de Boussinesq dados por (1)-(2) num domínio $\Omega = [-L, L]$ com condições de contorno periódicas em Ω. Obtivemos os seguintes resultados:

Teorema 2.1. *O sistema discretizado obtido a partir de (1)-(2) é estável se $\Delta t \leq CN^{-\ell}$ para alguma constante positiva C e $\ell \in \{0, 1, 2, 3\}$, onde Δt é o tamanho do passo de tempo e N é o número de pontos considerados na discretização espacial. A relação entre o valor de ℓ e os parâmetros (a, b, c, d) podem ser observados na Tabela 1.*

$\ell = 0$	$\ell = 1$
$a < 0, b > 0, c = 0, d > 0$	$a < 0, b > 0, c < 0, d > 0$
$a = 0, b > 0, c < 0, d > 0$	$a < 0, b > 0, c = 0, d = 0$
$a = 0, b > 0, c = 0, d = 0$	$a = 0, b > 0, c < 0, d = 0$
$a = 0, b > 0, c = 0, d > 0$	$a < 0, b = 0, c = 0, d > 0$
$a = 0, b = 0, c = 0, d > 0$	$a = 0, b = 0, c < 0, d > 0$
	$a = 0, b = 0, c = 0, d = 0$
	$a = c > 0, b > 0, d > 0$
$\ell = 2$	$\ell = 3$
$a < 0, b > 0, c < 0, d = 0$	$a < 0, b = 0, c < 0, d = 0$
$a < 0, b = 0, c < 0, d > 0$	$a = c > 0, b = 0, d = 0$
$a < 0, b = 0, c = 0, d = 0$	
$a = 0, b = 0, c < 0, d = 0$	
$a = c > 0, b = 0, d > 0$	
$a = c > 0, b > 0, d = 0$	

Tabela 1: Relação entre ℓ e a, b, c, d.

A Tabela 2 mostra o erro cometido e a taxa de convergência esperada ao se aproximar a solução exata do sistema do tipo Bona-Smith pela solução aproximada obtida pelos métodos numéricos utilizados nesse trabalho. A taxa de convergência mostrada na Tabela 2 foi calculada com respeito à discretização temporal, e seus valores estão de acordo com a ordem de convergência determinada no Teorema 2.2. Isso sugere que a discretização espacial fornece uma aproximação bastante precisa e pouco altera a precisão global da solução.

	η		u	
Δt	Erro H^1	Taxa Conv.	Erro L^2	Taxa Conv.
5e-2	6.33e-5	-	1.06e-5	-
2.5e-2	3.53e-6	4.17	5.97e-7	4.16
1.25e-2	2.07e-7	4.09	3.50e-8	4.08
6.25e-3	1.25e-8	4.04	2.12e-9	4.04

Tabela 2: Erros e taxas de convergência para o sistema do tipo Bona-Smith

Teorema 2.2. *Sejam $s \geq 4$ e $(\eta, u) \in C(0, T; H^s(\Omega) \times H^{s+2}(\Omega))$ solução do sistema (1) para uma determinada região de parâmetros (a, b, c, d) e para algum $0 < T < \infty$, com dado inicial $(\eta_0, u_0) \in H^s(\Omega) \times H^{s+2}(\Omega)$. Suponha que exista uma constante $\hat{M}$ tal que $\max_{t\in[0,T]} \left(\|\partial_t^5 \eta(t)\|_1 + \|\partial_t^5 u(t)\|_1\right) \leq \hat{M}$. Seja (H^n, U^n) solução do problema totalmente discreto associado. Então, para N suficientemente grande e Δt suficientemente pequeno tais que $\Delta t \leq cN^{-1}$ para alguma constante $C > 0$, existe uma outra constante, também denotada por C, independente de N, tal que:*

$$\max_{0\leq n\leq M} (\|\eta(t^n) - H^n\|_1 + \|u(t_n) - U^n\|_1) \leq C \left(\Delta t^4 + N^{1-s}\right) \tag{1}$$

3 Conclusões

Através dessa análise, identificamos quais regiões de parâmetros (a, b, c, d) resultam em sistemas do tipo Boussinesq que podem ser resolvidos numericamente de maneira mais eficiente. Os exemplos numéricos implementados em MATLAB comprovaram os resultados teóricos obtidos sobre a precisão e eficácia dos métodos numéricos utilizados, e mostraram que essa classificação é satisfatória para problemas não lineares do tipo de Boussinesq.

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EXISTENCE OF GROUND STATE SOLUTIONS TO DIRAC EQUATIONS WITH VANISHING POTENTIALS AT INFINITY

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Abstract

In this work we study the existence of ground-state solutions of Dirac equations with potentials which are allowed to vanish at infinity. The approach is based on minimization of the energy functional over a generalized Nehari set. Some conditions on the potentials are given in order to overcome the lack of compactness.

1 Introduction

In this work we study the following version of the Dirac equation

$$-i\boldsymbol{\alpha}\nabla u + a\beta u + V(x)u = K(x)f(|u|)u, \quad \text{in } \mathbb{R}^3, \tag{1}$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, α_i and β are four-dimensional complex matrices given by

$$\beta = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix},$$

for $k = 1, 2, 3$ and σ_k given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

and where the potentials $V, K : \mathbb{R}^3 \to \mathbb{R}$ are continuous in $\mathbb{R}^3$ and are assumed to satisfy the following general conditions. We say that $(V, K) \in \mathcal{K}$ if

(VK_0) $V(x), K(x) > 0$ for all $x \in \mathbb{R}^3$, $V, K \in L^\infty(\mathbb{R}^3)$, with $\|V\|_\infty < a$.

(VK_1) If $(A_n) \subset \mathbb{R}^3$ is a sequence of Borel sets such that its Lebesgue measure $|A_n| \leq R$, for all $n \in \mathbb{N}$ and some $R > 0$, then

$$\lim_{r\to+\infty} \int_{A_n \cap B_r^c(0)} K(x) = 0, \quad \text{uniformly in } n \in \mathbb{N}.$$

Furthermore, one of the below conditions occurs

(VK_2) $\dfrac{K}{V} \in L^\infty(\mathbb{R}^3)$

or

(VK_3) there exists $q \in (2, 3)$ such that

$$\frac{K(x)}{V(x)^{3-q}} \to 0 \quad \text{as } |x| \to +\infty.$$

Moreover, we assume the following growth conditions at the origin and at infinity for the continuous function $f : \mathbb{R}^+ \to \mathbb{R}$:

(f_1) $\lim_{s\to 0^+} f(s) = 0$;

(f_2) there exist $c_1, c_2 > 0$ and $p \in (2,3)$ such that $|f(s)s| \leq c_1|s| + c_2|s|^{p-1}$, for all $s \in \mathbb{R}^+$;

(f_3) $\lim_{t\to\infty} \frac{F(t)}{t^2} = +\infty$;

(f_4) f is increasing in $\mathbb{R}^+$.

2 Main Results

Theorem 2.1. *Suppose that $(V,K) \in \mathcal{K}$ and $f \in C^0(\mathbb{R})$ verifies $(f_1) - (f_4)$. Then, problem (1) possesses a ground state solution, namely, a solution with the lowest energy among all nontrivial ones.*

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SINGULAR PROBLEMS IN ORLICZ-SOBOLEV SPACES

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Abstract

This paper deals with existence of positive solutions of singular elliptic problems on a smooth and bounded domain with Dirichlet boundary condition to the Φ-Laplacian operator. The proofs are based on kind of generalized Galerkin method that we developed inspired on ideas of Browder [3], comparison principle and in the use of apropriate test functions. Besides this, by using a kind of Moser's scheme, we proved $L^\infty(\Omega)$-regularity under an apropriate potential a and power $\alpha > 0$.

1 Introduction

We consider the quasilinear problem with a singular nonlinearity

$$\begin{cases} -\Delta_\Phi u &= \frac{a(x)}{u^\alpha} + b(x)u^\gamma \ \text{ in } \ \Omega, \\ \quad u &> \ 0, \text{ in } \Omega \\ \quad u &= \ 0, \text{ on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, α, γ are positive constants, a, b are measurable and non-negative functions, and $\phi : (0,\infty) \to (0,\infty)$ is of class C^1 that satisfies:

(ϕ_1) (i) $t\phi(t) \to 0$ *as* $t \to 0$, (ii) $t\phi(t) \to \infty$ *as* $t \to \infty$,

(ϕ_2) $t\phi(t)$ *is strictly increasing in* $(0,\infty)$,

(ϕ_3) *there exist* $\ell, m \in (1, N)$ *such that*

$$\ell - 1 \leq \frac{(t\phi(t))'}{\phi(t)} \leq m - 1, \ t > 0.$$

We extend $s \mapsto s\phi(s)$ to $\mathbb{R}$ as an odd function. Due to the nature of the operator

$$\Delta_\Phi u = \operatorname{div}(\phi(|\nabla u|)\nabla u), \text{ where } \Phi(t) = \int_0^t s\phi(s)ds, t \in \mathbb{R},$$

we shall work in the framework of Orlicz and Orlicz-Sobolev spaces, denoted by $L_\Phi(\Omega)$ and $W_0^{1,\Phi}(\Omega)$ (cf. [1]). The conditions $(\phi_1) - (\phi_3)$ imply that those spaces are Banach, reflexive and saparable.

The operator Δ_Φ appears in Plasticity, nonlinear elasticity and generalized Newtonian fluids. For details, see [6] and singular terms appears in fluid mechanics pseudoplastics flow, chemical heterogeneous catalysts and non-Newtonian fluids (see for example [4]).

In the past years, singular problems have been considered in a number of works, for instance, Karlin & Nirenberg [7] studied singular integral equations; Giacomoni, Schindler & Takac [5] studied quasilinear equations whose model is (1), in this case $\phi(t) = t^{p-2}$, $1 < p < N$; and Bocardo & Orsina [2] studied this problems where $\phi(t) = 1$.

Example to present a Theorem

2 Main Results

Our main results are.

Theorem 2.1. *Suppose* $(\phi_1)-(\phi_3)$ *hold. Assume in addition that* $a \in L^{\widetilde{\Psi}}_{loc}(\Omega) \cap L^1(\Omega)$ *for some N-function* $\Phi < \Psi << \Phi_*$, *and* $b=0$. *Then there is* $u \in W^{1,\Phi}_{loc}(\Omega)$ *such that* $u^{(\alpha-1+\ell)/\ell} \in W^{1,\ell}_0(\Omega)$ *(this is the sense of* $u=0$ *on* $\partial\Omega$*) and* $u(x) \geq Cd(x)$ *a.e.* Ω *for some* $C>0$, *solution of problem* (1), *that is,*

(*i*) *either the equation*

$$\int_\Omega \phi(|\nabla u|)\nabla u \nabla \varphi dx = \int_\Omega \frac{a(x)}{u^\alpha}\varphi dx, \text{ for all } \varphi \in W^{1,\Phi}_0(\Omega), \tag{1}$$

holds and $u \in W^{1,\Phi}_0(\Omega)$ *if* $ad^{-\alpha} \in L_{\widetilde{\Psi}}(\Omega)$, *where* $d(x) = \inf\{|x-y| \ / \ y \in \partial\Omega\}$ *for* $x \in \Omega$,

(*ii*) *or the equation*

$$\int_\Omega \phi(|\nabla u|)\nabla u \nabla \varphi dx = \int_\Omega \frac{a(x)}{u^\alpha}\varphi dx, \text{ for all } \varphi \in W^{1,\Phi}_0(U),$$

holds for each $U \subset\subset \Omega$ *given if* $\alpha \geq 1+(m-\ell)-m\ell N^{-1}$.

Corollary 2.1. *Under the conditions of above Theorem, there exists at most one solution to Problem* (1) *in* $W^{1,\Phi}_0(\Omega)$ *in the sence of* (1). *Besides this, the solution* $u \in W^{1,\Phi}_{loc}(\Omega)$, *given by above theorem, is such that:*

(*i*) $u \in C(\overline{\Omega})$ *if* $a \in L^\infty(\Omega)$,

(*ii*) $u \in L^\infty(\Omega)$ *if either* $a \in L^q(\Omega) \cap L^{\ell^*/(\ell^*+\alpha-1)}(\Omega)$ *and* $0<\alpha<1$ *or* $a \in L^q(\Omega)$ *and* $\alpha \geq 1$ *for some* $N/\ell < q < q(\alpha)$, *where*

$$q(s) := \begin{cases} \ell^*/s \text{ if } 0 < s \leq 1, \\ (\ell^* + (\alpha-1)\ell^*/\ell)/s \text{ if } s > 1. \end{cases} \tag{2}$$

Theorem 2.2. *Suppose* $(\phi_1)-(\phi_3)$ *and* $0 \leq \gamma < \ell-1$ *hold. Assume in addition that* $ad^{-\alpha} \in L_{\widetilde{\Psi}}(\Omega)$, *for some N-function* $\Phi < \Psi << \Phi_*$, *and* $0 \leq b \in L^\sigma(\Omega)$ *for some* $\sigma > \ell/(\ell-\gamma-1)$. *Then the problem* (1) *admits a weak solution* $u \in W^{1,\Phi}_0(\Omega)$ *such that* $u(x) \geq Cd(x)$ *a.e.* Ω *for some* $C>0$. *Besides this,* $u \in L^\infty(\Omega)$ *if* $b \in L^\infty(\Omega)$, *and either* $a \in L^q(\Omega) \cap L^{\ell^*/(\ell^*+\alpha-1)}(\Omega)$ *and* $0<\alpha<1$ *or* $a \in L^q(\Omega)$ *and* $\alpha \geq 1$ *for some* $N/\ell < q < q(\alpha+\gamma)$, *where* $q(s)$ *was defined at* (2).

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EXISTENCE OF SOLUTIONS FOR A CLASS OF $P(X)$ KIRCHHOFF TYPE EQUATION WITH DEPENDENCE ON GRADIENT

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Dedicated to Raul Moises Izaguirre Maguiña(1950-2015), (in memorium)

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Abstract

In our research we will study the existence of weak solutions for a class of $p(x)$ - Kirchhoff type equation with dependence on the gradient and Dirichlet boundary data.We establish our result by using the degree theory and working on the variable exponent Lebesgue-Sobolev spaces,

1 Introduction

In this work we consider the problem

$$-M\Big(\int_{\Omega} A(x,\nabla u)dx\Big)div\Big(a(x,\nabla u)\Big) = f(x,u,\nabla u) + u, \ \ in\ \Omega$$
$$u = 0, \ \ on\ \Gamma$$

where is Ω a bounded domain with smooth boundary Γ in $\mathbb{R}^n$, $(n \geq 3)$.
(A_1) $M : [0,+\infty[\longrightarrow [m_0,+\infty[$ is a continuous and nondecreasing functions, $m_0 > 0$.
(A_2) $a(x,\xi) : \overline{\Omega} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is the continuous derivate with respect to ξ of the continuous mapping $A : \overline{\Omega} \times \mathbb{R}^n \longrightarrow \mathbb{R}, A = A(x,\xi)$, i,e. $a(x,\xi) = \nabla_\xi A(x,\xi)$; there exists two positive constants $c_1 \leq c_2$ such that $c_1|\xi|^{p(x)} \leq a(x,\xi)\xi$, for all $x \in \Omega$, $\xi \in \mathbb{R}^n$ and $a(x,\xi) \leq c_2|\xi|^{p(x)-1}$. Also $A(x,0) = 0$, for all $x \in \Omega$ and $A(x,.)$ is strictly convex in $\mathbb{R}^n$, p is a continuous function on $\overline{\Omega}$.
(A_3) $f : \overline{\Omega} \times \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$ is a Caratheodory function such that

$$f(x,\eta,\xi) \leq c_3\Big(k(x) + |\eta|^{q(x)-1} + |\xi|^{q(x)-1}\Big)$$

for almost all $x \in \Omega$ and all $(\eta,\xi) \in \mathbb{R} \times \mathbb{R}^n$, $c_3 > 0$, q is a continuous function such that $1 > q(x) < p(x)$ and $k \in L^{p'(x)}(\Omega)$.
Recently, the studies of the Kirchhoff equations and the Kirchhoff systems have been considered by variational method in the case involving the p - Laplacian operator.
Moreover, due to the increasing amount of attention towards partial differential equations with nonstandard growth conditions, it was extended to the $p(x)$ - Laplacian operator $\Delta_{p(x)}$ [1, 3, 2]. More recently F. Faraci et al. [4] showed, via sub-super solution techniques the existence of weak solutions, but with $M = 1$ and $p(x) = p$.

2 Main Results

Our main result is as follows.

Theorem 2.1. *Under the assumptions $(A_1)-(A_3)$, there exists at least one solutions $u \in W_0^{1,p(x)}(\Omega)$ of the problem* (1).

Proof We transform the corresponding integral equation to problem (1) in the form of abstract Hammerstein equation, then we apply the Berkovits degree theory for demicontinuous operators of generalized (S_+) type in real reflexive Banach spaces[5].□

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HÉNON TYPE EQUATIONS WITH ONE-SIDE EXPONENTIAL GROWTH FOR HIGH EIGENVALUES

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Abstract

In this work, we prove existence of at least two solutions for a Ambrosetti-Prodi problem to Hénon type equations with exponential growth using variational methods. We get a first solution using the Fredholm alternative and a second solution is obtained via the Mountain Pass Theorem or Linking Theorem, depending on the relation between the nonlinearity and the first eigenvalue of $(\Delta, H_0^1(B_1))$. We explore two distinguishable cases: the subcritical and the critical ones. In the subcritical case, the Palais-Smale condition is satisfied by the associated functional for all levels of energy. On the other hand, in the critical case, the levels where we can retrieve compactness conditions lie below some constants depending on the critical exponent. Moreover, under certain conditions, we can see that we obtain a first solution that is radially symmetric, so we search for a second, which is also radial. This discussion becomes interesting in the critical case, where we can increase the level for the (PS_c) condition and thus, modifying the proof used in the subcritical framework, we obtain a second radially symmetric solution for the problem.

1 Introduction

In this paper we study the solvability of problems of the type

$$\begin{cases} -\Delta u = \lambda u + |x|^\alpha g(u_+) + f(x) & \text{in} \quad B_1, \\ u = 0 & \text{on} \quad \partial B_1, \end{cases} \tag{1}$$

where λ is a positive parameter which is not an eigenvalue of $(\Delta, H_0^1(B_1))$, $B_1 = \{x \in \mathbb{R}^2 : |x| < 1\}$ and g will treat the so-called the subcritical and critical cases.

We have $g \in C(\mathbb{R}, \mathbb{R})$ such that, in a neighborhood of 0, it behaves as a polynomial with $g(0) = 0$ and satisfies

(g_1) There exist s_0 and $M > 0$ such that $0 < G(s) = \int_0^s g(s)ds \leq M|g(s)|$ for all $|s| > s_0$.
(g_2) $0 < G(s) \leq \frac{1}{\sigma} g(s)s$ for all $s \in \mathbb{R} \setminus \{0\}$ and $\sigma > 2$.

Under certain conditions, we can get a negative solution ψ for (1). A second solution is given by $v + \psi$, where v is a non-trivial solution of the following problem:

$$\begin{cases} -\Delta u = \lambda u + |x|^\alpha g(u + \psi)_+ & \text{in} \quad B_1; \\ u = 0 & \text{on} \quad \partial B_1, \end{cases} \tag{2}$$

So we can apply the Mountain Pass Theorem or the Linking Theorem in order to get a solution for (2) and, consequently, a second solution for (1). In the critical case, we have

$$\lim_{|t|\to\infty} \frac{|g(t)|}{e^{\beta t^2}} = 0, \quad \forall \beta > \beta_0; \quad \lim_{|t|\to\infty} \frac{|g(t)|}{e^{\beta t^2}} = +\infty, \quad \forall \beta < \beta_0. \tag{3}$$

In two dimensions, some types of exponential growth can be handled variationally due to the well-known Trudinger-Moser inequality, given below

$$\sup_{\|u\|=1} \int_{B_1} e^{\beta u^2} \begin{cases} < +\infty & if \quad \beta < 4\pi; \\ = +\infty & if \quad \beta > 4\pi. \end{cases} \tag{4}$$

for $u \in H_0^1(B_1)$. This fact guarantee that the $(PS)_c$ condition is satisfied by the associated functional J_λ for $c < \dfrac{4\pi}{\beta_0}$.

Working with a type Hénon problem, we observe the weight $|x|^\alpha$ can change this fact. If we consider the subspace $H^1_{0,rad}(B_1)$, we have

$$\sup_{\|u\|=1} \int_{B_1} |x|^\alpha e^{\beta u^2} \begin{cases} < +\infty & if \quad \beta < 2\pi(2+\alpha); \\ = +\infty & if \quad \beta > 2\pi(2+\alpha). \end{cases} \tag{5}$$

So the $(PS)_c$ condition is satisfied by the associated functional J_λ for $c < \dfrac{2(2+\alpha)\pi}{\beta_0}$.

2 Main Results

Our main results concern the existence of two solutions for the above problem, where we considered the subcritical, critical and radially symmetric cases. The radial case only becomes interesting when we have critical growth, since there is a significant change of arguments among the proofs of the results. We state the main theorems below.

Theorem 2.1. *(The subcritical case) Let ψ be a negative (radially symmetric) solution of* (1)*. Considering $\lambda \neq \lambda_j$ for all $j \in \mathbb{N}$ and g as above and with subcritical growth. Then, the problem* (1) *has a second solution (which is also radially symmetric).*

Theorem 2.2. *(The critical case) Let ψ be the negative solution of* (1)*. Assume $\lambda \neq \lambda_j$ for all $j \in \mathbb{N}$ and take g as above satisfying* (3) *and for $\gamma \geq 0$ there exists $c_\gamma \geq 0$ such that*

$$g(s)s \geq \gamma e^{(\beta_0 s^2)} h(s) \tag{6}$$

for all $s > c_\gamma$, where $h : \mathbb{R} \to \mathbb{R}^+$ is a Carathéodory function satisfying

$$\liminf_{s \to +\infty} \frac{\log(h(s))}{s} > 0. \tag{7}$$

Then, the problem (1) *has a second solution.*

Theorem 2.3. *(The critical and radial case) Let ψ be a solution of* (1) *is negative and radially symmetric. Assume $\lambda \neq \lambda_j$ for all $j \in \mathbb{N}$ and g as above,* (3) *and* (6) *with $h : \mathbb{R} \to \mathbb{R}^+$ is a Carathéodory function satisfying*

$$\liminf_{s \to +\infty} \frac{\log(h(s))}{s} = +\infty. \tag{8}$$

Then, problem (1) *has a second solution that is radially symmetric.*

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MULTIPLICITY OF SOLUTIONS FOR FOURTH SUPERLINEAR ELLIPTIC PROBLEMS UNDER NAVIER CONDITIONS

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Abstract

It is establish existence and multiplicity of solutions for a class of superlinear elliptic involving a fourth elliptic problem under Navier condition on the boundary. Here we do not consider the well known Ambrosetti-Rabinowitz condition. Here we suppose that the nonlinear term is a nonlinear function which is nonquadratic at infinity..

1 Introduction

In this talk we shall consider the fourth elliptic problem

$$\begin{cases} \alpha\Delta^2 u + \beta\Delta u = f(x,u) & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Delta^2 = \Delta \circ \Delta$ is the biharmonic operator, $N \geq 4, \Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $\alpha > 0, \beta \in (-\infty, \alpha\lambda_1)$. Here and throughout this paper λ_1 denotes the first eigenvalue problem on $(-\Delta, H_0^1(\Omega))$. The nonlinear term f is a continuous function which is superlinear at infinity and at the origin.

The weak solutions for problem (1) are precisely the critical points for the functional of C^1 class $I : \mathcal{H} \to \mathbb{R}$ given by

$$I(u) = \frac{1}{2}\int_\Omega \alpha|\Delta u|^2 - \beta|\nabla u|^2\,dx - \int_\Omega F(x,u)\,dx, \tag{2}$$

where the primitive for f is denoted by $F(x,u) = \displaystyle\int_0^u f(x,t)dt\ x \in \Omega,\ t \in \mathbb{R}$.

Throughout this work we shall consider the following hypotheses

(f_0) There exist $a_1 > 0$ and $p \in (2, 2_*)$ such that

$$|f(x,s)| \leq a_1(1 + |s|^{p-1}),\ \text{for any },\ (x,s) \in \Omega \times \mathbb{R}.$$

(f_1) $\displaystyle\lim_{|t|\to\infty} \frac{f(x,t)}{t} = \infty$ uniformly in Ω;

(f_2) $\displaystyle\lim_{|t|\to 0} \frac{f(x,t)}{t} = f_0 < \lambda_1(\alpha\lambda_1 - \beta)$ uniformly in Ω;

We will consider the following condition nonquadraticity condition

(NQ) setting $H(x,s) := f(x,s)s - 2F(x,s)$, we have that

$$\lim_{|s|\to\infty} H(x,s) = +\infty,\ \text{uniformly for } x \in \Omega.$$

2 Main Results

In this talk we shall consider the existence of a nontrivial solution for problem (1) via the mountain pass theorem. Applying the strong maximum principle we ensure the existence of one positive and one negative solution. Our main first result can be stated as

Theorem 2.1. *Suppose that f satisfies $(f_0), (f_1)$, (f_2) and (NQ). Then the problem* (1) *admits at least one nontrivial solution. Furthermore, assuming that $f(x,t)t \geq 0, x \in \Omega, t \in \mathbb{R}$, problem* (1) *admits at least three two nontrivial solutions u_1, u_2 satisfying $u_1 > 0$ in Ω and $u_2 < 0$ in Ω.*

It is worthwhile to mention that the functional energy associated with the problem (1) admits the mountain pass geometry. Besides that, this functional possesses the compactness condition given by the Cerami condition. So that there exists a nontrivial critical point of the functional which is one weak and nontrivial solution for problem (1)

Our second result can be written in the following form

Theorem 2.2. *Suppose that f satisfies $(f_0), (f_1)$ and (f_2). Assume also that $t \to f(x,t)$ is an odd function for any $x \in \Omega$ fixed. Then the problem* (1) *admits infinitely many nontrivial solutions.*

The theorem just above can be proven using the symmetric mountain pass which permit us to find an unbounded sequence of nontrivial solution for the elliptic problem (1). For early results on fourth elliptic problem we refer the reader to [2, 3, 4].

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STRICTLY POSITIVE DEFINITE MULTIVARIATE COVARIANCE FUNCTIONS ON COMPACT TWO-POINT HOMOGENEOUS SPACES

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Abstract

An l-variate covariance function on a compact two-point homogeneous space M is a positive definite real matrix function $(x, y) \in M \times M \to [f_{\mu\nu}(x, y)]_{\mu\nu=1,2,\ldots,l}$. In this work we provide a characterization for the continuous and isotropic l-variate covariance functions on M, thus complementing a similar result on spheres usually attributed to either Hannan ([3]) or Yaglom ([4]). We also present an abstract necessary and sufficient condition for the strict positive definiteness of a continuous and isotropic l-variate covariance function on M, in the case in which M is not a sphere. Finally, in the case $l = 2$, we present an alternative necessary and sufficient condition for strict positive definiteness based upon the diagonal elements f_{11} and f_{22} in the matrix representation for the covariance function.

1 Introduction

Scattered data sets collected in many observations from a variety of geophysical and atmospheric studies may be interpreted as realizations of random fields on a specific manifold. As so, the construction of multivariate covariance functions satisfying practical demands is a constant need in a number of models. Applications are found in applied statistics, atmospheric data assimilation, stochastic processes on spheres and manifolds, approximation theory, etc.

In this work, we deal with real, continuous and isotropic multivariate covariance functions on a (connected) compact two-point homogeneous space $\mathbb{M}^d$ of dimension d. These spaces were characterized in [5] and the d-dimensional unit sphere is the most illustrious example in this category of spaces. Projective spaces of various types also belong to this category of spaces. For a fixed positive integer l, denote by $M_l(\mathbb{R})$ the set of all $l \times l$ matrices with real entries. An *l-variate covariance function* on $\mathbb{M}^d$ is a matrix function $F : \mathbb{M}^d \times \mathbb{M}^d \to M_l(\mathbb{R})$ with the following features:

$$f_{\mu\nu}(x, y) = f_{\mu\nu}(y, x) = f_{\nu\mu}(y, x), \quad \mu, \nu = 1, 2, \ldots, l, \quad x, y \in \mathbb{M}^d,$$

and, if

$$F(x, y) = [f_{\mu\nu}(x, y)], \quad x, y \in \mathbb{M}^d, \quad \mu, \nu = 1, 2, \ldots, l,$$

then for each positive integer n and for each choice of n distinct points $x_1, x_2, \ldots, x_n$ on $\mathbb{M}^d$, the $ln \times ln$ block matrix $[f_{\mu\nu}(x_i, x_j)]$ is nonnegative definite in the sense that

$$\sum_{i,j=1}^{n} B_i^t F(x_i, y_j) B_j = \sum_{i,j=1}^{n} \sum_{\mu,\nu=1}^{l} B_i^{\mu} B_j^{\nu} f_{\mu\nu}(x_i, y_j) \geq 0,$$

for $B_1, B_2, \ldots, B_n$ in $\mathbb{R}^l$, in which $B_i^t = (B_i^1, B_i^2, \ldots, B_i^l)$, $i = 1, 2, \ldots, n$.

The main objective of this work is to characterize the *strict positive definiteness* of a real, continuous and isotropic l-variate covariance function on $\mathbb{M}^d$, in the case in which $\mathbb{M}^d$ is not a sphere. The strict positive definiteness of a real, continuous and isotropic l-variate covariance function F on $\mathbb{M}^d$ demands that the previous inequalities be strict when at least one vector B_i is nonzero. Univariate covariance functions on compact two-point homogeneous spaces are studied in [1] and references therein.

2 Main Results

The continuous and isotropic multivariate covariance functions on a compact two-point homogeneous space can be described as follows. The symbol $P_k^{(d-2)/2,\beta}$ stands for the Jacobi polynomial of degree k associated to the pair $((d-2)/2, \beta)$. We assume the spaces are normalized so that their diameter is always 2π.

Theorem 2.1. *Let $F : \mathbb{M}^d \times \mathbb{M}^d \to M_l(\mathbb{R})$ be real, continuous and isotropic. It is an l-variate covariance function on $\mathbb{M}^d$ if, and only if, it has the matrix representation*

$$F(x,y) = \sum_{k=0}^{\infty} A_k P_k^{(d-2)/2,\beta}(\cos(|xy|/2)), \quad x, y \in \mathbb{M}^d,$$

where each $A_k = [A_k^{\mu\nu}]$ is a nonnegative definite element of $M_l(\mathbb{R})$ and

$$\sum_{k=0}^{\infty} A_k^{\mu\nu} P_k^{(d-2)/2,\beta}(1) < \infty, \quad \mu, \nu = 1, 2, \ldots, l.$$

Theorem 2.2. *Let F be a real, continuous and isotropic l-variate covariance function on $\mathbb{M}^d$. Assume $\mathbb{M}^d$ is not a sphere. The following assertions are equivalent:*
(i) F is strictly positive definite;
(ii) For every $B \in \mathbb{R}^l \setminus \{0\}$, the set $\{k : B^t A_k B > 0\}$ is infinite.

For 2-variate covariance functions, we have this explicit result.

Theorem 2.3. *Let $F = [f_{\mu\nu}]$ be a 2-variate covariance function on $\mathbb{M}^d$ fitting the description provided by Theorem 2.1. Assume $\mathbb{M}^d$ is not a sphere and F is not of the form*

$$F(x,y) = \left[\sum_{k \in K_1} A_k + A \sum_{k=0}^{\infty} a_k \right] P_k^{(d-2)/2,\beta}(\cos(|xy|/2)), \quad x, y \in \mathbb{M}^d,$$

in which K_1 is a finite set, A is a fixed nonnegative definite, but not positive definite, element of $M_2(\mathbb{R})$, the a_k are nonnegative real numbers and $\sum_{k=0}^{\infty} a_k P_k^{(d-2)/2,\beta}(1) < \infty$. In order that F be strictly positive definite it is necessary and sufficient that both univariate diagonal covariance functions f_{11} and f_{22} be strictly positive definite.

It remains open whether a result of the same type of the one above can be deduced for l-variate covariance functions, $l \geq 2$. These three results are detailed in [2].

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TRACEABILITY OF POSITIVE INTEGRAL OPERATORS: A REPRODUCING KERNEL POINT OF VIEW

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Abstract

Let $K : X \times X \to \mathbb{C}$, where X is a nonempty set endowed with a measure ν. It is usual to find papers dealing with properties of the associated integral operator $\mathcal{K}$ generated by K and acting on $L^2(X, \nu)$, if the later is defined. An important one concerns the summability of singular values of $\mathcal{K}$ (trace-class, nuclearity), under specific assumptions. Here, we consider this very same problem under a reproducing kernel standpoint.

1 Introduction

Let X be a nonempty set and $K : X \times X \to \mathbb{C}$ a *positive definite kernel* on X, that is, a function satisfying the inequality

$$\sum_{i,j=1}^{n} \overline{c_i} c_j K(x_i, x_j) \geq 0,$$

whenever $n \geq 1$, $\{x_1, x_2, \ldots, x_n\}$ is a subset of X and $c_1, c_2, \ldots, c_n$ are complex numbers.

Let $\mathcal{H}_K$ be the unique Hilbert space of functions on X for which K is a reproducing kernel. If $\langle \cdot, \cdot \rangle_K$ is the inner product of $\mathcal{H}_K$ and K^x, $x \in X$, denotes the function $y \in X \to K(y, x) \in \mathbb{C}$, then

$$\langle K^x, K^y \rangle_K = K(y, x), \quad x, y \in X. \tag{1}$$

Given a convenient measure ν on X and conditions on K (please see [3, 4]), we can define the integral operator $\mathcal{K} : L^2(X, \nu) \to L^2(X, \nu)$, through the formula

$$\mathcal{K}(f)(x) = \int_X K(x, y) f(y)\, d\nu(x), \quad f \in L^2(X, \nu), \quad x \in X.$$

And its positivity, that is, self-adjointness and the property

$$\langle \mathcal{K}(f), f \rangle_{L^2} \geq 0, \qquad f \in L^2(X, \nu),$$

is one of its most desirable attributes.

Usually, positivity and many other properties for $\mathcal{K}$ are achieved via operator theory as one can check in [1]. Papers like [2] and [5] analyze the nuclearity of $\mathcal{K}$ by using Maximal functions. Here, we will seek the nuclearity of $\mathcal{K}$ through the reproducing kernel Hilbert space theory.

2 Main Results

We will say that $\mathcal{H}_K$ is embedded into $L^2(X, \nu)$ if $\mathcal{H}_K \subset L^2(X, \nu)$ and the inclusion map $i : \mathcal{H}_K \hookrightarrow L^2(X, \nu)$ is bounded. The main result is based upon the lemma below (please see [4]).

Lemma 2.1. *Assume that every function in $\mathcal{H}_K$ is ν-measurable. If $\mathcal{H}_K$ is embedded into $L^2(X,\nu)$, then the integral operator $\mathcal{K}$ is bounded, self-adjoint and positive. Moreover, the adjoint $i^* : L^2(X,\nu) \to \mathcal{H}_K$ of i is given by*

$$i^*(f) = \mathcal{K}(f), \qquad f \in L^2(X,\nu),$$

and we have the factorization

$$\begin{array}{ccc} & \mathcal{H}_K & \\ {\scriptstyle i^*}\nearrow & & \searrow{\scriptstyle i} \\ L^2(X,\nu) & \overset{}{\underset{\mathcal{K}}{- - - \to}} & L^2(X,\nu) \end{array} \tag{2}$$

Proof Since,

$$\langle f, g\rangle_{L^2} = \langle i(f), g\rangle_{L^2} = \langle f, i^*(g)\rangle_K, \quad f \in \mathcal{H}_K, \quad g \in L^2(X,\nu),$$

it follows that

$$i^*(g)(x) = \langle i^*(g), K^x\rangle_K = \langle g, K^x\rangle_{L^2} = \int_X K(x,y)g(y)\,d\nu(y), \quad x \in X, \quad g \in L^2(X,\nu),$$

and Diagram (2) holds true. Hence, $\mathcal{K} = i \circ i^*$ is bounded, self-adjoint, and the inequality

$$\langle \mathcal{K}(f), f\rangle_{L^2} = \langle i \circ i^*(f), f\rangle_{L^2} = \langle i^*(f), i^*(f)\rangle_K \geq 0$$

holds. □

Theorem 2.1. *If $\mathcal{H}_K$ is embedded into $L^2(X,\nu)$, then the inclusion map is Hilbert-Schmidt like if, and only if, $\int_X K(x,x)d\nu(x) < \infty$ if, and only if, $\mathcal{K}$ is trace-class.*

Proof First note that

$$\int_X |f(x)|^2 d\nu(x) = \int_X |\langle f, K^x\rangle|^2 d\nu(x) \leq \int_X \|f\|_K^2 \|K^x\|_K^2 d\nu(x) = \|f\|_K^2 \int_X K(x,x)d\nu(x), \quad f \in \mathcal{H}_K.$$

In order to see that i and i^* are Hilbert-Schmidt like and that $\mathcal{K} = i \circ i^*$ is trace-class, let $\{\phi_i\}_{i\in I}$ be an orthonormal basis of $\mathcal{H}_K$. The equalities

$$\sum_{j\in I} \|i(\phi_j)\|_{L^2}^2 = \sum_{j\in I} \int_X |\phi_j(x)|^2 d\nu(x) = \int_X \sum_{j\in I} |\langle \phi_j, K^x\rangle_K|^2 d\nu(x) = \int_X \|K^x\|_K^2 d\nu(x) = \int_X K(x,x)d\nu(x)$$

follow from monotone convergence theorem, Parseval's identity and Equality (1). That implies the result. □

Note that, if ν is σ-finite, K is $\nu \times \nu$-measurable and $\int_X K(x,x)d\nu(x) < \infty$, then Fubini's theorem implies that $K \in L^2(X \times X, \nu \times \nu)$ and $\mathcal{H}_K$ is embedded into $L^2(X,\nu)$. In particular, $\mathcal{K}$ is compact and Hilbert-Schmidt like.

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FROM SCHOENBERG COEFFICIENTS TO SCHOENBERG FUNCTIONS: STRICT POSITIVE DEFINITENESS

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Abstract

Let S^d be the unit sphere of $\mathbb{R}^{d+1}$ and $\cdot$ the usual inner product in $\mathbb{R}^{d+1}$. For a locally compact group $(G,*)$ and a continuous function $f : G \times [-1,1] \to \mathbb{C}$, it was presented in [1] a necessary and sufficient condition on f in order that the kernel

$$((u,x),(v,y)) \in (G \times S^d) \times (G \times S^d) \to f(u^{-1} * v, x \cdot y)$$

be positive definite, an extension of result of I. J. Schoenberg ([4]) to $G \times S^d$. The main issue here is to obtain an necessary and sufficient condition for the strict positive definiteness of the kernel.

1 Introduction

A function K mapping a product space $X \times X$ into $\mathbb{C}$ is termed a *positive definite kernel* on X if

$$\sum_{\mu,\nu=1}^{n} c_\mu \overline{c_\nu} K(x_\mu, x_\nu) \geq 0,$$

for $n \geq 1$, distinct points $x_1, x_2, \ldots, x_n$ in X and complex scalars $c_1, c_2, \ldots, c_n$. The strict positive definiteness of a positive definite kernel K demands that the inequalities above be strict when at least one of the c_i is nonzero.

In 1942, I. J. Schoenberg published his seminal paper characterizing the continuous and isotropic positive definite kernels on spheres: if $f : [-1,1] \to \mathbb{C}$ is continuous and $\cdot$ is the usual inner product in $\mathbb{R}^{d+1}$, then the kernel $(x,y) \in S^d \times S^d \to f(x \cdot y)$ is positive definite if, and only if,

$$f(t) = \sum_{k=0}^{\infty} a_k^d P_k^d(t), \quad t \in [-1,1],$$

where all the coefficients a_k^d are nonnegative, P_k^d denotes the usual Gegenbauer polynomial of degree k attached to the rational number $(d-1)/2$ and $\sum_k a_k^d P_k^d(1) < \infty$. Usually, f is referred to as the *isotropic part* of the kernel $(x,y) \in S^d \times S^d \to f(x \cdot y)$. Schoenberg's result can also be described via the geodesic distance δ on S^d if one recalls that

$$cos\, \delta(x,y) = x \cdot y, \quad x, y \in S^d.$$

The need for characterizing the strictly positive definite kernels from Schoenberg's class came years later. Indeed, after the so-called radial basis function interpolation method was adapted to hold in manifolds endowed with metrics, it became clear that the spherical version of this problem would be uniquely solved if the solutions were generated by strictly positive definite kernels in the sense of Schoenberg. The characterization for strict positive definiteness in the case $d \geq 2$ appeared in [5]: a positive definite kernel from Schoenberg's class is strictly positive definite if, and only if, the index set $\{k : a_k^d > 0\}$ extracted from the series representation for the isotropic part of the kernel contains infinitely many even and infinitely many odd integers. Recently, motivated by problems from probability theory and stochastic processes, Porcu and Berg ([1]) investigated positive definite kernels on $G \times S^d$, in which $(G, *)$ is an arbitrary locally compact group, keeping both, the group structure of G and the isotropy of S^d in the setting. For a continuous function $f : G\times[-1,1] \to \mathbb{C}$, they showed that kernel $((u,x),(v,y)) \in (G\times S^d)\times(G\times S^d) \to f(u^{-1}*v, x\cdot y)$ is positive definite if, and only if, f has a representation in the form

$$f(u,t) = \sum_{k=0}^{\infty} f_k^d(u) P_k^d(t), \quad (u,t) \in G \times [-1,1],$$

in which $\{f_k^d\}$ is a sequence of continuous functions on G for which $(u,v) \in G \to f_k^d(u^{-1} * v)$ is positive definite and $\sum_k f_k^d(e)P_k^d(1) < \infty$, with uniform convergence of the series for $(u,t) \in G \times [-1,1]$. Their proof also implied the following integral representation for the f_k^d:

$$f_k^d(u) = c(d,k) \int_{-1}^{1} f(u,t) P_k^d(t)(1-t^2)^{(d-2)/2} dt, \quad k \in \mathbb{Z}_+, \quad u \in G,$$

in which $c(d,k)$ is a positive constant depending upon d and k.

2 Main Results

Our contribution to the results described in the introduction is the following upgrade of the result in [1] to strict positive definiteness.

Theorem 2.1. $(d \geq 2)$ *Let* $f : G \times [-1,1] \to \mathbb{C}$ *be a continuous function and assume the kernel* $((u,x),(v,y)) \in (G \times S^d) \times (G \times S^d) \to f(u^{-1} * v, x \cdot y)$ *is positive definite. The following assertions are equivalent;*
(i) The kernel is strictly positive definite;
(ii) If q *is a positive integer,* $u_1, u_2, \ldots, u_q$ *are distinct points in* G *and* c *is a nonzero vector in* $\mathbb{C}^q$*, then the set*

$$\{k \in \mathbb{Z}_+ : c^t \left[f_k^d(u_i^{-1} * u_j)\right]_{i,j=1}^q \overline{c} > 0\}$$

contains infinitely many even and infinitely many odd integers.

The paper [5] describes a characterization for strict positive definiteness for functions in Schoenberg's class in S^1. In this case, the index set $\{k : a_{|k|}^1 > 0\}$ needs to intersect each full arithmetic progression in $\mathbb{Z}$. In view of this, we conjecture that Condition (ii) in the previous theorem must be replaced with the following one in the case $d = 1$: "If q is a positive integer, $u_1, u_2, \ldots, u_q$ are distinct points in G and c is a nonzero vector in $\mathbb{C}^q$, then the set

$$\left\{k \in \mathbb{Z} : c^t \left[f_{|k|}^1(u_i^{-1} * u_j)\right]_{i,j=1}^q \overline{c} > 0\right\}$$

intersects each full arithmetic progression in $\mathbb{Z}$".

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CARACTERIZAÇÕES DA PROPRIEDADE DE SCHUR EM ESPAÇOS DE BANACH

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Resumo

O objetivo deste trabalho é enunciar e demonstrar várias caracterizações da propriedade de Schur em espaços de Banach. Apresentamos também uma aplicação de uma das caracterizações enunciadas.

1 Introdução

Dizemos que uma sequência $(x_n)_n$ converge fracamente para x em um espaço de Banach E, e denotamos este fato por $x_n \overset{w}{\longrightarrow} x$, se $\varphi(x_n) \longrightarrow \varphi(x)$ para todo $\varphi \in E'$, onde E' é o dual topológico de E. É fácil observar que a convergência em norma implica na convergência fraca, porém a reciproca nem sempre é verdadeira, um exemplo clássico deste fato é a sequência dos vetores canônicos no espaço c_0 das sequências de escalares que convergem para zero.

Em 1921, o matemático J. Schur demonstrou, em [4], que no espaço ℓ_1 a convergência fraca implica na convergência em norma, assim ℓ_1 foi o primeiro espaço em que tal implicação foi observada. O fato é que esta implicação também vale em outros espaços de Banach, e por isso passou-se a dizer que um espaço de Banach E possui a *propriedade de Schur*, ou que E é um *espaço de Schur*, se em E a convergência fraca implica na convergência em norma. Muitos resultados na teoria dos espaços de Banach valem para espaços de Schur, e por isso é importante verificar se um determinado espaço é de Schur ou não. Assim como ocorre com outras propriedades em matemática, em vez de testar a definição, às vezes é mais fácil verificar a validade, ou não, de uma condição que é equivalente à definição da propriedade de Schur. Daí a importância de se conhecer propriedades que são equivalentes à propriedade de Schur. Neste trabalho apresentamos algumas caracterizações da propriedade de Schur que são ferramentas bastante úteis em algumas demonstrações envolvendo tais espaços.

2 Resultados Principais

Os espaços de Banach E e F são considerados sobre o corpo $\mathbb{K}$, onde $\mathbb{K} = \mathbb{C}$ ou $\mathbb{K} = \mathbb{R}$, e $\mathcal{L}(E, F)$ denota o espaço de Banach dos operadores lineares contínuos de E em F munida da norma usual do supremo. Um subconjunto A de um espaço de Banach E é *fracamente compacto* se A é compacto na topologia fraca de E. Uma sequência $(x_n)_n$ no espaço de Banach E é *fracamente de Cauchy* se a sequência de escalares $(\varphi(x_n))_n$ é convergente qualquer que seja $\varphi \in E'$. Um operador $T \in \mathcal{L}(E; F)$ é *completamente contínuo* se $T(x_n) \longrightarrow T(x)$ em F sempre que $x_n \overset{w}{\longrightarrow} x$ em E. Um subconjunto de um espaço topológico é *separável* se admite um subconjunto enumerável e denso.

Teorema 2.1. *As seguintes afirmações acerca de um espaço de Banach E são equivalentes.*

(a) E é um espaço de Schur.

(b) Toda sequência $(x_n)_n$ fracamente nula ($x_n \overset{w}{\longrightarrow} 0$) em E converge para zero em norma.

(c) Todo subconjunto fracamente compacto de E é compacto.

(d) *Toda sequência fracamente de Cauchy em E é de Cauchy.*

(e) *Toda sequência fracamente de Cauchy em E converge em norma.*

(f) *Para qualquer espaço de Banach F, todo operador linear e contínuo definido em E ou tomando valores em E é completamente contínuo.*

(g) *Todos os subespaços separáveis e fechados de E são espaços de Schur.*

(h) *Para toda sequência $(x_n)_n$ em E com $\|x_n\| = 1$ para todo $n \in \mathbb{N}$, tem-se $x_n \overset{w}{\not\longrightarrow} 0$.*

Mencionamos que conhecemos as caracterizações (c) e (g) no artigo [6] de B. Tanbay. A caracterização (h) foi inspirada na dissertação [3] e as caracterizações (d) e (e) foram inspiradas na dissertação [5].

Como ℓ_1 é de Schur, é natural questionar se $L_1[0,1]$ também é de Schur. Vejamos que isso não é verdade usando uma das caracterizações acima:

Proposição 2.1. *$L_1[0,1]$ não é de Schur.*

Prova: Pela Desigualdade de Khintchine [1, 1.10] sabemos que $L_1[0,1]$ contém uma cópia isomorfa de ℓ_2. Como ℓ_2 é separável e não é de Schur, segue da equivalência (g) do Teorema acima que $L_1[0,1]$ não é de Schur. □

Existem outras caracterizações da propriedade de Schur, como por exemplo, um recente resultado obtido em 2012 por Dowling, Freeman, Lennard, Odell, Randrianantoanina e Turett [2] que relaciona a propriedade de Schur em espaços de Banach com um resultado análogo ao enunciado que é conhecido como *Princípio da Compacidade de Grothendieck*, quando a topologia da norma é trocada pela topologia fraca. Neste trabalho os autores mostram que um espaço de Banach E é de Schur se, e somente se, todo subconjunto fracamente compacto de E está contido na envoltória convexa fechada de uma sequência fracamente nula.

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UPPER BOUNDS FOR EIGENVALUES OF POSITIVE INTEGRAL OPERATORS ON THE SPHERE

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Abstract

We obtain upper bounds for the eigenvalues of positive integral operators generated by infinitely many times Laplace-Beltrami differentiable kernels on the unit sphere in $\mathbb{R}^{m+1}$, $m \geq 2$.

1 Introduction

Let m be a positive integer at least 2 and S^m the unit sphere in $\mathbb{R}^{m+1}$ endowed with the induced Lebesgue measure. In this work, we consider compact integral operators defined by

$$\mathcal{K}(f) = \int_{S^m} K(\cdot, y) f(y)\, d\sigma_m(y), \tag{1.1}$$

in which the generating kernel $K : S^m \times S^m \to \mathbb{C}$ is an element of $L^2(S^m \times S^m)$.

If K is positive definite in the sense that

$$\int_{S^m} \int_{S^m} K(x, y) f(x) \overline{f(y)}\, d\sigma_m(x) d\sigma_m(y) \geq 0, \quad f \in L^2(S^m), \tag{1.2}$$

then $\mathcal{K}$ is also self-adjoint and the standard spectral theorem for compact and self-adjoint operators yields the following expansion:

$$\mathcal{K}(f) = \sum_{n=0}^{\infty} \lambda_n(\mathcal{K}) \langle f, f_n \rangle_2 f_n, \quad f \in L^2(S^m), \tag{1.3}$$

in which $\{\lambda_n(\mathcal{K})\}$ is a sequence of nonnegative reals (possibly finite) decreasing to 0 and $\{f_n\}$ is an orthonormal basis of $L^2(S^m)$. The numbers $\lambda_n(\mathcal{K})$ are the eigenvalues of $\mathcal{K}$ and the sequence $\{\lambda_n(\mathcal{K})\}$ takes into account repetitions implied by the algebraic multiplicity of each eigenvalue. Orthogonality refers to the usual normalized inner product on $L^2(S^m)$. The positive definiteness of K means nothing but the positivity of the integral operator $\mathcal{K}$ and since it relates to the inner product then we call it L^2-*positive definiteness*.

The addition of continuity to K implies that $\mathcal{K}$ is also *trace-class* (nuclear), that is,

$$\sum_{f \in B} \langle \mathcal{K}^* \mathcal{K}(f), f \rangle_2^{1/2} < \infty, \tag{1.4}$$

whenever B is an orthonormal basis of $L^2(S^m)$. In particular,

$$\sum_{n=1}^{\infty} \lambda_n(\mathcal{K}) = \int_{S^m} K(x, x)\, d\sigma_m(x) < \infty, \tag{1.5}$$

from where we observe that exists a positive real c such that

$$\lambda_n(\mathcal{K}) \leq c\, n^{-1}, \quad n \in \mathbb{N}. \tag{1.6}$$

2 Main Result

The Laplace-Beltrami derivative is a variation of the usual derivative on S^m when, in the definition of the later, we replace the usual translation operator with the *spherical shifting operator*, which is defined by the formula

$$T_\epsilon^m(f)(x) := \frac{1}{\sigma_{m-1}(1-\epsilon^2)^{(m-1)/2}} \int_{x\cdot y=\epsilon} f(y)\, dy, \quad x \in S^m. \tag{2.1}$$

Here, $\epsilon \in (-1,1)$, " $\cdot$ " is the usual inner product of $\mathbb{R}^{m+1}$, σ_{m-1} stands for the surface measure of S^{m-1} and dy denotes the measure element of the rim $\{y \in S^m : x \cdot y = \epsilon\}$ of the spherical cap $\{y \in S^m : x \cdot y \geq \epsilon\}$. If we write $\Delta_\epsilon := I - T_\epsilon^m$, in which I denotes the identity operator, a function $f \in L^2(S^m)$ is said to be *differentiable in the sense of Laplace-Beltrami* if there exists $\mathcal{D}f \in L^2(S^m)$ such that

$$\lim_{\epsilon \to 1^-} \left\|(1-\epsilon)^{-1}\Delta_\epsilon(f) - \mathcal{D}f\right\|_2 = 0. \tag{2.2}$$

The function $\mathcal{D}f$ is then called the *Laplace-Beltrami derivative* of f. Higher order derivatives are defined by the formulas $\mathcal{D}^1 = \mathcal{D}$ and

$$\mathcal{D}^r := \mathcal{D}^1 \circ \mathcal{D}^{r-1}, \quad r = 2, 3, \ldots. \tag{2.3}$$

We now introduce basic Sobolev-type spaces for functions on S^m.

Definition 2.1. *The space of all complex functions on S^m which are differentiable, up to order r, in the sense explained above, will be denoted by W_2^r. If the function has LB-derivatives of all orders in $L^2(S^m)$ then we say it belongs to W_2^∞.*

The action of the Laplace-Beltrami derivative on kernels is done separately: we keep one variable fixed and differentiate with respect to the other. The symbol $K_{0,r}$ indicate the kernel obtained after K be r times differentiated with respect to the second variable y. The integral operator associated with $K_{0,r}$ is written as $\mathcal{K}_r$.

Definition 2.2. *A kernel $K \in L^2(S^m \times S^m)$ belongs to W_2^r, $1 \leq r \leq \infty$, when $K(x,\cdot) \in W_2^r$, $x \in S^m$ a.e.*

It is known that if $K \in L^2(S^m \times S^m)$ is a L^2-positive definite kernel that belongs to W_2^r, $1 \leq r < \infty$, and $\mathcal{K}_r$ belongs to the Schatten p-class $\mathcal{S}_p$, then the eigenvalues λ_n of $\mathcal{K}$ satisfy

$$\lambda_n(\mathcal{K}) \leq c\, n^{-(1/p)-(2r/m)}, \quad n \in \mathbb{N}, \tag{2.4}$$

for some positive constant c.

We intend to present the following result.

Theorem 2.3. *If $K \in L^2(S^m \times S^m) \cap W_2^\infty$ is L^2-positive definite then*

$$\lambda_n(\mathcal{K}) \leq c\, n^{-1-n/m}, \quad n \in \mathbb{N}, \tag{2.5}$$

for some positive real number c.

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HIPERCICLICIDADE DE OPERADORES DE CONVOLUÇÃO SOBRE ESPAÇOS DE FUNÇÕES HOLOMORFAS DEFINIDAS EM ESPAÇOS DE DIMENSÃO INFINITA

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Resumo

Neste trabalho provaremos que não existem operadores de convolução hipercíclicos em $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$. Provaremos também resultados de hiperciclicidade frequente para operadores de convolução sobre certos espaços de funções holomorfas definidas em espaços vetoriais topológicos complexos de dimensão infinita.

1 Introdução

Se X é um espaço vetorial topológico, dizemos que um operador linear contínuo $T\colon X \to X$ é *hipercíclico* se sua *órbita* $\{x, T(x), T^2(x), \ldots\}$ é densa em X, para algum $x \in X$.

Godefroy e Shapiro [7] provaram que todo operador de convolução não trivial sobre o espaço $\mathcal{H}(\mathbb{C}^n)$ das funções inteiras definidas em $\mathbb{C}^n$ é hipercíclico. Por um *operador de convolução* em $\mathcal{H}(\mathbb{C}^n)$ (ou, de maneira análoga, em qualquer espaço de funções inteiras), estamos nos referindo a um operador linear e contínuo $L\colon \mathcal{H}(\mathbb{C}^n) \to \mathcal{H}(\mathbb{C}^n)$ que comuta com translações, ou seja, $L(\tau_a f) = \tau_a(Lf)$ para toda $f \in \mathcal{H}(\mathbb{C}^n)$ e $a \in \mathbb{C}^n$, onde $(\tau_a f)(x) = f(x-a)$ para todo $x \in \mathbb{C}^n$.

Por um *operador de convolução não trivial* estamos nos referindo a um operador de convolução que não é múltiplo escalar da identidade. Este resultado generaliza resultados clássicos de Birkhoff [4] e MacLane [8] sobre hiperciclicidade dos operadores translação e diferenciação sobre $\mathcal{H}(\mathbb{C})$. Em contraste com estes resultados mostraremos que, sobre o espaço $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ das funções inteiras definidas em $\mathbb{C}^{\mathbb{N}}$, **nenhum** operador de convolução é hipercíclico. Este resultado foi provado em [6] e melhora um resultado apresentado pelo primeiro autor no IX ENAMA de que nenhum operador translação sobre $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ é hipercíclico.

Em [3], os autores provaram um resultado geral de hiperciclicidade para operadores de convolução sobre o espaço $\mathcal{H}_{\Theta b}(E)$ das funções inteiras de Θ-tipo limitado definidas sobre o espaço de Banach complexo E. Os resultados de [3] e [7] são baseados no critério de Kitai (também conhecido como critério de hiperciclicidade). Atualmente, sabe-se que o critério de Kitai prova mais do que hiperciclicidade, prova mixing. Dizemos que T é *mixing* se dados abertos não-vazios $U, V \subset X$, existe $n_0 \in \mathbb{N}$ tal que $T^n(U) \cap V \neq \emptyset$, para todo $n \geq n_0$. É fácil ver que todo operador mixing é hipercíclico. Logo as demonstrações dos resultados de [3] e [7] garantem que os referidos operadores de convolução são mixing.

Um outro conceito importante na dinâmica linear e amplamente estudado nos últimos anos é o de operador frequentemente hipercíclico, que é uma noção mais forte do que hiperciclicidade. De maneira informal, um operador linear contínuo $T\colon X \longrightarrow X$ é *frequentemente hipercíclico* se existe um vetor $x \in X$, cuja órbita intersecta cada conjunto aberto não vazio uma quantidade de vezes igual a cardinalidade de um conjunto de densidade inferior positiva.Tal conceito foi introduzido por Bayart e Grivaux em [1]. Como os conceitos de operador mixing e frequentemente hipercíclico são independentes (i.e., existem operadores mixing que não são frequentemente hipercíclicos e vice-versa), é natural se perguntar se os operadores de convolução definidos em

$\mathcal{H}_{\Theta b}(E)$ são frequentemente hipercíclicos. Neste trabalho provaremos que todo operador de convolução não trivial sobre $\mathcal{H}_{\Theta b}(E)$ é de fato frequentemente hipercíclico. A demonstração deste resultado é baseada num critério de hiperciclicidade de Bayart e Matheron [2] e em idéias de Muro, Pinasco e Savransky [9].

Neste trabalho provaremos também que se E é um espaço DFN arbitrário, então todo operador de convolução não trivial sobre o espaço de Fréchet $\mathcal{H}(E)$ é frequentemente hipercíclico. Relembre que um *espaço DFN* é um espaço que é o dual forte de um espaço de Fréchet nuclear. Nossa prova combina os resultados de hiperciclicidade frequente para $\mathcal{H}_{\Theta b}(E)$ (com E normado) com um método de fatoração introduzido por Colombeau e Matos [5] para o espaço $\mathcal{H}_{uNb}(E)$ (com E localmente convexo).

2 Resultados

Teorema 2.1. *Todo operador de convolução não trivial sobre $(\mathcal{H}(\mathbb{C}^{\mathbb{N}}), \tau)$ não é hipercíclico, onde τ é qualquer uma das 3 topologias usuais em $\mathcal{H}(\mathbb{C}^{\mathbb{N}})$ (compacto-aberta, bornológica, ou portada de Nachbin).*

Teorema 2.2. *Sejam E um espaço normado com dual separável e $(\mathcal{P}_{\Theta}(^{m}E))_{m=0}^{\infty}$ um π_1-tipo de holomorfia. Se L é um operador de convolução não trivial sobre $\mathcal{H}_{\Theta b}(E)$, então L é frequentemente hipercíclico.*

Teorema 2.3. *Seja E um espaço DFN e L um operador de convolução não trivial sobre $\mathcal{H}(E)$. Então L é frequentemente hipercíclico.*

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CARACTERIZAÇÃO DO DUAL TOPOLÓGICO DO ESPAÇO DOS POLINÔMIOS HIPER-(S, R)-NUCLEARES VIA TRANSFORMADA DE BOREL

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Resumo

Neste trabalho caracterizamos o dual topológico do espaço dos polinômios homogêneos hiper-$(s; r)$-nucleares entre espaços de Banach via transformada de Borel.

1 Definições e Resultados

Sejam E e F espaços de Banach e E' o dual topológico de E. Uma aplicação $P\colon E \to F$ é chamada de polinômio n-homogêneo se existe uma aplicação n-linear $A\colon E \times \cdots \times E \to F$ tal que $P(x) = A(x^n) = A(x, \ldots, x)$ para todo $x \in E$. O espaço vetorial dos polinômios n-homogêneos de E em F é denotado por $\mathcal{P}(^nE; F)$. Quando $F = \mathbb{R}$ ou $\mathbb{C}$, escrevemos simplesmente $\mathcal{P}(^nE)$ ao invés de $\mathcal{P}(^nE; F\mathbb{C})$.

Vejamos agora a definição de polinômios hiper-nucleares introduzida em [1].

Definição 1.1. *Sejam $s \in (0, \infty)$ e $r \in [1, \infty]$ tais que $1 \leq 1/s + 1/r$. Um polinômio n-homogêneo $P \in \mathcal{P}(^nE; F)$ é dito hyper-(s, r)-nuclear se existem escalares $(\lambda_j)_{j=1}^{\infty} \in \ell_s$, polinômios $(P_j)_{j=1}^{\infty} \in \ell_r^w(\mathcal{P}(^nE))$ e vetores $(y_j)_{j=1}^{\infty} \in \ell_\infty(F)$ tais que*

$$P(x) = \sum_{j=1}^{\infty} \lambda_j P_j \otimes y_j(x) = \sum_{j=1}^{\infty} \lambda_j P_j(x) y_j, \tag{1}$$

para todo $x \in E$. Denotamos o espaço vetorial de tais polinômios por $\mathcal{P}_{\mathcal{HN}_{(s,r)}}(^nE; F)$. Chamando

$$\|P\|_{\mathcal{HN}_{(s,r)}} = \inf\{\|(\lambda_j)_{j=1}^{\infty}\|_s \cdot \|(P_j)_{j=1}^{\infty}\|_{w,r} \cdot \|(y_j)_{j=1}^{\infty}\|_\infty\},$$

onde o ínfimo é tomado sobre todas as representações de P como em (1), temos uma norma em $\mathcal{P}_{\mathcal{HN}_{(s,r)}}(^nE; F)$.

Definição 1.2. *(Pietsch [2]) Sejam $s \in (0, \infty)$ e $r \in [1, \infty]$ com $1 \leq \frac{1}{r} + \frac{1}{s}$ ande $r \leq s'$. Um operador $u \in \mathcal{L}(E; F)$ é dito (s', r)-somante, se existe uma constante $C > 0$ tal que*

$$\left(\sum_{j=1}^{k} \|u(x_j)\|^{s'}\right)^{\frac{1}{s'}} \leq C \cdot \sup_{\varphi \in B_{E'}} \left(\sum_{j=1}^{k} |\varphi(x_j)|^r\right)^{\frac{1}{r}}, \tag{2}$$

onde $x_j \in E$, $j = 1, \ldots, k$. Denotamos o espaço vetorial de tais operadores por $\Pi_{(s',r)}(E; F)$. Chamando

$$\|u\|_{\Pi_{(s',r)}} = \inf\{C; C \text{ satisfazem } (2)\},$$

temos uma norma em $\Pi_{(s',r)}(E; F)$.

Definimos a transformada de Borel por:

$$\begin{aligned}\beta\colon \left[\mathcal{P}_{\mathcal{HN}_{(r,s)}}({}^nE;F), \|\cdot\|_{\mathcal{HN}_{(r,s)}}\right]' &\longrightarrow \left[\Pi_{(s',r)}(\mathcal{P}({}^nE),F'), \|\cdot\|_{\Pi_{(s',r)}}\right]\\ \psi &\longmapsto \beta(\psi)\colon \mathcal{P}({}^nE) \longrightarrow F'\\ &\qquad P \mapsto \beta(\psi)(P)\colon F \longrightarrow \mathbb{K}\\ &\qquad\qquad \beta(\psi)(P)(y) = \psi(P\otimes y)\end{aligned}$$

onde $P\otimes y\colon E \longrightarrow F$ é definido por $P\otimes y(x) = P(x)\cdot y$.

O objetivo deste trabalho é provar que a transformada de Borel é um isomorfismo isométrico, quando o espaço $\mathcal{P}({}^nE)$ tem a propriedade de aproximação λ-limitada. Relembre que um espaço de Banach G tem a propriedade de aproximação λ-limitada se para todo subconjunto compacto $K\subset G$ e $\varepsilon > 0$, existe um operador linear contínuo de tipo finito T de G em G tal que $\|T\| \leq \lambda$ e $\|x - T(x)\| < \varepsilon$ para todo $x \in K$.

A injetividade da transformada de Borel não depende de $\mathcal{P}({}^nE)$ ter a propriedade de aproximação λ-limitada e segue do seguinte resultado.

Proposição 1.1. *β é injetora se, e somente se, $\overline{\mathcal{P}_{\mathcal{F}}}^{\mathcal{HN}_{(r,s)}}(E;F) = \mathcal{P}_{\mathcal{HN}_{(r,s)}}(E;F)$ (isto é, $\mathcal{P}_{\mathcal{F}}(E;F)$ é denso em $\mathcal{P}_{\mathcal{HN}_{(r,s)}}(E;F)$ na norma hiper-(r,s)-nuclear). Aqui $\mathcal{P}_{\mathcal{F}}(E;F)$ denota o espaço dos polinômios de posto finito de E em F.*

Finalmente temos o resultado principal deste trabalho.

Teorema 1.1. *Se $\mathcal{P}({}^nE)$ tem a propriedade de aproximação λ-limitada, então a transformada de Borel*

$$\beta\colon \left[\mathcal{P}_{\mathcal{HN}_{(r,s)}}\right]' \longrightarrow \Pi_{(s',r)}(\mathcal{P}({}^nE),F')$$

é um isomorfismo isométrico.

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WEIGHTED SPHERICAL MEANS OPERATORS AND THEIR CONNECTION WITH K-FUNCTIONALS

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Abstract

We design a new type of spherical mean operators, depending on a real number as parameter, given by a convolution with certain translations of dilated measures, and employ them to approximate L_p class functions. Asymptotic relations between the rate of approximation of the new operator and the K-functional of fractional order are stablished. When the parameter we work with is taken as a natural number the general type of spherical mean operator, the K-funcional and also the result relating such objects turn out the same as in Dai & Ditzian (2004) which introduces a class of "multi-layered" spherical mean operators.

1 Introduction

We work in $\mathbb{R}^{d+1}$, $d \geq 1$, the $(d+1)$-dimensional Euclidian space equipped with the Lebesgue measure. For $x, y \in \mathbb{R}^{d+1}$, we denote $x \cdot y$ the usual inner product of x and y, and L_p the regular Banach spaces consisting of all Lebesgue measurable functions which are p - integrable in $\mathbb{R}^{d+1}$, and we denote $\|\cdot\|_p$ its norm.

For $t \in [0, \infty)$, the standard spherical mean operator is defined by

$$V_t(f)(x) = \frac{1}{\omega_d} \int_{S^d} f(x + t\omega) d\omega, \qquad f \in L^p(\mathbb{R}^{d+1}), \quad x \in \mathbb{R}^{d+1},$$

where S^d is the unit sphere in $\mathbb{R}^{d+1}$, ω_d its volume and $d\omega$ is induced on S^d by the Lebesgue measure in $\mathbb{R}^{d+1}$.

Let $0 < a < d+1$ and $A_{a,d}$ the constant such that $x \to A_{a,d}|x|^{-a}$ is a L^1 - unit distribution which is denoted by $\phi_{a,d}$. Being L^1 - unit distribution means that $\|\phi_{a,d} * \psi\|_1 \leq \|\psi\|_1$, for all test functions and $\|\phi_{a,d} * \psi\|_1 = \|\psi\|_1$ for some ψ test function. Here $*$ denotes the usual convolution on the Euclidean space.

Let $r, t > 0$ be real numbers and $\ell := \lceil r \rceil$, where $\lceil \cdot \rceil$ is the ceiling function. We define the *weighted spherical mean operator* $T_r : L_p \longrightarrow L_p$ as the following convolution

$$T_{r,t}(f) = f * (\mu_\ell - \tau_r^t), \quad f \in L_p,$$

where μ_ℓ is defined in [2] and given by

$$\mu_\ell = 2\binom{2\ell}{\ell}^{-1} \sum_{j=1}^{\ell} (-1)^{j+1} \binom{2\ell}{\ell - j} \omega_k,$$

with ω_k being the dilation of the measure $\omega_d^{-1}\omega$ by k that is

$$\int_{S^d} f(x) d\omega_k(x) = \frac{1}{\omega_d} \int_{S^d} f(k\omega) d\omega, \quad f \in L_1(\mathbb{R}^{d+1});$$

and

$$\tau_r^t(x) = \binom{2\ell}{\ell}^{-1} \left[\binom{2\ell}{\ell} \phi_{a,d}(|x - y|) + 2 \sum_{k=1}^{\ell} (-1)^k \binom{2\ell}{\ell - k} \int_{S_k} \phi_{a,d}(|x - ty|) d\omega_k(y) \right], \quad x \in \mathbb{R}^{d+1},$$

for $a = -2\ell + 2r + d + 1$, S_k denotes the sphere centered ate the origin with radius k.

We define a K-functional on L_p associated with the "fractional order" Laplacian Δ^r:

$$K_r(f, \Delta, t)_p := \inf_g \{\|f - g\|_p + t^{2r}\|\Delta^r g\|_p : \Delta^r g \in L^p\},$$

where $(\Delta^r g)(x) := (|2\pi\xi|^{2r}\,\widehat{g}(\xi))\check{}(x)$. Here the hat over de function is the Fourier transform defined by

$$\widehat{f}(\xi) = \int_{\mathbb{R}^{d+1}} f(x) e^{-2\pi i \xi \cdot x} dx, \quad f \in L^1(\mathbb{R}^{d+1}),$$

and the notation $\check{f}$ refer to the inverse Fourier transform of f.

When r is taken a natural number, let us say ℓ, the weighted spherical mean operator and also the K-funcional turn out being the same operator and K-functional in Dai and Ditzian [3] introduced a class of "multi-layered" spherical mean operators $V_{\ell,t}$ defined by

$$V_{\ell,t}(f)(x) = \frac{-2}{\binom{2\ell}{\ell}} \sum_{k=1}^{\ell} (-1)^k \binom{2\ell}{\ell - k} V_{kt}(f)(x), \qquad f \in L^p, \quad x \in \mathbb{R}^{d+1}.$$

The authors showed the following equivalence

$$\|V_{\ell,t}(f) - f\|_p \asymp K_\ell(f, \Delta, t^{2\ell}), \quad 1 \le p \le \infty, \tag{1}$$

where $\asymp$ means the same asymptotic behavior in terms of variable t. Belinsky, Dai and Ditzian [1], and Dai and Ditzian [3] also studied the counterparts of the operators $V_{\ell,t}$ on S^d, the unit sphere in $\mathbb{R}^{d+1}$, and obtained comparably interesting results.

2 Main Result

We establish the following asymptotic relation:

$$\|T_{r,t}(f) - f\|_p \asymp K_r(f, \Delta, t^{2r}), \quad f \in L_p, \quad r > 1, \quad 1 \le p < \infty. \tag{2}$$

This equivalence gives us a new characterisation for the K-functional having fractional order. If $r = \ell$ is a natural number, then $T_{\ell,t}$ and $V_{\ell,t}$ coincide, as do the asymptotic relation (1.1).

The method to obtain such equivalence makes use, mainly, of multipliers operators technics as Hörmander multiplier theorem and its consequences. Similar results, characterizing the K-functional with fractional order, can be found here [4] where the authors make use of an operator given a infinite sum (series expansion) of spherical mean operator in order to characterize the same K-functional as here. Even if the operator defined in [4] is an improvement of what we have on the literature it does seem to worth in applicable problems (computational implementations, for example) that is why the weighted spherical mean operator came up as a option for the K-functional of fractional order characterization.

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SCHAUDER DECOMPOSITION AND LINEARIZATION OF HOLOMORPHIC FUNCTIONS OF BOUNDED TYPE

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Abstract

We show that the preduals of the spaces of continuous m-homogeneous polynomials on a complex Banach space E form a Schauder decomposition for the predual of the space of holomorphic functions of bounded type on a balanced open subset of E.

1 Introduction

If E and F are (complex) Banach spaces and $m \in \mathbb{N}_0$, then $\mathcal{P}(^mE;F)$ denotes the Banach space of all continuous m-homogeneous polynomials from E into F. If U is a nonempty open subset of E, then $\mathcal{H}_b(U;F)$ denotes the Fréchet space of all holomorphic mappings of bounded type from U into F.

If E and F are (complex) locally convex spaces, then $\mathcal{L}(E;F)$ denotes the vector space of all continuous linear mappings from E into F, and $cs(E)$ denotes the set of all continuous seminorms on E. Let $\mathbb{N}$ denote the set of all positive integers, and let $\mathbb{N}_0$ denote the set $\mathbb{N} \cup \{0\}$.

We then have the following linearization theorems.

Theorem 1.1. *(see [5]) Let E be a Banach space. Then for each $m \in \mathbb{N}_0$ there are a Banach space $Q(^mE)$ and a mapping $\delta_E^m \in \mathcal{P}(^mE;Q(^mE))$ with the following universal property: For each Banach space F and each $P \in \mathcal{P}(^mE;F)$, there is a unique $T_P \in \mathcal{L}(Q(^mE);F)$ such that $P = T_P \circ \delta_E^m$.*

Theorem 1.2. *(see [2]) Let U be a balanced open subset of a Banach space E. Then there are an (LB)-space $G_b(U)$ and a mapping $\delta_U \in \mathcal{H}_b(U;G_b(U))$ with the following universal property: For each Banach space F and each $f \in \mathcal{H}_b(U;F)$, there is a unique $T_f \in \mathcal{L}(G_b(U);F)$ such that $f = T_f \circ \delta_U$.*

2 Main Results

We prove that the spaces $Q(^mE)$ and $G_b(U)$ are related as follows.

Theorem 2.1. *Let U be a balanced open subset of a Banach space E. Then $(Q(^mE))_{m=0}^{\infty}$ is an absolute $\mathcal{S}$-Schauder decomposition for $G_b(U)$.*

Next we present the relevant definitions. If E is a locally convex space, then a sequence $(E_m)_{m=0}^{\infty}$ of subspaces of E is said to be a *decomposition* for E if each $x \in E$ can be uniquely written as a series

$$x = \sum_{m=0}^{\infty} x_m, \quad \text{with } x_m \in E_m \text{ for every } m \in \mathbb{N}_0.$$

A decomposition $(E_m)_{m=0}^{\infty}$ for E induces a sequence of projections $\pi_m : E \to E$, defined by

$$\pi_m\left(\sum_{k=0}^{\infty} x_k\right) = \sum_{k=0}^{m} x_k.$$

A decomposition $(E_m)_{m=0}^{\infty}$ for E is said to be a *Schauder decomposition* if all the projections π_m are continuous. A Schauder decomposition $(E_m)_{m=0}^{\infty}$ for E is said to be *equi-Schauder* if the sequence of projections $(\pi_m)_{m=0}^{\infty}$ is equicontinuous. A Schauder decomposition $(E_m)_{m=0}^{\infty}$ for E is said to be an *absolute Schauder decompositon* if for each $p \in cs(E)$, the formula

$$q\left(\sum_{m=0}^{\infty} x_m\right) = \sum_{m=0}^{\infty} p(x_m)$$

also defines a continuous seminorm on E. Every absolute Schauder decomposition is equi-Schauder. Let

$$\mathcal{S} = \{\sigma = (\sigma_m)_{m=0}^{\infty} \subset \mathbb{C} : \limsup_{m\to\infty} |\sigma_m|^{1/m} \leq 1\}.$$

A Schauder decomposition $(E_m)_{m=0}^{\infty}$ for E is said to be an *$\mathcal{S}$-Schauder decomposition* if $\sum_{m=0}^{\infty} \sigma_m x_m \in E$ for each $x = \sum_{m=0}^{\infty} x_m \in E$ and $\sigma = (\sigma_m)_{m=0}^{\infty} \in \mathcal{S}$. An $\mathcal{S}$-Schauder decomposition $(E_m)_{m=0}^{\infty}$ for E is said to be an *absolute $\mathcal{S}$-Schauder decomposition* if for each $p \in cs(E)$ and $\sigma = (\sigma_m)_{m=0}^{\infty} \in \mathcal{S}$, the formula

$$q\left(\sum_{m=0}^{\infty} x_m\right) = \sum_{m=0}^{\infty} |\sigma_m| p(x_m)$$

also defines a continuous seminorm on E.

Theorem 2.1 is a version for holomorphic functions of bounded type of a result of Boyd [1] for holomorphic functions, and the proof of Theorem 2.1 is an adaptation of Boyd's proof. The proof of Theorem 2.1 rests also on the description of $Q(^m E)$ given in [3] and the description of $G_b(U)$ given in [4].

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THE BOREL TRANSFORM AND THE DUAL OF THE SPACE OF $\sigma(P)$-NUCLEAR LINEAR AND MULTILINEAR OPERATORS

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Abstract

When presenting the concepts of τ-summing and σ-nuclear operators in [3], Pietsch stablishes a relation between the adjoint of σ-nuclear operators and τ-summing operators. However, we realized there also is a duality relation between the dual of σ-nuclear operators $[\mathcal{L}_{\sigma(p)}(E;F)]'$ and τ-summing operators $\mathcal{L}_{\tau(p)}(E';F')$, through the Borel transform, as long as F is reflexive Banach space. However, if we dismiss reflexivity and consider the Borel transform from $[\mathcal{L}_{\sigma(p)}(E;F)]'$ into $\mathcal{L}(E';F')$, what will its image be? We extend σ-nuclear operators to n-linear operators and answer the question above: the solution is a new class of operators, the quasi-$\tau(p)$-summing operators, which are extremely close to the $\tau(p)$-summing (the latter have been studied in [2]).

1 Introduction

The following definition generalizes the class of σ-nuclear linear operators of Pietsch [3] to the multilinear setting and introduces the classes of $\sigma(p)$-nuclear linear and multilinear operators for $p > 1$:

Definition 1.1. For $p \geq 1$, we say that an n-linear operator $A\colon E_1 \times \cdots \times E_n \to F$ is $\sigma(p)$-*nuclear* if there are sequences $(\lambda_j)_{j=1}^{\infty} \in \ell_{p'}$, $(x'_{ij})_{j=1}^{\infty} \subseteq E'_i$ for $i = 1, \ldots, n$, and $(y_j)_{j=1}^{\infty} \subseteq F$ such that

$$A(x_1, \ldots, x_n) = \sum_{j=1}^{\infty} \lambda_j x'_{1j}(x_1) \cdots x'_{nj}(x_n) y_j$$

for all $x_1 \in E_1, \ldots, x_n \in E_n$,

$$\sup_{\substack{x_i \in B_{E_i} \\ y' \in B_{F'}}} \left(\sum_{j=1}^{\infty} |x'_{1j}(x_1) \cdots x'_{nj}(x_n) y'(y_j)|^p \right)^{1/p} < \infty \tag{1}$$

and

$$\lim_{m\to\infty} \sup_{\substack{x_i \in B_{E_i} \\ y' \in B_{F'}}} \left(\sum_{j=m}^{\infty} |x'_{1j}(x_1) \cdots x'_{nj}(x_n) y'(y_j)|^p \right)^{1/p} = 0. \tag{2}$$

In this case we write $A = \sum\limits_{j=1}^{\infty} \lambda_j x'_{1j} \otimes \cdots \otimes x'_{nj} \otimes y_j$, say that this is a $\sigma(p)$-nuclear representation of A and define

$$\|A\|_{\sigma(p)} = \inf\left\{ \|(\lambda_j)_{j=1}^{\infty}\|_{p'} \cdot \sup_{\substack{x_i \in B_{E_i} \\ y' \in B_{F'}}} \left(\sum_{j=1}^{\infty} |x'_{1j}(x_1) \cdots x'_{nj}(x_n) y'(y_j)|^p \right)^{1/p} \right\},$$

where the infimum runs over all $\sigma(p)$-nuclear representations of A. The set of all such n-linear operators is denoted by $\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F)$.

Of course the case $n = p = 1$ recovers the Banach ideal of σ-nuclear linear operators from [3, Section 23.1].

In order to answer which is the subset of $\mathcal{L}(E_1', \ldots, E_n'; F')$ that is isometrically isomorphic to the dual of the space $\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F)$ of $\sigma(p)$-nuclear n-linear operators, via the Borel transform, we introduce the class of *quasi-$\tau(p)$-summing* n-linear operators:

Definition 1.2. For $1 \leq q \leq p$, an n-linear operator $S \in \mathcal{L}(E_1, \ldots, E_n; F')$ is said to be *quasi-$\tau(p;q)$-summing* if there is a constant $\mathsf{C} \geq 0$ such that

$$\left(\sum_{j=1}^{m} |S(x_{1j}, \ldots, x_{nj})(y_j)|^p\right)^{1/p} \leq \mathsf{C} \sup_{\substack{x_i' \in B_{E_i'} \\ y' \in B_{F'}}} \left(\sum_{j=1}^{m} |x_1'(x_{1j}) \ldots x_n'(x_{nj}) y'(y_j)|^q\right)^{1/q}, \tag{3}$$

for all $m \in \mathbb{N}$, $x_{ij} \in E_i$, $y_j \in F$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. The infimum of all such constants C is denoted by $\|S\|_{q\tau(p;q)}$. We denote this class of operators by $\mathcal{L}_{q\tau(p;q)}(E_1, \ldots, E_n; F')$. Routine computations show that $[\mathcal{L}_{q\tau(p;q)}(E_1, \ldots, E_n; F'), \|\cdot\|_{q\tau(p;q)}]$ is a Banach space.

Whenever $p = q$, we simply write $\mathcal{L}_{q\tau(p)}$, $\|S\|_{q\tau(p)}$ and say S is *quasi-$\tau(p)$-summing.* If $p = q = 1$, we write $\mathcal{L}_{q\tau}$, $\|S\|_{q\tau}$ and say S is *quasi-τ-summing.*

This class is closely related to the class $\mathcal{L}_{\tau(p)}$ of $\tau(p)$-summing operators studied in [2]. Using the results of [1], it can be proved that the class of $\mathcal{L}_{q\tau(p)}$ enjoys a Pietsch-type domination theorem.

2 Main Results

The Borel transform

$$\mathcal{B}\colon [\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F), \|\cdot\|_{\sigma(p)}]' \longrightarrow \mathcal{L}(E_1', \ldots, E_n'; F'),$$
$$\mathcal{B}(\varphi)(x_1', \ldots, x_n')(y) := \varphi(x_1' \otimes \cdots x_n' \otimes y),$$

is a well defined linear operator. The question is to identify the range of $\mathcal{B}$ and a norm on it that makes $\mathcal{B}$ an isometric isomorphism. The answer is the class of quasi-$\tau(p)$-summing operators:

Theorem 2.1. *Let $p \geq 1$, $E_1, \ldots, E_n, F$ be Banach spaces where $E_1', \ldots, E_n'$ have the bounded approximation property. Then the spaces*

$$[\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F)]' \quad \text{and} \quad \mathcal{L}_{q\tau(p)}(E_1', \ldots, E_n'; F')$$

are isometrically isomorphic via the Borel transform.

As $\mathcal{L}_{q\tau(p)}(E_1', \ldots, E_n'; F') = \mathcal{L}_{\tau(p)}(E_1', \ldots, E_n'; F')$ whenever F is reflexive, we have:

Corollary 2.1. *Let $p \geq 1$, $E_1, \ldots, E_n, F$ be Banach spaces where $E_1', \ldots, E_n'$ have the bounded approximation property and F is reflexive. Then the spaces*

$$[\mathcal{L}_{\sigma(p)}(E_1, \ldots, E_n; F)]' \quad \text{and} \quad \mathcal{L}_{\tau(p)}(E_1', \ldots, E_n'; F')$$

are isometrically isomorphic via the Borel transform.

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ISOLATED SINGULARITIES OF SOLUTIONS FOR A CRITICAL ELLIPTIC SYSTEM

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Abstract

In this paper we study the asymptotic behavior of local solutions for strongly coupled critical elliptic systems near an isolated singularity, when the dimension is less than or equal to five and the potential of the operator is less, in the sense of bilinear forms, than the geometric threshold potential of the conformal Laplacian. We prove a sharp result on the removability of the isolated singularity for all components of the solutions.

1 Introduction

Our main goal in this paper is to study, when the dimension is less than or equal five, the asymptotic behavior of positive local solutions for the following coupled system

$$-\Delta_g u_i + \sum_{j=1}^{p} A_{ij}(x) u_j = |\mathcal{U}|^{2^*-2} u_i \tag{1}$$

in the punctured ball $\Omega = B_1^n(0)\backslash\{0\} \subset \mathbb{R}^n, n \geq 3$. Here Δ_g denotes the Laplace-Beltrami operator of the metric g, $|\mathcal{U}|^2 = \sum_{i=1}^{p} u_i^2$ and A is a C^1 map from Ω to $M_p^s(\mathbb{R})$, where $M_p^s(\mathbb{R})$ denote the vector space of symmetrical $p \times p$ real matrices for $p \geq 1$ integer. We write that $A = (A_{ij})_{i,j}$, where A_{ij}'s are C^1 real-valued functions in Ω.

To study the asymptotic behavior of the local solutions of the system, we will need a fundamental assumption on the map A. We will suppose that the potential of the operator is less or equal, in the sense of bilinear forms, than the geometric threshold potential of the conformal Laplacian, that is

$$A \leq \frac{n-2}{4(n-1)} R_g Id_p \tag{2}$$

in the sense of bilinear forms, where R_g denotes de scalar curvature, and Id_p is the identity matrix in $M_p^s(\mathbb{R})$.

Coupled systems of nonlinear Schorödinger equations are part of several important branches on mathematical physics. They appear in the Hartree-Fock theory for Bose-Einstein double condensates, in the theory of Langmuir waves in plasma physics, in fiber-optic theory and in the behavior of deep water waves and freak waves in the ocean. From the viewpoint of conformal geometry, our systems are pure extensions of Yamabe type equations since inherits a conformal structure.

2 Main Results

The following theorem establishes an upper bound near the isolated singularity.

Theorem 2.1. *Suppose* $3 \leq n \leq 5$. *Assume that* $\mathcal{U} = (u_1, ..., u_p)$ *is a positive solution of* (1) *in* $\Omega = B_1^n(0)$ *and the* C^1 *map* A *satisfies* (2). *There exists a constant* $c > 0$ *such that*

$$u_i(x) \leq c d_g(x,0)^{\frac{2-n}{2}}, \tag{3}$$

for $0 < d_g(x,0) < \frac{1}{2}$ *and for all* $i \in \{1, ..., p\}$.

As a consequence of the upper bound we get the following spherical Harnack inequality around the singularity

Corollary 2.1. *Suppose $\mathcal{U}$ is a positive smooth solution of* (1) *in $\Omega = B_1^n(0)\backslash\{0\}$, $3 \leq n \leq 5$ and the C^1 map A satisfies* (2)*.. Then there exists a constant $c_1 > 0$ such that*

$$\max_{|x|=r} u_i \leq c_1 \min_{|x|=r} u_i \tag{4}$$

for every $0 < r < \frac{1}{4}$. Moreover, $|\nabla u_i| \leq c_1|x|^{-1}u_i$ and $|\nabla^2 u_i| \leq c_1 r^{-2} u_i$.

Given $\mathcal{U}$ a positive solution to the system (1), define

$$P(r;\mathcal{U}) = \sum_{i=1}^{p} \int_{\partial B_r} \left[\frac{n-2}{2} u_i \frac{\partial u_i}{\partial \nu} - \frac{r}{2}|\nabla u_i|^2 + r\left|\frac{\partial u_i}{\partial \nu}\right|^2 \right] + \int_{\partial B_r} \frac{r}{2^*}|\mathcal{U}|^{2^*} \tag{5}$$

Due the Pohozaev identity we can define the *Pohozaev invariant* as the limit

$$P(\mathcal{U}) := \lim_{r\to 0} P(r;\mathcal{U}).$$

Our main result here is the following removable singularity theorem:

Theorem 2.2. *Assume $3 \leq n \leq 5$ and let $\mathcal{U}$ be a positive solution to the system* (1) *in $B_1^n(0)\backslash\{0\}$ and the C^1 map A satisfies* (2)*. Then $P(\mathcal{U}) \leq 0$. Moreover, $P(\mathcal{U}) = 0$ if and only if each u_i is smooth on the origin.*

As a consequence of the removable singularity theorem, we can now establish a fundamental lower bound.

Corollary 2.2. *Assume $3 \leq n \leq 5$ and let $\mathcal{U}$ a positive solution to the system* (1) *in $B_1^n(0)\backslash\{0\}$ and the C^1 map A satisfies* (2)*. If 0 is a nonremovable singularity, then there exists $c > 0$ such that*

$$u_i(x) \geq c d_g(x,0)^{\frac{2-n}{2}}$$

for each $i = 1, ..., p$ and $0 < d_g(x,0) < \frac{1}{2}$.

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ON A GENERALIZED KIRCHHOFF EQUATION WITH SUBLINEAR NONLINEARITIES

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Abstract

In this paper we consider a generalized Kirchhoff equation in a bounded domain under the effect of a sublinear nonlinearity. Under suitable assumptions on the data of the problem we show that, with a simple change of variable, the equation can be reduced to a classical semilinear equation and then studied with standard tools.

1 Introduction

In this article we study the existence of solutions $u : \Omega \to \mathbb{R}$ for the following nonlocal problem in divergence form

$$\begin{cases} -\mathrm{div}\left(m\left(u, |\nabla u|_2^2\right)\nabla u\right) = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary,

$$m : \mathbb{R} \times [0,\infty) \to \mathbb{R}, \quad f : \Omega \times \mathbb{R} \to \mathbb{R}$$

are given functions satisfying suitable conditions which will be given later. Hereafter we denote with $|\cdot|_p$ the usual $L^p(\Omega)-$norm. By a solution of the above problem, we mean a function $u_* \in H_0^1(\Omega) \cap L^\infty(\Omega)$ such that

$$\forall \varphi \in H_0^1(\Omega) : \quad \int_\Omega m(u_*(x), |\nabla u_*|_2^2)\nabla u_* \nabla \varphi \, dx = \int_\Omega f(x,u_*)\varphi \, dx,$$

whenever the integrals make sense.

When the function m does not depend on u we have the classical problem

$$\begin{cases} -m\left(|\nabla u|_2^2\right)\Delta u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

which is the N-dimensional version, in the stationary case, of the *Kirchhoff equation* introduced in [2]. We do not list here the huge amount of papers concerning this equation. On the other hand in many physical problems, rather then on $|\nabla u|_2$, the function m depends on the unknown u, or even on quantities related to u, as its L^1-norm (see e.g. [3]). Problem (1) studied in this paper can be considered as a slight generalization of the Kirchhoff equation. In this paper we will treat the situations in which $f = f(x)$ and $f = f(x,u)$ with sublinear growth in u. Here we say that the function m will satisfy quite general assumptions. In particular, in contrast to the case in which m depend only on $|\nabla u|_2$, here there is no restriction in the growth at infinity of m with respect to $|\nabla u|_2^2$. The main novelty of our approach is that the proofs are based on a simple "change of variable" device which seems not to have been used for these kind of nonlocal equations. With the use of the change of variable, the equation (1) is reduced to a "local" semilinear equation, for which various tools are available to solve it.

2 Main Results

We suppose that $m : \mathbb{R} \times [0, \infty) \to (0, \infty)$ is a function satisfying the following conditions:

(m_0) $m : \mathbb{R} \times [0, +\infty) \to (0, +\infty)$ is a continuous function;

(m_1) there is $\mathfrak{m} > 0$ such that $m(t, r) \geq \mathfrak{m}$ for all $t \in \mathbb{R}$ and $r \in [0, \infty)$;

(m_2) for each $r \in [0, +\infty)$ the map $m_r : \mathbb{R} \to (0, +\infty)$ is strictly decreasing in $(-\infty, 0)$ and strictly increasing in $(0, +\infty)$.

Moreover $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a function satisfying the following condition

(C) f is a Carathéodory function and

$$\exists c > 0 : \quad |f(x, t)| \leq c(1 + |t|^p), \quad (x, t) \in \Omega \times \mathbb{R},$$

where $1 < p < 2^* - 1$ and $2^* = 2N/(N - 2)$.

Teorema 2.1. *If (m_0)-(m_2) hold, $0 \not\equiv f \in L^q(\Omega)$ and $q > N/2$, then problem (1), with $f(x, u) = f(x)$, has a nontrivial weak solution u_*.*

Teorema 2.2. *Let m and f satisfy (m_0)-(m_2) and (C), respectively. Suppose, additionally, that for each $r \geq 0$, problem*

$$\begin{cases} -div(m_r(u)\nabla u) = f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P_r}$$

where

$$m_r : t \in \mathbb{R} \mapsto m(t, r) \in \mathbb{R},$$

has a unique nontrivial weak solution $v_r \in H_0^1(\Omega) \cap L^\infty(\Omega)$ and the map

$$V : r \in [0, +\infty) \longmapsto \int_\Omega |\nabla v_r|^2 dx \in [0, +\infty)$$

is in $L^\infty([0, +\infty))$. Then (1) possesses a nontrivial weak solution.

The last theorem above is quite general and can be applied in several particular situations. In this paper we provide solutions for two special problems by using theorem 2.2.

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SOME BERNSTEIN-LIOUVILLE TYPE RESULTS FOR FULLY NONLINEAR ELLIPTIC EQUATIONS AND THEIR CONSEQUENCES

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Abstract

In this work we prove Bernstein-Liouville type Theorems to fully nonlinear equations of degenerate/singular or uniformly elliptic type, i.e., entire viscosity solutions to

$$\mathfrak{F}\left(x, Du, D^2u\right) = 0 \quad \text{in} \quad \mathbb{R}^n, \tag{1}$$

are polynomial functions of certain degree, provided their growth at infinity can be controlled in an appropriated manner. Inhomogeneous problems with strong absorption condition (dead core problems, see [1]) will also be treated. As application, we classify some Blow-up and Half-space solutions coming from free boundary problems.

1 Introduction

In the current literature, Bernstein-Liouville type Theorems are well known by yielding classification results to global profiles for certain solutions, e.g., an entire solution to a uniformly elliptic operator $\mathcal{L}[u](x) = 0$ (in divergence or non-divergence form) such that $|u(x)| = o(|x|^{\alpha})$ as $|x| \to \infty$, for some $\alpha \in (0,1)$, must be constant. Furthermore, such type results play a significant role in the modern theory of elliptic PDE's and mathematical analysis due to their several applications in Nonlinear equations, Free boundary problems and Geometry, just to mention a few (cf. [1, Section 5], [3] and [4, Section 3]).

Our studies on Bernstein-Liouville type results have been motivated by understanding some global profiles coming from free boundary problems (such as Blow-up or Half-space type solutions). For this very reason, our approach consist in analysing the relationship between growth rate of solutions and their regularity estimates (cf. [3, Section 1] and [4, Section 3]).

Hereafter, $\mathfrak{F} : \mathbb{R}^n \times \mathbb{R}^n_* \times Sym(n) \to \mathbb{R}$ is a fully nonlinear elliptic operator satisfying the structural properties:

(F1) [**γ-Ellipticity condition**] There exist constants $\Lambda \geq \lambda > 0$ such that for $\gamma > -1$ there holds

$$|\overrightarrow{p}|^{\gamma}\|P\| \leq \mathfrak{F}\left(x, \overrightarrow{p}, M+P\right) - \mathfrak{F}\left(x, \overrightarrow{p}, M\right) \leq |\overrightarrow{p}|^{\gamma}\|P\|, \ \forall \ (x, \overrightarrow{p}, M, P) \in \mathbb{R}^n \times \mathbb{R}^n_* \times \mathrm{Sym}(n) \times \mathrm{Sym}^+(n) \tag{2}$$

(F2) [$(\gamma, 1)$**-Homogeneity condition**] For all $(s, t, x, \overrightarrow{p}, M) \in \mathbb{R}^* \times \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n_* \times Sym(n)$ there holds

$$\mathfrak{F}\left(x, s\overrightarrow{p}, tM\right) = |s|^{\gamma} t . \mathfrak{F}\left(x, \overrightarrow{p}, M\right). \tag{3}$$

(F3) [**Continuity condition**] There exists a continuous function $\tau : [0, \infty) \to [0, \infty)$ with $\tau(0) = 0$ such that

$$|\mathfrak{F}\left(x, \overrightarrow{p}, M\right) - \mathfrak{F}\left(y, \overrightarrow{p}, M\right)| \leq \tau\left(|x-y|\right) |\overrightarrow{p}|^{\gamma}\|M\|, \quad \forall \ \ (x, y, \overrightarrow{p}, M) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n_* \times Sym(n)$$

Next, the appropriated definition of viscosity solutions to our context.

Definition 1.1 (**Viscosity solutions**)**.** *$u \in C^0(\mathbb{R}^n)$ is said a viscosity super-solution (resp. sub-solution) to*

$$\mathfrak{F}\left(x, Du, D^2u\right) = g(x,u) \quad in \quad \mathbb{R}^n$$

if for every $x_0 \in \mathbb{R}^n$ we have the following

1. *Either $\forall\ \phi \in C^2_{loc}(\mathbb{R}^n)$ such that $u - \phi$ has a local minimum at x_0 and $|D\phi(x_0)| \neq 0$ holds*

$$\mathfrak{F}\left(x_0, D\phi(x_0), D^2\phi(x_0)\right) \leq g(x_0, \phi(x_0)) \quad (resp.\ \geq g(x_0, \phi(x_0)))$$

2. *Or there exists an open ball $B(x_0, \varepsilon)$ where u is constant, $u = \mathfrak{C}$ and holds*

$$g(x, \mathfrak{C}) \geq 0 \ \ \forall \ \ x \in B(x_0, \varepsilon) \quad (resp.\ g(x, \mathfrak{C}) \leq 0)$$

Finally, u is said to be a viscosity solution if it is simultaneously a viscosity super-solution and sub-solution.

2 Main Results

Our main results in this work are described below.

Theorem 2.1 (**Bernstein-Liouville type theorem**, [3])**.** *Let u be an entire viscosity solution to (1). If there exist an $h_{\mathfrak{F}} : [0, \infty) \to [0, \infty)$ and $\alpha = \alpha(n, \lambda, \Lambda, \gamma) \in (0, 1)$ such that $h(t) \leq t^{1+\alpha}$ (resp. $\leq t^{2+\alpha}$) for all $t \gg 1$ and*

$$|u(x)| = o\left(h_{\mathfrak{F}}(|x|)\right) \ \ as \ |x| \to \infty$$

then u is an affine function (resp. at most a quadradic function).

Corollary 2.1 (**Classifying Half-space solutions**, [3])**.** *Let $u \in C^0(\overline{\mathbb{R}^n_+})$ be a viscosity solution to*

$$\begin{cases} \mathfrak{F}\left(x, Du, D^2u\right) & = & 0 & in & \mathbb{R}^n_+ \\ u(x) & = & 0 & on & \partial\mathbb{R}^n_+ \\ \displaystyle\sup_{\substack{x,y \in \overline{\mathbb{R}^n_+} \\ x \neq y}} \frac{|u(x) - u(y)|}{|x-y|} & = & \mathfrak{L}_0 & & \end{cases}$$

Then u is a linear function.

Theorem 2.2 (**Liouville type theorem**, [1, 3])**.** *Let u be a viscosity solution to*

$$\mathfrak{F}\left(x, Du, D^2u\right) = u_+^{\mu}(x) \quad in \quad \mathbb{R}^n$$

such that u(0) = 0 and $0 \leq \mu < \gamma + 1$. If $\displaystyle\limsup_{|x| \to \infty} \frac{u(x)}{|x|^{\frac{\gamma+2}{\gamma+1-\mu}}} = 0$, then $u \equiv 0$.

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BOA COLOCAÇÃO GLOBAL DAS EQUAÇÕES DE EULER COM FORÇA DE CORIOLIS EM ESPAÇOS DE BESOV

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Resumo

Apresentamos um resultado de existência e unicidade de soluções (locais no tempo) para o problema de valor inicial das equações de Euler Coriolis com dado inicial em espaços de Besov. Também, mostramos que se a velocidade de rotação for suficientemente grande, o tempo de existência de soluções pode ser tomado arbitrariamente grande usando estimativas do tipo Strichartz e um critério de blow-up do tipo Beale-Kato-Madja.

1 Introdução

Para este trabalho, consideramos o problema de valor inicial para as equações de Euler com força de Coriolis em $\mathbb{R}^3$, as quais descrevem o movimento de fluidos incompressíveis perfeitos sob efeito de rotação,

$$\begin{cases} \dfrac{\partial u}{\partial t} + \mathbb{P}\Omega e_3 \times u + \mathbb{P}\,(u \cdot \nabla)\, u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ \text{div } u = 0 & \text{in } \mathbb{R}^3 \times (0,\infty), \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^3, \end{cases} \tag{EC$_\Omega$}$$

onde $u = u(x,t) = (u_1(x,t), u_2(x,t), u_3(x,t))$ denota o campo de velocidade do fluido no ponto $(x,t) \in \mathbb{R}^3 \times (0,\infty)$, $\mathbb{P}$ é o projetor de Helmholtz e $u_0 = u_0(x) = (u_{0,1}(x), u_{0,2}(x), u_{0,3}(x))$ denota a velocidade inicial que satisfaz a condição de compatibilidade div $u_0 = 0$. Aqui, $\Omega \in \mathbb{R}$ é chamado o parâmetro de Coriolis e este representa a velocidade de rotação em torno do vetor unitário $e_3 = (0,0,1)$.

Consideramos o operador $e^{\pm i\Omega t \frac{D_3}{|D|}}$, definido por

$$e^{\pm i\Omega t \frac{D_3}{|D|}} f := \frac{1}{(2\pi)^3} \int_{\mathbb{R}^3} e^{ix\cdot\xi \pm i\Omega t \frac{\xi_3}{|\xi|}} \widehat{f}(\xi)\; d\xi, \quad (t,x) \in (0,\infty) \times \mathbb{R},$$

onde $\widehat{f}$ denota a transformada de Fourier de f.

Observando que a regularidade do semigrupo $e^{\pm i\Omega t \frac{D_3}{|D|}}$ não cobre a perda de derivadas no termo não linear $\mathbb{P}(u\cdot\nabla)u$, em contraste com o caso viscoso (veja [2] et al.), não podemos provar a existência global de soluções para (EC$_\Omega$) com $|\Omega|$ grande via um argumento de ponto fixo para a equação integral

$$u(t) = e^{\pm i\Omega t \frac{D_3}{|D|}} u_0 - \int_0^t e^{\pm i\Omega (t-\tau) \frac{D_3}{|D|}} \mathbb{P}(u(\tau)\cdot\nabla)u(\tau)\; d\tau.$$

Embora o método de energia clássico e as estimativas do comutador nos permitem lidar com esta dificuldade, nestes métodos não é possível derivar um efeito de dispersão da força de Coriolis $\Omega e_3 \times u$ por causa da anti-simetria

$$\int_{\mathbb{R}^3} \Omega e_3 \times u(x,t) \cdot u(x,t)\; dx = 0.$$

Recentemente, Koh, Lee e Takada [2] provaram que a solução local no tempo de (EC_Ω) pode ser estendida para um intervalo de tempo arbitrariamente grande desde que a velocidade de rotação seja suficientemente grande em espaços de Sobolev $H^s(\mathbb{R}^n)$ usando um critério de blow-up do tipo Beale-Kato-Majda [1] e estimativas do tipo Strichartz [2, 3], contornando as dificuldades descritas acima. Aqui apresentamos uma extensão desses resultados em espaços de Besov.

2 Resultados Principais

Começamos mostrando o seguinte resultado sobre a existência local de soluções para todo $\Omega \in \mathbb{R}$ no espaço de Besov $B^s_{2,q}(\mathbb{R}^3)$.

Teorema 2.1. *Seja $s > 3/2+1$ com $1 \leq q < \infty$, ou $s = 3/2+1$ com $q = 1$. Então, para $u_0 \in B^s_{2,q}(\mathbb{R}^3)$ satisfazendo div $u_0 = 0$, existe um tempo positivo $T = T(s, \|u_0\|_{B^s_{2,q}})$ tal que (EC_Ω) possui uma única solução u satisfazendo*

$$u \in C^1([0,T]; B^{s-1}_{2,q}(\mathbb{R}^3))^3 \cap C([0,T]; B^s_{2,q}(\mathbb{R}^3))^3. \tag{1}$$

Prova: Basicamente consideramos a regularização parabólica do problema em questão, obtemos a existência de soluções do problema parabólico e finalmente passamos o limite para obter uma solução do problema. ∎

Agora, mostramos que a solução local construída no Teorema 1 pode ser continuada para qualquer intervalo $[0,T]$ desde que a velocidade de rotação seja suficientemente grande.

Teorema 2.2. *Seja $s > 3/2+1$ com $1 \leq q < \infty$, ou $s = 3/2+1$ com $q = 1$. Então, para $0 < T < \infty$ e $u_0 \in B^s_{2,q}(\mathbb{R}^3)$ satisfazendo div $u_0 = 0$, existe um parâmetro positivo $\Omega_0 = \Omega_0(s, T, \|u_0\|_{B^{s+1}_{2,q}})$ tal que se $|\Omega| \geq |\Omega_0|$ então (EC_Ω) possui uma única solução u satisfazendo*

$$u \in C^1([0,T]; B^s_{2,q}(\mathbb{R}^3))^3 \cap C([0,T]; B^{s+1}_{2,q}(\mathbb{R}^3))^3. \tag{2}$$

En particular, para $2 < r < \infty$ existem constantes positivas C_0 e C_1 tais que o parâmetro Ω_0 pode ser tomado tal que

$$\Omega_0 \geq C_0 \left[1 + \|u_0\|_{B^{s+1}_{2,q}} T \exp\left(C_1 T^{1-\frac{1}{r}} \|u_0\|_{B^{s+1}_{2,q}}\right)\right]^r. \tag{3}$$

Prova: Basicamente os principais ingredientes para a prova deste teorema são Basicamente os principais ingredientes para a prova deste teorema são estimativas do tipo Strichartz e um critério de blow-up do tipo Beale-Kato-Madja em nossos espaços. ∎

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SOBRE UMA INEQUAÇÃO ASSOCIADA A UM SISTEMA DE EQUAÇÕES DE UM FLUIDO MICROPOLAR NÃO NEWTONIANO

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Resumo

Neste trabalho vamos investigar uma inequação associada a um problema envolvendo um sistema acoplado de EDP's associado a uma modelagem para fluidos micropolares não-newtonianos. O problema é considerado em um domínio suave e limitado do $\mathbb{R}^3$ com condições de Dirichlet na fronteira. O operador tensor de stress extra é dado por $\tau(e(u)) = \mu_0[(1 + |e(u)|^2)e(u)]$. Para provar a existência de soluções fracas usamos o método de Penalização, teoria dos operadores monótonos e argumentos de compacidade. Unicidade de soluções também são consideradas.

1 Introdução

Neste trabalho estudamos o seguinte sistema de equações diferenciais que entrou em desequilibrio devido a uma força externa

$$\left|\begin{array}{lcll} u' - \nabla \cdot \left[(\nu + \nu_0 M(|e(u)|^2))e(u)\right] + (u.\nabla)u + \nabla p & \geq & \mathrm{rot\,w} + \mathrm{f} & \text{in } \mathrm{Q_T} \\ w' - \nu_1 \nabla \cdot e(w) \;\; + \;\; (u.\nabla)w + \lambda_1 w & \geq & \lambda_2 \mathrm{rot\,u} + \mathrm{g} & \text{in } \mathrm{Q_T} \\ \mathrm{div\,u} & = & 0 & \text{in } Q_T \\ u & = & 0 & \text{on } \Sigma_T \\ w & = & 0 & \text{on } \Sigma_T \\ u(x,0) & = & u_0(x) & \text{in } \Omega \\ w(x,0) & = & w_0(x) & \text{in } \Omega \end{array}\right. \tag{1}$$

o qual é um modelo para um fluido micropolar com viscosidade variável caracterizada pelo tensor de estresse $\tau(e(u)) = (\nu+\nu_0+M(|e(u)|^2))e(u)$ e os símbolos $\nu, \nu_0, \nu_r, \lambda_1 e \lambda_2$ são constantes positivas. Nesse estudo estabelecemos resultados de existência e unicidade.

Em relação as notações usadas: vamos considerar Ω limitado em $\mathbb{R}^n$, $n > 1$, com fronteira suave $\partial\Omega$, sendo $T > 0$, denotamos o nosso domínio por Q_T o cilindro espaço-temporal $I \times \Omega$, com fronteira lateral $\Sigma = I \times \partial\Omega$, em que $I = (0, T)$ é um intervalo de tempo. Nesse contexto, os vetores $u = (u_1, ..., u_d)$ e $w = (w_1, ..., w_d)$ representam, respectivamente, a velocidade linear e microrrotacional de um fluido contido em Q_T. Essas velocidades são as variáveis de nosso problema. A pressão desse fluido é representada por p e $f = (f_1, ..., f_d)$ será a resultante das forças externas aplicadas a ele. A aplicação $\tau : \mathbb{R}^{d^2}_{sym} \to \mathbb{R}^{d^2}_{sym}$ é o tensor de estresse, onde $e : R^d \to R^{d^2}_{sym}$ leva cada vetor $u \in R^d$ na parte simétrica do gradiente da velocidade, dada pela expressão $e(u) = \frac{1}{2}\left[\nabla u + (\nabla u)^T\right]$, a aplicação $M : (0, \infty) \to (0, \infty)$ também respeita algumas hipóteses.

Para fazer o estudo de existência e unicidade do problema (1) vamos precisar utilizar os operadores de penalização $\beta : V \to V'$ e $\widetilde{\beta} : \mathbf{H}^1_0(\Omega) \to \mathbf{H}^{-1}(\Omega)$

O problema penalizado associado com a desiguldade variacional (1) é dado por:

$$\left| \begin{array}{lll} u_\epsilon' - \nabla\cdot\left[(\nu+\nu_0 M(|e(u_\epsilon)|^2))e(u_\epsilon)\right] + (u_\epsilon.\nabla)u_\epsilon + \frac{1}{\epsilon}\beta u_\epsilon + \nabla p & = & \mathrm{rot\,w_\varepsilon + f} \quad \text{in } \mathrm{Q_T} \\ w_\varepsilon' - \nu_1\nabla\cdot e(w_\varepsilon) \; + \; (u_\epsilon.\nabla)w_\varepsilon + \lambda_1 w_\varepsilon + \frac{1}{\varepsilon}\widetilde{\beta} w_\varepsilon & = & \lambda_2 \mathrm{rot\,u_\epsilon + g} \quad \text{in } \mathrm{Q_T} \\ \mathrm{div\,u_\epsilon} & = & 0 \quad \text{in } Q_T \\ u_\epsilon & = & 0 \quad \text{on } \Sigma_T \\ w_\varepsilon & = & 0 \quad \text{on } \Sigma_T \\ u_\epsilon(x,0) & = & u_{\epsilon 0}(x) \quad \text{in } \Omega \\ w_\varepsilon(x,0) & = & w_{\varepsilon 0}(x) \quad \text{in } \Omega \end{array} \right. \qquad (2)$$

2 Resultados Principais

Definição 2.1. *Sejam* $u_{\epsilon 0} \in V, w_{\varepsilon 0} \in \boldsymbol{H}_0^1(\Omega)$, *bem como* $f \in L^{4/3}(I, V')$ *e* $g \in \L^2(I; \boldsymbol{H}^{-1}(\Omega))$. *Uma solução fraca para (2) consiste de um par de funções* $\{u_\epsilon, w_\varepsilon\}$, *tal que* $u_\epsilon \in L^4(I; V_4) \cap L^2(I; V) \cap L^\infty(I; H)$, $w_\varepsilon \in L^\infty(I; \L^2(\Omega)) \cap L^2(I; \boldsymbol{H}_0^1(\Omega))$, *e o seguinte sistema seja satisfeito*

$$\left| \begin{array}{l} (u_\epsilon', \varphi) + \nu a(u_\epsilon, \varphi) + b(u_\epsilon, u_\epsilon, \varphi) + \nu_0 \langle K_{u_\epsilon} u_\epsilon, \varphi\rangle + \frac{1}{\epsilon}(\beta u_\epsilon, \varphi) \\ = (\mathrm{rot\,w_\epsilon}, \varphi) + (\mathrm{f}, \varphi), \forall \varphi \in \mathcal{D}(0, \mathrm{T}; \mathrm{V}) \\ \\ (w_\epsilon', \phi) + \nu_1 a(w_\epsilon, \phi) + \nu_1(\nabla(\nabla\cdot w_\varepsilon), \phi) + b(u_\epsilon, w_\varepsilon, \phi) \\ +\lambda_1(w_\varepsilon, \phi) + \frac{1}{\varepsilon}(\widetilde{\beta} w_\varepsilon, \phi) = \lambda_2(\mathrm{rot\,u_\epsilon}, \phi) + (\mathrm{g}, \phi), \forall \phi \in \mathcal{D}(0, \mathrm{T}; \mathcal{D}(\Omega)) \\ \\ u_\epsilon(0) = u_\epsilon 0, \quad w_\varepsilon(0) = w_\varepsilon 0 \end{array} \right. \qquad (1)$$

Teorema 2.1. *Se* $f \in L^{4/3}(I; V')$, $g \in L^2(I; \boldsymbol{H}^{-1}(\Omega))$, $u_{\epsilon 0} \in V$ *e* $w_{\varepsilon 0} \in \boldsymbol{H}_0^1(\Omega)$, *então para cada* $0 < \epsilon, \varepsilon < 1$ *existe uma solução para o problema (2) no sentido da definição (2.1).*

Teorema 2.2. *Assumindo que* $n = 2$ *e* $f, f', g, g' \in L^2(I; \L^2(\Omega))$. *Então para cada* $0 < \epsilon, \varepsilon < 1$, $u_{\epsilon 0} \in V$ *e* $w_{\varepsilon 0} \in H_0^1$, *existe um único par de funções* $(u_\epsilon, w_\varepsilon)$ *definidas para* $(x, t) \in Q_T$, *soluções para o problema (2) no sentido da definição (2.1).*

Sabendo que $\widetilde{\beta}, \beta \to 0$ quando $\epsilon, \varepsilon \to 0$, a partir dos teoremas (2.1) e (2.2) poderemos passar o limite no problema (2), com $\epsilon, \varepsilon \to 0$, recaindo assim no problema (1). A partir de então, o problema (1) possui existência de solução para $n \leq 3$ e unicidade para $n = 2$, no sentido da definição de solução fraca.

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IMERSÃO DE ATRATORES GLOBAIS: REGULARIDADE E DINÂMICA EM DIMENSÃO FINITA

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Resumo

Em continuidade ao projeto de mestrado (FAPESP, n.º do processo: 2012/23783-8), estudamos o comportamento assintótico dos processos de evolução respaldado na teoria das dimensões. Para tanto, resultados de Mallet-Paret [4] e Mañé [5] têm um papel decisivo junto às propriedades da dimensão de Hausdorff e a da dimensão fractal. Idealmente, dada uma equação em um espaço de Banach de dimensão infinita, buscamos uma equação diferencial ordinária em algum espaço euclidiano N-dimensional que copie, em algum sentido, a dinâmica da equação original.

1 Introdução

A evolução da taxa de juros de um país, a distribuição de calor de projétil em movimento e a dispersão de uma população controle de mosquitos numa região são exemplos de fenômenos observáveis (entre os numerosos) que evoluem com o tempo e que podem ser modelados por sistemas dinâmicos. É de fundamental importância predizer qual será o comportamento de longo prazo para tais eventos. Em outras palavras, do ponto de vista matemático, estudar o atrator global, quando existe.

Se $x(t,s;x_s)$ denota a solução em $(X, \|\cdot\|_X)$ de uma equação diferencial ordinária não autônoma

$$\begin{cases} \dot{x} = f(x,t) \\ x(s) = x_s \end{cases}$$

no instante de tempo t, a família de operadores solução, $\{S(t,s) : t \geqslant s \,\text{em}\, \mathbb{R}\}$, satisfaz as seguintes propriedades:

1) $S(t,t)$ é o operador identidade em X, para cada t;

2) $S(t,s)S(s,r) = S(t,r)$, para $t \geqslant s \geqslant r$; e

3) $(t,s,x_s) \mapsto S(t,s)x_s$ é contínua.

Tal família é dita um processo de evolução e escrevemos $T(t) := S(t,0)$ sempre que $S(t,s) = S(t-s,0)$.

Definição 1.1. *Dizemos que $\mathcal{A} \subset X$ é um* **atrator global** *do semigrupo quando é compacto; $T(\cdot)$-invariante, i.e., $T(t)\mathcal{A} = \mathcal{A}$, para todo $t \geqslant 0$; e atrai cada um dos subconjuntos limitados $B \subset X$, i.e.,* $\mathrm{dist}_{\mathrm{H}}\big(T(t)B, \mathcal{A}\big)t \to \infty \overset{t\to\infty}{\longrightarrow} 0$.

Como $\mathcal{A}$ é a reunião de todos os ω-limite de conjuntos limitados de X, o atrator global determina de que forma os pontos do espaço evoluem pela ação do semigrupo. Se pudermos "enxergá-lo" sob a perspectiva de algum espaço euclidiano finito dimensional - com o cuidado de não perder informações sobre a dinâmica -, então conquistamos todos os benefícios de se trabalhar em dimensão finita, entre eles a análise topológica e a intuição geométrica de órbitas que vemos em $\mathbb{R}^N$. O resultado meta de maior interesse é obter uma EDO que "copie" a dinâmica imersa em $\mathbb{R}^N$, e consequentemente reduzir o estudo de diversos fenômenos evolutivos à clássica teoria qualitativa de EDOs.

2 Resultados Principais

Segundo o Teorema da Imersão, [6], espaços métricos separáveis de dimensão topológica não maior que m admitem uma imersão topológica em $\mathbb{R}^{2m+1}$. Graças à maneira como as dimensões topológica, de Hausdorff e fractal estão relacionadas ([2, 3, 8]), este resultado inspira o seguinte, que é o principal estudado na dissertação de mestrado.

Teorema 2.1. (ver [2, 4, 5])
Se $N > 2\dim_{\mathrm{F}}(\mathcal{A}) + 1$, então existe uma projeção $P\colon X \to \mathbb{R}^N$ de modo que $P\big|_{\mathcal{A}} : \mathcal{A} \to \mathbb{R}^N$ é injetora.

A finitude de $\dim_{\mathrm{F}}(\mathcal{A})$ é tratada em diversos trabalhos e abrange uma larga classe de semigrupos ([1, 2, 8]).

Estudamos condições para existir uma **imersão não linear do atrator global** que admita inversa suficientemente **regular** e condições para que a dinâmica em dimensão finita **corresponda** a um sistema de equações diferenciais em algum espaço euclidiano finito dimensional. Dada uma equação

$$\frac{du}{dt} = f(u,t), \quad u \in X, \tag{1}$$

com um atrator global $\mathcal{A}$ em X, buscamos idealmente uma imersão não linear $P\colon X \to \mathbb{R}^N$ tal que

i) a inversa $P^{-1}\colon P(\mathcal{A}) \subset \mathbb{R}^N \to \mathcal{A}$ exista e seja uma aplicação Lipschitz contínua;

ii) a equação
$$\frac{dx}{dt} = P \circ f \circ P^{-1}(x), \quad x \in P(\mathcal{A}) \subset \mathbb{R}^N \tag{2}$$
admita uma única solução $t \mapsto x(t)$ dada como a solução $t \mapsto u(t)$ de (1) imersa em $\mathbb{R}^N$ por meio de P;

iii) o atrator global da equação (2) seja $P(\mathcal{A})$.

O Teorema 1.2 em [7] é um resultado parcial nesta direção: se $\mathcal{A}$ é o atrator global associado a uma EDP em H separável e $\mathcal{A} - \mathcal{A}$ tem dimensão de Assouad finita ([8]) e igual a m, então podemos escolher $N > m$ e uma EDO em $\mathbb{R}^{N+1}$ cuja única solução reproduz a dinâmica em $\mathcal{A}$ e de forma que o seu atrator esteja arbitrariamente próximo de $\mathcal{A}$, no sentido da distância simétrica de Hausdorff.

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ESTUDO DE UM MODELO DE CAMPO DE FASE NÃO ISOTÉRMICO COM COEFICIENTES TERMO-INDUZIDOS PARA DOIS FLUIDOS INCOMPRESSÍVEIS

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Abstract

Neste trabalho estudamos a existência e unicidade de solução do modelo de campo de fase não isotérmico com coeficientes de viscosidade e condutividade térmica termo-induzidos, o que é fisicamente importante no estudo de fluidos não isotérmicos. Tal modelo descreve a mistura de dois fluidos incompressíveis que possuem a mesma densidade.

1 Introdução

O modelo estudado consiste da equação de Navier-Stokes acoplada com a equação de campo de fase, que é do tipo Allen-Cahn, e com a equação para a temperatura. Além disso, estamos considerando condições de fronteira do tipo Neumann para o campo de fase e para a temperatura.

Um sistema não isotérmico do tipo Navier-Stokes-Allen-Cahn com condições de fronteira do tipo Dirichlet, cujos coeficientes de viscosidade e condutividade térmica são constantes, ou seja, não dependentes da temperatura, foi estudado em [3]. Nessa direção, em [2] o autor considera o mesmo problema estudado em [3] porém com os coeficientes de viscosidade e condutividade térmica termo-induzidos e com condições de fronteira do tipo Dirichlet. Os modelos estudados em [3, 2] foram propostos por Liu e outros autores em [1].

Nosso problema foi baseado em [2] utilizando outro tipo de acoplamento na equação de Navier-Stokes. Além disso, consideramos condições de fronteira do tipo Neumann para as equações de campo de fase e temperatura, as quais são fisicamente mais apropriadas neste tipo de problemas. Os principais desafios encontrados na análise matemática do nosso modelo são os acoplamentos altamente não lineares entre as equações.

Dado Ω um domínio suave e limitado em $\mathbb{R}^2$, e $T \in (0,\infty)$ um tempo finito e fixado, estudamos a existência e unicidade de solução do seguinte sistema não-linear: para $(x,t) \in \Omega \times (0,T)$

$$\left\{ \begin{array}{rcl} u_t + u\cdot\nabla u - \nabla\cdot(\nu(\theta)\nabla u) + \nabla P &=& (-\Delta\phi + F'(\phi))\nabla\phi - \Delta\theta\nabla\theta \\ \mathrm{div}\ u &=& 0 \\ \phi_t + u\cdot\nabla\phi &=& \gamma(\Delta\phi - F'(\phi)) \\ \theta_t + u\cdot\nabla\theta - \nabla\cdot(k(\theta)\nabla\theta) &=& 0 \end{array} \right. \tag{1}$$

em que, $F(\phi) = \dfrac{1}{4}(\phi^2-1)^2$.

Aqui, u denota a velocidade do fluido, P é a pressão, ϕ é o campo de fase e θ é a temperatura. Os coeficientes ν, γ e k são respectivamente a viscosidade do fluido, a relaxação elástica e a condutividade térmica.

Completamos o sistema (1) com as seguintes condições iniciais e de fronteira

$$\begin{cases} u(0,x) = u_0(x), \quad \phi(0,x) = \phi_0(x), \quad \theta(0,x) = \theta_0(x), \quad x \in \Omega, \\ u = 0, \quad \dfrac{\partial \phi}{\partial \eta} = 0, \quad \dfrac{\partial \theta}{\partial \eta} = 0, \quad (x,t) \in \partial\Omega \times (0,T), \end{cases} \tag{2}$$

em que η é o vetor normal exterior.

Vamos supor inicialmente, que as funções ν e k são de classe C^1, globalmente Lipschitz, e que existem constantes $\nu_0, k_0 > 0$ tais que

$$\nu_0 \leq \nu(s), \;\; k_0 \leq k(s),$$

para todo $s \in \mathbb{R}$. Podemos supor também que existe $s_0 > 0$, tal que

$$|k'(s)||s| \leq \frac{k_0}{16c},$$

para todo $s \in [-s_0, s_0]$, e c é uma constante que é dada pelas constantes da interpolação de Glagliardo Nirember, e pela desigualdade de regularidade eliptica.

2 Resultados Principais

Teorema 2.1. *Sejam* $u_0 \in \mathrm{H} = \{u \in \mathrm{L}^2(\Omega) : \; div\ u = 0 \text{ em } \Omega; \; u \cdot \eta = 0 \text{ em } \partial\Omega\}$, $\phi_0, \theta_0 \in \mathrm{H}^1(\Omega) \cap \mathrm{L}^\infty(\Omega)$ *com* $|\phi_0| \leq 1$ *q.t.p. em* Ω, *e* $\|\theta_0\|_{\mathrm{L}^\infty} \leq s_0$. *Então, existe pelo menos uma solução fraca global* (u, ϕ, θ) *do sistema* (1)-(2), *satisfazendo*

$$u \in \mathrm{L}^\infty(0,T;\mathrm{H}) \cap \mathrm{L}^2(0,T;\mathrm{V}),$$
$$\phi, \theta \in \mathrm{L}^\infty(0,T;\mathrm{H}^1 \cap \mathrm{L}^\infty) \cap \mathrm{L}^2(0,T;\mathrm{H}^2),$$

onde $\mathrm{V} = \{\mathrm{u} \in \mathrm{H}_0^1(\Omega) : \; div\ u = 0\}$.

Teorema 2.2. *Seja* $u_0 \in \mathrm{V}$, ϕ_0, $\theta_0 \in \mathrm{H}^2(\Omega) \cap \mathrm{L}^\infty(\Omega)$ *com* $|\phi_0| \leq 1$ *q.t.p. em* Ω, ν *e* k *sendo de classe* C^2, *globalmente Lipschitz, e* $\|\theta_0\|_{\mathrm{L}^\infty} \leq s_0$. *Então, existe uma única solução forte global* (u, ϕ, θ) *do sistema* (1)-(2), *satisfazendo*

$$u \in \mathrm{L}^\infty(0,T;\mathrm{V}) \cap \mathrm{L}^2(0,T;\mathrm{H}^2),$$
$$\phi, \theta \in \mathrm{L}^\infty(0,T;\mathrm{H}^2 \cap \mathrm{L}^\infty) \cap \mathrm{L}^2(0,T;\mathrm{H}^3).$$

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EXISTENCE OF SOLUTIONS FOR A PARABOLIC PROBLEM WITH $P(X)$-LAPLACIAN

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Abstract

In this work we will study the existence of weak solutions of the problem described below.

1 Introduction

Let Ω be an open bounded set of $\mathbb{R}^n$. Consider the problem:

$$u' + \mathcal{A}u + |u|^{\sigma} u = 0 \text{ in } \Omega \times (0, \infty) \tag{1}$$

with boundary and initial conditions

$$\begin{aligned} &u = 0 \text{ on } \partial\Omega \times]0, \infty[, \\ &u(x, 0) = u^0(x) \text{ in } \Omega. \end{aligned}$$

Here $\mathcal{A}$ is the operator $\mathcal{A} : W_0^{1,p(x)}(\Omega) \to W^{-1,p'(x)}(\Omega)$, $\mathcal{A}u = -\sum_{i=1}^{n} \left(\left| \frac{\partial u}{\partial x_i} \right|^{p(x)-2} \frac{\partial u}{\partial x_i} \right)$, and $\sigma \in \mathbb{R}$, $\sigma \geq 0$, is a given constant.

2 Notations and Main Results

Let $p \in L^{\infty}(\Omega)$ with $inf\ ess\ p(x) \geq 1$. The generalized Lebesgue space $L^{p(x)}(\Omega)$ is defined by

$$L^{p(x)}(\Omega) := \left\{ u : \Omega \to \mathbb{R} \text{ measurable}; \int_{\Omega} |u(x)|^{p(x)} dx < \infty \right\}.$$

For $u \in L^{p(x)}(\Omega)$, we define $\rho(x) = \int_{\Omega} |u(x)|^{p(x)} dx$. The norm in $L^{p(x)}(\Omega)$ is given by

$$\|u\|_{L^{p(x)}(\Omega)} = \inf \left\{ \lambda > 0 : \rho\left(\frac{u}{\lambda}\right) \leq 1 \right\}.$$

The generalized Sobolev space is defined by

$$W^{1,p(x)}(\Omega) := \left\{ u \in L^{p(x)}(\Omega); \frac{\partial u}{\partial x_j} \in L^{p(x)}(\Omega),\ j = 1, 2, \ldots, n \right\},$$

equipped with the norm

$$\|u\|_{W^{1,p(x)}(\Omega)} = \|u\|_{L^{p(x)}(\Omega)} + \sum_{j=1}^{n} \left\| \frac{\partial u}{\partial x_j} \right\|_{L^{p(x)}(\Omega)}.$$

All derivates considered in this paper are in the distributional sense.

We represent by $W_0^{1,p(x)}(\Omega)$ the closure of $C_0^{\infty}(\Omega)$ in $W^{1,p(x)}(\Omega)$. The dual space of $W_0^{1,p(x)}(\Omega)$ is denoted by $W^{-1,p'(x)}(\Omega)$. and by $L^{p'(x)}(\Omega)$ the dual space of $L^{p(x)}(\Omega)$.

Consider the following hypotheses:
(**H1**) $p \in C^0(\overline{\Omega})$, p lipschitzian, and $\min_{x\in\overline{\Omega}} p(x) > \sigma + 2$;
(**H2**) $u^0 \in W_0^{1,p(x)}(\Omega)$.

Theorem 2.1. *Assume that hypotheses $(H1) - (H2)$ are satisfied. Then there exists a function u in the class*

$$u \in L^\infty(0,\infty; W_0^{1,p(x)}(\Omega) \cap L^{\sigma+2}(\Omega)),\ \ u' \in L^2(0,\infty; L^2(\Omega));$$

such that u satisfies.

$$u' + \mathcal{A}u + |u|^\sigma u = 0 \ in \ L^2_{loc}(0,\infty; W^{-1,p'(x)}(\Omega) \cap L^{\frac{\sigma+2}{\sigma+1}}(\Omega));$$

$$u(0) = u^0 \ in \ \Omega.$$

The theorem follows by applying the Galerkin method, compactness argument and the theory of monotone operators.

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EXPOENTES CRÍTICOS PARA UM PROBLEMA PARABÓLICO SEMILINEAR COM TERMO DE REAÇÃO VARIÁVEL

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Resumo

Consideramos o problema parabólico semilinear $u_t - \Delta u = f(t)u^{p(x)}$ em $\Omega \times (0,T)$, com a condição de Dirichlet na fronteira. A função $p \in C(\Omega)$ é limitada, Ω é um domínio limitado ou ilimitado, $f \in C([0,\infty))$ e o dado inicial $u(0) = u_0$ pertencem ao espaço $C_0(\Omega)$. Encontramos condições que determinam quando uma solução do problema explode ou é global. Estas condições são expressadas em termos do comportamento assimptótico de $\|S(t)u_0\|_\infty$ e os valores extremos da função p.

1 Introdução

Seja $\Omega \subset \mathbb{R}^N$ um domínio regular(limitado ou ilimitado) com fronteira regular $\partial\Omega$. Consideramos o seguinte problema parabólico semilinear

$$\begin{cases} u_t - \triangle u &= f(t)u^{p(x)} \quad \text{em } \ \Omega \times (0,T), \\ u &= 0 \quad \text{em } \ \partial\Omega \times (0,T), \\ u(x,0) &= u_0(x) \geq 0 \quad \text{em } \Omega, \end{cases} \tag{1}$$

onde $p \in C(\Omega)$ é uma função limitada verificando $0 < p^- \leq p(x) \leq p^+$, para todo $x \in \Omega$, $p^- = \inf_{x\in\Omega} p(x)$, $p^+ = \sup_{x\in\Omega} p(x)$, $f \in C([0,\infty))$ e $u_0 \in C_0(\Omega)$.

O problema (1) possui uma solução local. A unicidade pode falhar já que a função p pode ser menor do que um em algum subdomínio de Ω, mas mesmo assim, temos uma solução maximal $u \in C([0,T_{max}), C_0(\Omega))$ definida no intervalo maximal $[0,T_{max})$ satisfazendo a igualdade $u(t) = S(t)u_0 + \int_0^t S(t-\sigma)f(\sigma)u(\sigma)^{p(x)}d\sigma$, para cada $t \in [0,T_{max})$. Mais ainda, ou $T_{\max} = \infty$ ou $T_{\max} < \infty$ e $\limsup_{t\to T_{\max}} \|u(t)\|_\infty = \infty$. Neste último caso dizemos que a solução explode num tempo finito. Denotamos por $(S(t))_{t\geq 0}$ o semigrupo do calor com a condição de Dirichlet na fronteira.

O problema (1), com $f \equiv 1$, foi considerado em [1], [3] e [4]. Em [1], assumindo $f = 1$ e $\Omega = \mathbb{R}$ foi encontrado o expoente de Fujita de (1). Mais precisamente, foi mostrado que se $p^- > 1 + 2/N$, então o problema (1) possui solução global não trivial. Se $1 < p^- < p^+ \leq 1 + 2/N$, então todas as soluções não triviais de (1) explodem em tempo finito. Agora, quando $p^- < 1 + 2/N < p^+$, existem funções $p \in C(\Omega)$ tal que o problema (1) solução global não trivial e funções $p \in C(\Omega)$ tal que toda solução não trvial explode num tempo finito. Em [1], foi considerado também o caso em que Ω é limitado e foi observado um fenómeno interessante: existem funções $p \in C(\Omega)$ de maneira que toda solução não trivial de (1) explode num tempo finito. Este resultado difere de forma significativa do caso em que p é constante, pois neste caso sempre existem soluções globais.

Diversas técnicas tem sido usadas para determinar quando uma solução explode ou é global. Destacamos o método da energia, o método das super e subsoluções, o método de Kaplan, argumentos de ponto fixo, etc. Em [2] foi mostrado o seguinte resultado.

Teorema [2] Suponha que $p(x) \equiv p$ é constante e $f \in C([0,\infty))$.

1. Se $\limsup_{t\to\infty} \|S(t)u_0\|_\infty^{p-1} \int_0^t f(\sigma)d\sigma = \infty$, para cada $u_0 \in C_0(\Omega)$, então cada solução não trivial do problema (1) explode num tempo finito.

2. Se existe $u_0 \in C_0(\Omega)$ tal que $\int_0^\infty f(\sigma)\|S(t)u_0\|_\infty^{p-1}d\sigma < \infty$, então existe uma solução global positiva para o problema (1).

Note que as condições no teorema anterior são dados em termos do comportamento assimptótico de $\|S(t)u_0\|$, que depende da geometria de Ω. Por exemplo, se $u_0 \in C_0(\Omega)$ sabemos que $\|S(t)u_0\|_\infty \sim t^{-N/2}$ for t grande e $\Omega = \mathbb{R}^N$. Já quando Ω é limitado $\|S(t)u_0\|_\infty \sim e^{-\lambda_1 t}$ para t suficientemente grande. Aqui $\lambda_1 > 0$ é o primeiro autovalor do operador Laplaciano em $H_0^1(\Omega)$. Se substituirmos estes estimativas no Teorema anterior obtemos os expoentes críticos do problema (1) em $\mathbb{R}^N$ e num domínio limitado respectivamente.

2 Resultados Principais

Nosso principal resultado estende o resultado obtido por Meier [2] no caso em que $p(x) = p$ é uma constante.

Teorema 2.1. *Suponha $p \in C(\Omega)$ e $f \in C([0,\infty))$.*

1. *Assuma que $p^+ > 1$ e $\limsup_{t\to\infty} \|S(t)u_0\|_\infty^{p^+-1} \int_0^t f(\sigma)d\sigma = \infty$ para cada $u_0 \in C_0(\Omega)$. Então toda solução não trivial de (1) explode em tempo finito ou infinito(i.e. a solução é global e $\limsup_{t\to\infty} \|u(t)\|_\infty = \infty^+$).*

2. *Assuma que $p^- > 1$ e existe $w_0 \in C_0(\Omega)$, $w_0 \geq 0$, $w_0 \neq 0$ verificando $\int_0^\infty f(\sigma)\|S(\sigma)w_0\|_\infty^{p^--1}d\sigma < \infty$. Então existe uma constante $\Lambda > 0$, dependendo de p^+, p^-, de modo que para $0 < \lambda < \Lambda$a solução u of (1)com dado inicial λw_0 é uma solu cão global. Mais ainda, existe uma constante $\gamma > 0$ tal que $u(t) \leq (1+\gamma)S(t)u_0$.*

Note que na conclusão do Teorema 2.1 temos a opção que a solução pode explodir em tempo infinito. Comparando com o resultado obtido em [1] observamos que isto não ocorre quando $\Omega = \mathbb{R}^N$. O seguinte resultado elimina a possibilidade da explosão no infinito em algumas situações.

Teorema 2.2. *Suponha que algumas das seguintes condições seja válida.*

1. *$f(t) \geq Ct^q (q > -1)$ para t grande, Ω contém um cone com vértice na origem $D \times (0,\infty) (D \subset S^{N-1})$ e $1 < p^- < p^+ < 1 + 2/(N+\gamma^+)$, onde γ^+ é a raiz positiva da equação $\gamma(\gamma + N - 2) = w_1$ e w_1 é o primeiro autovalor do operador de Laplace-Beltrami em D.*

2. *$f(t) = e^{\beta t} (\beta > 0)$ e $1 < p^- < p^+ < 1 + \beta/\lambda_1$.*

Então, cada solução do problema (1) explode num tempo finito.

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CONTROLABILIDADE LOCAL NULA DO SISTEMA N-DIMENSIONAL DE LADYZHENSKAYA-SMAGORINSKY COM TURBULÊNCIA E COM N-1 CONTROLES ESCALARES EM UM DOMINIO ARBITRARIO DO CONTROLE

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Resumo

Neste trabalho estudamos a controlabilidade local nula com N-1 controles escalares do sistema N-dimensional de Ladyzhenskaya-Smagorinsky no caso N=2 ou N=3, fazendo uso do teorema de Liusternik como é feito nos artigos [1] e [2].

1 Introdução

Seja $\Omega \subset \mathbb{R}^N$ (N=2 ou N=3), aberto, limitado, conexo, $\partial\Omega \in C^\infty$, $Q = \Omega \times]0,T[$ com $T > 0$ cuja fronteira lateral denotaremos por $\Sigma = \partial\Omega \times]0,T[$ e $\omega \subset \Omega$ é um aberto. Neste trabalho estudamos a controlabilidade local nula com N-1 controles escalares do seguinte sistema:

$$\begin{cases} y_t - \nabla \cdot ((\nu_0 + \nu_1(\|Dy\|^2))Dy) + (y \cdot \nabla)y + \nabla p = v1_\omega, & \text{em Q}, \\ \nabla \cdot y = 0, & \text{em Q}, \\ y = 0, & \text{sobre } \Sigma, \\ y(0) = y_0, & \text{em } \Omega, \end{cases} \tag{1}$$

onde $Dy = \left(\dfrac{\partial y_i}{\partial x_j} + \dfrac{\partial y_j}{\partial x_i} \right)_{1 \le i,j \le N}$, ν_0 é uma constante positiva e ν_1 é uma função de $C_b^1(\mathbb{R})$ não negativa. Definamos

$$\begin{aligned} H &= \{w \in L^2(\Omega)^N : \nabla \cdot w = 0 \text{ em } \Omega,\ \eta \cdot w = 0 \text{ sobre } \partial\Omega\}, \\ V &= \{w \in H_0^1(\Omega)^N : \nabla \cdot w = 0 \text{ em } \Omega\}. \end{aligned}$$

O operador $A : D(A) \longrightarrow H$ é o operador de Stokes, onde $D(A) = H^2(\Omega)^N \cap V$, $A(w) = P(-\Delta w),\ w \in D(A)$, com $P : L^2(\Omega)^N \longrightarrow H$ a projeção ortogonal.

Quando $N = 2$, para qualquer $y_0 \in V$ e qualquer $v \in L^2(\omega \times]0,T[)^N$, (1) possui exatamente uma solução forte (y,p), com $y \in L^2(0,T,D(A)) \cap C(0,T,V)$, $y_t \in L^2(0,T,H)$.

Quando $N = 3$, a última afirmação também é verdade sempre que y_0 e v sejam suficientemente pequenos em seus respectivos espaços.

2 Resultados Principais

Pelo fato de usar o teorema de Liusternik para garantir que o sistema (1) seja localmente nulo controlável com N-1 controles escalares num domínio arbitrário do controle, vamos estudar primeiro a controlabilidade do problema linear

$$\begin{cases} y_t - (\nu_0 + \nu_1(0))\Delta y + \nabla p = v1_\omega + f, & \text{em Q}, \\ \nabla \cdot y = 0, & \text{em Q}, \\ y = 0, & \text{sobre } \Sigma, \\ y(0) = y_0, & \text{em } \Omega. \end{cases} \tag{2}$$

Definição 2.1. *$\rho(t) = e^{5s\beta^*/2}(\gamma^*)^{-2}$, $\rho_0(t) = e^{3s\beta^*/2}$ e $\rho_1(t) = e^{s\widehat{\beta}+3s\beta^*/2}(\widehat{\gamma})^{-7/2}$ são funções positivas que explodem no tempo $t = T$, onde β^*, γ^*, $\widehat{\beta}$ e $\widehat{\gamma}$ são as funções construídas no artigo [1]*

Teorema 2.1. *Para $i \in \{1, ..., N\}$. Assumindo que $\rho f \in L^2(Q)^N$. Então é possível encontrar um controle v tal que a solução associada (y, p) para (2) satisfaz*

$$\iint_Q \rho_0^2 |y|^2 \, dxdt + \sum_{j=1, j\neq i}^{N} \iint_{\omega\times]0,T[} \rho_1^2 |v_j|^2 \, dxdt < +\infty.$$

Em particular $y(T) = 0$, $v_i = 0$ e $y \in L^2(0, T, V) \cap C(0, T, H)$

O resultado principal do trabalho é o seguinte:

Teorema 2.2. *Para $i \in \{1, ..., N\}$, $T > 0$ e $\omega \subset \Omega$ existe $\delta > 0$ tal que para todo $y_0 \in V$ satisfazendo*

$$\|y_0\|_V \leq \delta,$$

é possível encontrar um controle $v \in L^2(\omega\times]0, T[)^N$ com $v_i = 0$ e uma correspondente solução (y, p) para (1) tal que

$$y(T) = 0.$$

Isto é, o sistema (1) é localmente nulo controlável com N-1 controles escalares num domínio arbitrário do controle.

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CONTROLABILIDADE LOCAL NULA DA EQUAÇÃO DO CALOR COM TEMPERATURA DEPENDENDO DE PARÂMETROS

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Resumo

Consideremos a equação diferencial parcial não linear da seguinte forma

$$u' - div(a(u)\nabla u) + b(u)|\nabla u|^2 = f$$

que modela problemas de condução do calor com temperatura dependendo de parâmetros como o material, densidade, massa, calor específico e condutividade do calor. A controlabilidade local nula pelo método de Liusternik só no caso unidimensional será estudada.

1 Introdução

Materiais metálicos apresentam um comportamento complexo durante o tratamento do calor pois a mudança da temperatura induz uma transformação da estrutura metálica. De fato, quantidades específicas do calor e condutividade mostram uma dependência forte de parâmetros externos. Neste trabalho, consideramos a simples abordagem sem acoplamento termomecânico de deformações, mas considerando a dependência não linear da temperatura de parâmetros térmicos como o único efeito.
Seja $\Omega \subset \mathbb{R}$ um conjunto aberto não vazio, limitado, conexo com fronteira Γ de classe C^2 e $Q = \Omega \times (0,T)$ com $T > 0$ cuja fronteira lateral denotamos $\Sigma = \Gamma \times (0,T)$.
Consideremos o problema que descreve o comportamento acima

$$\left| \begin{array}{l} u_t - (a(u)u_x)_x + b(u)|u_x|^2 = v\mathbf{1}_\omega \quad \text{em } Q, \\ u = 0 \quad \text{sobre } \Sigma, \\ u(0) = u_0 \quad \text{em } \Omega, \end{array} \right. \tag{1}$$

onde v é o controle e $\omega \subset\subset \Omega$ não vazio é o domínio onde atua o controle.

Consideremos as seguintes hipóteses

H1. $a(s) \in C^2(\mathbb{R})$, $b(s) \in C^1(\mathbb{R})$ e existem constantes positivas a_0, a_1 tais que $a_0 \leq a(s) \leq a_1$ e $b(s)s \geq 0 \;\; \forall s \in \mathbb{R}$.

H2. Existe uma constante $M > 0$ tal que $\max_{s\in\mathbb{R}} \{|a'(s)|, |a''(s)|, |b'(s)|\} \leq M$.

H3. $a'(0) = 0$.

H4. $u_0 \in H_0^1(\Omega) \cap H^2(\Omega)$.

Teorema 1.1. *Se a, b, u_0 satisfazem $(H1)-(H4)$ e além disso $v \in L^2(\omega \times (0,T))$ com $|v|_{L^2(\omega\times(0,T))}$ suficientemente pequena, existe uma constante $\delta > 0$ tal que se u_0 satisfaz $\|u_0\|_{H_0^1} + |u_0|_{H^2} < \delta$, então o sistema (1) tem uma única solução u satisfazendo*

$$u \in L^\infty\left(0,T; H_0^1(\Omega)\right) \cap L^2(0,T; H^2(\Omega)),, \tag{2}$$

$$u_t \in L^2\left(0,T; L^2(\Omega)\right). \tag{3}$$

Prova Ver [4].

2 Resultados Principais

Pelo fato de aplicar o teorema de Liusternik, o primeiro a estudar será a controlabilidade do problema linearizado

$$\left| \begin{array}{l} u_t - a(0)u_{xx} = v\mathbf{1}_\omega + f \quad \text{em } Q, \\ u = 0 \quad \text{sobre } \Sigma, \\ u(0) = u_0 \quad \text{em } \Omega. \end{array} \right. \tag{1}$$

Definição 2.1. *Sejam $\rho(x,t)$ e $\rho_0(x,t)$ funções positivas que explodem exponencialmente no tempo $t = T$.*

Teorema 2.1. *A equação (1) é nula controlável no tempo T, isto é, suponhamos $u_0 \in H_0^1(\Omega) \cap H^2(\Omega)$, $\rho_0 f \in L^2(Q)$ então podemos encontrar um controle $v \in L^2(\omega \times (0,T))$ tal que o estado associado (u, v) solução de (1) satisfaz $\rho u \in L^2(Q)$ e $\rho_0 v \in L^2(\omega \times (0,T))$. Em particular $u(T) = 0$.*

Prova Utiliza-se a desigualdade de Carleman no sistema adjunto do sistema linearizado (1) e métodos de H. R. Clark, E. Fernández-Cara, J. Límaco e L. A. Medeiros [1].

O resultado principal do trabalho é o seguinte

Teorema 2.2. *Se a, b, u_0 satisfazem $(H1) - (H4)$, então a equação (1) é localmente nula controlável no tempo T, isto é, existe um $\epsilon > 0$ tal que se u_0 satisfaz $|||u_0||_{H_0^1} + |u_0|_{H^2} < \epsilon$. Então podemos encontrar um controle $v \in L^2(\omega \times (0,T))$ tal que a solução u de (1) satisfaz $u(T) = 0$.*

Prova Utiliza-se o teorema de Liusternik e métodos de E. Férnandez-Cara, J. Límaco e S. B. Menezes [2].

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EXISTÊNCIA GLOBAL PARA AS EQUAÇÕES α-NAVIER-STOKES-VLASOV

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Abstract

Neste trabalho mostramos a existência global de solução fraca para as equações α-Navier-Stokes-Vlasov para fluidos incompressíveis com acoplamento considerando a força de arrasto. Também provamos um resultado de regularidade para tais soluções.

1 Introdução

Existem vários modelos para o estudo de um spray, que é uma coleção de partículas pequenas (ou gotas) que se move dentro de um fluido. No caso de um fluido incompressível, um dos primeiros trabalhos é o de Hamdache [3] onde é estabelecida a existência de soluções para um sistema acoplado Stokes-Vlasov, considerando um domínio limitado e que as partículas do spray são refletidas pela fronteira seguindo a lei da reflexão especular. Em [3, 4] é estudado o sistema de Navier-Stokes-Vlasov, considerando soluções periódicas na variável espacial e satisfazendo a lei da reflexão especular sobre a fronteira, respectivamente. Neste trabalho estudamos o sistema α-Navier-Stokes-Vlasov, investigando a existência de soluções periódicas na variável espacial, obtendo soluções mais regulares.

Sejam $T > 0$ uma constante fixada e $\mathbb{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$. Consideramos que o fluido é governado pelas equações α-Navier-Stokes, com velocidades associadas $u : [0,T] \times \mathbb{T}^3 \to \mathbb{R}^3$ e $w : [0,T] \times \mathbb{T}^3 \to \mathbb{R}^3$ e suas respectivas pressões hidrostáticas $\pi : [0,T] \times \mathbb{T}^3 \to \mathbb{R}$ e $p : [0,T] \times \mathbb{T}^3 \to \mathbb{R}$, com $p = \pi + \|u\|_{H^2(\mathbb{T}^3)} - \alpha^2(u \cdot \Delta u)$. O parâmetro $\nu > 0$ denota a viscosidade do fluido, $\alpha > 0$ é uma constante dada e associada com a regularização da velocidade no modelo das equações α-Navier-Stokes. As partículas do spray são descritas pela função $f : [0,T] \times \mathbb{T}^3 \times \mathbb{R}^3 \to \mathbb{R}$ que satisfaz uma equação do tipo Vlasov. A quantidade $f(t,x,v)$ é a densidade das partículas localizadas $x \in \mathbb{T}^3$, no tempo t e que possuem velocidade $v \in \mathbb{R}^3$. As equações para o fluido e para o spray estão acopladas pela força de arrasto. Desta forma, o problema consiste em procurar funções f, u e p satisfazendo o seguinte sistema de equações:

$$\partial_t f + v \cdot \nabla_x f + \nabla_v \cdot [(u-v)f] = 0 \qquad \text{em } (0,T) \times \mathbb{T}^3 \times \mathbb{R}^3, \tag{1}$$

$$\partial_t w - \nu \Delta w - u \times (\nabla \times w) + \nabla p = -\int_{\mathbb{R}^3} f(u-v)\, dv \qquad \text{em } (0,T) \times \mathbb{T}^3, \tag{2}$$

$$w = u - \alpha^2 \Delta u \qquad \text{em } (0,T) \times \mathbb{T}^3, \tag{3}$$

$$\nabla_x \cdot u = 0 \qquad \text{em } (0,T) \times \mathbb{T}^3, \tag{4}$$

$$f(0,x,v) = f_{\texttt{in}}(x,v) \qquad \text{em } \mathbb{T}^3 \times \mathbb{R}^3, \tag{5}$$

$$u(0,x) = u_{\texttt{in}}(x) \qquad \text{em } \mathbb{T}^3, \tag{6}$$

onde $f_{\texttt{in}} \geq 0$, $u_{\texttt{in}}$ são funções dadas.

2 Resultados Principais

Teorema 2.1. *Sejam* $f_{in} \in L^\infty(\mathbb{T}^3 \times \mathbb{R}^3) \cap L^1(\mathbb{T}^3 \times \mathbb{R}^3)$, $|v|^2 f_{in} \in L^1(\mathbb{T}^3 \times \mathbb{R}^3)$ *e* $u_{in} \in V$ *então o sistema* (1)-(6) *tem pelo menos uma solução* (u, f) *satisfazendo*

$$\begin{aligned} u \in L^\infty(0,T;V) \cap L^2(0,T;H^2(\mathbb{T}^3) \cap V) \cap C([0,T];H) \\ f(t,x,v) \geq 0, (t,x,v) \in (0,T) \times \mathbb{T}^3 \times \mathbb{R}^3 \\ f \in L^\infty(0,T;L^\infty(\mathbb{T}^3 \times \mathbb{R}^3) \cap L^1(\mathbb{T}^3 \times \mathbb{R}^3)) \\ f|v|^2 \in L^\infty(0,T;L^1(\mathbb{T}^3 \times \mathbb{R}^3)) \end{aligned}$$

com $V = \{u \in H^1(\mathbb{T}^3);\, \nabla_x \cdot u = 0\}$ *e* $H = \{u \in L^2(\mathbb{T}^3);\, \nabla_x \cdot u = 0\}$. *Além disso,* (u, f) *satisfaz a seguinte desigualdade de energia, q.t.p em* $[0, T]$,

$$\begin{aligned} &\iint_{\mathbb{T}^3 \times \mathbb{R}^3} f(t)|v|^2 \, dx\, dv + \|u(t)\|_H^2 + \alpha^2 \|\nabla u(t)\|_H^2 + 2\nu \int_0^t \left(\|\nabla u(s)\|_H^2 + \alpha^2 \|\Delta u(s)\|_H^2 \right) ds \\ &+2 \int_0^t \iint_{\mathbb{T}^3 \times \mathbb{R}^3} f|u-v|^2 \, dx\, dv\, ds \leq \iint_{\mathbb{T}^3 \times \mathbb{R}^3} f_{in}|v|^2 \, dx\, dv + \|u_{in}\|_H^2 + \alpha^2 \|\nabla u_{in}\|_H^2 , \end{aligned}$$

e

$$\|f\|_{L^\infty((0,T)\times\mathbb{T}^3\times\mathbb{R}^3)} \leq e^{3T} \|f_{in}\|_{L^\infty(\mathbb{T}^3\times\mathbb{R}^3)} .$$

Prova: De forma similar a [3]: primeiro definimos um problema auxiliar regularizando o problema original. Em seguida, o problema auxiliar regularizado é desacoplado através de um método iterativo. O problema desacoplado é resolvido usando o método das curvas características e resultados para as equações α-Navier-Stokes [2]. No que segue, prova-se que a sequência de soluções do problema desacoplado converge para a solução do problema regularizado. Finalmente, usando resultados de compacidade mostra-se que a sequência de soluções do problema regularizado converge para uma solução fraca do problema original. ∎

Teorema 2.2. *Suponhamos que as hipóteses do Teorema 2.1 são satisfeitas e que* $f_{in}|v|^k \in L^1(\mathbb{T}^3 \times \mathbb{R}^3)$, $k \in \mathbb{N}$. *Então, a solução* f *encontrada no Teorema 2.1 satisfaz* $f|v|^k \in L^\infty(0,T;L^1(\mathbb{T}^3 \times \mathbb{R}^3))$.

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LOCAL NULL CONTROLLABILITY FOR A PARABOLIC-ELLIPTIC SYSTEM WITH LOCAL AND NONLOCAL NONLINEARITIES

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Abstract

This work deals with the null controllability of an initial boundary value problem for a parabolic-elliptic coupled system with nonlinear terms of local and nonlocal kinds. The control is distributed, locally in space and appears only in one PDE (the parabolic one). We first prove that, if the initial data is sufficiently small and the linearized system at zero satisfies an appropriate condition, the equations can be driven to zero.

1 Introduction

Let $\Omega \subset \mathbb{R}^N$, $(N \geq 1)$, a non-empty open bounded connected set, with locally Lipschitz-continuous boundary $\partial\Omega$. Let us set $Q = \Omega \times (0,T) \subset \mathbb{R}^{N+1}$, where $T > 0$. The lateral boundary of Q is defined by $\Sigma = \partial\Omega \times (0,T)$. Let $\mathcal{O} \subset\subset \Omega$ be a (small) non-empty open set.

We will be concerned with the null controllability of the following nonlinear system:

$$\begin{cases} y_t - \beta_1\left(\displaystyle\int_\Omega y\,dx, \int_\Omega z\,dx\right)\Delta y + F(y,z) = v1_{\mathcal{O}} & \text{in } Q,\\ \quad -\beta_2\left(\displaystyle\int_\Omega y\,dx, \int_\Omega z\,dx\right)\Delta z + f(y,z) = 0 & \text{in } Q,\\ y = z = 0 & \text{on } \Sigma,\\ y(x,0) = y_0(x) & \text{in } \Omega, \end{cases} \tag{1}$$

where v is the control, (y,z) is the state and y_0 denotes the initial state. Here, $1_{\mathcal{O}}$ the characteristic function of $\mathcal{O}$; $\beta_i = \beta_i(r,s)$ $(i = 1,\, 2)$, $F = F(r,s)$ and $f = f(r,s)$ are real C^1 functions (defined in $\mathbb{R}\times\mathbb{R}$) that possess bounded derivatives and satisfy

$$\begin{cases} \quad 0 < c_0 \leq \beta_i(r,s) \leq c_1, \quad i = 1,2, \quad \forall (r,s) \in \mathbb{R}\times\mathbb{R},\\ F(0,0) = f(0,0) = 0, \quad D_1 f(0,0) \neq 0, \quad |D_2 f(0,0)| < c_0\mu_1, \end{cases} \tag{2}$$

where μ_1 is the first eigenvalue of $-\Delta$.

Definition 1.1. *It will be said that* (1) *is locally null-controllable at time* $T > 0$ *if there exists* $\epsilon > 0$ *such that, for any* $y_0 \in H_0^1(\Omega)$ *with*

$$\|y_0\|_{H_0^1(\Omega)} < \epsilon,$$

there exists controls $v \in L^2(\mathcal{O}\times(0,T))$ *such that the associated states* (y,z) *satisfy*

$$y(x,T) = 0 \text{ in } \Omega \quad \text{and} \quad \limsup_{t\to T^-} |z(.,t)|_{L^2(\Omega)} = 0 \tag{3}$$

2 Main Result - Local Null Control Theorem

Theorem 2.1. *Let us assume that* (2) *holds. Then, the nonlinear system* (1) *is locally null-controllable in* $T > 0$*. In other words, there exists* $\epsilon > 0$ *such that, whenever* $y_0 \in H_0^1(\Omega)$ *and*

$$\|y_0\|_{H_0^1(\Omega)} < \epsilon,$$

there exists controls $v \in L^2(\mathcal{O} \times (0,T))$ *and associated states* (y, z) *satisfying* (3).

The main difficulties found in the proof are that (a) nonlinear terms appear in the main part of the partial derivative operators and (b) only one scalar control is used in the system (in the parabolic PDE).

We will employ a technique relying on the so called *Liusternik's Inverse Mapping Theorem* in Hilbert Spaces.

The arguments are inspired by the works of A. V. Fursikov and O. Yu. Imanuvilov in [4], E. Fernández-Cara, J. Límaco and S. B. Menezes in [3] and H. R. Clark, E. Fernández-Cara, J. Límaco and L. A. Medeiros in [1] and rely on some estimates already used by these authors for other similar problems.

Proof In a first step, we must consider a linearized system at zero

$$\begin{cases} y_t - \beta_1(0,0)\,\Delta y + ay + bz = v 1_{\mathcal{O}} + h & \text{in } Q, \\ \quad -\beta_2(0,0)\,\Delta z + cy + dz = k & \text{in } Q, \\ y = z = 0 & \text{on } \Sigma, \\ y(x,0) = y_0(x) & \text{in } \Omega. \end{cases} \tag{1}$$

where the coefficients a, b, c, and d are obtained from the partial derivatives of F and f at $(0,0)$, with $c \neq 0$, $|d| < c_0\mu_1$. Following well known ideas, the null controllability of (1) (for appropriate h and k) will be obtained as a consequence of suitable Carleman Estimates for the solutions of the associated *adjoint system*.

In a second step, we will rewrite the null controllability property of (1) as an equation for (y, z, v) in a well chosen space of "admissible" state-control triplets:

$$H(y, z, v) = (0, 0, y_0), \quad (y, z, v) \in Y. \tag{2}$$

The choice of the space Y is nontrivial and it motivates some preliminary estimates of the null controls and associated solutions to (1). We will apply Liusternik's Theorem to (2) and we will deduce the (local) desired result from a similar (global) property for the linear system (1).

An additional consequence of Liusternik's Theorem will be that the triplets (y, z, v) can be chosen in such a way that $\|(y, z, v)\|_Y \to 0$ as $\|y_0\|_{H_0^1(\Omega)} \to 0$. ∎

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ON THE CONVERGENCE OF SPECTRAL APPROXIMATIONS FOR THE HEAT CONVECTION EQUATIONS

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Abstract

In this talk, we focus on the convergence rate of solutions of spectral Galerkin approximations for the heat convection equations on a bounded domain. Estimates in H^2-norm for the velocity and temperature without compatibility conditions are obtained. Moreover, we give rates of convergence for the derivative of the velocity and temperature in L^2-norm.

1 Introduction

The following equations describe the heat convection motion of a fluid in a bounded domain $\Omega \subset \mathbb{R}^N$, $N = 2$ or 3, with smooth boundary, in the time interval $[0, T)$, $0 \le T \le \infty$, considering the Oberbeck-Boussinesq approximation (see Joseph [2]):

$$\left\{ \begin{array}{rcl} \dfrac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \nu \Delta \mathbf{u} + \nabla p & = & \mathbf{j} + \theta \mathbf{g}, \\ \text{div } \mathbf{u} & = & 0, \\ \dfrac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta - k \Delta \theta & = & f, \end{array} \right.$$

Here $\mathbf{u}(t,x) \in \mathbb{R}^N$, $\theta(t,x) \in \mathbb{R}$, and $p(t,x) \in \mathbb{R}$ denote respectively the unknowns velocity, temperature and pressure of a liquid at a point $x \in \Omega$ at time $t \in [0,T)$. The constants ν and k are respectively, the kinematic viscosity and thermal conductivity. The gravitational field, $\mathbf{g}(t,x)$, the coefficient of volume expansion, $\mathbf{j}(t,x)$, and the source function $f(t,x)$ are given.We have considered the viscosity coefficient and the thermal conductivity equal to 1, without loss of generality.

On the boundary Γ, we assume that

$$\mathbf{u}(t,x) = 0, \ \theta(t,x) = \theta_1, \tag{1}$$

where θ_1 is a known function, and the initial data conditions are expressed by

$$\mathbf{u}(0,x) = \mathbf{u}_0(x), \ \theta(0,x) = \theta_0(x), \tag{2}$$

where $\mathbf{u}_0$ and θ_0 are given functions on the variable $x \in \Omega$. To simplify the analysis, we consider $\nu = 1$, $k = 1$, and $\theta_1 = 0$. The nonhomogeneous case $\theta_1 \neq 0$, can be treated by using an appropriate lifting and only the obvious changes should be required in the statement of the results.

We use the standard notation of mathematical theory of the Navier-Stokes equations. Thus, A denote the Stokes operator and B denote the Laplacian operator. Also, $\{\lambda_k\}_{k=1}^{\infty}$ denote the eigenvalues of the Stokes operator and $\{\gamma_k\}_{k=1}^{\infty}$ the eigenvalues of the Laplacian operator.

Theorem 1.1. *If* $\mathbf{g}, \mathbf{j} \in C([0,T], \mathbf{H}^1(\Omega))$, $f \in C([0,T], H^1(\Omega))$, $\mathbf{g}_t, \mathbf{j}_t \in L^2(0,T; \mathbf{L}^2(\Omega)$, $f \in L^2(0,T; L^2(\Omega)$ *and* $\mathbf{u}_0 \in D(A^{1+\epsilon})$, $\theta_0 \in D(B^{1+\epsilon})$ *with* $\epsilon \in (0, \frac{1}{4})$, *then*

$$\|A\mathbf{u}(t) - A\mathbf{u}^n(t)\| + \|\mathbf{u}_t(t) - \mathbf{u}_t^n(t)\| \leq C \left[\frac{C(\epsilon)}{\lambda_{n+1}^{\epsilon}} + \left(\frac{1}{\lambda_{n+1}} + \frac{1}{\gamma_{n+1}} \right)^{1/2} \right], \tag{3}$$

$$\|B\theta(t) - B\theta^n(t)\| + \|\theta_t(t) - \theta_t^n(t)\| \leq C \left[\frac{C(\epsilon)}{\lambda_{n+1}^{\epsilon}} + \frac{1}{\gamma_{n+1}^{\epsilon}} + \left(\frac{1}{\gamma_{n+1}} + \frac{1}{\gamma_{n+1}} \right)^{1/2} \right], \tag{4}$$

where $C > 0$ *is a positive constant that depend only on data.*

Thus, we prove a pointwise convergence rate in the H^2-norm for the velocity and the temperature. Moreover, the pointwise convergence rate in the L^2-norm for the time-derivative of velocity and temperature is obtained. The innovation of our results is, again, that we do not need impose compatibility conditions on the initial data.

The complete proof can be found in [1]. This result complemented the findings above in [3] and [4].

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ALMOST PERIODIC MILD SOLUTIONS TO EVOLUTIONS EQUATIONS WITH STEPANOV ALMOST PERIODIC COEFFICIENTS

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Abstract

In this work we study sufficient conditions for the existence of almost periodic mild solutions to an autonomous fractional differential equation and an integro-differential equation.

1 Preliminaries

Set $\mathbb{X}$ a Banach space. We study sufficient conditions for the existence of almost periodic mild solutions to:

- the autonomous fractional differential equation:

$$D_t^{\alpha} u(t) = Au(t) + D_t^{\alpha-1} f(t, u(t)), \quad t \in \mathbb{R}, \tag{1}$$

  where the fractional derivative is in the Riemann-Liouville sense with $1 < \alpha < 2$, $A : D(A) \subseteq \mathbb{X} \to \mathbb{X}$ is a linear densely defined operator of sectorial type on $\mathbb{X}$.

- the integro-differential equation:

$$u'(t) = Au(t) + \int_{-\infty}^{t} a(t-s) Au(s)\, ds + f(t, u(t)), \quad t \in \mathbb{R}, \tag{2}$$

  where A is a closed linear operator defined in $\mathbb{X}$ and $a \in L^1_{loc}(\mathbb{R}^+)$ is an scalar-valued kernel.

In both cases $f : \mathbb{R} \times \mathbb{X} \to \mathbb{X}$ is Stepanov almost periodic in $t \in \mathbb{R}$ uniformly in $x \in \mathbb{X}$ (see Definition 1.2).

Definition 1.1. *The space $BS^p(\mathbb{X})$ of all Stepanov functions, with the exponent $p \in [1, \infty)$, consists of all measurable functions f on $\mathbb{R}$ with values in $\mathbb{X}$ such that $\|f\|_{S^p} := \sup_{t\in\mathbb{R}} \left(\int_t^{t+1} \|f(\tau)\|^p d\tau \right)^{\frac{1}{p}} < +\infty$.*

Definition 1.2.

(a) A function $f \in BS^p(\mathbb{X})$ is called Stepanov almost periodic if for each $\epsilon > 0$ there is a relatively dense set $P(\epsilon, f) \subseteq \mathbb{R}$ such that $\sup_{t\in\mathbb{R}} \left(\int_0^1 \|f(t+s+\tau) - f(t+s)\|^p ds \right)^{\frac{1}{p}} < \epsilon$, $\forall \tau \in P(\epsilon, f)$. We denote the space of such functions by $APS^p(\mathbb{X})$.

(b) A function $f : \mathbb{R} \times \mathbb{X} \to \mathbb{X}$ with $f(t, \cdot) \in C(\mathbb{R}, \mathbb{X})$ and $f(\cdot, x) \in BS^p(\mathbb{R}, \mathbb{X})$ for each $x \in \mathbb{X}$, is said to be Stepanov almost periodic in $t \in \mathbb{R}$ uniformly in $x \in \mathbb{X}$ if for each $\epsilon > 0$ and each compact set $\mathbb{K} \subseteq \mathbb{X}$, there is a relatively dense set $P(\epsilon, f, \mathbb{K}) \subseteq \mathbb{R}$ such that $\sup_{t\in\mathbb{R}} \left(\int_0^1 \|f(t+s+\tau, x) - f(t+s, x)\|^p ds \right)^{\frac{1}{p}} < \epsilon$, for each $\tau \in P(\epsilon, f, \mathbb{K})$ and each $x \in \mathbb{K}$. We denote the space of such functions by $APS^p(\mathbb{R} \times \mathbb{X}, \mathbb{X})$.

Lemma 1.1 ([4])**.** *Assume that $p > 2$ and that the following conditions hold:*

1. *Let $f \in APS^p(\mathbb{R}\times\mathbb{X},\mathbb{X})$ be S^p-Lipschitzian in the second variable, that is, there is a function $L \in BS^p(\mathbb{X})$ such that*

$$\|f(t,u) - f(t,v)\| \le L(t)\,\|u-v\|\,,\ \forall\, t\in\mathbb{R},\ \forall\, u,v\in\mathbb{X}. \tag{3}$$

2. *$x \in APS^p(\mathbb{X})$, and there is a set $\mathbb{E}\subseteq\mathbb{R}$ with $\mathrm{mes}(\mathbb{E}) = 0$ such that $\mathbb{K} = \overline{\{x(t):\ t\in\mathbb{R}\setminus\mathbb{E}\}}$ is compact in $\mathbb{X}$.*

Then $f(\cdot,x(\cdot)) \in APS^{\frac{p}{2}}(\mathbb{X})$.

2 Main Results

2.1 Autonomous Fractional Differential Equations

(H1) A is sectorial of type $\omega < 0$ and angle θ satisfying $0 \le \theta < \pi\left(1-\frac{\alpha}{2}\right)$, with $1<\alpha<2$.

Theorem 2.1. *Assume that A satisfies (H1). If f is Stepanov almost periodic with exponent $p > 1$, then the equation: $D_t^{\alpha}u(t) = Au(t) + D_t^{\alpha-1}f(t)\,,\quad t\in\mathbb{R}$, has an almost periodic mild solution given by $u(t) = \int_{-\infty}^{t} S_\alpha(t-s)\,f(s)\,ds$, where $S_\alpha(\cdot)$ is the solution operator generated by A.*

Definition 2.1. *Assume that A satisfies (H1). A function $u:\mathbb{R}\to\mathbb{X}$ is called a mild solution of (1) if the function $s\to S_\alpha(t-s)\,f(s,u(s))$ is integrable on $(-\infty,t)$ for each $t\in\mathbb{R}$ and $u(t) = \int_{-\infty}^{t} S_\alpha(t-s)\,f(s,u(s))\,ds$, for any $t\in\mathbb{R}$.*

Theorem 2.2. *Assume that A satisfies (H1). Let $f \in APS^p(\mathbb{R}\times\mathbb{X},\mathbb{X})$ with $p>2$ and that there is a function $L\in BS^p(\mathbb{X})$ such that (3) holds. If $\|L\|_{S^p} < \frac{\alpha\sin\left(\frac{\pi}{\alpha}\right)}{C(\theta,\alpha)M|\omega|^{-\frac{1}{\alpha}}\pi}$, where $C(\theta,\alpha)$ and M are the constants in [1], then the equation (1) has a unique almost periodic mild solution.*

2.2 Integro-Differential Equations

(H2) A generates an $(1, 1+(1*a))$ −regularized family $\{S(t)\}_{t\ge 0}$ on the Banach space $\mathbb{X}$ such that $\|S(t)\| \le \phi(t)$, $\forall t\ge 0$, where $\phi:\mathbb{R}^+\to\mathbb{R}^+$ is a decreasing function such that $\phi_0 = \sum\limits_{m=0}^{\infty}\phi(m) < +\infty$.

Theorem 2.3. *Assume that A satisfies (H2). If f is Stepanov almost periodic with exponent $p>1$, then the equation: $u'(t) = Au(t) + \int_{-\infty}^{t} a(t-s)\,Au(s)\,ds + f(t)\,,\quad t\in\mathbb{R}$, has a unique almost periodic mild solution.*

Theorem 2.4. *Assume that A satisfies (H2). Let $f\in APS^p(\mathbb{R}\times\mathbb{X},\mathbb{X})$ with $p>2$ and that there is a function $L\in BS^p(\mathbb{X})$ such that (3) holds. If $\phi_0\,\|L\|_{S^p} < 1$, then the equation (2) has a unique almost periodic mild solution.*

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ALMOST PERIODICITY FOR A NONAUTONOMOUS DISCRETE DISPERSIVE POPULATION MODEL

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Abstract

We study the almost periodic profile of solutions for nonautonomous difference equations in Banach spaces. We apply our results in population dynamics.

1 Introduction

In this work, we are concerned with the existence of discrete almost periodic solutions for the linear nonautonomous difference equation

$$u(n+1) = T(n)u(n) + f(n),\ n \in \mathbb{Z}, \tag{1}$$

and its perturbation

$$u(n+1) = T(n)u(n) + f(n, u(n)),\ n \in \mathbb{Z}, \tag{2}$$

where $\{T(n)\}_{n\in\mathbb{Z}}$ is a family of bounded linear operators on the complex Banach space $\mathbb{X}$ and f (in both cases) is almost periodic

We apply our abstract results to study the problem of dispersive populations described by the integrodifference equation

$$\varphi_{n+1}(x) = \int_\Omega k(x-y)F(n, y, \varphi(y))dy,\ n \in \mathbb{Z},\ x \in \Omega \tag{3}$$

and its perturbation

$$\varphi_{n+1}(x) = \int_\Omega k(x-y)\left[F(n, y, \varphi(y)) + g_n(y)\right]dy, n \in \mathbb{Z}, x \in \Omega, \tag{4}$$

where $\Omega \subseteq \mathbb{R}^N$ is a compact subset and the non negative growth function $F : \mathbb{Z} \times \Omega \times \mathbb{R}^+ \to \mathbb{R}^+$ is given by $F(n, y, r) = a_n r + f(n, r)$ where f is the Beverton-Holt function.

We begin by recalling the concept of almost periodic sequence.

Definition 1:

A sequence $f : \mathbb{Z} \to \mathbb{X}$ is said to be (discrete) almost periodic sequence if for any given number $\varepsilon > 0$, there is an integer $l = l(\varepsilon) \in \mathbb{Z}^+$ such that among any l consecutive integers, there is at least one integer p with the property

$$||f(t+p) - f(t)|| < \varepsilon, \text{ for all } t \in \mathbb{Z}. \tag{5}$$

The integer p is called an ε-translation number of f. We denote by $AP(\mathbb{Z}; \mathbb{X})$ the set of all such sequences.

For applications to nonlinear difference equations, the following concept of discrete almost periodic function depending on parameters will be useful.

Definition 2: A function $f : \mathbb{Z} \times \mathbb{X} \to \mathbb{X}$ is said to be discrete almost periodic if for every compact $K \subseteq \mathbb{X}$ $f(\cdot, x)$ is discrete almost periodic uniformly in $x \in K$. We denote by $AP(\mathbb{Z} \times \mathbb{X}; \mathbb{X})$ the set of all such functions.

2 Main Results

We will assume the following condition.

(H1) The discrete evolution family $\{U(n,s) : (n,s) \in \mathcal{P}\}$ associated to $\{T(n)\}_{n\in\mathbb{Z}}$ has an exponential dichotomy with data $(M, \omega, P(n))$.

and for the next result, we assume that the following hypothesis about the perturbation f is fulfilled:

(H_{loc}) The function $f : \mathbb{Z} \times \mathbb{X} \to \mathbb{X}$ is Lipschitz on bounded subsets of $\mathbb{X}$, with Lipschitz's function $L : \mathbb{R}^+ \to \mathbb{R}^+$, that is

$$||f(n,x) - f(n,y)|| \leq L(R)||x - y||,$$

for all $n \in \mathbb{Z}$ and for all $x, y \in \mathbb{X}$ satisfying $||x|| \leq R$ and $||y|| \leq R$.

Theorem 1: Suppose that $(H1)$ and (H_{loc}) are fulfilled and let $f \in AP(\mathbb{Z} \times \mathbb{X}; \mathbb{X})$. If there is $R > 0$ such that

$$M\left[L(R) + \frac{||f(\cdot,0)||_\infty}{R}\right] < \frac{1 - e^{-\omega}}{1 + e^{-\omega}}, \tag{6}$$

then the equation (2) has a unique bounded solution u satisfying $||u||_\infty \leq R$. In addition, $u \in AP(\mathbb{Z}; \mathbb{X})$ if, and only if, $Im(u) \subset \mathbb{X}$ is relatively compact.

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LOCAL NULL CONTROLLABILITY OF A FREE-BOUNDARY PROBLEM FOR THE SEMILINEAR 1D HEAT EQUATION

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Abstract

This presentation deals with the local null control of a free-boundary problem for the 1D semilinear heat equation with distributed controls. In the main result we verify that, if the final time T is fixed and the initial state is sufficiently small, there exists controls that drive the state exactly to rest at time $t = T$.

1 Introduction

Let $T > 0$ be given and let us assume that $f : \mathbb{R} \to \mathbb{R}$ is a globally Lipschitz continuous function. For any function $L \in C^1([0,T])$ with

$$0 < L_* \leq L(t) \leq B, \;\; t \in (0,T), \tag{1}$$

we will set $Q_L := \{(x,t) : x \in (0, L(t)), \;\; t \in (0,T)\}$.

We will consider free-boundary problems for semilinear parabolic systems of the form

$$\begin{cases} y_t - y_{xx} + f(y) = v1_\omega, & (x,t) \in Q_L, \\ y(0,t) = 0, \;\; y(L(t),t) = 0, & t \in (0,T), \\ y(x,0) = y^0(x), & x \in (0, L_0), \\ L(0) = L_0, \end{cases} \tag{2}$$

with the additional boundary condition

$$L'(t) = -y_x(L(t),t), \;\; t \in (0,T). \tag{3}$$

Here, $y = y(x,t)$ is the state and $v = v(x,t)$ is a control; it acts on the system at any time through the nonempty open set $\omega = (a,b)$ with $0 < a < b < L_*$; 1_ω denotes the characteristic function of the set ω; we assume that $y^0 \in H_0^1(0, L_0)$ and $L(0) = L_0$.

The main goal of this work is to analyze the null controllability of (2). It will be said that (2) is null-controllable at time T if, for each $y^0 \in H_0^1(0,T)$, there exists $v \in L^2(\omega \times (0,T))$, a function $L \in C^1([0,T])$ satisfying (1) and an associated solution $y = y(x,t)$ satisfying (2), (3) and

$$y(x,T) = 0, \;\; x \in (0, L(T)). \tag{4}$$

On the other hand, it will be said that (2) is approximately controllable in $L^2(0, L(T))$ at time T if, for any $y^0 \in H_0^1(0, L_0)$ and any $\varepsilon > 0$, there exists a control $v \in L^2(\omega \times (0,T))$, a function $L \in C^1([0,T])$ satisfying (1) and an associated state $y = y(x,t)$ satisfying (2), (3) and

$$\|y(\cdot,T)\|_{L^2(0,L(T))} \leq \varepsilon. \tag{5}$$

2 Main Results

Theorem 2.1. *Assume that f is globally Lipschitz continuous, $f(0) = 0$, $T > 0$ and $B > 0$. Also, assume that $0 < a < b < L_* < L_0 < B$. Then (2) is locally null-controllable. More precisely, there exists $\kappa > 0$ such that, if $\|y^0\|_{H_0^1(0,L_0)} \leq \kappa$ there exists triplets (L, v, y) with*

$$\begin{cases} L \in C^1([0,T]), \quad L_* \leq L(t) \leq B, \\ v \in L^2(\omega \times (0,T)), \quad y^* \in C^0([0,T]; H_0^1(0,B)), \end{cases} \tag{1}$$

where y^* is the extension of y by 0. satisfying (2), (3) *and* (4).

Proof The proof relies on the following argument:

1. First, for each $\varepsilon > 0$, we prove the existence of triplets $(y_\varepsilon, L_\varepsilon, v_\varepsilon)$ that are uniformly bounded in an appropriate space and satisfy (2), (3) and (5). To this purpose, we introduce a fixed point reformulation relying suitable linearized problems and we check that, if y_0 is sufficiently small, *Schauder's Theorem* can be applied.

   In particular, in order to get compactness, we rewrite (3) as an equation where the L in the right hand side is given and the L in the left hand side is obtained after integration in time. We use parabolic regularity theory to deduce that y_x is Hölder-continuous near the lateral boundary and, consequently, a C^1 function L in the right leads to a $C^{1+\alpha}$ function L in the left.

   It is not easy to prove this for $\varepsilon = 0$. Indeed, it becomes difficult to prove the continuity of the corresponding fixed point mapping.

2. Then, we take limits as $\varepsilon \to 0$ and we see that, at least for a subsequence, we get convergence to a solution to (2)–(4) ■

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BOUNDARY CONTROLLABILITY OF A ONE-DIMENSIONAL PHASE-FIELD SYSTEM WITH ONE CONTROL FORCE

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Abstract

We present some controllability results for linear and nonlinear phase field systems of Caginalp type considered in a bounded interval of $\mathbb{R}$ when the scalar control force acts on the temperature equation of the system by means of the Dirichlet condition on one of the endpoints of the interval. To prove the linear result we use the moment method providing an estimate of the cost. Using this estimate and following the methodology developed in [1], we prove a local exact boundary controllability result to constant trajectories of the nonlinear phase-field system.

1 Introduction

We deal with the boundary controllability properties of a phase-field system of Caginalp type (see [3]) which is a model describing the transition between the solid and liquid phases in solidification/melting processes of a material occupying an interval:

$$\begin{cases} \tilde\theta_t - \xi\tilde\theta_{xx} + \dfrac{1}{2}\rho\xi\tilde\phi_{xx} + \dfrac{\rho}{\tau}\tilde\theta + \dfrac{\rho}{4\tau}\left(\tilde\phi - \tilde\phi^3\right) = 0 & \text{in } Q_T, \\ \tilde\phi_t - \xi\tilde\phi_{xx} - \dfrac{2}{\tau}\tilde\theta - \dfrac{1}{2\tau}\left(\tilde\phi - \tilde\phi^3\right) = 0 & \text{in } Q_T, \\ \tilde\theta(0,\cdot) = v, \quad \tilde\phi(0,\cdot) = \tilde\theta(\pi,\cdot) = \tilde\phi(\pi,\cdot) = 0 & \text{on } (0,T), \\ \tilde\theta(\cdot,0) = \tilde\theta_0, \quad \tilde\phi(\cdot,0) = \tilde\phi_0 & \text{in } (0,\pi), \end{cases} \tag{1}$$

Here, $T > 0$ is some final time, $\tilde\theta = \tilde\theta(x,t)$ denotes the temperature of the material and $\tilde\phi = \tilde\phi(x,t)$ is the phase field function used to identify the solidification level of the material in such a way that $\tilde\phi = 1$ means that the material is in solid state and $\tilde\phi = -1$ in liquid state. The parameters ξ, ρ, τ are positive real constants with physical meaning and $c \in \{-1,1\}$. Finally, $v \in L^2(0,T)$ is the control force, which is exerted at point $x = 0$ by means of the boundary Dirichlet condition, and the initial data $\tilde\theta_0$, $\tilde\phi_0$ are given functions.

Our objective is to prove a null controllability result at time T for the temperature variable $\tilde\theta$ of system (1). If we consider the transition region associated to the temperature, i.e., the set

$$\Gamma(t) := \left\{x \in (0,\pi) : \tilde\theta(x,t) = 0\right\},$$

then, the problem under consideration consists of proving that there exists a control v such that the transition region associated to the temperature $\tilde\theta$ satisfies $\Gamma(T) = (0,\pi)$. It is interesting to underline that in this case the material could be in solid phase ($\tilde\phi(\cdot,T) = 1$), liquid phase ($\tilde\phi(\cdot,T) = -1$) or in an intermediate phase (mushy) which corresponds to $\tilde\phi(\cdot,T) = 0$. In this work we are interested in showing the null controllability result at time T for the temperature $\tilde\theta$ but keeping the material in solid state, $c = 1$, or liquid state, $c = -1$, at time T, that is to say, proving that there exists a control $v \in L^2(0,T)$ such that system (1) has a solution $\tilde y = (\tilde\theta, \tilde\phi)$ (in an appropriate space) such that

$$\tilde\theta(\cdot,T) = 0 \quad \text{and} \quad \tilde\phi(\cdot,T) = c \quad \text{in } (0,\pi). \tag{2}$$

2 Main Results

With the intent to apply a fixed point result, we will linearize the system (1). In fact, for simplicity, we perform the change of variable $(\theta, \phi) = (\tilde{\theta}, \tilde{\phi} - c)$ and then a linearization of system (1) around $(0,0)$, which gives us

$$\begin{cases} \theta_t - \xi\theta_{xx} + \frac{1}{2}\rho\xi\phi_{xx} - \frac{\rho}{2\tau}\phi + \frac{\rho}{\tau}\theta = f_1 & \text{in } Q_T, \\ \phi_t - \xi\phi_{xx} + \frac{1}{\tau}\phi - \frac{2}{\tau}\theta = f_2 & \text{in } Q_T, \\ \theta(0,\cdot) = v, \quad \phi(0,\cdot) = \theta(\pi,\cdot) = \phi(\pi,\cdot) = 0 & \text{on } (0,T), \\ \theta(\cdot,0) = \theta_0, \quad \phi(\cdot,0) = \phi_0 & \text{in } (0,\pi). \end{cases} \tag{1}$$

The controllability results will lie around the following conditions:

$$\xi^2\tau^2(\ell^2 - k^2)^2 - 2\xi\rho\tau(\ell^2 + k^2) - 2\rho - 1 \neq 0, \quad \forall k, \ell \geq 1, \quad \ell > k. \tag{2}$$

$$\xi \neq \frac{1}{j^2}\frac{\rho}{\tau}, \quad \forall j \geq 1. \tag{3}$$

Our main results are described in the following.

Theorem 2.1. *Let us consider ξ, ρ and τ three positive real numbers, $f_1 = f_2 = 0$ and let us fix $T > 0$. Then,*

1. *system (1) is approximately controllable in $H^{-1}(0,\pi;\mathbb{R}^2)$ at time $T > 0$ if and only if (2) holds.*
2. *system (1) is exactly controllable to zero in $H^{-1}(0,\pi;\mathbb{R}^2)$ at time $T > 0$ if (2) and (3) hold.*

Proof The proof is developed using the moment method. Such method requires an spectral analysis for the space operator associated to the system (1), i.e., the operator L such that (1) is written as $y_t + Ly = 0$. The hypotheses (3) and (2) are shown to be required so that the spectrum of L and L^* satisfies suitable properties that are used to apply the moment method. ∎

Theorem 2.2. *Let us consider ξ, τ and ρ three positive numbers satisfying (2) and (3), and let us fix $T > 0$ and $c = -1$ or $c = 1$. Then, there exist $\varepsilon > 0$ such that, for any $(\tilde{\theta}_0, \tilde{\phi}_0) \in H^{-1}(0,\pi) \times (c + H_0^1(0,\pi))$ fulfilling*

$$\|\tilde{\theta}_0\|_{H^{-1}} + \|\tilde{\phi}_0 - c\|_{H_0^1} \leq \varepsilon,$$

there exists $v \in L^2(0,T)$ for which system (1) has a unique solution $(\tilde{\theta}, \tilde{\phi}) \in L^2(Q_T) \times C^0(\overline{Q}_T)$ which satisfies (2).

Proof Following the methodology developed in [1] we use Theorem 1 to prove the null controllability result for system (1) in the case where (f_1, f_2) are not necessarily zero, but are in a L^2 space with weight. This allow us to apply a fixed point theorem for contractions, proving the controllability of the nonlinear system (1). ∎

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ON AN INVERSE PROBLEM IN THE SYSTEM MODELLING THE BIOCONVECTIVE FLOW

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Abstract

This work deals with the analysis of the inverse problem of determining the density function $\mathbf{F}$ modeling the vector external source for the linear momentum of particles, in a model for a bioconvective flow. In the case of the inverse problem we assume that a overspecification condition is given. The proof of local inverse problem uniqueness is given by characterizing the inverse problem solutions by using an operator equation of second kind and introducing several apriori estimates. Then we apply the Tikhonov fixed point Theorem.

1 Introduction

This work deals with the analysis of the inverse problem of determining the density function $\mathbf{F}$ modelling the vector external source for the linear momentum of particles, in a model for a bioconvective flow. If the fluid is contained on a bounded and regular domain $\Omega \subset \mathbb{R}^3$, with boundary $\partial\Omega$, then a model (direct problem) for a bioconvective flow in a finite time $T > 0$ is given by the following initial boundary value problem:

$$
\begin{aligned}
&\frac{\partial \mathbf{u}}{\partial t} - 2\mathrm{div}(\mu(c)D(\mathbf{u})) + \mathbf{u}.\nabla\mathbf{u} + \nabla p = -g(1+\rho c)\chi + \mathbf{F}, \quad \text{in} \quad \Omega,\\
&\mathrm{div}(\mathbf{u}) = 0, \quad \text{in} \quad \Omega,\\
&\frac{\partial c}{\partial t} - \theta\Delta c + \mathbf{u}.\nabla c + U\frac{\partial c}{\partial x_3} = 0, \quad \text{in} \quad (0,T)\times\Omega,\\
&\mathbf{u} = 0, \quad \text{in} \quad (0,T)\times S,\\
&\mathbf{u}.\mathbf{n} = 0, \quad \text{in} \quad (0,T)\times\Gamma,\\
&\mu(c)[D(\mathbf{u})\mathbf{u} - \mathbf{u}.(D(\mathbf{u})\mathbf{n})\mathbf{n}] = b_1, \quad \text{in} \quad (0,T)\times\Gamma,\\
&\theta\frac{\partial m}{\partial n} - Un_3 m = 0, \quad \text{in} \quad (0,T)\times\partial\Omega,\\
&\int_\Omega m\, dx = \alpha, \quad \text{in} \quad \Omega,
\end{aligned}
$$

where $\mathbf{u}, c$ and p denotes the velocity field, the concentration of microorganisms and the pressure distribution, respectively. The constant g is the gravity acceleration and θ, μ, b_1 and α are given. The strain tensor is defined as follows

$$D(\mathbf{u}) := \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^t\right).$$

The inverse problem is defined as follows: "Given θ, μ, b_1 and α determining the density function $\mathbf{F}$ such that $\{\mathbf{u}, c, m\}$ is solution of the direct problem and the overspecification condition

$$\int_\Omega \mathbf{u}(\mathbf{x},t)\cdot\boldsymbol{\psi}(\mathbf{x})d\mathbf{x} = \phi(t), \quad t\in[0,T],$$

is satisfied".

2 Main Results

The main result of the paper is the following theorem :

Theorem 2.1. *Let us consider that $H(\Omega), J_0(\Omega)$ and Y are defined as follows:*

- $H(\Omega)$ *is the closure of* $\dot{H}(\Omega) = \Big\{\mathbf{u} \in C^\infty(\overline{\Omega}) \,:\, \mathbf{u}/_S = 0, \mathbf{u}/_\Gamma = 0\Big\}$ *with the topology induced by the norm* $\|\mathbf{u}\|_{H(\Omega)} := \|\nabla \mathbf{u}\|_{L^2(\Omega)}$,
- $J_0(\Omega)$ *is the closure of* $\dot{J}(\Omega) = \Big\{\mathbf{u} \in \dot{H}(\Omega) \,:\, \mathrm{div}(\mathbf{u}) = 0\Big\}$ *with the topology induced by the norm* $\|\|_{H(\Omega)}$,
- $Y = \Big\{f \in L^2((\Omega)) \quad ; \quad \int_\Omega f(\mathbf{x})d\mathbf{x} = 0\Big\}$.

Assume that the following hypotheses

$$\nu \in C^1(\overline{\Omega}), \quad u_0 \in J_0(\Omega), \quad m_0 \in H^2(\Omega) \cap Y, \quad \psi, \phi \in [H^2(\Omega)]^3,$$

are satisfied. Moreover, assume that the $\mathbf{F}$ *admit the Helmholtz descomposition* $\mathbf{F}(\mathbf{x}, t) = f(t)\mathbf{h}(\mathbf{x}, t)$ *with* $\mathbf{h}$ *a know function and* f *and unknown function such that there exist* $r \in \mathbb{R}^+$ *satisfying* $\left|\int_\Omega u_0(\mathbf{x})h(\mathbf{x}, 0)d\mathbf{x}\right| \geq 2r^\varepsilon$. *Then there exist a unique solution* $\{\mathbf{u}, m, c, f\}$ *of the inverse problem defined on a small enough time* T_*.

The proof of local uniqueness of the inverse problem is given by characterizing the inverse problem solutions by using an operator equation of second kind, introducing several apriori estimates and then applying the Tikhonov fixed point Theorem.

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LOCAL NULL CONTROLLABILITY OF A NONLINEAR PARABOLIC SYSTEM IN DIMENSION 1

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Abstract

This paper deals with the internal and bounday null controllability of a 1D parabolic PDE with nonlinear diffusion. In the proofs of the main results, we use Liusternik's Inverse Function Theorem, together with some appropriate estimates.

1 Introduction

In this work we are concerned in studying parabolic problems with nonlinear diffusion, i.e., of the type

$$y_t - div(a(y)\nabla y) = f.$$

Here, we assume the function a to be a generic function depending on the solution y and show that, if a is suitably smooth, the controllability of a system with an equation of this kind holds in the one dimensional case.

Let $I \subset \mathbb{R}$ be an open bounded interval and let us denote by Q the cylinder $Q := I \times (0,T)$, with lateral boundary $\Sigma := \partial I \times (0,T)$. Also, we consider a non-empty open set of $\omega \subset I$. As usual, 1_ω denotes the characteristic function of ω. We will be concerned with the null controllability of the nonlinear systems

$$\begin{cases} y_t - (a(y)y_x)_x = v_1 1_\omega & \text{in} \quad Q \\ y(x,t) = 0 & \text{on} \quad \Sigma \\ y(x,0) = y_0(x) & \text{in} \quad I \end{cases} \tag{1}$$

and

$$\begin{cases} y_t - (a(y)y_x)_x = 0 & \text{in} \quad (0,1)\times(0,T) \\ y(0,t) = v_2(t), \;\; y(1,t) = 0 & \text{on} \quad (0,T) \\ y(x,0) = y_0(x) & \text{in} \quad (0,1), \end{cases} \tag{2}$$

where v_1 and v_2 are the controls and y is the associated state. It will be assumed that the real function $a = a(r)$ is of class C^1, possesses bounded derivatives and satisfies

$$0 < m \le a(r) \le M \quad \forall r \in \mathbb{R}.$$

The main goal of this paper is to analyze the null controllability of (1). It will be said that (1) (resp. (2)) is locally null-controllable at time T if there exists $\epsilon > 0$ such that, for any $y_0 \in H_0^1(I)$ with $\|y_0\|_{H_0^1}(I) \le \epsilon$, there exist controls $v_1 \in L^2(\omega \times (0,T))$ (resp., $v_2 \in L^2(0,T)$) such that the associated states y satisfy

$$y(x,T) = 0 \;\text{ in }\; I. \tag{3}$$

2 Main Results

Our main results are the following:

Theorem 2.1. *Under the previous assumptions on a, the nonlinear system* (1) *is locally null-controllable at any time $T > 0$.*

Theorem 2.2. *Under the previous assumptions on a, the nonlinear system* (2) *is locally null-controllable at any time $T > 0$.*

Proof The proof of Theorem 2.1 relies on an application of *Liusternik's Inverse Function Theorem* in Banach spaces. We follow the ideas of [1]; this paper is in turn inspired by the works of Fursikov and Imanuvilov [2] and Imanuvilov and Yamamoto [3]. We give now an idea of the proof.

In order to prove Theorem 2.1 we will use the following strategy: linearize the original system and use a result providing existence of a function under appropriate hypotheses. Thus, in a first step, we consider the following linearized system at zero

$$\begin{cases} y_t - a(0)y_{xx} = v_1 1_\omega + h & \text{in} \quad Q \\ y(x,t) = 0 & \text{on} \quad \Sigma \\ y(x,0) = y_0(x), & \text{in} \quad I. \end{cases} \tag{4}$$

The adjoint of (4) is given by

$$\begin{cases} -\varphi_t - a(0)\varphi_{xx} = F & \text{in} \quad Q \\ \varphi = 0 & \text{on} \quad \Sigma \\ \varphi(x,T) = \varphi_0(x), & \text{in} \quad I. \end{cases} \tag{5}$$

The null controllability of (5) (for appropriate h) will be obtained as a consequence of a suitable Carleman inequality for the solutions to (5). Indeed, we apply a Carleman inequality with weight for the system (4) using suitably weight functions. As a consequence, some estimates concerning the solutions of (4) are shown to hold if h decay to zero faster than such weights.

In a second step, we define a space Y of "admissible" state-controls constructed based on the estimates with weight obtained previously for solution y of the system (4). We consider then the mapping $H : Y \to Z$ given by

$$H(y,v) = (y_t - (a(y)y_x)_x - v1_\omega, y(\cdot,0)),$$

where $Z = \mathcal{F} \times H_0^1(I)$, $\mathcal{F}$ being the space $L^2(Q)$ with weight, and use Liusternik's Theorem to prove that there exists $\epsilon > 0$ such that, if $(h, y_0) \in Z$ and $\|(h, y_0)\|_Z \leq \epsilon$, then the equation

$$H(y,v) = (h, y_0), \ (y,v) \in Y,$$

possesses at least one solution. In particular, this will show that (1) is locally null-controllable, with controls v and associated states y satisfying $(y,v) \in Y$. ■

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EQUAÇÕES DE SEGUNDA ORDEM DE EVOLUÇÃO EM TEMPO DISCRETO EM ESPAÇOS PONDERADOS VIA TEORIA DE REGULARIDADE MAXIMAL DISCRETA

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Abstract

Estudamos equações de evolução em tempo discreto em espaços com a propriedade “unconditional martingale differences” (espaços UMD), um caso linear e outro semilinear dependendo do tempo, estado e velocidade. Estamos interessados em discutir acerca de existência e unicidade de solução e algumas estimativas e também teoremas de caracterização para o nosso problema. Em particular, centralizamos nossa atenção nos recentes conceitos de Regularidade Maximal Discreta envolvendo também os conceitos de R-limitação, Teoria de Multiplicadores do tipo l_p e Teoria de Equações bem-postas.

1 Introdução

Em 2001 (ver [1]), Blunck caracteriza o problema de primeira ordem. Em 2007, Claudio Cuevas e Carlos Lizama consideraram (ver [2]) as problemáticas análogas às estudadas por Blunck para o problema de segunda ordem. Em 2010, Airton Castro, Claudio Cuevas e Carlos Lizama (ver [3]) seguem com a caracterização em espaços ponderados.

Definition 1.1. *Sejam X e Y espaços de Banach, dizemos que o subconjunto $\mathcal{T} \subset B(X,Y)$ é R-limitado se existe uma constante $C > 0$ tal que dados $n \in \mathbb{Z}$, $T_1, ..., T_n \in \mathcal{T}$ e $x_1, ..., x_n \in X$,*

$$\int_0^1 \left|\left| \sum_{i=1}^{n} r_i(u) T_i(x_i) \right|\right| du \leq C \int_0^1 \left|\left| \sum_{i=1}^{n} r_i(u) x_i \right|\right| du \tag{1}$$

Onde $(r_n)_{n\geq 1}$ é uma sequência de variáveis aleatórias simétricas com valores em $\{-1,1\}$ que no nosso caso iremos utilizar as funções de Rademacher. A menor constante C para a qual a desigualdade (1) é válida será denotada por $\mathcal{R}(\mathcal{T})$.

Para cada $i \in \mathbb{N}$, a i-ésima função de Rademacher é definida por:

$$\begin{array}{rccc} r_i: & [0,1] & \to & \{-1,1\} \\ & x & \mapsto & sgn\{\sin(2^i \pi x)\} \end{array}$$

onde sgn é a função sinal.
Vamos introduzir algumas definições a fim de caracterizar o seguinte o problema:

$$\begin{cases} \Delta_r^2 x_n - r^2(I-T)x_n = f_n, \\ x_0 = \Delta_r x_0 = 0 \end{cases} \tag{2}$$

$n \in \mathbb{Z}_+$ e $f : \mathbb{Z}_+ \longrightarrow X$.

Definition 1.2. *Seja $p \in (1, +\infty)$. A equação (2) tem regularidade maximal discreta se $\mathcal{K}^r f := (I-T) r^{\bullet+1} \mathcal{S} * f$ define um operador linear limitado $\mathcal{K}^r \in B(l_p(\mathbb{Z}_+; X))$.*

Considere os seguintes espaços:

$$l^1_{p,r}(\mathbb{Z}_+;X) := \{y = (y_n)/y_0 = 0, (\Delta_r y_n) \in l_p(\mathbb{Z}_+;X)\}$$
$$l^2_{p,r}(\mathbb{Z}_+;X) := \{y = (y_n)/y_0 = y_1 = 0, (\Delta^2_r y_n) \in l_p(\mathbb{Z}_+;X)\}$$
$$l_{p,I-T}(\mathbb{Z}_+;X) := \{y = (y_n)/((I-T)y_n) \in l_p(\mathbb{Z}_+;X)\}$$

Definition 1.3. *Dizemos que* $\{Q(z)\}_{z\in\mathbb{T}^\alpha_r}$ *é um* $l_p - l^i_{p,r}$*-multiplicador,* $i = 1,\ 2$*, se cada* $f = (f_n) \in l_p(\mathbb{Z}_+;X)$*, existe uma sequência* $y = (y_n) \in l^i_{p,r}(\mathbb{Z}_+;X)$ *tal que* $\hat{y}(z) = Q(z)\hat{f}(z)$*,* $z \in \mathbb{T}^\alpha_r$*.*

Definition 1.4. *Dizemos que o problema (2) é bem-posto se para cada* $f = (f_n) \in l_p(\mathbb{Z}_+;X)$ *existe uma única solução* $x = (x_n) \in l^2_{p,r}(\mathbb{Z}_+;X) \cap l_{p,I-T}(\mathbb{Z}_+;X)$ *da equação (2).*

2 Resultados Principais

Seja X um espaço de Banach. Denotamos por Δ_r o operador r-diferença de primeira ordem, ou seja, para cada $n \in \mathbb{Z}_+$, $\Delta_r x_n = x_{n+1} - r x_n$, $r \in \mathbb{R}_+$. Além disso, denotaremos $\Delta^2_r x_n = \Delta_r(\Delta_r x_n)$.

Considere, para todo $n \in \mathbb{Z}_+$, o problema:

$$\begin{cases} \Delta^2_r x_n - r^2(I-T)x_n = f_n \quad , \\ x_0 = a, \Delta_r x_0 = b \end{cases} \tag{3}$$

onde $f : \mathbb{Z}_+ \longrightarrow X$ e $T \in B(X)$.

Theorem 2.1. *[Existência e Unicidade] A única solução do problema (3) é da forma*

$$x_{m+1} = r^{m+1}\mathcal{C}(m+1)x + r^m\mathcal{S}(m+1)y + (r^{\bullet-1}\mathcal{S}(\bullet) * f)_m.$$

Além disso,

$$\Delta_r x_{m+1} = r^{m+2}(I-T)\mathcal{S}(m+1)x + r^{m+1}\mathcal{C}(m+1)y + (r^\bullet\mathcal{C}(\bullet) * f)_m.$$

Theorem 2.2. *Sejam* X *um espaço UMD e* $T \in B(X)$ *um operador analítico. Assuma que* $\alpha = 1+\sqrt{2}$*,* $r \geq r_0$ *e* $\frac{1}{1+\sqrt{2}} < r_0 < 1$*. Então as seguintes afirmações são equivalentes:*
(i) O problema (2) é bem-posto;
(ii) $\{M(z) := (z-r)^2((z-r)^2 - r^2(I-T))^{-1} : z \in \mathbb{T}^\alpha_r, z \neq \alpha r\}$ *é* $l_p - l_p$*- multiplicador;*
(iii) A família $\{M(z) : z \in \mathbb{T}^\alpha_r, z \neq \alpha r\}$ *é R-limitada;*
(iv) O problema (2) tem regularidade maximal discreta.

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FUNÇÕES DE GREEN NO SISTEMA DE COORDENADAS DA FRENTE DE LUZ PARA UM BÓSON LIVRE

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Abstract

O objetivo deste trabalho é mostrar como se comporta a função de Green no sistema de coordenadas da Frente de Luz em um Estado Ligado Bosônico para um bóson livre, justificando sua importância para a Mecânica Quântica. Através disso, será possível identificar uma transformação do propagador quântico covariante no tempo da Frente de Luz obtendo a função de Green para partículas escalares, sem interação, nestas coordenadas. Este operador propaga a função onda de $x^+ = 0$ até $x^+ > 0$, correspondendo à definição da operação de ordenação temporal na coordenada x^+. Fazemos o cálculo da função de Green na Frente de Luz para 1 bóson não interagente se propagando para frente no tempo x^+.

1 Introdução

Em mecânica quântica a função de onda define completamente o estado do sistema e satisfaz a equação de Schrödinger:

$$H\left|\Psi\right\rangle = i\frac{d}{dt}\left|\Psi\right\rangle, \tag{1}$$

onde H é a Hamiltoniana do sistema e $|\Psi\rangle$ é um vetor de estado. Em (1), estamos considerando $\hbar = 1$. A função de onda , Ψ, evolui no tempo e sua evolução pode ser descrita pelo operador unitário $U\left(t - t'\right)$:

$$\left|\Psi\right\rangle = U\left(t - t'\right)\left|\Psi'\right\rangle. \tag{2}$$

Introduzindo a equação (2) na equação (1) temos:

$$HU\left(t - t'\right) = i\frac{d}{dt}U\left(t - t'\right), \tag{3}$$

onde $U(0) = 1$. Através do operador evolução, podemos introduzir o propagador da equação de Schrödinger:

$$S\left(t - t'\right) = U\left(t - t'\right)\theta\left(t - t'\right), \tag{4}$$

onde $\theta(t) = 1$ para $t > 0$ e $\theta(t) = 0$ para $t < 0$. Derivando a equação (4) em relação a t, temos:

$$\frac{d}{dt}S\left(t - t'\right) = \left(\frac{d}{dt}U\left(t - t'\right)\right)\theta\left(t - t'\right) + U\left(t - t'\right)\frac{d}{dt}\theta\left(t - t'\right). \tag{5}$$

Usando a equação (3), temos:

$$i\frac{d}{dt}S\left(t - t'\right) - HS\left(t - t'\right) = i\delta\left(t - t'\right), \tag{6}$$

onde usamos que $\frac{d}{dt}\theta(t) = \delta(t)$ e $U(0) = 1$. O propagador associado à função de Green da equação de Schrödinger é definido por: $G\left(t - t'\right) = -iS\left(t - t'\right)$. A função de Green ou o propagador descreve completamente a evolução do sistema quântico. Neste caso estamos usando o propagador para "tempos futuros". Poderiamos igualmente definir o propagador para "trás" no tempo. A propagação de um bóson livre de spin nulo no espaço quadridmensional é representada pelo propagador de Feynman covariante

$$S(x^\mu) = \int \frac{d^4k}{(2\pi)^4} \frac{ie^{-ik^\mu x_\mu}}{k^2 - m^2 + i\varepsilon}, \quad \text{com} \quad \mu = 0, 1, 2, 3, \tag{7}$$

onde a coordenada x^0 representa o tempo e k^0 a energia. Esse processo também pode ser encontrado em [1].

2 Resultados Principais

Fazemos a transformação do propagador de um bóson livre no tempo associado à Frente de Luz reescrevendo as coordenadas em termos da coordenada temporal x^+ e das coordenadas de posição (x^- e $\overrightarrow{x}_\perp$). Com isto os momentos são dados por k^-, k^+ e $\overrightarrow{k}_\perp$, e portanto, teremos

$$S(x^-, x^+, x^\perp) = \frac{1}{2} \int \frac{dk_1^- dk_1^+ d^2 k_1^\perp}{(2\pi)} \frac{ie^{-i\left[\frac{k_1^- x^+ + k_1^+ x^-}{2} - k_1^\perp x^\perp\right]}}{k_1^+ \left(k_1^- - \frac{k_{1\perp}^2 + m^2 - i\varepsilon}{k_1^+}\right)}. \tag{8}$$

Como nosso interesse é estudar a propagação apenas para o tempo x^+ na Frente de Luz, então simplesmente iremos resolver a equação

$$S(x^+) = \frac{1}{2} \int \frac{dk_1^-}{(2\pi)} \frac{ie^{\frac{-i}{2} k_1^- x^+}}{k_1^+ \left(k_1^- - \frac{k_{1\perp}^2 + m^2 - i\varepsilon}{k_1^+}\right)}, \tag{9}$$

em que na mudança de coordenada para Frente de Luz estamos considerando a convenção Lepage-Brodsky (veja em [3]), ou seja, $\alpha = 1$. O jacobiano da transformação k^0, $\overrightarrow{k} \rightarrow k^-, k^+, \overrightarrow{k}_\perp$ é igual a $\frac{1}{2}$ e k^+, $k_\perp$ são operadores de momento. Calculando uma transformada de Fourier, onde aplicamos $\delta\left(\frac{K^- - k_1^-}{2}\right) = \frac{1}{2\pi} \int dx^+ e^{\frac{i}{2}\left(K^- - k_1^-\right)x^+}$ utilizando a propriedade do delta de Dirac $\delta\left(ax\right) = \frac{1}{a}\delta\left(x\right)$ obtemos,

$$\widetilde{S}(K^-) = \frac{i}{k_1^+ \left(K^- - \frac{k_{1\perp}^2 + m^2 - i\varepsilon}{k_1^+}\right)}, \tag{10}$$

que descreve a propagação de uma partícula para o futuro e de uma antipartícula para o passado. Isso pode ser observado em (2.1) pelo denominador que nos indica que para $x^+ > 0$ e $k^+ > 0$ temos a partícula se propagando para frente no tempo da Frente de Luz. Caso contrário, para $x^+ < 0$ e $k^+ < 0$ teremos a antipartícula propagando-se para o passado. No caso de um bóson livre, a função de Green, para a propagação de partícula é dada pelo operador:

$$G_0^{(1p)}(k^-) = \frac{\theta(k^+)}{k^- - k_{on}^- + i\varepsilon} \ ; \tag{11}$$

onde $k_{on}^- = \frac{k_\perp^2 + m^2}{k^+}$ é a energia da partícula. Para propagação de antipartícula, temos

$$G_0^{(1a)}(k^-) = \frac{\theta(-k^+)}{k^- + k_{on}^- - i\varepsilon} \ . \tag{12}$$

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ALMOST AUTOMORPHIC SOLUTIONS OF VOLTERRA EQUATIONS ON TIME SCALES

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Abstract

The existence and uniqueness of almost automorphic solutions for linear and semilinear nonconvolution Volterra equations on time scales is studied. Examples that illustrate our results are given.

1 Introduction

Several important and interesting models in diverse applied areas are described using differential and integral equations, or even difference and summation equations. However, in the last years, there are some recent studies which have been showed that these equations are not the best choices to describe most of the existent models. It happens, because most of phenomena in the environment do not involve only continuous aspects or only discrete aspects, but they feature elements of both the continuous and the discrete. These phenomena are called *hybrid processes*.

By these reasons, Stefan Hilger and Bernd Aulbach in 1988 introduced a theory to study in a unified way large classes of time scales. This theory encompasses the study of the differential equations, integral equations, difference equations, summation equations, among others. Therefore, using this theory, it is possible to describe in a more precise way the real-world problems, obtaining a more detailed analysis and description of specifics problems. For more details about the time scales theory, see [1, 3].

On the other hand, it is well known that Volterra integral equations play an important role in applications since they can describe several interesting phenomena. Some typical examples are provided by viscoelastic fluids and heat flow in materials of fading memory type.

Motivated by this fact, our goal in this paper is to search for the existence and uniqueness of almost automorphic solutions for the class of nonconvolution Volterra integral equation on time scales given by:

$$u(t) = \int_{t_0}^{t} a(t,\sigma(s))[u(s) + f(s,u(s))]\Delta s, \tag{1}$$

where $a : \mathbb{T} \times \mathbb{T} \to \mathbb{R}^{n\times n}$ is almost automorphic in both variables and $f : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ is almost automorphic with respect to the first variable and satisfies a Lipschitz condition in the second variable.

We find a practical condition that should be verified for the kernel in equation (1), namely

(H1) There exist positive constants $K, \gamma \in \mathbb{R}$ such that

$$\int_{-\infty}^{u} \|a(t,\sigma(s))\|\Delta s \le \int_{-\infty}^{u} K e_{\ominus\gamma}(t,\sigma(s))\Delta s,$$

for every $t, u \in \mathbb{T}$.

Under this condition, we prove that if $\mathbb{T}$ is an invariant under translations time scale, then there exists a unique almost automorphic solution of (1), provided $\dfrac{\gamma}{K(1+\tilde{\mu}\gamma)} > 2(1+L)$, where L is the Lipschitz constant of f. Moreover, if f does not depend on the second variable, then L can be chosen to be 0. Finally, it is worthwhile to remark that the results are essentially new when $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$.

2 Main Results

Definition 2.1 ([4]). *A time scale* $\mathbb{T}$ *is called* invariant under translations *if*

$$\Pi := \{\tau \in \mathbb{R} : t \pm \tau \in \mathbb{T}, \ \forall t \in \mathbb{T}\} \neq \{0\}. \tag{1}$$

Definition 2.2 ([4]). *Let* X *be a (real or complex) Banach space and* $\mathbb{T}$ *be an invariant under translations time scale. Then, an rd-continuous function* $f : \mathbb{T} \times X \to X$ *is called* almost automorphic at $t \in \mathbb{T}$ *for each* $x \in X$*, if for every sequence* $(\alpha'_n) \in \Pi$*, there exists a subsequence* $(\alpha_n) \subset (\alpha'_n)$ *such that*

$$\lim_{n\to\infty} f(t+\alpha_n, x) = \bar{f}(t,x) \tag{2}$$

exists and is well defined for each $t \in \mathbb{T}$, $x \in X$ *and*

$$\lim_{n\to\infty} \bar{f}(t-\alpha_n, x) = f(t,x) \tag{3}$$

exists and is well-defined for every $t \in \mathbb{T}$ *and* $x \in X$.

Definition 2.3. *Let* X *be a (real or complex) Banach space and* $\mathbb{T}$ *be an invariant under translations time scale. Then, an rd-continuous function* $f : \mathbb{T} \times \mathbb{T} \to X$ *is called* almost automorphic with respect to both variables *if for every sequence* $(\alpha'_n) \in \Pi$*, there exists a subsequence* $(\alpha_n) \subset (\alpha'_n)$ *such that*

$$\lim_{n\to\infty} f(t+\alpha_n, s+\alpha_n) = \bar{f}(t,s) \tag{4}$$

exists and is well defined for each $t,\ s \in \mathbb{T}$ *and*

$$\lim_{n\to\infty} \bar{f}(t-\alpha_n, s-\alpha_n) = f(t,s) \tag{5}$$

exists and is well-defined for every $t,\ s \in \mathbb{T}$.

Theorem 2.1. *Suppose that the time scale* $\mathbb{T}$ *is invariant under translations, the function* $f : \mathbb{T} \times \mathbb{R}^n \to \mathbb{R}^n$ *is almost automorphic on time scales in* t *for each* $x \in \mathbb{R}^n$ *and satisfies a Lipschitz condition in* x *uniformly in* t*, that is,*

$$\|f(t,x) - f(t,y)\| \leq L\|x-y\|,$$

for all $x,\ y \in \mathbb{R}^n$*, the function* $a : \mathbb{T} \times \mathbb{T} \to \mathbb{R}^{n\times n}$ *is almost automorphic with respect to both variables and satisfies hypothesis (H1) with the positive constants* γ *and* K *being such that* $\dfrac{\gamma}{K(1+\tilde{\mu}\gamma)} > 2(1+L)$*, where* L *is the Lipschitz constant. Then, the equation* (1) *possesses a unique almost automorphic solution.*

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CONTROLABILIDADE NULA LOCAL DE UM PROBLEMA DE FRONTEIRA LIVRE PARA A EQUAÇÃO DO CALOR EM DOMÍNIOS 2D ESTRELADOS

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Abstract

Neste trabalho, consideramos um problema de controlabilidade nula local para a equação do calor 2D com fronteira livre em domínios estrelados. No resultado principal, provamos que, se o tempo final T é fixado e a condição inicial é suficientemente pequena, então existe um controle que leva o estado a zero no tempo final T.

1 Introdução

Dado $y_0 \in H_0^1(\Omega)$, $T > 0$ e uma função suficientemente regular 2π-periodica $\rho_0 = \rho_0(\phi)$, considere o domínio estrelado

$$\Omega := \{x \in \mathbb{R}^2 : \ x_1 = r\cos\phi, x_2 = r\sin\phi, 0 \leq r < \rho_0(\phi), \phi \in [0, 2\pi)\}.$$

Estamos interessados na controlabilidade nula do seguinte problema de fronteira livre:

$$\begin{cases} y_t - \Delta y = v1_\omega, & (x,t) \in Q_\rho, \\ y = 0, & (x,t) \in \Sigma_\rho, \\ y(x,0) = y_0(x), & x \in \Omega, \end{cases} \tag{1}$$

juntamente com,

$$\rho(\phi, 0) = \rho_0(\phi), \ \ \phi \in [0, 2\pi), \tag{2}$$

$$(V - \nu\Delta V) \cdot n = -\frac{\partial y}{\partial n}, \ \ (x,t) \in \Sigma_\rho, \tag{3}$$

onde V é a velocidade da fronteira livre $\partial\Omega_\rho(t)$, $\omega \subset\subset \Omega$ um aberto não-vazio o qual é chamado *domínio de controle*, $\rho : \mathbb{R} \times [0,T] \to \mathbb{R}$, $\rho = \rho(\phi, t)$ é uma função 2π-periodic em ϕ tal que $\rho > 0$ e

$$\Omega_\rho(t) := \{x \in \mathbb{R}^2 : \ x_1 = r\cos\phi, x_2 = r\sin\phi, 0 \leq r < \rho(\phi, t), \ \phi \in [0, 2\pi)\},$$
$$Q_\rho := \{(x,t) : \ x \in \Omega_\rho(t), \ t \in (0,T)\}, \quad \text{e} \quad \Sigma_\rho(t) := \{(x,t) : \ x \in \partial\Omega_\rho(t), \ t \in (0,T)\}.$$

Problemas de fronteira livre como este são motivados por diversas aplicações como processos de solidificação, escoamento de fluídos em meios porosos e crescimento de tumor com pode ser visto em [1, 2] e suas referências.

2 Resultados Principais

Se definirmos $\zeta(\phi,t) := \rho_t(\phi,t)$, podemos escrever a condição de Stokes (3) da seguinte forma:

$$\begin{cases} -\zeta_{\phi\phi} + \dfrac{2\rho_\phi}{\rho}\zeta_\phi + \left(\dfrac{\rho^2}{\nu}+1\right)\zeta = f_0(\phi,t)\rho\sqrt{\rho^2+\rho_\phi^2} & \text{em } \mathbb{R}\times(0,T) \\ \zeta(\phi+2\pi,t) = \zeta(\phi,t),\ \zeta_\phi(\phi+2\pi,t) = \zeta_\phi(\phi,t) & \text{em } \mathbb{R}, \end{cases} \tag{1}$$

onde $f_0(\phi,t) := \dfrac{1}{\nu}|\nabla y(\rho(\phi,t)\cos\phi, \rho(\phi,t)\sin\phi, t)|$.

Teorema 2.1. *Seja $\rho_0 \in C^{2,1}(\mathbb{R})$ uma função 2π-periodica tal que $0 < \nu_0 \leq \rho_0(\phi)$ para todo $\phi \in \mathbb{R}$. Assuma que $T > 0$ e um conjunto aberto não-vazio $\omega \subset\subset B_{\nu_0}$ são dados. Então o problema (1) é localmente nulamente controlável. Mais precisamente, existe $\varepsilon > 0$ tal que, se $y_0 \in C^1(\overline{\Omega})$ e $\|y_0\|_{C^1(\overline{\Omega})} \leq \varepsilon$, existem $v \in L^2(\omega\times(0,T))$ e uma solução associada (ρ, y) para (1)-(3) satisfazendo*

(i) $y, \partial_i y, \partial_i\partial_j y, y_t \in L^2(Q_\rho)$ $(i,j = 1,2)$,

(ii) $\rho, \rho_\phi \in C^1(\mathbb{R}\times(0,T))$ *são 2π-periodicas em ϕ,*

(iii) $\rho(\phi,t) > 0, \forall(\phi,t) \in \mathbb{R}\times[0,T]$.

e

$$y(x,T) = 0,\ x \in \Omega_\rho(T)$$

Prova: Primeiramente, provamos um resultado de controlabilidade nula para o problema não-cilíndrico para uma função fixada $\rho = \rho(\phi,t)$ satisfazendo uma condição adequada. Para isso, seguindo as ideias em [3], construímos um difeomorfismo que transforma do problema não-cilíndrico (1) em um problema parabólico em um domínio cilíndrico, o qual podemos estabelecer resultados de existência, unicidade, regularidade e controlabilidade nula local.

Em seguida, assim como em [3], provamos a existência, unicidade e regularidade da solução para uma versão linearizada da condição de Stokes (1). Isto permite-nos definir uma aplicação Λ_δ que associa a cada $\rho = \rho(\phi,t)$ e $\delta > 0$ uma função

$$P(\phi,t) = \int_0^t \zeta_\delta(\phi,s)ds + \rho_0(\phi),$$

onde ζ_δ é a solução do problema (1) associado à solução (y_δ, v_δ) do problema (1).

Finalmente, provamos que se $\|y_0\|_{H_0^1(\Omega)} \leq \varepsilon$, para ε suficientemente pequeno,a aplicação Λ_δ satisfaz as hipóteses do *Shcauder's Fixed-Point Theorem*, o que garante o resultado final. ■

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LOCALLY DAMPED KAWAHARA EQUATION POSED ON THE LINE

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Abstract

We prove the exponential decay of the energy related to a locally damped Kawahara equation posed on the whole real line with the initial datum from a bounded set of L^2. A local smoothing effect in H^2 is established, which is essential to obtain all necessary a priory estimates.

1 Introduction

The Kawahara equation

$$u_t + u_x + uu_x + u_{xxx} - u_{xxxxx} = 0 \tag{1}$$

is the nonlinear dispersive PDE that appears in the theory of magneto-acoustic waves in plasma, [10], and in modeling of gravity-capillary water waves, [7]. This equation sometimes is also mentioned as a fifth-order KdV, [8, 14], as well as a special version of the Benney-Lin equation, [2].

It is well-known that dispersive equations provide soliton-like solutions whose profiles do not change in time. In certain situations, however, it is of interest to control such physical phenomena; the decay of solutions is an essential tool for this purpose. There are various dissipative mechanisms which can be added into the model: second and forth-order "viscous" terms, nonlocal integral terms, "frictional" damping terms, etc. All these instruments are usually introduced for the KdV or Schrödinger equation; there are few results regarding the Kawahara equation (1). Moreover, the most part of these studies is mainly concerned with a bounded spatial interval, [1, 15, 16, 17].

The main aim of the present paper is to prove the exponential decay of the energy related to the Kawahara equation with so-called "localized" damping term. This kind of dissipation was proposed in [12] to control the KdV equation posed on a bounded interval, and later it was considered for models involved unbounded domains of waves propagation (see, for instance, [5, 11] and the references therein).

2 Main result

We are concerned here with the IVP consisting in a locally damped Kawahara equation subject to initial data posed on the whole real line:

$$u_t + u_x + uu_x + u_{xxx} - \gamma u_{xxxxx} + a(x)u = 0, \quad (x,t) \in \mathbb{R} \times \mathbb{R}^+, \tag{2}$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}, \tag{3}$$

where $u : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$, $\gamma > 0$, and $a(x)$ satisfies the following assumptions:

$$a \in L^\infty(\mathbb{R}) \text{ is a nonnegative function and } a(x) \geq \alpha_0 > 0 \text{ for } |x| \geq R, \quad R > 0. \tag{4}$$

Our main result is to establish local and global (in time) well-posedness of (2), (3), the smoothing (hidden regularity) effect, and necessary bounds to prove that $E(t) = \frac{1}{2}\|u\|^2_{L^2(\mathbb{R})}(t)$ is exponentially decreasing as $t \to +\infty$. Technically, we mainly follow [5] and [11], with new issues regarding the unboundness (in both directions) of the spatial domain, and the higher order of the differential operator.

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DECAIMENTO NA NORMA L^2 PARA AS SOLUÇÕES DAS EQUAÇÕES DOS FLUIDOS MICROPOLARES HOMOGÊNEOS

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Abstract

Provamos, formalmente, uma taxa de decaimento temporal em $\mathbf{L^2}$ para as soluções das equações dos fluidos micropolares homogêneos, com velocidades iniciais em $\mathbf{L^1}$ e $\mathbf{L^2}$.

1 Introdução

Consideraremos o seguinte sistema de equações na região do espaço-tempo $\mathbb{R}^3 \times (0, \infty)$:

$$\left\{\begin{array}{rcl} \mathbf{u_t} + (\mathbf{u} \cdot \nabla)\mathbf{u} - \mathbf{\Delta u} + \nabla \mathbf{p} & = & \operatorname{rot} \mathbf{w} + \mathbf{f}, \\ \operatorname{div} \mathbf{u} & = & 0, \\ \mathbf{w}_t + (\mathbf{u} \cdot \nabla)\mathbf{w} - \nabla(\operatorname{div} \mathbf{w}) + \mathbf{w} & = & \Delta \mathbf{w} + \operatorname{rot} \mathbf{u} + \mathbf{g}, \\ \mathbf{u}(\mathbf{x}, \mathbf{0}) & = & \mathbf{u_0}(\mathbf{x}), \\ \mathbf{w}(\mathbf{x}, \mathbf{0}) & = & \mathbf{w_0}(\mathbf{x}), \end{array}\right. \tag{1}$$

Além disso, vamos supor

$$\lim_{|\mathbf{x}| \to \infty} \big(\mathbf{u}(\mathbf{x}, \mathbf{t}),\, \mathbf{w}(\mathbf{x}, \mathbf{t})\big) = (\mathbf{0}, \mathbf{0}), \quad \forall\, \mathbf{t} \in (\mathbf{0}, \infty). \tag{2}$$

Este sistema descreve o movimento de um fluido micropolar, viscoso, incompressível com densidade constante. As funções $\mathbf{u}(\mathbf{x}, \mathbf{t}) \in \mathbb{R}^\mathbf{3}$, $p(\mathbf{x}, \mathbf{t}) \in \mathbb{R}$ e $\mathbf{w}(\mathbf{x}, \mathbf{t}) \in \mathbb{R}^\mathbf{3}$ são as incógnitas e representam, respectivamente, a velocidade linear, a pressão e a velocidade angular de rotação das partículas do fluido em um ponto $\mathbf{x} \in \mathbb{R}^\mathbf{3}$ no tempo $t \in [0, \infty)$. A primeira equação corresponde a lei de conservação do momento linear, a segunda equação representa a incompressibilidade do fluido e a terceira equação é a lei de conservação do momento angular do fluido. Por sua vez, $\mathbf{f}$ e $\mathbf{g}$ são forças externas conhecidas do momento linear e angular das partículas do fluido, respectivamente. Nosso objetivo é "estender" os resultados de Schonbek [1] para o caso não-Newtoniano, através das equações 3D dos fluidos micropolares (1). As principais ferramentas que utilizaremos são a transformada de Fourier e o método desenvolvido por Schonbek que consiste em dividir a integração no espaço em dois domínios depedentes do tempo ("Fourier splitting method"). Aqui a transformada de Fourier de φ será denotada por

$$\widehat{\varphi}(\xi) = \int_{\mathbb{R}^3} e^{-i\,\xi \cdot \mathbf{x}} \varphi(\mathbf{x})\, \mathbf{dx},$$

onde $i = \sqrt{-1}$ é a unidade imaginária e $\xi \cdot \mathbf{x}$ denota o produto escalar usual em $\mathbb{R}^3$.

2 Resultados Principais

Os principais resultados obtidos foram

Teorema 2.1. *Suponha que* $(\mathbf{u},\mathbf{p},\mathbf{w})$ *é uma solução suave do problema* (1) *com* $\mathbf{f}=\mathbf{g}=\mathbf{0}$. *Se* $\mathbf{u_0},\mathbf{w_0}\in\mathbf{L^1}(\mathbb{R}^\mathbf{3})\cap\mathbf{L^2}(\mathbb{R}^\mathbf{3})$, *com* $\operatorname{div}\mathbf{u_0}=0$, *então, para todo* $t\geq 0$, *vale que*

$$\|\mathbf{u}(\cdot,\mathbf{t})\|_\mathbf{2}^\mathbf{2}+\|\mathbf{w}(\cdot,\mathbf{t})\|_\mathbf{2}^\mathbf{2}\leq\mathbf{C}(\mathbf{t}+\mathbf{1})^{-\mathbf{1/2}}, \tag{3}$$

onde $C\in\mathbb{R}^+$ *é uma constante que depende somente das normas de* $\mathbf{u_0}$ *e* $\mathbf{w_0}$ *em* $\mathbf{L^1}$ *e* $\mathbf{L^2}$.

Para provar o Teorema (2.1), usamos a seguinte Proposição

Proposição 2.1. *Sob as mesmas hipóteses no Teorema 2.1, temos que se* $\mathcal{K}\subset\mathbb{R}^3$ *é um conjunto compacto, então, para todos* $t\geq 0$ *e* $\xi\in\mathcal{K}$, *com* $\xi\neq 0$, *vale que*

$$|\widehat{\mathbf{u}}(\xi,t)|+|\widehat{\mathbf{w}}(\xi,t)|\leq C|\xi|^{-1}, \tag{4}$$

onde $C\in\mathbb{R}^+$ *é uma constante que depende apenas das normas de* $\mathbf{u_0}$ *e* $\mathbf{w_0}$ *em* $\mathbf{L^1}$ *e* $\mathbf{L^2}$.

Assumindo que as forças $\mathbf{f},\mathbf{g}$ satisfazem as condições

$$\mathbf{f},\,\mathbf{g}\in\mathbf{L^1}(\mathbf{0},\infty;\mathbf{L^2}(\mathbb{R}^\mathbf{3})),\ \operatorname{div}\mathbf{f}(\mathbf{t})=\mathbf{0},\,\forall\,\mathbf{t}\geq\mathbf{0}, \tag{5}$$

$$\|\mathbf{f}(\cdot,\mathbf{t})\|_\mathbf{2}+\|\mathbf{g}(\cdot,\mathbf{t})\|_\mathbf{2}\leq\mathbf{K_1}(\mathbf{t}+\mathbf{1})^{-\mathbf{3/2}},\,\forall\,\mathbf{t}\geq\mathbf{0} \tag{6}$$

$$|\widehat{\mathbf{f}}(\xi,t)|+|\widehat{\mathbf{g}}(\xi,t)|\leq K_2|\xi|,\,\forall\,t\geq 0\ \text{ e }\ \xi\in\mathbb{R}^3, \tag{7}$$

onde K_1 e K_2 são constantes positivas, obtemos o seguinte

Teorema 2.2. *Sejam* $(\mathbf{u},\mathbf{p},\mathbf{w})$ *solução suave de* (1) *e* $\mathbf{f},\,\mathbf{g}$ *satisfazendo* (5)-(7). *Se* $\mathbf{u_0},\mathbf{w_0}\in\mathbf{L^1}(\mathbb{R}^\mathbf{3})\cap\mathbf{L^2}(\mathbb{R}^\mathbf{3})$, *com* $\operatorname{div}\mathbf{u_0}=0$, *então*

$$\|\mathbf{u}(\cdot,\mathbf{t})\|_\mathbf{2}^\mathbf{2}+\|\mathbf{w}(\cdot,\mathbf{t})\|_\mathbf{2}^\mathbf{2}\leq\mathbf{C}(\mathbf{t}+\mathbf{1})^{-\mathbf{1/2}}, \tag{8}$$

onde C *depende das normas de* $\mathbf{u_0}$ *e* $\mathbf{w_0}$ *em* $\mathbf{L^1}$ *e* $\mathbf{L^2}$, *das normas de* $\mathbf{f}$ *e* $\mathbf{g}$ *em* $\mathbf{L^1}(\mathbf{0},\infty;\mathbf{L^2}(\mathbb{R}^\mathbf{3}))$ *e de* K_1 *e* K_2.

Corolário 2.1. *Suponha que* $(\mathbf{u},\mathbf{p},\mathbf{w})$ *é uma solução suave de* (1) *e que os dados* $\mathbf{u_0}$, $\mathbf{w_0}$, $\mathbf{f}$ *e* $\mathbf{g}$ *satisfazem as hipóteses do Teorema 2.2. Se* $\|\mathbf{u}(\cdot,\mathbf{t})\|_\mathbf{1}+\|\mathbf{w}(\cdot,\mathbf{t})\|_\mathbf{1}\leq\mathbf{K_3}$, *então*

$$\|\mathbf{u}(\cdot,\mathbf{t})\|_\mathbf{2}^\mathbf{2}+\|\mathbf{w}(\cdot,\mathbf{t})\|_\mathbf{2}^\mathbf{2}\leq\mathbf{C}(\mathbf{t}+\mathbf{1})^{-\mathbf{1/2}}, \tag{9}$$

onde C *depende das normas de* $\mathbf{u_0}$ *e* $\mathbf{w_0}$ *em* $\mathbf{L^1}$ *e* $\mathbf{L^2}$, *das normas de* $\mathbf{f}$ *e* $\mathbf{g}$ *em* $\mathbf{L^1}(\mathbf{0},\infty;\mathbf{L^2}(\mathbb{R}^\mathbf{3}))$ *e das constantes* K_1, K_2 *e* K_3.

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A GENERAL PERIODIC 1D-MODEL WITH NONLOCAL VELOCITY VIA MASS TRANSPORT

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Abstract

In this work, we consider a generalized periodic 1D-model of that considered in [2]. In fact, we take a repulsive periodic interaction potential and we define an interaction energy functional on the set of periodic probability measures. The first result assert that this functional is finite in singular probability measures with respect to the Legesgue measure. Using the general metric theory to gradient flows of [1] and under suitable conditions on the interaction potential we get the existence of a minimizing curve to the interaction energy functional. We show that this curve is a distributional solution and converges asymptotically to the minimum of the interaction energy functional. Moreover, we show that in presence of viscosity the minimizing curve is a bounded variation function.

1 Introduction

We are concerned with the following general equation:

$$\partial_t \mu - \partial_x ((W' * \mu)\mu) = 0, \tag{1}$$

where $\mu : (0,\infty) \times \mathbb{R} \to \mathbb{R}$ is a $2\pi-$periodic function in the spatial variable and $W : \mathbb{R} \to (-\infty, +\infty]$ is a $2\pi-$periodic function satisfying the following assumptions:

(i) W is even, i.e. $W(-t) = W(t)$, lower semicontinuous, convex in the interval $(0, \pi)$ and $W(0) = +\infty$.

(ii) W is differentiable in $(0, \pi]$ with $W'(\pi) = 0$. Also, there is a constant $\alpha \in (0, 1)$ such that

$$\sup_{t\in(0,\pi)} t^{\alpha} W(t) < \infty. \tag{2}$$

(iii) If α^* denote the infimum of the $\alpha' s$ satisfying (2), then we assume that

$$\sup_{t\in(0,\pi)} |t^{1+\alpha^*} W'| < +\infty \text{ and } \inf_{t\in(0,\epsilon)} t^* W(t) > 0, \tag{3}$$

for some $\epsilon > 0$.

For more generality we consider the model

$$\partial_t \mu - \partial_x ((W' * \mu)\mu) = \nu \partial_{xx} \mu, \tag{4}$$

where $\nu \geqslant 0$ is the viscosity.

Let us recall that $\mathcal{P}(\mathbb{S})$ denote the set of probability measures in the circle $\mathbb{S} = \mathbb{R}/2\pi\mathbb{Z} \equiv (-\pi, \pi]$. We define the interaction functional:

$$\mathcal{W}[\mu] = \frac{1}{2} \int\int_{(-\pi,\pi]^2} W(x-y)\ d(\mu \otimes \mu)(x,y), \tag{5}$$

for each $\mu \in \mathcal{P}(\mathbb{S})$ and the entropy functional

$$\mathcal{U}[\mu] = \int_{(-\pi,\pi]} \rho(x) \log(\rho(x))\ dx, \tag{6}$$

if $d\mu = \rho\ dx$ is absolutely continuous with respect to the Lebesgue measure and $\mathcal{U}[\mu] = +\infty$ in another case.

2 Main Results

With respect to the interaction functional $\mathcal{W}$ we have

Theorem 2.1. *Assume* **(i)-(iii)***. Then the functional* $\mathcal{W}$ *is finite in some singular measures that not given mass to sets with Hausdorf dimension less or equal than* α^**.*

Proof This result can be achieved by using tools of potential theory, see [3, pag. 111]. ■

Now consider the functional $\mathcal{E}[\mu] = \nu\mathcal{U}[\mu] + \mathcal{W}[\mu]$.

Theorem 2.2. *Assume* **(i)-(iii)***. Given* $\mu_0 \in \mathcal{P}(\mathbb{S})$*, there exist* $\mu_t : (0, \infty) \to \mathcal{P}(\mathbb{S})$ *a curve weakly continuous that is a gradient flow of the functional* $\mathcal{E}$*. Moreover:*

a) If $\nu > 0$*,* μ_t *is absolutely continuous with respect to the Lebesgue measure. If* $\nu = 0$*,* μ_t *not give mass to sets with Hausdorff dimension less or equal than* α^**.*

b) μ_t *converges weakly to* μ_0 *as* $t \to 0$ *and converges weakly to* $\bar{\mu}$ *as* $t \to \infty$*, where* $\bar{\mu}$ *is the minimum of the functional* $\mathcal{E}$*.*

c) The curve μ_t *is stable with respect to the parameter* ν *at* $\nu = 0$*.*

d) μ_t *is a distributional solution of the equation* (4)*. For* $\nu > 0$*,* $\mu_t \in L^1_{loc}((0, \infty); BV(\mathbb{S}, dx))$*.*

Proof After to show the lower semicontinuity, convexity, coercivity we can apply the general metric theory developed in [1, chapter 4], it follows the existence result and the item *b)*. The item *a)* is a consequence of Theorem 2.1. The item *c)* follows by show a $\Gamma-$ convergence result to the functional $\mathcal{E}$ with respect to the parameter ν at 0, this result is some technical. Finally, the item *d)* follows by using the sub-differential calculus in the space $\mathcal{P}(\mathbb{S})$ induced by the Wasserstein metric, then we use the equation and the assumptions **(i)-(iii)** to show the BV-regularity asserted. ■

Taking respectively $W = -\log\left|\sin\left(\frac{x}{2}\right)\right|$ and $W = \left|\csc\left(\frac{x}{2}\right)\right|^s$, where $s \in (0, 1)$ we get distributional solutions of the equations

$$\partial_t\mu + \partial_x(H(\mu)\mu) = \nu\partial_{xx}\mu$$
$$\partial_t\mu - \partial_x\left(\left(s\left|\csc\left(\frac{x}{2}\right)\right|^s \cot\left(\frac{x}{2}\right) * \mu\right)\mu\right) = \nu\partial_{xx}\mu,$$

Where H denotes the periodic Hilbert transform.

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SISTEMAS ACOPLADOS COM RETARDAMENTO

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Resumo

Consideramos problemas lineares de valor inicial-fronteira que são acoplados como é o caso da termoelasticidade de segunda ordem. A dificuldade é que há um termo de atraso dado em uma parte do sistema acoplado, e demonstraremos que o amortecimento inerente esperado não vai impedir de que o sistema seja não estável; De fato, mostraremos que o sistema em estudo é mal colocado: uma sequência de dados iniciais que permanecem limitados podem levar a explodir as soluções (em qualquer tempo fixado).

1 Introdução

Para resultados de boa colocação para equações de onda com termos de atraso (no interior), há uma série de trabalhos como o de Nicaise e co-autores Ammari, Fridman, Pignotti, e Valein [1], em [4] também com resultados de instabilidade. Para trabalhos com termos de atraso nas condições de contorno, consulte as referências no trabalho [3].

Para o sistema de equações da onda do tipo Timoshenko com termos de atraso do tipo

$$\rho_1\partial_t^2\phi(t,\,x) - K\partial_x(\partial_x\phi + \psi)(t,\,x) = 0 \tag{1}$$

$$\rho_2\partial_t^2\psi(t,\,x) - b\partial_x^2\psi(t,\,x) + K(\partial_x\phi + \psi) + \mu_1\partial_t\psi(t,\,x) + \mu_2\partial_t\psi(t-\tau,\,x) = 0; \tag{2}$$

a boa colocação (sob certas condições em nos coeficientes μ_1 e μ_2) foi investigada por Said-Houari & Laskri [5] e estendida para um termo de atraso variando no tempo; substituindo $\partial_t\psi(t-\tau,\,x)$ por $\partial_t\psi(t-\tau(t),\,x)$ no trabalho de Kirane, Said-Houari & Anwar [3].

Nesta ocasião, consideramos os sistemas acoplados de diferentes tipos. Um primeiro exemplo típico é o acoplamento originado na termoelasticidade. Em uma dimensão, temos o sistema parabólico hiperbólico dado por,

$$\partial_t^2u(t,\,x) - a\partial_x^2u(t-\tau,\,x) + b\partial_x\theta(t,\,x) = 0; \tag{3}$$

$$\partial_t\theta(t,\,x) - d\,\partial_x^2\theta(t,\,x) + b\partial_{tx}^2u(t,\,x) = 0; \tag{4}$$

onde u descreve o deslocamento, e θ é a diferença da temperatura, e onde $t \geq 0$, $x \in\,]0,\,L[\, \subset \mathbb{R}$ com $L > 0$.

Para completar o problema de valor inicial-fronteira a resolver, consideramos as condições de contorno

$$u(t,\,x) = \partial_x\theta(t,\,x) = 0 \quad \text{para} \quad t \geq 0 \quad \text{e} \quad x \in \{0,\,L\} \tag{5}$$

e condições iniciais para $\partial_tu(0,\,\cdot)$, $u(s,\,\cdot)$ para $-\tau \leq s \leq 0$, e para $\theta(0,\,\cdot)$. O amortecimento através de condução de calor essencialmente dada na (4) é para termoelasticidade clássica; para $\tau = 0$; o suficiente forte para constituir um sistema exponencialmente estável (funções θ de módulo constante devido a condição de contorno), assim, impactando fortemente na parte oscilante da equação de onda (pura ou livre),

$$\partial_t^2u(t,\,x) - a\partial_x^2u(t,\,x) = 0,$$

veja Racke, ou Jiang & Racke em [2] para pesquisas extensivas.

Demonstraremos que o sistema com atraso (3)-(4) não está bem colocado e instável, isto é, o amortecimento através da condução de calor acaba por não ser suficientemente forte; a parte do sistema instável

$$\partial_t u(t,\cdot) - a\partial_x^2 u(t-\tau,\cdot) = 0,$$

predominará.

Então, ainda o mais esperado, o mesmo irá acontecer para o sistema, onde um atraso é dado na equação para a temperatura, isto é, para

$$\partial_t^2 u(t,x) - a\partial_x^2 u(t,x) + b\partial_x \theta(t,x) = 0; \tag{6}$$

$$\partial_t \theta(t,x) - d\,\partial_x^2 \theta(t-\tau,x) + b\partial_{tx}^2 u(t,x) = 0; \tag{7}$$

2 Resultados Principais

Consideramos o sistema termoelástico com atraso dado em (3), (4) e o sistema relacionado (6), (7). Os parâmetros a, b, d que aparecem são constantes positivas, e $\tau > 0$ é o parâmetro de relaxamento (em aplicações muitas vezes relativamente pequena)

Ambos sistemas são complementados com a condição de contorno (5) e com as condições iniciais,

$$u(s,\cdot) = u^0(s), \quad (-\tau \le s \le 0), \quad \partial_t u(0,\cdot) = u^1, \quad \theta(0,\cdot) = \theta^0 \tag{1}$$

$$u(0,\cdot) = u^0, \quad \partial_t u(0,\cdot) = u^1, \quad \theta(s,0) = \theta^0(s); \quad (-\tau \le s \le 0), \tag{2}$$

respectivamente.

Estes sistemas enunciados anteriormente são mostrados ser não bem-colocados, ou seja, provamos o resultado principal,

Teorema 2.1. *Sejam*

(i) *O problema de valor inicial-fronteira com atraso* (3), (4), (5), (1) *não está bem colocado. Existe uma sequência* $((u_j,\theta_j))_j$ *de soluções com* $\|u_j(t,\cdot)\|$ *norma em* L^2 *tendendo ao infinito (quando* $j \to \infty$*) para qualquer* $t > 0$ *fixado, enquanto que as normas para os dados iniciais*

$$\sup_{-\tau \le s \le 0} \|(u_j^0(s), u_j^1, \theta_j^0)\| \quad \textit{permanecem limitadas.}$$

(ii) *A correspondente afirmação sobre a não boa colocação também é válida para o problema de valor inicial-fronteira, com atraso* (6), (7), (5), (2).

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FUNCIONAIS QUE ASSUMEM A NORMA E REFLEXIVIDADE

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Abstract

É uma consequência simples do Teorema de Extensão de Hahn-Banach que se X é um espaço reflexivo, então todo funcional $x^* \in X^*$ atinge a norma. A respeito da recíproca deste fato, veremos o Teorema de James: se X é um espaço de Banach tal que todo funcional $x^* \in X^*$ atinge a norma, então X é reflexivo. Veremos ainda um importante contra-exemplo, também devido a R. C. James, que mostra que a completude é essencial no Teorema de James. Contudo, temos o seguinte resultado: se X é um espaço normado tal que todo elemento de X^* atinge a norma, então o completamento de X é reflexivo. Finalmente, veremos como aplicação do Teorema de James uma outra caracterização de reflexividade para espaços de Banach.

1 Introdução

Seja X um espaço normado. Para cada $x \in X$, definimos um funcional linear $\tilde{x}$ sobre X^* por $\tilde{x}(x^*) = x^*(x) \quad (x^* \in X^*)$. Decorre do Teorema de Extensão de Hahn-Banach que $\|\tilde{x}\| = \|x\|$ para todo $x \in X$. Assim, a aplicação $x \in X \mapsto \tilde{x} \in X^{**}$ é um isomorfismo isométrico de X sobre um subespaço $\tilde{X}$ de X^{**}. O espaço X é dito **reflexivo** quando esta aplicação é sobrejetiva. Uma propriedade básica dos espaços reflexivos é a seguinte:

Proposição 1.1. *Se X é um espaço reflexivo, então todo elemento de X^* atinge a norma.*

Prova: Seja $x^* \in X^*$. Segue do Teorema de Hahn-Banach que existe um elemento $x^{**} \in B_{X^{**}}$ tal que $|x^{**}x^*| = \|x^*\|$. Como X é reflexivo, existe $x \in X$ tal que $x^{**} = \tilde{x}$. Logo, $\|x^*\| = |x^{**}x^*| = |\tilde{x}(x^*)| = |x^*x|$. ■

O Teorema de James consiste na recíproca deste fato para espaços de Banach: se X é um espaço de Banach tal que todo funcional $x^* \in X^*$ atinge a norma, então X é reflexivo. Esse é um resultado profundo que levou um longo tempo para ser estabelecido. Apresentaremos aqui a demonstração dada por James [3] em 1972. Apresentaremos ainda o contra-exemplo de James [1]: um espaço normado não completo X (logo, não reflexivo) tal que todo $x^* \in X^*$ atinge a norma. Com este exemplo James mostrou que a hipótese da completude do espaço é essencial para a validade do seu teorema.

2 Resultados Principais

O Teorema de James é estabelecido primeiro para o caso em que X é um espaço de Banach real, através das equivalências abaixo.

Teorema 2.1. *Seja X um espaço de Banach real. As seguintes afirmações são equivalentes:*

1. *X não é reflexivo.*
2. *Se $\theta \in (0,1)$ então existem um subespaço fechado M de X e uma sequência (x_n^*) em B_{X^*} tais que $d(M^\perp, co(\{x_n^* : n \in \mathbb{N}\})) \geq \theta$ e $\lim_{n\to\infty} x_n^* x = 0$ para todo $x \in M$.*

3. *Se $\theta \in (0,1)$ e (β_n) é uma sequência de reais positivos com $\Sigma_{n=1}^{\infty}\beta_n = 1$, então existem $\alpha \in [\theta, 2]$ e (y_n^*) sequência em B_{X^*} tais que, para cada $w^* \in L(y_n^*) := \{x^* \in X^* : x^*x \leq \limsup_{n\to\infty} y_n^*x \ \forall x \in X\}$, tem-se:*
   - $\left\|\Sigma_{j=1}^{\infty}\beta_j(y_j^* - w^*)\right\| < \alpha$;
   - $\left\|\Sigma_{j=1}^{n}\beta_j(y_j^* - w^*)\right\| < \alpha\left(1 - \theta\Sigma_{j=n+1}^{\infty}\beta_j\right)$ *para cada n.*
4. *Existe $z^* \in X^*$ que não atinge a norma.*

Teorema 2.2. *(Teorema de James) Seja X um espaço de Banach. Se todo elemento de X^* atinge a norma, então X é reflexivo.*

Prova: O caso real foi considerado no teorema anterior. Suponha que X seja um espaço de Banach complexo e seja X_r o espaço de Banach real obtido a partir de X restringindo a operação de multiplicação por escalar a $\mathbb{R} \times X$. Tome $u^* \in (X_r)^*$ e seja $x^* \in X^*$ tal que $Rex^* = u^*$. Como x^* atinge a norma, então u^* atinge a norma. Segue então do teorema anterior que X_r é reflexivo, donde X é reflexivo. ■

A completude é essencial no Teorema de James, entretanto temos o seguinte resultado:

Corolário 2.1. *Seja X um espaço normado tal que todo elemento de X^* atinge a norma. Então o completamento de X é reflexivo.*

Prova: Sejam Y o completamento de X e $y^* \in Y^*$. Como $y^*|_X \in X^*$, temos que existe $x \in B_X$ tal que $\|y^*|_X\| = |y^*|_X(x)| = |y^*(x)|$. Mas como X é denso em Y, $\|y^*|_X\| = \|y^*\|$. Pelo Teorema de James, concluímos que Y é reflexivo. ■

Vejamos o contra-exemplo de James: um espaço normado incompleto X tal que todo elemento de X^* atinge a norma. Seja $B := (\bigoplus_n \mathbb{R}^n)_2$, onde os espaços $\mathbb{R}^n$ estão todos com a norma do sup. Então, os elementos de B são do tipo

$$x = (x_1^{(1)}, x_1^{(2)}, x_2^{(2)}, x_1^{(3)}, x_2^{(3)}, x_3^{(3)}, \ldots).$$

O contra-exemplo de James é o espaço X gerado por todos os elementos de B tais que

$$|x_1^{(n)}| = |x_2^{(n)}| = \ldots = |x_n^{(n)}|, \quad \text{para todo} \quad n \geq 1.$$

Finalmente, como aplicação do Teorema de James veremos a seguinte caracterização de reflexividade:

Teorema 2.3. *Um espaço de Banach X é reflexivo se, e somente se, a seguinte propriedade se verifica: se A e B são subconjuntos convexos, fechados e não-vazios de X, com A limitado, tais que $d(A, B) = 0$, então $A \cap B \neq \emptyset$.*

Prova: Suponha X reflexivo e tome (a_n) e (b_n) sequências em A e B, respectivamente, tais que $\|a_n - b_n\| \to 0$. Existem subsequências (a_{n_k}) e (b_{n_k}) que convergem fracamente para $a \in A \cap B$. Provemos agora a recíproca. Seja $x^* \in X^*\backslash\{0\}$ e consideremos os conjuntos $A = B_X$ e $B = \{x \in X; x^*x = \|x^*\|\}$, que são convexos, fechados e não-vazios, sendo A limitado, e $d(A, B) = 0$. Então, $A \cap B \neq \emptyset$, o que significa que x^* assume a norma. Pelo Teorema de James, concluímos que X é reflexivo. ■

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STRICTLY POSITIVE DEFINITE KERNELS ON $S^1 \times S^M$ $(M \geq 2)$

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Abstract

We present a necessary and sufficient condition for the strict positive definiteness of an isotropic and positive definite kernel on the cartesian product of a circle and a higher dimensional sphere.

1 Introduction

The theory of positive definite and strictly positive definite kernels on manifolds and groups cannot be separated from the seminal paper of I. J. Schoenberg ([7]), where the following characterization for the continuous and isotropic positive definite functions on the unit sphere S^m in $\mathbb{R}^{m+1}$ was obtained: a real and continuous kernel $(x, y) \in S^m \times S^m \to f(x \cdot y)$ is positive definite if, and only if, f has a series representation in the form

$$f(t) = \sum_{k=0}^{\infty} a_k^m P_k^m(t), \quad t \in [-1, 1],$$

where $a_k^m \geq 0$, for all k, P_k^m denotes the usual Gegenbauer polynomial of degree k attached to the rational number $(m-1)/2$ and $\sum_k a_k^m P_k^m(1) < \infty$. Recall that the positive definiteness of a general real kernel $(x, y) \in X^2 \to K(x, y)$ on a nonempty set X, demands that $K(x, y) = K(y, x)$, $x, y \in X$, and

$$\sum_{\mu,\nu=1}^{n} c_\mu c_\nu K(x_\mu, x_\nu) \geq 0,$$

whenever n is a positive integer, $x_1, x_2, \ldots, x_n$ are n distinct points on X and $c_1, c_2, \ldots, c_n$ are real scalars. The strict positive definiteness requires add that the previous inequalities be strict whenever at least one c_μ is nonzero.

Some fifty years later, the very same positive definite functions were found useful for solving scattered data interpolation problems on spheres. But that demanded *strictly* positive definite functions, and thus a characterization of these functions was needed at the start. The strict positive definiteness on spheres was an issue for some time until Schoenberg's result was complemented by a result of Debao Chen et all in 2003 ([2]) and by Menegatto et all ([6]): the kernel $(x, y) \in S^m \times S^m \to f(x \cdot y)$ is strictly positive definite if, and only if,

- $(m = 1)$ the set $\{k \in \mathbb{Z} : a_{|k|}^1 > 0\}$ intersects each full arithmetic progression in $\mathbb{Z}$.
- $(m \geq 2)$ the set $\{k : a_k^m > 0\}$ contains infinitely many even and infinitely many odd integers.

In the past two years, the attention shifted all the way to positive definiteness on a product of spaces, the main motivation coming from problems involving random fields on spaces across time. In [1], the authors investigated positive definite kernels on a product of the form $G \times S^m$, in which G is an arbitrary locally compact group, keeping both, the group structure of G and the isotropy of S^m in the setting. Let us denote by e the neutral element of G, $*$ the operation of the group G and by u^{-1} the inverse of $u \in G$ with respect to $*$. The main contribution in the paper states that a continuous kernel of the form $((u, x), (v, y)) \in (G \times S^m)^2 \to f(u^{-1} * v, x \cdot y)$ is positive definite if, and only if, the generating function f has a representation in the form

$$f(u, t) = \sum_{n=0}^{\infty} f_n^m(u) P_n^m(t), \quad (u, t) \in G \times [-1, 1],$$

in which $\{f_n^m\}$ is a sequence of continuous functions on G for which $\sum f_n^m(e) < \infty$, with uniform convergence of the series for $(u,t) \in G \times [-1,1]$. As a matter of fact, the functions f_n^m are positive definite on G in the sense that the kernel $(u,v) \in G^2 \to f_n^m(u^{-1} * v)$ is positive definite as previously defined.

Simultaneously, positive definiteness and strict positive definiteness on a product of spheres was investigated in [3, 4, 5]. A real, continuous and isotropic kernel $((x,z),(y,w)) \in (S^m \times S^M)^2 \to f(x \cdot y, z \cdot w)$ is positive definite if, and only if, the generating function f has a double series representation in the form

$$f(t,s) = \sum_{k,l=0}^{\infty} a_{k,l}^{m,M} P_k^m(t) P_l^M(s), \quad t,s \in [-1,1],$$

in which $a_{k,l}^{m,M} \geq 0$, $k,l \in \mathbb{Z}_+$ and $\sum_{k,l=0}^{\infty} a_{k,l}^{m,M} P_k^m(1) P_l^M(1) < \infty$.

One of the main theorems in [3] reveals that, in the case in which $m, M \geq 2$, a positive definite kernel as above is strictly positive definite if, and only if, the set $J_f := \{(k,l) : a_{k,l}^{m,M} > 0\}$ contains sequences from each one of the sets $2\mathbb{Z}_+ \times 2\mathbb{Z}_+$, $2\mathbb{Z}_+ \times (2\mathbb{Z}_+ + 1)$, $(2\mathbb{Z}_+ + 1) \times 2\mathbb{Z}_+$, and $(2\mathbb{Z}_+ + 1) \times (2\mathbb{Z}_+ + 1)$, all of them having both component sequences unbounded. In the case $m = M = 1$, the condition becomes this one ([5]): the set $\{(k,l) : a_{|k|,|l|}^{1,1} > 0\}$ intersects all the translations of each subgroup of $\mathbb{Z}^2$ having the form $\{(pa, qb) : q, p \in \mathbb{Z}\}$, $a, b > 0$.

2 Main Result

Our main result that complements those mentioned in the introduction is:

Theorem 2.1. *The isotropic and positive definite kernel* $((x,z),(y,w)) \in (S^1 \times S^m)^2 \to f(x \cdot y, z \cdot w)$ *is strictly positive definite if, and only if, for each* $\gamma \geq 0$, *the sets*

$$\{k \in \mathbb{Z} : \{l : (|k|, l) \in J_f\} \cap \{\gamma, \gamma+1, \ldots\} \cap (2\mathbb{Z}+1) \neq \emptyset\}$$

and

$$\{k \in \mathbb{Z} : \{l : (|k|, l) \in J_f\} \cap \{\gamma, \gamma+1, \ldots\} \cap 2\mathbb{Z}_+ \neq \emptyset\}$$

intersect every arithmetic progression in $\mathbb{Z}$.

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ÍNDICE DAUGAVETIANO POLINOMIAL DE UM ESPAÇO DE BANACH

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Abstract

Neste trabalho introduzimos o índice daugavetiano polinomial de um espaço de Banach, generalizando para polinômios o índice definido para operadores por M. Martín em 2003. Também apresentamos algumas propriedades do índice introduzido.

1 Introdução

Dado um espaço de Banach X sobre $\mathbb{K} = \mathbb{R}$ ou $\mathbb{C}$, denotaremos por X^* o dual topológico de X, por $K(X)$ o espaço dos operadores lineares compactos em X, por $\mathcal{P}_K(X)$ o espaço dos polinômios compactos em X e, por S_X e S_{X^*} as esferas unitárias de X e X^*, respectivamente. Escreveremos $\Pi(X)$ para denotar o subconjunto de $X \times X^*$ dado por

$$\Pi(X) = \big\{(x, x^*) : \ x \in S_X, \ x^* \in S_{X^*}, \ x^*(x) = 1\big\}.$$

Dada uma função limitada $\Phi : S_X \to X$, sua *imagem numérica* é o conjunto

$$V(\Phi) = \big\{x^*(\Phi(x)) : \ (x, x^*) \in \Pi(X)\big\}$$

e seu *raio numérico* é o valor

$$v(\Phi) = \sup\big\{|\lambda| : \ \lambda \in V(\Phi)\big\}.$$

Em 2003 M. Martín [5] definiu o conceito de *índice de Daugavet* de um espaço de Banach X de dimensão infinita da seguinte forma

$$\mathrm{daug}(X) = \max\{m \geq 0 : \|Id + T\| \geq 1 + m\|T\| \ \text{ para todo } \ T \in K(X)\}.$$

Observemos que $0 \leq \mathrm{daug}(X) \leq 1$ e que o valor extremo $\mathrm{daug}(X) = 1$ indica que o espaço X tem a propriedade de Daugavet. Lembremos que um espaço de Banach X tem a *propriedade de Daugavet* [4] se

$$\|Id + T\| = 1 + \|T\|$$

para todo operador linear contínuo de posto um T em X. Escrevendo $\omega(T) = \sup \mathrm{Re} V(T)$, M. Martín provou que

$$\mathrm{daug}(X) = \inf\{\omega(T) : T \in K(X), \|T\| = 1\} \tag{1}$$

e obteve diversas propriedades sobre tal índice, entre as quais destacamos a seguinte propriedade de estabilidade. Dada uma família arbitrária $(X_\lambda)_{\lambda\in\Lambda}$ de espaços de Banach e denotando por $\big[\bigoplus_{\lambda\in\Lambda} X_\lambda\big]_{c_0}$ (resp. $\big[\bigoplus_{\lambda\in\Lambda} X_\lambda\big]_{\ell_1}$, $\big[\bigoplus_{\lambda\in\Lambda} X_\lambda\big]_{\ell_\infty}$) a soma c_0 (resp. soma ℓ_1, soma ℓ_∞) da família, tem-se

$$\mathrm{daug}\left(\left[\bigoplus_{\lambda\in\Lambda} X_\lambda\right]_{c_0}\right) = \mathrm{daug}\left(\left[\bigoplus_{\lambda\in\Lambda} X_\lambda\right]_{\ell_1}\right) = \mathrm{daug}\left(\left[\bigoplus_{\lambda\in\Lambda} X_\lambda\right]_{\ell_\infty}\right) = \inf\{\mathrm{daug}(X_\lambda) : \lambda \in \Lambda\}.$$

O objetivo deste trabalho é introduzir o índice daugavetiano polinomial de um espaço de Banach, generalizando o índice daugavetiano apresentado por M. Martín [5].

2 Resultados Principais

Motivados pela definição e pelos resultados obtidos por M. Martín [5], definimos o *índice daugavetiano polinomial* de um espaço de Banach X como o valor

$$\operatorname{daug}_p(X) = \inf \{\omega(P) : P \in \mathcal{P}_K(X), \|P\| = 1\},$$

onde $\omega(P) = \sup \operatorname{Re} V(P)$. Pela igualdade (1), temos que $\operatorname{daug}_p(X) \leq \operatorname{daug}(X)$.

Observemos que neste caso, o valor extremo $\operatorname{daug}_p(X) = 1$ indica que $\sup \operatorname{Re} V(P) = 1 = \|P\|$ para todo $P \in \mathcal{P}_K(X)$ com $\|P\| = 1$. De [2, Proposition 1.3], segue que P satisfaz a *equação de Daugavet* para todo $P \in \mathcal{P}_K(X)$ com $\|P\| = 1$, isto é,

$$\|Id + P\| = 1 + \|P\|.$$

Fazendo uso das ideias de M. Martín [5], é possível provar o seguinte resultado.

Proposição 2.1. *Seja X um espaço de Banach. Então*

$$\begin{aligned}\operatorname{daug}_p(X) &= \sup \{m : \omega(P) \geq m\|P\| \ \text{ para todo } \ P \in \mathcal{P}_K(X)\} \\ &\leq \sup \{m : \|Id + P\| \geq 1 + m\|P\| \ \text{ para todo } \ P \in \mathcal{P}_K(X)\}.\end{aligned}$$

Com uma demonstração similar as demonstrações de [1, Proposition 2.8] e [6, Proposition 2.3], é possível verificar a validade da seguinte proposição.

Proposição 2.2. *Seja $(X_\lambda)_{\lambda \in \Lambda}$ uma família de espaços de Banach. Então*

(i) $\operatorname{daug}_p\left(\left[\bigoplus_{\lambda \in \Lambda} X_\lambda\right]_{c_0}\right) \leq \inf\left\{\operatorname{daug}_p(X_\lambda) : \lambda \in \Lambda\right\}$;

(ii) $\operatorname{daug}_p\left(\left[\bigoplus_{\lambda \in \Lambda} X_\lambda\right]_{\ell_1}\right) \leq \inf\left\{\operatorname{daug}_p(X_\lambda) : \lambda \in \Lambda\right\}$;

(iii) $\operatorname{daug}_p\left(\left[\bigoplus_{\lambda \in \Lambda} X_\lambda\right]_{\ell_\infty}\right) \leq \inf\left\{\operatorname{daug}_p(X_\lambda) : \lambda \in \Lambda\right\}$.

Além disso, adaptando a demonstração de [3, Proposition 2.3], provamos a afirmação a seguir.

Proposição 2.3. *Seja $(X_\lambda)_{\lambda \in \Lambda}$ uma família de espaços de Banach complexos. Então*

(i) $\operatorname{daug}_p\left(\left[\bigoplus_{\lambda \in \Lambda} X_\lambda\right]_{c_0}\right) \geq \inf\left\{\operatorname{daug}_p(X_\lambda) : \lambda \in \Lambda\right\}$;

(ii) $\operatorname{daug}_p\left(\left[\bigoplus_{\lambda \in \Lambda} X_\lambda\right]_{\ell_\infty}\right) \geq \inf\left\{\operatorname{daug}_p(X_\lambda) : \lambda \in \Lambda\right\}$.

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TAXAS ÓTIMAS PARA UMA EQUAÇÃO TIPO PLACAS COM INÉRCIA ROTACIONAL GENERALIZADA E DISSIPAÇÃO FRACIONÁRIA

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Abstract

Nesse trabalho mostramos existência e unicidade de solução e taxas de decaimento para a norma L^2 da solução para uma equação do tipo placas com dissipação fracionária e um termo de inércia rotacional generalizada. Mostramos que as taxas de decaimento dependem das potências fracionárias dos operadores de Laplace envolvidos. Também mostramos que as taxas de decaimento são ótimas para algumas potências fracionárias usando uma expansão assintótica da solução e $n \geq 3$.

1 Introdução

Consideramos neste trabalho o seguinte problema de Cauchy para uma equação do tipo placas com um generalizado termo de inércia rotacional e uma dissipação fracinária em $\mathbb{R}^n$:

$$\begin{cases} u_{tt}(t,x) + (-\Delta)^{\delta} u_{tt}(t,x) + \alpha\Delta^2 u(t,x) - \Delta u(t,x) + (-\Delta)^{\theta} u_t(t,x) = 0, \\ u(0,x) = u_0(x), \\ u_t(0,x) = u_1(x) \end{cases} \tag{1}$$

com $(t,x) \in (0,\infty) \times \mathbb{R}^n$, $\alpha > 0$, $\delta \geq 0$ e $\theta \geq 0$ constantes.

A função $u = u(x,t)$ descreve o deslocamento transversal da placa enquanto o termo $(-\Delta)^{\theta} u_t(t,x)$, representa uma dissipação fracionária na placa. Quando $\delta = 1$ o termo $(-\Delta)^{\delta} u_{tt}(t,x)$ é conhecido na literatura como o termo de inércia rotacional e é devido a efeitos rotacionais no ponto (t,x) da placa.

Problemas do tipo (1) com $\delta = 0$ e $\delta = 1$ têm sido extensivamente estudados para os casos $\theta = 0$ e $\theta = 1$. Recentemente, vários autores estudaram equações de evolução com operador Laplacian fracionário $(-\Delta)^{\theta}$. No caso $\delta = \alpha = 0$ e $\theta \in [0,1]$ Ikehata-Natsume [4] obtiveram estimativas explicitas de decaimento para a norma da energia total e para a norma L^2 das soluções com base no método de energia no espaço de Fourier. Uma melhoria dos resultados de [4] foi dada por Charão-da Luz-Ikehata [1] através da introdução de um novo método de energia no espaço de Fourier. No caso $\delta = \alpha = 0$, podemos citar 3 importantes papers, Matsumura [6] ($\theta = 0$), Ponce [7] e Shibata [8] ($\theta = 1$), no qual eles derivam estimativas para a norma L^p-L^q da solução. Quando $\delta = 1$ podemos citar o recente paper de Charão-da Luz-Ikehata [2], no qual eles encontraram taxas quase ótimas para a Energia total e para a norma L^2 da solução. Para a equação de placas podemos ainda citar o recente trabalho de Ikehata-Soga [5] que obtiveram uma expansão assintótica e taxa ótima para a norma L^2 da solução no caso $\delta = 0$ e $\theta = 1$.

Observamos que nossos resultados melhoram resultados prévios citados nas referências deste trabalho. Também temos resultados de decaimento para $0 \leq \theta \leq 1/2$ e $n > 4\theta$, mas que não são mostrados neste resumo.

2 Resultados Principais

Usando Teoria de Semigrupo mostramos que existe única solução forte para o Problema (1). Usando um melhoramento do método da energia no espaço de Fourier apresentados pelos autores nas referências deste trabalho encontramos taxas de decaimento para a solução do problema (1). Por fim, usando uma expansão assintótica para a solução do problema (1) mostramos que as taxas encontradas são ótimas.

Teorema 2.1. *Seja* $n \geq 1$, $0 \leq \delta \leq 2$ *e* $0 \leq \theta \leq \dfrac{2+\delta}{2}$. *Se* $u_0 \in H^{4-\delta}(\mathbb{R}^n)$ *e* $u_1 \in H^2(\mathbb{R}^n)$ *então o problema de Cauchy* (1) *tem uma única solução* u *satisfazendo*

$$u \in C^2\left([0,\infty); H^{\delta}(\mathbb{R}^n)\right) \cap C^1\left([0,\infty); H^2(\mathbb{R}^n)\right) \cap C\left([0,\infty); H^{4-\delta}(\mathbb{R}^n)\right).$$

Teorema 2.2. *Seja* $n \geq 3$, $0 \leq \delta \leq 2$ *e* $\dfrac{1}{2} < \theta \leq \dfrac{2+\delta}{2}$. *Então as seguintes estimativas para norma* L^2 *da solução* $u(t,x)$ *do problema (1) são validas.*

a) Considerando $\delta < \theta$, $u_0 \in L^2(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$ *e* $u_1 \in L^2(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$ *tem-se*

$$\|u(t,x)\|^2 \leq Ce^{-\frac{\varepsilon}{10}t}\Big(\|u_0\|^2 + \|u_1\|^2\Big) + K\big(\|u_0\|_1^2 + \|u_1\|_1^2\big)t^{-\frac{n-2}{2\theta}}.$$

b) Considerando $\theta \leq \delta$, $u_0 \in H^{\frac{(\delta-\theta)(n-2)}{2\theta}}(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$ *e* $u_1 \in H^{\frac{(\delta-\theta)(n-2)}{2\theta}+\delta-2}(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)$ *tem-se*

$$\|u(t,x)\|^2 \leq Ct^{-\frac{n-2}{2\theta}}\left(\|u_0\|^2_{H^{\frac{(\delta-\theta)(n-2)}{2\theta}}} + \|u_1\|^2_{H^{\frac{(\delta-\theta)(n-2)}{2\theta}+\delta-2}}\right) + K\|u_0\|_1^2 t^{-\frac{n}{2\theta}} + K\|u_1\|_1^2 t^{-\frac{n-2}{2\theta}}.$$

Teorema 2.3. *Seja* $P_1 = \displaystyle\int_{\mathbb{R}^n} u_1(x)dx \neq 0$ *com* $1/2 < \theta < \min\left\{\frac{3}{2}, \delta + \frac{1}{2}\right\}$ *e* $0 < \delta < \theta$ *e ainda* $[u_0, u_1] \in \left(L^2(\mathbb{R}^n) \cap L^1(\mathbb{R}^n)\right) \times \left(L^2(\mathbb{R}^n) \cap L^1(\mathbb{R}^n) \cap L^{1,\gamma}(\mathbb{R}^n)\right)$. *Então existem* $C_1, C_2 > 0$ *e* $t_0 > 0$ *tal que para* $t \geq t_0$ *vale*

$$C_1|P_1|t^{-\frac{n-2}{4\theta}} \leq \|u(t,\cdot)\|_2 \leq C_2 t^{-\frac{n-2}{4\theta}},$$

onde $u(t,x)$ *é a única solução do problema* (1).

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EXISTENCE OF SOLUTIONS FOR THE WAVE EQUATION WITH INTEGRAL BOUNDARY CONDITION AND BOUNDARY SOURCE TERM

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Abstract

We study a wave equation with integral boundary condition and boundary source term by the use of Faedo - Galerkin procedure and the potential well method, we prove the existence of weak solutions.

1 Introduction

Let Ω be a bounded domain of $\mathbb{R}^n$, $n \geq 1$ with a smooth boundary $\Gamma = \Gamma_0 \bigcup \Gamma_1$, $\overline{\Gamma}_0 \bigcup \overline{\Gamma}_1 = \emptyset$, where Γ_0 , Γ_1 have positive measures. In this work, we are concerned with the following problem

$$\begin{aligned} u_{tt} - \Delta u + \alpha u_t = f(u) \quad \text{in } \Omega \times]0, \infty[, \\ u = 0 \quad \text{on } \Gamma_0 \times]0, \infty[, \\ \frac{\partial u}{\partial \nu} + \int_\Omega K(x,y) u(y,t) dy = \int_0^t g(t-\tau) h(\tau, u(\tau))\, d\tau \quad \text{on } \Gamma_1 \times]0, \infty[, \\ u(x,0) = u^0(x) \quad u_t(x,0) = u^1(x) \quad \text{in } \Omega \end{aligned} \tag{1}$$

where ν represents the unit outwend normal to Γ and f, K, g and h are functions satisfying some general properties; $\alpha \geq 0$.
Nonlocal problems with integral conditions have been actively studed by many authors [1,2]. Motivated by the above works, we are devoted to study problem (1).

2 Assumptions and main results

First, we define

$$V = \{u \in H^1(\Omega); u = 0 \quad \text{on} \quad \Gamma_0\} \quad , \|u\|_{p,\Gamma}^p = \int_\Gamma |u(x)|^p \, d\Gamma$$

For simplicity we denote $\|\cdot\|_{L^2(\Omega)}$ and $\|\cdot\|_{2,\Gamma}$ by $\|\cdot\|$ and $\|\cdot\|_\Gamma$
Now, we make the following assumptions
(A1) $f \in C(\mathbb{R})$ $f(s)s \geq 0$ and there exists $k_1 > 0$ such that

$$|f(s)| \leq k_1 |s|^p; \quad \text{if } 1 < p < \infty N = 2, 1 < p \leq \frac{n}{n-2} \quad \text{if }, n \geq 3$$

(A2) $k \in L^2(\Gamma \times \Omega)$
(A3) $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a continuous differentiable function verifying

$$g'(t) \leq -k_2 g(t) \quad , \quad \forall t > 0 \quad , \quad g(0) > 0 \quad , \quad 1 - u_1^2 \int_0^\infty g(\zeta)\, d\zeta \equiv L > 0$$

where $k_2 > 0$, $\|u\|_\Gamma \leq \mu_1 \|\nabla u\|, \forall u \in V$

(A4) $h(\tau, s)$ is measurable with τ and continuous with s, and if satisfies

$$|h(\tau, s) - s| \leq \sqrt{\frac{g(\tau)}{g(0)}} |s| \forall s \in \mathbb{R}, \tau \geq 0$$

(A5) Let us consider $U^0 \in V \cap H^2(\Omega)$ and $U^{'} \in V$ verifying the compability conditions

$$-\Delta U^0 + \alpha U^{'} = f(U^0) \quad \text{in} \quad \Omega, \quad U^0 = U^{'} = 0 \quad \text{on} \quad \Gamma_0, \quad \frac{\partial U^0}{\partial \nu} + \int_\Omega K(x,y) U^0(y) dy = 0, \quad \text{on} \quad \Gamma_1$$

(A6) $\lambda_0 = \left[\frac{L}{(p+1)k_1 B_0} \right]^{\frac{1}{p-1}}$; $B_0 = \sup\limits_{\substack{v \in V \\ v \neq 0}} \left(\frac{(1/p+1)\|v\|_{p+1}^{p+1}}{|\nabla v|^{p+1}} \right)$,

Now, we define

$J(U) = \frac{L}{2} \|\nabla U\|^2 - \frac{k_1}{p+1} \|U\|_{p+1}^{p+1}$, $d = \inf\limits_{\substack{v \in V \\ v \neq 0}} \{\sup\limits_{\lambda > 0} J(\lambda U)\}$

$$E(t) = \frac{1}{2}\|U_t\|^2 + \frac{1}{2}\|\nabla U\|^2 - \int_\Omega F(u)\, dx - \frac{1}{2}\left(\int_0^t g(\tau)\, d\tau\right) \|U\|_{\Gamma_1}^2 + \frac{1}{2}(g \diamond U) + \int_\Gamma \left(\int_\Omega K(x,y) U(y,t)\, dy \right) U(y,t)\, d\Gamma$$

where $F(s) = \int_0^s f(\zeta) d\zeta$, $(g \diamond U)(t) = \int_0^t g(t-\tau) \|h(\tau, U(\tau)) - U(t)\|_{\Gamma_1}^2 d\tau$

Theorem 2.1. *Let the assumptions **(A1)**- **(A6)**. If in addition, the initial data satisfy $\|\nabla u^0\| < \lambda_0$ and $E(0) < d$, then problem (1) admits a solution $U \in L^\infty(0,T;V)$, $U_t \in L^\infty(0,T;L^2(\Omega))$ y $U_{tt} \in L^2(0,T;L^2(\Omega))$*

Proof. We apply the Faedo - Galerkin approximation and the potential method.

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EXISTENCE AND ASYMPTOTIC BEHAVIOR FOR THE KELLER-SEGEL SYSTEM WITH SINGULAR DATA

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Abstract

This work considers the Keller-Segel system of parabolic-parabolic type in $\mathbb{R}^n$ for $n \geq 2$ with time fractional differential operator of order $\alpha \in (0,1)$. We prove existence results in a new framework and with initial data in $\mathcal{N}^{-\alpha b}_{r,\lambda,\infty} \times \dot{B}^0_{\infty,\infty}$. The kind of solution involves a family of opperators called Mittag-Leffler, which arises naturally in the abstract theory of fractional calculus.

1 Introduction

We study the large time behavior of bounded solutions to the Cauchy problem for the following system of partial differential equations in $\mathbb{R}^n$, $n \geq 2$ (see [2]):

$$\begin{cases} cD^\alpha_t u &= \nabla \cdot (\nabla u - u\nabla v), \quad x \in \mathbb{R}^n,\ t > 0, \\ cD^\alpha_t v &= \Delta v - \gamma v + \kappa u, \quad x \in \mathbb{R}^n,\ t > 0, \\ u(x,0) &= u_0, \quad v(x,0) = v_0, \quad x \in \mathbb{R}^n. \end{cases} \tag{1}$$

where $\alpha \in (0,1]$, cD^α_t is Caputo's fractional derivative, $n \geq 2$, $u(x,t)$ represents the density or population of a biological species, which could be a cell, a germ, or an insect and so on. While $v(x,t)$ represents an attractive resource of the species. The parameters $\gamma \geq 0$ and $\kappa \geq 0$ denote the decay and production rate of attractant, respectively.

For $1 \leq p < \infty$ and $0 \leq \lambda < n$, the local Morrey space $\mathcal{M}_{p,\lambda} = \mathcal{M}_{p,\lambda}(\mathbb{R}^n)$ is defined as

$$\mathcal{M}_{p,\lambda} = \left\{ f \in L^p_{loc}(\mathbb{R}^n) : \|f\|_{p,\lambda} < \infty \right\}, \quad \text{where} \quad \|f\|_{p,\lambda} = \sup_{x_0 \in \mathbb{R}^n, R>0} \left(R^{-\frac{\lambda}{p}} \|f\|_{L^p(B_R(x_0))} \right) \tag{2}$$

and $B_R(x_0) \subset \mathbb{R}^n$ is the open ball with center x_0 and radius R. The space $\mathcal{M}_{p,\lambda}$ endowed with $\|\cdot\|_{p,\lambda}$ is a Banach space. Recalling the notation $\mathcal{M}^s_{p,\lambda} = (-\Delta)^{-s/2}\mathcal{M}_{p,\lambda}$ for Sobolev-Morrey spaces, the inhomogeneous Besov-Morrey space $\mathcal{N}^s_{p,\lambda,q}$ is the following interpolation space

$$\left(\mathcal{M}^{s_1}_{p,\lambda}, \mathcal{M}^{s_2}_{p,\lambda} \right)_{\theta,q} = \mathcal{N}^s_{p,\lambda,q}, \tag{3}$$

where $\theta \in (0,1)$ and $s = (1-\theta)s_1 + \theta s_2$ with $s_1 \neq s_2$. In view of (3), the reiteration theorem implies that $\left(\mathcal{N}^{s_1}_{p,\lambda,q_1}, \mathcal{N}^{s_2}_{p,\lambda,q_2} \right)_{\theta,q} = \mathcal{N}^s_{p,\lambda,q}$, where $1 \leq q, q_1, q_2 \leq \infty$ with $q^{-1} = (1-\theta)q_1^{-1} + \theta q_2^{-1}$ and $\theta \in (0,1)$. Also, $\mathcal{N}^s_{\infty,\lambda,\infty} \equiv \dot{B}_{\infty,\infty}$

We introduce the Mittag-Leffler operators. To this end, let X be a Banach space and $-A : D(A) \subset X \to X$ be the infinitesimal generator of an analytic semigroup $\{T(t) : t \geq 0\}$. Then, for each $\alpha \in (0,1)$, we define the Mittag-Leffler families $\{E_\alpha(-t^\alpha A) : t \geq 0\}$ and $\{E_{\alpha,\alpha}(-t^\alpha A) : t \geq 0\}$ by

$$E_\alpha(-t^\alpha A) = \int_0^\infty \Phi_\alpha(s) T(st^\alpha) ds, \quad \text{and} \quad E_{\alpha,\alpha}(-t^\alpha A) = \int_0^\infty \alpha s \Phi_\alpha(s) T(st^\alpha) ds. \tag{4}$$

Definition 1.1. *In according to Duhamel's principle, we will work with the following integral formulation for (1):*

$$u(t) = E_\alpha(-t^\alpha G)u_0 - \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(-(t-s)^\alpha \nabla G)(u\nabla v)(s)ds \tag{5}$$

$$v(t) = E_\alpha(-t^\alpha \widetilde{G})v_0 + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(-(t-s)^\alpha \widetilde{G})u(s)ds \tag{6}$$

with $\widetilde{G}(t) = e^{-t\gamma}G(t)v_0$. *A pair* $[u,v]$ *satisfying (5)-(6) is called a mild solution for the Cauchy problem (1).*

2 Main Results

We will look for global-in-time solutions $[u(x,t), v(x,t)]$ in the scaling-invariant Kato class:

$$X_1 = \left\{ u \in BC\left((0,\infty); \mathcal{N}^{-\alpha b}_{r,\lambda,\infty}\right) : t^{\alpha b/2}u \in BC\left((0,\infty); (\mathcal{M}_{r,\lambda})\right) \text{ and } t^{\alpha a/2}u \in BC\left((0,\infty); (\mathcal{M}_{q,\lambda})\right) \right\}. \tag{1}$$

$$X_2 = \left\{ v \in BC\left((0,\infty); \dot{B}^0_{\infty,\infty}\right) : t^{\alpha/2}\nabla_x v \in BC\left((0,\infty); L^\infty\right) \right\}, \tag{2}$$

which are Banach space with norms, where $a = 2 - \frac{n-\lambda}{q}$ with $q \neq r$.

$$\|u\|_{X_1} = \sup_{0<t<T} \|u(\cdot,t)\|_{\mathcal{N}^{-\alpha b}_{r,\lambda,\infty}} + \sup_{0<t<T} t^{\alpha b/2}\|u(\cdot,t)\|_{r,\lambda} + \sup_{0<t<T} t^{\alpha a/2}\|u(\cdot,t)\|_{q,\lambda} + \tag{3}$$

$$\|v\|_{X_2} = \sup_{0<t<T} \|v(\cdot,t)\|_{\dot{B}^0_{\infty,\infty}} + \sup_{0<t<T} t^{\alpha/2}\|\nabla v(\cdot,t)\|_{L^\infty} \tag{4}$$

Theorem 2.1. *Assume that* $n \geq 2$, $0 \leq \lambda \leq n-2$, $p_0 = \frac{n-\lambda}{2}$ *and* $p_0 < r < 2p_0 < q < \infty$, *and suppose that the* $(u_0, v_0) \in \mathcal{N}^{-\alpha b}_{r,\lambda,\infty} \times \dot{B}^0_{\infty,\infty}$. *Let* τ *and* K *constants, and* $0 < \epsilon < \frac{1}{2K(1+2\tau)}$.

There exists $\delta_1 = \delta_1(\epsilon)$ *and* $\delta_2 = \delta_2(\epsilon)$ *such that system (5)-(6) has a global mild solution* $[u,v] \in X_1 \times X_2$, *provided that* $\|u_0\|_{\mathcal{N}^{-\alpha b}_{r,\lambda,\infty}} \leq \delta_1$ *and* $\|v_0\|_{\dot{B}^0_{\infty,\infty}} \leq \delta_2$. *The solution is the unique one satisfying* $\|u_0\|_{X_1} \leq 2\epsilon$ *and* $\|v_0\|_{X_2} \leq (1+2\tau)\epsilon$. *Moreover,* $[u(t), v(t)] \rightharpoonup [u_0, v_0]$ *in* $\mathcal{S}'(\mathbb{R}^n)$ *as* $t \to 0^+$.

Corollary 2.1. *Under the hypotheses of Theorem 2.1. Assume that* $u_0 \in \mathcal{N}^{-\alpha b}_{r,\lambda,\infty}$ *and* $v_0 \in \dot{B}^0_{\infty,\infty}$ *are homogeneous functions with degree* -2 *and* 0, *respectively. Then the solution* $[u,v]$ *obtained through Theorem 2.1 is self-similar, that is,*

$$u(x,t) = \sigma^2 u(\sigma x, \sigma^2 t) \text{ and } v(x,t) = v(\sigma x, \sigma^2 t)$$

for all $\sigma > 0$, $t > 0$ *and* $x \in \mathbb{R}^n$.

We also provide an asymptotic stability result for solutions of system (1). In addition to refining the time decays obtained in Theorem 2.1, next theorem shows that certain perturbations of the initial data vanish as $t \to \infty$.

Theorem 2.2. *Under the hypotheses of Theorem 2.1. Assume that* $[u,v]$ *and* $[\bar{u},\bar{v}]$ *are two solutions given by Theorem 2.1 corresponding to the initial data* $[u_0,v_0]$ *and* $[\bar{u}_0,\bar{v}_0]$, *respectively. We have that*

$$\lim_{t\to+\infty} \left(t^{\frac{\alpha b}{2}} \|G(t)(u_0-\bar{u}_0)\|_{r,\lambda} + t^{\frac{\alpha a}{2}} \|G(t)(u_0-\bar{u}_0)\|_{q,\lambda} + t^{\frac{\alpha}{2}} \left\|\nabla\widetilde{G}(t)(v_0-\bar{v}_0)\right\|_{L^\infty} \right) = 0 \tag{5}$$

if only if

$$\lim_{t\to+\infty} t^{\frac{\alpha b}{2}} \|(u(\cdot,t)-\bar{u}(\cdot,t))\|_{r,\lambda} = \lim_{t\to+\infty} t^{\frac{\alpha a}{2}} \|u(\cdot,t)-\bar{u}(\cdot,t)\|_{q,\lambda} = \lim_{t\to+\infty} t^{\frac{\alpha}{2}} \|\nabla(v-\bar{v})(\cdot,t)\|_{L^\infty} = 0 \tag{6}$$

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QUASILINEAR HYPERBOLIC–PARABOLIC PROBLEM WITH NONLOCAL BOUNDARY DAMPING AND SOURCE TERM

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Abstract

In this article we study the existence of global solutions for a quasilinear hyperbolic-parabolic problem with nonlocal boundary damping and source term.Moreover, we consider the uniform decay of energy for the problem.

1 Introduction

In this work we consider the problem

$$\begin{gathered} K(x,t)u_{tt} - M\Big(\int_\Omega |\nabla u|^2 dx\Big)\Delta u + \alpha u_t + f(u) = 0 \quad \text{in } \Omega\times]0,\infty[, \\ u = 0 \quad \text{on } \Gamma_0\times]0,\infty[, \\ M\Big(\int_\Omega |\nabla u|^2 dx\Big)\frac{\partial u}{\partial \nu} + N\Big(\int_{\Gamma_1} |u|^2 d\sigma\Big)g(u_t) - |u|^{\gamma-2}u = 0 \quad \text{on } \Gamma_1\times]0,\infty[, \\ u(x,0) = u^0(x) \quad u_t(x,0) = u^1(x) \quad \text{in } \Omega \end{gathered} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ $(N \geq 1)$with a smooth boundary $\partial\Omega$ such that $\Gamma = \Gamma_0 \bigcup \Gamma_1$, $\overline{\Gamma_0} \bigcup \overline{\Gamma_1} = \emptyset$ and Γ_0 ,Γ_1 have positive measures, $\vec{\nu}$ is the unit outward normal on $\partial\Omega$ and $\frac{\partial}{\partial \nu}$ is the outward normal derivative on $\partial\Omega$;K, M, f and N are given fuctions, $\alpha, \gamma > 0$. The precise hypotheses on the above system will be given in the next section.

2 Notations and Main Results

Throughout this work we define

$$V = \{u \in H^1(\Omega) : u = 0 \quad \text{on} \quad \Gamma_0\} \quad ,2 < \gamma < \frac{2(N-1)}{2(N-2)} + 1 \ (2 < \gamma < \infty \ if N = 2)$$

$$(u,v) = \int_\Omega u(x)v(x)\,dx, (u,v)_{\Gamma_1} = \int_{\Gamma_1} u(x)v(x)\,dx, \|u\|^p_{p,\Gamma_1} = \int_{\Gamma_1} |u(x)|^p\,d\Gamma$$

To simplify we denote $\|u\|_{L^2(\Omega)} := \|u\|$, $\quad \|u\|_{p,\Gamma_1} := \|u\|_{\Gamma_1}$. In the sequel we state the general hypotheses
(A_1) $f : \mathbb{R} \to \mathbb{R}$ is a piecewise $C^1(\mathbb{R})$ function with $f(s)s \geq 0$ for $s \in \mathbb{R}$. There exist $C_1, C_2 > 0$ such that

$$|f'(s)| \leq C_1(1 + |s|^{p-1}),\ 1 < p \leq \frac{n}{n-2},\ s \in \mathbb{R} \quad and \quad |f(x) - f(y)| \leq C_2(|x|^{p-1} + |y|^{p-1})|x - y| \quad \text{for } x, y \in \mathbb{R}$$

Setting $F(s) = \int_0^s f(\lambda)d\lambda$, there exist $C_3, C_4 > 0$ satisfying

$$C_3|s|^{p+1} \leq F(s) \leq C_4 s f(s) \quad \text{for } s \in \mathbb{R}.$$

(A_2) $M, N \in C^1([0,\infty[;\mathbb{R}^+)$ are functions such that $M(s) \geq m_0, N(s) \geq n_0$ for all $s \geq 0$, for some $m_0, n_0 > 0$.
Also, we define $\widehat{M}(s) = \int_0^s M(s)\,ds$

(A_3) g is a function of C^1–class and $g(0) = 0,\ g'(s) \geq \tau$ with
a) $g(s)s \geq \beta|s|^2$ and b) $|g(s)| \leq \rho|s|, \quad \beta, \rho > 0\ , \quad \forall s \in \mathbb{R}$
(A_4) $K \in W^{1,\infty}(]0,\infty[; C^1(\overline{\Omega}))$, $K \geq 0$ in $\Omega\times]0,\infty[$, $\alpha - \frac{1}{2}|K_t| \geq \delta,\ K(x,0) \geq \eta > 0$ $a.e.x \in \Omega$, $\|K\|_\infty = esssup_{t\geq 0}|K(t)|_{C(\overline{\Omega})}$
Now,we define the energy $E(t)$ of the problem (1) by

$$E(t) = \frac{1}{2}\|K^{1/2}u_t(t)\|^2 + J(u(t))$$

where $J(u(t)) = \frac{1}{2}\widehat{M}(\|\nabla u(t)\|^2) + \int_\Omega F(u)\,dx - \frac{1}{\gamma}\|u(t)\|^\gamma_{\gamma,\Gamma_1}$
We define the potential well by

$$W = \{u \in V | I(u(t)) = \widehat{M}(\|\nabla u(t)\|^2) - \|u(t)\|^\gamma_{\gamma,\Gamma_1} > 0\} \bigcup \{0\}$$

and $d = \inf\limits_{u\in V\setminus\{0\}} \{\sup\limits_{\lambda>0} J(\lambda u)\}$.
Recently, damping and source terms (of attractive forces) have attracted a lot of attention and various results are available (see [1],[2],[3] among many others). However, there are few results for quasilinear hyperbolic-parabolic problems with nonlocal boundary damping and source term. Motivated by the above researches, in this paper, we study the global existence and uniform decay of solution to problem (1).

Theorem 2.1. *Let $u^0 \in W \cap H^2(\Omega)$ and $u^1 \in V \cap H^2(\Omega)$ with the compatibility conditions:*

$$\frac{\partial u^0}{\partial \nu} + N(\|u^0\|_{\Gamma_1})u^1 - |u^0|^\gamma u^0 = 0 \quad on \quad \Gamma_1$$

Under assumptions $(A_1) - (A_4)$ and if u^0 and u^1 are sufficiently small, problem (1) has a unique solution u such that

$$u \in L^\infty\left(0,\infty; V \cap H^2(\Omega)\right),\ K^{1/2}u_t \in L^\infty(0,\infty; V),\ u_t \in L^\infty(0,\infty; V),$$
$$u_{tt} \in L^2\left(0,\infty; L^2(\Omega)\right), \quad K^{1/2}u_{tt} \in L^\infty\left(0,\infty; L^2(\Omega)\right).$$

Furthermore , there exist positive constants μ and C_0 such that

$$E(t) \leq C_0 \exp(-\mu t) \qquad \forall t \geq 0.$$

Proof. To obtain the existence of solutions we use Faedo-Galerkin's method.Meanwhile, we get the energy decay of global solutions by use of the lemma of V. Komornik.□

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LONGITUDINAL VIBRATIONS OF A BAR

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Abstract

It is deduced a mathematical model for the longitudinal vibrations of a bar which is clamped at one end and submitted to the action of a force at the other end. The existence and uniqueness of bounded global solutions of the above mathematical model with an internal dissipation is obtained

1 Introduction

Consider an elastic homogeneous cylindrical bar of length L such that in its equilibrium position coincides with the segment $[0, L]$ of the Ox axis of a Cartesian system of coordinates. Suppose that the end $x = 0$ of the bar is clamped and on the other end $x = L$ acts a force of mass M in the negative direction of the Ox axis. We denote by by $u(x, t)$ the displacement of the cross section x of the bar of its equilibrium position at the time $t > 0$.

The objective in this paper is to deduce a mathematical model that describes the above small longitudinal vibrations of the cross sections of the bar and then to find a solution of this mathematical model.

2 Notations and Main Results

Theorem 2.1. *The small longitudinal vibrations of the cross sections of the bar in the above condition can be described by the following problem:*

$$\left| \begin{array}{l} \rho A \frac{\partial^2 u}{\partial t^2}(x,t) - \frac{\partial}{\partial x}(\sigma(u_x(x,t)) = 0 \ , \ 0 < x < L \ , \ t > 0; \\ u(0,t) = 0 \ , \ M\frac{\partial^2 u}{\partial t^2}(L,t) + \sigma(u_x(L,t)) = 0 \ , \ t > 0; \\ u(x,0) = u^0(x) \ , \ \frac{\partial u}{\partial t}(x,0) = u^1(x) \ , \ 0 < x < L. \end{array} \right. \tag{1}$$

where ρ is the constant density and A, the constant area of the cross section of the bar, respectively.

To obtain (2.1), we have used the D'Alembert principle (see [5]) and the nonlinear Hooke's law $\tau(x, t) = \sigma(u_x(x, t))$ where $\sigma(s)$ is a nonnegative regular function, $\tau(x, t)$ denotes the tension at he point (x, t) of the bar and $u_x(x, t)$, its deformation at (x, t).

In order to state the second result, we introduce some spaces. We denote by $W_l^{1,p+2}$ $(p \in \mathbb{R}, p \geq 0)$ the Banach space

$$W_l^{1,p+2} = \{u \in W^{1,p+2}(0, L) \ ; \ u(0) = 0\}$$

equipped wih the norm

$$||u||_{W_l^{1,p+2}} = (\int_0^L |\frac{du}{dx}|^{p+2} \ dx)^{1/p+2}$$

and by $H_l^2(0, L)$, the Hilbert space

$$H_l^2(0,L) = \{u \in H^2(0,L) \ ; \ u(0) = 0, \frac{du}{dx}(0) = 0\}$$

provided with the scalar product

$$(u,v)_{H_l^2(0,L)} = (u,v)_{L^2(0,L)} + (\frac{d^2u}{dx^2}, \frac{d^2v}{dx^2})_{L^2(0,L)}.$$

Consider $\rho = A = M = 1$ and $\sigma(s) = |s|^\rho s$ in 2.1. In these conditions we determine a solution u of (2.1) with an internal dissipation, more precisely,

Theorem 2.2. *Let* $p \in \mathbb{R}, p \geq 0$. *Consider*

$$u^0 \ , \ u^1 \in H_l^2(0,L) \cap H^4(0,L)$$

with

$$\frac{du^0}{dx}(L) = 0 \ , \ \frac{d^2u^1}{dx^2}(L) = 0 \ , \ \frac{d^3u^1}{dx^3}(L) = 0.$$

Then there exists a function u *in the class*

$$u \in L^\infty(0,\infty; W_l^{1,p+2}) \cap L^\infty_{loc}(0,\infty; H_l^2(0,L) \cap H^4(0,L));$$

$$u' \in L^\infty(0,\infty; L^2(0,L)) \cap L^\infty_{loc}(0,\infty; H_l^2(0,L) \cap H^4(0,L));$$

$$u'' \in L^\infty_{loc}(0,\infty; L^2(0,L)) \cap L^2_{loc}(0,\infty; H_l^2(0,L))$$

such that

$$u'' - \tfrac{\partial}{\partial x}(|\tfrac{\partial u}{\partial x}|^p \tfrac{\partial u}{\partial x}) + \tfrac{\partial^4 u'}{\partial x^4} = 0 \ in \ L^\infty_{loc}(0,\infty; L^2(0,L));$$

$$u''(L,.) + |\tfrac{\partial u}{\partial x}(L,.)|^p \tfrac{\partial u}{\partial x}(L,.) - \tfrac{\partial^3 u'}{\partial x^3}(L,.) = 0 \ in \ L^\infty_{loc}(0,\infty);$$

$$\tfrac{\partial^2 u'}{\partial x^2}(L,.) = 0 \ in \ L^\infty_{loc}(0,\infty);$$

$$u(0) = u^0 \ , \ u'(0) = u^1.$$

In the proof of the preceding theorem, we use the Faedo-Galerkin method with a special basis, compactness arguments and Trace Theorems. The uniqueness of solution is derived for $p \geq 1$.

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AN ATTRACTOR FOR A KIRCHHOFF WAVE MODEL WITH NONLOCAL NONLINEAR DAMPING

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Abstract

This paper is concerned with well-posedness and asymptotic behavior of solutions for a quasi-linear Kirchhoff wave model with nonlocal nonlinear damping like $\sigma\left(\int_\Omega |\nabla u|^2\,dx\right) g(u_t)$, where σ and g are nonlinear functions under proper conditions.

1 Introduction

In this paper we discuss on well-posedness and long-time behavior to the following quasi-linear Kirchhoff wave model with nonlocal nonlinear damping

$$u_{tt} - \phi(\|\nabla u(t)\|_2^2)\Delta u + \sigma\left(\|\nabla u(t)\|_2^2\right) g(u_t) + f(u) = h \quad \text{in} \quad \Omega \times (0,\infty), \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary Γ, ϕ and σ are scalar functions defined on $\mathbb{R}^+ = [0,+\infty)$, f and g are nonlinear functions on $\mathbb{R}$ corresponding to source and damping terms, and h is a external force. Here, $\|\cdot\|_2$ stands for L^2-norm. The following initial-boundary conditions are coupled to equation (1)

$$u(\cdot,0) = u_0(\cdot),\ u_t(\cdot,0) = u_1(\cdot) \ \text{ in } \ \Omega \quad \text{and} \quad u = 0 \ \text{ on } \ \Gamma \times \mathbb{R}^+. \tag{2}$$

Our analysis is given in the Hilbert phase spaces $\mathcal{H} = H_0^1(\Omega) \times L^2(\Omega)$ and $\mathcal{H}_1 = \left[H^2(\Omega) \cap H_0^1(\Omega)\right] \times H_0^1(\Omega)$, equipped with their respective norms $\|(u,v)\|_{\mathcal{H}}^2 = \|\nabla u\|_2^2 + \|v\|_2^2$ and $\|(u,v)\|_{\mathcal{H}_1}^2 = \|\Delta u\|_2^2 + \|\nabla v\|_2^2$.

We assume the following hypotheses on ϕ, σ, f and g.

(H1) The stiffness factor $\phi \in C^1(\mathbb{R}^+)$ satisfies

$$\phi(s) \geq \phi_0 \quad \text{and} \quad \phi(s)s \geq \widehat{\phi}(s), \quad \forall\ s \in \mathbb{R}^+,$$

for some constant $\phi_0 > 0$, where we denote $\widehat{\phi}(s) := \int_0^s \phi(\tau)d\tau$.

(H2) The damping coefficient $\sigma \in C^1(\mathbb{R}^+)$ is a positive function, namely,

$$\sigma(s) > 0, \quad \forall\ s \in \mathbb{R}^+.$$

(H3) The source $f \in C^1(\mathbb{R})$ fulfills $f(0) = 0$, $\quad |f'(s)| \leq C_{f'}(1+|s|^\rho), \quad \forall\ s \in \mathbb{R}$, where we consider $C_{f'} > 0$ and the growth ρ satisfying $\rho \geq 0$ if $n = 1,2$ or $0 \leq \rho \leq \frac{2}{n-2}$ if $n \geq 3$. In addition, for some $\eta \in (0,\lambda_1)$ and $C_f \geq 0$, where $\lambda_1 > 0$ is the first eigenvalue of $-\Delta$, we assume that

$$f(s)s \geq \widehat{f}(s) - C_f \geq -\frac{\phi_0 \eta}{2}s^2 - 2C_f, \quad \forall\ s \in \mathbb{R},$$

where $\widehat{f}(s) := \int_0^s f(\tau)d\tau$.

(H4) The damping $g \in C^1(\mathbb{R})$ satisfies $g(0) = 0$, $\quad \kappa_1|s|^\gamma \leq g'(s) \leq \kappa_2(1+|s|^\gamma)$, $\forall\ s \in \mathbb{R}$, for some κ_1, $\kappa_2 > 0$, and $\gamma \geq 0$.

2 Main Results

Theorem 2.1. *Let us assume that assumptions* (**H1**)-(**H4**) *hold and take* $h \in H_0^1(\Omega)$. *If* $(u_0, u_1) \in \mathcal{H}_1$, *then there exists a* $T > 0$ *such that the problem* (1)-(2) *has a unique solution* $u = u(x,t)$ *in the class*

$$u \in L^\infty\big(0,T; H^2(\Omega) \cap H_0^1(\Omega)\big) \cap W^{1,\infty}(0,T; H_0^1(\Omega)),$$

$$u_t \in L^{\gamma+2}\big(0,T; L^{\gamma+2}(\Omega)\big) \quad \textit{and} \quad u_{tt} \in L^{\frac{\gamma+2}{\gamma+1}}\big(0,T; L^{\frac{\gamma+2}{\gamma+1}}(\Omega)\big).$$

Besides, if we set $S(t)(u_0, u_1) = (u(t), u_t(t))$, *then the evolution operator* $S(t)$ *is continuous with respect to* $\mathcal{H}$-*topology for any bounded set* $B \subset \mathcal{H}_1$.

The proof on existence relies on Faedo-Galerkin method which was extensively applied in the study of wave equations, see for instance Lions [3].

We recall that a bounded set $\mathfrak{A} \subset B$ is a $(\mathcal{H}_1, \mathcal{H})$ "local" attractor associated with the semigroup $S(t)$ iff the following conditions are fulfilled:

(i) $dist_{\mathcal{H}}(S(t)D, \mathfrak{A}) \to 0$ as $t \to \infty$ for any bounded set $D \subset B$, where $dist_{\mathcal{H}}(D, \mathfrak{A}) = \sup_{d \in D} \inf_{a \in \mathfrak{A}} \|d - a\|_{\mathcal{H}}$,

(ii) $S(t)\mathfrak{A} = \mathfrak{A}$ for any $t \geq 0$,

(iii) $\mathfrak{A}$ is compact in $\mathcal{H}$.

Theorem 2.2. *Let us assume that* ϕ, σ, f *and* g *satisfy the following hypotheses:* $\phi \in C^2(\mathbb{R}^+)$ *is a positive monotonous nondecreasing function,* σ *satisfies* (**H2**), f *fulfills condition* (**H5**), *and* g *satisfies* (**H4**) *with* $g'(0) > 0$ *and* γ *fulfilling*

$$\gamma \geq 0 \quad \textit{if} \quad n = 1, 2 \quad \textit{or} \quad 0 \leq \gamma \leq \frac{4}{n-2} \quad \textit{if} \quad n \geq 3.$$

Then

(i) There exists an open set $\mathcal{B} \subset \mathcal{H}_1$ *such that if* $(u_0, u_1) \in \mathcal{B}$ *the solution* $u(t)$ *of the problem* (1)-(2) *exists on* $[0, \infty)$ *and* $S(t)(u_0, u_1) \in \mathcal{B}$

(ii) The semigroup $S(t)$ *corresponding to problem* (1)-(2) *has a* $(\mathcal{H}_1, \mathcal{H})$ *"local" attractor* $\mathfrak{A} \subset V$, *where* V *is a neighborhood of* $(0,0)$ *in* $\mathcal{H}_1$.

The proof is based in the works of Nakao [4, 5] along with some results which can be found in Chueshov and Lasiecka [1, 2].

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EXTENSÕES DE POLINÔMIOS E FUNÇÕES ANALÍTICAS EM ESPAÇOS DE BANACH

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Resumo

Dado um espaço de Banach E, apresentaremos a extensão de Aron-Berner para polinômios m-homogêneos em E para o seu bidual E''. Posteriormente, apresentaremos uma maneira de estender funções analíticas de tipo limitado. Esta extensão generaliza aquela apresentada por Aron e Berner. Por fim apresentaremos uma caracterização desta extensão em termos de continuidade fraca-estrela.

1 Introdução

O problema de estender polinômios de um subespaço de um espaço de Banach para todo o espaço foi estudado pela primeira vez por R. Aron e P. Berner em [1], em 1978. Neste trabalho os autores mostraram que, em contraste com o Teorema de Hahn-Banach, estas extensões nem sempre existem. Porém, foi provado que cada polinômio m-homogêneo em um espaço de Banach E se estende a um polinômio m-homogêneo em seu bidual E''. Este trabalho tornou-se bastante influente e as propriedades das extensões de polinômios foram estudadas em muitos outros trabalhos. Por exemplo, onze anos mais tarde, Davie e Gamelin mostraram que a extensão de Aron-Berner preserva a norma e tem uma importante relação com a tolopogia fraca-estrela em [2]. Este e muitos outros resultados referentes à extensões de polinômios homogêneos foram reunidos por I. Zalduendo no survey [4].

A extensão de Aron-Berner representa um ponto importante para o desenvolvimento da teoria de holomorfia em dimensão infinita, sobretudo no problema de estender funções analíticas em espaços de Banach. Este problema é tratado por I. Zalduendo em [3], onde é apresentada uma extensão canônica para esta classe de funções e uma caracterização da extensão em termos da continuidade fraca-estrela do seu operador diferencial de primeira ordem.

2 Resultados Principais

Ao longo deste trabalho E denota um espaço de Banach sobre um corpo $\mathbb{K}$, onde $\mathbb{K} = \mathbb{R}$ ou $\mathbb{K} = \mathbb{C}$, e E'' o seu bidual. O espaço das aplicações multilineares contínuas $A : E^m \to \mathbb{K}$ será denotado por $L(^mE)$ enquanto que o espaço das multilineares simétricas contínuas será denotado por $L^s(^mE)$. Denotaremos por $P^m(E)$ o espaço dos polinômios m-homogêneos $P : E \to \mathbb{K}$ e por $H_b(E)$ o espaço das funções holomorfas de tipo limitado $f : E \to \mathbb{K}$.

Definição 2.1 (Aron-Berner)**.** *Sejam $A \in L(^mE)$ e $x_1, \dots, x_m \in E$. Definimos para cada $1 \leq k \leq m$ a aplicação $A_{x_1,\dots,x_k} : E^{m-k} \to \mathbb{K}$ por $A_{x_1,\dots,x_k}(x_{k+1}, \dots, x_m) \doteq A(x_1, \dots, x_m)$. Deste modo, $A_{x_1,\dots,x_k} \in L(^{m-k}E)$ e vale $\|A_{x_1,\dots,x_k}\| \leq \|A\|$.*

Dados $z_1, \dots, z_m \in E''$, para cada $1 \leq k \leq m$, definimos $\overline{z_k} : L(^kE) \to L(^{k-1}E)$ por

$$\overline{z_k}(A)(x_1, \dots, x_{k-1}) \doteq z_k(A_{x_1,\dots,x_{k-1}}).$$

Deste modo, cada $\overline{z_k}$ é um operador linear e contínuo com $\|\overline{z_k}\| \leq \|z_k\|$, assim podemos definir uma extensão $\overline{A} \in L(^mE'')$ por:

$$\overline{A}(z_1, \dots, z_m) \doteq \overline{z_1} \circ \overline{z_2} \circ \dots \circ \overline{z_m}(A)$$

Observe que definindo a extensão desta maneira vale que $\|\overline{A}\| = \|A\|$ e que, variando a ordem das composições acima, exitem $m!$ maneiras de se estender uma aplicação m-linear. Como estamos interessados em estender polinômios, definimos a extensão canônica de $P \in P^m(E)$ da seguinte maneira:

Definição 2.2. *Dado $P \in P^m(E)$, seja $A \in L^s({}^mE)$ tal que $P(x) = A(x \ldots, x)$ para todo $x \in E$. Definimos a extensão $\overline{P} \in P^m(E'')$ por:*

$$\overline{P}(z) \doteq \overline{z} \circ \cdots \circ \overline{z}(A).$$

Note que, para a extensão definida acima, vale a seguinte desigualdade:

$$\|\overline{P}\| \leq \|\overline{A}\| = \|A\| \leq \frac{m^m}{m!}\|P\|,$$

porém o próximo teorema nos garante que esta extensão preserva a norma de P.

Teorema 2.1 (Davie-Gamelin)**.** *A extensão canônica $\overline{P}$ de um polinômio $P \in \mathcal{P}({}^mE)$ satisfaz $\|\overline{P}\| = \|P\|$.*

Uma consequência da demonstração apresentada por Davie e Gamelin é a seguinte generalização do Teorema de Goldstine:

Teorema 2.2. *Seja E um espaço de Banach e E'' seu bidual. Para cada $z \in B_{E''}$ existe uma rede $(x_\alpha) \subset B_E$ tal que $P(x_\alpha) \to P(z)$ para todo polinômio homogêneo em E.*

Após o estudo de extensões de aplicações multilineares, como é feito em [3], definimos a extensão de uma função analítica $f : E \to \mathbb{K}$, de tipo limitado da seguinte maneira:

Definição 2.3. *Dada $f \in H_b(E)$, definimos a extensão $\overline{f} \in H_b(E'')$ por:*

$$\overline{f}(z) = \sum_{k=0}^{\infty} \frac{1}{k!}\overline{D^k f(0)}(z, \ldots, z).$$

Teorema 2.3. *Sejam $f \in H_b(E)$ e $g \in H_b(E'')$ tal que $g|_E = f$. São equivalentes:*

(a) *Para todo $x \in E$, $Dg(x)$ é w^*-contínuo e para todo $z \in E''$ e toda rede $(x_\alpha) \subset E$ w^*-convergente a z, $Dg(z)(x_\alpha) \to Dg(z)(z)$.*

(b) *$g = \overline{f}$.*

Observação 2. *Este artigo é baseado na dissertação de mestrado do aluno Victor dos Santos Ronchim, orientado pela professora Daniela Mariz Silva Vieira.*

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ON THE MIXED (ℓ_1,ℓ_2)-LITTLEWOOD INEQUALITIES AND INTERPOLATION

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Abstract

It is well-known that the optimal constant of the bilinear Bohnenblust–Hille inequality (i.e., Littlewood's 4/3 inequality) is obtained by interpolating the bilinear mixed (ℓ_1,ℓ_2)-Littlewood inequalities. We remark that this cannot be extended to the 3-linear case and, in the opposite direction, we show that the asymptotic growth of the constants of the m-linear Bohnenblust–Hille inequality is the same of the constants of the mixed $\left(\ell_{\frac{2m+2}{m+2}},\ell_2\right)$-Littlewood inequality. This means that, contrary to what the previous works seem to suggest, interpolation does not play a crucial role in the search of the exact asymptotic growth of the constants of the Bohnenblust–Hille inequality. We also use mixed Littlewood type inequalities to obtain the optimal cotype constants of certain sequence spaces.

1 Introduction

The mixed (ℓ_1,ℓ_2)-Littlewood inequality for $\mathbb{K}=\mathbb{R}$ or $\mathbb{C}$ asserts that

$$\sum_{j_1=1}^{\infty}\left(\sum_{j_2,\dots,j_m=1}^{\infty}\left|U(e_{j_1},\dots,e_{j_m})\right|^2\right)^{\frac{1}{2}}\leq\left(\sqrt{2}\right)^{m-1}\|U\|, \tag{1}$$

for all continuous m-linear forms $U: c_0\times\cdots\times c_0\to\mathbb{K}$, where $(e_i)_{i=1}^{\infty}$ denotes the sequence of canonical vectors of c_0. It is well-known that arguments of symmetry combined with an inequality due to Minkowski yields that for each $k\in\{2,\dots,m\}$ we have

$$\left(\sum_{j_1,\dots,j_{k-1}=1}^{\infty}\left(\sum_{j_k=1}^{\infty}\left(\sum_{j_{k+1},\dots,j_m=1}^{\infty}\left|U(e_{j_1},\dots,e_{j_m})\right|^2\right)^{\frac{1}{2}\times 1}\right)^{\frac{1}{1}\times 2}\right)^{\frac{1}{2}}\leq\left(\sqrt{2}\right)^{m-1}\|U\|, \tag{2}$$

which is also called mixed (ℓ_1,ℓ_2)-Littlewood inequality. For the sake of simplicity we can say that we have m inequalities with "multiple" exponents $(1,2,2,\dots,2),\dots,(2,\dots,2,1)$. These inequalities are in the heart of the proof of the famous Bohnenblust–Hille inequality for multilinear forms ([3]) which states that there exists a sequence of positive scalars $\left(B_m^{\mathbb{K}}\right)_{m=1}^{\infty}$ in $[1,\infty)$ such that

$$\left(\sum_{i_1,\dots,i_m=1}^{\infty}\left|U(e_{i_1},\dots,e_{i_m})\right|^{\frac{2m}{m+1}}\right)^{\frac{m+1}{2m}}\leq B_m^{\mathbb{K}}\|U\| \tag{3}$$

for all continuous m-linear forms $U: c_0\times\cdots\times c_0\to\mathbb{K}$.

In the paper [2], Albuquerque *et al.* have shown that the Bohnenblust–Hille inequality is a very particular case of the following theorem:

Theorem 1.1. *Let $1 \leq k \leq m$ and $n_1, \ldots, n_k \geq 1$ be positive integers such that $n_1 + \cdots + n_k = m$, let $q_1, \ldots, q_k \in [1,2]$. The following assertions are equivalent:*

(A) There is a constant $C^{\mathbb{K}}_{k,q_1\ldots q_k} \geq 1$ such that

$$\left(\sum_{i_1=1}^{\infty}\left(\sum_{i_2=1}^{\infty}\left(\cdots\left(\sum_{i_{k-1}=1}^{\infty}\left(\sum_{i_k=1}^{\infty}\left|A\left(e_{i_1}^{n_1},\ldots,e_{i_k}^{n_k}\right)\right|^{q_k}\right)^{\frac{q_{k-1}}{q_k}}\right)^{\frac{q_{k-2}}{q_{k-1}}}\cdots\right)^{\frac{q_2}{q_3}}\right)^{\frac{q_1}{q_2}}\right)^{\frac{1}{q_1}} \leq C^{\mathbb{K}}_{k,q_1\ldots q_k}\|A\| \tag{4}$$

for all continuous m-linear forms $A : c_0 \times \cdots \times c_0 \to \mathbb{K}$.

(B) $\frac{1}{q_1} + \cdots + \frac{1}{q_k} \leq \frac{k+1}{2}$.

The inequalities (4) when $k = m$, $q_j = 2$ and $q_l = \frac{2m-2}{m}$ for all $l \in \{1, ..., j-1, j+1, ..., m\}$ can be called mixed $\left(\ell_{\frac{2m-2}{m}}, \ell_2\right)$-Littlewood inequality for short. The best constants $C^{\mathbb{K}}_{\frac{2m}{m+1}\cdots\frac{2m}{m+1}}$ ($C^{\mathbb{K}}_m$ for short) are unknown (even its asymptotic growth is unknown). We stress that it is even unknown if the sequence $\left(C^{\mathbb{K}}_m\right)_{m=1}^{\infty}$ is increasing. The theorema below tells us that to the search of the precise asymptotic growth of the best constants of the Bohnenblust–Hille inequality is equivalent to the search of the precise asymptotic growth of, for instance, the sequence $\left(C^{\mathbb{K}}_{2,\frac{2m-2}{m},\ldots,\frac{2m-2}{m}}\right)_{m=1}^{\infty}$.

Theorem 1.2. *The growth of the best constants of the Bohnenblust–Hille inequality is asymptotically equivalent the growth of the sequence $\left(C^{\mathbb{K}}_{2,\frac{2m-2}{m},\ldots,\frac{2m-2}{m}}\right)_{m=1}^{\infty}$,*

$$C^{\mathbb{K}}_m \sim C^{\mathbb{K}}_{2,\frac{2m-2}{m},\ldots,\frac{2m-2}{m}} \sim \cdots \sim C^{\mathbb{K}}_{\frac{2m-2}{m},\ldots,\frac{2m-2}{m},2}, \tag{5}$$

i.e., $\lim_{m\to\infty} \frac{C^{\mathbb{K}}_{2,\frac{2m-2}{m},\ldots,\frac{2m-2}{m}}}{C^{\mathbb{K}}_m} = \cdots = \lim_{m\to\infty} \frac{C^{\mathbb{K}}_{\frac{2m-2}{m},\ldots,\frac{2m-2}{m},2}}{C^{\mathbb{K}}_m} = 1.$

Another interesting result that can be obtained using mixed inequality is the optimal cotype constant of ℓ_p spaces. From now on, p_0 is the solution of the following equality

$$\Gamma\left(\frac{p_0+1}{2}\right) = \frac{\sqrt{\pi}}{2}.$$

Let $2 \leq q < \infty$ and $0 < s < \infty$. A Banach space X has cotype q (see [1, page 138]) if there is a constant $C_{q,s} > 0$ such that, no matter how we select finitely many vectors $x_1, \ldots, x_n \in X$,

$$\left(\sum_{k=1}^{n}\|x_k\|^q\right)^{\frac{1}{q}} \leq C_{q,s}\left(\int_{[0,1]}\left\|\sum_{k=1}^{n} r_k(t)x_k\right\|^s dt\right)^{1/s}, \tag{6}$$

where r_k denotes the k-th Rademacher function. The smallest of all of these constants will be denoted by $C_{q,s}(X)$.

It is well-known that for all $p \geq 1$, the sequence space ℓ_p has cotype $\max\{p, 2\}$. The optimal values of $C_{2,s}(\ell_p)$ for $1 \leq p \leq p_0$ are perhaps known or at least folklore, but we were not able to find in the literature.

Theorem 1.3. *If $1 \leq p \leq p_0 \approx 1.84742$, then $C_{2,p}(\ell_p) = 2^{\frac{1}{p}-\frac{1}{2}}$.*

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OPTIMAL EXPONENTS FOR HARDY–LITTLEWOOD INEQUALITIES

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Abstract

The Hardy–Littlewood inequalities on ℓ_p spaces provide optimal exponents for some classes of inequalities for bilinear forms on ℓ_p spaces. In this work we investigate in detail the exponents involved in Hardy–Littlewood type inequalities and provide several optimal results that were not achieved by the previous approaches. More precisely, we generalize to the m-linear setting one of the classical Hardy–Littlewood inequalities for bilinear forms. Our result is sharp in a very strong sense: the constants and exponents are optimal, even if we consider mixed sums.

1 Introduction

Let $\mathbb{K}$ be the real or complex scalar field. In 1934 Hardy and Littlewood proved several theorems on the summability of bilinear forms on $\ell_p \times \ell_q$ (here, and henceforth, when $p = \infty$ we consider c_0 instead of ℓ_∞).

For any function f we shall consider $f(\infty) := \lim_{s\to\infty} f(s)$ and for any $s \geq 1$ we denote the conjugate index of s by s^*, i.e., $\frac{1}{s} + \frac{1}{s^*} = 1$.

For all $p, q \in (1, \infty]$, such that $\frac{1}{p} + \frac{1}{q} < 1$, let us define

$$\lambda := \frac{pq}{pq - p - q}.$$

From now on, $(e_k)_{k=1}^{\infty}$ denotes the sequence of canonical vectors in ℓ_p. The following theorem is one of the classical Hardy–Littlewood inequalities for bilinear forms:

Theorem 1.1. *(See Hardy and Littlewood [3, Theorem 3]) Let $1 < q \leq 2 < p$, with $\frac{1}{p} + \frac{1}{q} < 1$. There is a constant $C_{p,q} \geq 1$ such that*

$$\left(\sum_{j_1, j_2 = 1}^{\infty} |A(e_{j_1}, e_{j_2})|^{\lambda} \right)^{\frac{1}{\lambda}} \leq C_{p,q} \|A\|,$$

for all continuous bilinear forms $A : \ell_p \times \ell_q \to \mathbb{K}$.

The exponent in the above inequality was improved in [4]:

Theorem 1.2. *(See Osikiewicz and Tonge [4]) Let $1 < q \leq 2 < p$, with $\frac{1}{p} + \frac{1}{q} < 1$. If $A : \ell_p \times \ell_q \to \mathbb{K}$ is a continuous bilinear form, then*

$$\left(\sum_{j_1=1}^{\infty} \left(\sum_{j_2=1}^{\infty} |A(e_{j_1}, e_{j_2})|^{q^*} \right)^{\frac{\lambda}{q^*}} \right)^{\frac{1}{\lambda}} \leq \|A\|.$$

Hardy–Littlewood type inequalities were extensively investigated in recent years, but despite much progress there are still several open questions concerning the optimality of exponents and constants.

Our main results generalize Theorems 1.1 and 1.2 with optimal exponents and constants, to the multilinear setting. Furthermore, we show that the optimal constant is precisely 1.

2 Main Results

Let $p_1, ..., p_m > 1$, such that $\frac{1}{p_1} + \frac{1}{p_2} + ... + \frac{1}{p_m} < 1$. For all positive integers m and $k = 1, ..., m$, let us define

$$\delta_{m-k+1}^{p_k,...,p_m} := \frac{1}{1 - \left(\frac{1}{p_k} + ... + \frac{1}{p_m}\right)}.$$

Theorem 2.1. *Let $m \geq 2$, $q_1, ..., q_m > 0$, and $1 < p_m \leq 2 < p_1, ..., p_{m-1}$, with*

$$\frac{1}{p_1} + \frac{1}{p_2} + ... + \frac{1}{p_m} < 1.$$

The following assertions are equivalent:

(a) There is a constant $C_{p_1,...,p_m} \geq 1$ such that

$$\left(\sum_{j_1=1}^{\infty} \left(\sum_{j_2=1}^{\infty} \cdots \left(\sum_{j_m=1}^{\infty} |A(e_{j_1}, ..., e_{j_m})|^{q_m} \right)^{\frac{q_{m-1}}{q_m}} \cdots \right)^{\frac{q_1}{q_2}} \right)^{\frac{1}{q_1}} \leq C_{p_1,...,p_m} \|A\|$$

for all continuous m-linear operators $A : \ell_{p_1} \times \cdots \times \ell_{p_m} \to \mathbb{K}$.

(b) The exponents $q_1, ..., q_m > 0$ satisfy

$$q_1 \geq \delta_m^{p_1,...,p_m}, q_2 \geq \delta_{m-1}^{p_2,...,p_m}, ..., q_{m-1} \geq \delta_2^{p_{m-1},p_m}, q_m \geq \delta_1^{p_m}.$$

Theorem 2.2. *Let $m \geq 2$, $q_1, ..., q_m > 0$, and $1 < p_m \leq 2 < p_1, ..., p_{m-1}$, with*

$$\frac{1}{p_1} + \frac{1}{p_2} + ... + \frac{1}{p_m} < 1.$$

In Theorem 2.1, if (b) is true then the optimal constat $C_{p_1,...,p_m}$ in (a) is 1.

Proofs The proofs of the Theorems 2.1 and 2.2 are contained in: [1, Theorem 3.2, Theorem 4.3, and Remark 4.6].

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APLICAÇÕES MULTILINEARES FORTEMENTE FATORÁVEIS EM ESPAÇOS DE BANACH

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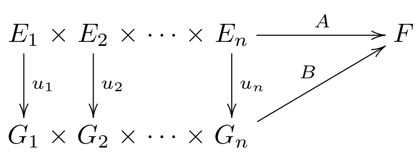
Abstract

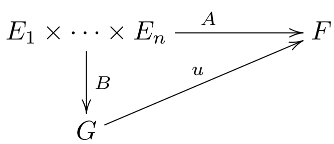
Nesse trabalho nosso objetivo é mostrar que para qualquer ideal de aplicações multilineares $\mathcal{M}$, existe um ideal $\mathcal{M}^{fat}$ de modo que toda aplicação A em $\mathcal{M}^{fat}$ é fortemente $\mathcal{M}$-fatorável no seguinte sentido: A se fatora por aplicações multilineares em $\mathcal{M}$ com relação a qualquer partição do conjunto $\{1, \ldots, n\}$. Em seguida exibimos o ideal $\mathcal{M}^{fat}$ obtido a partir de importantes ideais $\mathcal{M}$. Por fim mostramos que a passagem de $\mathcal{M}$ para $\mathcal{M}^{fat}$ é vantajosa pois agrega propriedades que $\mathcal{M}$ pode não possuir.

1 Introdução

A teoria de ideais de operadores multilineares (multi-ideais) em espaços de Banach foi iniciada por Pietsch em [5] e uma quantidade expressiva de trabalhos foram realizados nesse tema (veja, por exemplo, as referências em [3]). Na teoria linear, existem muitos procedimentos que associam um ideal de operadores $\mathcal{I}$ a um outro ideal $\mathcal{I}^{new}$ que satisfaz certas propriedades que $\mathcal{I}$ pode não satisfazer(veja [4, Chapter 4]) Uma dessas tecnicas, chamada *método da fatoração*, introduzido por Pietsch [5], considera as aplicações multilineares $A \colon E_1 \times \cdots \times E_n \longrightarrow F$ que admitem uma fatoração



onde $u_1, \ldots, u_n$ são operadores lineares em $\mathcal{I}$. Por outro lado, por propriedades de multi-ideal sabemos que um multi-ideal $\mathcal{M}$ coincide com a classe das aplicações multilineares $A \colon E_1 \times \cdots \times E_n \longrightarrow F$ que admitem uma fatoração



onde B é uma aplicação n-linear que pertence a $\mathcal{M}$. Buscando generalizar esse conceito para qualquer partição de $\{1, \ldots, n\}$ propomos a seguinte definição:

Definição 1.1. Uma aplicação n-linear $A \in \mathcal{L}(E_1, \ldots, E_n; F)$ *se fatora por um multi-ideal p-Banach* $\mathcal{M}$ *com relação a partição* $\Pi = \{j_1^1, \ldots, j_{k_1}^1\} \cup \cdots \cup \{j_1^m, \ldots, j_{k_m}^m\}$ *de* $\{1, \ldots, n\}$ se existem espaços de Banach $G_1, \ldots, G_m$, e aplicações $B_i \in \mathcal{M}(E_{j_1^i}, \ldots, E_{j_{k_i}^i}; G_i)$, $i = 1, \ldots, m$, e $C \in \mathcal{L}(G_1, \ldots, G_m; F)$ tais que

$$A(x_1, \ldots, x_n) = C(B_1(x_{j_1^1}, \ldots, x_{j_{k_1}^1}), \ldots, B_m(x_{j_1^m}, \ldots, x_{j_{k_m}^m})),$$

para quaisquer $x_1 \in E_1, \ldots, x_n \in E_n$. Nesse caso escrevemos

$$A \overset{\Pi}{=} C \circ (B_1, \ldots, B_m),$$

e dizemos que $C \circ (B_1, \ldots, B_m)$ é uma *fatoração de A por $\mathcal{M}$ com relação a partição* Π. Para que essa notação faça sentido, a ordem na qual os conjuntos da partição estão escritos importa.

2 Resultados Principais

Teorema 2.1. *Seja $\mathcal{M}$ um multi-ideal p-Banach. Então existe o maior multi-ideal $(\frac{p}{n})_{n=1}^{\infty}$-Banach $\mathcal{M}^{fat}$ contido em $\mathcal{M}$ onde todos os seus elementos são fortemente $\mathcal{M}$-fatoráveis.*

Exemplo 2.1. (1)$\mathcal{L}_{\mathcal{W}}^{fat} \subsetneq \mathcal{L}_{\mathcal{W}}$, onde $\mathcal{L}_{\mathcal{W}}$ é o multi-ideal das aplicações fracamente compactas.

(2)$\mathcal{M}^{fat} = \mathcal{L}(\mathcal{M}^1)$ se e somente se $\mathcal{L}(\mathcal{M}^1) \subseteq \mathcal{M}$, onde $\mathcal{L}(\mathcal{M}^1)$ denota o método da fatoração clássico, mencionado acima, com respeito ao ideal formado por todos os operadores lineares de $\mathcal{M}$, indicado por $\mathcal{M}^1$.

(3)$\mathcal{L}(\mathcal{I})^{fat} = [\mathcal{I}]^{fat} = \mathcal{L}(\mathcal{I})$, onde $\mathcal{I}$ é um ideal de operadores e $[\mathcal{I}]^{fat}$ denota o método da linearização (que pode ser encontrado em [5]).

Definição 2.1. Sejam $A \in \mathcal{L}(E_1, \ldots, E_n; F)$ e $x_j \in E_j$, para $j \in \{1, \ldots, n\}$. Considere a aplicação $(n-1)$-linear $A_{x_j} \colon E_1 \times \cdots \times E_{j-1} \times E_{j+1} \times \cdots \times E_n \longrightarrow F$ definida por

$$A_{x_j}(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_n) = A(x_1, \ldots, x_n).$$

Um multi-ideal $\mathcal{M}$ é chamado *coerente para baixo* se $A_{x_j} \in \mathcal{M}(E_1, \ldots, E_{i-1}, E_{i+1}, \ldots, E_n; F)$ para todo $A \in \mathcal{M}(E_1, \ldots, E_n; F)$, $j \in \{1, \ldots, n\}$ e $x_j \in E_j$.

Bons motivo para passar de $\mathcal{M}$ para $\mathcal{M}^{fat}$ são dados nas proposições seguintes.

Proposição 2.1. *Seja $\mathcal{M}$ um multi-ideal p-Banach. Então $\mathcal{M}^{fat}$ é coerente para baixo e, para todo $n \in \mathbb{N}$, $A \in \mathcal{M}(E_1, \ldots, E_n; F)$, $j \in \{1, \ldots, n\}$ e $x_j \in E_j$,*

$$\|A_{x_j}\|_{\mathcal{M}^{fat}} \leq \left(\frac{C_n^n}{C_{n-1}^{n-1}} \right)^{\frac{1}{p}} \cdot \|A\|_{\mathcal{M}^{fat}} \cdot \|x_j\|.$$

Definição 2.2 (veja [1])**.** Seja S_n o grupo das permutações de $\{1, \ldots, n\}$. Sendo $A \in \mathcal{L}(^n E; F)$ e $\sigma \in S_n$, definimos $A_\sigma \in \mathcal{L}(^n E; F)$ por

$$A_\sigma(x_1, \ldots, x_n) = A(x_{\sigma(1)}, \ldots, x_{\sigma(n)}).$$

Um multi-ideal quasi-Banach $\mathcal{M}$ é dito *fortemente simétrico* se $A_\sigma \in \mathcal{M}(^n E; F)$ e $\|A_\sigma\|_{\mathcal{M}} = \|A\|_{\mathcal{M}}$ para quaisquer $A \in \mathcal{M}(^n E; F)$, $\sigma \in S_n$ e $n \in \mathbb{N}$.

Proposição 2.2. *$\mathcal{M}^{fat}$ é fortemente simétrico para todo multi-ideal p-Banach $\mathcal{M}$.*

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ESPAÇABILIDADE DO CONJUNTO DE FUNÇÕES $\mathfrak{A}$ QUE ADMITEM A-PRIMITIVAS E QUE NÃO ADMITEM C-PRIMITIVAS

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Abstract

Em [4] prova-se que o conjunto das funções Kurzweil integráveis e que não são integráveis segundo Lebesgue é espaçável. O estudo da integral de Kurzweil leva à definição de funções com a-primitivas, c-primitivas e f-primitivas, como feito em [1], e assim surge naturalmente a questão - Será espaçável o conjunto das funções com a-primitivas, mas que não possuem c-primitivas? - Este trabalho tem a intenção de demonstrar que a resposta a essa pergunta é afirmativa.

1 Introdução

A integral de Kurzweil é uma medida capaz de mensurar as funções Lebesgue integráveis, assim como aquelas que possuem integrais impróprias (segundo Riemann). Além disso, pode-se provar que o Teorema Fundamental do Cálculo continua válido desde que seja feita uma adaptação do conceito de primitiva. Bartle em [1] faz essa adaptação da seguinte forma.

Definição 1.1. *Seja F uma função real contínua, dizemos que F é uma a-primitiva (respectivamente c-primitiva) de $f : [a,b] \longrightarrow \mathbb{R}$ desde que $F'(x) = f(x), \quad \forall x \in [a,b] \setminus A$ em que A é um conjunto de medida nula ($F'(x) = f(x), \quad \forall x \in [a,b] \setminus C$ em que C é enumerável).*

Utilizando-se os conceitos de primitiva acima define-se $\mathfrak{A}$ o conjunto das funções que admitem a-primitivas, mas que não admitem c-primitivas. O objetivo deste trabalho é mostrar que $\mathfrak{A}$ é espaçável, isto é, existe em $\mathfrak{A}$ um espaço vetorial fechado de dimensão infinita. Para provar tal fato será feita a construção de uma sequência $\{f_n\}$ de funções linearmente independentes tal que o fecho do espaço gerado por elas, denotado por $\overline{Span\{f_n\}}$, está contido em $\mathfrak{A}$.

2 Resultados Principais

A fim de construir uma sequência de funções $\{f_n\}$ tais que $\overline{Span\{f_n\}} \subset \mathfrak{A}$, divide-se, assim como em [3], o intervalo $[0,1)$ em sub intervalos I_n, de forma que os intervalos I_n sejam dois a dois disjuntos e $\bigcup I_n = [0,1)$. Posteriormente utilizam-se cópias do Conjunto de Cantor, denotado por $\mathcal{C}$, em cada um dos I_n, pois este é um subconjunto de $[0,1]$ que possui medida nula e é não enumerável.

Teorema 2.1. *O conjunto $\mathfrak{A}$ das funções que possuem a-primitva, mas não admitem c-primitivas é espaçável.*

Prova: Considere a seguinte partição do intervalo $[0,1)$,

$$[0,1) = [0, 1-1/2) \cup [1-1/2, 1-1/4) \cdots \left[1 - \frac{1}{2^{n-1}}, 1 - \frac{1}{2^n}\right) \cup \cdots = \bigcup_{n \in \mathbb{N}} I_n.$$

A fim de exibir, em cada intervalo I_n, um conjunto não enumerável e de medida nula, utiliza-se uma cópia do conjunto de Cantor da seguinte forma. Dados $a_n < b_n$ em I_n, define-se

$$A_n = a_n + (b_n - a_n)\mathcal{C} = \{a_n + (b_n - a_n)x : x \in \mathcal{C}\}.$$

Cabe observar que $A_n \subset I_n$, que a medida de A_n é igual à medida de $\mathcal{C}$, ou seja, nula ($\mathrm{m}(A_n) = (b_n - a_n)\mathrm{m}(\mathcal{C}) = 0$) e que A_n é não enumerável. Com isso, para cada n natural, define-se $f_n : [a, b] \longrightarrow \mathbb{R}$ dada por

$$f_n(x) = \begin{cases} 1 \text{ , se } x \in A_n \\ 0 \text{ , caso contrário.} \end{cases}$$

Como A_n tem medida nula então $F(x) := 0$, $\forall x \in [a, b]$, é uma a-primitiva de f_n, porém f_n não admite uma c-primitiva já que A_n é não enumerável. Além disso o conjunto $\{f_n\}$ é linearmente independentes pois, conforme construído, as funções possuem suportes disjuntos. Fica então provada a lineabilidade de $\mathfrak{A}$.

Dada $f \in \overline{Span\{f_n\}}$, então, considerando a topologia gerada pela norma do supremo essencial, $f = \sum_{i=1}^{\infty} \alpha_i f_i$ e portanto

$$f(x) = \begin{cases} \alpha_n \text{ , se } x \in A_n \text{ para algum } n \\ 0\text{, caso contrário .} \end{cases}$$

Lembrando que $\bigcup A_n$ é não enumerável e possui medida nula, temos que F, a função identicamente nula, é uma a-primitiva de f, pois $f(x) = 0 = F'(x)$ para todo x fora de um conjunto de medida nula, isto é, $x \notin \bigcup A_n$.

Por outro lado, como $\bigcup A_n$ é não enumerável f não pode admitir uma c-primitiva já que esta não teria derivada igual a f que difere desta apenas fora de um conjunto enumerável. Sendo assim, $f \in \mathfrak{A}$ e, portanto, $\mathfrak{A}$ é espaçável.

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A NOTE ON THE BOHNENBLUST-HILLE INEQUALITY FOR MULTILINEAR FORMS

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Abstract

The general versions of the Bohnenblust–Hille inequality for m-linear forms are valid for exponents $q_1, ..., q_m \in [1,2]$. In this note we show that a slightly different characterization is valid with no restrictions for the range of the parameters, i.e., for $q_1, ..., q_m \in (0, \infty)$.

1 Introduction

The Bohnenblust-Hille inequality [3] asserts that for all positive integers $m \geq 1$ there is a constant $C = C(\mathbb{K}, m) \geq 1$ such that

$$\left(\sum_{i_1,...,i_m=1}^{\infty} \left| T(e_{i_1}, ..., e_{i_m}) \right|^{\frac{2m}{m+1}} \right)^{\frac{m+1}{2m}} \leq C \left\| T \right\| \tag{1}$$

for all continuous m-linear forms $T : c_0 \times \cdots \times c_0 \to \mathbb{K}$, where $\mathbb{K}$ denotes $\mathbb{R}$ or $\mathbb{C}$. This result can be generalized in some different directions. A very interesting and far reaching generalization is the following (below, $e_j^{n_j}$ means $(e_j, ..., e_j)$ repeated n_j times):

Theorem 1.1 (Albuquerque, Araújo, Nuñes, Pellegrino and Rueda)**.** *[2] Let $1 \leq k \leq m$ and $n_1, \ldots, n_k \geq 1$ be positive integers such that $n_1 + \cdots + n_k = m$, and let $q_1, \ldots, q_k \in [1,2]$. The following assertions are equivalent:*
(I) There is a constant $C^{\mathbb{K}}_{k,q_1 \ldots q_k} \geq 1$ such that

$$\left(\sum_{i_1=1}^{\infty} \left(\sum_{i_2=1}^{\infty} \left(\cdots \left(\sum_{i_{k-1}=1}^{\infty} \left(\sum_{i_k=1}^{\infty} \left| T\left(e_{i_1}^{n_1}, \ldots, e_{i_k}^{n_k} \right) \right|^{q_k} \right)^{\frac{q_{k-1}}{q_k}} \right)^{\frac{q_{k-2}}{q_{k-1}}} \cdots \right)^{\frac{q_2}{q_3}} \right)^{\frac{q_1}{q_2}} \right)^{\frac{1}{q_1}} \leq C^{\mathbb{K}}_{k,q_1 \ldots q_k} \left\| T \right\|$$

for all continuous m-linear forms $T : c_0 \times \cdots \times c_0 \to \mathbb{K}$.
(II) $\frac{1}{q_1} + \cdots + \frac{1}{q_k} \leq \frac{k+1}{2}$.

When $k = m$ we recover a characterization of Albuquerque et al. [2], and finally when $q_1 = \cdots = q_m = \frac{2m}{m+1}$ we recover the Bohnenblust-Hille inequality (1).

In this note we present an extension of the above theorem to $q_1, ..., q_m \in (0, \infty)$. In particular, we remark that in general the condition $\frac{1}{q_1} + \cdots + \frac{1}{q_k} \leq \frac{k+1}{2}$ is not enough to prove (I) for $q_1, ..., q_m \in (0, \infty)$. For instance, if $m = k = 3$ and $(q_1, q_2, q_3) = \left(1, \frac{18}{10}, 3\right)$ we have $\frac{1}{q_1} + \frac{1}{q_2} + \frac{1}{q_3} < \frac{3+1}{2}$ but, as we shall see, (I) is not true. We prove the following:

2 The Main Result

Theorem 2.1. *Let $1 \leq k \leq m$ and $n_1, \ldots, n_k \geq 1$ be positive integers such that $n_1 + \cdots + n_k = m$, and let $q_1, \ldots, q_k \in (0, \infty)$. The following assertions are equivalent:*

(i) There is a constant $C^{\mathbb{K}}_{k,q_1\ldots q_k} \geq 1$ such that

$$\left(\sum_{i_1=1}^{\infty}\left(\sum_{i_2=1}^{\infty}\left(\cdots\left(\sum_{i_{k-1}=1}^{\infty}\left(\sum_{i_k=1}^{\infty}\left|T\left(e_{i_1}^{n_1},\ldots,e_{i_k}^{n_k}\right)\right|^{q_k}\right)^{\frac{q_{k-1}}{q_k}}\right)^{\frac{q_{k-2}}{q_{k-1}}}\cdots\right)^{\frac{q_2}{q_3}}\right)^{\frac{q_1}{q_2}}\right)^{\frac{1}{q_1}} \leq C^{\mathbb{K}}_{k,q_1\ldots q_k}\|T\|$$

for all continuous m-linear forms $T : c_0 \times \cdots \times c_0 \to \mathbb{K}$.

(ii) $\sum_{j\in A} \frac{1}{q_j} \leq \frac{card(A)+1}{2}$ for all $A \subset \{1, ..., k\}$.

Proof (sketch). The route to a proof is to invoke the characterization of Albuquerque et al. [3, Theorem 1.1] (as we mentioned before, this is precisely Theorem 2.1 with $k = m$). Using a lemma and the Kahane-Salem-Zygmund inequality (see [3, Lemma 6.1]), we extend that characterization as follows:

Theorem 2.2. *Let $m \geq 1$, let $q_1, ..., q_m \in (0, \infty)$. The following assertions are equivalent:*

(A) There is a constant $C^{\mathbb{K}}_{q_1\ldots q_m} \geq 1$ such that

$$\left(\sum_{i_1=1}^{\infty}\left(\sum_{i_2=1}^{\infty}\left(\cdots\left(\sum_{i_{m-1}=1}^{\infty}\left(\sum_{i_m=1}^{\infty}\left|T\left(e_{i_1},\ldots,e_{i_m}\right)\right|^{q_m}\right)^{\frac{q_{m-1}}{q_m}}\right)^{\frac{q_{m-2}}{q_{m-1}}}\cdots\right)^{\frac{q_2}{q_3}}\right)^{\frac{q_1}{q_2}}\right)^{\frac{1}{q_1}} \leq C^{\mathbb{K}}_{q_1\ldots q_m}\|T\|$$

for all continuous m-linear forms $T : c_0 \times \cdots \times c_0 \to \mathbb{K}$.

(B) $\sum_{j\in A} \frac{1}{q_j} \leq \frac{card(A)+1}{2}$ for all $A \subset \{1, ..., m\}$.

Finally, the proof is now a consequence from the main result of [1] and theorems 2.2 and 2.1. ■

Remark 2.1. *It is worth mentioning that, using canonical arguments of the theory of absolutely summing operators, our main result can be translated to the theory of (generalized) multiple summing operators sketched in [1, Remark2.6]. For recent results on multiple summing operators we refer to [4] and references therein.*

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CONTINUIDADE EM CADEIA EM SISTEMAS INDUZIDOS

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Abstract

O conceito de continuidade em cadeia - noção mais forte do que a equicontinuidade - é analisado em dois tipos - o coletivo e o probabilístico - de sistemas induzidos por uma transformação contínua ou um homeomorfismo definido sobre um espaço métrico compacto. Mostramos que uma mesma transformação pode induzir sistemas com comportamento totalmente distinto com respeito à continuidade em cadeia.

1 Introdução

Sejam M um espaço métrico compacto com métrica d e $\mathcal{B}_M$ o conjunto de todos os subconjuntos de Borel de M. Denotamos por $\mathcal{C}(M)$ (resp. $\mathcal{H}(M)$) o espaço de todas as transformações contínuas de M em M (resp. de todos os homeomorfismos de M sobre M) munido da métrica

$$\tilde{d}(f,g) := \max_{x \in M} d(f(x), g(x)).$$

Além disso, denotamos por $\mathcal{K}(M)$ o hiperespaço de todos os subconjuntos fechados e não-vazios de M munido da métrica de Hausdorff

$$d_H(X,Y) := \max\big\{\max_{x \in X} d(x,Y), \max_{y \in Y} d(y,X)\big\},$$

e por $\mathcal{M}(M)$ o espaço de todas as medidas de Borel probabilísticas sobre M munido da métrica de Prohorov

$$d_P(\mu,\nu) := \inf\{\delta > 0 : \mu(X) \leq \nu(X^\delta) + \delta \text{ para todo } X \in \mathcal{B}_M\},$$

onde $X^\delta := \{x \in M : d(x,X) < \delta\}$ é a δ-vizinhança de X ($X \subset M$). Os espaços $\mathcal{K}(M)$ e $\mathcal{M}(M)$ são métricos compactos. Agora, dada $f \in \mathcal{C}(M)$, as aplicações induzidas $\overline{f} : \mathcal{K}(M) \to \mathcal{K}(M)$ and $\widetilde{f} : \mathcal{M}(M) \to \mathcal{M}(M)$ são as aplicações contínuas definidas por

$$\overline{f}(X) := f(X) \qquad (X \in \mathcal{K}(M))$$

e

$$(\widetilde{f}(\mu))(X) := \mu(f^{-1}(X)) \qquad (\mu \in \mathcal{M}(M), X \in \mathcal{B}_M).$$

Se f é um homeomorfismo, também o são $\overline{f}$ e $\widetilde{f}$.

$(\overline{f}, \mathcal{K}(M))$ é dito *sistema coletivo induzido por* (f,M) e $(\widetilde{f}, \mathcal{M}(M))$ é dito *sistema probabilístico induzido por* (f,M).

Dado um espaço de Baire Z, dizemos que "o elemento genérico de Z tem uma certa propriedade P" se o conjunto de todos os elementos de Z que não satisfazem a propriedade P é de primeira categoria em Z.

Dada $f : M \to M$, dizemos que f é *contínua em cadeia* no ponto $x \in M$ [1, 3] se para todo $\epsilon > 0$, existe $\delta > 0$ tal que para qualquer escolha de pontos

$$x_0 \in B(x;\delta),\ x_1 \in B(f(x_0);\delta),\ x_2 \in B(f(x_1);\delta), \ldots,$$

tem-se

$$d(x_n, f^n(x)) < \epsilon \quad \text{para todo } n \geq 0.$$

Claramente, continuidade em cadeia é uma propriedade mais forte que equicontinuidade.

2 Resultados Principais

Teorema 2.1. *Para toda $f \in \mathcal{C}(M)$, são equivalentes:*

(i) $\widetilde{f}$ *é contínua em cadeia em algum ponto;*

(ii) $\widetilde{f}$ *é contínua em cadeia em todo ponto;*

(iii) $\bigcap_{n=1}^{\infty} f^n(M)$ *é um conjunto unitário.*

Prova: Veja [5]. ∎

Teorema 2.2. *Suponha que M tem pelo menos dois pontos distintos. Então, o sistema probabilístico induzido $(\widetilde{h}, \mathcal{M}(M))$ não possui pontos de continuidade em cadeia, qualquer que seja $h \in \mathcal{H}(M)$.*

Prova: Decorre imediatamente do teorema anterior. ∎

Em forte contraste com o caso probabilístico, temos o seguinte resultado válido para o caso onde M é o espaço de Cantor $\{0,1\}^{\mathbb{N}}$:

Teorema 2.3. *Para o homeomorfismo genérico $h \in \mathcal{H}(\{0,1\}^{\mathbb{N}})$, o sistema coletivo induzido $(\overline{h}, \mathcal{K}(\{0,1\}^{\mathbb{N}})$ é contínuo em cadeia em todo ponto de um subconjunto aberto e denso em $\mathcal{K}(\{0,1\}^{\mathbb{N}})$.*

Prova: Veja [4]. ∎

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DECAIMENTO DE ENERGIA E CONTROLE PARA UM SISTEMA DE N-CORDAS ACOPLADAS PARALELAMENTE

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Resumo

O propósito deste trabalho consiste em apresentar uma estimativa de decaimento local de energia para a solução de um problema de Cauchy envolvendo um sistema unidimensional de n equações de onda lineares acopladas. Em seguida, tal estimativa é aplicada na resolução de um problema de controle na fronteira envolvendo tal sistema. O controle obtido é do tipo Neuman, age em toda fronteira e é de quadrado integrável para dados iniciais com energia finita. O tempo de controle obtido é qualquer instante maior que o comprimento do intervalo onde os dados iniciais estão definidos.

1 Introdução

Sejam a, b e T números reais tais que $a < b$ e $T > 0$. Por simplicidade, denote o intervalo $]a, b[$ por Ω. Sejam $U = (u^1, u^2, \cdots, u^n)^T$, (T=transposta), $U_{tt} = (u^1_{tt}, u^2_{tt}, \cdots, u^n_{tt})^T$, $U_{xx} = (u^1_{xx}, u^2_{xx}, \cdots, u^n_{xx})^T$ e $A = [a_{ij}]_{n\times n}$ uma matriz diagonalizável com auto-valores não negativos. Considere o sistema

$$\begin{cases} U_{tt} - U_{xx} + AU = 0 & \text{in} \quad \Omega\times]0, T[\\ U(\cdot, 0) = U_0, \quad U_t(\cdot, 0) = U_1 & \text{in} \quad \Omega \\ U_x(a, t) = F(t), \quad U_x(b, t) = G(t) & \text{on} \quad]0, T[. \end{cases} \tag{1}$$

com $F(t) = (f_1(t), f_2(t), \cdots, f_n(t))^T$ e $G(t) = (g_1(t), g_2(t), \cdots, g_n(t))^T$. O sistema (1) descreve vibrações transversais de n cordas homogêneas acopladas, paralelamente por um meio elástico. Tais modelos, bem como outros em dimensões maiores são de grande importância no campo da engenharia e matemática, (veja [3], [5], [2]).

Seja $\mathcal{O}$ um intervalo aberto. Denotemos $(L^2(\mathcal{O}), \|\cdot\|_0)$ e $(H^1(\mathcal{O}), \|\cdot\|_1)$ os espaços de Lebesgue e Sobolev munido com suas normas usuais. Considere os espaços $\mathcal{L}^2(\mathcal{O}) = [L^2(\mathcal{O})]^n$ e $\mathcal{H}^1(\mathcal{O}) = [H^1(\mathcal{O})]^n$ munidos com as normas $\|(u^1, \cdots, u^n)\|_0 = [\|u^1\|_0^2 + \cdots + \|u^n\|_0^2]^{\frac{1}{2}}$ e $\|(u^1, \cdots, u^n)\|_1 = [\|u^1\|_1^2 + \cdots + \|u^n\|_1^2]^{\frac{1}{2}}$ respectivamente. Defina o seguinte espaço de Hilbert $\mathcal{H}(\mathcal{O}) = [H^1(\mathcal{O}) \times L^2(\mathcal{O})]^n$ com a norma natural

$$\|(u_0^1, u_1^1, u_0^2, u_1^2, \cdots, u_0^n, u_1^n)\|^2_{\mathcal{H}(\mathcal{O})} = \|u_0^1\|^2_{H^1(\mathcal{O})} + \|u_1^1\|^2_{L^2(\mathcal{O})} + \cdots + \|u_0^n\|^2_{H^1(\mathcal{O})} + \|u_1^n\|^2_{L^2(\mathcal{O})}.$$

Definimos também $\mathcal{H}_0(\mathcal{O}) = [H_0^1(\mathcal{O}) \times L^2(\mathcal{O})]^n$ munido com a norma do espaço $\mathcal{H}(\mathcal{O})$. Considere o seguinte problema de Cauchy

$$\begin{cases} U_{tt} - U_{xx} + AU = 0 & \text{in} \quad \mathbb{R} \times \mathbb{R} \\ U(\cdot, 0) = U_0, \quad U_t(\cdot, 0) = U_1 & \text{in} \quad \mathbb{R}. \end{cases} \tag{2}$$

A energia da solução U de (2), confinado em $\mathcal{O}$, no instante t, é definido por
$E(U, \mathcal{O}, t) = \frac{1}{2}\|(u^1(\cdot, t), u^1_t(\cdot, t), \cdots, u^n(\cdot, t), u^n_t(\cdot, t))\|^2_{\mathcal{H}(\mathcal{O})}$. É importante saber como a energia se comporta quando $t \to +\infty$, e quando ocorre $E(U, \mathcal{O}, t) \to 0$ para $t \to +\infty$, temos uma importante aplicação na teoria de controle.

2 Resultados Principais

Teorema 2.1. *Dado intervalo aberto $\mathcal{O}$, existem constantes positivas $T_0 > diam(\mathcal{O})$ e $\mathcal{K}$, tais que para quaisquer dados iniciais $(U_0, U_1) \in \mathcal{H}_0(\mathbb{R})$ com $suppU_0, suppU_1 \subset \mathcal{O}$ a energia da solução U de (2) satisfaz*

$$E(U, \mathcal{O}, t) \leq \frac{\mathcal{K}}{t} E(U, \mathcal{O}, 0), \qquad para \qquad t \geq T_0.$$

Teorema 2.2. *Sejam $\Omega =]a, b[$, $T > b - a$ e $(U_0, U_1) \in \mathcal{H}(\Omega)$. Existem funções controle $F, G \in \mathcal{L}^2(]0, T[)$ tais que a solução $U \in \mathcal{H}^1(\Omega \times]0, T[)$ do problema(1) satisfaz a condição final $U(\cdot, T) = 0 = U_t(\cdot, T)$ em Ω.*

Note que o Teorema 2.2 apresenta o $b - a$ como um limitante inferior para o tempo de controle. O argumento decisivo para obtermos tal estimativa para o tempo de controle é o resultado contido no lema abaixo.

Lema 2.1. *Se $U(\cdot, t)$ é a solução do problema de Cauchy (2) com dado inicial $(U_0, U_1) \in \mathcal{H}_0(\mathcal{O})$, então a aplicação $t \longmapsto S_t$, $t \geq \widetilde{T}_0$, em que $S_t : \mathcal{H}_0(\mathcal{O}) \longrightarrow \mathcal{H}(\mathbb{R})$ é dada por $S_t(U_0, U_1) = (U(\cdot, t), U_t(\cdot, t))$, estende-se analiticamente ao setor do plano complexo $\Sigma_0 = \{\zeta = \widetilde{T}_0 + z, \ |arg(z)| \leq \frac{\pi}{4}\}$. Aqui $\widetilde{T}_0$ é uma constante arbitrária maior que $diam(\Omega)$.*

Existe na literatura alguns artigos que lidam com problemas de controle e estabilização para um sistema de equações de ondas acopladas (veja [1], [6]) com os mais diversos tipos de acoplamento e em espaços de dimensão maior que um. Aqui, como em [1], é usado método de controlabilidade apresentado por Russell em [7] e melhorado por Lagnese em [4]. Tal método requer do sistema; lineridade, reversabilidade no tempo, decaimento local de energia e teoremas de traço para obtenção das funções de controle, com a regularidade desejada. Embora o método pareça requerer muito do sistema ele tem vantagem em simulações computacionais de problemas de controle na fronteira (veja [8]).

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CONTROLABILIDADE E ESTRATÉGIAS DO TIPO STACKELBERG-NASH

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Abstract

Neste trabalho provamos novos resultados relacionados a um problema de controle multi-objetivo. Fixada uma equação parabólica, queremos determinar a existência de controles que atuem no sistema de tal modo que a solução se comporte da maneira desejada cumprindo um, ou vários, objetivos pré-estabelecidos. Como os objetivos são, possivelmente, conflitivos um conceito de otimização multi-critério deve ser aplicado, neste caso, aplicamos estratégias do tipo Stackelberg-Nash. Consideramos um controle (o líder), responsável por controlar o sistema a zero e outros dois controles (os seguidores) que vão estar associados a dois objetivos secundários.

1 Introdução

Seja Ω um domínio limitado de $\mathbb{R}^n$ e $T > 0$. Definimos $Q = \Omega \times (0,T)$ e para $i = 1,2$ sejam $\mathcal{O}, \mathcal{O}_i$ subconjuntos abertos de Ω. Nestas condições, consideremos a equação

$$\begin{cases} y_t - \Delta y = f 1_{\mathcal{O}} + v^1 1_{\mathcal{O}_1} + v^2 1_{\mathcal{O}_2} & \text{in} \quad Q, \\ y = 0 & \text{on} \quad \Sigma, \\ y(\cdot,0) = y^0 & \text{in} \quad \Omega, \end{cases} \tag{1}$$

onde y^0 é o dado inicial, f é o controle líder e (v^1, v^2) o par de controles seguidores. Associado a este problema introduzimos os seguintes funcionais custo:

$$J_i(f; v^1, v^2) := \frac{\alpha_i}{2} \iint_{\mathcal{O}_{i,d} \times (0,T)} \left| y - y_{i,d} \right|^2 \, dx\, dt + \frac{\mu_i}{2} \iint_{\mathcal{O}_i \times (0,T)} |v^i|^2 \, dx\, dt, \; i = 1,2. \tag{2}$$

Para este problema, o par (v^1, v^2) será definido como um equilíbrio de Nash no seguinte sentido:

$$J_1(f; v^1, v^2) = \min_{\hat{v}^1} \; J_1(f; \hat{v}^1, v^2), \quad J_2\left(f; v^1, v^2\right) = \min_{\hat{v}^1} \; J_2(f; v^1, \hat{v}^2). \tag{3}$$

Com isso, obtemos o seguinte sistema de otimalidade:

$$\begin{cases} y_t - \Delta y = f 1_{\mathcal{O}} - \dfrac{1}{\mu_1} \phi^1 1_{\mathcal{O}_1} - \dfrac{1}{\mu_2} \phi^2 1_{\mathcal{O}_2} & \text{in} \quad Q, \\ -\phi^i_t - \Delta \phi^i = \alpha_i (y - y_{i,d}) 1_{\mathcal{O}_{i,d}} & \text{in} \quad Q, \\ \quad y = 0, \quad \phi^i = 0 & \text{on} \quad \Sigma, \\ y(\cdot,0) = y^0, \quad \phi^i(\cdot,T) = 0 & \text{in} \quad \Omega. \end{cases} \tag{4}$$

Neste ponto o líder deve ser escolhido de forma a cumprir uma propriedade de controle a zero, ou seja, devemos determinar f de tal forma que

$$y(\cdot,T) = 0. \tag{5}$$

Citamos [3] e [4] como os trabalhos pioneiros neste tipo de abordagem.

Em [1], supondo que $\mathcal{O}_{1,d} = \mathcal{O}_{2,d}$, os autores provaram a existência de um controle f tal que a solução y de (4) satisfaz (5). Neste trabalho provamos um resultado mais geral no sentido que permite situações onde $\mathcal{O}_{1,d} \neq \mathcal{O}_{2,d}$.

2 Resultado principal

Teorema 2.1. *Suponha que*

$$\mathcal{O}_{i,d} \cap \mathcal{O} \neq \emptyset \quad (i = 1, 2) \tag{6}$$

e μ_i *(*$i = 1, 2$*) são suficientemente grandes. Suponha que alguma das duas condições sejam verdadeiras:*

$$\mathcal{O}_{1,d} = \mathcal{O}_{2,d} \quad \text{ou} \quad \mathcal{O}_{1,d} \cap \mathcal{O} \neq \mathcal{O}_{2,d} \cap \mathcal{O}. \tag{7}$$

Então existe uma função positiva $\hat{\rho} = \hat{\rho}(t)$ *com* $\lim_{t \to T} \hat{\rho} = +\infty$ *tal que se*

$$\iint_{\mathcal{O}_{i,d} \times (0,T)} \hat{\rho}^2 |y_{i,d}|^2 \, dx \, dt < +\infty, \quad i = 1, 2, \tag{8}$$

existe um controle $f \in L^2(\mathcal{O} \times (0, T))$ *e um par de Nash associado* (v^1, v^2) *tal que a solução correspondente* y *de* (4) *satisfaz* (5).

Prova: O caso $\mathcal{O}_{1,d} = \mathcal{O}_{2,d}$ foi provado em [1]. O caso em que $\mathcal{O}_{1,d} \cap \mathcal{O} \neq \mathcal{O}_{2,d} \cap \mathcal{O}$ é provado como segue: o resultado de controle do Teorema 2.1 é equivalente à seguinte *desigualdade de observabilidade*

$$\iint_{\Omega} |\psi(x, 0)|^2 \, dx + \sum_{i=1}^{2} \int_Q \hat{\rho}^{-2} |\gamma^i|^2 \, dx \, dt \leq C \int_{\mathcal{O} \times (0,T)} |\psi|^2 \, dx \, dt. \tag{9}$$

Esta desigualdade é provada por meio das desigualdades de Carleman. A dificuldade neste caso foi superada por uma escolha cuidadosa das funções peso. □

Observação 3. *É interessante também o estudo quando pedimos outras condições de equilíbrio para os seguidores. Neste caso, o sistema otimalidade muda, sendo necessário outros argumentos para provar o resultado de controle.*

Observação 4. *É relevante o estudo de estratégia de Stackelberg Nash para outras equações em derivadas parciais. Mencionamos as equações de Navier-Stokes onde grandes dificuldades técnicas são observadas. Neste caso, resultado preliminares foram obtidos.*

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CONTROLE INTERNO PARA UM SISTEMA DE TERMODIFUSÃO LINEAR

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Abstract

Neste trabalho, serão apresentados resultados de controlabilidade para um sistema de termodifusão (modelado segundo a Lei de Cattaneo) em um domínio unidimensional. A controlabilidade obtida é do tipo exato-aproximada, onde será aplicado o Método HUM (Hilbert Uniqueness Method) sem fazer uso do Teorema da Unicidade de Holmgren.

1 Introdução

Esse trabalho foi motivado pelos resultados obtidos por [2], [1] e [4]. Um aspecto relevante a ser mencionado é o fato que, quando a lei de Fourier para propagação de calor, a uma velocidade infinita, é substituída pela lei de Cattaneo o sistema torna-se mais realista do ponto de vista físico.

Estamos interessados no sistema de termodifusão linear dado por

$$\begin{cases} \rho u_{tt} - (\lambda + 2\mu) u_{xx} + \gamma_1 \theta_{1x} + \gamma_2 \theta_{2x} = 0, \ (f_1 1_{(a,b)}) \\ c\theta_{1t} + \sqrt{k} q_{1x} + \gamma_1 u_{tx} + d\theta_{2t} = 0, \\ n\theta_{2t} + \sqrt{D} q_{2x} + \gamma_2 u_{tx} + d\theta_{1t} = 0, \\ \tau_1 q_{1t} + \alpha_1(x) q_1 + \sqrt{k}\theta_{1x} = 0, \ (f_2 1_{(a,b)}) \\ \tau_2 q_{2t} + \alpha_2(x) q_2 + \sqrt{D}\theta_{2x} = 0, \ (f_3 1_{(a,b)}) \end{cases} \tag{1}$$

em $(0, L) \times (0, \infty)$, com condições iniciais

$$\begin{cases} u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \\ \theta_1(x,0) = \theta_{10}(x), \quad \theta_2(x,0) = \theta_{20}(x), \\ q_1(x,0) = q_{10}(x), \quad q_2(x,0) = q_{20}(x), \end{cases} \tag{2}$$

de modo que na fronteira se verifique

$$u(0,t) = u(L,t) = \theta_1(0,t) = \theta_1(L,t) = \theta_2(0,t) = \theta_2(L,t) = 0, \tag{3}$$

onde assumimos que $\alpha_i \in L^\infty(0, L)$, $(i = 1, 2)$ são funções não negativas tais que $supp(\alpha_i) \subset (a, b)$. A expressão $1_{(a,b)}$, denotará a função característica sob o intervalo (a, b).

Para o sistema (1), será estabelecido um resultado de controlabilidade do tipo exato-aproximada, conforme definição abaixo:

Definição 1.1. *O sistema (1) é exato-aproximadamente controlável se, para cada dados iniciais* $(u_0, u_1, \theta_{10}, \theta_{20}, q_{10}, q_{20}) \in H_0^1(0, L) \times L^2(0, L) \times L^2(0, L) \times L^2(0, L) \times L^2(0, L) \times L^2(0, L)$*, dados finais* $(z_0, z_1, \eta_{10}, \eta_{20}, \xi_{10}, \xi_{20}) \in H_0^1(0, L) \times L^2(0, L) \times L^2(0, L) \times L^2(0, L) \times L^2(0, L) \times L^2(0, L)$ *e* $\epsilon > 0$*, podemos encontrar um tempo* $T > 0$*, controles* f_1*,* f_2*, e* f_3 *tal que a solução de* (1)*, com controles* $f_1 1_{(a,b)}$*,* $f_2 1_{(a,b)}$*,* $f_3 1_{(a,b)}$ *atuando na primeira, quarta e quinta equação de 1, satisfaz*

$$\begin{cases} u(T) = z_0, \quad u_t(T) = z_1 \\ |\theta_1(T) - \eta_{10}|_{L^2(0,L)} \le \epsilon, \ |\theta_2(T) - \eta_{20}|_{L^2(0,L)} \le \epsilon, \\ |q_1(T) - \xi_{10}|_{L^2(0,L)} \le \epsilon, \ |q_2(T) - \xi_{20}|_{L^2(0,L)} \le \epsilon \end{cases} \tag{4}$$

2 Resultados Principais

Teorema 2.1. *O sistema (1) é exato-aproximadamente controlável.*

Prova: Para a demonstração desse resultado, considere o sistema adjunto associado ao sistema (1), o qual é dado por

$$\begin{cases} \rho v_{tt} - (\lambda + 2\mu) v_{xx} + \gamma_1 \beta_{1xt} + \gamma_2 \beta_{2xt} = 0, \\ c\beta_{1t} + \sqrt{k} p_{1x} + \gamma_1 v_x + d\beta_{2t} = 0, \\ n\beta_{2t} + \sqrt{D} p_{2x} + \gamma_2 v_x + d\beta_{1t} = 0, \\ \tau_1 p_{1t} - \alpha_1(x) p_1 + \sqrt{k}\beta_{1x} = 0, \\ \tau_2 p_{2t} - \alpha_2(x) p_2 + \sqrt{D}\beta_{2x} = 0, \end{cases} \tag{1}$$

sujeito a

$$\begin{cases} v(0,t) = v(L,t) = \beta_1(0,t) = \beta_1(L,t) = \beta_2(0,t) = \beta_2(L,t) = 0 \\ v(T) = v_0,\ v_t(T) = v_1,\ \beta_1(T) = \beta_{10},\ \beta_2(T) = \beta_{20},\ p_1(T) = p_{10},\ p_2(T) = p_{20}. \end{cases} \tag{2}$$

Para o sistema (1), usando técnicas de multiplicadores e adaptando as ideias feitas em [3], temos

Teorema 2.2. *Seja $(v, \beta_1, \beta_2, p_1, p_2)$ solução de (1) com dados finais $(v_0, v_1, \beta_{10}, \beta_{20}, p_{10}, p_{20}) \in \widetilde{\mathcal{H}}$. Então tem-se a seguinte desigualdade de observabilidade*

$$\| (v_0, \rho v_1 + \gamma_1 \beta_{10x} + \gamma_2 \beta_{20x}, \beta_{10}, \beta_{20}, p_{10}, p_{20}) \|_{\widetilde{\mathcal{H}}} \leq C \int_0^T \int_a^b (v^2 + p_1^2 + p_2^2) dx dt \tag{3}$$

para $T > T_0$ suficientemente grande e com $\widetilde{\mathcal{H}} = L^2(0,L) \times H^{-1}(0,L) \times L^2(0,L) \times L^2(0,L) \times L^2(0,L) \times L^2(0,L)$

Do Teorema 2.2 e adaptando as ideias de [4], tem-se o resultado desejado. ∎

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AN ANALYSIS OF AN OPTIMAL CONTROL PROBLEM FOR MOSQUITO POPULATIONS

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Abstract

We analyze a system of nonlinear partial differential equations modelling the dynamics of certain mosquito populations by taking in consideration the iterations among the immature (aquatic) subpopulation, the adult winged subpopulation and the environment resources; the immature subpopulation is assumed to be age-structured and the model also considers the action of control mechanisms on these subpopulations.

After a first analysis on the existence and uniqueness of solutions for the model, we use the obtained results to prove the existence of an optimal solution of a given optimal control problem.

1 Introduction

We analyse an optimal control problem associated to the following system of partial differential equations related to the one studied in Calsina e Elidrissi in [1]:

$$\begin{cases} u_t(a,t) + u_a(a,t) + m_1(r(t))u(a,t) + \mu_1(c(a,t))u(a,t) = 0, \\ v'(t) + m_2(r(t))v(t) + \mu_2(L_1(c))v(t) = u(l,t), \\ r'(t) - [g(r(t)) - h(L_2(u,v))]r = 0, \\ u(0,t) = b(t)v(t), \\ u(a,0) = u_0(a), \\ v(0) = v_0, \\ r(0) = r_0. \end{cases} \tag{1}$$

This system models the dynamics of a mosquito populations by taking in consideration the interaction among the immature (aquatic) form, the adult (winged) form of the mosquitos and the amount of available resources for survival. The first equation is of Gurtin-MacCamy type and governs the age-structured dynamics of the immature (aquatic) mosquito population, $u = u(a,t)$, where a represents age, and t, time; here, $0 \leq a \leq l$, where $l > 0$ is given and denotes the maturation age, that is, the age when the an aquatic individual becomes adult; $T > 0$ is given and denotes the maximum time of interest; we denote $Q = (0,l) \times (0,T)$. We remark that, for simplicity of the model, eggs, pupae and larval forms were lumped together in a unique aquatic population. The second equation governs the dynamics of the adult (winged) mosquito population, $v = v(t)$, which in the present model, as in Calsina e Elidrissi in [1], is considered non age-structured. The third equation governs the variation of the amount of available resources denoted by $r(t)$. The fourth equation in (1) is the standard renewal condition that requires that new immature individuals enter in the system due to reproduce mechanism of the adults; in this equation, $b(\cdot) \in L^\infty(0,T)$ is a positive function related to the adult fertility rate. Moreover, as in Calsina e Elidrissi in [1], in the first equation of (1), $m_1(r(t))$ is the natural mortality rate of aquatic individuals, while $m_2(r(t))$ in the second equation is the corresponding natural mortality rate of the adults; both such mortalities may depend on the available amount of resources. In the third equation, $g(\cdot)$ is a Verhurst type function associate to the possibility of recovery of the available resources, while the degradation rate of the resources by the populations is given by $h(\cdot)$, which is a known nonnegative function; such degradation is mediated by a linear integral operator $L_2(u,v)(t) = \int_0^l [u(a,t)H_1(a,t) + v(t)H_2(a,t)]da$, where $H_1, H_2 \in L^\infty(Q)$ and $H_1(\cdot), H_2(\cdot) \geq 0$.

We consider the action of an external control variable $c = c(a,t)$, $(a,t) \in Q$, associated for instance with the use of chemical agents, that act by increasing the mortalities rates of the populations; in the case of immature individuals, such action may depend on their respective maturity level. Thus, in the first equation of (1), $\mu_1(c(a,t))$ is an additional mortality rate of the immature form caused by the external control c, while in the second equation $\mu_2(L_1(c)(t)$ is the respective additional mortality rate for the adult form; the control action in this case is mediated by a linear integral operator $L_1(c)(t) = \int_0^l c(a,t)H_0(a,t)da$, where $H_0 \in L^\infty(Q)$ and $H_0(\cdot) \geq 0$. The initial conditions are as follows: u_0 is a nonnegative function in $L^\infty(0,l)$; v_0 and r_0 are nonnegative numbers.

We consider the question of existence of optimal controls associated to (1). This problem is the following: we want to show the existence of a control $c^* \in U$ such that

$$\mathcal{F}(c^*) = \min\{\mathcal{F}(c) : \ c \in U\}. \tag{2}$$

Here, the functional $\mathcal{F}(c)$ to be minimized is

$$\begin{aligned}\mathcal{F}(c) \ &= \rho_0 \int_0^T \int_0^l G(a,u(a,t))dadt + \rho_1 \int_0^T \int_0^l \ |c(a,t)|^{p_1} dadt \\ &+\tilde{\rho}_1 \int_0^T \int_0^l \ |c_t(a,t)|^{\tilde{p}_1} dadt + \overline{\rho}_1 \int_0^T \int_0^l \ |c_a(a,t)|^{\overline{p}_1} dadt \\ &+\rho_2 \int_0^T \ |v(t)|^{p_2} dt + \rho_3 \int_0^T \ |r(t)|^{p_3} dt,\end{aligned} \tag{3}$$

where (u,v,r) is the solution of (1) associated to the c being considered; $G : (0,l) \times \mathbb{R} \to \mathbb{R}$ is a given lower bounded function such that $G(a,y)$ is measurable in a for each fixed y and continuous and convex in y for each fixed a; the weighting constants $\rho_1, \ \tilde{\rho}_1, \ \overline{\rho}_1 > 0$ and $\rho_2, \ \rho_3 \geq 0$ and the exponents $p_1, \ \tilde{p}_1, \ \overline{p}_1, \ p_2, \ p_3 \geq 1$ are given.

The set of admissible controls, U, is defined as:

$$U \text{ is a closed convex set in } L^{p_1}(Q), \text{ with } p_1 \geq 1, \tag{4}$$

2 Main Results

Theorem 2.1. *Suppose that conditions described hold true; then problem* (2)*, where* $\mathcal{F}$ *is defined in* (3)*, the admissible controls are in* U *defined in* (4)*, and dynamics governed by* (1)*, has a solution* $c^* \in U$*.*

Proof To prove the existence of optimal controls for the present problem, we will start with the usual argument using minimization sequences; to proceed with this technique the difficulties are then those of finding suitable estimates that allow us to pass certain limits in the nonlinear terms. Some of these required estimates are exactly the ones given in our existence and uniqueness theorem for (1); next, based on these estimates, we then are able to obtain further ones that allow us to complete the task.

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HIPOELITICIDADE GLOBAL DE OPERADORES INVARIANTES EM VARIEDADES COMPACTAS

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Abstract

Neste trabalho obtemos novos resultados sobre a investigação da Hipoeliticidade Global de operadores lineares do tipo

$$\mathcal{L} \doteq D_t + C(t, x, D_x), (t, x) \in \mathbb{T} \times M,$$

sendo $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ o toro plano, M uma variedade compacta e sem bordo e $C(t, x, D_x)$ um operador pseudo-diferencial de primeira ordem sobre M o qual depende suavemente da variável $t \in \mathbb{T}$.

1 Introdução

Neste trabalho (ver [1]) investiga-se a Hipoelicidade Global de operadores lineares pertencentes a classe

$$\mathcal{L} \doteq D_t + C(t, x, D_x), (t, x) \in \mathbb{T} \times M, \tag{1}$$

sendo que $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ representa o toro plano, M uma variedade suave e fechada munida de uma medida positiva dx e $C(t, x, D_x)$ um operador pseudo-diferencial sobre M, o qual depende suavemente da variável $t \in \mathbb{T}$.

Lembramos que o operador L é globalmente hipoelítico em $\mathbb{T} \times M$ (GH por simplicidade) quando as condições

$$\mathcal{L} \cdot u \in C^\infty(\mathbb{T} \times M) \quad \text{e} \quad u \in \mathcal{D}'(\mathbb{T} \times M)$$

implicam em $u \in C^\infty(\mathbb{T} \times M)$.

Neste trabalho é proposta uma nova abordagem para o estudo da hipoeliticidade global, cujas principais inspirações são as seguintes:

i) os resultados obtidos por J. Hounie (Trans AMS, 1979, ver [4]) a respeito dos operadores do tipo $\partial_t + b(t, A)$, sendo $t \in \mathbb{T}$ e A um operador linear, auto-adjunto densamente definido num espaço de Hilbert H;

ii) o estudo feito por S. Greenfield e N. Wallach (Trans AMS, 1973, ver [5]) no qual se investiga a hipoeliticidade global de operadores invariantes com respeito aos auto-espaços de um operador elíptico E previamente fixado;

iii) a recente generalização para a noção de invariância para operadores pseudodiferenciais em variadedades compactas obtidas por J. Delgado e M. Ruzhansky (C.R. Math. Acad. Sci.,2014, ver [3]), na qual considera-se a discretização dos operadores através da expansão de Fourier obtida por operadores elípticos;

iv) a redução a forma normal para operadores de priemira ordem sobre o toro, no sentido dado por D. Dickinson, T. Gramchev e M. Yoshino (Proc. Edinb. Math. Soc., 2002, ver [2]);

Para esta exposição consideramos o caso da separação de variáveis, isto é, para a classe de operadores

$$\mathcal{L} \doteq D_t + a(t)p(x, D_x) + ib(t)q(x, D_x), \quad (t, x) \in \mathbb{T} \times M. \tag{2}$$

Mostremos então como os trabalhos de (i) a (iv) podem ser combinados para se introduzir o conceito de sequências Diofantinas, as quais se tornam condições necessárias e suficientes para se obter hipoeliticidade global.

2 Resultados Principais

Sejam $E = E(x, D_x)$ um operador pseudo-diferencial, normal e elíptico definido na variedade M e $p(x, D_x)$, $q(x, D_x)$ operadores pseudo-diferenciais, auto-adjuntos de primeira ordem pertencentes ao centralizador de E, isto é,

$$[p(x, D_x), E(x, D_x)] = 0 \ \text{ e } \ [q(x, D_x), E(x, D_x)] = 0.$$

Assuma que os auto-espaços E_j de E sejam unidimensionais, para cada $j \in \mathbb{N}$, e considere as sequências reais $\{\mu_j\}_{j\in\mathbb{N}}$ e $\{\nu_j\}_{j\in\mathbb{N}}$ dadas por

$$p(x, D_x) \cdot \varphi_j(x) = \mu_j \varphi_j(x) \ \text{ e } \ q(x, D_x) \cdot \varphi_j(x) = \nu_j \varphi_j(x),$$

sendo $\{\varphi_j(x)\}_{j\in\mathbb{N}}$ uma base ortonormal de autofunções de $L^2(M)$ determinada por E.

Considere ainda as médias

$$a_0 = (2\pi)^{-1} \int_0^{2\pi} a(\tau) d\tau, \quad b_0 = (2\pi)^{-1} \int_0^{2\pi} b(\tau) d\tau. \tag{3}$$

e o conjunto $\Gamma_{a_0} = \{j \in \mathbb{N};\ a_0\mu_j \in \mathbb{Z}\}$.

Theorem 2.1. *Seja L o operador (2), tal que zero não é ponto de acumulação de $\{\nu_j\}$.*

i. se $b \equiv 0$, então L é (GH) se, e somente se, o conjunto Γ_{a_0} é finito e vale a condição Diofantina

$$\inf_{\ell\in\mathbb{Z}} |a_0\mu_j + \ell| = O(j^{-\delta}),\ j \to \infty,\ \text{para algum } \delta \geqslant 0. \tag{4}$$

ii. se $b \not\equiv 0$ e b não muda de sinal, então L é (GH);

iii. se b muda de sinal, então L não é (GH), caso exista $\{\nu_{j_k}\}$ satisfazendo

$$\lim_{k\to\infty} \frac{|\nu_{j_k}|}{\log(j_k)} = +\infty.$$

Entretanto, caso

$$\limsup_{j\to\infty} \frac{|\nu_j|}{\log(j)} = \kappa < +\infty,$$

então L é (GH) se, e somente se, $L_{a_0,b_0} \doteq D_t + a_0 p(x, D_x) + ib_0 q(x, D_x)$ é (GH), ou seja,

a. se $b_0 \neq 0$, então L_{a_0,b_0} is (GH);

b. se $b_0 = 0$, então L_{a_0,b_0} é (GH) se, e somente se Γ_{a_0} é finito e vale (4).

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ALGEBRABILIDADE DE CERTOS SUBCONJUNTOS DA ÁLGEBRA DE DISCO

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Abstract

Neste trabalho estudamos o subconjunto das funções pertencentes à álgebra de disco que não pertecem a certas álgebras de Dales-Davie. Mostramos que o conjunto $\left\{f \in \mathcal{A}(D) : \sum_{n=0}^{\infty} \frac{\|f^{(n)}\|}{n!} = +\infty\right\}$ é algebrável.

1 Introdução

Na última década, diversos autores têm buscado por estruturas algébricas em espaços de funções contínuas possuindo alguma propriedade especial. Se um espaço vetorial V possui um subconjunto M tal que $M \cup \{0\}$ contém um espaço vetorial de dimensão infinita, então M é chamado de *lineável*. Este conceito é devido Gurariy [6], também [7], que provou que existe um espaço vetorial de dimensão infinta contido no conjunto das funções que não são diferenciáveis em nenhum ponto do intervalo $[0, 1]$. Esta definição sugere naturalmente o conceito de *algebrabilidade*, neste caso busca-se por álgebras infinitamente geradas contidas em subconjuntos de alguma álgebra. Este conceito foi também estudado por Gurariy e primeiramente definido em [2], tendo sido extensivamente estudado por outros autores, como por exemplo [2, 3, 4].

Em 1973, Dales e Davie [5] introduziram e estudaram certas álgebras de funções diferenciáveis em subconjuntos compactos e perfeitos do plano complexo $X \subset \mathbb{C}$. Tais álgebras foram chamadas de *álgebras de Dales-Davie* por Abtahi e Honary em [1] e denotadas por $\mathcal{D}(X, M)$. Se $D \subset \mathbb{C}$ denota o disco aberto unitário e $X = \overline{D}$, então $\mathcal{D}(\overline{D}, M)$ é uma subálgebra da álgebra de disco $\mathcal{A}(D)$. Neste trabalho estudamos o quão grande é a diferença $\mathcal{A}(D) \setminus \mathcal{D}(\overline{D}, M)$. Neste sentido, mostramos que o conjunto $\mathcal{A}(D) \setminus \mathcal{D}(\overline{D}, M)$ contém uma álgebra infinitamente gerada, sendo portanto algebrável.

2 Resultados Principais

Seja $X \subset \mathbb{C}$ um subconjunto compacto e perfeito do plano complexo. Uma função $f : X \longrightarrow \mathbb{C}$ é **diferenciável em um ponto** $z_0 \in X$ se o seguinte limite existe:

$$f'(z_0) = \lim \left\{ \frac{f(z) - f(z_0)}{z - z_0} \; : \; z \in X \, , z \to z_0 \right\}.$$

A função f é **diferenciável em** X se ela for diferenciável em todos os pontos de X. A álgebra das funções em X com n-ésimas derivadas contínuas será denotada por $\mathcal{D}^n(X)$, e $\mathcal{D}^\infty(X)$ denotará a álgebra $\cap_{n=0}^{\infty} \mathcal{D}^n(X)$.

Uma sequência $(M_n)_{n\in\mathbb{N}}$ de números positivos é chamada de **sequência de álgebra** se $M_0 = 1$ e para cada $n \geq 1$, $\frac{M_n}{M_k \, M_{n-k}} \geq \binom{n}{k}$ $\quad (0 \leq k \leq n)$.

As **álgebras de Dales-Davie** em X são definidas por

$$\mathcal{D}(X, M) = \left\{ f \in D^\infty(X) \; : \; \sum_{n=0}^{\infty} \frac{\|f^{(n)}\|_X}{M_n} < +\infty \right\}.$$

A norma em $\mathcal{D}(X, M)$ é dada por $\|f\| = \sum_{n=0}^{\infty} \dfrac{\|f^{(n)}\|_X}{M_n}$. Quando (M_n) é uma sequência de álgebra, então $\mathcal{D}(X, M)$ é uma álgebra normada.

Quando $X = \overline{D}$ temos que $\mathcal{D}(\overline{D}, M)$ é uma subálgebra de $\mathcal{A}(D)$, a clássica álgebra de disco.

Neste trabalho, queremos investigar a algebrabilidade do conjunto $\mathcal{A}(D) \setminus \mathcal{D}(\overline{D}, M)$. Para uma sequência de álgebra fixada $(M_n)_{n \in \mathbb{N}}$, denotaremos

$$\mathcal{H}(M) = \Big\{ f \in \mathcal{A}(D) \, : \, \sum_{n=0}^{\infty} \frac{\|f^{(n)}\|_X}{M_n} = +\infty \Big\}.$$

Quando $M_n = n!$, para todo $n \in \mathbb{N}$, escrevemos $\mathcal{H}$ ao invés de $\mathcal{H}(M)$.

Nossos principais resultados podem ser resumidos da seguinte forma:

Teorema 2.1. *(1) O conjunto $\mathcal{H}$ é não vazio;*

(2) O conjunto $\mathcal{H}$ é espaçável (possui um espaço vetorial fechado *de dimensão infinita);*

(3) O conjunto $\mathcal{H}$ é algebrável.

Para a demonstração dos resultados, alguns lemas técnicos são necessários. A natureza de certas subálgebras infinitamente geradas de $\mathbb{C}[z_1, z_2]$, a álgebra dos polinômios em duas variáveis complexas, também é muito útil para a prova do item (3) do Teorema 2.1. Neste sentido, o conceito de *funções algebricamente independentes* aparece. Como também temos que trabalhar com derivadas n-ésimas de funções compostas, a fórmula de Hoppe, uma variação da Fórmula de Faá di Bruno [8] é de grande utilidade para nossos cálculos.

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