The PDEs of Mathematical Finance

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Of concern is the parabolic partial differential equation

$$\frac{\partial u}{\partial t} = \alpha x^k \frac{\partial^2 u}{\partial x^2} + (\beta + \gamma x) \frac{\partial u}{\partial x} + (\delta + \varepsilon x) u$$

for $t > 0$, $x \in J$. Here the Greek letters denote real constants with $\alpha > 0$. The generalized heat equation corresponds to $J = \mathbb{R}$, $k = 0$, $\gamma = \varepsilon = 0$; the generalized Black-Scholes equation corresponds to $J = (0, \infty)$, $k = 2$, $\beta = \varepsilon = 0$. The generalized Cox-Ingersoll-Ross equation corresponds to $J = (0, \infty)$, $k = 1$, $\delta = 0$, and β, γ both nonzero. These are deterministic equations having stochastic backgrounds in the mathematical finance context. These equations are studied on weighted sup norm spaces with various positive weights w,

$$Y_w = \{ f \in C(J) : wf \in C_0(J) \} .$$

Results to be presented include semigroup geneation, chaos for the generalized heat and Black-Scholes equations, and a new Feynman-Kac type formula for the Cox-Ingersoll-Ross equation. This work is joint with G. R. Goldstein, H. Emamirad, R. Mininni, Ph. Rogeon and S. Romanelli.