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Dynamics of semigroups of operators

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Linear dynamics is a rapidly evolving branch of functional analysis. In this series of lectures, I will give a short account on the dynamics of groups or semigroups of operators. Let Γ be a group (resp. a semigroup), let X be a Banach or Fréchet space and let $(T_\gamma)_{\gamma \in \Gamma}$ be a group (resp. a semigroup) of operators. We will say that $(T_\gamma)_{\gamma \in \Gamma}$ is hypercyclic provided there exists a vector $x \in X$ such that $\{T_\gamma x; \gamma \in \Gamma\}$ is dense in X . We will concentrate ourselves on the cases $\Gamma = \mathbb{Z}_+$, $\Gamma = \mathbb{Z}$, $\Gamma = (0, +\infty)$ or $\Gamma = \mathbb{R}^n$.

We will first show how to prove that a group (T_γ) is hypercyclic and we will give classical examples coming from operator theory or from the theory of partial differential equations. Next, we will investigate structural properties of operator groups. In particular, we will show that if a C_0 -semigroup $(T_t)_{t>0}$ is hypercyclic, then any nontrivial subsemigroup of it is hypercyclic. The last part will be devoted to the problem of the existence of common hypercyclic vectors for operators in a semigroup.

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