

# **Random Fields on the Sphere of Planet Earth, and their connections to Mathematical Analysis and Approximation Theory.**

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## Abstract

An increasing attention from several branches of sciences has been given to natural or anthropogenic phenomena that evolve temporally on a global scale, that is over the whole sphere representing our planet. Amongst them, climate change, environmental and atmospheric phenomena, etc.

On a mathematical and statistical viewpoint, it is often convenient to imagine that such phenomena are realizations of some underlying random field. If also Gaussianity is assumed, then covariance functions cover a central importance for both inference and prediction.

Covariance functions are positive definite, a non-trivial requirement that has attracted the interest of several scientists from mathematical analysis, probability theory, and numerical analysis, amongst others.

In this talk we review the characterization of positive definite functions on spheres, from the celebrated work of Iso Schoenberg (1942). We then present the more recent result in mathematical analysis and statistics, with special emphasis on a personal selection of works where we have collaborated recently. Special attention will be put on characterization of such functions in product spaces  $\mathbf{d}\mathbf{x}\mathbf{G}$ , for  $\mathbf{G}$  being any locally compact group.

Some connections with mathematical and numerical analysis will then be introduced, in the light of PDE's approaches related to Gaussian fields, and in the context of interpolation through radial basis functions.