

VIII ENAMA

O Encontro Nacional de Análise Matemática e Aplicações (ENAMA) é um evento científico anual com propósito de criar um fórum de debates entre alunos, professores e pesquisadores de instituições de ensino e pesquisa, tendo como áreas de interesse: Análise Funcional, Análise Numérica, Equações Diferenciais Parciais, Ordinárias e Funcionais.

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ADAMS INEQUALITIES AND EXTREMAL FUNCTIONS ON UNBOUNDED DOMAINS

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1 Introduction

This work is concerned on the problem of finding optimal Sobolev inequalities and the attainability to the associated variational problem for the borderline case known nowadays as Trudinger-Moser-Adams case (cf [1, Theorem 1]). The main purpose is three-fold: First we obtain a scaling invariant inequality for the higher order Sobolev space of radially symmetric functions and prove the existence of extremal to the associated variacional problem. Second we prove a result about nonexistence of extremals for Adams type inequality [5, Theorem 1.4] and [3, Theorem 1.1] in the Hilbert case. Third, in line with the Concentration Compactness Principle due to P.-L. Lions [4], we will obtain an improvement for Adams exponent in certain classes of sequence on $W_0^{m,n/m}(\Omega)$, for any arbitrary domain.

2 Mathematical Results

First we establish the following Adams type inequality of the scaling invariant form.

Theorem 2.1. *Let $n > m \geq 2$ be integers. Then given $\beta \in (0, \beta_0)$ there exists $C_{\beta,m,n} = C(\beta, m, n)$ depending only on β , m and n such that*

$$\mu_{\beta,n,m} := \sup_{\substack{u \in W_{rad}^{m,n/m}(\mathbb{R}^n) \setminus \{0\} \\ \|\nabla^m u\|_{\frac{n}{m}} = 1}} \frac{1}{\|u\|_{n/m}^{n/m}} \int_{\mathbb{R}^n} \Phi\left(\beta(|u|)^{n/(n-m)}\right) dx \leq C_{\beta,m,n}. \quad (2.1)$$

for all $u \in W_{rad}^{m,n/m}(\mathbb{R}^n) \setminus \{0\}$, where

$$\Phi(t) := e^t - \sum_{j=0}^{j_{m,n}-2} \frac{t^j}{j!}, \quad j_{m,n} := \min\{j \in \mathbb{N} : j \geq n/m\}, \quad (2.2)$$

$$\beta_0 = \beta_0(m, n) = \begin{cases} \frac{n}{\omega_{n-1}} \left[\frac{\pi^{\frac{n}{2}} 2^m \Gamma(\frac{m+1}{2})}{\Gamma(\frac{n-m+1}{2})} \right]^{n/(n-m)}, & m \text{ odd}, \\ \frac{n}{\omega_{n-1}} \left[\frac{\pi^{\frac{n}{2}} 2^m \Gamma(\frac{m}{2})}{\Gamma(\frac{n-m}{2})} \right]^{n/(n-m)}, & m \text{ even}, \end{cases} \quad (2.3)$$

and $W_{rad}^{m,n/m}(\mathbb{R}^n)$ denote the space of the radial $W^{m,n/m}(\mathbb{R}^n)$ -functions. Moreover, for $\beta \in [\beta_0, \infty)$ inequality (2.1) fail and $\mu_{\beta,n,m}$ is attained for all $\beta \in (0, \beta_0)$.

Secondly, for $n, m \geq 2$ integers and $1 < q < \infty$ a real number, we consider the Sobolev space $W^{m,q}(\mathbb{R}^n)$ endowed with the norm

$$\|u\|_{m,n,q}^q = \begin{cases} \|\nabla(-\Delta + I)^k u\|_q^q + \|(-\Delta + I)^k u\|_q^q, & \text{for } m = 2k + 1; \\ \|(-\Delta + I)^k u\|_q^q, & \text{for } m = 2k, \end{cases}$$

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which is equivalent to the usual Sobolev norm in $W^{m,q}(\mathbb{R}^n)$. Now we denote the extremal constant for the Adams type inequalities, [5, Theorem 1.4] and [3, Theorem 1.1], by

$$\eta_{\beta,n,m} := \sup_{\substack{u \in W^{m,\frac{n}{m}}(\mathbb{R}^n) \\ \|u\|_{m,n}, \frac{n}{m} \leq 1}} \int_{\mathbb{R}^n} \Phi(\beta|u|^{n/(n-m)}) \, dx. \quad (2.4)$$

By [5, Theorem 1.4] and [3, Theorem 1.1], we know that $\eta_{\beta,n,m}$ is bounded for $\beta \leq \beta_0$ and infinite for $\beta > \beta_0$. So we prove the following theorem

Theorem 2.2. $\eta_{\beta,n,m}$ is not attained when $n/m = 2$ and $0 < \beta \ll (4\pi)^m m! = \beta_0 = \beta_0(m, 2m)$.

In the last direction,

Theorem 2.3. Let $\Omega \subset \mathbb{R}^n$ be any arbitrary domain. Assume that $u_i, u \in W_0^{m,n/m}(\Omega)$, $\|u_i\|_{m,n} \leq 1$, $u \neq 0$ and $u_i \rightarrow u$ in $W_0^{m,n/m}(\Omega)$. Then, given $\gamma \in [1, \eta]$, there exists a constant $C = C(\gamma, \Omega) > 0$ such that

$$\sup_i \int_{\Omega} \Phi\left(\beta_0 \gamma |u_i|^{n/(n-m)}\right) \, dx \leq C,$$

where $\eta = \eta_{m,n}(u) := \left(1 - \|(I - \Delta)^k u\|_{n/m}^{n/m}\right)^{-m/(n-m)}$ if $m = 2k + 1$ or $m = 2k$ for some $k \in \mathbb{N}$.

Proof of Theorem 2.1: Considering the following operators $I_t(u)(x) := t^{m/n} u(t^{1/n}x)$ and $J_s(u)(x) := u(s^{1/n}x)$, we can $\|u\|_{n/m} = 1$ and $\|\nabla^m u\|_{n/m} = 1$ in (2.1). Then given $u \in W_{rad}^{m,n/m}(\mathbb{R}^n)$ satisfying $\|u\|_{n/m} = 1$ and $\|\nabla^m u\|_{n/m} = 1$ we divide de integral $\int_{\mathbb{R}^n} \Phi(\beta|u|^{n/(n-m)}) \, dx$ inside and outside a some ball B_{R_0} . To show the boundedness outside of the ball we use a radial lemma and to show the boundedness inside the ball we use some Adams type inequality proved by C. Tarsi (cf [6]).

To prove the existence of extremal we consider some maximizer sequence $(u_i) \subset W_{rad}^{m,n/m}(\mathbb{R}^n)$ to $\mu_{\beta,n,m}$ such that $\|u_i\|_{n/m} = 1$ and $\|\nabla^m u_i\|_{n/m} = 1$ and the week limit $u \in W_{rad}^{m,n/m}(\mathbb{R}^n)$. Then we proceed with the study of compactness of the functional. We show that $u \not\equiv 0$ and then, for $u \not\equiv 0$, the sequence (u_i) converge in the functional.

Proof of Theorem 2.2: First, using some comparison result, we prove that if (2.4) has an extremal function then must have some extremal function $u \in W_{rad}^{m,n/m}(\mathbb{R}^n)$. Then, by contradiction, using the inequality (2.1) we prove that $\eta_{\beta,n,m}$ can not have an extremal function $u \in W_{rad}^{m,2}(\mathbb{R}^n)$ for $\beta > 0$ sufficiently small.

Proof of Theorem 2.3: The argument used here follows the ideas used to prove a similar result for a bounded domain in [2]. Using the comparison result and [2, Lemma 2] we can change the sequence (u_i) by a suitable sequence and prove the boundedness using inequalities [5, Theorem 1.4] and [3, Theorem 1.1].

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EXISTÊNCIA E NÃO EXISTÊNCIA DE SOLUÇÃO GLOBAL PARA UMA CLASSE DE EQUAÇÕES DE ONDAS NÃO LINEARES DE SEXTA ORDEM

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Apresentaremos, neste trabalho, um estudo acerca de existência e unicidade de solução para o problema de Cauchy envolvendo uma classe de equações de onda não lineares de sexta ordem. Uma vez estabelecida, sob as hipóteses adequadas, a existência e unicidade de solução para tal problema de Cauchy apresentaremos, através do método do poço potencial (*potential well*), um estudo acerca da existência e não existência de solução global para tal problema de Cauchy.

1 Introdução

Vamos considerar o seguinte problema de Cauchy para a equação da onda não linear de sexta ordem

$$u_{tt} - au_{xx} + u_{xxxx} + u_{xxxxtt} = \varphi(u_x)_x \quad (1.1)$$

$$u(x, 0) = u_0, \quad u_t(x, 0) = u_1(x) \quad (1.2)$$

onde $a > 0$, $\varphi(z) = \alpha|z|^p$, $p > 1$ inteiro e $\alpha \neq 0$. Além disso, consideramos os dados iniciais $u_0, u_1 \in H^s(\mathbb{R})$, $s > 0$. Tal equação, introduzida por Roseneu [1], foi estudada, na forma do problema de Cauchy (1.1) e (1.2), em [2 – 3].

Em nosso trabalho estudaremos, em primeiro lugar, a existência e unicidade de solução local para o problema de Cauchy (1.1) e (1.2). Para este fim, faremos uso do Teorema do ponto fixo de Banach. Posteriormente, fazendo uso do método do poço potencial, abordaremos o problema de existência e não existência de solução global.

2 Resultados principais

Teorema 2.1. *Suponha que $\frac{3}{2} < s < p + 1$, $u_0, u_1 \in H^s(\mathbb{R})$. Deste modo, o problema de Cauchy (1.1),(1.2) admite uma única solução local $u(x, t)$, definida sobre um intervalo de tempo maximal $[0, T_0]$ com $u \in C^1([0, T_0]; H^s(\mathbb{R}))$. Além disso, se*

$$\sup_{t \in [0, T_0]} (\|u(t)\|_{H^s(\mathbb{R})} + \|u_t(t)\|_{H^s(\mathbb{R})}) < \infty$$

então, $T_0 = \infty$.

Supondo $2 \leq s < p + 1$, $u_0, u_1 \in H^s(\mathbb{R})$ e $[0, T_0]$ o intervalo de tempo maximal de existência da solução do problema de Cauchy (1.1) e (1.2) vamos considerar, para cada $t \in [0, T_0]$, o seguinte funcional de energia associado a este problema de Cauchy,

$$E(t) = \frac{1}{2} \left(\|u_t(t)\|_{L^2(\mathbb{R})}^2 + a\|u_x(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xx}(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xxt}(t)\|_{L^2(\mathbb{R})}^2 \right) + \frac{\alpha}{p+1} \int_{\mathbb{R}} |u_x(x, t)|^p u_x(x, t) dx.$$

Denotando por,

$$C_0 = \sup_{0 \neq u \in \dot{H}^2(\mathbb{R})} \frac{\|u_x(t)\|_{L^{p+1}}}{(a\|u_x(t)\|_{L^2}^2 + \|u_{xx}(t)\|_{L^2}^2)^{\frac{1}{2}}}$$

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$$d = \frac{p-1}{2(p+1)} |\alpha|^{-\frac{2}{p-1}} C_0^{-\frac{2(p+1)}{p-1}}$$

podemos definir os conjuntos estável (poço potencial) e instável como,

$$\begin{aligned} W &= \left\{ u(t) \in H^2(\mathbb{R}); a\|u_x(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xx}(t)\|_{L^2(\mathbb{R})}^2 < \frac{2(p+1)}{p-1} d \right\} \\ V &= \left\{ u(t) \in H^2(\mathbb{R}); a\|u_x(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xx}(t)\|_{L^2(\mathbb{R})}^2 > \frac{2(p+1)}{p-1} d \right\}. \end{aligned}$$

Teorema 2.2. Assuma que $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$. Se $E(0) < d$ e $u_0 = u(0) \in W$ então o problema de Cauchy (1.1) e (1.2) tem uma única solução global $u \in C^1([0, \infty); H^s(\mathbb{R}))$ e $u(t) \in W$ para todo $t \in [0, \infty)$.

Teorema 2.3. Suponha que $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$. Se $E(0) \leq d$ e $a\|u_{0x}\|_{L^2(\mathbb{R})}^2 + \|u_{0xx}\|_{L^2(\mathbb{R})}^2 \leq \frac{2(p+1)}{p-1} d$, então o problema de Cauchy (1.1) e (1.2) admite uma única solução global $u \in C^1([0, \infty); H^s(\mathbb{R}))$.

Teorema 2.4. Assuma que $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$. Se $E(0) \leq d$, $u_0 = u(0) \in V$ e $(u_0, u_1)_{L^2(\mathbb{R})} + (u_{0xx}, u_{1xx}) \geq 0$, quando $E(0) = d$, então a solução do problema de Cauchy (1.1) e (1.2) explode em tempo finito.

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THE EFFECT OF SIGNAL ON THE SCALAR CHEN-SIMONS EQUATION

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1 Introduction

We investigate the effect of admitting signed measures on the scalar Chern-Simons equation

$$-\Delta u + e^u(e^u - 1) - 1 = \mu \quad \text{em } \Omega \quad (1.1)$$

with Dirichlet condition, in terms of stability of solutions. In a previous work [2], approximating μ in the weak*-topology by a nonnegative sequence of Radon measures, we show that the sequence of solutions converges to largest subsolution of the Dirichlet problem, that is the limit of solutions satisfies the scalar Chern-Simons with the largest measure less than μ such that (1.1) has a solution, denoted by μ^* . We are now interested in analysing the approximating scheme by sequences of signed measures Radon. Unlike the former case, the difference of the convergence speed between negative and positive parts of the sequence of measures will produce extra Dirac measures, that is if the sequence of solutions converges, then the limit solves (1.1) with μ^* suffering a mass loss given by a Dirac measures sum. Furthermore, all measures obtained from μ^* by decreasing a such type of sum can arise as data in the scalar Chern-Simon problem for a limit solution.

2 Mathematical Results

In a couple of results, we give a complete characterization of the limit solutions of scalar Chern-Simons problems.

Theorem 2.1. *Let $(\mu_n)_{n \in \mathbb{N}}$ be a sequence of Radon measures in Ω such that*

$$\mu_n(\{x\}) \leq 2\pi, \quad \forall x \in \Omega, \quad \forall n \in \mathbb{N}$$

and let u_n be solution of

$$\begin{cases} -\Delta u_n + e^{u_n}(e^{u_n} - 1) = \mu_n & \text{in } \Omega, \\ u_n = 0 & \text{on } \partial\Omega. \end{cases} \quad (2.2)$$

If the sequence $(\mu_n)_{n \in \mathbb{N}}$ converges weakly to μ and the sequence $(u_n)_{n \in \mathbb{N}}$ converges to u in $L^1(\Omega)$. Then there exist $c_1, \dots, c_m \geq 0$ and $r_1, \dots, r_m \in \Omega$ such that u solves (1.1) with datum $\mu^ - \sum_{i=1}^m c_i \delta_{r_i}$, where μ^* is the largest measure less than or equal to μ satisfying*

$$\mu^*(\{x\}) \leq 2\pi \quad \forall x \in \Omega.$$

Theorem 2.2. *Let μ be a Radon measure in Ω , $c_1, \dots, c_m \geq 0$ and $r_1, \dots, r_m \in \Omega$. Then there exists a sequence $(\mu_n) \subset \mathcal{C}_c^\infty(\Omega)$ such that $(\mu_n)_{n \in \mathbb{N}}$ converges weakly to μ and the sequence $(u_n)_{n \in \mathbb{N}}$ of the solution of the scalar Chern-Simons problem (2.2) associated to $(\mu_n)_{n \in \mathbb{N}}$ converges to the solution of (1.1) with datum $\mu^* - \sum_{i=1}^m c_i \delta_{r_i}$.*

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ESTABILIZAÇÃO, ANÁLISE E SIMULAÇÃO NUMÉRICA DA EQUAÇÃO DE ONDAS COM CONDIÇÃO DA ACÚSTICA NA FRONTEIRA

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1 Introdução

Considere $\Omega \subset \mathbb{R}^n$ aberto e limitado com fronteira $\partial\Omega = \Gamma_0 \cup \Gamma_1$ de classe $C^2(\Omega)$ e $\Gamma_0 \cap \Gamma_1 = \emptyset$.

Os objetivos principais deste trabalho são determinar a existência e unicidade de soluções, e a simulação numérica do sistema acoplado

$$\left\{ \begin{array}{l} u_{tt}(x, t) - \Delta u(x, t) = 0 \quad \text{em } Q = \Omega \times (0, T), \\ u(x, t) = 0 \quad \text{sobre } \Sigma_0 = \Gamma_0 \times (0, T), \\ u_t(x, t) + f_1(x)z_{tt}(x, t) + f_2(x)z_t(x, t) + f_3(x)z(x, t) = 0 \quad \text{sobre } \Sigma_1 = \Gamma_1 \times (0, T), \\ \frac{\partial u}{\partial \nu}(x, t) - h(x)z_t(x, t) + g(x)u_t(x, t) = 0 \quad \text{sobre } \Sigma_1 = \Gamma_1 \times (0, T), \\ z_t(x, 0) = \frac{1}{h(x)} \left(\frac{\partial u_0}{\partial \nu}(x) + g(x)u_1(x) \right) \quad \text{sobre } \Gamma_1, \\ z(x, 0) = z_0(x) \quad \text{sobre } \Gamma_1, \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) \quad \text{em } \Omega, \end{array} \right. \quad (1.1)$$

onde o par (u, z) representam, respectivamente, o deslocamento de ondas e a propagação de ondas de som, e as funções f_i , g , $h : \bar{\Gamma}_1 \rightarrow \mathbb{R}$ são contínuas e positivas em $\bar{\Gamma}_1$, para $i = 1, 2, 3$ e $h(x) = 0 \forall x \in \bar{\Gamma}_0 \cap \bar{\Gamma}_1$.

Sobre a fronteira Γ_1 temos as equações (1.1)₃ e (1.1)₄, que foram introduzidas em [2] e representam as equações da Acústica. Além disso, na equação (1.1)₄ é considerado o termo de amortecimento, $g(x)u_t(x, t)$, o qual produz o decaimento assintótico da energia total associada a solução do sistema (1.1). Sobre Γ_0 , temos a condição clássica de Dirichlet, na qual há absorção total do som. Este trabalho foi motivado pelo artigo [1].

2 Resultados

Será usado a notação usual dos espaços funcionais de Lebesgue e de Sobolev, em particular, necessitamos dos espaços funcionais $V = \{\varphi \in H^1(\Omega), \gamma_0(\varphi) = 0 \text{ q.s em } \Gamma_0\}$ e $H_\Delta = \{\varphi \in H^1(\Omega), \Delta\varphi \in L^2(\Omega)\}$.

Teorema 2.1. *Suponha $u_0 \in V \cap H_\Delta(\Omega)$, $u_1 \in V$ e $z_0 \in L^2(\Gamma_1)$ tais que $\frac{\partial u_0}{\partial \nu} - hz_t(x, 0) + gu_1 = 0$ e assumindo as hipóteses, acima fixadas, sobre as funções f_i , g , h , então existe um único par de funções, (u, z) , solução do problema misto (1.1) na classe*

$$\left| \begin{array}{l} u \in L_{loc}^\infty(0, \infty; V \cap H_\Delta(\Omega)), \quad u' \in L_{loc}^\infty(0, \infty; V), \quad u'' \in L_{loc}^\infty(0, \infty; L^2(\Omega)), \\ \gamma_0(u'), \gamma_0(u'') \in L_{loc}^2(0, \infty; L^2(\Gamma_1)), \\ z, z_t, z_{tt} \in L_{loc}^\infty(0, \infty; L^2(\Gamma_1)), \end{array} \right.$$

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e satisfazem as equações integrais

$$\int_0^T \int_{\Omega} [u''\varphi + \nabla u \cdot \nabla \varphi] dx dt = \int_0^T \int_{\Gamma_1} [hz_t - gu'] \varphi d\Gamma_1 dt, \quad e \quad \int_0^T \int_{\Gamma_1} [u' + f_1 z_{tt} + f_2 z_t + f_3 z] \psi d\Gamma_1 dt = 0,$$

$\forall \varphi \in L^2(0, T; V) \quad e \quad \forall \psi \in L^2(0, T; L^2(\Gamma_1))$. Além disso, u e z satisfazem as condições iniciais

$$u(x, 0) = u_0(x), \quad u'(x, 0) = u_1(x) \quad em \quad \Omega, \quad z(x, 0) = z_0(x), \quad z_t(x, 0) = \frac{1}{h} \left(\frac{\partial u_0}{\partial \nu} + g u_1(x) \right) \quad sobre \quad \Gamma_1.$$

A demonstração do Teorema 2.1 é feita via método de Faedo-Galerkin. Além disso, é realizada simulações numéricas para os casos unidimensional e bidimensional do modelo (1.1) por meio do método de elementos finitos na parte espacial e na parte de evolução usamos o método das diferenças finitas, particularmente o método de Crank Nicolson e método da diferença centrada, como podemos ver em [3]. Para validar o modelo (1.1), foram construídos problemas cuja solução exata é conhecida, para com isso calcularmos o erro norma $L^\infty(0, T; L^2(\Omega))$.

A Figura 1, é um exemplo do caso unidimensional do modelo acima, onde foi construído um par (u, z) de solução exata, tal qual $u(x, t) = x^2 \exp(2t)$ e $z(1, t) = \left(\frac{1+g(1)}{h(1)} \right) (\exp(2t) - 1)$ com $\Omega = (0, 1)$, $\Gamma_0 = 0$, $\Gamma_1 = 1$ e $t \in (0, 1)$.

| | Δx | Δt | Erro $L^\infty(0, T; L^2(\Omega))$ |
|-----------|------------|------------|------------------------------------|
| $u(x, t)$ | 200^{-1} | 350^{-1} | 0.0010918 |
| $z(1, t)$ | - | 350^{-1} | 0.0006570 |

Tabela 1: Método de Crank-Nicolson e Dif. Centrada

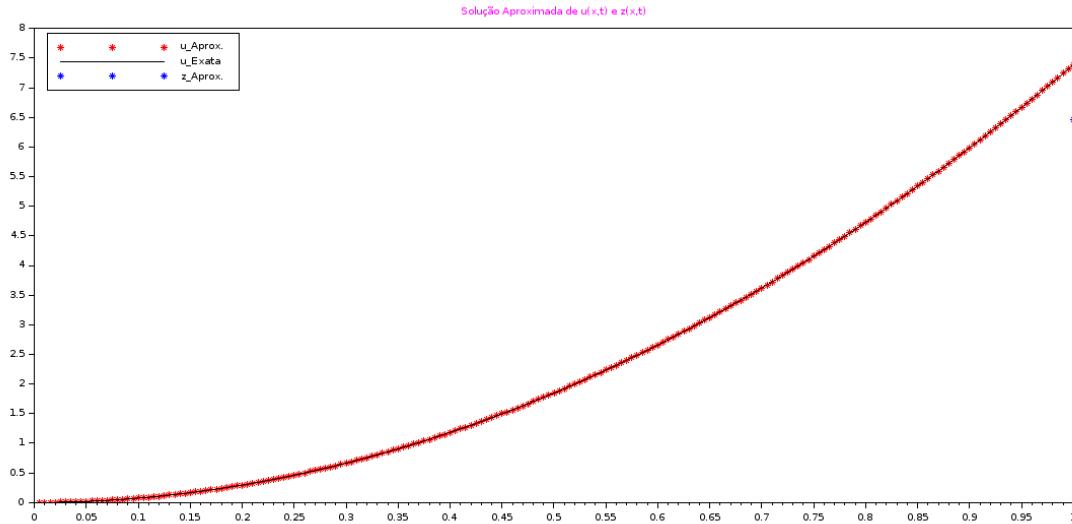


Figura 1: u e z no instante $t = 1$.

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LOCAL SOLUTION OF KIRCHHOFF EQUATIONS WITH NEGATIVE TERMS

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1 Introduction

Let Ω be a bounded open set of \mathbb{R}^n with a C^2 boundary Γ , Γ constituted of two disjoint closed parts Γ_0 and Γ_1 . By $\nu(x)$ is denoted the exterior unit normal at $x \in \Gamma_1$.

Motivated by the papers [1] and [2], we investigate the existence of local solution of the following problem:

$$\begin{cases} u'' - M(\cdot, \|u\|^2) \Delta u + |u|^\rho = f & \text{in } \Omega \times [0, T_0); \\ u = 0 & \text{on } \Gamma_0 \times (0, T_0); \\ \frac{\partial u}{\partial \nu} + \delta h(u') = 0 & \text{on } \Gamma_1 \times (0, T_0); \\ u(x, 0) = u^0(x), \quad u'(x, 0) = u^1(x), \quad x \in \Omega \end{cases}$$

2 Main Results

Let Ω be bounded open set of \mathbb{R}^n with boundary Γ of class C^2 . It is assumed that Γ is constituted by two disjoint parts Γ_0 and Γ_1 , Γ_0 and Γ_1 with positive measures, such that $\overline{\Gamma_0} \cap \overline{\Gamma_1} = \emptyset$. By $\nu(x)$ it is represented the unit normal vector at $x \in \Gamma_1$.

We denote by (u, v) and $|u|$, the scalar product and norm, respectively, of $L^2(\Omega)$. By V it is represented the Hilbert space

$$V = \{v \in H^1(\Omega); \quad v = 0 \text{ on } \Gamma_0\},$$

equipped with the scalar product

$$((u, v)) = \sum_{i=1}^n \int_{\Omega} \frac{\partial u}{\partial x_i}(x) \frac{\partial v}{\partial x_i}(x) dx$$

and norm $\|u\|^2 = ((u, u))$. All scalar function considered in this paper will be real-valued.

In order to state our main result, we introduce the necessary hypotheses.

We consider the functions $M(t, \lambda)$, $h(s)$ and $\delta(x)$ satisfying the following conditions:

$$(H1) \quad \begin{cases} M \in C^1([0, \infty[^2); \\ M(t, \lambda) \geq m_0 > 0, \quad \forall \{t, \lambda\} \in ([0, \infty[)^2, \quad (m_0 \text{ constant}). \end{cases}$$

$$(H2) \quad \begin{cases} h \text{ is a Lipschitz continuous function with } h(0) = 0; \\ h \text{ is a strongly monotone function, that is,} \\ (h(r) - h(s))(r - s) \geq d_0(r - s)^2, \quad \forall r, s \in \mathbb{R}, \quad (d_0 \text{ positive constant}). \end{cases}$$

$$(H3) \quad \delta \in W^{1,\infty}(\Gamma_1), \quad \delta(x) \geq \delta_0, \quad \forall x \in \Gamma_1, \quad (\delta_0 \text{ positive constant});$$

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(H4) The real number ρ has the following restrictions:

$$\rho > 1 \text{ if } n = 1, 2; \frac{n+1}{n} \leq \rho \leq \frac{n}{n-2} \text{ if } n \geq 3. \quad (2.1)$$

In this condition we have $V \hookrightarrow L^{\rho+1}(\Omega)$. Use the notation $\|v\|_{L^{\rho+1}} \leq k_0 \|v\|$, $v \in V$.

Introduce the real number

$$\lambda^* = \left(\frac{m_0}{2k_0^{\rho+1}} \right)^{\frac{1}{\rho-1}}.$$

Theorem 2.1. Assume that hypotheses (H1) – (H4) are satisfied. Consider $\{u^0, u^1\}$ in $V \cap H^2(\Omega) \times V$, satisfying the compatibility conditions,

$$\frac{\partial u^0}{\partial \nu} + \delta h(u^1) = 0 \text{ in } \Gamma_1 \quad (2.2)$$

and a function f in the class

$$f \in L^1(0, T; L^2(\Omega)), \quad f' \in L^1(0, T; L^2(\Omega)).$$

Then there exists a real number T_0 with $0 < T_0 \leq \min\{1, T\}$ such that under the restrictions on u^0, u^1 and T_0

$$\begin{cases} \|u^0\| < \lambda_1^*, \\ \left(\frac{2}{m_0} \right)^{1/2} \left[(2N)^{1/2} + \int_0^{T_0} |f(t)| dt \right] \exp \left(\frac{1}{m_0} K(1 + 2R^2) T_0 \right) < \lambda^* \end{cases} \quad (2.3)$$

where

$$N = \frac{1}{2} |u^1|^2 + \frac{1}{2} M(0, \|u^0\|^2) \|u^0\|^2 + \frac{k_0^{\rho+1}}{\rho+1} \|u^0\|^{\rho+1}$$

and K and R are positive constants that depend on $u^0, u^1, M(t, \lambda), \rho$ and T_0 , we obtain a unique function u in the class

$$\begin{cases} u \in L^\infty(0, T_0; V \cap H^2(\Omega)), \\ u' \in L^\infty(0, T_0; V), \\ u'' \in L^\infty(0, T_0; L^2(\Omega)) \cap L^2(0, T_0; L^2(\Gamma_1)), \end{cases} \quad (2.4)$$

such that u satisfies

$$u'' - M(\cdot, \|u\|^2) \Delta u + |u|^\rho = f \text{ in } L^\infty(0, T_0; L^2(\Omega)), \quad (2.5)$$

$$\begin{cases} \frac{\partial u}{\partial \nu} + \delta h(u') = 0 \text{ in } L^2(0, T_0; H^{\frac{1}{2}}(\Gamma_1)), \\ \frac{\partial u'}{\partial \nu} + \delta h'(u') u'' = 0 \text{ in } L^2(0, T_0; L^2(\Gamma_1)), \end{cases} \quad (2.6)$$

$$u(0) = u^0, \quad u'(0) = u^1. \quad (2.7)$$

Proof. The existence of solutions follows by applying a special basis of $V \cap H^2(\Omega)$, using Galerkin Method, the Tartar's method and the Banach Fixed-Point Theorem. \square

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O LAPLACIANO MAGNÉTICO DE DIRICHLET EM TUBOS LIMITADOS

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Considerando uma sequência de regiões tubulares limitadas no espaço e o operador Laplaciano magnético de Dirichlet restrito a esses domínios, encontramos o operador efetivo quando o diâmetro de tais regiões tende à zero. Essa convergência é no sentido uniforme dos resolventes dos operadores auto adjuntos associados ao problema.

1 Introdução

Seja S um círculo de comprimento $l > 0$ e $r : S \rightarrow \mathbb{R}^3$ uma curva simples, fechada e de classe C^3 em \mathbb{R}^3 parametrizada pelo seu comprimento de arco s . Denotamos por $k(s)$ e $\tau(s)$ a curvatura e a torção no ponto $r(s)$, respectivamente. Seja Q um subconjunto aberto, simplesmente conexo, com fronteira suave e não vazio de \mathbb{R}^2 . Construímos um tubo em \mathbb{R}^3 movendo a região Q ao longo de $r(s)$. Em cada ponto a região Q pode apresentar uma rotação de ângulo a qual denotamos por $\alpha(s)$. Suponhamos que α é de classe C^2 e impomos a condição de Dirichlet na fronteira $\partial\Omega$. Consideremos o campo magnético $\mathbf{A} = (A_1, A_2, A_3)$, em que $A_j : \Omega \rightarrow \mathbb{R}$, $j = 1, 2, 3$, são funções reais de forma que, supondo \mathbf{A} diferenciável, $\mathbf{B} = \nabla \times \mathbf{A}$ é o campo magnético correspondente. Consideremos agora a família de operadores

$$(H_\varepsilon \psi)(x) := [(-i\partial - \mathbf{A})^2 \psi](x) \quad (0 < \varepsilon < 1),$$

$\text{dom } H_\varepsilon = \mathcal{H}^2(\Omega_\varepsilon) \cap \mathcal{H}_0^1(\Omega_\varepsilon)$ (Ω_ε é a região obtida movendo-se $\varepsilon\Omega$ ao longo de $r(s)$ e, para cada $0 < \varepsilon < 1$, estamos considerando \mathbf{A} restrito à Ω_ε).

O objetivo é estudar a sequência H_ε no limite $\varepsilon \rightarrow 0$. Para isso é necessário algumas renormalizações, por exemplo, precisamos controlar as oscilações transversas quando $\varepsilon \rightarrow 0$. Um ponto interessante é que mesmo com a presença do potencial vetorial, controlamos essas oscilações subtraindo λ_0/ε^2 de H_ε , em que λ_0 é o primeiro autovalor (ou seja, o menor) do operador Laplaciano de Dirichlet (sem potencial magnético) restrito à Q . A saber,

$$-\Delta u_0 = \lambda_0 u_0, \quad u_0 \in \mathcal{H}_0^1(Q), \quad u_0 \geq 0, \quad \int_Q |u_0|^2 dy = 1.$$

u_0 denota a autofunção normalizada associada ao autovalor λ_0 (λ_0 é um autovalor simples).

Consideremos o operador unidimensional

$$(G_0 w)(s) := (-i\partial_s - \langle \mathbf{A}(r(s)), T(s) \rangle)^2 w(s) + \left[C(Q)(\tau + \alpha')^2(s) - \frac{k^2(s)}{4} \right] w(s),$$

$\text{dom } G_0 = \mathcal{H}^2(S)$, em que $C(Q)$ é uma constante real que depende somente da região Q e $T(s)$ é o vetor tangente à curva no ponto $r(s)$; $\langle \cdot, \cdot \rangle$ denota o produto interno usual em \mathbb{R}^3 . Nossa principal resultado diz que

$$H_\varepsilon - \frac{\lambda_0}{\varepsilon^2} \mathbf{1} \longrightarrow G_0,$$

quando $\varepsilon \rightarrow 0$, no sentido uniforme dos resolventes (veja Teorema 4 de [4] para uma formulação mais precisa). Para provar essa convergência, usamos a técnica variacional de Γ -convergência [1, 2, 4].

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2 Resultados

Ao aplicarmos a técnica de Γ -convergência passamos ao estudo da sequência de formas quadráticas associada à sequência de operadores $H_\varepsilon - (\lambda_0/\varepsilon^2)\mathbf{1}$:

$$\int_{\Omega_\varepsilon} \left(|(-i\partial_x - \mathbf{A})\psi|^2 - \frac{\lambda_0}{\varepsilon^2} |\psi|^2 \right) dx, \quad \psi \in \mathcal{H}_0^1(\Omega_\varepsilon).$$

No entanto, é conveniente uma mudança de variável e algumas renormalizações (veja Seção 3 de [4] para mais detalhes) ao analizarmos o limite $\varepsilon \rightarrow 0$ na expressão acima. Obtemos assim uma nova sequência de formas quadráticas, denotada por $\{g_\varepsilon\}_{0 < \varepsilon < 1}$ (por simplicidade a definição de g_ε será omitida aqui; novamente, veja Seção 3 de [4]). Ainda mais, com a mudança de variável passamos a trabalhar no espaço de Hilbert $L^2(S \times Q)$ e $\text{dom } g_\varepsilon = \mathcal{H}_0^1(S \times Q)$ (observemos a independencia do parâmetro ε nos domínios das formas quadráticas). Usamos a notação $(s, y) := (s, y_1, y_2)$ para um ponto de $S \times Q$ e G_ε para o operador auto adjunto associado à g_ε .

Agora, denotamos por g_0 a forma quadrática associada ao operador G_0 definido na introdução. Definimos também, a partir de g_0 , uma nova forma quadrática em todo o espaço $L^2(S \times Q)$:

$$g(v) := \begin{cases} g_0(w), & \text{se } v(s, y) = w(s)u_0(y) \text{ com } w \in \text{dom } g_0, \\ +\infty, & \text{caso contrário} \end{cases}.$$

Denotamos por G o operador auto adjunto à g . Enunciamos um dos principais resultados de [4].

Teorema: A sequência de operadores G_ε converge à G no sentido uniforme dos resolventes quando $\varepsilon \rightarrow 0$. Mais precisamente,

$$\|(G_\varepsilon - i\mathbf{1})^{-1} - (G - i\mathbf{1})^{-1}\|_{L^2(S \times Q)} \rightarrow 0,$$

quando $\varepsilon \rightarrow 0$.

Observações: (i) A motivação para o estudo acima surgiu dos trabalhos [1] e [3].

(ii) A técnica de Γ -convergência em espaços de Hilbert reais pode ser encontrada em [2]. No entanto, para demonstrarmos o teorema acima, precisamos de uma versão complexa do Teorema 13.6 desta mesma referência. A generalização para espaços de Hilbert complexos foi demonstrada em [4].

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MULTIPLICIDADE DE SOLUÇÕES PARA UM PROBLEMA QUASELINEAR ELÍTICO

ALEX J. BECKER * & MÁRCIO L. MIOTTO † & TAÍSA J. MIOTTO ‡

1 Introdução

Neste trabalho pretende-se obter resultados de existência e multiplicidade de soluções para a seguinte classe de problemas

$$(P_t) \quad \begin{cases} -\Delta_p u = h(x)|u|^{q-2}u + f(x, u) + t\phi(x) + g(x) & \text{em } \Omega, \\ u = 0 & \text{sobre } \partial\Omega, \end{cases}$$

onde $\Delta_p = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ é o operador p-Laplaciano, $\Omega \subset \mathbb{R}^N$ um domínio limitado com fronteira $\partial\Omega$ suave, os expoentes satisfazem $1 < q < p$ e $2 \leq p < N$, $h, \phi, g \in L^\infty(\Omega)$, com $h \not\equiv 0, g_+ \not\equiv 0, g_- \not\equiv 0$ e $\phi > 0$ em Ω , t um parâmetro real e $f \in C(\bar{\Omega} \times \mathbb{R}, \mathbb{R})$ com $f(x, 0) = 0$ e satisfazendo

$$(f_1) \quad \eta_o \leq \liminf_{s \rightarrow -\infty} \frac{f(x, s)}{|s|^{p-2}s} \leq \limsup_{s \rightarrow -\infty} \frac{f(x, s)}{|s|^{p-2}s} \leq \eta_1 < \lambda_1 < \eta_2 \leq \liminf_{s \rightarrow \infty} \frac{f(x, s)}{|s|^{p-2}s} \leq \limsup_{s \rightarrow \infty} \frac{f(x, s)}{|s|^{p-2}s} \leq \eta_3,$$

q.t.p. $x \in \Omega$, onde $\eta_o, \eta_1, \eta_2, \eta_3 \in \mathbb{R}$ e λ_1 é o primeiro autovalor do operador $(-\Delta_p, W_o^{1,p}(\Omega))$.

Atualmente, autores têm se preocupado em garantir resultados de multiplicidade de soluções para problemas semelhantes a (P_t) . No caso em que $g \equiv 0, t = 0$ e $p = 2$, [5] justificou a existência de dois resultados sobre multiplicidade de soluções quando a não linearidade f satisfaz a condição (f_1) . No primeiro resultado, utilizando o Teorema do Passo da Montanha, o Princípio Variacional de Ekeland, juntamente com a Teoria do Grau de Leray-Schauder, ele obteve a existência de quatro soluções não-triviais, sendo uma não-negativa, uma não-positiva, uma positiva e a última passível de mudança de sinal. Para o segundo, combinando o método da sub-supersolução, com Teorema do Passo da Montanha, Princípio Variacional de Ekeland e a Teoria do Grau de Leray-Schauder, garantiu a existência de quatro soluções não-triviais para o problema, as quais não são possíveis de se conhecer o sinal.

Ainda para $h \equiv 0$ e com condições similares sobre f , combinando o método da sub e supersolução, técnica blow up e Teoria do Grau de Leray-Schauder, [12] garantiu a existência de constantes $-\infty < t_* \leq t^* < \infty$ de modo que (P_t) possui ao menos duas soluções quando $t < t_*$, ao menos uma solução para $t \leq t^*$ e não possui solução quando $t > t^*$. Outros resultados para este tipo de problema podem ser vistos em [2], [3], [4], [7], [8], [10], [11].

2 Resultados

Utilizando o Teorema do Passo da Montanha juntamente com o Princípio Variacional de Ekeland, pode-se justificar o seguinte resultado:

Teorema. *Assumindo as condições sobre a função f e se existem $\mu_1, \mu_2 \in (0, \lambda_1)$ tais que*

$$\mu_1 \leq \liminf_{|s| \rightarrow 0} \frac{f(x, s)}{|s|^{p-2}s} \leq \limsup_{|s| \rightarrow 0} \frac{f(x, s)}{|s|^{p-2}s} \leq \mu_2,$$

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então existem constantes $\alpha_1, \alpha_2 > 0$ e $t_* < 0 < t_{**}$ tais que se $\|h\|_\infty < \alpha_1$ e $\|g\|_\infty < \alpha_2$, decorre que:

- (P_t) admite ao menos duas soluções não-triviais para todo $t < t_{**}$;
- (P_t) admite ao menos três soluções não-triviais para todo $t \in (t_*, t_{**})$.

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TAXAS DE DECAIMENTO PARA UM MODELO VISCOELÁSTICO DE PLACAS

ALTAIR SANTOS DE OLIVEIRA TOSTI *

1 Introdução

Neste trabalho estudamos a boa colocação e o comportamento assintótico para a seguinte equação viscoelástica da placa

$$u_{tt} + \Delta^2 u - \int_0^t g(t-s) \Delta^2 u(s) ds + M(\|\nabla u(t)\|_2^2) u_t + f(u) = 0 \text{ em } \Omega \times (0, \infty), \quad (1.1)$$

com condição de fronteira

$$u = \frac{\partial u}{\partial \nu} = 0 \text{ sobre } \partial\Omega \times [0, \infty), \quad (1.2)$$

e condições iniciais

$$u(\cdot, 0) = u_0, \quad u_t(\cdot, 0) = u_1 \text{ em } \Omega, \quad (1.3)$$

onde Ω é um domínio limitado de \mathbb{R}^n com fronteira $\partial\Omega$ bem regular e ν é o vetor normal unitário exterior a $\partial\Omega$. Aqui, $\Delta^2 = \Delta(\Delta)$ denota o operador biharmônico, g uma função real chamada de *núcleo da memória*, M e $f(u)$ funções não lineares e $\|\cdot\|_2$ a norma em $L^2(\Omega)$.

Para determinar a existência e unicidade de solução usamos o método de Faedo-Galerkin assim como nos trabalhos [1, 2], mas com hipóteses menos restritivas sobre g e $f(u)$. Para estudar as taxas de decaimento da energia associada ao problema (1.1)-(1.3) usamos hipóteses menos restritivas para M e g do que as utilizadas em [1, 2]. Para tal finalidade, usamos as ideias introduzidas em [3, 4] primeiramente para equações de ondas, a qual nos dá um decaimento geral dependendo do decaimento do núcleo da memória.

2 Resultados

Iniciamos com as hipóteses sobre g , M e f .

Núcleo da memória. Suponhamos que $g : [0, \infty) \rightarrow \mathbb{R}^+$ é uma função de classe C^1 com $g(0) > 0$ e tal que

$$l := 1 - \int_0^\infty g(s) ds > 0 \quad \text{e} \quad g'(t) \leq -\xi(t)g(t), \quad \forall t > 0, \quad (2.4)$$

onde $\xi : [0, \infty) \rightarrow \mathbb{R}^+$ é uma função de classe C^1 satisfazendo

$$\xi(t) > 0, \quad \xi'(t) \leq 0, \quad \left| \frac{\xi'(t)}{\xi(t)} \right| \leq L < \infty, \quad \forall t \geq 0. \quad (2.5)$$

Termo não local. Suponhamos que $M : [0, \infty) \rightarrow \mathbb{R}$ é de classe C^1 com

$$M(s) \geq 0, \quad \forall s \in [0, \infty). \quad (2.6)$$

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Termo não linear. Consideramos $f : \mathbb{R} \rightarrow \mathbb{R}$ de classe C^1 com $f(0) = 0$ e tal que

$$|f'(u)| \leq k_1(1 + |u|^{\rho/2}), \quad \forall u \in \mathbb{R}, \quad (2.7)$$

com $k_1 > 0$ e ρ satisfazendo

$$0 < \rho < \frac{8}{N-4} \quad \text{se } N \geq 5 \quad \text{e} \quad \rho > 0 \quad \text{se } 1 \leq N \leq 4. \quad (2.8)$$

Além disso, suponhamos que

$$\frac{-l\beta}{2}|u|^2 \leq \hat{f}(u) := \int_0^u f(s)ds \leq f(u)u + \frac{l\beta}{2}|u|^2, \quad \forall u \in \mathbb{R}, \quad (2.9)$$

onde $\beta \in [0, \mu_1)$ e $\mu_1 > 0$ corresponde a constante de imersão $\mu_1\|u\|_2^2 \leq \|\Delta u\|_2^2$.

A energia (modificada) $\mathcal{E}(t) := \mathcal{E}(u(t), u_t(t))$ associada a (1.1)-(1.3) é dada por

$$\mathcal{E}(t) = \frac{1}{2}\|u_t(t)\|_2^2 + \frac{1}{2} \left(1 - \int_0^t g(s) ds \right) \|\Delta u(t)\|_2^2 + \frac{1}{2} \int_0^t g(t-s) \|\Delta u(t) - \Delta u(s)\|_2^2 ds + \int_{\Omega} \hat{f}(u(t)) dx.$$

O principal resultado deste trabalho se resume no seguinte teorema:

Teorema 2.1. Seja $T > 0$ dado. Sob as hipóteses (2.4)-(2.9), temos:

1. Se $(u_0, u_1) \in H^4(\Omega) \times H_0^2(\Omega)$, então existe uma única solução forte para (1.1)-(1.3) na classe

$$(u, u_t) \in L^\infty(0, T; H^4(\Omega) \times H_0^2(\Omega)) \cap C([0, T], H_0^2(\Omega) \times L^2(\Omega)).$$

2. Se $(u_0, u_1) \in H_0^2(\Omega) \times L^2(\Omega)$, então existe uma única solução fraca para (1.1)-(1.3) na classe

$$(u, u_t) \in C([0, T], H_0^2(\Omega) \times L^2(\Omega)).$$

3. Em ambos os casos, a energia $\mathcal{E}(t)$ satisfaz

$$\mathcal{E}(t) \leq C\mathcal{E}(0) e^{-\gamma \int_0^t \xi(s) ds}, \quad \forall t \geq 0,$$

onde $C > 0$ e $\gamma > 0$ são constantes que podem depender dos dados iniciais em $H_0^2(\Omega) \times L^2(\Omega)$.

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ESTABILIZAÇÃO UNIFORME PARA EQUAÇÃO DA ONDA COM CONDIÇÕES DE FRONTEIRA DA ACÚSTICA

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Seja $\Omega \subset \mathbb{R}^n$ um conjunto aberto e limitado com fronteira suave Γ , $n \geq 2$. Sejam Γ_0 e Γ_1 subconjuntos fechados e disjuntos de Γ com $\Gamma = \Gamma_0 \cup \Gamma_1$, Γ_0 e Γ_1 com medida positiva. Neste trabalho provaremos a existência, unicidade de solução global e estabilização uniforme da energia associada ao seguinte problema não linear:

$$u'' - \Delta u = 0 \quad \text{em } \Omega \times (0, \infty); \quad (1)$$

$$u = 0 \quad \text{em } \Gamma_0 \times (0, \infty); \quad (2)$$

$$\frac{\partial u}{\partial \nu} + \varphi(u') = \frac{\delta'}{M \left(\int_{\Gamma_1} |\delta|^2 d\Gamma \right)} \quad \text{em } \Gamma_1 \times (0, \infty); \quad (3)$$

$$f\delta'' - M \left(\int_{\Gamma_1} |\delta|^2 d\Gamma \right) (\Delta_\Gamma \delta - \delta) + g\delta' = -u' \quad \text{em } \Gamma_1 \times (0, \infty); \quad (4)$$

$$u(x, 0) = u_0(x), \quad u'(x, 0) = u_1(x), \quad x \in \Omega; \quad (5)$$

$$\delta(x, 0) = \delta_0(x), \quad x \in \Gamma_1, \quad (6)$$

$$\delta'(x, 0) = M \left(\int_{\Gamma_1} |\delta_0|^2 d\Gamma \right) \frac{\partial u_0}{\partial \nu}(x), \quad x \in \Gamma_1, \quad (7)$$

onde $' = \frac{\partial}{\partial t}$; $\Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$ e Δ_Γ são os operadores de Laplace e Laplace-Beltrami, respectivamente; ν é o vetor unitário normal a Γ_1 ; $f, g : \overline{\Gamma_1} \rightarrow \mathbb{R}$, $\varphi : \mathbb{R} \rightarrow \mathbb{R}$, $M : [0, \infty) \rightarrow \mathbb{R}$, $u_0, u_1 : \Omega \rightarrow \mathbb{R}$ e $\delta_0 : \Gamma_1 \rightarrow \mathbb{R}$ são funções conhecidas.

Quando $\varphi \equiv 0$, $M \equiv 1$ e, além disso, a equação (??) é considerada sem o operador de Laplace-Beltrami, as condições de fronteira (??)–(??) são conhecidas como condições de fronteira da acústica e foram introduzidas por Beale e Rosencrans [2]. Neste caso o modelo físico tem como motivação o estudo do movimento de ondas acústicas em fluidos onde considera-se que parte da fronteira, Γ_1 , reage localmente a pressão que o fluido exerce sobre ela. Resultados sobre existência, bem como comportamento de soluções, tem sido obtido por diversos autores ao longo dos últimos anos, ver [5,6,8,9,10,11] e suas referências. Um trabalho interessante é [1], devido à Beale, no qual o autor prova que, sem a presença de termo dissipativo na equação definida sobre Ω , mesmo que $g > 0$, não é possível obter taxas uniforme de decaimento.

Por outro lado, quando o operador de Laplace-Beltrami é introduzido na equação (??) pensa-se que parte de fronteira reage como uma membrana elástica à pressão que o fluido exerce sobre ela e dizemos que a fronteira é não localmente reagente. Neste caso resultados sobre existência e unicidade de solução bem como sobre comportamento assintótico podem ser encontrados em [7,13]. Com a introdução do operador de Laplace-Beltrami obtém-se mais regularidade na função δ definida sobre Γ_1 o que permite considerar o termo não linear envolvendo a função M .

Neste trabalho, usando as técnicas desenvolvidas por Lasiecka e Tataru [12], provaremos um resultado de estabilização uniforme para a solução do problema (??)–(??). Esta técnica permite obter resultados envolvendo o comportamento da solução sem a imposição de hipóteses de crescimento da função φ próximo da origem. Recentemente tem surgido vários estudos usando os resultados de Lasiecka e Tataru, ver [3,4,10,14] e referências.

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Destacamos aqui os trabalhos de Graber [10] e Wu [14] os quais fazem estudos, usando as técnicas de [12], para problemas envolvendo as condições de fronteira da acústica. O trabalho aqui apresentado estende, num certo sentido, os resultados obtidos por Graber e Wu e também Vicente, Frota e Medeiros [13].

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AN UNIQUENESS RESULT FOR AN INVERSE PROBLEM ARISING IN A MASS DIFFUSION PROBLEM

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This work deals with the analysis of the inverse problem of determining the density function \mathbf{F} modelling the vector external source for the linear momentum of particles, in a model for a viscous incompressible fluid with mass diffusion. If the fluid is contained on a bounded and regular domain $\Omega \subset \mathbb{R}^3$, with boundary $\partial\Omega$, then a model for mass diffusion problem in a finite time $T > 0$ is given by the following initial-value problem:

$$\rho \mathbf{u}_t + \left((\rho \mathbf{u} - \lambda \nabla \rho) \cdot \nabla \right) \mathbf{u} - \mu \Delta \mathbf{u} - \lambda (\mathbf{u} \cdot \nabla) \nabla \rho + \nabla p + \lambda^2 \left(\nabla \cdot \left(\frac{1}{\rho} \nabla \rho \otimes \nabla \rho \right) \right) = \rho \mathbf{F} \quad \text{in } Q_T, \quad (0.1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } Q_T, \quad \rho_t - \lambda \Delta \rho + \mathbf{u} \cdot \nabla \rho = 0 \quad \text{in } Q_T, \quad (0.2)$$

$$\mathbf{u}(x, t) = 0 \quad \text{on } \Gamma_T, \quad \mathbf{u}(x, 0) = \mathbf{u}_0(x) \quad \text{on } \Omega, \quad \rho(x, 0) = \rho_0(x) \quad \text{on } \Omega, \quad (0.3)$$

$$\int_{\Omega} \rho(x, t) \mathbf{u}(x, t) \psi(x) dx = \phi(t), \quad t \in [0, T], \quad (0.4)$$

where $Q_T := \Omega \times [0, T]$ and $\Gamma_T := \partial\Omega \times [0, T]$. Here, \mathbf{u}, ρ and p denotes the velocity field, the mass density and the pressure distribution, respectively. The constant $\mu > 0$ is the usual Newtonian viscosity and the positive constant λ is a density coefficient related with the diffusion of mass. The functions ψ and ϕ in the integral overdetermination condition (0.4) are given and satisfy some restrictions which will be specified later. The differential notation is the standard one, i.e. the symbols \mathbf{u}_t and ρ_t denotes the time derivatives and ∇, Δ and $\nabla \cdot$ denotes the gradient, Laplacian and divergence operators, respectively. Now, by applying the Helmholtz decomposition, the vector field \mathbf{F} is representable for $(x, t) \in \Omega_T$ by the following relations

$$\mathbf{F}(x, t) = f(t)(\nabla h(x, t) - \mathbf{m}(x, t)), \quad (0.5)$$

where \mathbf{m} is a given functions and f and h are unknown functions such that

$$\operatorname{div}(\rho \nabla h) = \operatorname{div}(\rho \mathbf{m}) \quad \text{in } \Omega, \quad \frac{\partial h}{\partial \mathbf{n}} = h \cdot \mathbf{n} \quad \text{on } \partial\Omega, \quad \int_{\Omega} h(x, t) dx = 0, \quad t \in [0, T], \quad (0.6)$$

where \mathbf{n} is the outward unit normal vector to $\partial\Omega$.

1 Main Results

We consider the following assumptions:

$$\mathbf{u}_0 \in \mathbf{V} = \left\{ \mathbf{u} : \mathbf{u} \in \mathbf{H}^1(\Omega), \nabla \cdot \mathbf{u} = 0, \mathbf{u} = 0 \text{ on } \partial\Omega \right\}, \quad (1.7)$$

$$\rho_0(x) \in [\alpha, \beta] \subset \mathbb{R}^+, \quad \rho_0 \in H_N^2(\Omega) = \left\{ \rho_0 \in H^2(\Omega) : \frac{\partial \rho}{\partial \mathbf{n}} = 0 \text{ on } \partial\Omega, \int_{\Omega} \rho(x) dx = \int_{\Omega} \rho_0(x) dx \right\}, \quad (1.8)$$

$$\psi \in H_0^1(\Omega) \cap W^{1,\infty}(\Omega) \cap H^2(\Omega), \quad \operatorname{div}(\psi) = 0 \text{ in } \Omega, \quad \phi \in H^2(\Omega), \quad h \in C^1([0, T]), \quad (1.9)$$

$$\left| \int_{\Omega} \rho_0(x) (\nabla h(x, 0) - \mathbf{m}(x, 0)) dx \right| \geq 2\epsilon_0 > 0. \quad (1.10)$$

Then, our main results are the following:

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Theorem 1.1. Assume that ρ_0 and \mathbf{u}_0 satisfy the assumptions (1.7)-(1.8) and $f \in H^1(\Omega)$. The direct problem (0.1)-(0.3) and (0.5)-(0.6) possesses a unique solution $\{\mathbf{u}, \rho, p, h\}$ defined on a maximal interval $[0, T_1] \subset [0, T]$.

Theorem 1.2. Let (1.7)-(1.10) be satisfied and $T_2 \leq T_1$, then there exists a unique collection of functions $\{\mathbf{u}, \rho, p, f\}$ solution of the inverse problem (0.1)-(0.6) defined on a maximal interval $[0, T_2]$.

The main guidelines to the proof of Theorems 1.1 and 1.2 are the following: First, the proof of existence in Theorem 1.1 is based on the results of [5]. Second, the proof uniqueness in Theorem 1.1 is given by introducing and proving a continuous dependence estimates like the results obtained by [6] for nonhomogeneous Navier-Stokes equations and recently by [4] for a nonhomogeneous asymmetric fluid. Third, the proof of theorem 1.2 is given by characterizing the inverse problem solutions using an operator equation of second kind and introducing several estimates. Here, the cornerstone of the estimates is the verification of the hypothesis of the Tikhonov fixed point Theorem.

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ANALYTICAL CONSTRUCTION OF THE SOLUTION OF THE RIEMANN PROBLEM FOR BURGERS EQUATION WITH DISCONTINUOUS SOURCE

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This work is concerned with the explicit construction of the solution for the Riemann problem

$$u_t + \left(\frac{u^2}{2} \right)_x = g(x), \quad (x, t) \in \mathbb{R} \times \mathbb{R}^+, \quad (0.1)$$

$$u(x, 0) = u_0(x), \quad x \in \mathbb{R}, \quad (0.2)$$

where the source term and the initial condition are given by

$$g(x) = g_R H(x) + g_L H(-x), \quad u_0(x) = u_R H(x) + u_L H(-x), \quad (g_L, g_R, u_L, u_R) \in \mathbb{R}^4, \quad (0.3)$$

where H is the Heaviside function defined by $H(x) = 0$ for $x \in \mathbb{R}^-$ and $H(x) = 1$ for $x \in \mathbb{R}_0^+$. The analysis of the problem (0.1)-(0.3) is motivated by the theory of Radiation Hydrodynamics [1, 3, 6, 7]. The system for modeling the radiation hydrodynamics is given by four partial differential equations having as unknowns the mass density, the velocity, the total energy and the spectral intensity. The coupling of the radiative transport equation and the equations of hydrodynamics is given by the source term for the linear momentum and energy equations. This system is complex. Then, assuming that the velocity of the fluid particles is small with respect to the velocity of the light, a Burger equation with discontinuous source term is obtained.

1 Main result

The main result of this work is the following theorem

Theorem 1.1. *Consider the Riemann Problem (0.1)-(0.3) and denote by u_i for $i \in \{1, \dots, 43\}$ each of the possible entropic solutions. Then, there exists a partition $\{\mathbb{U}_1, \dots, \mathbb{U}_{43}\}$ of \mathbb{R}^4 such that u_i is the entropic solution of (0.1)-(0.3) if and only if $(u_L, u_R, g_L, g_R) \in \mathbb{U}_i$.*

The proof of Theorem 1.1 is developed in sixty Lemmas. First, we apply the characteristics method and introduce a classification of the different types of waves. A systematic discussion of all possible types of waves at $t = 0$ implies the existence of sixty types of solutions. Then, we analyze in detail the analytic construction of these solution types. Basically, and in a broad sense, a shock or a rarefaction wave are formed at $t = 0$. The evolution of the shock curve is completely characterized by analyzing the initial value problem obtained by the Rankine-Hugoniot condition. The rarefaction wave solution is explicitly obtained by the characteristics method. Here, a subcase of rarefaction wave, called “vacuum wave”, requires a regularization of the source term and the initial condition before applying the characteristics method. Finally, unifying the sixty Lemmas, we obtain the partition $\{\mathbb{U}_1, \dots, \mathbb{U}_{43}\}$.

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EXISTENCE AND UNIQUENESS FOR A LOTKA-VOLTERRA SYSTEM IN BESOV SPACES

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This work is devoted to study local existence and uniqueness of weak solutions in the integral sense for a diffusive non-autonomous Lotka-Volterra system. Initial data are taken in the Besov space $B_{p,q,\mathcal{N}}^\sigma$.

1 Introduction

Since its first formulation, Lotka-Volterra systems have been deeply studied. Nowadays, this system has several variations due to the efforts to find a more realistic model. See [2, 3, 4, 5, 6]. In particular, taking account of effects of dispersal in a continuous environment, and crowding effects both in the predator population and in the prey population, the diffusive Lotka-Volterra equations are formulated by

$$\begin{aligned} u_t &= d_1 \Delta u + u(b - cu - kv), \\ v_t &= d_2 \Delta v + v(-\lambda + \delta u - av). \end{aligned} \quad (1.1)$$

Instead, we are interested in the case when the crowding effect of the predator depends on the entire time from the beginning until the time t . Namely, we consider the following initial-boundary value problem Lotka-Volterra system with crowding effects both in the prey population and in the predator population subject to the zero flux boundary condition:

$$u_t = d_1 \Delta u + u(1 - u - kv), \text{ in } \Omega \times (0, \infty); \quad (1.2)$$

$$v_t = d_2 \Delta v + v(-\lambda + u) - \int_0^t a(t-s)v^2(s)ds, \text{ in } \Omega \times (0, \infty); \quad (1.3)$$

$$\partial_{\vec{n}} u = \partial_{\vec{n}} v = 0, \text{ in } \partial\Omega \times (0, \infty); \quad (1.4)$$

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \text{ in } \Omega; \quad (1.5)$$

where $\Omega \subset \mathbb{R}^N$ is a bounded sufficiently regular domain, \vec{n} is the outward normal vector in $\partial\Omega$, u and v are densities of the prey population and the predator population, respectively. Furthermore, $d_i > 0$, $i = 1, 2$, are the diffusion coefficients, λ and k are positive constants, and $a : [0, \infty) \rightarrow [0, \infty)$ is a suitable function. We take initial data (u_0, v_0) in the space $B_{p,q,\mathcal{N}}^\sigma \times B_{p,q,\mathcal{N}}^\sigma$, for $1 < p, q < \infty$ and some $\sigma \in (0, 2)$. Here, $B_{p,q,\mathcal{N}}^\sigma$ denotes the Besov space in Ω with the Neumann boundary conditions. Instead of its intrinsic definition we rely on the interpolation theory to regard the Besov space with Neumann boundary conditions as the space $(L^p(\Omega), W_{\mathcal{N}}^{2,p})_{\frac{\sigma}{2},q}$, where

$$W_{\mathcal{N}}^{2,p} = \{\varphi \in W^{2,p}(\Omega) : \partial_{\vec{n}} \varphi = 0 \text{ on } \partial\Omega\}.$$

It is well-known that $-\Lambda_1 = -d_1 \Delta$ and $-\Lambda_2 = \lambda I - d_2 \Delta$ are sectorial operators from $W_{\mathcal{N}}^{2,p}$ into $L^p(\Omega)$. Therefore, the analytic semigroup generated by them enjoy the following lift property:

$$\|e^{\Lambda_i t} \psi\|_{B_{p,q,\mathcal{N}}^\sigma} \leq M t^{-\frac{\sigma}{2}} \|\psi\|_{L^p}, \quad i = 1, 2, \quad (1.6)$$

where $M \geq 1$ and $\sigma \neq 1 + \frac{1}{p}$. See [1, Chapter V].

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We are interested in the weak solution in the integral sense for (1.2)-(1.5), that is, a function $\begin{pmatrix} u \\ v \end{pmatrix} \in C([0, \tau]; B_{p,q,\mathcal{N}}^\sigma \times B_{p,q,\mathcal{N}}^\sigma)$ such that,

$$u(t) = e^{\Lambda_1 t} u_0 + \int_0^t e^{\Lambda_1(t-s)} u(s)(1 - u(s) - kv(s))ds, \quad (1.7)$$

$$v(t) = e^{\Lambda_2 t} v_0 + \int_0^t e^{\Lambda_2(t-s)} u(s)v(s)ds - \int_0^t e^{\Lambda_2(t-s)} \int_0^s a(s-r)v^2(s)ds, \quad (1.8)$$

$$u(0) = u_0 \in B_{p,q,\mathcal{N}}^\sigma, \quad (1.9)$$

$$v(0) = v_0 \in B_{p,q,\mathcal{N}}^\sigma, \quad (1.10)$$

for $t > 0$.

2 Mathematical Results

The aim of this work is the following result.

Theorem 2.1. *Let $1 < p < \infty$, $\frac{N}{2p} \leq \sigma < 2$, $1 \leq q \leq 2p$ and $\sigma \neq 1 + \frac{1}{p}$. Then, given $\begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \in B_{p,q,\mathcal{N}}^\sigma \times B_{p,q,\mathcal{N}}^\sigma$, there exist $\tau > 0$ and $r > 0$ such that for all $\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} \in B_{B_{p,q,\mathcal{N}}^\sigma \times B_{p,q,\mathcal{N}}^\sigma}(\begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}, r)$ the problem (1.2)-(1.5) has a unique weak solution in the integral sense $\begin{pmatrix} u \\ v \end{pmatrix} : [0, \tau] \longrightarrow B_{p,q,\mathcal{N}}^\sigma \times B_{p,q,\mathcal{N}}^\sigma$.*

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ON A BI-NONLOCAL $p(x)$ -KIRCHHOFF EQUATION WITH CRITICAL EXPONENT

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1 Introduction

In this work we are going to study questions of existence and multiplicity of solutions of the $p(x)$ -Kirchhoff equation with critical growth, with an additional nonlocal term,

$$\begin{cases} -M \left(\int_{\Omega} \frac{1}{p(x)} |\nabla u|^{p(x)} \right) \Delta_{p(x)} u = \lambda f(x, u) \left[\int_{\Omega} F(x, u) \right]^r + |u|^{q(x)-2} u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $p, q \in C(\bar{\Omega})$, $f : \bar{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ and $M : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ are continuous functions enjoying some conditions which will be stated later, $F(x, u) = \int_0^u f(x, \xi) d\xi$, $\lambda, r > 0$ are real parameter and $\Delta_{p(x)}$ is the $p(x)$ -Laplacian operator, that is,

$$\Delta_{p(x)} u = \sum_{i=1}^N \frac{\partial}{\partial x_i} \left(|\nabla u|^{p(x)-2} \frac{\partial u}{\partial x_i} \right), \quad 1 < p(x) < N.$$

We assume the following hypotheses: there are positive constants A_1, A_2 and a function $\beta(x) \in C_+(\bar{\Omega}) = \{h; h \in C(\bar{\Omega}), h(x) > 1 \text{ for all } x \in \bar{\Omega}\}$, such that

$$A_1 t^{\beta(x)-1} \leq f(x, t) \leq A_2 t^{\beta(x)-1}, \quad (1.2)$$

for all $t \geq 0$ and for all $x \in \bar{\Omega}$, with $f(x, t) = 0$ for all $t < 0$. Furthermore,

$$1 < p^- = \min_{x \in \bar{\Omega}} p(x) \leq \max_{x \in \bar{\Omega}} p(x) = p^+ < N, \quad (1.3)$$

and

$$1 < \beta^+(r+1) < q(x) \leq p^* = \frac{Np(x)}{N-p(x)}, \quad (1.4)$$

with $\{x \in \bar{\Omega}; q(x) = p^*(x)\} \neq \emptyset$.

The multiplicity will be studied considering (1.2) for all $t \geq 0$ and for all $x \in \bar{\Omega}$, with

$$f(x, t) = -f(x, -t), \quad (1.5)$$

for all $t \in \mathbb{R}$ and for all $x \in \bar{\Omega}$.

2 Mathematical Results

Theorem 2.1.

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- (i) Assume (1.2), (1.3) and (1.4). Moreover, assume there exists $0 < m_0$ and m_1 such that $m_0 \leq M(\tau) \leq m_1$, with $\frac{m_1 p^+}{m_0} < \left(\frac{A_1}{A_2}\right)^{r+1} \frac{(\beta^-)^{r+1}(r+1)}{(\beta^+)^r}$ and $p^+ < \beta^-(r+1)$. Then there exists $\bar{\lambda} > 0$ such that for all $\lambda > \bar{\lambda}$ there exists a nontrivial solution to (1.1) in $W_0^{1,p(x)}(\Omega)$.
- (ii) Assume (1.2), (1.3), (1.4) and $M(\tau) = a + b\tau^\eta$, with $a \geq 0, b > 0, \tau \geq 0$ and $\eta \geq 1$. Moreover, assume $(\eta+1)p^+ < \beta^-(r+1)$ and $\frac{(\eta+1)(p^+)^{\eta+1}}{(p^-)^\eta} < \left(\frac{A_1}{A_2}\right)^{r+1} \frac{(\beta^-)^{r+1}(r+1)}{(\beta^+)^r}$. Then there exists $\tilde{\lambda} > 0$ such that for all $\lambda > \tilde{\lambda}$ there exists a nontrivial solution to (1.1) in $W_0^{1,p(x)}(\Omega)$.

We use the concentration-compactness principle of Lions [3] to the variable exponent spaces, extended by Bonder and Silva [2] to the generalized Lebesgue-Sobolev spaces, to prove this result.

Theorem 2.2.

- (i) Assume (1.2), (1.3), (1.4) and (1.5). Moreover, assume there exists $0 < m_0$ and m_1 such that $m_0 \leq M(\tau) \leq m_1$, with $\frac{p^+ m_1}{m_0} < q^-$ and $\beta^-(r+1) < p^-$. Then there exists $\tilde{\lambda} > 0$ such that for all $0 < \lambda < \tilde{\lambda}$ there exists infinitely many solutions to (1.1) in $W_0^{1,p(x)}(\Omega)$.
- (ii) Assume (1.2), (1.3), (1.4), (1.5) and $M(\tau) = a + b\tau^\eta$, with $a \geq 0, b > 0, \tau \geq 0$ and $\eta \geq 1$. Moreover, assume $\beta^+(r+1) < p^-$ and $\frac{(\eta+1)(p^+)^{\eta+1}}{(p^-)^\eta} < q^-$. Then there exists $\bar{\lambda} > 0$ such that for all $0 < \lambda < \bar{\lambda}$ there exists infinitely many solutions to (1.1) in $W_0^{1,p(x)}(\Omega)$.

We use a truncation argument and the concentration-compactness principle of Lions [3], to the variable exponent spaces, extended by Bonder and Silva [2], to prove this result.

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ANÁLISE E SIMULAÇÃO NUMÉRICA DE UM SISTEMA DISSIPATIVO DO TIPO TERMOELÁSTICO

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1 Introdução

Neste trabalho estudamos um sistema acoplado não-linear do tipo termoelástico descrito pelas equações

$$\begin{cases} u''(x, t) - \alpha(t)\Delta u(x, t) + (a \cdot \nabla)\theta(x, t) + \lambda|u(x, t)|^\rho u(x, t) = 0 & \text{em } Q, \\ \theta'(x, t) - \beta \left(\int_{\Omega} \theta(x, t) dx \right) \Delta \theta(x, t) + (a \cdot \nabla)u'(x, t) + \gamma(\theta(x, t)) = 0 & \text{em } Q, \end{cases} \quad (1.1)$$

com condições iniciais e de fronteira

$$\begin{cases} u(x, 0) = u_0(x), \quad u'(x, 0) = u_1(x), \quad \theta(x, 0) = \theta_0(x) & \text{em } \Omega, \\ u(x, t) = 0 \quad \text{sobre } \Gamma_0 \times]0, T[, \\ \frac{\partial u}{\partial \nu}(x, t) + \eta(x)u'(x, t) = 0 \quad \text{sobre } \Gamma_1 \times]0, T[, \\ \theta(x, t) = 0 \quad \text{sobre } \Gamma \times]0, T[, \end{cases} \quad (1.2)$$

onde u e θ são respectivamente, o deslocamento e a temperatura. O domínio Q é definido por $\Omega \times]0, T[$, sendo Ω um conjunto aberto do \mathbb{R}^n com fronteira suave $\Gamma = \overline{\Gamma_0 \cup \Gamma_1}$, que satisfaz as propriedades de $\Gamma_0 \cap \Gamma_1 = \emptyset$. O vetor normal unitário exterior a Γ é dado por ν e a é um vetor constante do \mathbb{R}^n . Para estabelecer resultados de existência e unicidade de solução, considere as seguintes hipóteses:

$$\begin{cases} \alpha \in W_{loc}^{1, \infty}([0, \infty); \mathbb{R}) \text{ com } \alpha' \in L^1(0, \infty) \text{ e } \alpha(t) \geq \alpha_0 > 0; \\ \beta \in W_{loc}^{1, \infty}(\mathbb{R}) \text{ e } \beta(t) \geq \beta_0 > 0; \quad \gamma \in W_{loc}^{1, \infty}(\mathbb{R}) \text{ e } \gamma(0) = 0; \\ 0 < \rho < \infty \text{ se } n = 1, \quad \frac{1}{2} \leq \rho < \infty \text{ se } n = 2 \text{ ou} \\ \frac{1}{n} < \rho \leq \frac{2}{n-2} \text{ se } n \geq 3; \quad \eta \in W^{1, \infty}(\Gamma_1) \text{ e } \eta(x) \geq \eta_0 > 0 \end{cases} \quad (1.3)$$

Nessas condições temos o seguinte resultado de existência e unicidade de solução:

Teorema 1.1. *Sob as hipóteses (1.3) e $u_0 \in V \cap H^2(\Omega)$, $u_1 \in V$, $\theta_0 \in H_0^1(\Omega)$ e $\frac{\partial u_0}{\partial \nu} + u_1 = 0$ em Γ_1 . Então existe um único par de funções $\{u, \theta\}$ solução de (1.1) - (1.2), satisfazendo*

$$\begin{aligned} u &\in L_{loc}^\infty(0, \infty; V), \quad u' \in L_{loc}^\infty(0, \infty; V) \cap L_{loc}^2(0, \infty; L^2(\Gamma_1)), \\ u'' &\in L_{loc}^\infty(0, \infty; L^2(\Omega)) \cap L_{loc}^2(0, \infty; L^2(\Gamma_1)), \\ \theta &\in L_{loc}^2(0, \infty; H^2(\Omega)), \quad \theta' \in L_{loc}^\infty(0, \infty; H_0^1(\Omega)). \end{aligned} \quad (1.4)$$

O objetivo do trabalho é desenvolver um programa computacional e implementar a solução numérica do problema (1.1) com as condições iniciais e de fronteira (1.2) para os casos em que o domínio Ω seja uni ou bidimensional.

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Aplicando o método de Faedo-Galerkin para determinação da solução aproximada foi desenvolvida a formulação variacional do sistema de equações. Utiliza-se o método dos elementos finitos na parte espacial, resultando em um sistema de equações diferenciais ordinárias. Como o problema é não linear, então foram desenvolvidas duas estratégias para a resolução do sistema não linear associado. Em ambas aplicamos o método de Crank-Nicolson, contudo, na primeira, foi feita uma linearização do sistema não linear e na segunda, o sistema não-linear foi resolvido pelo método de Newton. Os métodos numéricos aplicados podem ser encontrados em [1] -[2].

2 Resultados

Com o intuito de validar a implementação da solução numérica, foram realizadas simulações para casos em que conhecemos a solução exata, possibilitando analisar o erro entre a solução exata e a solução numérica obtida. Para facilitar a busca por soluções exatas, acrescentamos as funções f e g do lado direito das equações em (1.1). No presente trabalho, apresentamos a implementação e validação do caso em que Ω é um intervalo da reta.

Nas simulações avaliadas a seguir, consideramos $\Omega =]0, 1[, T = 1$ e soluções $u(x, t) = \exp^{\frac{2\pi}{\eta}t} (\cos(2\pi(x+1)-1))$ e $\theta(x, t) = \exp^{\frac{2\pi}{\eta}t} \sin(\pi x)$. Substituindo as soluções u e θ no sistema, foram construídos as funções f , g e as condições iniciais. Para as demais constantes e funções, tomamos $\rho = 2$, $a = 1$, $\eta = 1$, $\gamma(t) = t$ e $\alpha(t) = \beta(t) = t + 1$.

Para avaliar o comportamento do erro, consideramos diversas discretizações para o espaço e para o tempo, obtendo para cada discretização o erro para o caso linearizado(L) e para o caso não-linear(NL). Devido ao alto custo computacional não foi calculado o erro quando $h = 10^{-3}$ e $\Delta t = 10^{-6}$. Para calcular o erro, mostrados nas Tabelas 1 e 2, foi considerado a norma discreta $L^\infty(0, 1; L^2(0, 1))$.

| | | $\Delta t = 10^{-2}$ | $\Delta t = 10^{-3}$ | $\Delta t = 10^{-4}$ | $\Delta t = 10^{-5}$ | $\Delta t = 10^{-6}$ |
|---------------|----|----------------------|----------------------|----------------------|----------------------|----------------------|
| $h = 10^{-2}$ | L | 0.78958195 | 0.08215268 | 0.00867689 | 0.00179555 | 0.00141990 |
| | NL | 0.47024741 | 0.04937007 | 0.00556031 | 0.00163955 | 0.00141526 |
| $h = 10^{-3}$ | L | 0.78925823 | 0.08178397 | 0.00821046 | 0.00082469 | — |
| | NL | 0.47001245 | 0.04884987 | 0.00490339 | 0.00049520 | — |

Tabela 1: Erro em $u(x, t)$

| | | $\Delta t = 10^{-2}$ | $\Delta t = 10^{-3}$ | $\Delta t = 10^{-4}$ | $\Delta t = 10^{-5}$ | $\Delta t = 10^{-6}$ |
|---------------|----|----------------------|----------------------|----------------------|----------------------|----------------------|
| $h = 10^{-2}$ | L | 0.25121620 | 0.02535452 | 0.00254997 | 0.00027816 | 0.00008952 |
| | NL | 0.02800842 | 0.00289163 | 0.00036074 | 0.00010892 | 0.00008472 |
| $h = 10^{-3}$ | L | 0.25120370 | 0.02534208 | 0.00253638 | 0.00025377 | — |
| | NL | 0.02795822 | 0.00281656 | 0.00028395 | 0.00002909 | — |

Tabela 2: Erro em $\theta(x, t)$

Analizando as tabelas de erro, observamos o decaimento linear do erro para ambas as metodologias utilizadas, entretanto, como esperado, os melhores resultados foram obtidos resolvendo o sistema não-linear pelo método de Newton. Resultados análogos foram obtidos para outras soluções que analisamos. O próximo passo é fazer a implementação computacional para o problema bidimensional. Utilizamos o Matlab como linguagem computacional.

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MULTIPLICIDADE DE SOLUÇÕES SIMÉTRICAS RADIAIS PARA PROBLEMAS ELÍPTICOS EM UM CILINDRO ILIMITADO

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Em [1], Chung e Toan estudaram um problema elíptico do tipo p-Laplaciano envolvendo um domínio cilíndrico simétrico limitado e provaram sob certas hipóteses a existência de pelo menos duas soluções.

Neste trabalho, provaremos a existência de pelo menos duas soluções para o seguinte problema elíptico

$$-\Delta_p u = |u|^{q-2}u + g \text{ em } \Omega, \quad u = 0 \text{ sobre } \partial\Omega \quad (P1)$$

em um domínio cilíndrico ilimitado

$$\Omega := \{x = (y, z) \in \mathbb{R}^{m+1} \times \mathbb{R}^{N-m-1} : 0 < a < |y| < b\},$$

quando $p \geq 2$, $q \in \left(p, \frac{p(N-m)}{N-m-p}\right)$, $|\nabla u|^p$, $|u|^q \in L^1(\Omega)$ e a função g pertence ao dual do subespaço das funções invariantes, denotado por $W_{0,G}^{1,p}(\Omega)$.

Verificamos que funções $O(m+1)$ -invariantes $u(y, z) = v(|y|, z)$ são soluções de (P1) se, e somente se, $v := v(r, z)$ com $r = |y|$ é solução do seguinte problema

$$-div(r^m |\nabla v|^{p-2} \nabla v) = r^m |v|^{q-2} v + r^m g \text{ em } S, \quad v = 0 \text{ sobre } \partial S \quad (P2)$$

onde $S := (a, b) \times \mathbb{R}^{N-m-1}$, $\partial S := \{a, b\} \times \mathbb{R}^{N-m-1}$ e $|\nabla v|^p$, $|v|^q \in L^1(S)$.

Seja $W_0^{1,p}(S)$ o espaço de Sobolev usual, com a norma $\|v\|_{1,p} = \left(\int_S |\nabla v|^p dx\right)^{\frac{1}{p}}$. Esta norma $\|\cdot\|_{1,p}$ é equivalente a norma padrão em $W_0^{1,p}(S)$ (veja [5]).

Defina o subespaço

$$W_{0,G}^{1,p}(S) = \{v \in W_0^{1,p}(S) : v(r, gz) = v(r, z) \text{ para toda } g \in G, \text{ onde } G := O(N-m-1)\}.$$

Com $a < r < b$, definimos as normas

$$\|v\|_{m,p} := \left(\int_S r^m |\nabla v|^p dx\right)^{\frac{1}{p}} \quad \text{e} \quad |v|_{m,q} := \left(\int_S r^m |v|^q dx\right)^{\frac{1}{q}} \quad (*)$$

equivalentes as normas usuais de $W_0^{1,p}(S)$ e $L^q(S)$, respectivamente.

Ao introduzirmos o espaço de Banach $X := \{v \in W_{0,G}^{1,p}(S) : \int_S r^m |\nabla v|^p dx < \infty\}$ considerando a norma $\|\cdot\|_{m,p}$, definimos $I : X \rightarrow \mathbb{R}$ sendo o funcional associado ao problema (P2) dado por

$$I(v) = \frac{1}{p} \int_S r^m |\nabla v|^p dx - \frac{1}{q} \int_S r^m |v|^q dx - \int_S r^m g v dx$$

cuja derivada é dada por

$$I'(v)\varphi = \int_S r^m |\nabla v|^{p-2} \nabla v \nabla \varphi - \int_S r^m |v|^{q-2} v \varphi - \int_S r^m g \varphi \quad \text{para toda } \varphi \in X.$$

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Segue do Princípio de Criticalidade Simétrica (veja [4]), que pontos críticos de I sobre X são também pontos críticos de I sobre $W_0^{1,p}(S)$. Assim, soluções fracas para o problema $(P2)$ são exatamente pontos críticos do funcional I .

Além disso, assuma $N - p > m \geq 1$ e $q \in \left(p, \frac{p(N-m)}{N-m-p}\right)$. Em [3], Lions mostrou para esses valores de m e q que $W_{0,G}^{1,p}(S)$ está compactamente imerso em $L^q(S)$. Então $W_{0,G}^{1,p}(S)$ está compactamente imerso em $L_G^q(S)$ para as normas definidas em (*).

Portanto, provamos a existência de uma solução para o problema $(P1)$ aplicando o Teorema do Passo da Montanha de Ambrosetti-Rabinowitz (veja [2]).

A existência de uma segunda solução é provada utilizando o Princípio Variacional de Ekeland (veja [6]).

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Boa e Má Colocação Para a Equação de Schrodinger Não Linear

CARLOS GUZMAN JIMENEZ *

Nesse trabalho estudaremos a Equação de Schrödinger não linear

$$i\partial_t u + \Delta u + \lambda|u|^\alpha u = 0, \quad t \in \mathbb{R}, \quad x \in \mathbb{R}^N, \quad (\text{NLS})$$

O propósito da apresentação é mostrar o trabalho desenvolvido na minha dissertação de mestrado. Especificamente, estudaremos boa e má colocação do Problema de Cauchy associado a equação (NLS) em $H^s(\mathbb{R}^N)$ (Espaço de Sobolev de ordem $s \in \mathbb{R}$). Além disso, falarei da pesquisa que estou desenvolvendo atualmente no doutorado.

1 Introdução

A equação de Schrödinger não linear, denotado simplesmente por NLS, recebeu esse nome em homenagem ao Físico Austríaco Erwin Schrödinger, pelos trabalhos desenvolvidos em 1926, em sua famosa teoria chamada: Mecânica Quântica. Essa equação tem sido o motivo de várias pesquisas e recebeu uma grande atenção pelos Matemáticos, em particular devido às aplicações para óptica não-linear.

O propósito deste trabalho é estudar existência, unicidade, persistência e dependência contínua da solução com relação aos dados iniciais, isto é, boa colocação do problema de valor inicial (PVI) associado a equação NLS

$$\begin{cases} i\partial_t u + \Delta u + \lambda|u|^\alpha u = 0, & t > 0, \quad x \in \mathbb{R}^N, \\ u(0, x) = u_0(x). \end{cases} \quad (1.1)$$

em que, $\lambda = \pm 1$, $0 < \alpha \leq 4/N$ e $u_0 \in H^s(\mathbb{R}^N)$.

Na primeira parte, mostraremos boa colocação local e global do PVI acima em $L^2(\mathbb{R}^N)$ e $H^1(\mathbb{R}^N)$. Além disso, proveremos alguns resultados de má colocação para s negativo.

Na segunda parte, apresentarei o trabalho que estamos desenvolvendo no doutorado.

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EXPONENTIAL DECAY AND NUMERICAL SOLUTION FOR A TIMOSHENKO SYSTEM WITH DELAY TERM IN THE INTERNAL FEEDBACK

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1 Introduction

In this paper we consider the following Timoshenko system

$$\rho_1 \varphi_{tt}(x, t) - K(\varphi_x + \psi)_x(x, t) + \mu_1 \varphi_t(x, t) + \mu_2 \varphi_t(x, t - \tau) = 0, \quad (1.1)$$

$$\rho_2 \psi_{tt}(x, t) - b\psi_{xx}(x, t) + K(\varphi_x + \psi)(x, t) + \mu_3 \psi_t(x, t) + \mu_4 \psi_t(x, t - \tau) = 0, \quad (1.2)$$

where φ is the transverse displacement of the beam, ψ is the rotation angle of the filament of the beam, $(x, t) \in (0, L) \times (0, \infty)$, $\tau > 0$ represents the time delay and $\rho_1, \rho_2, b, K, \mu_i$, $i = 1, 2, 3, 4$, are positive constants. This beam, of length L is subjected to the following boundary conditions

$$\varphi(0, t) = \varphi(L, t) = \psi(0, t) = \psi(L, t) = 0, \quad t > 0, \quad (1.3)$$

and initial conditions $(\varphi_0, \varphi_1, \psi_0, \psi_1, f_0, g_0)$ belongs to a suitable functional space, defined for all $x \in (0, L)$ by

$$\begin{aligned} \varphi(x, 0) &= \varphi_0(x), & \varphi_t(x, 0) &= \varphi_1(x), \\ \psi(x, 0) &= \psi_0(x), & \psi_t(x, 0) &= \psi_1(x), \end{aligned} \quad (1.4)$$

and for $(x, t) \in (0, L) \times [0, \tau]$, that implies past history with $t - \tau \leq 0$, by

$$\varphi_t(x, t - \tau) = f_0(x, t - \tau), \quad \psi_t(x, t - \tau) = g_0(x, t - \tau). \quad (1.5)$$

Note that $f_0(x, 0) = \varphi_1(x)$ and $g_0(x, 0) = \psi_1(x)$.

In the study of the asymptotic behavior, we use the result due to Gearhart. See [2, 3, 4].

Theorem 1.1. *Let $S(t) = e^{\mathcal{A}t}$ be a C_0 -Semigroup of contractions on a Hilbert space. Then $S(t)$ is exponentially stable if and only if*

$$\rho(\mathcal{A}) \supseteq i\beta, \beta \in \mathbb{R} \quad (1.6)$$

and

$$\lim_{|\beta| \rightarrow \infty} \|(i\beta I - \mathcal{A})^{-1}\| < \infty \quad (1.7)$$

hold.

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Certainly, this approach is very different from other works in the literature, especially for problems with delay, where the exponential decay is made by the method of energy, see, for example [5], and references therein. The method of energy, in general, imposes a additional condition on the wave speeds, that is, $K\rho_2 = b\rho_1$. Here we do not use any additional condition for the coefficients of the system. Our work improves the result obtained in [6] in the sense that delays has been introduced in the control (damping terms). The delays $\mu_2\varphi_t(x, t - \tau)$, $\mu_4\psi_t(x, t - \tau)$ makes the problem different from that considered in the literature. It is well known that small delays in the controls might turn such well-behaving system into a wild one. In recent years, the PDEs with time delay effects have become an active area of research.

The plan of this work is: to introduce the Energy Space and to prove that the full energy of the system decay. In the sequel to give the semigroup representation for the system and to prove that \mathcal{A} the infinitesimal generator of the semigroup is dissipative, and more, that \mathcal{A} generates a $e^{\mathcal{A}t}$, C_0 -semigroup of contractions, that implies, to prove the existence and regularity of solution. Finally by Theorem of Gearhart to prove that $e^{\mathcal{A}t}$ is exponentially stably.

2 Mathematical Results

Now we are in position to present our principal result

Theorem 2.1. *The semigroup $e^{\mathcal{A}t}$ is exponentially stably.*

In this work, we have demonstrated the well-posedness and asymptotic behavior solution of the Timoshenko system. Thus, it also was obtained numerically the asymptotic behavior of the solution confirming the theory developed.

Proof We now use Theorem 1.1 and we use a contradiction argument.

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ON THE GLOBAL SOLVABILITY FOR SYSTEMS OF VECTOR FIELDS ON THE TORUS

CLEBER DE MEDEIRA *

1 Introduction

In this work we study the global solvability of a system of complex vector fields on the torus $\mathbb{T}^{n+1} \simeq (\mathbb{R}/2\pi\mathbb{Z})^{n+1}$ given by

$$L_j = \frac{\partial}{\partial t_j} + (a_j(t) + ib_j(t)) \frac{\partial}{\partial x}, \quad j = 1, \dots, n, \quad (1.1)$$

where $a_j, b_j \in C^\infty(\mathbb{T}^n; \mathbb{R})$ and $(t, x) = (t_1, \dots, t_n, x)$ are the coordinates on the torus \mathbb{T}^{n+1} .

We assume that the system (1.1) is involutive (see [6]) or equivalently that the associated 1-form $c = \sum_{j=1}^n (a_j(t) + ib_j(t)) dt_j \in \bigwedge^1 C^\infty(\mathbb{T}^n)$ is closed. Also, for each j we consider a_j or b_j identically zero (see [8]).

The system (1.1) gives rise to a complex of differential operators \mathbb{L} which at the first level acts in the following way

$$\mathbb{L}u = d_t u + c(t) \wedge \frac{\partial}{\partial x} u, \quad u \in C^\infty(\mathbb{T}^{n+1}) \text{ (or } u \in \mathcal{D}'(\mathbb{T}^{n+1})),$$

where d_t denotes the exterior differential on the torus \mathbb{T}_t^n .

Our aim is to carry out a study of the global solvability at the first level of this complex. In other words, we study the global solvability of the equation $\mathbb{L}u = d_t u + c(t) \wedge \frac{\partial}{\partial x} u = f$ where $u \in \mathcal{D}'(\mathbb{T}^{n+1})$ and $f \in \bigwedge^1 C^\infty(\mathbb{T}_t^n \times \mathbb{T}_x^1)$. If there exists u such that $\mathbb{L}u = f$ then f must be of the form $f = \sum_{j=1}^n f_j(t, x) dt_j$. If so u is a solution of the linear partial differential equations $L_j u = f_j$, $j = 1, \dots, n$.

The local solvability of this complex of operators was treated by Treves in [9]. When the 1-form $c = a + ib$ is exact the problem was solved by Cardoso and Hounie in [7]. We are interested in global solvability when the 1-form $b = \sum_{j=1}^n b_j dt_j$ is exact however the 1-form $a = \sum_{j=1}^n a_j dt_j$ is not exact.

We prove that the global solvability of this class of systems involves Liouville forms (see [5]) and it is closely related to the property of all the sublevel and superlevel sets of a global primitive of b being connected in \mathbb{T}^n .

The articles [1], [2], [4] and [5] deal with similar questions.

2 Mathematical Results

If $f \in C^\infty(\mathbb{T}_t^n \times \mathbb{T}_x^1; \wedge^{1,0})$ we consider the x -Fourier series

$$f(t, x) = \sum_{\xi \in \mathbb{Z}} \hat{f}(t, \xi) e^{i\xi x},$$

where $\hat{f}(t, \xi) = \sum_{j=1}^n \hat{f}_j(t, \xi) dt_j$ and $\hat{f}_j(t, \xi)$ denotes the Fourier transform with respect to x . If there exists $u \in \mathcal{D}'(\mathbb{T}^{n+1})$ such that $\mathbb{L}u = f$ then, since \mathbb{L} defines a differential complex, $\mathbb{L}f = 0$ also

$$\hat{f}(t, \xi) e^{i(\psi_\xi(t) + \xi C(t))} \text{ is exact when } \xi a_0 \text{ is integral,} \quad (2.2)$$

where $\psi_\xi \in C^\infty(\mathbb{R}^n; \mathbb{R})$ is such that $d\psi_\xi = \Pi^*(\xi a_0)$ and $\Pi : \mathbb{R}^n \rightarrow \mathbb{T}^n$ denotes the universal covering of \mathbb{T}^n .

We define the following set

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$$\mathbb{E} = \{f \in C^\infty(\mathbb{T}_t^n \times \mathbb{T}_x^1; \Lambda^{1,0}); \mathbb{L}f = 0 \text{ and (2.2) holds}\}.$$

Definition 2.1. The operator \mathbb{L} is said to be globally solvable on \mathbb{T}^{n+1} if for each $f \in \mathbb{E}$ there exists $u \in \mathcal{D}'(\mathbb{T}^{n+1})$ such that $\mathbb{L}u = f$.

We consider the following two sets $J = \{j \in \{1, \dots, n\}; b_j \equiv 0\}$ and $K = \{k \in \{1, \dots, n\}; a_k \equiv 0\}$. If $J \neq \emptyset$ we write $a_J \doteq \sum_{j \in J} a_j(t)dt_j$.

Under the previous notation the main result of this work is the following theorem.

Theorem 2.1. Let $c = \sum_{j=1}^n (a_j + ib_j)dt_j$ be a smooth closed 1-form where $b = \sum_{j=1}^n b_jdt_j$ is exact. If B is a global primitive of b and $J \cup K = \{1, \dots, n\}$ then the operator $\mathbb{L} = dt + c(t) \wedge \frac{\partial}{\partial x}$ is globally solvable if and only if one of the following two conditions holds:

(I) $J \neq \emptyset$ and a_J is non-Liouville.

(II) The sublevels $\Omega_s = \{t \in \mathbb{T}^n, B(t) < s\}$ and superlevels $\Omega^s = \{t \in \mathbb{T}^n, B(t) > s\}$ are connected for every $s \in \mathbb{R}$ and a_J is rational if $J \neq \emptyset$.

When $J = \{1, \dots, n\}$ we have that $b = 0$, hence any primitive of b has only connected sublevels and superlevels on \mathbb{T}^n . In this case Theorem 2.1 says that \mathbb{L} is globally solvable if and only if either a_J is non-Liouville or a_J is rational, which was proved in [3].

If $J = \emptyset$ then, since $J \cup K = \{1, \dots, n\}$ by hypothesis, $K = \{1, \dots, n\}$. In this case each $a_k \equiv 0$ and Theorem 2.1 says that \mathbb{L} is globally solvable if and only if all the sublevels and superlevels of B are connected in \mathbb{T}^n , which is according to [7]. Thus, in order to prove Theorem 2.1 it suffices to consider the following situation $\emptyset \neq J \neq \{1, \dots, n\}$, in other words, the system (1.1) is made up of two blocks, one formed by a number of real vector fields and the other by some complex ones.

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ARONSSON'S EQUATION WITH STRONG ABSORPTIONS

DAMIÃO J. ARAÚJO *

1 Introduction

The study of reaction-diffusion problems involving strong absorption terms has been a central theme of research in physics, material sciences, industry, biotechnology, chemical engineering, etc. It appears for instance in models involving porous catalysis or enzymatic processes. Some of those models require the study of equations ruled by very degenerated operators, for which the infinite Laplacian,

$$\Delta_\infty f := \sum_{i,j} \partial_i f \partial_{ij} f \partial_j f,$$

is an important prototype. This present work concerns the study of reaction-diffusion models governed by highly degenerate operators of the infinite Laplace type. The equation we treat is the following:

$$\begin{cases} \Delta_\infty u \approx (u^+)^{\gamma} & \text{in } \Omega \\ u = \phi & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

for $0 \leq \gamma < 3$. A decisive aspect of the mathematical formulation of such problems is the existence of dead-cores, i.e., regions where the density of the given substance (or gas) vanishes.

The first step in the program is to establish comparison principle, existence and uniqueness of viscosity solutions to the Dirichlet problem (1.1). The key, main qualitative result established in our work is an improved regularity estimate of solution along the free boundary $\partial\{u > 0\}$. Recall that infinite harmonic functions are locally of class $C^{0,1}$. This is the best regularity estimate available in the literature, at least if $n \geq 3$. In this present work we show that, along the free boundary $\partial\{u > 0\}$, solutions to (1.1) are locally of class $C^{\beta,\alpha}$, for

$$\beta = \left\lfloor \frac{4}{3-\gamma} \right\rfloor \quad \text{and} \quad \alpha = \frac{4}{3-\gamma} - \left\lfloor \frac{4}{3-\gamma} \right\rfloor$$

where $\lfloor \kappa \rfloor$ represents the largest natural number strictly smaller than κ . It is important to notice that for each $0 \leq \gamma < 3$ we have $4/3 \leq \beta(\gamma) + \alpha(\gamma)$ and $\beta(3^-) + \alpha(3^-) = +\infty$. In particular, for $\gamma \geq 1$, solutions are twice differentiable along such a set, so they solve the equation in the classical sense.

The strategy to prove such a regularity estimate is based on a new, striking flatness improvement technique recently developed by Teixeira in [2]. The geometric insights for Teixeira's breakthrough use maximum principle methods combined with tangential regularity theories. Such an improved regularity allows us to establish geometric-measure estimates of the contact set. We prove for any point Y_0 at the free boundary, we have

$$c \cdot r^{\beta+\alpha} \leq \sup_{B_r(Y_0)} u \leq C \cdot r^{\beta+\alpha},$$

for positive dimensional constants $0 < c, C < \infty$. With such a geometric control on u near free boundary points, we can then obtain Bernstein type theorems and Hausdorff estimates of the free boundary. We further show that when $\gamma = 3$, when positive solutions cannot vanish.

This lecture is based on recent joint work with *R. Leitão and E. Teixeira* [1].

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CONTINUIDADE ÓTIMA DO GRADIENTE PARA EQUAÇÕES ELÍTICAS DEGENERADAS

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Neste presente trabalho estudaremos importantes propriedades analíticas de soluções para equações diferenciais parciais elípticas totalmente não-lineares do tipo

$$|\nabla u|^\gamma F(X, D^2u) = f(X).$$

Iremos obter a regularidade ótima para este tipo de equação, as quais tem como principal característica a degenerescência do seu gradiente ao longo do conjunto em que tal taxa de variação se anula. Faremos aplicações relacionando os resultados obtidos à teoria do ∞ -laplaciano.

1 Introdução

A teoria de regularidade, na qual busca propriedades intrínsecas de soluções de equações diferenciais parciais, se torna relevante desde a fundação da teoria de análise moderna de EDPs, por volta do século XVIII. Esse estudo se torna necessário na modelagem matemática de fenômenos físicos e sociais, por exemplo, que são governados por equações diferenciais parciais elípticas de segunda ordem.

Suavidade de soluções fracas para equações uniformemente elípticas de segunda ordem,tanto da forma divergente quanto não-divergência, é hoje em dia bastante bem estabelecida.A parte central no desenvolvimento desta teoria, destacamos a busca de um módulo de continuidade universal de soluções de tais equações. Podemos destacar a teoria de DeGiorgi-Nash-Moser para equações da forma divergente, onde encontramos a obtenção do módulo universal de continuidade para soluções da equação linear homogênea $Lu = 0$ e a desigualdade de Harnack Krylov-Safonov para operadores da forma não divergente.

Apesar da distinta importância das obras supra-citadas acima, um grande número de modelos matemáticos,envolvem operadores cuja a elipticidade se degenera ao longo de uma região desconhecida *a priori*, que depende de sua própria solução. Tais modelos são chamados de *problemas de fronteira livre*.Este fato diminui a eficácia das características de difusão próximo dessa região,e portanto a teoria de regularidade para soluções se torna,para este caso,mais sofisticada do ponto de vista matemático.

Outro avanço destacável, está na teoria de regularidade para soluções de viscosidade de equações uniformemente elípticas não lineares,

$$F(D^2u) = 0 \tag{1.1}$$

que atraiu a atenção da comunidade matemática nessas últimas três décadas. É sabido que soluções da equação homogênea (1.1), são localmente de classe C^{1,α_0} para um expoente universal α_0 , isto é, que depende apenas das constantes d -dimensão e λ, Λ -constantes de elipticidade, veja [1]. Caso nenhuma hipótese estrutural seja imposta para o operador F , a regularidade C^{1,α_0} é de fato ótima, veja [4, 5] e [6]. Sob hipótese de concavidade ou convexidade em F , um teorema devido a Evans e Krylov, estabelece que soluções são $C^{2,\alpha}$. A teoria de regularidade para o caso não-homogêneo

$$F(X, D^2u) = f(X) \tag{1.2}$$

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naturalmente se torna um caso mais delicado. Como consequência, soluções de tais equações podem não ser tão regulares quanto as do caso homogêneo. Em um trabalho de bastante importância, Caffarelli [2], estabelece estimativas $W^{2,p}$ *a priori* para soluções de (1.2), onde $f \in L^p$, com $p < d$ -dimensão, adicionando ao operador F uma hipótese do tipo VMO para seus coeficientes. Recentemente, E. Teixeira em [8], fornece um módulo de continuidade universal ótimo para equações totalmente não-lineares de coeficientes variáveis, baseado nas propriedades de fraca-integrabilidade que o potencial f em (1.2) pode assumir.

O trabalho tem como objetivo obter estimativas ótimas interiores para equações elípticas degeneradas da forma:

$$\mathcal{H}(X, \nabla u)F(X, D^2u) = f(X) \quad \text{em } B_1 \subset \mathbb{R}^d, \quad (1.3)$$

onde $f \in L^\infty(B_1)$ e o operador $\mathcal{H}: B_1 \times \mathbb{R}^d \rightarrow \mathbb{R}$ degenera com uma taxa comparável a uma potência da magnitude do gradiente, isto é,

$$\lambda \|\vec{p}\|^\gamma \leq \mathcal{H}(X, \vec{p}) \leq \Lambda \|\vec{p}\|^\gamma, \quad (1.4)$$

para algum $\gamma > 0$. O operador de 2ª ordem $F: B_1 \times \text{Sym}(N) \rightarrow \mathbb{R}$ na equação (1.3) é o responsável pela difusão, ou seja, F é um operador uniformemente elíptico totalmente não-linear.

Em diversos modelos matemáticos, a degenerescência elíptica ocorre ao longo do conjunto singular:

$$\mathcal{S}(u) := \{X : \nabla u(X) = 0\},$$

de uma solução existente. De fato, um certo número de equações elípticas degeneradas tem seus graus de degenerescência comparados a

$$f(\nabla u)|D^2u| \approx 1, \quad (1.5)$$

para alguma função $f: \mathbb{R}^d \rightarrow \mathbb{R}$, com $\text{Zero}(f) = \{0\}$. Assim, para compreendermos o efeito preciso sobre a falta de suavidade imposta pela equação modelo (1.5) nos guia a uma melhor compreensão sobre a teoria de regularidade ótima para uma quantidade razoável de operadores elípticos degenerados.

Notemos que pela estrutura da equação (1.3) nenhuma teoria de regularidade universal para tal equação pode ir além da regularidade C^{1,α_0} , para α_0 em (1.1), ou seja, essa regularidade é ótima. De fato, o termo de degenerescência $\mathcal{H}(X, \nabla u)$ força as soluções a serem menos regulares que soluções do problema uniformemente elíptico próximo de seu conjunto singular. Esta característica particular indica que a obtenção de estimativas de regularidade ótima para soluções de (1.3) não deve seguir de técnicas de perturbação. De fato, isto exige novas ideias envolvendo uma interação de equilíbrio entre a teoria de regularidade universal para equações uniformemente elípticas e o efeito de degenerescência atribuídos pelo operador difusão (1.4).

Sob esta análise estrutural, mostraremos que o gradiente de uma solução de viscosidade u , de (1.3), é localmente de classe $C^{0,\min\{\alpha_0^-, \frac{1}{1+\theta}\}}$. O expoente ótimo de Hölder-continuidade para o gradiente de soluções,

$$\beta := \min \left\{ \alpha_0^-, \frac{1}{1+\theta} \right\}, \quad (1.6)$$

nos dá precisamente a regularidade ótima e universal de equações degeneradas do tipo (1.3). O resultado nos fornece uma aquisição extra-qualitativa em relação ao recente resultado de Imbert e Silvestre, [3], onde foi provado que soluções de viscosidade de (1.3) são continuamente diferenciáveis.

2 Resultados

Iremos apresentar o principal resultado acerca da teoria que foram desenvolvidas no trabalho. Tal resultado fornecerá uma estimativa de regularidade ótima para funções u , satisfazendo

$$|\nabla u|^\gamma \cdot |F(D^2u)| \lesssim 1, \quad \gamma > 0, \quad (2.7)$$

no sentido da viscosidade, para algum operador uniformemente elíptico F . Como comentado no início, para o caso não-degenerado, isto é, $\gamma = 0$, a melhor regularidade possível é $C_{loc}^{1+\alpha_0^-}$. O ponto delicado, está em obter uma estimativa universal precisa o suficiente para que o grau de singularidade $\gamma > 0$, que surge da equação (2.7) seja explicitamente sentido, ao longo do conjunto singular $(\nabla u)^{-1}(0)$. Devido aos comentários feitos acima, estamos aptos a formular o principal resultado o qual foi dedicado no início.

Teorema 2.1. *Seja u uma solução de viscosidade de*

$$\mathcal{H}(X, \nabla u)F(D^2u) = f(X) \quad \text{em } B_1. \quad (2.8)$$

Assuma $f \in L^\infty(B_1)$, \mathcal{H} satisfaz (1.4) e $F: S(d) \rightarrow \mathbb{R}$ é uniformemente elítico. Fixado um expoente

$$\alpha \in (0, \alpha_0) \cap \left(0, \frac{1}{1+\gamma}\right],$$

existe uma constante $C(d, \lambda, \Lambda, \gamma, \|f\|_\infty, \alpha) > 0$, dependente apenas de $d, \lambda, \Lambda, \gamma, \|f\|_\infty$ e α , tal que

$$\|u\|_{C^{1,\alpha}(B_{1/2})} \leq C(d, \lambda, \Lambda, \gamma, \|f\|_\infty, \alpha) \cdot \|u\|_{L^\infty}.$$

Como consequênciá imediata do Teorema 2.1 obtemos

Corolário 2.1. *Nas hipóteses do Teorema 2.1, se F for um operador côncavo (ou convexo) então soluções de (2.8) são de classe $C_{loc}^{1, \frac{1}{1+\gamma}}$*

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Operadores Múltiplo N -Separadamente Somantes

Daniel Núñez Alarcón

Introduziremos a noção de *operadores múltiplo N -separadamente somantes*. Nossa abordagem estende e unifica alguns resultados recentes; por exemplo, recuperamos a melhor estimativa conhecida para as constantes da desigualdade multilinear de Bohnenblust-Hille, dada por F. Bayart, D. Pellegrino and J. Seoane-Sepúlveda em [2].

1 Introdução

A ℓ_1 -norma fraca de vetores x_1, \dots, x_N num espaço de Banach X , é definida por:

$$\| (x_i)_{i=1}^N \|_{w,1} := \sup_{\|x'\|_{X'} \leq 1} \sum_{i=1}^N |x'(x_i)|.$$

Seja X, X_1, \dots, X_m, Y espaços de Banach, e $\mathcal{L}(X_1, \dots, X_m; Y)$ o espaço de Banach de todos os operadores (limitados) m -lineares $U : X_1 \times \dots \times X_m \rightarrow Y$. Para $1 \leq r < \infty$, $U \in \mathcal{L}(X_1, \dots, X_m; Y)$ é chamado *múltiplo $(r, 1)$ -somante*, se existir uma constante $C > 0$ tal que:

$$\left(\sum_{i_1, \dots, i_m=1}^N \|U(x_{i_1}^{(1)}, \dots, x_{i_m}^{(m)})\|_Y^r \right)^{\frac{1}{r}} \leq C \prod_{k=1}^m \left\| \left(x_i^{(k)} \right)_{i=1}^N \right\|_{w,1}$$

para qualquer escolha finita de vetores $x_i^{(k)} \in X_k$, $1 \leq i \leq N$, $1 \leq k \leq m$. O espaço vetorial de todos os operadores múltiplo $(r, 1)$ -somantes é denotado por $\Pi_{(r,1)}^m(X_1, \dots, X_m; Y)$. O ínfimo, $\pi_{(r,1)}^m(U)$, de todas as possíveis constantes C que satisfazem a desigualdade anterior, define um norma completa em $\Pi_{(r,1)}^m(X_1, \dots, X_m; Y)$. Usaremos $\mathcal{P}_k(m)$ para denotar o conjunto de todos os subconjuntos de $\{1, \dots, m\}$, com cardinalidade k , $k = 1, \dots, m$.

Para X_1, \dots, X_m e um subconjunto próprio $D \subset \{1, \dots, m\}$, seja X^D o produto cartesiano $\prod_{k \in D} X_k$. Um vetor $x_D \in X^D$ pode ser visto como um elemento $\widetilde{x_D} \in X_1 \times \dots \times X_m$, com $\widetilde{x_D^i} = x_D^i$, se $i \in D$, e $\widetilde{x_D^i} = 0$, caso contrário.

Dado $U \in \mathcal{L}(X_1, \dots, X_m; Y)$, definimos a aplicação

$$\begin{aligned} U^D : X^{\widehat{D}} &\rightarrow \mathcal{L}(X^D; Y) \\ x_{\widehat{D}} &\mapsto U_{x_{\widehat{D}}}^D : X^D \rightarrow Y \\ y_D &\mapsto U_{x_{\widehat{D}}}^D(y_D) := U(\widetilde{x_D} + \widetilde{y_D}), \end{aligned}$$

onde \widehat{D} denota o complementar de D em $\{1, \dots, m\}$. U^D é claramente bem definida e $|\widehat{D}|$ -linear. Além disso, notemos que para cada $x_{\widehat{D}} \in X^{\widehat{D}}$, $U_{x_{\widehat{D}}}^D$ é a restrição de U às D -coordenadas, com as \widehat{D} -coordenadas fixadas. A seguinte definição foi introduzida em [3].

Definição 1.1. Seja $1 \leq r < \infty$, e D um subconjunto próprio de $\{1, \dots, m\}$. Dizemos que $U \in \mathcal{L}(X_1, \dots, X_m; Y)$ é múltiplo $(r, 1)$ -somante nas coordenadas de D (ou múltiplo $(r, 1)$ -somante em D) quando U^D tem sua imagem em $\Pi_{(r,1)}^{|D|}(X^D; Y)$. Além disso, U será dito separadamente $(r, 1)$ -somante se U é múltiplo $(r, 1)$ -somante em cada subconjunto unitário de $\{1, \dots, m\}$.

O seguinte resultado, dado em [3], pode ser visto como uma versão vetorial da desigualdade multilinear de Bohnenblust-Hille:

Teorema 1.1 ([3]; Corollary 5.2). Seja Y um espaço de Banach com cotipo q , e $1 \leq r < q$. Então, existe uma constante $\sigma_m \geq 1$ tal que cada operador separadamente $(r, 1)$ -somante $U \in \mathcal{L}(X_1, \dots, X_m; Y)$, é múltiplo

$\left(\frac{qrm}{q+(m-1)r}, 1\right)$ -somante, e

$$\pi_{\left(\frac{qrm}{q+(m-1)r}, 1\right)}^m(U) \leq \sigma_m \prod_{k=1}^m \left\| U^{\{k\}} : X^{\widehat{\{k\}}} \rightarrow \Pi_{r,1}(X^{\{k\}}; Y) \right\|^{\frac{1}{m}}$$

onde σ_m , depende somente de m , r , q e a constante de cotipo $C_q(Y)$.

A seguinte definição é uma variação da definição 1.1.

Definição 1.2. Seja $1 \leq r < \infty$. Dizemos que $U \in \mathcal{L}(X_1, \dots, X_m; Y)$ é N -separadamente $(r, 1)$ -somante, se U for múltiplo $(r, 1)$ -somante em cada subconjunto de $\{1, \dots, m\}$ com cardinalidade N .

Seja Y um espaço de Banach com cotipo q . O seguinte resultado generaliza o Teorema 1.1:

Teorema 1.2. Seja $1 \leq r \leq q$, e $1 \leq n < m$. Se $U \in \mathcal{L}(X_1, \dots, X_m; Y)$ é n -separadamente $(r, 1)$ -somante, então U é N -separadamente $(r_N, 1)$ -somante, para todo $n < N \leq m$, com $r_N := \frac{qrN}{nq+(N-n)r}$. Além disso, se $N < m$, temos, para cada $D \in \mathcal{P}_N(m)$,

$$\pi_{(r_N, 1)}^N(U_{x_{\widehat{D}}}) \leq \sigma_N \left(\prod_{S \in \mathcal{P}_n(N)} \left\| \left(U_{x_{\widehat{D}}}^D \right)^S : X^{\widehat{S}} \rightarrow \Pi_{(r, 1)}^n(X^S; Y) \right\| \right)^{\frac{1}{\binom{N}{n}}},$$

para todo $x_{\widehat{D}} \in X^{\widehat{D}}$, onde σ_N , depende somente de N , r , q e a constante de cotipo $C_q(Y)$. A estimativa para $N = m$ é

$$\pi_{(r_m, 1)}^m(U) \leq \sigma_m \left(\prod_{S \in \mathcal{P}_n(m)} \left\| U^S : X^{\widehat{S}} \rightarrow \Pi_{(r, 1)}^n(X^S; Y) \right\| \right)^{\frac{1}{\binom{m}{n}}},$$

onde σ_m , depende somente de m , r , q e a constante de cotipo $C_q(Y)$.

Para o caso em que $Y = \mathbb{K}$, e $X_1 = \dots = X_m = c_0$, este resultado recupera a desigualdade multilinear de Bohnenblust-Hille e fornece as mesmas estimativas dadas por F. Bayart, D. Pellegrino and J. Seoane-Sepúlveda em [2].

Este trabalho encontra-se contido em [1].

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PERIODIC SOLUTIONS FOR NONLINEAR DIFFERENTIAL EQUATIONS OF SECOND ORDER

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1 Introduction

From qualitative theory of ordinary differential equations, the problem of existence of periodic solutions for various nonlinear differential equations of higher order continues to attract the attentions of many specialists despite its long history. In few works, several mathematics dealt with the problems by using Lyapunovs functions and Greens functions, and obtained criteria for the existence of periodic solutions. In particular, for the use of Greens functions, ones can refer to Cronin [1] and Shadman [2].

In [3], the author used Leray-Schauder degree to show the existence of periodic solutions to second order nonlinear differential equations of the form

$$x'' + c(x)x' + f(x) = e(t).$$

Now, consider the real second order nonlinear differential equations of the type:

$$x'' + c(t)x' + f(t, x) = p(t, x, x'). \quad (1.1)$$

in which, $p(t, x, x')$, $f(t, x)$ and $c(t)$ are continuous functions in their respective domains $[0, L] \times \mathbb{R}^2$, $[0, L] \times \mathbb{R}$ e $[0, L]$, respectively. Further, it is assumed that all initial value problems corresponding to equation (1.1) can be extended to $[0, L]$.

2 Mathematical Results

The main result is obtained.

Theorem 2.1. *We assume that the following conditions hold:*

1. $|f(t, x)| \leq \gamma|x| + \beta$ for all $t \in [0, L]$ and $x| < \infty$, where γ and β are some non-negative constants.

2. $|p(t, x, x')| \leq |e(t)|$ for all t, x and x' , and $e(t)$ is a continuous function for all $t \in [0, L]$.

3. $\gamma \left(\frac{L}{\pi}\right)^2 + \gamma_1 \left(\frac{L}{\pi}\right) < 1$, $\gamma_1 = \max |c(t)|$.

Then equation (1.1) possesses a solution satisfying

$$x^i(0) + x^i(L) = 0, \quad (i = 0, 1). \quad (2.2)$$

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Proof We mention that the proof of theorem will use the Leary-Schauder principle and the complete continuity of the operator

$$(Lx)(t) = \mu \int_0^T g(t, s) [p(s, x(s), x'(s)) - f(s, x(s))] ds, \quad (2.3)$$

ones can conclude that equation (1.1) has at least a solution in the open sphere $\{x : \|x\|_{c^2} < R\}$.

First of all, we show an estimate on the magnitude of the solutions of problem:

$$\begin{aligned} x'' + c(t)x' &= \mu [p(t, x, x') - f(t, x)], \quad \mu \in [0, 1], \\ x^i(0) + x^i(L) &= 0, \quad (i = 0, 1). \end{aligned} \quad (2.4)$$

We assume that $x(t)$ is a function of class $C^{n-1}[0, L]$, such that $x(t+L) + x(t) = 0$ for all t , and we use Wirtingers inequalities in the following from:

$$\begin{aligned} \|x^{(i-1)}(t)\|_2 &\leq \left(\frac{L}{\pi}\right)^{n-i+1} \|x^{(n)}(t)\|_2, \quad (i = 1, 2, \dots, n), \\ \|\cdot\|_2 &= \left[\int_0^L |.|^2 dt \right]^{\frac{1}{2}}. \end{aligned} \quad (2.5)$$

Now, we suppose that is a solution of the problem given by (2.4). In view of the assumptions of the theorem, it is easily followed from (2.4) that

$$|x''(t)| \leq \gamma_1 |x'| + \mu [|e(t)| + \gamma |x(t)| + \beta].$$

Hence, by using the Minkowskis inequality

$$\|x''(t)\|_2 \leq \gamma_1 \|x'\|_2 + \mu \left\{ \|e(t)\|_2 + \gamma \|x(t)\|_2 + \beta \sqrt{L} \right\},$$

it can be seen from Wirtingers inequality that

$$\|x''(t)\|_2 \leq \gamma_1 \left(\frac{L}{\pi}\right) \|x'\|_2 + \mu \left\{ \|e(t)\|_2 + \gamma \left(\frac{L}{\pi}\right)^2 \|x''(t)\|_2 + \beta \sqrt{L} \right\},$$

where

$$[1 - \gamma_1 \left(\frac{L}{\pi}\right) - \mu \gamma \left(\frac{L}{\pi}\right)^2] \|x''(t)\|_2 \leq \mu \left\{ \|e(t)\|_2 + \beta \sqrt{L} \right\}.$$

Making use of assumption 3 of the theorem and in view of the fact $0 \leq \leq 1$, we obtain

$$\|x''(t)\|_2 \leq \frac{\|e(t)\|_2 + \beta \sqrt{L}}{1 - \gamma_1 \left(\frac{L}{\pi}\right) - \gamma \left(\frac{L}{\pi}\right)^2}. \quad (2.6)$$

Now, we write

$$x^{(i-1)}(t) = x^{(i-1)}(0) + \int_0^t x^{(i)}(\tau) d\tau, \quad (i = 1, 2).$$

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MAGNETOHYDRODYNAMIC FLOW TYPE: GRADE-TWO FLUID MODEL

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1 Introduction

A fluid of grade two is a non-Newtonian fluid of differentiable type introduced by Rivlin and Ericksen in [6]. An analysis in [1] shows that the equation of a fluid of grade two is given by

$$\begin{aligned} \frac{\partial}{\partial t}(\mathbf{u} - \alpha\Delta\mathbf{u}) - \nu\Delta\mathbf{u} + \sum_j (\mathbf{u} - \alpha\Delta\mathbf{u})_j \nabla u_j - \mathbf{u} \cdot \nabla(\mathbf{u} - \alpha\Delta\mathbf{u}) &= -\nabla p + \mathbf{f} \\ \operatorname{div} \mathbf{u} &= 0 \end{aligned}$$

where $\alpha \geq 0$ is a constant of material, $\nu > 0$ is the viscosity of the fluid, \mathbf{u} is the velocity field, and p is pressure. For $\alpha = 0$ the classics Navier-Stokes equations is obtained.

On the other hand, in several situations the motion of an incompressible electrical conducting fluid can be modelled by the magnetohydrodynamic equation, which corresponds to the Navier-Stokes equations coupled with the Maxwell equations, see [7, 5]. In the case when the MHD equation is coupled with the equation of an incompressible second grade fluid, the model can be written as

$$\begin{aligned} \frac{\partial}{\partial t}(\mathbf{u} - \alpha\Delta\mathbf{u}) - \nu\Delta\mathbf{u} + \operatorname{curl}(\mathbf{u} - \alpha\Delta\mathbf{u}) \times \mathbf{u} - (\mathbf{h} \cdot \nabla) \mathbf{h} &= \mathbf{f} - \nabla(p^* + \mathbf{h}^2) \\ \frac{\partial \mathbf{h}}{\partial t} - \Delta \mathbf{h} + (\mathbf{u} \cdot \nabla) \mathbf{h} - (\mathbf{h} \cdot \nabla) \mathbf{u} &= -\operatorname{grad} w \\ \operatorname{div} \mathbf{u} = \operatorname{div} \mathbf{h} &= 0 \\ \mathbf{u} = \mathbf{h} &= 0 \quad \text{on } \partial\Omega \times (0, T) \\ \mathbf{u}(0) = \mathbf{u}_0; \quad \mathbf{h}(0) = \mathbf{h}_0 & \quad \text{in } \Omega. \end{aligned} \tag{1.1}$$

Where we considered the physical constants $\mu = \sigma = \eta = 1$.

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The system (1.1) can be rewritten by introducing the auxiliary variable $z = \operatorname{curl} (\mathbf{u} - \alpha \Delta \mathbf{u})$, as

$$\begin{aligned} \frac{\partial}{\partial t}(\mathbf{u} - \alpha \Delta \mathbf{u}) - \nu \Delta \mathbf{u} + z \times \mathbf{u} - (\mathbf{h} \cdot \nabla) \mathbf{h} &= \mathbf{f} - \nabla(p^* + \mathbf{h}^2) \\ \frac{\partial \mathbf{h}}{\partial t} - \Delta \mathbf{h} + (\mathbf{u} \cdot \nabla) \mathbf{h} - (\mathbf{h} \cdot \nabla) \mathbf{u} &= -\operatorname{grad} w \\ \operatorname{div} \mathbf{u} = \operatorname{div} \mathbf{h} &= 0 \\ \mathbf{u} = \mathbf{h} &= 0 \quad \text{on } \partial\Omega \times (0, T) \\ \mathbf{u}(0) = \mathbf{u}_0; \quad \mathbf{h}(0) = \mathbf{h}_0 &\quad \text{in } \Omega \end{aligned} \tag{1.2}$$

In this work we discuss the MHD flow of a second grade fluid [4], in particular we prove the existence of a weak solution of a time-dependent grade two fluid model in a two-dimensional Lipschitz domain. We follow the methodology of [3], i.e., we use a constructive method which can be adapted to the numerical analysis of finite-element schemes for solving this problem numerically.

2 Mathematical Results

Theorem 2.1. *Let Ω be a bounded Lipschitz-continuous domain in two dimensions. Then for any $\alpha > 0, \nu > 0, \mathbf{f} \in L^2(0, T; H(\operatorname{curl}; \Omega))$ and $\mathbf{u}_0, \mathbf{h}_0 \in V$ with $\operatorname{curl}(\mathbf{u}_0 - \alpha \Delta \mathbf{u}_0) \in L^2(\Omega)$, problem (1.2) has at least one solution $\mathbf{u}, \mathbf{h} \in L^\infty(0, T; V)$ with $\frac{\partial \mathbf{u}}{\partial t}, \frac{\partial \mathbf{h}}{\partial t} \in L^2(0, T; V)$ and $p, w \in L^2(0, T; L_0^2(\Omega))$.*

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PROPRIEDADE POLINOMIAL ALTERNATIVA DE DAUGAVET

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Neste trabalho investigamos uma versão mais fraca da propriedade polinomial de Daugavet denominada propriedade polinomial alternativa de Daugavet. Realizamos um estudo sobre a estabilidade desta propriedade sobre somas c_0 , ℓ_∞ e ℓ_1 de espaços de Banach. Deste estudo obtemos exemplos de espaços de Banach com a propriedade polinomial alternativa de Daugavet, a saber $L_\infty(\mu, X)$ e $C(K, X)$, onde X tem a propriedade polinomial alternativa de Daugavet.

1 Introdução

Seja X um espaço de Banach. Dizemos que um operador linear limitado $T : X \rightarrow X$ satisfaz a *equação de Daugavet* se

$$\|\text{Id} + T\| = 1 + \|T\|, \quad (\text{DE})$$

e dizemos que T satisfaz a *equação alternativa de Daugavet* se

$$\max_{|w|=1} \|\text{Id} + wT\| = 1 + \|T\|. \quad (\text{ADE})$$

Estas definições foram apresentadas por I. K. Daugavet [3] e por J. Duncan et al. [4], respectivamente. Desde seu surgimento, a validade destas equações tem sido verificada por diversos autores para várias classes de operadores em diferentes espaços de Banach.

Dizemos que um espaço de Banach X tem a *propriedade de Daugavet* (resp. *propriedade alternativa de Daugavet*) se todo operador de posto um em X satisfaz (DE) (resp. (ADE)). Segundo P. Wojtaszczyk [7], se os espaços $(X_j)_{j=1}^\infty$ possuem a propriedade de Daugavet, então $[\bigoplus_{j=1}^\infty X_j]_{\ell_1}$ e $[\bigoplus_{j=1}^\infty X_j]_{\ell_\infty}$ possuem a propriedade de Daugavet. E de acordo com M. Martín e T. Oikhberg [5], $[\bigoplus_{j=1}^\infty X_j]_{\ell_\infty}$ (ou $[\bigoplus_{j=1}^\infty X_j]_{c_0}$, ou $[\bigoplus_{j=1}^\infty X_j]_{\ell_1}$) tem a propriedade alternativa de Daugavet se, e somente se, todo X_j tem a propriedade alternativa de Daugavet.

Y. S. Choi et al. [1] generalizaram as definições da equação de Daugavet e da equação alternativa de Daugavet para funções limitadas em um espaço de Banach X da seguinte forma: se $\ell_\infty(B_X, X)$ é o espaço de Banach de todas funções limitadas de B_X para X , munido com a norma do supremo, dizemos que uma função $\Phi \in \ell_\infty(B_X, X)$ satisfaz a *equação de Daugavet* se

$$\|\text{Id} + \Phi\| = 1 + \|\Phi\|, \quad (\text{DE})$$

e dizemos que Φ satisfaz a *equação alternativa de Daugavet* se

$$\max_{|w|=1} \|I + w\Phi\| = 1 + \|\Phi\|. \quad (\text{ADE})$$

Estas equações tem sido estudadas em particular para polinômios. No caso em que todo polinômio de posto um em um espaço de Banach X satisfaz (DE) (resp. (ADE)), dizemos que X tem a *propriedade polinomial de Daugavet* (resp. *propriedade polinomial alternativa de Daugavet*). Segundo Y. S. Choi et al. [2], se $(X_j)_{j=1}^\infty$ é uma sequência de espaços de Banach, então $[\bigoplus_{j=1}^\infty X_j]_{\ell_\infty}$ (ou $[\bigoplus_{j=1}^\infty X_j]_{c_0}$) tem a propriedade polinomial de Daugavet se, e somente se, todo X_j tem a propriedade polinomial de Daugavet.

O principal objetivo deste trabalho é apresentar resultados sobre a estabilidade da propriedade polinomial alternativa de Daugavet sobre somas c_0 , ℓ_∞ e ℓ_1 de espaços de Banach.

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2 Resultados

Motivados pelos resultados de P. Wojtaszczyk [7], M. Martín e T. Oikhberg [5], e Y. S. Choi et al. [2] mostramos que a propriedade polinomial alternativa de Daugavet também é estável sobre somas c_0 e ℓ_∞ .

Proposição 2.1. *Seja $(X_j)_{j=1}^\infty$ uma sequência de espaços de Banach. Então $[\bigoplus_{j=1}^\infty X_j]_{\ell_\infty}$ (ou $[\bigoplus_{j=1}^\infty X_j]_{c_0}$) tem a propriedade polinomial alternativa de Daugavet se, e somente se, todo X_j tem a propriedade polinomial alternativa de Daugavet.*

Esta proposição implica que, para um espaço de Banach X , $c_0(X)$ e $\ell_\infty(X)$ têm a propriedade polinomial alternativa de Daugavet se, e somente se, X tem a propriedade polinomial alternativa de Daugavet.

Embora a propriedade alternativa de Daugavet seja estável sobre somas ℓ_1 de espaços de Banach, infelizmente o mesmo não é válido para a propriedade polinomial alternativa de Daugavet. De fato, \mathbb{C} tem a propriedade polinomial alternativa de Daugavet, segundo ([1], Example 2.1.b), enquanto que $\ell_1(\mathbb{C})$ não possui propriedade polinomial alternativa de Daugavet, por ([1], Example 3.12). Entretanto, temos o seguinte resultado.

Proposição 2.2. *Seja $(X_j)_{j=1}^\infty$ uma sequência de espaços de Banach. Se $[\bigoplus_{j=1}^\infty X_j]_{\ell_1}$ tem a propriedade polinomial de Daugavet (resp. a propriedade polinomial alternativa de Daugavet), então todo X_j tem a propriedade polinomial de Daugavet (resp. a propriedade polinomial alternativa de Daugavet).*

Fazendo uso das proposições 2.1 e 2.2 obtemos a proposição a seguir.

Proposição 2.3. *Sejam X um espaço de Banach, K um espaço de Hausdorff compacto e (Ω, Σ, μ) um espaço de medida σ -finita. São válidas as seguintes afirmações:*

- (a) *$C(K, X)$ tem a propriedade polinomial alternativa de Daugavet se, e somente se, K não possui pontos isolados ou X tem a propriedade polinomial alternativa de Daugavet;*
- (b) *$L_\infty(\mu, X)$ tem a propriedade polinomial alternativa de Daugavet se, e somente se, μ é não atômica ou X tem a propriedade polinomial alternativa de Daugavet;*
- (c) *Se $L_1(\mu, X)$ tem a propriedade polinomial de Daugavet (resp. propriedade polinomial alternativa de Daugavet), então μ é não atômica ou X tem a propriedade polinomial de Daugavet (resp. propriedade polinomial alternativa de Daugavet).*

As demonstrações dos resultados enunciados acima estão disponíveis em [6].

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SEMICONTINUIDADE SUPERIOR DE ATRATORES PULLBACK PARA PROBLEMAS PARABÓLICOS COM $p_\epsilon(x)$ -LAPLACIANO

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Neste artigo provamos a semicontinuidade superior da família de atratores pullback de equações de evolução não-autônomas governadas por uma perturbação do operador maximal monótono.

1 Introdução

Neste trabalho analisamos, por meio da Teoria de Atrator Pullback [2] e [3], o comportamento assintótico da seguinte família de problemas não-autônomos:

$$\begin{cases} u_t - \operatorname{div}(|\nabla u|^{p_\epsilon(x)-2}\nabla u) = B(t, u) \\ u(\tau) = u_0 \in L^2(\Omega), \end{cases} \quad (1.1)$$

onde $\Omega \subset \mathbb{R}^n$ é um domínio limitado com fronteira $\partial\Omega$ suave, $n \geq 1$. Sob uma pequena variação a função $p_\epsilon(x) := p(x) + \epsilon \in C(\bar{\Omega})$, com $\epsilon \in [0, 1]$, satisfaz $2 + \delta \leq p_\epsilon(x) \leq 3 - \delta$, para $\delta > 0$ q.t.p. $x \in \Omega$ e $B : \mathbb{R} \times L^2(\Omega) \rightarrow L^2(\Omega)$ satisfaz:

i) Existe uma aplicação $L \in C(\mathbb{R}; L^\infty(\mathbb{R}^n))$ não decrescente e absolutamente contínua tal que

$$\|B(t, u_1) - B(t, u_2)\|_{L^2(\Omega)} \leq L(t, x)\|u_1 - u_2\|_{L^2(\Omega)}, \quad \forall t \in \mathbb{R}, \forall u_1, u_2 \in L^2(\Omega);$$

ii) $B(t, 0) = 0$;

Podemos citar trabalhos recentes, como [8] e [9], sobre o comportamento assintótico de problemas com o $p(x)$ -Laplaciano, onde os autores estudam a existência de um atrator global. Porém, neste trabalho, o problema que consideramos é não-autônomo e portanto a Teoria de Atrator Pullback se faz necessária.

Antes de apresentarmos os resultados principais deste trabalho consideremos o espaço generalizado de Lebesgue

$$L^{p_\epsilon(x)}(\Omega) = \left\{ u : \Omega \rightarrow \mathbb{R} : u \text{ é mensurável, } \int_{\Omega} |u(x)|^{p_\epsilon(x)} dx < \infty \right\},$$

sendo $p_\epsilon \in L^\infty(\Omega)$ e $p_\epsilon \geq 1$. Definimos

$$\rho(u) = \int_{\Omega} |u(x)|^{p_\epsilon(x)} dx.$$

Com esta definição e da mesma forma que [4], [5] e [6], podemos garantir que $L^{p_\epsilon(x)}(\Omega)$ é um espaço de Banach com a norma $\|u\|_{L^{p_\epsilon(x)}(\Omega)} = \inf \left\{ \lambda > 0 ; \rho \left(\frac{u}{\lambda} \right) \leq 1 \right\}$ e que $W^{1,p_\epsilon(x)}(\Omega)$ é um espaço de Banach com a norma $\|u\|_* := \|u\|_{L^{p_\epsilon(x)}(\Omega)} + \|\nabla u\|_{L^{p_\epsilon(x)}(\Omega)}$.

Como o operador principal $A_\epsilon u := -\operatorname{div}(|\nabla u|^{p_\epsilon(x)-2}\nabla u)$ é maximal monótono (veja [7]) e as condições sobre a B a deixa globalmente Lipschitz, podemos garantir, via Proposição 3.13 [1] que, para todo dado inicial $u_0^\epsilon \in L^2(\Omega)$ existe uma única solução $u^\epsilon(\cdot) := u(\cdot, \tau)u_0^\epsilon \in C([\tau, \infty), L^2(\Omega))$ do problema (1.1).

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2 Resultados

Lema 1: Para cada parâmetro $\epsilon \in [0, 1]$, o problema (1.1) tem um atrator pullback $\{\mathcal{A}_\epsilon(t)\}_{t \in \mathbb{R}}$ em $L^2(\Omega)$.

Prova: Para cada parâmetro $\epsilon \in [0, 1]$ definimos o processo $U_\epsilon(\cdot, \tau) := u^\epsilon(\cdot)$. Por demonstrações adaptadas de Lemas de [8] fazemos estimativas, uniformes em ϵ , da solução de (1.1) em $L^2(\Omega)$ e em $W^{1,p_\epsilon(x)}(\Omega)$, assim, garantimos a existência de um atrator pullback para cada parâmetro ϵ e portanto, de uma família de atratores pullback $\{\mathcal{A}_\epsilon(t)\}_{t \in \mathbb{R}}$.

A seguir enunciamos o resultado principal deste trabalho:

Teorema 1: Para cada valor do parâmetro $\epsilon \in [0, 1]$ o problema (1.1) tem um processo $\{U_\epsilon(t, \tau)\}_{t \geq \tau}$ em $L^2(\Omega)$ associado, o qual tem um atrator pullback $\{\mathcal{A}_\epsilon(t)\}_{t \in \mathbb{R}}$. Além disso, esta família de atratores pullback é semicontínua superiormente em $\epsilon = 0$, desde que $u_0^\epsilon \xrightarrow{\epsilon \rightarrow 0} u_0$ em $L^2(\Omega)$.

Prova Vamos provar que a família de atratores pullback $\{\mathcal{A}_\epsilon(t)\}_{t \in \mathbb{R}}$ é semicontínua superiormente em $\epsilon = 0$, isto é, que

$$\lim_{\epsilon \rightarrow 0} \text{dist}_{L^2(\Omega)} (\mathcal{A}_\epsilon(t), \mathcal{A}_0(t)) := \lim_{\epsilon \rightarrow 0} \sup_{a_\epsilon \in \mathcal{A}_\epsilon(t)} \inf_{b \in \mathcal{A}_0(t)} \|a_\epsilon - b\|_{L^2(\Omega)} = 0$$

A idéia da prova consiste em observar que dado $\delta > 0$, existe $\epsilon_0 > 0$ tal que

$$\text{dist} (\mathcal{A}_\epsilon(t), \mathcal{A}_0(t)) \leq \text{dist} (U_\epsilon(t, \tau) \mathcal{A}_\epsilon(\tau), U_0(t, \tau) \mathcal{A}_\epsilon(\tau)) + \text{dist} (U_0(t, \tau) \mathcal{A}_\epsilon(\tau), U_0(t, \tau) \mathcal{A}_0(\tau)) < \delta,$$

para todo $\epsilon < \epsilon_0$, o que prova a semicontinuidade superior da família de atratores.

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CLASSIFICAÇÃO DE SOLUÇÕES DE ALGUMAS EQUAÇÕES ELÍPTICAS NÃO LINEARES

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1 Introdução

Neste trabalho, classificamos algumas das soluções de equações tipo $\Delta u + f e^u = 0$ em \mathbb{R}^2 ou \mathbb{R}_+^2 . Para isso, utilizamos basicamente o Método dos Planos Móveis e o Método das Esferas Móveis, garantindo, sob certas condições a monotonicidade e a simetria radial da solução. O primeiro método foi usado para estudarmos o caso $f \equiv 1$, em \mathbb{R}^2 com $\int_{\mathbb{R}^2} e^u$ finito. O outro foi utilizado para verificar que a equação não tem solução quando f é uma função contínua, radialmente simétrica e monótona na região em que tem imagem positiva e não constante. Este último método também foi aplicado no estudo do problema

$$\begin{cases} \Delta u + \alpha e^u = 0 & \text{em } \mathbb{R}_+^2; \\ \frac{\partial u}{\partial t} = ce^{u/2} & \text{sobre } \partial\mathbb{R}_+^2; \end{cases}$$

para $\alpha = 1, \alpha = -1$ ou $\alpha = 0$, modificando as condições em relação a finitude das integrais $\int_{\mathbb{R}_+^2} e^u$ e $\int_{\partial\mathbb{R}_+^2} e^{u/2}$. Na maioria dos casos em que a equação tem solução, verificamos que esta era a radialmente simétrica. A partir dessa simetria, transformamos nas equações diferenciais parciais em equações diferenciais ordinárias e podemos classificar suas soluções.

2 Resultados

2.1 Método dos Planos Móveis

Descrevemos o Métodos dos Planos Móveis e o aplicamos para obter o seguinte resultado:

Teorema 2.1. *Seja $u \in C^2(\mathbb{R}^2)$ solução de*

$$\begin{cases} \Delta u + e^u = 0, & x \in \mathbb{R}^2 \\ \int_{\mathbb{R}^2} e^{u(x)} < +\infty. \end{cases} \quad (2.1)$$

então u é radialmente simétrica com respeito a algum ponto x^0 de \mathbb{R}^2 e, portanto, assume a forma

$$\phi_{\lambda,x^0}(x) = \ln \left(\frac{32\lambda^2}{(4 + \lambda^2|x - x^0|^2)^2} \right), \lambda > 0, x^0 \in \mathbb{R}^2. \quad (2.2)$$

2.2 Método das Esferas Móveis

Descrevemos o Método das Esferas Móveis e o aplicamos para obter o seguinte resultado:

Teorema 2.2. *Seja $u \in C^2(\mathbb{R}^2)$ satisfazendo*

$$\Delta u(x) + R(|x|)e^u = 0 \text{ para } x \in \mathbb{R}^2. \quad (2.3)$$

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com

$$\lim_{|x| \rightarrow \infty} \frac{u(x)}{4 \log |x|} < \infty$$

e R uma função contínua e radialmente simétrica, se R é monótona na região em que $R > 0$ e R não constante. Então, (2.3) não tem solução.

2.3 Classificação da Métricas Conformais sobre \mathbb{R}_+^2 com curvatura gaussiana constante e curvatura geodésica sobre a fronteira com diversas condições de finitude da integral

Teorema 2.3. Seja $u \in C^2(\mathbb{R}_+^2) \cap C^1(\overline{\mathbb{R}_+^2})$ uma solução de

$$\begin{cases} \Delta u + \alpha e^u = 0 & \text{em } \mathbb{R}_+^2 \\ \frac{\partial u}{\partial t} = ce^{u/2} & \text{sobre } \partial \mathbb{R}_+^2 \end{cases} \quad (2.4)$$

- Para $\alpha = 1$ e $\int_{\mathbb{R}_+^2} e^u < \infty$. Então u é da forma

$$u(x, t) = \log \left(\frac{8\lambda^2}{(\lambda^2 + (x - x_0)^2 + (t - t_0)^2)^2} \right) \quad (2.5)$$

para algum $\lambda > 0$, $x_0 \in \mathbb{R}$ e $t_0 = c\lambda/\sqrt{2}$.

- Para $\alpha = -1$ e $\int_{\mathbb{R}_+^2} e^u < \infty$ ou $\int_{\partial \mathbb{R}_+^2} e^{u/2} < \infty$, então $c < -\sqrt{2}$ e u é da forma

$$u(s, t) = \log \frac{8\lambda^2}{((s - s_0)^2 + (t - t_0)^2 - \lambda^2)^2} \quad (2.6)$$

onde $s_0 \in \mathbb{R}$, $t_0 = -c\lambda/\sqrt{2}$, $\lambda > 0$.

- E para $\alpha = 0$ com $\int_{\mathbb{R}_+^2} e^u < \infty$. Então $c < 0$ e

$$u(s, t) = 2 \log \frac{2t_1}{(s - s_1)^2 + (t - t_1)^2} + 2 \log \frac{4}{|c|} \quad (2.7)$$

onde s_1 é um número real e t_1 é algum número positivo.

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THE KIRCHHOFF EQUATION WITH VARIABLE EXPONENT OF NONLINEARITY AND BOUNDARY DAMPING

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Abstract

In this work we are concerned with the existence of strong solutions and exponential decay of the energy for the initial boundary value problem associated with the quasilinear wave equation with variable exponent of nonlinearity and boundary damping. The results are proved by means of the multiplier technique and careful estimations within the framework of variable exponent spaces.

1 Introduction

Consider the system

$$\begin{cases} u_{tt} - \left[a + b \int_0^1 u_x^2 dx \right] u_{xx} + \mu |u|^{r(x)-2} u = 0 & , \quad \text{in }]0, 1[\times]0, +\infty[\\ u(0, t) = 0 & , \quad \forall t > 0 \\ \left[a + b \int_0^1 u_x^2 dx \right] u_x(1, t) = -|u_t(1, t)|^{\rho-2} u_t(1, t) & , \quad \forall t > 0 \\ u(x, 0) = u^0(x), \quad u_t(x, 0) = u^1(x) & , \quad \forall x \in]0, 1[\end{cases} \quad (1.1)$$

where a, b, μ, ρ are positive constants and the exponent $r(x)$ is continuous in Ω with logarithmic module of continuity:

$$1 < r^- = \inf_{x \in \Omega} r(x) \leq r(x) \leq r^+ = \sup_{x \in \Omega} r(x) < \infty$$

$$\forall z, \xi \in \Omega, |z - \xi| < 1, |r(z) - r(\xi)| \leq \omega(|z - \xi|)$$

where

$$\limsup_{\tau \rightarrow 0^+} \omega(\tau) \ln \frac{1}{\tau} = C < \infty.$$

For $r = \text{constant} > 2$, system (1.1) and related problems have been studied by many authors (See for instance [1, 2, 3, 4]). It seems that nonconstant power is new in the literature. In this article, our main objective is to study the exponential decay of the solutions of (1.1).

2 Mathematical Results

Consider the Hilbert space

$$V = \{v \in H^1(0, 1) : v(0) = 0\}$$

To get the global existence and regularity for the system (1.1) it is natural to deal first with the local existence and uniqueness which can be proved by the contraction mapping theorem. By similar arguments used in [2] we have the

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Theorem 2.1. Let the initial data $u^0 \in H^2(0, 1) \cap V$, $u^1 \in V$ be sufficiently small and satisfy the compatibility condition

$$\left(a + b \int_0^1 |u_x^0|^2 dx \right) u_x^0(1) + |u^1(1)|^{\rho-2} u^1(1) = 0.$$

Then the system (1.1) has a unique solution in the class

$$u \in C([0, \infty[; H^2(0, 1) \cap V) \cap C^1([0, \infty[; V)). \blacksquare$$

The energy related to problem (1.1) is given by

$$E(t) = \frac{1}{2} \int_0^1 |u_t|^2 dx + \frac{a}{2} \int_0^1 |u_x|^2 dx + \frac{b}{4} \left[\int_0^1 |u_x|^2 dx \right]^2 + \mu \int_0^1 \frac{|u|^{r(x)}}{r(x)} dx$$

The main result is the following theorem

Theorem 2.2. (Exponential stability) Let all the conditions of theorem (2.1) be satisfied. Then there exist positive constants M and γ such that

$$E(t) \leq M e^{-\gamma t} \quad \text{for all } t \geq 0$$

Proof We get our result applying the multiplier method, suitable estimations of the energy $E(t)$ and a Gronwall-type lemma due to Komornik [5]. \blacksquare

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INDEFINITE SEMILINEAR ELLIPTIC PROBLEMS

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1 Introduction

In this article we apply variational and sub-supersolution methods to study the existence and multiplicity of nonnegative solutions for a class of semilinear elliptic problem involving a weight function. More precisely we consider the Dirichlet Problem

$$\begin{cases} -\Delta u = \lambda_1 u + \mu g(x, u) + W(x)f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where Ω is a bounded smooth domain in \mathbb{R}^N , $N \geq 3$, μ is a nonnegative parameter, λ_1 is the first eigenvalue of the operator $-\Delta$ under Dirichlet boundary conditions, $W \in C(\overline{\Omega}, \mathbb{R})$ is a weight function, $g : \overline{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ is a Carathéodory locally bounded function, and $f \in C(\mathbb{R}, \mathbb{R})$ satisfies a subcritical growth condition.

Our main motivation is to study the existence, multiplicity and nonexistence of nonnegative and nonzero weak solutions for (1.1) in the space $H_0^1(\Omega) \cap L^\infty(\Omega)$ when the Problem (1.1) is indefinite, i.e. when the weight function W changes sign. We emphasize that problems involving indefinite nonlinearities have been the object of intense research in the last two decades (see [5, 6, 7, 8, 9, 11, 12] and references therein).

The existence of a solution for (1.1) is established without imposing any restriction on the growth of g at infinity. For this reason the associated functional may not be well defined in the Sobolev space $H_0^1(\Omega)$. In order to overcome the difficulty caused by this fact, we establish a related result for an auxiliary problem that enables us to find a positive supersolution for Problem (1.1) when the parameter $\mu > 0$ is sufficiently small. Then, using a truncation argument and a minimization method, we derive the existence of a nonnegative and nonzero solution for (1.1). Assuming that f is locally Lipschitz at the origin and that g is a nonnegative function, we apply a sub-supersolution method combined with variational methods to verify that the set of values of μ for which (1.1) has a positive solution is a subinterval of the nonnegative real line.

The particular case of the indefinite Problem (1.1) with $g(\cdot, s) \equiv s$ was considered in the articles [2, 3, 4] when the nonlinearity f satisfies the following superlinear growth condition at infinity:

$$\lim_{s \rightarrow +\infty} \frac{f(s)}{s^{p-1}} = 1, \quad 2 < p < 2^* = \frac{2N}{N-2}, \quad N \geq 3. \quad (1.2)$$

As observed in earlier works (see e.g. [2, 3]), a delicate issue faced when applying variational methods to study indefinite problems is to establish a necessary compactness condition for the associated functional. The condition (1.2), introduced in [3], has played a central role in verifying the Palais-Smale condition for this class of problems. One of the main features in this article is to provide sufficient conditions for the existence and multiplicity of nonnegative solutions for (P_μ) under hypotheses more general than (1.2). In particular, for proving the existence of two solutions for the Problem (P_μ) , we verify that the Palais-Smale condition holds when the nonlinearity f satisfies the famous Ambrosetti-Rabinowitz superlinear condition and the weight function W has a thick zero set. More specifically, for the existence of two solutions for (1.1), we suppose that $g(x, s)$ satisfies a linear growth condition,

(W_1) $\Omega^\pm \neq \emptyset$ and $\overline{\Omega^+} \cap \overline{\Omega^-} = \emptyset$,

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and

(AR) there are $\theta > 2$ and $R > 0$ such that

$$sf(s) \geq \theta F(s) > 0, \text{ for every } |s| \geq R;$$

where $\Omega^\pm := \{x \in \Omega : \pm W(x) > 0\}$ and $F(s) = \int_0^s f(t) dt$.

We address the interested reader to [10] for related results and the proofs of the results cited above.

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PROBLEMA ELÍPTICO SUPERLINEAR RESSONANTE

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Os resultados aqui apresentados assim como as demonstrações encontram-se em [2], Cuesta-Figueiredo-Srikanth. Vamos discutir a resolvibilidade do problema superlinear (1.1). Utilizamos desigualdades do tipo Hardy para obter limitações a priori para soluções desses problema e argumentos de grau topológico para garantir a existência de soluções.

1 Introdução

Vamos discutir a resolvibilidade do problema superlinear

$$-\Delta u = \lambda_1 u + u_+^p + f(x) \quad \text{em } \Omega, \quad (1.1)$$

com $u = 0$ em $\partial\Omega$. Em que Ω é um domínio limitado suave em \mathbb{R}^N , com $N \geq 3$, f é um função não nula, tal que

$$f \in L^r \quad \text{para } r > N \quad (1.2)$$

e $1 < p < \frac{N+1}{N-1}$.

Consideramos $\lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_n \leq \dots$ os autovalores de $(-\Delta, H_0^1(\Omega))$ e $\phi_1, \phi_2, \dots, \phi_n, \dots$ as correspondentes autofunções.

2 Resultados

Teorema 2.1. *Seja $1 < p < \frac{N+1}{N-1}$, assumindo que f satisfaz a condição (1.2) e que*

$$\int f\phi_1 < 0 \quad (2.3)$$

então, dada $u \in H_0^1(\Omega)$ solução de (1.1), existe uma função contínua crescente $\rho : \mathbb{R}^+ \rightarrow \mathbb{R}^+$, dependendo somente de p e Ω , tal que $\rho(0) = 0$ e

$$\|u\|_{C_0^1(\Omega)} \leq \rho(\|f\|_r). \quad (2.4)$$

Em que $\|f\|_r$ denota a norma L^r de f .

Teorema 2.2. *Assumindo as mesmas hipóteses do Teorema (2.1), temos que o problema (1.1) tem pelo menos uma solução em $W^{2,r}(\Omega) \cap H_0^1(\Omega)$.*

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VERSÕES FRACAS DE ESPAÇOS DE BANACH DE FUNÇÕES COM VALORES VETORIAIS

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1 Introdução

Neste trabalho, consideramos μ uma medida finita, $\mathbb{D} = \{z \in \mathbb{C}; |z| < 1\}$, $\mathbb{T} = \{z \in \mathbb{C}; |z| = 1\}$, F um espaço de Banach (real ou complexo), B_F a respectiva bola unitária fechada e F' seu dual (topológico). O objetivo deste trabalho é estudar as versões fracas dos espaços das funções com valores vetoriais que são integráveis segundo Lebesgue-Bochner, denotados por $L^p(\mu, F)$, e as versões fracas dos espaços de Hardy com valores vetoriais, denotados por $H^p(\mathbb{D}, F)$. Tal investigação é motivada pela que Mujica [2] fez da versão fraca do espaço das funções holomorfas de \mathbb{D} em F , denotado por $\mathcal{H}(\mathbb{D}; F)$. Estudamos as versões fracas dos espaços de Lebesgue e Hardy como casos particulares do seguinte quadro:

Seja A um conjunto e assuma que para todo espaço de Banach F nós definimos um espaço normado $\mathcal{F}(A; F)$ de (classes) de funções de A em F . Para simplificar, quando $F = \mathbb{C}$ escrevemos $\mathcal{F}(A) = \mathcal{F}(A; \mathbb{C})$. A versão fraca de $\mathcal{F}(A; F)$ é o conjunto da (classe de) funções

$$\mathcal{F}(A; F)_w := \left\{ f: A \longrightarrow F : \varphi \circ f \in \mathcal{F}(A) \text{ para todo } \varphi \in F' \text{ e } \sup_{\varphi \in B_{F'}} \|\varphi \circ f\|_{\mathcal{F}(A)} < +\infty \right\},$$

em que duas funções f e g são identificadas se $f = g$ $\mathcal{F}(A; F)$ -fracamente, ou seja, $\varphi \circ f = \varphi \circ g$ como elementos de $\mathcal{F}(A)$ para todo $\varphi \in F'$. Desta definição, é imediato que a correspondência

$$f \in \mathcal{F}(A; F)_w \mapsto \|f\|_{\mathcal{F}(A; F)}^w := \sup_{\varphi \in B_{F'}} \|\varphi \circ f\|_{\mathcal{F}},$$

define uma norma $\mathcal{F}(A; F)_w$. Para simplificar escrevemos $\|f\|_{\mathcal{F}}^w$ ao invés de $\|f\|_{\mathcal{F}(A; F)}^w$.

Por exemplo, se denotarmos por $\mathcal{C}(F; F)$ o espaço de todas as funções contínuas $f: F \longrightarrow F$, então $\mathcal{C}(F; F)_w = \mathcal{C}(F; F)$ se, e somente se, F tem a propriedade de Schur.

Mujica [2] demonstra que para todo espaço de Banach F , $\mathcal{H}(\mathbb{D}; F) = \mathcal{H}(\mathbb{D}; F)_w$.

Para os espaços $L^p(T; F)$ e $H^p(\Delta; F)$, veremos que as igualdades $L^p(T; F) = L^p(T; F)_w$ e $H^p(\mathbb{D}; F) = H^p(\mathbb{D}; F)_w$ ocorrem em casos bem restritos.

2 Resultados

São indicados, na sequência, os dois principais resultados do trabalho, bem como comentários sobre as demonstrações dos mesmos.

Teorema 2.1. Suponha que exista uma sequência $(A_n)_{n=1}^\infty$ de conjuntos mensuráveis de medida positiva dois a dois disjuntos tais que $\Omega = \bigcup_{n=1}^\infty A_n$. As seguintes condições são equivalentes:

- (a) $L^p(\mu; F) = L^p(\mu; F)_w$ para todo $1 \leq p < \infty$.
- (b) $L^p(\mu; F) = L^p(\mu; F)_w$ para algum $1 \leq p < \infty$.
- (c) F tem dimensão finita.

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Notamos que a condição imposta sobre μ no Teorema 2.1 é satisfeita por grande parte das medidas finitas usuais, como por exemplo, a medida de Lebesgue em qualquer intervalo limitado de \mathbb{R} . Para a demonstração deste teorema, utilizamos do resultado que diz que se toda sequência fracamente p -somável em F é absolutamente p -somável ([1, Theorem 2.18]), então F tem dimensão finita.

Quanto aos espaços de Hardy, para enunciar o resultado principal, precisamos de alguns conceitos.

Definição 2.1. A projeção analítica f^a de uma função $f \in L^p(\mathbb{T}; F)$ é uma função cujos coeficientes de Fourier negativos são nulos e, os outros coincidem com os respectivos coeficientes de Fourier de f . A correspondência $f \mapsto f^a$ é denominada *projeção analítica*.

Definição 2.2. Dada $f \in H^p(\mathbb{D}; F)$, definimos $\tilde{f} \in L^p(\mathbb{T}; F)$ como a função com valores de fronteira de f se

$$\tilde{f}(e^{i\theta}) = \lim_{r \rightarrow 1} f(re^{i\theta}),$$

sendo que tais limites existem θ -quase-sempre.

Quanto aos espaços UMD (*unconditional martingale difference*), espaços cujas diferenças de martingale são incondicionais, apenas damos uma caracterização dos mesmos:

Definição 2.3. Um espaço de Banach F tem a *propriedade UMD* (ou F é UMD ou, ainda, F é um UMDP-espaço) se a projeção analítica $f \in L^p(\mathbb{T}; F) \mapsto f^a \in H^p(\mathbb{T}; F)$, é um transformação linear limitada para todo $1 < p < \infty$, em que f^a é a projeção analítica de f .

Teorema 2.2. Sejam $1 < p, q < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$, e F um espaço de Banach. Se $H^p(\mathbb{D}; F)_w = H^p(\mathbb{D}; F)$, então F é UMD. Em particular, $H^p(\mathbb{D}; F)'$ e $H^q(\mathbb{D}; F)'$ são canonicamente topologicamente isomorfos, ou seja, vale o seguinte isomorfismo:

$$\begin{aligned} \Psi_p: \quad H^q(\mathbb{D}; F)' &\longrightarrow H^p(\mathbb{D}; F)' \\ g &\longmapsto \quad \Psi_p(g): \quad H^p(\mathbb{D}; F) &\longrightarrow \mathbb{C} \\ && f &\longmapsto \quad \Psi_p(g)(f) = \frac{1}{2\pi} \int_0^{2\pi} \langle \tilde{f}(e^{i\theta}), \tilde{g}(e^{-i\theta}) \rangle d\theta, \end{aligned}$$

em que \tilde{f} e \tilde{g} são as funções valores de fronteira associadas a $f \in H^p(\mathbb{D}; F)$ e a $g \in H^q(\mathbb{D}; F)'$, respectivamente.

A implicação contrária do Teorema 2.2 não é válida. De fato, é válido, para $1 < p, r < \infty$, que $L^r(\nu)$ e $H^r(\mathbb{D})$ são UMD e temos o

Exemplo 2.1. Para $1 < p, r < \infty$ e ν uma medida σ -finita que não é puramente-atômica (cf. Rosenthal [3, p. 225]) são verdadeiras as afirmações:

- a) $H^p(\mathbb{D}; L^r(\nu))_w \neq H^p(\mathbb{D}; L^r(\nu))$.
- b) $H^p(\mathbb{D}; H^r(\mathbb{D}))_w \neq H^p(\mathbb{D}; H^r(\mathbb{D}))$.

Os únicos exemplos de espaços de Banach F que obtemos satisfazendo $H^p(\mathbb{D}; F)_w = H^p(\mathbb{D}; F)$, para $1 < r < \infty$, são os espaços de Banach de dimensão finita.

A junção de todos os fatos anteriores nos levou a conjecturar o seguinte:

“Se $1 < p < \infty$, então $H^p(\mathbb{D}; F) = H^p(\mathbb{D}; F)_w$ se, e somente se, F tem dimensão finita.”

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CONTROLABILIDADE EXATA PARA EQUAÇÃO DO CALOR SEMILINEAR POR ESTRATÉGIAS DO TIPO STACKELBERG-NASH

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Seja Ω um conjunto aberto de \mathbb{R}^n e $T > 0$. Definimos $Q = \Omega \times (0, T)$ e para $i = 1, 2$ sejam $\mathcal{O}, \mathcal{O}_i$ subconjuntos abertos de Ω . Nestas condições, consideremos a equação

$$\begin{cases} y_t - \Delta y + a(x, t)y = F(y) + f1_{\mathcal{O}} + v^1 1_{\mathcal{O}_1} + v^2 1_{\mathcal{O}_2} & \text{in } Q, \\ y = 0 & \text{on } \Sigma, \\ y(x, 0) = y^0(x) & \text{in } \Omega, \end{cases} \quad (0.1)$$

onde $F : \mathbb{R} \rightarrow \mathbb{R}$ é localmente lipschitziana. Para $i = 1, 2$, sejam $\mathcal{O}_{i,d}$ subconjuntos abertos de Ω e consideremos os seguintes funcionais (secundários):

$$J_i(f; v^1, v^2) := \frac{\alpha_i}{2} \iint_{\mathcal{O}_{i,d} \times (0, T)} |y - y_{i,d}|^2 dx dt + \frac{\mu_i}{2} \iint_{\mathcal{O}_i \times (0, T)} |v^i|^2 dx dt. \quad (0.2)$$

A estrutura do processo de controle pode ser dividida em dois passos principais

Passo 1: Fixado $f \in L^2(\mathcal{O} \times (0, T))$, procuramos por controles $v^i \in L^2(\mathcal{O}_i \times (0, T))$ que satisfazem

$$J_1(f; v^1, v^2) = \min_{\hat{v}^1} J_1(f; \hat{v}^1, v^2), \quad J_2(f; v^1, v^2) = \min_{\hat{v}^1} J_2(f; v^1, \hat{v}^2). \quad (0.3)$$

O par satisfazendo (0.3) será chamado *equilíbrio de Nash* para J_1 e J_2 . Observemos que, se os funcionais J_i ($i = 1, 2$) são convexos, então (v^1, v^2) é um equilíbrio de Nash se, e somente se,

$$D_i J_i(f; v^1, v^2) = 0, \quad \forall \hat{v}^i \in L^2(\mathcal{O}_i \times (0, T)); \quad i = 1, 2. \quad (0.4)$$

Denominamos um par (v^1, v^2) satisfazendo (0.4) por *quase equilíbrio de Nash*.

Passo 2: Fixemos uma trajetória suficientemente regular, isto é, solução do problema:

$$\begin{cases} \bar{y}_t - \Delta \bar{y} + a(x, t)\bar{y} = F(\bar{y}) & \text{in } Q, \\ \bar{y} = 0 & \text{on } \Sigma, \\ \bar{y}(x, 0) = \bar{y}^0(x) & \text{in } \Omega. \end{cases} \quad (0.5)$$

Uma vez que o equilíbrio de Nash foi determinado para cada f , procuramos um controle óptimo $\hat{f} \in L^2(\mathcal{O} \times (0, T))$ tal que

$$\hat{f} = \min_f \int_{\mathcal{O} \times (0, T)} |f|^2 dx dt, \quad (0.6)$$

sujeito à restrição

$$y(x, T) = \bar{y}(x, T) \quad \text{em } \Omega. \quad (0.7)$$

O principal resultado do trabalho segue

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Teorema 0.1. Suponhamos que $F \in W^{1,\infty}(\mathbb{R})$ e que $\mu_i > 0$ são suficientemente grandes. Seja \bar{y} a única solução de (0.5) com dado inicial $\bar{y}^0 \in L^2(\Omega)$. Assumimos que as funções $y_{i,d}$ satisfazem a seguinte propriedade de compatibilidade: existe uma função positiva $\hat{\rho} = \hat{\rho}(x,t)$ que explode em $t = T$ tal que

$$\iint_{\mathcal{O}_d \times (0,T)} \hat{\rho}^2 |\bar{y} - y_{i,d}|^2 dx dt < +\infty, \quad i = 1, 2. \quad (0.8)$$

Para cada $y_0 \in L^2(\Omega)$, existem controles $f \in L^2(\mathcal{O} \times (0,T))$ e um quase equilíbrio de Nash (v^1, v^2) tal que a solução de (0.1) satisfaz (0.7).

Demonstração. O par de Nash é caracterizado pelo seguinte sistema de optimalidade

$$\begin{cases} y_t - \Delta y + a(x,t)y = F(y) + f1_{\mathcal{O}} - \frac{1}{\mu_1}\phi^1 1_{\mathcal{O}_1} - \frac{1}{\mu_2}\phi^2 1_{\mathcal{O}_2} & \text{in } Q, \\ -\phi_t^i - \Delta \phi^i + a(x,t)\phi^i = F'(y)\phi^i + \alpha_i(y - y_{i,d})1_{\mathcal{O}_{i,d}} & \text{in } Q, \\ y = 0, \quad \phi^i = 0 & \text{on } \Sigma, \\ y(x,0) = y^0(x), \quad \phi^i(x,T) = 0 & \text{in } \Omega. \end{cases} \quad (0.9)$$

Linearizamos o sistema (0.9) e em seguida consideramos o seguinte sistema adjunto

$$\begin{cases} -\psi_{z,t} - \Delta \psi_z + a(x,t)\psi_z = G(x,t;z)\psi_z + (\alpha_1\gamma_z^1 + \alpha_2\gamma_z^2)1_{\mathcal{O}_d} & \text{in } Q, \\ \gamma_{z,t}^i - \Delta \gamma_z^i = F'(z + \bar{y})\gamma_z^i - \frac{1}{\mu_i}\psi_z 1_{\mathcal{O}_i} & \text{in } Q, \\ \psi_z = 0, \quad \gamma_z^i = 0 & \text{on } \Sigma, \\ \psi_z(x,T) = \psi^T, \quad \gamma_z^i(x,0) = 0 & \text{in } \Omega. \end{cases}$$

Em seguida obtemos a seguinte desigualdade de observabilidade

$$\int_{\Omega} |\psi_z(x,0)|^2 dx + \sum_{i=1}^2 \iint_Q \hat{\rho}^{-2} |\gamma_z^i|^2 dx dt \leq C \iint_{\mathcal{O} \times (0,T)} |\psi_z|^2 dx dt. \quad (0.10)$$

Para finalizar, aplicamos o método de unicidade de Hilbert combinado com o teorema do ponto fixo de Schauder. \square

Resultados adicionais

- Se $F \in W^{2,\infty}$, $n \leq 14$ (resp. $n \leq 12$) e $y_0 \in H_0^1(\Omega)$ (resp. $y_0 \in L^2(\Omega)$), então as condições de equilíbrio de Nash e quase equilíbrio de Nash são equivalentes,
- Resultados análogos aos do Teorema 0.1 são obtidos com (v^1, v^2) sujeitos a restrições locais.

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OPTIMAL CONTROL FOR A SECOND GRADE FLUID SYSTEM

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1 Introduction

In this work we consider the following second grade fluid system

$$\left\{ \begin{array}{ll} \frac{\partial}{\partial t}(\mathbf{u} - \alpha\Delta\mathbf{u}) - \nu\Delta\mathbf{u} + \operatorname{curl}(\mathbf{u} - \alpha\Delta\mathbf{u}) \times \mathbf{u} + \nabla q &= \mathbf{f} + \mathbf{v} \quad \text{in } \Omega \times]0, T[, \\ \operatorname{div} \mathbf{u} &= 0 \quad \text{in } \Omega \times]0, T[\\ \mathbf{u} &= 0 \quad \text{on } \partial\Omega \times]0, T[, \\ \mathbf{u}(0) &= \mathbf{u}_0, \quad \text{in } \Omega \end{array} \right. \quad (1.1)$$

The study of this kind of fluids was initiated by Dunn and Fosdick in [?] and by Fosdick and Rajapogal in [?]. The first successful mathematical analysis of (??) was done by Cioranescu and Ouazar in [?]. More recently Cioranescu and Girault in [?] established existence, uniqueness and regularity of a global weak solution of (??) with small data \mathbf{f} and $\mathbf{u}(0)$ and the same result on some interval for arbitrary data. The existence is obtained by applying Galerkin's method with a special basis.

In this work, let us introduce the non-empty subset $\omega \subseteq \Omega$ and a velocity \mathbf{u}_1 defined on ω is given. The problem is to find a external force \mathbf{v} so that the associated velocity \mathbf{u} minimize the functional

$$J(\mathbf{u}, \mathbf{v}) = \int_0^T \int_{\omega} |\mathbf{u} - \mathbf{u}_1|^2 dx + \int_0^T \int_{\Omega} |\mathbf{v}|^2 dx dt. \quad (1.2)$$

2 Mathematical Results

Let Ω be a simply-connected bounded domain of \mathbb{R}^3 with boundary $\partial\Omega$ which is al least of class $C^{3,1}$. In what follows, the spaces in bold face represent spaces of tri-dimensional vector functions. We define the Hilbert spaces \mathbf{H} and \mathbf{V} in the following manner:

$$\begin{aligned} \mathbf{H} &= \{\Psi \in \mathbf{L}^2(\Omega) : \operatorname{div} \Psi = 0, \Psi \cdot \mathbf{n} = 0 \text{ on } \partial\Omega\} \\ \mathbf{V} &= \{\mathbf{v} \in \mathbf{H}^1(\Omega) : \operatorname{div} \mathbf{v} = 0, \mathbf{v} = \mathbf{0}, \text{ on } \partial\Omega\} \\ H(\operatorname{curl}; \Omega) &= \{\mathbf{v} \in \mathbf{L}^2(\Omega) : \operatorname{curl} \mathbf{v} \in \mathbf{L}^2(\Omega)\} \end{aligned}$$

For $\alpha \in \mathbb{R}^+$, we introduce the space $\mathbf{V}_2 = \{\mathbf{v} \in \mathbf{V} : \operatorname{curl}(\mathbf{v} - \alpha\Delta\mathbf{v}) \in \mathbf{L}^2(\Omega)\}$ equipped with the scalar product $(\mathbf{u}, \mathbf{v})_{V_2} = (\mathbf{u}, \mathbf{v}) + \alpha(\nabla\mathbf{u}, \nabla\mathbf{v}) + (\operatorname{curl}(\mathbf{u} - \alpha\Delta\mathbf{u}), \operatorname{curl}(\mathbf{v} - \alpha\Delta\mathbf{v}))$ and associated norm and semi-norm $\|\mathbf{v}\|_{V_2} = (\mathbf{v}, \mathbf{v})_{V_2}^{1/2}$, $|\mathbf{v}|_{V_2} = \|\operatorname{curl}(\mathbf{v} - \alpha\Delta\mathbf{v})\|_{\mathbf{L}^2(\Omega)}$.

Define

$$\begin{aligned} W_1 &= \{\mathbf{w} \in L^\infty(0, T; \mathbf{V}_2) : \mathbf{w}' \in L^\infty(0, T; \mathbf{V})\}, \\ W_2 &= L^2(0, T, H(\operatorname{curl}; \Omega)) \cap L^\infty(0, T, \mathbf{L}^2(\Omega)), \\ W &= W_1 \times W_2. \end{aligned}$$

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Now, the set of the admissible controls is given by:

$$\mathcal{U} = \left\{ \mathbf{v} \in W_2 : \left(\int_0^\infty (\|\mathbf{v}(t)\|_{\mathbf{L}^2(\Omega)}^2 + \|\operatorname{curl}(\mathbf{v}(t))\|_{\mathbf{L}^2(\Omega)}^2) dt \right)^{1/2} < \delta \right\}.$$

The above condition, given in definition of \mathcal{U} , is necessary in order to have solution for the system (??) (see [?]). Note that W_1 is a Hilbert space for the norm

$$\|\mathbf{w}\|_{W_1} = \left(\|\mathbf{w}(t)\|_{L^\infty(0,T;\mathbf{V}_2(\Omega))}^2 + \|\mathbf{w}'\|_{L^\infty(0,T;\mathbf{V}(\Omega))}^2 \right)^{1/2}$$

In the same manner, W_2 is a Hilbert space for the norm

$$\|\mathbf{g}\|_{W_2} = \left(\|\mathbf{g}\|_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))}^2 + \|\operatorname{curl}\mathbf{g}\|_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))}^2 \right)^{1/2}$$

Define $M : W \rightarrow \widetilde{W}$, by $M(\mathbf{w}, \mathbf{v}) = (\psi_1, \psi_2)$ and $\widetilde{W} = \mathbf{L}^\infty(0, T; \mathbf{H}) \times \mathbf{V}_2$.

$$\begin{cases} \frac{\partial}{\partial t}(\mathbf{w} - \alpha A \mathbf{w}) - \nu A \mathbf{w} + P(\operatorname{curl}(\mathbf{w} - \alpha \Delta \mathbf{w}) \times \mathbf{w}) - \mathbf{f} - \mathbf{v} &= \psi_1 \\ \mathbf{w}(0) - \mathbf{u}_0 &= \psi_2, \end{cases} \quad (2.3)$$

where $P : \mathbf{L}^2(\Omega) \rightarrow \mathbf{V}(\Omega)$ is the orthogonal projection and A is the Stokes operator.

The optimal control problem is the following: Find \mathbf{u} and \mathbf{v} such that

$$J(\mathbf{u}, \mathbf{v}) = \inf_{(\mathbf{w}, \tilde{\mathbf{v}}) \in \mathcal{G}} J(\mathbf{w}, \tilde{\mathbf{v}}) \quad (2.4)$$

where \mathcal{G} is the non-empty set $\mathcal{G} = \{(\mathbf{w}, \tilde{\mathbf{v}}) \in W : \tilde{\mathbf{v}} \in \mathcal{U}, M(\mathbf{w}, \tilde{\mathbf{v}}) = 0\}$.

Theorem 2.1. *Problem (??) has at least one solution. Furthermore, the following minimum principle is satisfied:*

$$\int_0^T \int_\Omega (-\xi + \mathbf{v})(\tilde{\mathbf{v}} + \mathbf{v}) dx dt < 0,$$

for all $\tilde{\mathbf{v}} \in \mathcal{U}$, $\mathbf{v} \in \mathcal{U}$. Here ξ is solution of the adjoint problem.

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ASYMMETRIC INCOMPRESSIBLE FLUIDS WITH VARIABLE DENSITY: SEMI-STRONG SOLUTIONS IN UNBOUNDED DOMAINS

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We prove, via the spectral semi-Galerkin method, the existence and uniqueness of semi-strong solutions for the equations of nonhomogeneous asymmetric (micropolar) fluids in $\mathbf{L}^2(\Omega) = (L^2(\Omega))^3$, in the case of unbounded three-dimensional domains with boundary uniformly of class C^3 .

1 Introduction

Let $\Omega \subseteq \mathbb{R}^3$ be a domain (not necessarily bounded), with boundary uniformly of class C^3 , and let T be a positive real number. We study, in an open set $\Omega \times (0, T)$, where $(0, T)$ is a time interval, the equations for the motion of a non-homogeneous viscous incompressible asymmetric fluid. The governing equations are the following:

$$\left\{ \begin{array}{lcl} \rho \mathbf{u}_t + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} - (\mu + \mu_r) \Delta \mathbf{u} + \nabla p & = & 2\mu_r \operatorname{curl} \mathbf{w} + \rho \mathbf{f}, \\ \operatorname{div} \mathbf{u} & = & 0, \\ \rho \mathbf{w}_t + \rho (\mathbf{u} \cdot \nabla) \mathbf{w} - (c_0 + c_d - c_a) \nabla (\operatorname{div} \mathbf{w}) + 4\mu_r \mathbf{w} & = & (c_a + c_d) \Delta \mathbf{w} + 2\mu_r \operatorname{curl} \mathbf{u} + \rho \mathbf{g}, \\ \rho_t + \mathbf{u} \cdot \nabla \rho & = & 0. \end{array} \right. \quad (1.1)$$

The symbols $\nabla, \Delta, \operatorname{div}$ and curl denote the *gradient*, *Laplacian*, *divergence* and *rotational* operators, respectively, and $\mathbf{u}_t, \mathbf{w}_t$ and ρ_t stand for the time derivatives of \mathbf{u}, \mathbf{w} and ρ .

For the derivation of equations (1.1) and a discussion on their physical meaning, see [1] and [2]. Physically, the first equation in system (1.1) corresponds to the conservation of linear momentum; the second one is the incompressibility of the fluid; the third corresponds to the conservation of angular momentum, and the fourth one corresponds to the conservation of mass. In system (1.1), the unknowns are $\mathbf{u}(\mathbf{x}, t) \in \mathbb{R}^3$, $\mathbf{w}(\mathbf{x}, t) \in \mathbb{R}^3$, $\rho(\mathbf{x}, t) \in \mathbb{R}$ and $p(\mathbf{x}, t) \in \mathbb{R}$. They represent, respectively, the velocity field, the angular velocity of rotation of the fluid particles, the mass density and the pressure distribution of the fluid as functions of position \mathbf{x} and time t . Here, \mathbf{f} and \mathbf{g} are known density functions of external sources for the linear and the angular momentum of particles, respectively. The positive constants μ, μ_r, c_0, c_a and c_d characterize the physical properties of the fluid. Thus, μ is the usual Newtonian viscosity; μ_r, c_0, c_a and c_d are additional viscosities related to the lack of symmetry of the stress tensor and, consequently, to the fact that the internal rotation field \mathbf{w} does not vanish. These constants must satisfy the inequality $c_0 + c_d > c_a$. We complement system (1.1) with initial and boundary conditions

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}), \quad \mathbf{w}(\mathbf{x}, 0) = \mathbf{w}_0(\mathbf{x}) \quad \text{and} \quad \rho(\mathbf{x}, 0) = \rho_0(\mathbf{x}) \quad \text{in } \Omega, \quad (1.2)$$

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{0}, \quad \mathbf{w}(\mathbf{x}, t) = \mathbf{0} \quad \text{on } \partial\Omega \times (0, T), \quad (1.3)$$

where the functions $\mathbf{u}_0, \mathbf{w}_0$ and ρ_0 are given. If Ω is unbounded, also impose the following condition on the velocities at infinity:

$$\lim_{|\mathbf{x}| \rightarrow \infty} \mathbf{u}(\mathbf{x}, t) = \mathbf{0}, \quad \lim_{|\mathbf{x}| \rightarrow \infty} \mathbf{w}(\mathbf{x}, t) = \mathbf{0}, \quad t \in (0, T). \quad (1.4)$$

Observe that this system includes as a particular case the classical Navier-Stokes equations ($\mathbf{w} = \mathbf{0}$ and $\mu_r = 0$).

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2 Mathematical Results

We adapt the techniques used in [4] by J. G. Heywood for the classical Navier-Stokes equations and in [6] by P. Braz e Silva, M. A. Rojas-Medar and E. J. Villamizar-Roa for Navier-Stokes equations with variable density, along with arguments used in [3, 5, 7] to show existence and uniqueness of local and global in time solutions. In what follows, $\mathbf{V}(\Omega)$ will represent the closure of $\mathcal{V}(\Omega)$ in $\mathbf{H}_0^1(\Omega)$, where $\mathcal{V}(\Omega) := \{\mathbf{u} \in \mathbf{C}_0^\infty(\Omega) / \operatorname{div} \mathbf{u} = 0 \text{ in } \Omega\}$, i.e., $\mathbf{V}(\Omega) := \overline{\mathcal{V}(\Omega)}^{\mathbf{H}_0^1(\Omega)} = \{\mathbf{v} \in \mathbf{H}_0^1(\Omega); \operatorname{div} \mathbf{v} = 0\}$ (see [8]). Our main result is the following

Theorem 2.1. *Let Ω be a domain of \mathbb{R}^3 , with boundary uniformly of class C^3 . Assume that $\mathbf{u}_0 \in \mathbf{V}(\Omega)$, $\mathbf{w}_0 \in \mathbf{H}_0^1(\Omega)$ and $\rho_0 \in L^\infty(\Omega)$, with $0 < \alpha \leq \rho_0(\mathbf{x}) \leq \beta < \infty$ in Ω . Also, assume that $\mathbf{f}, \mathbf{g} \in L^2(0, T; \mathbf{L}^2(\Omega))$. Then, there exists a semi-strong solution $(\rho, \mathbf{u}, \mathbf{w})$ of (1.1)-(1.3) in $\Omega \times (0, T^*)$, for some $T^* \in (0, T]$. The functions \mathbf{u} , \mathbf{w} , ρ and p , defined on $(0, T^*)$, are such that*

$$\begin{aligned} \mathbf{u} &\in L^\infty(0, T^*; \mathbf{V}(\Omega)), \\ \mathbf{w} &\in L^\infty(0, T^*; \mathbf{H}_0^1(\Omega)), \\ \mathbf{u}_t, \mathbf{w}_t, D^2\mathbf{u}, D^2\mathbf{w}, \nabla p &\in L^2(0, T^*; \mathbf{L}^2(\Omega)), \\ \rho &\in L^\infty(0, T^*; L^\infty(\Omega)), \\ \rho_t &\in L^\infty(0, T^*; L_{loc}^\infty(\Omega)), \\ \nabla \rho &\in L^\infty(0, T^*; \mathbf{L}_{loc}^\infty(\Omega)), \\ \|\nabla \mathbf{u}(t) - \nabla \mathbf{u}_0\| &\rightarrow 0 \text{ and } \|\nabla \mathbf{w}(t) - \nabla \mathbf{w}_0\| \rightarrow 0 \text{ as } t \rightarrow 0^+. \end{aligned}$$

Remark 2.1. *If Ω is either bounded or $\Omega = \mathbb{R}^3$, the regularities for the density are $\rho_t \in L^\infty(0, T^*; L^\infty(\Omega))$ and $\nabla \rho \in L^\infty(0, T^*; \mathbf{L}^\infty(\Omega))$ (see [6], Remarks 2 and 13).*

Remark 2.2. *Considering that $\mathbf{f}, \mathbf{g} \in L^\infty(0, \infty; \mathbf{L}^2(\Omega))$ and that the norms $\|\mathbf{u}_0\|_{\mathbf{H}^1}$, $\|\mathbf{w}_0\|_{\mathbf{H}^1}$, $\|\mathbf{f}\|_{L^\infty(0, \infty; \mathbf{L}^2(\Omega))}$ and $\|\mathbf{g}\|_{L^\infty(0, \infty; \mathbf{L}^2(\Omega))}$ are sufficiently small, one can show that there exists a semi-strong solution $(\rho, \mathbf{u}, \mathbf{w})$, global in time, to the problem (1.1)-(1.3).*

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EXISTÊNCIA DE SOLUÇÕES PARA UM PROBLEMA COM PERTURBAÇÃO ENVOLVENDO O OPERADOR p -LAPLACIANO

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1 Introdução

Considere o seguinte problema:

$$(P_\epsilon) \quad \begin{cases} -\Delta_p u = \lambda u^\alpha - (a(x) + \epsilon)u^q & , \text{ em } \Omega \\ u = 0 & , \text{ em } \partial\Omega \end{cases}$$

onde $\Omega \subset \mathbb{R}^N$ ($N \geq 2$) é um domínio limitado suave, $\Delta_p = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ denota o operador p-Laplaciano, $p > 1$, $\epsilon > 0$, λ é um parâmetro positivo e α, q são constantes dadas tais que $\alpha < q$. Além disso, $a(x)$ denota uma função contínua e não-negativa que se anula num subdomínio de Ω . Assumiremos que a função $a(x)$ pertence a $C^\beta(\overline{\Omega})$ ($0 < \beta < 1$) e o conjunto

$$\overline{\Omega}_0 = \{x \in \overline{\Omega} : a(x) = 0\}$$

satisfaz $\overline{\Omega}_0 \subset \Omega$ e Ω_0 é um domínio não vazio com fronteira C^2 .

Objetivamos garantir que o problema (P_ϵ) possui ao menos duas soluções positivas.

Em 2003, Du e Guo estudaram o problema sem perturbação no caso em que $\alpha = p + 1$, (ver [4]) e mostraram que este possuía uma única solução se $\lambda \in (\lambda_1(\Omega), \lambda_1(\Omega_0))$ e nenhuma outra solução positiva. Em 2005, Du e Li estudaram em [5] o problema perturbado para os valores $p = 2$ e $\alpha = 1$. Em tal caso mostrou-se que o mesmo possui apenas uma única solução positiva v_ϵ quando $\lambda > \lambda_1(\Omega)$, sendo $\lambda_1(\Omega)$ o primeiro autovalor do operador p-Laplaciano em Ω . Além disso, garantiu-se que quando $\epsilon \rightarrow 0$, v_ϵ se comporta como uma das soluções encontradas para (P_ϵ) enunciado neste trabalho.

O resultado proposto neste trabalho foi obtido por Dong em 2005, através da utilização do grau topológico de Leray-Schauder ([1],[2]) e o Teorema do Passo da Montanha [7].

2 Resultados

Enunciamos o seguinte resultado

Teorema 2.1. *Para qualquer $\lambda > 0$, existe um ϵ_λ tal que para $0 < \epsilon \leq \epsilon_\lambda$, o problema (P_ϵ) possui ao menos duas soluções positivas distintas \underline{u}_ϵ e u_ϵ satisfazendo $\underline{u}_\epsilon \leq u_\epsilon$ e $\underline{u}_\epsilon \neq u_\epsilon$.*

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SOBRE UM PROBLEMA DE ZEROS DE POLINÔMIOS ORTOGONALIS

FERNANDO R. RAFAELI *

1 Resumo do trabalho

Este trabalho foi motivado por um problema aberto proposto por M.E.H. Ismail em sua monografia *Classical and quantum orthogonal polynomials in one variable* (Cambridge University Press, 2005), Problema 24.9.1.

Seja $d\mu$ uma medida de Borel positiva com suporte em um subconjunto (a, b) da reta real tal que

$$\int_a^b |x|^n d\mu(x) < \infty, \quad n = 0, 1, \dots$$

Aplicando o processo de Gram-Schmidt a $1, x, x^2, \dots$ (conjunto linearmente independente no espaço de Hilbert $L^2((a, b), d\mu)$) obtemos uma sequência de polinômios $\{p_n\}_{n \geq 0}$ tal que

$$\int_a^b p_n(x) p_m(x) d\mu(x) = h_n \delta_{nm}, \quad (1.1)$$

onde as constantes h_n são positivas e δ_{nm} é o delta de Kronecker. Um resultado bem conhecido na literatura é que os zeros dos polinômios ortogonais com relação a (1.1) são todos reais, distintos e pertencem ao intervalo (a, b) .

Seja $\{p_n(x; \tau)\}$ uma sequência de polinômios ortogonais com relação a uma medida da forma $d\mu(x; \tau) = \omega(x; \tau) dx$ com $\tau \in (\tau_1, \tau_2)$. Uma questão natural que surge é como os zeros do polinômio $p_n(x; \tau)$ se comportam como função de τ . Um resultado clássico é o seguinte:

Teorema [Markov] (ver [2] ou [3, Theorem 6.12.1]): *Suponha que $\omega(x; \tau)$, definida no intervalo (a, b) , possui derivada contínua, $\omega_\tau(x; \tau)$, com relação τ , para todo $\tau \in (\tau_1, \tau_2)$ e $x \in (a, b)$. Assuma também que as integrais*

$$\int_a^b x^j \omega_\tau(x; \tau) dx, \quad j = 0, 1, \dots, 2n - 1,$$

convirjam uniformemente para τ em cada subconjunto compacto de (τ_1, τ_2) . Então os zeros de $p_n(x; \tau)$ são funções crescentes (decrescentes) do parâmetro τ se $\partial \ln \omega(x; \tau) / \partial \tau$ é uma função crescente (decrescente) de x em (a, b) .

Para exemplificar, considere os polinômios clássicos de Jacobi $\{P_n^{(\alpha, \beta)}(x)\}$ que são ortogonais em $(-1, 1)$ com relação a função $\omega(x; \alpha, \beta) = (1-x)^\alpha (1+x)^\beta$, $\alpha, \beta > -1$. As derivadas logarítmicas com relação a α e β são

$$\frac{\partial \ln \omega(x; \alpha, \beta)}{\partial \alpha} = \ln(1-x) \quad \text{e} \quad \frac{\partial \ln \omega(x; \alpha, \beta)}{\partial \beta} = \ln(1+x),$$

respectivamente. Como $\ln(1-x)$ é uma função decrescente de x e $\ln(1+x)$ é uma função crescente de x , para $x \in (-1, 1)$, concluímos, pelo Teorema de Markov, que os zeros de $P_n^{(\alpha, \beta)}(x)$ são funções decrescentes de α e crescentes de β , para $\alpha, \beta > -1$.

Suponha agora que adicionamos a medida $d\mu$ uma massa positiva λ em um ponto $c \notin (a, b)$, isto é,

$$d\mu(x; \lambda, c) = d\mu(x) + \lambda \delta_c, \quad \lambda > 0.$$

Seja $\{P_n(x; \lambda, c)\}$ a sequência de polinômios que é ortogonal com relação $d\mu(x; \lambda, c)$. Denotemos por $x_{n,k}(\lambda, c)$, $k = 1, 2, \dots, n$, os zeros de $P_n(x; \lambda, c)$. Observe que os zeros dependem de λ e c . Uma questão natural que surge é

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sobre o comportamento dos zeros com relação a esses parâmetros. Em [4] provou-se resultados de monotonicidade e assintótica dos zeros $x_{n,k}(\lambda, c)$ com relação a λ . Um estudo aponta o seguinte resultado que está sob avaliação: os zeros $x_{n,k}(\lambda, c)$ são funções crescentes de c para $c \notin (a, b)$. Além disso $n - 1$ zeros de $P_n(x; \lambda, c)$ convergem para os zeros de $P_{n-1}(x) = P_{n-1}(x; 0, c)$ quando c tende a infinito.

Para ilustrar nosso resultado usamos o polinômio clássico de Jacobi $P_n^{(\alpha, \beta)}(x)$. Fornecemos duas figura usando a função JacobiP[n,α,β,x] implementada no Wolfram Mathematica 9.0. Considere o polinômio $P_5^{(0.5,1)}(x; 0.2, c)$ associado com a modificação de $d\mu(x; \alpha, \beta) = (1-x)^\alpha(1+x)^\beta dx$ por adicionar uma massa $\lambda = 0.2$ em $c \notin (-1, 1)$. A Figura 1 mostra o correspondente polinômio para diferentes valores de c , a saber $c = 1.1$ (linha contínua), $c = 1.2$ (linha pontilhada) e $c = 1.3$ (linha tracejada). De acordo com a Figura 1, os zeros desse polinômio são funções crescentes de c . A Figura 2 mostra a convergência dos quatro zeros de $P_5^{(0.5,1)}(x; 0.2, c)$ (linha contínua) aos zeros de $P_4^{(0.5,1)}(x)$ (linha pontilhada) quando $c \rightarrow \infty$.

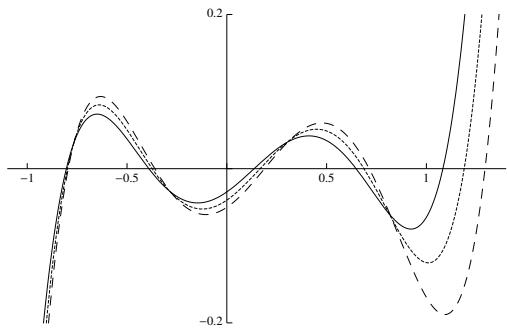


Figura 1: Monotonicidade dos zeros do caso Jacobi para diferentes valores de c .

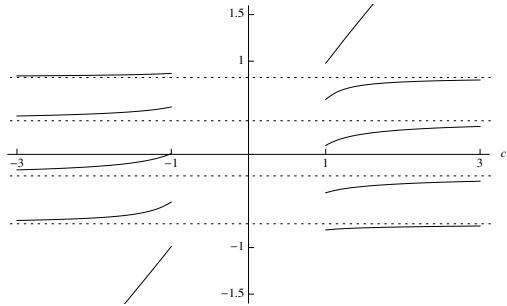


Figura 2: Monotonicidade e convergência dos zeros.

Este trabalho foi desenvolvido em conjunto com o Prof. Kenier Castillo.

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EXISTÊNCIA E ESTABILIDADE ASSINTÓTICA PARA UM SISTEMA ACOPLADO DE EQUAÇÕES DE ONDA COM MEMÓRIA

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1 Introdução

Nos últimos anos, Equações Diferenciais Parciais que envolvem termos de memória tem sido alvo de estudos de vários pesquisadores. Em tais problemas, é importante observar que a falta de uma estrutura de semigrupos torna a análise do mesmo um pouco mais delicada. Nesse sentido, estamos interessados em obter resultados de existência e estabilidade para o sistema acoplado com memória (1.1).

Para isso, considere Ω um aberto de \mathbf{R}^3 , com fronteira regular $\Gamma = \Gamma_0 \cup \Gamma_1$, onde Γ_0 e Γ_1 são fechadas e disjuntas e o sistema:

$$\left| \begin{array}{l} u_{tt} - \Delta u + \int_0^t g(t-s) \Delta u ds + \beta uv^2 = 0 \text{ em } \Omega \times (0, \infty) \\ v_{tt} - \Delta v + \int_0^t h(t-s) \Delta v ds + \beta vu^2 = 0 \text{ em } \Omega \times (0, \infty) \\ u = 0 \quad \text{em } \Gamma_0 \times (0, \infty) \\ v = 0 \quad \text{em } \Gamma_0 \times (0, \infty) \\ \frac{\partial u}{\partial \nu} - \int_0^t g(t-s) \frac{\partial u}{\partial \nu}(s) ds + h_1(., u') = 0 \text{ sobre } \Gamma_1 \times (0, \infty) \\ \frac{\partial v}{\partial \nu} - \int_0^t h(t-s) \frac{\partial v}{\partial \nu}(s) ds + h_2(., v') = 0 \text{ sobre } \Gamma_1 \times (0, \infty) \\ u(0) = u_0, \quad u'(0) = u_1 \text{ em } \Omega \\ v(0) = v_0, \quad v'(0) = v_1 \text{ em } \Omega \end{array} \right. \quad (1.1)$$

onde g e h são funções tomadas em $C^2(0, \infty) \cap W^{2,\infty}(0, \infty)$ satisfazendo as seguintes condições:

$$g(s) \geq 0, \quad 1 - \int_0^\infty g(s) ds = l_1 > 0$$

e

$$h(s) \geq 0, \quad 1 - \int_0^\infty h(s) ds = l_2 > 0.$$

As funções h_i , $i = 1, 2$ são tomadas em $C^0(\mathbf{R}, L^\infty(\Gamma_1))$, com $h_i(x, s)$ não decrescente em s para q.t.p. x em Γ_1 , $h_i(x, 0) = 0$ para q.t.p. x em Γ_1 e ainda h_i é fortemente monótona em s , para q.t.p. x em Γ_1 .

2 Existência e Estabilidade de Soluções

Com o objetivo de encontrar soluções e taxas de decaimento para as mesmas, lançaremos mão de algumas técnicas conhecidas na literatura, dentre as quais podemos destacar o "método de Faedo-Galerkin" combinado com o "método de compacidade", "aproximação de Strauss", "método de Lyapounov" e/ou técnicas desenvolvidas em [6].

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Tomando os dados iniciais em um espaço adequado e assumindo uma condição de compatibilidade para esses dados, esperamos mostrar que a solução de (1.1) verifique:

$$\{u, v\} \in (L_{loc}^\infty(0, \infty); H_{\Gamma_0}^1(\Omega))^2,$$

$$\{u', v'\} \in (L_{loc}^\infty(0, \infty); H_{\Gamma_0}^1(\Omega))^2,$$

$$\{u'', v''\} \in (L_{loc}^\infty(0, \infty); L^2(\Omega))^2,$$

onde

$$H_{\Gamma_0}^1(\Omega) = \{v \in H^1(\Omega); v = 0 \text{ sobre } \Gamma_0\},$$

e satisfaça as equações

$$u_{tt} - \Delta u + \int_0^t g(t-s) \Delta u ds + \beta uv^2 = 0 \text{ em } L_{loc}^\infty(0, \infty); L^2(\Omega)$$

$$v_{tt} - \Delta v + \int_0^t h(t-s) \Delta v ds + \beta vu^2 = 0 \text{ em } L_{loc}^\infty(0, \infty); L^2(\Omega)$$

e as condições de fronteira,

$$\frac{\partial u}{\partial \nu} - \int_0^t g(t-s) \frac{\partial u}{\partial \nu}(s) ds + h_1(., u') = 0 \text{ em } L_{loc}^1(0, \infty; L^1(\Gamma_1))$$

$$\frac{\partial v}{\partial \nu} - \int_0^t h(t-s) \frac{\partial v}{\partial \nu}(s) ds + h_2(., v') = 0 \text{ em } L_{loc}^1(0, \infty; L^1(\Gamma_1))$$

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Sharp constant and extremal function for weighted Trudinger-Moser type inequalities in \mathbb{R}^2 ^{*}

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In this work, we prove the sharpness and the existence of extremal function for a Trudinger-Moser type inequality in weighted Sobolev spaces established by F. S. B. Albuquerque, C. O. Alves and E. S. Medeiros in 2014 (see [3, Theorem 1.1]).

1 Introduction

We recall that if Ω is a bounded domain in \mathbb{R}^2 , the classical Trudinger-Moser inequality (see [6, 8]) asserts that $e^{\alpha u^2} \in L^1(\Omega)$ for all $u \in H_0^1(\Omega)$ and $\alpha > 0$. Moreover, there exists a constant $C = C(\Omega) > 0$ such that

$$\sup_{\|u\|_{H_0^1(\Omega)} \leq 1} \int_{\Omega} e^{\alpha u^2} dx \leq C, \quad \text{if } \alpha \leq 4\pi, \quad (1.1)$$

where $\|u\|_{H_0^1(\Omega)} = (\int_{\Omega} |\nabla u|^2 dx)^{1/2}$. Furthermore, (1.1) is sharp in the sense that if $\alpha > 4\pi$ the supremum (1.1) is $+\infty$. Related inequalities for unbounded domains have been proposed by D. M. Cao [4] and B. Ruf [7]. In [7], the author proved that there exists a constant $d > 0$ such that for any domain $\Omega \subset \mathbb{R}^2$,

$$\sup_{\|u\|_S \leq 1} \int_{\Omega} (e^{4\pi u^2} - 1) dx \leq d, \quad (1.2)$$

where $\|u\|_S = (\int_{\Omega} (|\nabla u|^2 + |u|^2) dx)^{1/2}$. Moreover, the inequality (1.2) is sharp in the sense that for any growth $e^{\alpha u^2}$ with $\alpha > 4\pi$ the supremum (1.2) is $+\infty$. Furthermore, he proved that the supremum (1.2) is attained whenever it is finite. On the other hand, Adimurthi and K. Sandeep [1] extended the Trudinger-Moser inequality (1.1) for singular weights. More precisely, they proved that if Ω is a bounded domain in \mathbb{R}^2 containing the origin, $u \in H_0^1(\Omega)$ and $\beta \in [0, 2)$, then

$$\sup_{\|u\|_{H_0^1(\Omega)} \leq 1} \int_{\Omega} \frac{e^{\alpha u^2} - 1}{|x|^\beta} dx < +\infty \Leftrightarrow 0 < \alpha \leq 4\pi(1 - \beta/2). \quad (1.3)$$

Later, J. M. do and M. de Souza in [5] investigated the Trudinger-Moser type inequality also with a singular weight for any domain $\Omega \subset \mathbb{R}^2$ containing the origin as well as some applications.

Throughout the work, we consider weight functions $V(|x|)$ and $Q(|x|)$ satisfying the following assumptions:

(V) $V \in C(0, \infty)$, $V(r) > 0$ and there exist $a, a_0 > -2$ such that

$$\limsup_{r \rightarrow 0} \frac{V(r)}{r^{a_0}} < \infty \quad \text{and} \quad \liminf_{r \rightarrow +\infty} \frac{V(r)}{r^a} > 0.$$

(Q) $Q \in C(0, \infty)$, $Q(r) > 0$ and there exist $b < (a - 2)/2$ and $-2 < b_0 \leq 0$ such that

$$0 < \liminf_{r \rightarrow 0} \frac{Q(r)}{r^{b_0}} \leq \limsup_{r \rightarrow 0} \frac{Q(r)}{r^{b_0}} < \infty \quad \text{and} \quad \limsup_{r \rightarrow +\infty} \frac{Q(r)}{r^b} < \infty.$$

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In order to establish our main result, we need to recall some notation.

- $C_0^\infty(\mathbb{R}^2)$ denotes the set of smooth functions with compact support.
- $C_{0,rad}^\infty(\mathbb{R}^2) = \{u \in C_0^\infty(\mathbb{R}^2) : u \text{ is radial}\}$.
- $D_{rad}^{1,2}(\mathbb{R}^2)$ denotes $\overline{C_{0,rad}^\infty(\mathbb{R}^2)}$ under the norm $\|\nabla u\|_{L^2(\mathbb{R}^2)}$.
- If $1 \leq p < \infty$ we define $L^p(\mathbb{R}^2; Q) \doteq \{u : \mathbb{R}^2 \rightarrow \mathbb{R} : u \text{ is measurable, } \int_{\mathbb{R}^2} Q(|x|)|u|^p dx < \infty\}$. Similarly we define $L^2(\mathbb{R}^2; V)$. Then we set $E \doteq D_{rad}^{1,2}(\mathbb{R}^2) \cap L^2(\mathbb{R}^2; V)$, which is a Hilbert space with the norm $\|u\| \doteq (\int_{\mathbb{R}^2} (|\nabla u|^2 + V(|x|)|u|^2) dx)^{1/2}$.

2 Mathematical Results

With the aid of inequalities (1.1), (1.3) and inspired by similar arguments developed in [3, 4, 7], we obtain what the title of this work states.

Theorem 2.1. *Assume that (V) – (Q) hold. Then there holds*

$$S_\alpha = \sup_{u \in E; \|u\| \leq 1} \int_{\mathbb{R}^2} Q(|x|)(e^{\alpha u^2} - 1) dx < +\infty \quad (2.4)$$

if and only if $0 < \alpha \leq \alpha' \doteq 2(b_0 + 2)$. Moreover, the supremum (2.4) is attained provided $0 < \alpha < \alpha'$.

Remark 2.1. *In [2, 3], the authors also used estimate (2.4) to study the existence and multiplicity of solutions for some classes of nonlinear Schrödinger elliptic equations (and systems of equations) with unbounded, singular or decaying radial potentials and involving nonlinearities with exponential critical growth of Trudinger-Moser type. In the argument, they combined the inequality (2.4) and variational methods.*

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A DIRICHLET PROBLEM UNDER INTEGRAL BOUNDARY CONDITION VIA SUB-SUPERSOLUTION METHOD

FRANCISCO JULIO S.A. CORRÊA * & JOELMA MORBACH †

1 Introduction

In this paper we deal with the Dirichlet problem under integral boundary condition

$$\begin{cases} -A \left(\int_{\Omega} u(y) dy \right) \Delta u = f(x, u) & \text{in } \Omega, \\ u = K \int_{\Omega} u(y) dy & \text{on } \partial\Omega \end{cases} \quad (1.1)$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$ is a bounded smooth domain, $K > 0$ is a real parameter satisfying $1 - K|\Omega| > 0$, $A : \mathbb{R} \rightarrow \mathbb{R}$ and $f : \bar{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$ are given functions whose properties will be timely introduced. This kind of problem has been studied by several authors. For example, in Wang [3], the author is concerned, among other things, with the problem

$$\begin{cases} -\Delta \varphi = \lambda \varphi & \text{in } \Omega, \\ \varphi = K \int_{\Omega} \varphi(y) dy & \text{on } \partial\Omega \end{cases} \quad (1.2)$$

Under suitable relations between λ , K and $|\Omega|$ the author makes a spectral analysis of problem (1.2). A semilinear counterpart of this problem is studied through classical sub and supersolution method.

We consider an equivalent form of the problem (1.1) by considering $v = u - K \int_{\Omega} u(y) dy$ and so $-\Delta v = -\Delta u$ and

$$\int_{\Omega} u(y) dy = \frac{1}{1 - K|\Omega|} \int_{\Omega} v(y) dy. \quad (1.3)$$

Consequently, u is a solution of problem (1.1) if, and only if, $v = u - K \int_{\Omega} u(y) dy$ is a solution of

$$\begin{cases} -A \left(\frac{1}{1 - K|\Omega|} \int_{\Omega} v(y) dy \right) \Delta v = f \left(x, v + \frac{1}{1 - K|\Omega|} \int_{\Omega} v(y) dy \right) & \text{in } \Omega, \\ v = 0 & \text{on } \partial\Omega. \end{cases} \quad (1.4)$$

We assume the following assumptions:

(H₁) There is a pair of ordered sub and supersolution, respectively, \underline{u} and \bar{u} satisfying $0 \leq \underline{u} \leq \bar{u}$ a.e. in Ω .

(H₂) $0 \leq f(x, t) \leq \mathcal{K}(x) \in L^2(\Omega)$ for a.e. $x \in \Omega$, $\forall t$ satisfying $\underline{u} + \frac{1}{1 - K|\Omega|} \int_{\Omega} \underline{u} dx \leq t \leq \bar{u} + \frac{1}{1 - K|\Omega|} \int_{\Omega} \bar{u} dx$.

(H₃) $f(x, \cdot)$ is nondecreasing on the interval $\left[\underline{u} + \frac{1}{1 - K|\Omega|} \int_{\Omega} \underline{u} dx, \bar{u} + \frac{1}{1 - K|\Omega|} \int_{\Omega} \bar{u} dx \right]$.

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2 Results

These results and other related with problem (1.1) may be found in Morbach's Doctoral Thesis [2].

Theorem 2.1. *Under assumptions (H_1) , (H_2) and (H_3) , problem (1.1) possesses solutions $U, V \in W_0^{1,2}(\Omega)$ with $\underline{u} \leq U \leq V \leq \bar{u}$. Moreover, any solution u of (1.1) with $\underline{u} \leq u \leq \bar{u}$ is such that $U \leq u \leq V$, ie, U is minimal solution and V is maximal solution in the range $[\underline{u}, \bar{u}]$.*

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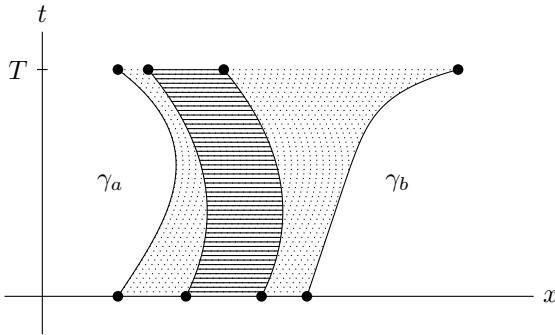
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REMARKS ON CARLEMAN INEQUALITY IN MOVING BOUNDARY DOMAINS

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1 Introduction

Consider $a, b, c, d, e \in C^2([0, T])$ satisfying $a(t) < c(t) < e(t) < d(t) < b(t)$ for all $t \in [0, T]$ and define the domain: $Q = \{(x, t) \in \mathbb{R}^2 : a(t) < x < b(t), 0 < t < T\}$, $\omega = \{(x, t) \in \mathbb{R}^2 : c(t) < x < d(t), 0 < t < T\}$, $\gamma_a = \{(a(t), t) : 0 \leq t \leq T\}$ and $\gamma_b = \{(b(t), t) : 0 \leq t \leq T\}$. (Dotted region Q and dashed region ω).



The goal of this work is to obtain, for given functions $A \in C^{1,2}(\overline{Q})$, $B \in C(\overline{Q})$, $C \in C(\overline{Q})$, a Carleman inequality (see (2.4) below) for the solutions of the boundary value problem

$$\begin{cases} L^* z = -z_t - Az_{xx} - Bz_x - Cz = g, & (x, t) \in Q, \\ z|_{\gamma_a \cup \gamma_b} = 0, \\ z(x, T) = z_0(x), & x \in [a(T), b(T)]. \end{cases} \quad (1.1)$$

It is worth to mention that Carleman estimates are applied in various ways to produce remarkable results in inverse problems and in control theory.

The weights appearing in inequality (2.4) are of the form $\phi_\lambda^m e^{2s\tilde{\phi}_\lambda}$, where $m \in \mathbb{Z}$, λ and s are positive parameters, and

$$\phi_\lambda(x, t) = \frac{e^{\lambda\psi(x, t)}}{[t(T-t)]^\kappa}, \quad \tilde{\phi}_\lambda(x, t) = \frac{e^{\lambda\psi(x, t)} - e^{2\lambda||\psi||_\infty}}{[t(T-t)]^\kappa}. \quad (1.2)$$

The function ψ appearing above is defined on \overline{Q} by

$$\psi(x, t) = \begin{cases} \frac{x-a(t)}{c(t)-a(t)}, & a(t) \leq x \leq c(t), \\ 1 + P\left(\frac{x-c(t)}{e(t)-c(t)}, \frac{e(t)-c(t)}{c(t)-a(t)}\right), & c(t) \leq x \leq e(t), \\ 1 + P\left(\frac{d(t)-x}{d(t)-e(t)}, \frac{d(t)-e(t)}{b(t)-d(t)}\right), & e(t) \leq x \leq d(t), \\ \frac{b(t)-x}{b(t)-d(t)}, & d(t) \leq x \leq b(t), \end{cases} \quad (1.3)$$

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where $P(w, z) = zw + (10 - 6z)w^3 + (8z - 15)w^4 + (6 - 3z)w^5$. It is no difficult to see that ψ belongs to $C^2(\overline{Q})$, is strictly positive in Q and vanishes at $\gamma_a = \{(a(t), t) : 0 \leq t \leq T\}$ and $\gamma_b = \{(b(t), t) : 0 \leq t \leq T\}$. Moreover, there exists $k > 0$ such that $|\psi_x| \geq k > 0$ in Q/ω .

Once defined the weights we obtain Theorem 2.1 using Leibnitz's formula and a slight modification of the usual tecnicas developed in [3]. Another possible approach in order to obtain the desired Carleman's inequality, used to prove Theorem 2.2, is to construct a C^2 diffeomorphism between Q and $[0, 1] \times [0, T]$ which maps ω into $(\alpha, \beta) \times (0, T)$, and then obtain the desired Carleman estimate from the usual one (fixed boundary).

Given real numbers $a < c < d < b$, $0 < \alpha < \beta < 1$, fix $\delta > 0$ small enough and let $g_{a,c,d,b,\delta}$ be the linear interpolation of the points $(a - \delta, -\delta)$, $(a + \delta, \delta)$, $(c - \delta, \alpha - \delta)$, $(c + \delta, \alpha + \delta)$, $(d - \delta, \beta - \delta)$, $(d + \delta, \beta + \delta)$, $(b - \delta, 1 - \delta)$, $(b + \delta, 1 + \delta)$. Consider $g_{a,c,d,b,\delta}$ as defined on the whole line by extending the end segments toward infinity. On the other hand, let φ be a nonnegative even smooth function with compact support contained in $(-\delta, \delta)$ which integral on this interval is equal to one.

It can be seen that $f_{a,c,d,b,\delta}$, defined as the convolution of $g_{a,c,d,b,\delta}$ with φ , is a C^∞ diffeomorphism which takes values $0, \alpha, \beta, 1$ at a, c, d, b , respectively. It is not difficult to see that $H(x_1, \dots, x_5) = f_{x_2, \dots, x_5, \delta}(x_1)$, defined on $D = \{(x_1, \dots, x_5) : x_2 + \delta < x_3, x_3 + \delta < x_4, x_4 + \delta < x_5\}$, is smooth.

The desired diffeomorphism is constructed by fixing $\delta = \frac{1}{4} \min\{\delta_1, \delta_2\}$, where $\delta_1 = \min_{t \in [0, T]} \{c(t) - a(t), d(t) - c(t), b(t) - d(t)\}$ and $\delta_2 = \min\{\alpha, \beta - \alpha, 1 - \beta\}$, and defining $F(x, t) = (H(x, a(t), c(t), d(t), b(t)), t)$ on Q .

2 Mathematical Results

Theorem 2.1. *Let A, B, C, g satisfying the preceding conditions and ψ the function defined in (1.3). Suppose the existence of $a_0 > 0$ such that $|A| \geq a_0$ on \overline{Q} . Then, there exist $\lambda > 0$ and positive constants C and s_0 such that*

$$\int_Q \left[\frac{1}{s\phi_\lambda} (z_t^2 + z_{xx}^2) + s\lambda^2 \phi_\lambda z_x^2 + s^3 \lambda^4 \phi_\lambda^3 z^2 \right] e^{2s\tilde{\phi}_\lambda} \leq C \left(\int_Q g^2 e^{2s\tilde{\phi}_\lambda} + \int_\omega z^2 + 2 \int_0^T \{a'(t) Aw_x^2|_{(a(t), t)} - b'(t) Aw_x^2|_{(b(t), t)}\} \right), \quad (2.4)$$

for every solution $z \in C^{1,2}(\overline{Q})$ of (1.1) and every $s > s_0$. Finally, $w = e^{s\tilde{\phi}} z$ and $\phi_\lambda, \tilde{\phi}_\lambda$ ($\kappa = 2$) are given in (1.2).

Theorem 2.2. *Under the same assumptions of Theorem 2.1 we have that there exist positive constants $\lambda > 0$, $s_1 > 0$ and C such that*

$$\int_Q \left[\frac{1}{s\Phi_\lambda} (z_t^2 + z_{xx}^2) + s\lambda^2 \Phi_\lambda z_x^2 + s^3 \lambda^4 \Phi_\lambda^3 z^2 \right] e^{2s\tilde{\Phi}_\lambda} \leq \int_Q g^2 e^{2s\tilde{\Phi}_\lambda} + \int_\omega z^2,$$

for every $s > s_1$, where $\Phi_\lambda = \phi_\lambda \circ F$ and $\tilde{\Phi}_\lambda = \tilde{\phi}_\lambda \circ F$.

Note that the preceding results are equivalent on increasing domains Q .

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A NONLOCAL ELLIPTIC TRANSMISSION PROBLEM OF $p(x)$ -KIRCHHOFF TYPE WITH NONLINEAR BOUNDARY CONDITION

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Abstract

In this research, by means of the monotone operator theory we show the existence of weak solutions for a transmission problem given by a system of two nonlinear elliptic equations of $p(x)$ -Kirchhoff type which is the generalization of a mathematical model arising of an electrolysis process.

1 Introduction

Let $\Omega, \Omega_1, \Omega_2 \subseteq \mathbb{R}^2$ be a bounded polygonal domains with their boundaries $\partial\Omega, \partial\Omega_1, \partial\Omega_2$ and closures $\overline{\Omega}, \overline{\Omega}_1, \overline{\Omega}_2$ satisfying the relations $\overline{\Omega} = \overline{\Omega}_1 \cup \overline{\Omega}_2, \Omega_1 \cap \Omega_2 = \emptyset$.

We denote

$$\Gamma_3 = \partial\Omega_1 \cap \partial\Omega_2, \quad \Gamma_i = \partial\Omega_i \setminus \Gamma_3, \quad i = 1, 2. \quad (\text{See Fig.1})$$

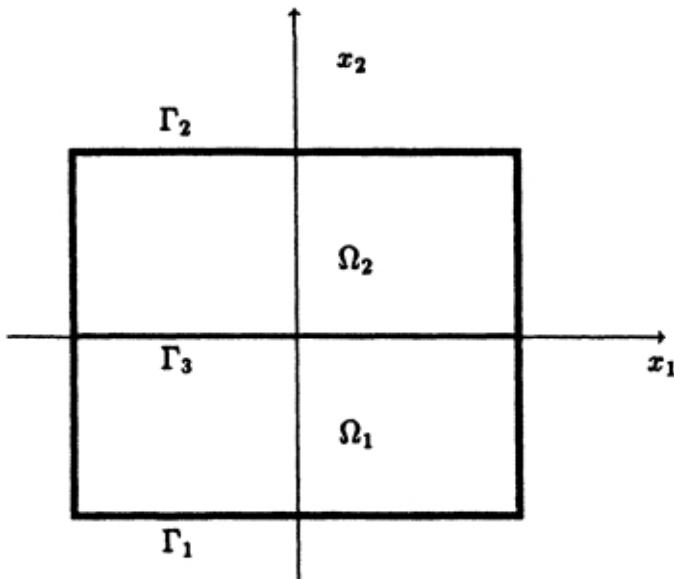


Figura 1.

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We are concerned with the existence of solutions to the following system of nonlinear elliptic equations:

$$\begin{aligned} -M_i \left(\int_{\Omega_i} \frac{1}{p(x)} |\nabla u_i|^{p(x)} dx \right) \operatorname{div}(|\nabla u_i|^{p(x)-2} \nabla u_i) &= \operatorname{div} \vec{f} \quad \text{in } \Omega_i, \quad i=1, 2. \\ M_i \left(\int_{\Omega_i} \frac{1}{p(x)} |\nabla u_i|^{p(x)} dx \right) \frac{\partial u_i}{\partial \nu} + k |u_i|^{\alpha(x)-2} u_i &= \vec{f} \cdot \vec{\nu} \quad \text{on } \Gamma_i, \quad i=1, 2. \\ M_1 \left(\int_{\Omega_1} \frac{1}{p(x)} |\nabla u_1|^{p(x)} dx \right) \frac{\partial u_1}{\partial \nu^1} &= -M_2 \left(\int_{\Omega_2} \frac{1}{p(x)} |\nabla u_2|^{p(x)} dx \right) \frac{\partial u_2}{\partial \nu^2} = k |u_2 - u_1|^{\alpha(x)-2} (u_2 - u_1) \quad \text{on } \Gamma_3 \end{aligned} \quad (1.1)$$

where

$$(M_0) \quad M_i : [0, +\infty[\rightarrow [m_{0i}, +\infty[\quad , \quad \text{are non decreasing Lipschitz continuous functions, } i = 1, 2 ,$$

k , m_{0i} , $i = 1, 2$ are positive numbers, p and α are continuous functions on $\bar{\Omega}$ satisfying appropriate conditions, $\vec{f} = (f_1, f_2)$ is a given vector field (determined from Maxwell's equations), $\vec{\nu} = (\nu_1, \nu_2)$ and $\vec{\nu}^i = (\nu_1^i, \nu_2^i)$ denote a unit outer normal to $\partial\Omega$ and to $\partial\Omega_i$, respectively; $\frac{\partial}{\partial \nu}$ and $\frac{\partial}{\partial \nu^i}$ denote the derivative in the direction $\vec{\nu}$ and $\vec{\nu}^i$ respectively of course $\vec{\nu}^1 = -\vec{\nu}^2$ and $\frac{\partial}{\partial \nu^1} = -\frac{\partial}{\partial \nu^2}$ on Γ_3 , $\vec{\nu} = \vec{\nu}^i$ on Γ_i , $i = 1, 2$. We confine ourselves to the case where $M_1 = M_2$ with $m_{01} = m_{02} = m_0$ for simplicity. Notice that the results of this work remain valid for $M_1 \neq M_2$. Transmission problems arise in several applications of physics and biology (see [2]). Our work is motivated by the one of Feisthauer et al [1].

2 Mathematical Results

We shall deal with the Lebesgue-Sobolev Spaces with variable exponent $L^{p(x)}(\Omega)$, $L^{p(x)}(\Omega_i)$, $L^{p(x)}(\partial\Omega)$ and $W_0^{1,p(x)}(\Omega_i)$. We refer to the book of Musielak [4].

Set $C_+(\bar{\Omega}) = \{h : h \in C(\bar{\Omega}) : h(x) > 1, \forall x \in \bar{\Omega}\}$; for any $h \in C_+(\bar{\Omega})$, we define $h^+ = \sup_{x \in \Omega} h(x)$ and $h^- = \inf_{x \in \Omega} h(x)$

Let $p(x), m(x) \in C_+(\bar{\Omega})$ with $\alpha(x) > 2$ for any $x \in \bar{\Omega}$. We have the following result.

Teorema 2.1. *If $\vec{f} \in [L^{p(x)}(\Omega)]^2$ and (M_0) hold, problem (1.1) has a weak solution*

$$\{u_1, u_2\} \in E = W^{1,p(x)}(\Omega_1) \times W^{1,p(x)}(\Omega_2)$$

Proof: We can write problem (1.1) as operator form

$$Au = F$$

Then, we prove that A is coercive, strictly monotone and locally Lipschitz continuous on E , so by the well known results from the monotone operator theory the proof is achieved. \square

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ARE INJECTIONS ALWAYS INJECTIVE?

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By definition, an injection is an injective map between two sets. But mathematical terminology can be misleading, and sometimes mathematicians call injection a non-necessarily injective map. This may cause problems: imagine you have just found a result you need in a paper, written by a well known expert, stating that a certain injection satisfies a certain desired property. Be honest: are you going to use the result at once, including the injectivity of the map, or are you going to check the proof and the injectivity of the map? In this work I shall describe a situation where we used the injectivity of a so-called injection which is not always injective.

1 Introduction

Given a Banach space E , we shall always consider the closed unit ball $B_{E'}$ of its dual E' endowed with the weak-star topology. By $W(B_{E'}, w^*)$ we mean the set of all regular Borel probability measures on $B_{E'}$. Given $0 < p < +\infty$ and a regular Borel probability measure μ on $B_{E'}$, let

$$j_p: C(B_{E'}) \longrightarrow L_p(\mu)$$

be the canonical map. Considering the Banach space $C(B_{E'})$ of continuous functions on $B_{E'}$ with the sup-norm, the map

$$e: E \longrightarrow C(B_{E'}) , \quad e(x)(\varphi) = \varphi(x),$$

is a linear isometry. By j_p^e we denote the restriction of j_p to $e(E)$. Quite often – and even in the most important case, that is, when μ is a Pietsch measure for an absolutely summing operator (cf. Section 2) – the map j_p is called *the canonical injection*. Even very important experts use this terminology in this context, see, e.g., Maurey [5] and Rosenthal [7]. And, sometimes, misled by the terminology, people believe that, at least in the context of Pietsch measures for summing operators, this map is always injective and use this property to *prove* results (see, e.g., [1, 2, 6]). In this talk we discuss the injectivity of the map j_p . As we shall see, sometimes this map fails to be injective, so the *proofs* depending on its injectivity are wrong and something must be done about that. In the cases mentioned above, in [3] we fortunately managed to fix the proofs of [1, 2, 6] that depend on the injectivity of j_p .

2 Results

Let us describe how the map j_p and its restriction j_p^e play a central role in the theory of absolutely summing operators. By definition, an operator $u: E \longrightarrow F$ is p -summing if $(u(x_n))_{n=1}^\infty$ is weakly p -summable in F whenever $(x_n)_{n=1}^\infty$ is absolutely p -summable in E . The celebrated Pietsch Factorization Theorem [4, Theorem 2.13] asserts that u is p -summing if and only if there are a measure $\mu \in W(B_{E'})$, a (closed) subspace X_p of $L_p(\mu)$ containing $(j_p \circ e)(E)$ and a continuous operator $\hat{u}: X_p \longrightarrow F$ such that $u = \hat{u} \circ j_p^e \circ e$. In other words, p -summing operators are exactly those which factor through j_p^e . Any such measure μ is called a Pietsch measure for u .

First we characterize when j_p^e is injective:

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Proposition 2.1. Let $\mu \in W(B_{E'}, w^*)$ and $1 \leq p < \infty$ be given. Then j_p^e is injective if and only if μ is a Pietsch measure for some injective p -summing linear operator defined on E .

There are many examples of injective p -summing operators, for example the formal inclusions $\ell_1 \hookrightarrow \ell_2$ and $L_\infty(\mu) \hookrightarrow L_p(\mu)$, where μ is a finite measure. In the latter case, even $j_p: C(B_{L_\infty(\mu)}) \longrightarrow L_p(\mu)$ is injective.

Next we show that, if $(B_{E'}, w^*)$ is separable, then p -summing operators on E always have a Pietsch measure for which the canonical map j_p is injective.

Proposition 2.2. Assume that $(B_{E'}, w^*)$ is separable. There exists a measure $\mu_0 \in W(B_{E'}, w^*)$ such that $j_q: C(B_{E'}) \longrightarrow L_q(\frac{\mu+\mu_0}{2})$ is injective for any $1 \leq q < \infty$ and any $\mu \in W(B_{E'}, w^*)$. In particular, any p -summing linear operator on E admits a Pietsch measure for which the canonical map j_p is injective.

Our next purpose is to show that, even in the context of Pietsch measures of summing operators, the canonical map j_p can fail to be injective. With the help of Uryshon's Lemma we have:

Proposition 2.3. Let K be a compact Hausdorff space containing at least two elements and $0 < p < \infty$. Then there is a regular probability measure μ on the Borel sets of K such that the canonical mapping $j_p: C(K) \longrightarrow L_p(\mu)$ fails to be injective.

For every Banach space $E \neq \{0\}$, $(B_{E'}, w^*)$ is a compact Hausdorff space containing more than two elements, so there is always a regular probability measure μ on $(B_{E'}, w^*)$ such that $j_p: C(B_{E'}) \longrightarrow L_p(\mu)$ fails to be injective. And, of course, μ is a Pietsch measure for the p -summing operator j_p . It is well known that the operator e is bijective whenever E is a $C(K)$ space, so in this case j_p^e is not injective either.

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HIPER-IDEAIS E APLICAÇÕES MULTILINEARES HIPER-NUCLEARES

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1 Introdução

Ao propormos uma definição de uma subclasse da classe das aplicações multilineares contínuas entre espaços de Banach, que chamamos de hiper-ideal, visando generalizar o conceito de multi-ideal apresentado por Pietsch em [3], observamos que as aplicações nucleares, um notável exemplo de multi-ideal, não é um hiper-ideal. Encontramos em [5] uma classe que, além de ser um hiper-ideal, satisfaz uma propriedade que mostra que esta classe está para os hiper-ideais assim como as nucleares estão para os multi-ideais.

A definição de hiper-ideais, descrita a seguir, foi inspirada por casos particulares tratados em [2] e [4].

Definição 1.1. Um *hiper-ideal de aplicações multilineares* \mathcal{H} , ou simplesmente *hiper-ideal*, é uma subclasse da classe das aplicações multilineares contínuas entre espaços de Banach tal que, para quaisquer $m \in \mathbb{N}$ e espaços de Banach E_1, \dots, E_m, F , as componentes $\mathcal{H}(E_1, \dots, E_m; F) := \mathcal{L}(E_1, \dots, E_m; F) \cap \mathcal{H}$ satisfazem:

(1) $\mathcal{H}(E_1, \dots, E_m; F)$ é um subespaço vetorial de $\mathcal{L}(E_1, \dots, E_m; F)$ que contém as aplicações m -lineares de tipo finito;

(2) Propriedade de hiper-ideal: Dados $1 < n_1 < \dots < n_m$ números naturais, $G_1, \dots, G_{n_m}, E_1, \dots, E_m, F, H$ espaços de Banach, se $B_1 \in \mathcal{L}(G_1, \dots, G_{n_1}; E_1), \dots, B_m \in \mathcal{L}(G_{1+n_{m-1}}, \dots, G_{n_m}; E_m)$ $A \in \mathcal{H}(E_1, \dots, E_m; F)$ e $t \in \mathcal{L}(F; H)$, então a composição $t \circ A \circ (B_1, \dots, B_m)$ pertence a $\mathcal{H}(G_1, \dots, G_{n_m}; H)$.

Se existe uma função $\|\cdot\|_{\mathcal{H}}: \mathcal{H} \rightarrow [0, \infty)$ tal que:

(a) A função $\|\cdot\|_{\mathcal{H}}$ restrita à componente $\mathcal{H}(E_1, \dots, E_n; F)$ é uma norma para quaisquer espaços de Banach E_1, \dots, E_n, F e todo $n \in \mathbb{N}$;

(b) A aplicação n -linear $I_n: \mathbb{K}^n \rightarrow \mathbb{K}$, dada por $I_n(\lambda_1, \dots, \lambda_n) = \lambda_1 \cdots \lambda_n$ é tal que $\|I_n\|_{\mathcal{H}} = 1$;

(c) Se $A \in \mathcal{H}(E_1, \dots, E_m; F)$, $B_k \in \mathcal{L}(G_{1+n_{k-1}}, \dots, G_{n_k}; E_k)$, $k = 1, \dots, m$ e $t \in \mathcal{L}(F; H)$, então

$$\|t \circ A \circ (B_1, \dots, B_m)\|_{\mathcal{H}} \leq \|t\| \cdot \|A\|_{\mathcal{H}} \cdot \|B_1\| \cdots \|B_m\|,$$

então $(\mathcal{H}; \|\cdot\|_{\mathcal{H}})$ é um *hiper-ideal normado*.

Mais ainda, se todas as componentes $\mathcal{H}(E_1, \dots, E_m; F)$ são subespaços completos relativamente à topologia gerada por $\|\cdot\|_{\mathcal{H}}$, dizemos que $(\mathcal{H}; \|\cdot\|_{\mathcal{H}})$ é um *hiper-ideal de Banach*.

2 Resultados

Uma conhecida caracterização de multi-ideais de Banach pode ser obtida para hiper-ideais:

Proposição 2.1 (Critério da Série). *Seja $(\mathcal{H}, \|\cdot\|_{\mathcal{H}})$ uma subclasse da classe das aplicações multilineares contínuas entre espaços de Banach munida de uma função $\|\cdot\|_{\mathcal{H}}: \mathcal{H} \rightarrow [0, +\infty)$. Então $(\mathcal{H}, \|\cdot\|_{\mathcal{H}})$ é um hiper-ideal de Banach se, e somente se, as seguintes condições estão satisfeitas:*

(i) $I_n \in \mathcal{H}(\mathbb{K}^n; \mathbb{K})$ e $\|I_n\|_{\mathcal{H}} = 1$, para todo $n \in \mathbb{N}$;

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(ii) Se $(A_n)_n \subset \mathcal{H}(E_1, \dots, E_m; F)$ e $\sum_{n=1}^{\infty} \|A_n\|_{\mathcal{H}} < \infty$, então

$$A = \sum_{n=1}^{\infty} A_n \in \mathcal{H}(E_1, \dots, E_m; F) \text{ e } \|A\|_{\mathcal{H}} \leq \sum_{n=1}^{\infty} \|A_n\|_{\mathcal{H}};$$

(iii) Se $1 < n_1 < \dots < n_m$, $G_1, \dots, G_{n_m}, E_1, \dots, E_m, F, H$ são espaços de Banach, $B_1 \in \mathcal{L}(G_1, \dots, G_{n_1}; E_1), \dots, B_m \in \mathcal{L}(G_{1+n_{m-1}}, \dots, G_{n_m}; E_m)$, $A \in \mathcal{H}(E_1, \dots, E_m; F)$ e $t \in \mathcal{L}(F; H)$, então

$$t \circ A \circ (B_1, \dots, B_m) \in \mathcal{H}(G_1, \dots, G_{n_m}; H) \text{ e } \|t \circ A \circ (B_1, \dots, B_m)\|_{\mathcal{H}} \leq \|t\| \cdot \|A\|_{\mathcal{H}} \cdot \|B_1\| \cdots \|B_m\|.$$

Por outro lado, notemos que nem todos os conceitos de multi-ideais podem ser transferidos para os hiper-ideais, como sugere o exemplo a seguir.

Exemplo 2.1. As aplicações nucleares não formam um hiper-ideal.

Buscamos então um hiper-ideal que desempenhe o papel das aplicações nucleares nos multi-ideais, isto é, que seja o menor hiper-ideal de Banach (visto que o hiper-ideal das aplicações de posto finito não é completo). Tal classe foi introduzida originalmente em [5] e é descrita a seguir:

Definição 2.1. Dizemos que uma aplicação n -linear contínua $A: E_1 \times \dots \times E_n \rightarrow F$ é *hiper-nuclear* se existem uma sequência $(\lambda_i)_i \in \ell_1$ e sequências limitadas $(A_i)_i \subset \mathcal{L}(E_1, \dots, E_n; \mathbb{K})$ e $(y_i)_i \in F$ tais que

$$A = \sum_{i=1}^{\infty} \lambda_i A_i \otimes y_i.$$

Toda série dessa forma é chamada de *representação hiper-nuclear de A*. Denotamos o espaço dessas aplicações hiper-nucleares por $\mathcal{HN}(E_1, \dots, E_n; F)$.

Definimos ainda a função $\|\cdot\|_{\mathcal{HN}}: \mathcal{HN} \rightarrow \mathbb{R}$ por

$$\|A\|_{\mathcal{HN}} = \inf \left\{ \sum_{i=1}^{\infty} |\lambda_i| \|A_i\| \|y_i\| \right\},$$

onde o infimo é tomado sobre todas as representações hiper-nucleares de A .

Proposição 2.2. $(\mathcal{HN}, \|\cdot\|_{\mathcal{HN}})$ é o menor hiper-ideal de Banach. Isso significa que se $(\mathcal{H}, \|\cdot\|_{\mathcal{H}})$ é um hiper-ideal de Banach, então $\mathcal{HN} \subset \mathcal{H}$ e $\|\cdot\|_{\mathcal{H}} \leq \|\cdot\|_{\mathcal{HN}}$.

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DISTINGUISHED EXAMPLES OF MULTILINEAR OPERATORS OF TYPE (p_1, \dots, p_n)

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We introduce the class of multilinear operators of type (p_1, \dots, p_n) and show that this class is quite “large” in the sense that it contains all multi-ideals constructed by composition, factorization and linearization methods from the class of linear operator of type p . Some examples are provided to show that, although quite large, this new class is a proper ideal of multilinear operators and, moreover, it does not coincide with the aforementioned ideals.

1 Introduction

The concepts of type and cotype of Banach spaces were introduced mainly by J. Hoffmann-Jørgensen [4] and by B. Maurey [5] in the study of Banach spaces-valued random variables. Since then the theory of type and cotype have found several applications and became a central part of the geometry of Banach spaces [7], and of the linear and multilinear operator theory and operator ideals (see [2, 3, 6] for instance).

Our principal aim throughout our recent research has been to define and study these concepts in a multilinear scenario setting up the relationships of new class with the linear and multilinear theory already established.

Let us denote by $l_p(E)$ the space of absolutely p -summing sequences in a Banach space E , i.e. sequences satisfying $\sum_{i=1}^{\infty} \|x_i\|^p < \infty$, and by $\text{Rad}(E)$ the Banach space of almost unconditionally summable E -valued sequences, namely the sequences such that $\left\| \sum_{j=1}^{\infty} r_j x_j \right\|_{L_2(E)} < \infty$, where $(r_j)_{j=1}^{\infty}$ are the Rademacher functions (see [3, p. 10]). From now on, p, p_1, \dots, p_n are positive real numbers and E, E_1, \dots, E_n, F are (real or complex) Banach spaces.

Definition 1.1. A continuous n -linear operator $T \in \mathcal{L}(E_1, \dots, E_n; F)$ has *type* (p_1, \dots, p_n) , $\frac{1}{2} \leq \frac{1}{p_1} + \dots + \frac{1}{p_n} \leq 1$, if there is a constant $C > 0$ such that, however we choose finitely many vectors $(x_j^{(1)}, \dots, x_j^{(n)})$ in $E_1 \times \dots \times E_n$, $j \in \{1, \dots, k\}$,

$$\left(\int_0^1 \left\| \sum_{j=1}^k r_j(t) T(x_j^{(1)}, \dots, x_j^{(n)}) \right\|^2 dt \right)^{1/2} \leq C \left(\sum_{j=1}^k \|x_j^{(1)}\|^{p_1} \right)^{1/p_1} \dots \left(\sum_{j=1}^k \|x_j^{(n)}\|^{p_n} \right)^{1/p_n}. \quad (1.1)$$

The set of all n -linear operators of type (p_1, \dots, p_n) from $E_1 \times \dots \times E_n$ to F is denoted by $\tau_{(p_1, \dots, p_n)}^n(E_1, \dots, E_n; F)$. This set, provided with the usual operations, is a normed subspace of $\mathcal{L}(E_1, \dots, E_n; F)$ equipped with the norm $\|\cdot\|_{\tau_{(p_1, \dots, p_n)}^n} := \inf\{C > 0, \text{ such that (1.1) holds}\}$, even further it is a Banach multi-ideal.

Although it is not present in this paper, we have established similar definitions and results that extend the concept of cotype of linear operators to the multilinear framework.

2 Results

A characterization of multilinear operators of type (p_1, \dots, p_n) is provided by the next theorem.

Theorem 2.1. Let (p_1, \dots, p_n) with $\frac{1}{2} \leq \frac{1}{p_1} + \dots + \frac{1}{p_n} \leq 1$. The following are equivalent for $T \in \mathcal{L}(E_1, \dots, E_n; F)$:

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i) T is an operator of type (p_1, \dots, p_n) ;

ii) $\left(T\left(x_j^{(1)}, \dots, x_j^{(n)}\right) \right)_{j=1}^{\infty} \in \text{Rad}(F)$ whenever $\left(x_j^{(i)}\right)_{j=1}^{\infty} \in l_{p_i}(E_i)$, $i \in \{1, \dots, n\}$.

A number of examples of continuous multilinear operators with proper type follows from the following result.

Proposition 2.1. Let $\left(x_j^{(i)}\right)_{j=1}^{\infty} \in l_{\infty}(E_i)$, $i \in \{1, \dots, n\}$ and $(T_j)_{j=1}^{\infty}$, where $T_j \in \mathcal{L}(E_1, \dots, E_n; F)$, $\forall j \in \mathbb{N}$. If $(T_j)_{j=1}^{\infty} \in \text{Rad}(\mathcal{L}(E_1, \dots, E_n; F))$, then $\left(T_j\left(x_j^{(1)}, \dots, x_j^{(n)}\right)\right)_{j=1}^{\infty} \in \text{Rad}(F)$.

For $(\lambda_j)_{j=1}^{\infty} \in l_2$ and $(x_j)_{j=1}^{\infty} \in l_{\infty}(E)$, defining the continuous linear operator $T_j : E \rightarrow E$ by $T_j(x) = \lambda_j x$, we notice that $(T_j)_{j=1}^{\infty} \in \text{Rad}(\mathcal{L}(E; E))$. Thus from Proposition 2.1 it follows that $(T_j(x_j))_{j=1}^{\infty} = (\lambda_j x_j)_{j=1}^{\infty} \in \text{Rad}(E)$. In other words,

$$l_2 \cdot l_{\infty}(E) \subseteq \text{Rad}(E).$$

Therefore, the following are examples of continuous multilinear operators with proper type:

Example 2.1. Let $\{i_1, \dots, i_k\}$ and $\{l_1, \dots, l_m\}$ be a partition of $\{1, \dots, n\}$, $A \in \mathcal{L}(E_{i_1}, \dots, E_{i_k})$ and $B \in \mathcal{L}(E_{l_1}, \dots, E_{l_m}; F)$. Defining $A \otimes B \in \mathcal{L}(E_1, \dots, E_n; F)$ by $A \otimes B := A(x_{i_1}, \dots, x_{i_k}) \cdot B(x_{l_1}, \dots, x_{l_m})$, we conclude that $A \otimes B \in \tau_{(p_1, \dots, p_n)}^n$ whenever $\frac{1}{2} \leq \frac{1}{p_{i_1}} + \dots + \frac{1}{p_{i_k}}$, with $\frac{1}{p_1} + \dots + \frac{1}{p_n} \leq 1$.

The next theorem establishes the relationship between $\tau_{(p_1, \dots, p_n)}^n$ and the multi-ideals generated from the linear ideal of operators of type p (cf. [1, 6]).

Theorem 2.2. If $\frac{1}{p} = \frac{1}{p_1} + \dots + \frac{1}{p_n}$, then $(\tau_p \circ \mathcal{L}) \cup [\tau_{p_1}, \mathcal{L}, \dots, \mathcal{L}] \cup \dots \cup [\mathcal{L}, \dots, \mathcal{L}, \tau_{p_n}] \subseteq \tau_{(p_1, \dots, p_n)}^n$.

Let us see that, despite the result above, the ideal $\tau_{(p_1, \dots, p_n)}^n$ is a proper ideal continuous multilinear operators and the inclusion above is strict:

1. The continuous bilinear operator $T : l_1 \times l_1 \longrightarrow l_1$ defined by $T((x_j)_{j=1}^{\infty}, (y_j)_{j=1}^{\infty}) = (x_j y_j)_{j=1}^{\infty}$, has no proper type (p_1, p_2) ;
2. If E and F are Banach spaces with no proper type, i.e. those for which id_E and id_F have only the trivial type $p = 1$, then the continuous bilinear operator $T : E \times F \longrightarrow E$ defined by $T(x, y) = \varphi(y)x$, where $0 \neq \varphi \in F'$, has a proper type (p_1, p_2) , with $\frac{1}{2} \leq \frac{1}{p_1}$ and $\frac{1}{p_1} + \frac{1}{p_2} < 1$, and T does not belong to $\tau_p \circ \mathcal{L}$ or $[\tau_{p_1}, \mathcal{L}]$.

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DIFERENCIAÇÃO DE TIPOS DE HOLOMORFIA

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Neste trabalho estudamos tipos de holomorfia entre espaços de Banach e, em especial, funções holomorfas que são associadas a um determinado tipo de holomorfia. O objetivo é provar que as derivadas de um tipo de holomorfia são ainda um tipo de holomorfia, e também que dada uma função holomorfa associada a um tipo de holomorfia, suas derivadas pertencem ao tipo de holomorfia correspondente.

1 Introdução

Introduzimos primeiramente a notação usual para espaços de polinômios homogêneos e funções holomorfas, que pode ser encontrada em [4, 3].

Definição 1.1. Sejam E e F espaços de Banach complexos e $m \in \mathbb{N}$.

(a) (Aplicação Multilinear Simétrica) Uma aplicação multilinear contínua $A: E^m \rightarrow F$ é dita *simétrica* se

$$A(x_1, \dots, x_m) = A(x_{\sigma(1)}, \dots, x_{\sigma(m)})$$

para todo $(x_1, \dots, x_m) \in E^m$ e toda $\sigma \in S_m$, onde S_m denota o conjunto das permutações do conjunto $\{1, \dots, m\}$. O conjunto das aplicações multilineares simétricas contínuas $A: E^m \rightarrow F$ será denotado por $\mathcal{L}^s(mE; F)$.

(b) (Polinômio m -Homogêneo) Uma aplicação $P: E \rightarrow F$ é denominado *polinômio m -homogêneo contínuo* se existe uma aplicação multilinear contínua $A: E^m \rightarrow F$ tal que $P(x) = Ax^m := A(x, \binom{m}{!}, x)$ para todo $x \in E$. O conjunto dos polinômios m -homogêneos contínuos de E em F será denotado por $\mathcal{P}(mE; F)$.

Dada $A \in \mathcal{L}^s(mE; F)$, denotamos por \widehat{A} o único polinômio m -homogêneo associado a A , que é definido por $\widehat{A}(x) = Ax^m$. Analogamente dado $P \in \mathcal{P}(mE; F)$, denotamos por \check{P} a única aplicação multilinear simétrica em $\mathcal{L}^s(mE; F)$ associada a P , definida por $\check{P}x^m = P(x)$ para todo $x \in E$.

(c) (Função Holomorfa) Seja U um subconjunto aberto de E . Uma função $f: U \rightarrow F$ é dita *holomorfa em U* se para cada $a \in U$ existem uma bola aberta $B(a, r) \subset U$ e uma sequência de polinômios $(P_m)_{m=0}^{\infty} \in \mathcal{P}(mE; F)$ tais que $f(x) = \sum_{m=0}^{\infty} P_m(x - a)$ uniformemente para $x \in B(a, r)$.

Denotamos por $\mathcal{H}(U; F)$ o espaço vetorial de todas as funções holomorfas de U em F . Quando $F = \mathbb{C}$, escrevemos simplesmente $\mathcal{H}(U)$. Denotaremos também $P_m = P^m f(a)$ para todos $m = 0, 1, \dots$, e $a \in E$. A série $\sum_{m=0}^{\infty} P_m(x - a)$ é chamada de *série de Taylor de f em a* .

Se $P_m \in \mathcal{P}(mE; F)$ é o polinômio correspondente a $A_m \in \mathcal{L}^s(mE; F)$ por $\widehat{P}_m = A_m$ para cada $m = 0, 1, \dots$, fixamos as seguinte notações:

$$d^m f(a) = m! A_m \quad \text{e} \quad \widehat{d}^m f(a) = m! P_m,$$

de modo a obter as aplicações diferenciais

$$d^m f : a \in U \rightarrow d^m f(a) \in \mathcal{L}^s(mE; F) \quad \text{e} \quad \widehat{d}^m f : a \in U \rightarrow \widehat{d}^m f(a) \in \mathcal{P}(mE; F).$$

O conceito de tipo de holomorfia foi introduzido por L. Nachbin no livro [4], e foi desenvolvido por vários autores (veja, por exemplo, [1, 2]).

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Definição 1.2. Sejam E, F espaços de Banach. Um *tipo de holomorfia* Θ de E em F é uma sequência de espaços de Banach $(\mathcal{P}_\Theta(^m E; F), \|\cdot\|_\Theta)_{m=0}^\infty$ para a qual são válidas as seguintes afirmações:

- (1) Cada $\mathcal{P}_\Theta(^m E; F)$ é um subespaço vetorial de $\mathcal{P}(^m E; F)$.
- (2) $\mathcal{P}_\Theta(^0 E; F)$ coincide com $\mathcal{P}(^0 E; F) = F$ como espaço vetorial normado.
- (3) Existe um número real $\sigma \geq 1$ tal que, dados $k \in \mathbb{N}_0$, $m \in \mathbb{N}_0$, $k \leq m$, $a \in E$ e $P \in \mathcal{P}_\Theta(^m E; F)$, vale que

$$\widehat{d}^k P(a) \in \mathcal{P}_\Theta(^k E; F) \text{ e } \left\| \frac{1}{k!} \widehat{d}^k P(a) \right\|_\Theta \leq \sigma^m \cdot \|P\|_\Theta \cdot \|a\|^{m-k}.$$

Nachbin também definiu o conceito de função holomorfa associada a um tipo de holomorfia, estudada por vários autores:

Definição 1.3. Seja $(\mathcal{P}_\Theta(^m E; F))_{m=0}^\infty$ um tipo de holomorfia entre os espaços de Banach complexos E e F . Uma função $f \in \mathcal{H}(U; F)$ é chamada de Θ -tipo de holomorfia em $\xi \in U$ se :

- (1) $\widehat{d}^m f(\xi) \in \mathcal{P}_\Theta(^m E; F)$ para todo $m \in \mathbb{N}$.
- (2) Existem números $C \geq 0$, $c \geq 0$ tais que $\left\| \frac{1}{m!} \widehat{d}^m f(\xi) \right\|_\Theta \leq C \cdot c^m$ para todo $m \in \mathbb{N}$.

Dizemos que f é de Θ -tipo de holomorfia em U se f é Θ -tipo de holomorfia em todo ponto de U . Denotaremos por $\mathcal{H}_\Theta(U; F)$ o subespaço vetorial de $\mathcal{H}(U; F)$ de todas as funções f Θ -tipo de holomorfia em U .

2 Resultados

O primeiro resultado estabelece que as derivadas de um tipo de holomorfia são ainda tipos de holomorfia:

Proposição 2.1. Sejam Θ um tipo de holomorfia de E em F e $l \in \mathbb{N}$. Então a sequência de espaços de Banach

$$\frac{1}{l!} \widehat{d}^l \mathcal{P}_\Theta(^{l+m} E; F) \quad (m \in \mathbb{N}_0),$$

é um tipo de holomorfia de E em $\mathcal{P}_\Theta(^l E; F)$ na norma τ , definida por

$$\left\| \frac{1}{l!} \widehat{d}^l P \right\|_\tau = \|P\|_\Theta.$$

Este tipo de holomorfia será denotado por $\frac{1}{l!} \widehat{d}^l \Theta$.

O segundo resultado diz respeito às derivadas de funções holomorfas associadas a um tipo de holomorfia:

Proposição 2.2. Sejam Θ um tipo de holomorfia de E em F e $\tau = \frac{1}{l!} \widehat{d}^l \Theta$ o correspondente tipo de holomorfia de E em $\mathcal{P}_\Theta(^l E; F)$ onde $l \in \mathbb{N}$. Se $f \in \mathcal{H}_\Theta(U; F)$, então

$$\widehat{d}^l f \in \mathcal{H}_\tau(U; \mathcal{P}_\Theta(^l E; F)).$$

Esses resultados são enunciados e demonstrados de forma muito compacta em [4]. O objetivo deste trabalho é detalhar e justificar as passagem das demonstrações.

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EQUAÇÕES DE SCHRÖDINGER QUASE LINEARES COM POTENCIAL PERIÓDICO E NÃO LINEARIDADE SUPERCRÍTICA

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1 Introdução

Recentemente vários matemáticos tem estudado equações do tipo

$$-\Delta u + W(x)u - k\Delta(u^2)u = p(x, u), \quad (1.1)$$

em \mathbb{R}^N , com $N \geq 3$, $W : \mathbb{R}^N \rightarrow \mathbb{R}$ uma função potencial e $p : \mathbb{R}^N \times \mathbb{R} \rightarrow \mathbb{R}$ função contínua.

As soluções de (1.1) estão relacionadas com a existência de ondas estacionárias para equações de Schrödinger quase lineares da forma

$$i\partial_t z = -\Delta z + W(x)z - f(|z^2|)z - k\Delta[g(|z^2|)]g'(|z^2|)z, \quad (1.2)$$

em que W é um potencial dado, k uma constante real, f, g são funções reais.

A fim de buscar solução para a equação (1.1) dois métodos variacionais vem sendo amplamente usados. O primeiro por meio de argumentos de minimização com vínculos, em [6] e estendidos em [4], os autores provaram a existência de soluções positivas usando um Multiplicador de Lagrange. No segundo método para contornar o problema de que o funcional associado a esta equação pode não estar bem definido, os autores em [5] introduziram uma mudança de variáveis, para transformar o problema quase linear em um semilinear.

2 Resultados

Iremos tratar da existência de solução, para a seguinte classe de problema:

$$-\Delta u - \Delta(u^2)u + V(x)u = p(u) \quad \text{em } \mathbb{R}^N, \quad N \geq 3, \quad (2.3)$$

onde V é uma função contínua que satisfaz as seguintes hipóteses:

(V_0) existe $\beta > 0$ tal que $V(x) \geq \beta > 0$, para todo $x \in \mathbb{R}^N$;

(V_1) $V(x) = V(x+y)$, $\forall x \in \mathbb{R}^N$, $y \in \mathbb{Z}^N$.

A função $p \in C(\mathbb{R}, \mathbb{R})$ pode ser escrita $p(s) = f_0(s) + \epsilon g(s)$, em que ϵ é um parâmetro real positivo, f_0 e g são funções localmente Hölder contínuas satisfazendo:

(F_1) $f_0(0) = g(0) = 0$ e $g(s) \geq 0$ para todo $s \neq 0$;

(F_2) $\lim_{|s| \rightarrow 0^+} \frac{f_0(s)}{s} = 0$ e $\lim_{|s| \rightarrow 0^+} \frac{g(s)}{s} = 0$;

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(F₃) Existe $q \in (4, 2(2^*))$ tal que $|f_0(s)| \leq C|s|^{q-1}$, para todo $s \in \mathbb{R}$;

(F₄) $\lim_{|s| \rightarrow \infty} \frac{F_0(s)}{s^4} = \infty$, onde $F_0(s) = \int_0^s f_0(t)dt$;

(F₅) Existe uma sequência de números reais positivos, (M_n) convergindo para $+\infty$ tal que

$$\frac{g(s)}{s^{q-1}} \leq \frac{g(M_n)}{M_n^{q-1}} \text{ para todo } s \in [0, M_n], \quad n \in \mathbb{N}.$$

(F₆) Para $\alpha > 0$ dado por (V₀) existem $l > 2$ e $\sigma \in (0, (\frac{l}{2} - 1)\alpha)$ tal que

$$\frac{1}{2}s f_0(s) - l F_0(s) \geq -\sigma s^2 \text{ e } \frac{1}{2}s g(s) - l G(s) \geq 0, \text{ para todo } s \neq 0,$$

onde $G(s) = \int_0^s g(t)dt$.

Para garantir a existência de solução positiva consideramos $p : \mathbb{R} \rightarrow \mathbb{R}$ satisfazendo (F₁) – (F₆) sobre $[0, +\infty)$ e definida como zero sobre $(-\infty, 0]$. Obtendo o seguinte resultado:

Teorema 2.1. *Suponhamos que V e p satisfazem (V₀), (V₁) e (F₁) – (F₆) respectivamente. Então existe $\epsilon_0 > 0$ tal que (2.3) tem uma solução positiva para todo $0 < \epsilon \leq \epsilon_0$.*

A ideia para provarmos o resultado acima é motivada pelos argumentos usados em [1] e [3]. Primeiro, usamos a mudança de variáveis e reduzimos nosso problema a encontrar solução para uma equação semilinear, lembrando que com isso perdemos a homogenidade do problema. Depois disso, provamos que o problema periódico envolvendo expoente subcrítico possui uma solução positiva. Para isso consideramos o funcional associado ao problema modificado e usamos uma versão do Teorema do Passo da Montanha, sem condição de compacidade (veja [7]), a fim de garantir a existência de uma sequência de Cerami limitada associada ao nível minimax. Em seguida, utilizamos esta sequência e um resultado técnico, devido a Lions (veja [2]), para obtermos um ponto crítico não trivial do funcional associado ao problema periódico modificado. Finalmente, construímos uma sequência de funções corte e modificamos a não linearidade para satisfazer o crescimento subcrítico, obtendo assim uma família de funcionais de classe C^1 . Utilizando um argumento de iteração de Moser, fornecemos uma estimativa envolvendo a norma L^∞ para a solução relacionada ao problema subcrítico.

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STATIONARY SOLUTIONS FOR THE KDV EQUATION POSED ON ARBITRARY INTERVALS

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1 Introduction

It is well-known [1] that the KdV equation

$$v_t + vv_x + v_{xxx} = 0 \quad (1.1)$$

possesses spatially periodic cnoidal-wave solutions which are determined to be stable to perturbation of the same period. They can be written explicitly as

$$v(x, t) = a + b \operatorname{cn}^2(d(x - ct); k) \quad (1.2)$$

in terms of the Jacobi elliptic function $\operatorname{cn}(x; k)$ where the elliptic modulus k and the parameters a , b , c and d are related by a system of nonlinear transcendental equations (see [9]).

Equation (1.1) has been deduced to describe long waves of a small amplitude propagating in a dispersive media that occupies all the spatial domain ($x \in \mathbb{R}$). Numerical needs, however, require to cut-off the infinite domains of wave propagation [2]. (One considers $x \in (0, L) \subset \mathbb{R}$, for instance). The correct equation in this case (see [2, 14]) should be written as

$$v_t + v_x + vv_x + v_{xxx} = 0. \quad (1.3)$$

Once bounded intervals are considered as a spatial region of waves propagation, their length $L > 0$ appears to be restricted by certain critical conditions. An important result in this context is the countable critical set (see *e.g.* [11])

$$\mathcal{N} = \frac{2\pi}{\sqrt{3}} \sqrt{k^2 + kl + l^2}; \quad k, l \in \mathbb{N}. \quad (1.4)$$

While studying the controllability and stabilization of solutions for (1.3), the set \mathcal{N} provides qualitative difficulties when the length of a spatial interval coincides with some of its elements. In fact, the function $v(x) = 1 - \cos x$ is a stationary (not decaying) solution for linearized (1.3) posed on $(0, 2\pi)$, and $2\pi \in \mathcal{N}$. However, if the transport term v_x is neglected, then (1.3) becomes (1.1), and the exponential decay rate of small solutions for (1.1) posed on any bounded interval is known to be held [7]. For (1.3) the same result has been shown if $L \notin \mathcal{N}$ (see [10]). The following questions arise:

- Are there solutions of (1.3) which do not decay for $L \in \mathcal{N}$? If so is, what is a “nonlinear analog” of \mathcal{N} ?

Despite the valuable advances in [4, 5, 6], the question whether solutions of undamped problems associated to nonlinear KdV decay as $t \rightarrow \infty$, for all finite interval lengths, is open (up to our knowledge).

In the present note we construct explicitly the stationary solutions to homogeneous IBVP for nonlinear KdV (1.3) posed on a bounded interval $(0, L) \subset \mathbb{R}$ with some (critical) values of $L > 0$. These solutions clearly do not decay in time and can be viewed as nontrivial periodic solutions of (1.3) with spatial period L that are different from (1.2), as well as, from example of [8].

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2 Main results

We provide the existence of explicit stationary solutions of (1.3) of the form $v(x, t) = \phi(x)$ related to the following initial boundary value problem posed in $[0, L] \times [0, \infty)$:

$$v_t + v_x + vv_x + v_{xxx} = 0, \quad v(0, t) = v(L, t) = 0, \quad v_x(L, t) = 0, \quad v(x, 0) = \phi(x). \quad (2.5)$$

Theorem 2.1. *For all $L \in (0, 2\pi)$, there exists a stationary solution $\phi \in C^\infty(\mathbb{R})$ to (2.5) satisfying, moreover,*

$$\phi' + \frac{1}{2}(\phi^2)' + \phi''' = 0, \quad \phi(x+L) = \phi(x), \quad \forall x \in \mathbb{R}, \quad \text{and} \quad \phi(0) = \phi'(0) = 0, \quad \phi''(0) \neq 0.$$

Proof. The solution is given by

$$\phi(x) = \left[\frac{3k^2(1-k^2)}{1-2k^2} \right] \operatorname{sd}^2 \left(\frac{x}{2\sqrt{1-2k^2}}; k \right),$$

where the elliptic function sd and the modulus k are defined in [3]. \square

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CONCENTRATED TERMS AND VARYING DOMAINS IN ELLIPTIC EQUATIONS - PART 1: UNIFORM LIPSCHITZ DEFORMATIONS

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1 Introduction

In this work, we analyze the convergence of the solutions of a concentrated elliptic equation with nonlinear boundary conditions of the type

$$\begin{cases} -\Delta u_\epsilon + u_\epsilon = \frac{1}{\epsilon} \mathcal{X}_{\omega_\epsilon} f(x, u_\epsilon), & \text{in } \Omega_\epsilon \\ \frac{\partial u_\epsilon}{\partial n} + g(x, u_\epsilon) = 0, & \text{on } \partial\Omega_\epsilon \end{cases} \quad (1.1)$$

when the boundary of the domain presents a highly oscillatory behavior, as the parameter $\epsilon \rightarrow 0$, and nonlinear terms are concentrated in a region of the domain neighboring the boundary. We consider a family of uniformly bounded smooth domains $\Omega_\epsilon \subset \mathbb{R}^N$, with $N \geq 2$ and $0 \leq \epsilon \leq \epsilon_0$, for some $\epsilon_0 > 0$ fixed, which satisfy both $\Omega_\epsilon \rightarrow \Omega \equiv \Omega_0$ and $\partial\Omega_\epsilon \rightarrow \partial\Omega$ in the sense of Hausdorff, that is, $\text{dist}(\Omega_\epsilon, \Omega) + \text{dist}(\partial\Omega_\epsilon, \partial\Omega) \rightarrow 0$ as $\epsilon \rightarrow 0$, where dist is the symmetric Hausdorff distance of two sets in \mathbb{R}^N . We will assume that $\Omega \subset \Omega_\epsilon$ and we will look at this problem from the perturbation of the domain point of view and we will refer to Ω as the unperturbed domain and Ω_ϵ as the perturbed domains. Now, for sufficiently small ϵ , ω_ϵ is the region between the boundaries of $\partial\Omega$ and $\partial\Omega_\epsilon$. Note that ω_ϵ shrinks to $\partial\Omega$ as $\epsilon \rightarrow 0$. Figure 1 illustrates the oscillating set $\omega_\epsilon \subset \overline{\Omega}_\epsilon$.

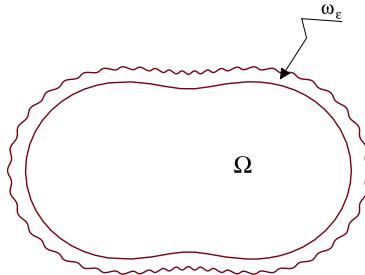


Figure 1: The set ω_ϵ .

We use the characteristic function $\mathcal{X}_{\omega_\epsilon}$ of the region ω_ϵ to express the concentration in ω_ϵ . We also assume that the nonlinearities $f, g : U \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous in both variables and C^2 in the second one, where U is a fixed and smooth bounded domain containing all $\overline{\Omega}_\epsilon$, for all $0 \leq \epsilon \leq \epsilon_0$.

Although the domains behave continuously as $\epsilon \rightarrow 0$, the way in which the boundary $\partial\Omega_\epsilon$ approach $\partial\Omega$ may not be smooth. We consider the case where the boundary $\partial\Omega_\epsilon$ presents an oscillatory behavior in which, up to a diffeomorphism, the period goes to zero in the same order as the amplitude. In this case, the measure of the deformation of $\partial\Omega_\epsilon$ with respect to $\partial\Omega$ is uniformly bounded for $\epsilon > 0$.

The boundary condition in the limit problem inherits the information about the behavior of the measure of the deformation of $\partial\Omega_\epsilon$ with respect to $\partial\Omega$ as $\epsilon \rightarrow 0$. Moreover, since ω_ϵ shrinks to $\partial\Omega$ as $\epsilon \rightarrow 0$, the family of solutions

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$\{u_\epsilon\}$ of (1.1) will converge to a solution of an equation with a nonlinear boundary condition on $\partial\Omega$ that also inherits the information about the concentration. We show that under certain conditions, the limiting equation of (1.1) is given by

$$\begin{cases} -\Delta u + u = 0, & \text{in } \Omega \\ \frac{\partial u}{\partial n} + \gamma(x)g(x, u) = \beta(x)f(x, u), & \text{on } \partial\Omega \end{cases} \quad (1.2)$$

where the function $\gamma \in L^\infty(\partial\Omega)$ is related to the behavior of the measure of the deformation of $\partial\Omega_\epsilon$ with respect to $\partial\Omega$ and $\beta \in L^\infty(\partial\Omega)$ is related to the average of the profile oscillatory boundary $\partial\Omega_\epsilon$.

The behavior of the solutions of nonlinear elliptic equations with nonlinear boundary conditions and rapidly varying boundaries was studied in [3], for the case of uniformly Lipschitz deformation of the boundary, but without concentration.

The behavior of the solutions of elliptic problems with terms concentrated in a neighborhood of the boundary of the domain was initially studied in [4], when the neighborhood is a strip of width ϵ and has a base in the boundary, without oscillatory behavior and inside of Ω .

Recently, in [1] some results of [4] were adapted to a nonlinear elliptic problem posed on an open square Ω in \mathbb{R}^2 , considering $\omega_\epsilon \subset \Omega$ and with highly oscillatory behavior in the boundary inside of Ω . The dynamics of the flow generated by a nonlinear parabolic problem posed on a C^2 domain Ω in \mathbb{R}^2 , when some reaction and potential terms are concentrated in a neighborhood of the boundary and the “inner boundary” of this neighborhood presents a highly oscillatory behavior, was studied in [2] where the continuity of the family of attractors was proved.

It is important to note that all previous works with terms concentrating in a neighborhood of the boundary deal with non varying domain since ω_ϵ is inside of Ω then all the equations are defined in the same domain. In our case, the region ω_ϵ is outside of Ω . We also generalize the domain that exist in the literature allowing $\Omega, \Omega_\epsilon \subset \mathbb{R}^N$ and Ω is a C^1 domain. Finally, we found a different limit boundary condition than the ones in [1, 2, 4].

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SOME PROPERTIES OF THE BEST CONSTANT IN SOBOLEV INEQUALITIES

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1 Introduction

Let Ω a bounded domain of \mathbb{R}^N , $N > 1$, $p > 1$ and

$$\lambda_q := \min \left\{ \frac{\|\nabla u\|_{L^p(\Omega)}^p}{\|u\|_{L^q(\Omega)}^p} : 0 \not\equiv u \in W_0^{1,p}(\Omega) \right\}$$

where

$$1 \leq q < p^* := \begin{cases} \frac{Np}{N-p} & \text{if } 1 < p < N \\ \infty & \text{if } p \geq N. \end{cases}$$

We present some recent results on the regularity of the function $q \in [1, p^*) \mapsto \lambda_q$ and also on its asymptotic behavior, as $q \rightarrow p^*$.

Moreover, we describe some known properties of λ_q related to its characterization as the first eigenvalue of the following eigenvalue problem

$$\begin{cases} -\operatorname{div}(|\nabla u|^{p-2} \nabla u) = \lambda \|u\|_{L^q(\Omega)}^{p-q} |u|^{q-2} u & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

and present a new result on the sign-definiteness of eigenfunctions in the case $p < q < p^*$.

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ON THE CHEEGER CONSTANT OF AN ANNULUS

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1 Introduction

Let $p > 1$ and Ω be a bounded domain of \mathbb{R}^N , $N > 1$. The *Cheeger constant* $h(\Omega)$ of Ω is defined by

$$h(\Omega) := \min_{E \subset \bar{\Omega}} \frac{|\partial E|}{|E|},$$

where $|\partial E|$ and $|E|$ denote, respectively, the $(N-1)$ -dimensional Lebesgue perimeter of ∂E in \mathbb{R}^N and the N -dimensional Lebesgue volume of E , the quotients being evaluated among all smooth subsets $E \subset \bar{\Omega}$. A subset E of $\bar{\Omega}$ is a *Cheeger set of Ω* if $h(\Omega) = \frac{|\partial E|}{|E|}$. When Ω is a Cheeger set of itself, one says that Ω is *calibrable*. Main contributions to the study of the Cheeger problem were made in a paper by Kawohl and Fridman [4].

It is easy to verify that a ball in \mathbb{R}^N is calibrable. Researchers of the area also know that the same happens with an N -dimensional annulus, but the only known proof that the N -dimensional annulus is calibrable was indirectly given by Demengel in [2], where approximation techniques were used to study the 1-Laplacian operator

$$\Delta_1 u := \operatorname{div}(|\nabla u|^{-1} \nabla u)$$

which is defined in $BV(\Omega)$, the space of bounded variation functions defined in Ω . As a byproduct of her work, one can infer that the annulus is calibrable; however, no connection with the Cheeger problem is explicitly stated in [2]. (In [5], Kawohl cites a numerical result to inform that the annulus are calibrable.)

In [1] we have proved that

$$\lim_{p \rightarrow 1^+} \frac{1}{\|u_p\|_\infty^{p-1}} = h(\Omega) = \lim_{p \rightarrow 1^+} \frac{1}{\|u_p\|_1^{p-1}},$$

where u_p is the solution of the p -torsional creep problem

$$\begin{cases} -\Delta_p u = 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

and $\Delta_p u := \operatorname{div}(|\nabla u|^{p-1} \nabla u)$ stands for the p -Laplacian operator. (Of course, $\|\cdot\|_1$ and $\|\cdot\|_\infty$ stand for the norms in $L^1(\Omega)$ and $L^\infty(\Omega)$, respectively.) The p -torsional creep problems were studied by Kawohl in [3].

2 Mathematical Results

In this paper we give a simple proof that the annulus $\Omega_{a,b} = \{x \in \mathbb{R}^N : a < |x| < b\}$ is calibrable by showing that

$$\lim_{p \rightarrow 1^+} \frac{1}{\|u_p\|_\infty^{p-1}} = \frac{|\partial\Omega_{a,b}|}{|\Omega_{a,b}|}.$$

Additionally, a second method of proof was given: we prove that, for all $\epsilon > 0$,

$$\lim_{p \rightarrow 1^+} \frac{u_p}{\|u_p\|_\infty} = 1 \quad \text{uniformly in } (a + \epsilon, b - \epsilon).$$

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By studying the behavior of u'_p , we also show that

$$\lim_{p \rightarrow 1^+} \frac{1}{\|u'_p\|_\infty} = h(\Omega_{a,b}).$$

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ANÁLISE DE UM SISTEMA PARABÓLICO SEMI-LINEAR COM NÃO-LINEARIDADE NÃO-LOCAL

ISIS GABRIELLA QUINTEIRO * & MIGUEL LOAYZA †

Estudamos o sistema parabólico não-local acoplado

$$u_t - \Delta u = \int_0^t (t-s)^{-\gamma_1} |v|^{p-1} v(s) ds, \quad v_t - \Delta v = \int_0^t (t-s)^{-\gamma_2} |u|^{q-1} u(s) ds$$

onde $0 \leq \gamma_1, \gamma_2 < 1$ e $p, q \geq 1$. Consideramos o problema em $(0, T) \times R^N$ e admitimos que os dados iniciais $u(0), v(0) \in C_0(R^N)$. Encontramos condições que garantem a existência de solução global e a explosão num tempo finito de qualquer solução do sistema em questão.

1 Introdução

Neste trabalho, consideramos o seguinte sistema parabólico semi-linear com uma não-linearidade não-local no tempo

$$\begin{cases} u_t - \Delta u = \int_0^t (t-s)^{-\gamma_1} |v|^{p-1} v(s) ds \text{ em } (0, T) \times R^N, \\ v_t - \Delta v = \int_0^t (t-s)^{-\gamma_2} |u|^{q-1} u(s) ds \text{ em } (0, T) \times R^N, \\ u = v = 0 \text{ em } (0, T) \times R^N, \\ u(0) = u_0, v(0) = v_0 \text{ em } R^N, \end{cases} \quad (1)$$

com $p, q \geq 1$, $0 \leq \gamma_1, \gamma_2 < 1$ e dados iniciais $u_0, v_0 \in C_0(R^N)$.

Estudamos a existência de soluções para o sistema (1) e, posteriormente, apresentamos condições que garantem a existência de solução global para o sistema (1).

O sistema (1) é equivalente, num sentido apropriado, ao sistema

$$\begin{cases} u(t) = S(t)u_0 + \int_0^t \int_0^s (s-\sigma)^{-\gamma_1} S(t-s) |v|^{p-1} v(\sigma) d\sigma ds, \\ v(t) = S(t)v_0 + \int_0^t \int_0^s (s-\sigma)^{-\gamma_2} S(t-s) |u|^{q-1} u(\sigma) d\sigma ds \end{cases} \quad (2)$$

para todo $t \in [0, T]$, onde $\{S(t)\}_{t \geq 0}$ é o semi-grupo do calor em R^N .

Nosso primeiro resultado trata da existência e unicidade de soluções para o sistema (2).

Teorema 1.1. *Considere $p, q \geq 1$, $\gamma_1, \gamma_2 \in [0, 1)$ e $u_0, v_0 \in C_0(R^N)$. Existe uma única solução $(u, v) \in \{C([0, T_{\max}), C_0(R^N))\}^2$ de (1) tal que*

1. $T_{\max} = \infty$ (a solução é global) ou

2. $T_{\max} < \infty$ e $\lim_{t \rightarrow T_{\max}} (\|u(t)\|_{\infty} + \|v(t)\|_{\infty}) = \infty$ (a solução explode num tempo finito).

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Além disso, se $(u_0, v_0) \neq (0, 0)$, $u_0, v_0 \geq 0$, então $u(t), v(t) \geq 0$ para todo $t \in (0, T_{\max})$. Se $0 < t \leq t + \tau < T_{\max}$, temos

$$u(t + \tau) \geq S(\tau)u(t), \quad v(t + \tau) \geq S(\tau)v(t). \quad (1.1)$$

Mais ainda, se $(u_0, v_0) \in L^{r_1}(R^N) \times L^{r_2}(R^N)$ com $1 \leq r_2 \leq pr_1$ e $1 \leq r_1 \leq qr_2$, então $(u, v) \in C([0, T_{\max}), L^{r_1}(R^N)) \times C([0, T_{\max}), L^{r_2}(R^N))$ e

$$\lim_{t \rightarrow T_{\max}} (\|u(t)\|_{\infty} + \|v(t)\|_{\infty} + \|u(t)\|_{r_1} + \|v(t)\|_{r_2}) = \infty, \quad (1.2)$$

quando $T_{\max} < \infty$.

2 Resultados sobre existência de solução global e blow-up

Apresentamos condições que determinam a explosão das soluções do sistema (1).

Teorema 2.1. Sejam $p, q \geq 1$, $pq > 1$, $0 \leq \gamma_1, \gamma_2 < 1$ e $u_0, v_0 \in C_0(R^N)$. Considere $(u, v) \in \{C([0, T_{\max}), C_0(R^N))\}^2$ a solução correspondente de (1). Suponha que

$$\begin{cases} 1 - p\gamma_2 + p(1 - q\gamma_1) + p(q + 1) \geq \frac{N}{2}(pq - 1) \\ \text{ou} \\ 1 - q\gamma_1 + q(1 - p\gamma_2) + q(p + 1) \geq \frac{N}{2}(pq - 1) \end{cases}$$

ou

$$\begin{cases} 1 - p\gamma_2 + p(1 - q\gamma_1) \geq 0 \\ \text{ou} \\ 1 - q\gamma_1 + q(1 - p\gamma_2) \geq 0 \end{cases}$$

Se $(u_0, v_0) \neq (0, 0)$ com $u_0, v_0 \geq 0$, então (u, v) explode num tempo finito.

Sobre a existência de solução global para o sistema (1), temos o seguinte resultado.

Teorema 2.2. Sejam $p, q \geq 1$, $pq > 1$, $0 \leq \gamma_1, \gamma_2 < 1$ e $u_0, v_0 \in C_0(R^N)$. Considere $(u, v) \in \{C([0, T_{\max}), C_0(R^N))\}^2$ a solução correspondente de (1). Suponha que as seguintes condições sejam válidas

$$\begin{cases} 1 - p\gamma_2 + p(1 - q\gamma_1) + p(q + 1) < \frac{N}{2}(pq - 1), \\ 1 - q\gamma_1 + q(1 - p\gamma_2) + q(p + 1) < \frac{N}{2}(pq - 1), \end{cases}$$

$$\begin{cases} 1 - p\gamma_2 + p(1 - q\gamma_1) < 0, \\ 1 - q\gamma_1 + q(1 - p\gamma_2) < 0 \end{cases}$$

e

$$\frac{p}{r_2} - \frac{4}{N} < \frac{1}{q}, \quad \frac{q}{r_1} - \frac{4}{N} < \frac{1}{p}.$$

Se $(u_0, v_0) \in L^{r_1}(R^N) \times L^{r_2}(R^N)$, onde r_1, r_2 são definidos por (8) e $(\|u_0\|_{\infty} + \|v_0\|_{\infty} + \|u_0\|_{r_1} + \|v_0\|_{r_2}) \leq \epsilon$ com $\epsilon > 0$ suficientemente pequeno, então (u, v) existe globalmente.

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ON $p_s(x)$ -LAPLACIAN PARABOLIC PROBLEMS WITH NON GLOBALLY LIPSCHITZ FORCING TERM*

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In the last seven years various researchers have spent efforts to obtain results on existence, uniqueness, blow-up, vanishing, local boundedness and localization of solutions for parabolic problems with variable exponents. However, until the moment few works have been appeared in the literature about global attractors. The theory of problems with variable exponents has application in electrorheological fluids (fluids characterized by the ability to drastically change the mechanical properties under the influence of exterior electromagnetic field), image processing and the models of porous medium equations with variable exponents of nonlinearity also was considered. The reader will find references on the subjects covered in this paragraph in [7], also we refer the reader to [5] for an overview of differential equations with variable exponents.

In [1], G. Akagi and K. Matsuura studied the limiting behavior of solutions for nonlinear diffusion equations driven by the $p(x)$ -Laplacian as $p(\cdot)$ diverges to the infinity.

In [4] P. Harjulehto, P. Hästö and M. Koskenoja considered Dirichlet energy integral minimizers in variable exponent Sobolev spaces. In the paper [2], B. Amaziane, L. Pankratov and V. Prytula studied homogenization of $p_\epsilon(x)$ -Laplacian elliptic equations and in [3], B. Amaziane, L. Pankratov and A. Piatnitski studied nonlinear flow through double porosity media in variable exponent Sobolev spaces, and the authors considered the following initial boundary value problem

$$\begin{cases} \omega^\epsilon(x) \frac{\partial u^\epsilon}{\partial t}(t) - \operatorname{div}(k^\epsilon(x) \nabla u^\epsilon |\nabla u^\epsilon|^{p_\epsilon(x)-2}) = g(t, x) & \text{in } Q \\ u^\epsilon = 0 \quad \text{on }]0, t[\times \partial\Omega, \\ u^\epsilon(0, x) = u_0(x) \quad \text{in } \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ ($n = 2, 3$) is a bounded domain, Q denotes the cylinder $]0, T[\times \Omega$, $T > 0$ is given, $g \in C([0, T]; L^2(\Omega))$ and $u_0 \in H^2(\Omega)$ are given functions. They studied the minimization problem for functionals in the limit of small ϵ and obtained the homogenized functional.

In [6] we considered the following one dimensional nonlinear PDE problem

$$\begin{cases} \frac{\partial u_s}{\partial t}(t) - \frac{\partial}{\partial x} \left(|\frac{\partial u_s}{\partial x}(t)|^{p_s(x)-2} \frac{\partial u_s}{\partial x}(t) \right) = B(u_s(t)), & t > 0 \\ u_s(0) = u_{0s}, \end{cases} \quad (0.1)$$

under Dirichlet homogeneous boundary conditions, where $u_{0s} \in H := L^2(I)$, $I := (c, d)$, $B : H \rightarrow H$ is a globally Lipschitz map with Lipschitz constant $L \geq 0$, $p_s(x) \in C^1(\bar{I})$, $p_s^- := \inf_{x \in I} p_s(x) > 2 \forall s \in \mathbb{N}$, and $p_s(\cdot) \rightarrow p$ in $L^\infty(I)$ ($p > 2$ constant) as $s \rightarrow \infty$ and proved the continuity of the flows and the upper semicontinuity of the family of global attractors $\{\mathcal{A}_s\}_{s \in \mathbb{N}}$ as s goes to infinity.

Let us consider the following nonlinear PDE problem

$$\begin{cases} \frac{\partial u_s}{\partial t}(t) - \operatorname{div}(|\nabla u_s(t)|^{p_s(x)-2} \nabla u_s(t)) + f(x, u_s(t)) = g, & t > 0 \\ u_s(0) = u_{0s}, \end{cases} \quad (0.2)$$

under Dirichlet homogeneous boundary conditions, where $u_{0s} \in H := L^2(\Omega)$, Ω is a bounded smooth domain in \mathbb{R}^n , $n \geq 1$, $g \in L^2(\Omega)$, $p_s(x) \in C^1(\bar{\Omega}) \forall s \in \mathbb{N}$, $2 < p \leq p_s(x) \leq a$, for all $x \in \Omega$ and for all $s \in \mathbb{N}$. and $p_s(\cdot) \rightarrow p$ in

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$L^\infty(\Omega)$ (p constant) as $s \rightarrow \infty$. We assume that $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ is a non globally Lipschitz Carathéodory mapping satisfying the following conditions: there exist positive constants ℓ, k, c_1 and $c_2 \geq 1$ such that

$$(f(x, s_1) - f(x, s_2))(s_1 - s_2) \geq -\ell|s_1 - s_2|^2, \quad \forall x \in \Omega \text{ and } s_1, s_2 \in \mathbb{R}, \quad (0.3)$$

$$c_2|s|^{q(x)} - k \leq f(x, s)s \leq c_1|s|^{q(x)} + k, \quad \forall x \in \Omega \text{ and } s \in \mathbb{R}, \quad (0.4)$$

where $q \in C(\overline{\Omega})$ with $2 < q^- := \inf_{x \in \Omega} q(x) \leq q^+ := \sup_{x \in \Omega} q(x)$. For example, if $\alpha_1 > 1$ and $r > 2$, we observe that the function $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x, u) = \alpha_1|u|^{r-2}u - u$ is not globally Lipschitz and satisfies the condition (0.3) with $\ell = 1$ and the condition (0.4) with $c_2 = 1$, $c_1 = \alpha_1$ and $q(x) = r$ for all $x \in I$.

In this work we investigate in what way the parameter $p_s(x)$ affects the dynamic of problem (0.2), analyzing the continuity properties of the flows and the global attractors with respect to the parameter $p_s(x)$, when f is a locally Lipschitz function. In what follows we state the main results of this work, whose proofs and more details can be found in [7].

Theorem 0.1. *For each $s \in \mathbb{N}$ let u_s be a solution of (0.2) with $u_s(0) = u_{0s}$. Suppose that there exists $C > 0$, independent of s , such that $\|u_{0s}\|_{X_s} \leq C$ for every $s \in \mathbb{N}$ and $u_{0s} \rightarrow u_0$ in H as $s \rightarrow \infty$. Then, for each $T > 0$, $u_s \rightarrow u$ in $C([0, T]; H)$ as $s \rightarrow \infty$, where u is a solution of*

$$\begin{cases} \frac{\partial u}{\partial t}(t) - \operatorname{div}(|\nabla u|^{p-2}\nabla u) + f(x, u) = g, & t > 0 \\ u(0) = u_0 \in H. \end{cases}$$

Theorem 0.2. *The family of global attractors $\{\mathcal{A}_s; s \in \mathbb{N}\}$ associated with problem (0.2) is upper semicontinuous on s at infinity, in the topology of H .*

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EQUAÇÕES DE EVOLUÇÃO ESTOCÁSTICAS COM RUIDO FRACIONÁRIO EM ESPAÇOS DE BANACH

JAMIL ABREU *

Neste trabalho desenvolvemos uma teoria de integração estocástica em espaços de Banach em relação a um ruído fracionário, com o intuito de aplicá-la na investigação de questões ligadas xistência, unicidade e regularidade de soluções para equações de evolução estocásticas (EEE) dirigidas por um ruído fracionário.

1 Introdução

Equações diferenciais parciais estocásticas constituem um importante campo de pesquisa da análise moderna, permitindo a modelagem de diversos fenômenos naturais nos quais os efeitos decorrentes de um ruído não são desprezíveis. A abordagem via semigrupos desenvolvida por Da Prato e Zabczyk ao longo dos anos 1980 consiste em reformular EDP's estocásticas como equações de evolução num espaço de estado de dimensão infinita e depende de se ter disponível uma boa teoria de integração estocástica. O estado da arte em torno de 1990 (no contexto Hilbertiano) se encontra bem documentada na monografia Da Prato & Zabczyk [1].

Por outro lado, desde os trabalhos de Mandelbrot e Van Ness [2] o movimento Browniano fracionário tem sido empregado na modelagem de diversos fenômenos naturais e da engenharia. Aplicações em telecomunicações, finanças, climatologia e tráfego de informações já são bem conhecidas. Sendo vários destes fenômenos modelados através de EDPs, isto motivou estudos em equações diferenciais parciais estocásticas dirigidas por ruído fracionário. Seguindo a abordagem de Da Prato e Zabczyk [1] de reformular EDP's estocásticas como equações de evolução num espaço de estado de dimensão infinita, alguns autores desenvolveram integrais estocásticas em relação a ruídos fracionários, notadamente em espaços de Hilbert (veja por exemplo Duncan et al [3] e referências lá encontradas). Uma teoria que permita integrar estocasticamente uma equação de evolução estocástica em seu espaço de estado natural, digamos num espaço $L^p(\mathcal{O})$, onde $\mathcal{O} \subset \mathbb{R}^d$ é um aberto e $p > 1$, pode ser naturalmente combinada com técnicas da teoria de interpolação fornecendo, via mergulhos de Sobolev, regularidade tipo Hölder no tempo e no espaço.

Seguindo a linha de desenvolvimentos iniciada em [4], e usando a abordagem via desacoplamento de [5, 6], desenvolvemos uma teoria de integração estocástica para processos $\Phi : (0, T) \times \Omega \rightarrow \mathcal{L}(H, X)$ em relação a um movimento Browniano cilíndrico fracionário de Liouville ($H\text{-LfBm}$), e a aplicamos na investigação de questões de existência, unicidade e regularidade de soluções de EEEs em espaços de Banach dirigidas por ruido fracionário, similares uelas tratadas em [7] no contexto Hilbertiano. Uma motivação concreta é a EDP estocástica

$$\frac{\partial u}{\partial t}(t, x) = \mathcal{A}u(t, x) + b(u(t, x)) \frac{\partial W^\beta(t, x)}{\partial t \partial x} \quad (1.1)$$

onde \mathcal{A} é um operador uniformemente elíptico de segunda ordem num aberto $\mathcal{O} \subset \mathbb{R}^d$, b é Lipschitz limitado e $\partial W^\beta(t, x)/\partial t \partial x$ é um ruído branco no espaço e Liouville fracionário no tempo. Queremos, em especial, investigar até que ponto os resultados nos trabalhos acima sobre existência de solução mild para $\frac{d}{4} < \beta < 1$ podem ser estendidos a este contexto.

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2 Resultados

Nosso primeiro resultado (cf. [8]) é uma extensão de resultados em [5, 6] referentes a EEEs dirigidas por um movimento Browniano usual.

Teorema 2.1. *Seja E um espaço de Banach com a propriedade UMD e seja $\Phi : (0, T) \times \Omega \rightarrow \mathcal{L}(H, E)$ um processo que pertence a $L^p(\Omega; H_{T-}^{1/2-\beta}(0, T; H))$ escalarmente. Então as seguintes afirmações são equivalentes:*

- (1) Φ representa um elemento $X \in L^p(\Omega; \gamma(H_{T-}^{1/2-\beta}(0, T; H), E))$;
- (2) Para quase todo $\omega \in \Omega$ a função $\Phi(\cdot, \omega)$ é estocasticamente integrável em relação a um H -LfBm independente $\widetilde{W}_H^{\beta'}$ e $\omega \mapsto \int_0^T \Phi(t, \omega) d\widetilde{W}_H^{\beta'}(t)$ pertence a $L^p(\Omega; L^p(\widetilde{\Omega}; E))$. Nesta situação temos a isometria

$$\mathbb{E} \left\| \int_0^T \Phi(t, \omega) d\widetilde{W}_H^{\beta'}(t) \right\|_{L^p(\widetilde{\Omega}; E)}^p \asymp \mathbb{E} \|X\|_{\gamma(H_{T-}^{1/2-\beta}(0, T; H), X)}^p. \quad (2.2)$$

O seguinte resultado de existência e unicidade estende resultados clássicos em [6]:

Teorema 2.2. *Seja E um espaço de Banach com a propriedade UMD e tipo $p \in (1, 2]$ e suponha que A seja o gerador de um semigrupo C_0 em E tal que $\{t^\theta S(t) : t \in [0, T]\}$ seja γ -limitado. Seja W_H^β um H -LfBm com $\frac{1}{p} < \beta < 1$ e suponha que $B : E \rightarrow \gamma(H, E)$ seja γ -Lipschitz e $u_0 \in L^q(\Omega, \mathcal{F}_0; E)$. Então*

$$dU(t) = AU(t) dt + B(U(t)) dW_H^\beta(t), \quad U(0) = 0.$$

tem uma única solução mild tal que

$$\|U\|_{V_\theta^q} \leq C_T(1 + \|u_0\|).$$

Discutiremos no final como este resultado pode ser empregado na análise de soluções de EDPs da forma (1.1).

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MÓDULO DE CONTINUIDADE PRECISO PARA SOLUÇÕES DE EQUAÇÕES PARABÓLICAS TOTALMENTE NÃO-LINEARES

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A proposta deste trabalho consiste em estudar soluções no sentido da viscosidade de equações parabólicas totalmente não-lineares da seguinte forma

$$u_t - F(D^2u, X, t) = f(X, t) \quad \text{em } Q_1 = B_1 \times (-1, 0] \quad (0.1)$$

onde F é um operador uniformemente parabólico e $f \in L^{p,q}(Q_1)$, com estes sendo definidos mais precisamente *a posteriori*. Tendo em vista tais condições estruturais e supondo que os coeficientes da não-linearidade F oscilem de forma controlada somos aptos a assegurar que soluções limitadas de (0.1) gozam de módulo de continuidade universal, o qual podemos descrever explicitamente. Vale ressaltar que tal determinismo está sujeito a duas peças pivotais: a primeira consiste na estimativa *a priori* adequada para cada cenário a ser analizado, a saber os regimes de regularidade C^α , $C^{1+\alpha}$ e $C^{2+\alpha}$ para a correspondente equação homogênea de coeficientes constantes, e, a segunda consiste na integrabilidade da fonte f em termos dos expoentes p e q , cuja relação está ligada a quantidade $\kappa(n, p, q) = \frac{n}{p} + \frac{2}{q}$. Finalmente, é importante enfatizarmos que este trabalho estende o recente artigo devido a Teixeira [4].

1 Introdução

Permita-nos focar nossa atenção em algumas definições importantes para os nossos propósitos.

Definição 1.1 (Parabolicidade uniforme). *Um operador $F : Sym(n) \times Q_1 \rightarrow \mathbb{R}$, normalizado como $F(0, X, t) = 0$, será dito (λ, Λ) uniformemente parabólico se existem constantes $\Lambda \geq \lambda > 0$ tais que para todo $(X, t) \in Q_1$ e $M, N \in Sym(n)$ com $N \geq 0$ tivermos*

$$\lambda \|M - N\| \leq F(M, X, t) - F(N, X, t) \leq \Lambda \|M - N\| \quad (1.2)$$

Definição 1.2 (Solução no sentido da viscosidade). *Para um operador como descrito acima diremos que uma função $u \in C^0(Q_1)$ é uma supersolução (resp. subsol.) no sentido da viscosidade para (0.1), se sempre que tocarmos o gráfico de u por baixo (resp. por cima) em um ponto $(Y, s) \in Q_1$ por uma função $\varphi \in C^2(Q_1)$, tivermos $F(D^2\varphi(Y, s), Y, s) \leq f(Y, s)$ (resp. $F(D^2\varphi(Y, s), Y, s) \geq f(Y, s)$). Finalmente, u é uma solução no sentido da viscosidade para (0.1) se a mesma é simultaneamente uma supersolução e subsolução.*

Em concordância com [5], veja também [1], mediremos a oscilação dos coeficientes do operador F em torno de (X_0, t_0) por

$$\Theta_F(X, t, X_0, t_0) = \sup_{M \in Sym(n) \setminus \{0\}} \frac{|F(M, X, t) - F(M, X_0, t_0)|}{\|M\|}. \quad (1.3)$$

Devemos destacar que o termo de força f em (0.1) se encontra no espaço de Lebesgue com norma mista $L^{p,q}(Q_1) = L^q((-1, 0]; L^p(B_1))$. A respectiva norma associada é dada por

$$\|f\|_{L^{p,q}(Q_1)} = \left(\int_{-1}^0 \left(\int_{B_1} |f(X, t)|^p dX \right)^{\frac{q}{p}} dt \right)^{\frac{1}{q}} = \|\|f(\cdot, t)\|_{L^p(B_1)}\|_{L^q((-1, 0])} \quad (1.4)$$

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2 Resultados principais

Uma vez definidos e conhecidos os entes da seção precedente poderemos enunciar os resultados principais contidos em nosso trabalho em forma de um Teorema geral.

Teorema 2.1. *Seja $u \in C^0(Q_1)$ uma solução no sentido da viscosidade para (0.1) e limitade. Então existe uma constante universal $\theta_0 > 0$ tal que se $\sup_{(X,t) \in Q_{1/2}} \|\Theta(X,t, X_0, t_0)\|_{L^{n+1}(Q_1)} \leq \theta_0$ então na métrica parabólica usual:*

- $u \in C^{0,\gamma}(Q_{1/2})$, onde $\gamma := \min \left\{ \alpha_0^-, 2 - \left(\frac{n}{p} + \frac{2}{q} \right) \right\}$, desde que $1 < \frac{n}{p} + \frac{2}{q} \leq \frac{n+2}{n+1-\varepsilon} < 2$, com $\varepsilon \in (0, \frac{n}{2})$ a constante de Escauriaza, veja [2], e, $0 < \alpha_0 < 1$ o expoente de Hölder continuidade proveniente da desigualdade de Harnack de Krylov-Safonov, veja [3];
- $u \in C^{0,\omega(r)}(Q_{1/2})$, onde $\omega(r) := r \log r^{-1}$, desde que $\frac{n}{p} + \frac{2}{q} = 1$, comparar com [4];
- $u \in C^{1,\sigma}(Q_{1/2})$, onde $\sigma := \min \left\{ \alpha_1^-, 1 - \left(\frac{n}{p} + \frac{2}{q} \right) \right\}$, desde que $0 < \frac{n}{p} + \frac{2}{q} < 1$, com $0 < \alpha_1 < 1$ o expoente de Hölder continuidade proveniente da estimativa a priori $C^{1+\alpha}$ para a equação homogênea de coeficientes constantes, veja [5];
- $u \in C^{1,\tau(r)}(Q_{1/2})$, onde $\tau(r) := r^2 \log r^{-1}$, desde que $f \in BMO(Q_1)$, tenhamos estimativas a priori $C^{2+\alpha}$ para o problema homogêneo com coeficientes constantes e $\Theta_F \in C^{0,\nu}(Q_1)$, comparar com [4] e consultar [5].

A intuição para a prova do Teorema 2.1 consiste em um sofisticado método de compacidade cuja centelha nasceu de [1], veja também [4] e [5]. Nós interpretaremos a equação homogênea de coeficientes constantes como a equação tangencial geométrica da variedade limite formada por operadores parabólicos totalmente não-lineares F_k cuja oscilação vai para zero quando $k \rightarrow \infty$. Sistematicamente, mostaremos que em determinada escala é possível encontrar uma função F -calórica¹, próxima na topologia L^∞ , de soluções de (0.1), desde que o termo de força f e a oscilação da não-linearidade Θ_F sejam universalmente² pequenos. Iterando um tal argumento em cubos ρ -ádicos mostra-se que o gráfico de uma solução de (0.1) pode ser aproximado em um ρ -nível por uma apropriada função polinomial cujo erro é da ordem de $O(\mu(\rho))$, onde $\mu : [0, \infty) \rightarrow [0, \infty)$ é o módulo de continuidade que gerará a regularidade almejada em cada respectivo caso acima tratado. Destacamos que a pré-compacidade de família de soluções de (0.1) (por exemplo desigualdade de Harnack, veja [2]) é essencial para tal propósito.

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¹Solução para a equação homogênea de coeficientes constantes.

²Um parâmetro é dito Universal se o mesmo depende somente das constantes de parabolicidade e dimensão.

ON A CLASS OF BI-ORTHOGONAL POLYNOMIALS ON THE UNIT CIRCLE

J. BORREGO-MORELL * & FERNANDO RODRIGO RAFAELI †

1 Introduction

In this work we are interested in asymptotic properties of a system of bi-orthogonal polynomials introduced by R. Askey [1, Vol. 1] in his discussions regarding the Szegő paper: *Beiträge zur Theorie der Toeplitzschen Formen*, 1921–1. More precisely, our concern is the two-parameter system $\{P_n, Q_n\}_{n \geq 0}$ of polynomials given by

$$\begin{aligned} P_n(z; \alpha, \beta) &= {}_2F_1(-n, \alpha + \beta + 1; 2\alpha + 1; 1 - z), \\ Q_n(z; \alpha, \beta) &= P_n(z; \alpha, -\beta), \end{aligned} \quad (1.1)$$

which is bi-orthogonal with respect to the complex valued weight $\omega(\theta) = (1 - e^{i\theta})^{\alpha+\beta}(1 - e^{-i\theta})^{\alpha-\beta} = (2 - 2\cos\theta)^{\alpha}(-e^{i\theta})^{\beta}, \theta \in [-\pi, \pi], \Re(\alpha) > -\frac{1}{2}$, that is

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} P_n(e^{i\theta}; \alpha, \beta) Q_m(e^{-i\theta}; \alpha, \beta) \omega(\theta) d\theta = \frac{\Gamma(2\alpha + 1)}{\Gamma(\alpha + \beta + 1)\Gamma(\alpha - \beta + 1)} \frac{n!}{(2\alpha + 1)_n} \delta_{n,m}.$$

In [2] we give a new uniform asymptotic expansion for the bi-orthogonal system (1.1) consisting of a sum of two inverse factorial series, for z and (α, β) varying in compact subsets of $\mathbb{C} \setminus \{1\}$ and $\Omega_0 = \{(\alpha, \beta) \in \mathbb{C}^2 : \Re(\alpha + \beta) > -1, \Re(\alpha - \beta) > 0\}$ respectively. We give the explicit expression of all the terms and bounds for the remainders as well. We also give a different solution from the one given by Temme in [4] for the explicit expression of the terms of an asymptotic formula given by Askey for this bi-orthogonal system. We also consider bounds for the remainder for our expansion, which turns out to be convergent.

We show also that the zeros of a class of para-orthogonal polynomials, introduced by Sri Ranga in [3], associated to the bi-orthogonal system (1.1) also obey an electrostatic model.

2 Mathematical Results

Our first result deals with a compound asymptotic expansion of Poincaré type involving two series of inverse factorials, with an accurate estimation of the remainders, which is given in the following

Theorem 2.1. *Assume that $n \in \mathbb{N} \cup \{0\}$, then*

$$\begin{aligned} P_n(z; \alpha, \beta) &= \frac{\Gamma(2\alpha + 1)}{\Gamma(\alpha + \beta + 1)} \frac{\Gamma(n + 1)}{\Gamma(n + \alpha - \beta + 1)} z^{n+\alpha-\beta} (z - 1)^{\beta-\alpha} \\ &\quad \times \left(\sum_{k=0}^{p_1} \binom{\alpha + \beta}{k} \left(\frac{z}{1-z} \right)^k \frac{\Gamma(k + \alpha - \beta)}{\Gamma(\alpha - \beta)} \frac{1}{(n + 1 + \alpha - \beta)_k} + \xi_{1,p_1} \right) \\ &\quad + \frac{\Gamma(2\alpha + 1)}{\Gamma(\alpha - \beta)} \frac{\Gamma(n + 1)}{\Gamma(n + \alpha + \beta + 2)} (1 - z)^{-\alpha-\beta-1} \\ &\quad \times \left(\sum_{k=0}^{p_2} \binom{\alpha - \beta - 1}{k} \frac{1}{(z - 1)^k} \frac{\Gamma(k + \alpha + \beta + 1)}{\Gamma(\alpha + \beta + 1)} \frac{1}{(n + 2 + \alpha + \beta)_k} + \xi_{2,p_2} \right), \end{aligned}$$

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uniformly in z and (α, β) varying in compact subsets of $\mathbb{C} \setminus \{1\}$ and Ω_0 respectively, where,

$$|\xi_{1,p_1}| \leq \frac{1}{(n+1+\Re(\alpha-\beta))_{p_1+1}} \frac{\Gamma(p_1 + \Re(\alpha-\beta))}{|\Gamma(\alpha-\beta)|} \left| \frac{z}{1-z} \right|^{p_1+1} \times \\ \begin{cases} m_1 \left(\frac{z}{1-z}; \alpha + \beta \right) + p_1 e^{|\alpha+\beta|^2 + \Re(\alpha+\beta)} + 1, & 0 \leq \Re(\alpha+\beta), \\ m_2(z; p_1 - 2\beta, \alpha + \beta) \frac{\Gamma(p_1 + 1 - 2\Re(\beta))}{\Gamma(p_1 + 1 + \Re(\alpha-\beta))} \frac{\Gamma(n + p_1 + 2 + \Re(\alpha-\beta))}{\Gamma(n + p_1 + 2 - 2\Re(\beta))} + \\ m_3(p_1, \alpha + \beta), & -1 < \Re(\alpha+\beta) < 0, \end{cases}$$

$$|\xi_{2,p_2}| < \frac{1}{(n+2+\Re(\alpha+\beta))_{p_2+1}} \frac{\Gamma(p_2 + 2 + \Re(\alpha+\beta))}{|\Gamma(\alpha+\beta+1)|} \frac{1}{|z-1|^{p_2+1}} \times \\ \begin{cases} m_1 \left(\frac{1}{z-1}; \alpha - \beta - 1 \right) + p_2 e^{|\alpha-\beta-1|^2 + \Re(\alpha-\beta)-1} + 1, & 1 \leq \Re(\alpha-\beta), \\ m_2(z^{-1}; p_2 + 2\beta + 2, \alpha - \beta - 1) \frac{\Gamma(p_2 + 2\Re(\beta) + 3)}{\Gamma(p_2 + \Re(\alpha+\beta) + 2)} \times \\ \frac{\Gamma(n + p_2 + 3 + \Re(\alpha+\beta))}{\Gamma(n + p_2 + 2\Re(\beta) + 4)} + m_3(p_2, \alpha - \beta + 1), & 0 < \Re(\alpha-\beta) < 1, \end{cases}$$

where m_1, m_2, m_3 are constants depending only of p_1, p_2, α, β ; see please [2].

For Askey's Problem, we prove that

Theorem 2.2. Assume that $(\alpha, \beta) \in \Omega_0$, then

$$P_n \left(e^{\frac{i\theta}{n}}; \alpha, \beta \right) = {}_1F_1(\alpha + \beta + 1; 2\alpha + 1; i\theta) + \sum_{j=1}^k \sum_{i_1+i_2+i_3=j} \frac{B_{i_1}^{(-\alpha-\beta)}(\alpha-\beta)}{i_1!} \frac{B_{i_2}^{(-\alpha+\beta+1)}(0)}{i_2!} \frac{B_{i_3}^{(2\alpha)}(0)}{i_3!} \times \\ \frac{(\alpha + \beta + 1)_{i_1} (\alpha - \beta)_{i_2}}{(2\alpha + 1)_{i_1+i_2}} {}_1F_1(\alpha + \beta + i_1; 2\alpha + 1 + i_2; i\theta) \left(\frac{i\theta}{n} \right)^j + R_{k,n}(\theta), \quad \theta \in [-\pi, \pi], \quad n \in \mathbb{N} \cup \{0\},$$

$$\text{where } |R_{k,n}(\theta)| \leq \frac{\Gamma(\Re(\alpha+\beta+1)\Gamma(\Re(\alpha-\beta))}{|\Gamma(\alpha+\beta+1)\Gamma(\alpha-\beta)|} \left(\frac{2\theta}{3n\pi - 2\theta} \right) \left(\frac{2\theta}{3n\pi} \right)^k \max_{|v|=\frac{3\pi}{2}} \left| \frac{e^{v(\alpha-\beta)} v}{e^v - 1} \right|.$$

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STANDING WAVES FOR A HAMILTONIAN SYSTEM OF SCHRÖDINGER EQUATIONS WITH CRITICAL GROWTH

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1 Introduction

The work focuses on the study of the existence of standing waves for the following system of time-dependent nonlinear Schrödinger equations

$$\begin{cases} i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \psi + W(x)\psi - F_\varphi(x, \psi, \varphi), & t \geq 0, \quad x \in \mathbb{R}^N, \\ i\hbar \frac{\partial \varphi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \varphi + W(x)\varphi - F_\psi(x, \psi, \varphi), & t \geq 0, \quad x \in \mathbb{R}^N, \\ \psi(x, t), \quad \varphi(x, t) \in \mathbb{C}, \end{cases} \quad (1.1)$$

where i denotes the imaginary unit, \hbar is the Plank constant, m is the particle's mass, $W(x)$ is a continuous potential, and the function $F : \mathbb{R}^N \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is of class C^1 . For the physical motivation of problems of this type, we refer to [2]. Suppose that

$$F_{u_j}(x, e^{i\theta}u_1, e^{i\theta}u_2) = e^{i\theta}F_{u_j}(x, u_1, u_2), \quad \forall u_j, \theta \in \mathbb{R}.$$

For system (1.1), a solution of the form

$$(\psi(x, t), \varphi(x, t)) = (u(x)e^{-iE/\hbar t}, v(x)e^{-iE/\hbar t}), \quad E \in \mathbb{R},$$

is called a *standing wave*. For this case, (ψ, φ) is a solution of (1.1) if and only if (u, v) solves the following system

$$\begin{cases} -\varepsilon^2 \Delta u + V(x)u = F_v(x, u, v) & \text{in } \mathbb{R}^N, \\ -\varepsilon^2 \Delta v + V(x)v = F_u(x, u, v) & \text{in } \mathbb{R}^N, \end{cases} \quad (1.2)$$

where $V(x) = W(x) - E \geq 0$ and $\varepsilon = \hbar/\sqrt{m}$ is small parameter.

In this work we are interested in the study of system (1.2) when $F(u, v)$ has critical growth, more precisely, $F(s, t) = (1/2^*)(|s|^{2^*} + |t|^{2^*})$, $s, t \in \mathbb{R}$, where $2^* = 2N/(N-2)$, $N \geq 3$, is the critical Sobolev exponent. Nonlinear elliptic problems involving critical growth have been considered by several authors since the seminal work of H. Brezis and L. Nirenberg [1], mainly when the domain is bounded. We mention in particular the work due to J. Hulshof, E. Mitidieri and R. Van der Vorst [4], where the authors studied a class of Hamiltonian system defined in a bounded domain $\Omega \subset \mathbb{R}^N$ and the nonlinearity have critical growth.

Our study complement the papers cited above in the sense that we are working with Hamiltonian systems involving critical growth in whole \mathbb{R}^N . One difficulty in the study of Hamiltonian system is the fact that the energy functional is strongly indefinite, in the sense that its leading in part is coercive and anti-coercive on infinite-dimensional subspace of the Sobolev space appropriated to study this class of problems. Moreover, another difficulty in dealing with system (1.2) is that possible lack of compactness since the equations in (1.2) are defined in the whole \mathbb{R}^N and the nonlinearities are in the critical growth range.

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2 Mathematical Results

In this work we consider system (1.2) when the potential $V : \mathbb{R}^N \rightarrow \mathbb{R}$ is a nonnegative locally Hölder continuous function satisfying

(V_0) The set $\mathcal{Z} = \{x \in \mathbb{R}^N : V(x) = 0\}$ has nonempty interior.

(V_1) There is $A > 0$ such that the level set $G_A = \{x \in \mathbb{R}^N : V(x) < A\}$ has finite Lesbegue measure.

We observe that (1.2) is in the variational form. In fact, we consider the subspace of $H^1(\mathbb{R}^N)$

$$H_V^1(\mathbb{R}^N) = \left\{ u \in H^1(\mathbb{R}^N) : \int_{\mathbb{R}^N} V(x)u^2 dx < \infty \right\},$$

which is a Hilbert space when endowed with the inner product

$$\langle u, v \rangle_\varepsilon = \int_{\mathbb{R}^N} [\varepsilon^2 \nabla u \nabla v + V(x)uv] dx, \quad u, v \in H_V^1(\mathbb{R}^N).$$

Note that, under assumptions (V_0) – (V_1), we have the continuous embedding

$$H_V^1(\mathbb{R}^N) \hookrightarrow H^1(\mathbb{R}^N) \hookrightarrow L^r(\mathbb{R}^N), \quad (2.3)$$

for all $2 \leq r \leq 2^*$ (see [5] for more details). Since we are interested in positive weak solution we will consider the functional $I_\varepsilon : H_V^1(\mathbb{R}^N) \times H_V^1(\mathbb{R}^N) \rightarrow \mathbb{R}$ defined by

$$I_\varepsilon(u, v) = \langle u, v \rangle_\varepsilon - \frac{1}{2^*} \int_{\mathbb{R}^N} (u^+)^{2^*} dx - \frac{1}{2^*} \int_{\mathbb{R}^N} (v^+)^{2^*} dx, \quad (2.4)$$

where $u^+ = \max\{u, 0\}$ (similarly v^+).

Here we are interested in finding a *ground state solution* of (1.2), that is, a solution $(u, v) \in H_V^1(\mathbb{R}^N) \times H_V^1(\mathbb{R}^N)$, with u, v positive functions, whose energy is minimal among the energy of all nontrivial solutions of (1.2) in $H_V^1(\mathbb{R}^N) \times H_V^1(\mathbb{R}^N)$.

Theorem 2.1. *Suppose that V satisfies (V_0) – (V_1) . Then:*

- i) *there exist $\varepsilon_0 > 0$ such that (1.2) has a ground state solution $(u_\varepsilon, v_\varepsilon) \in H_V^1(\mathbb{R}^N) \times H_V^1(\mathbb{R}^N)$, for all $\varepsilon \in (0, \varepsilon_0]$. Moreover, $u_\varepsilon, v_\varepsilon \in C_{loc}^{2,\alpha}(\mathbb{R}^N)$;*
- ii) *given $k \in \mathbb{N}$, there exists $\varepsilon_1 = \varepsilon_1(k) > 0$ such that (1.2) possesses k pairs of nontrivial solutions for all $\varepsilon \in (0, \varepsilon_1]$.*

See [3] for more details.

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CONTROLE ÓTIMO DO PROBLEMA DE INTERAÇÃO DE CÉLULAS TUMORAIS DO CÉREBRO E AGENTES CITOTÓXICOS

JOSÉ A. DÁVALOS *

1 Introdução

Estudamos o controle ótimo de um sistema parabólico acoplado semi-linear, o modelo descreve os efectos da terapia de um tumor no cérebro (glioblastoma). A densidade de células tumorais verifica uma equação diferencial parcial semi-linear parabólica acoplada com uma equação semelhante para um agente citotóxico. Consideramos $\Omega \subset \mathbb{R}^3$ aberto conexo limitado com fronteira regular $\Gamma = \Gamma_D \cup \Gamma_N$, ocupada pelo cérebro e $Q = \Omega \times (0, T)$, $T > 0$ domínio cilíndrico com fronteira lateral $\Sigma = \Sigma_D \cup \Sigma_N$ onde $\Sigma_D = \Gamma_D \times (0, T)$, $\Sigma_N = \Gamma_N \times (0, T)$. Como equação de estado é considerado o seguinte problema que descreve a evolução em Q de um tumor cerebral

$$(S) \begin{cases} c_t - \nabla \cdot (D(x) \nabla c) = a_1 c - b_1 c \beta - v 1_{\omega_1} & \text{em } Q, \\ \beta_t - \mu \Delta \beta = a_2 \beta - b_2 c \beta + w 1_{\omega_2} & \text{em } Q, \\ c(x, t) = \beta(x, t) = 0 & \text{sobre } \Sigma_D, \\ \frac{\partial c}{\partial \eta} = 0, \quad \frac{\partial \beta}{\partial \eta} = 0 & \text{sobre } \Sigma_N, \\ c(0) = c_0, \quad \beta(0) = \beta_0 & \text{em } \Omega, \end{cases}$$

onde $c(x, t)$, e $\beta(x, t)$ são respectivamente as concentrações de células tumorais e de agentes citotóxicos (anticorpos) gerados pelo corpo, 1_{ω_i} representa a função característica do conjunto aberto $\omega_i \subset \Omega$. $D(x)$, μ são os coeficientes de difusão de células tumorais e agentes citotóxicos respectivamente; a_1, a_2, b_1 e b_2 são constantes positivas. $c_0, \beta_0 \in V \cap L^\infty(\Omega)$, $c_0, \beta_0 \geq 0$ onde $V = H_{0, \Gamma_D}^1(\Omega) = \{z \in H^1(\Omega) \mid z = 0 \text{ sobre } \Gamma_D\}$. São obtidos resultados do seguinte problema de controle ótimo

$$\min_{\{\mathbf{v}, \mathbf{z}\} \in U_{ad} \times \mathbf{L}^2(0, T; V)} J(\mathbf{v}, \mathbf{z}), \quad (1.1)$$

onde $\mathbf{v} = (v, w) \in U_{ad} = \{v \in L^\infty(\omega_1 \times (0, T)) \mid 0 \leq v \leq K\} \times \{w \in L^\infty(\omega_2 \times (0, T)) \mid 0 \leq w \leq M\}$ e $\mathbf{z} = (c, \beta)$ é a solução do problema (S) .

$$J(\mathbf{v}, \mathbf{z}) = \frac{1}{2} \int_{\Omega} |c(x, T)|^2 dx + \frac{N}{2} \left[\int \int_{\omega_1 \times (0, T)} v(t)^2 dx dt + \int \int_{\omega_2 \times (0, T)} w(t)^2 dx dt \right],$$

Em concordância com [1], [4] e [5] consideramos o caso em que

$$D(x) = \begin{cases} D_\omega & \text{se } x \in \Omega_\omega, \\ D_g & \text{se } x \in \Omega_g, \end{cases}$$

onde $0 < D_\omega < D_g$; Ω_ω, Ω_g representam respectivamente as áreas do cérebro ocupadas pela matéria branca e cinza. Definimos $\Lambda : D(\Lambda) \subset L^2(\Omega) \rightarrow L^2(\Omega)$ o operador de difusão com domínio

$$D(\Lambda) = \{z \in V \mid \nabla \cdot (D(x) \nabla z) \in L^2(\Omega)\}, \quad \Lambda z = \nabla \cdot (D(x) \nabla z) \quad \forall z \in D(\Lambda).$$

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2 Resultados

Cada função v, w descreve uma terapia a ser aplicada ao longo do intervalo de tempo $(0, T)$. Os resultados esperados através desta terapia é que w determine um correto aumento do anticorpo, com o objetivo de fazer decrescer os valores de c através do termo $b_1 c \beta$. Assim mesmo, o fornecimento do agente citotóxico v na primeira equação de (S) permitirá também diminuir a taxa de crescimento da célula tumoral. Temos o seguinte resultado, cuja prova usa argumentos bem conhecidos

Teorema 2.1. *Se $c_0, \beta_0 \in V \cap L^\infty(\Omega)$ então o sistema de equações (S) tem uma única solução $\mathbf{z} = (c, \beta)$ satisfazendo*

$$c \in L^2(0, T; D(\Lambda)) \cap L^\infty(\Omega \times (0, T)), \quad c_t \in L^2(\Omega \times (0, T)), \quad (2.2)$$

$$\beta \in L^2(0, T; V \cap H^2(\Omega)) \cap L^\infty(\Omega \times (0, T)), \quad \beta_t \in L^2(\Omega \times (0, T)) \quad (2.3)$$

Teorema 2.2. *Existe ao menos uma solução do problema (??).*

Teorema 2.3. *Se (v^*, w^*, c^*, β^*) é solução do problema de controle (??). Então existe $(d, \eta) \in L^2(0, T; D(\Lambda)) \times L^2(0, T; V \cap H^2(\Omega))$ verificando.*

$$\left\{ \begin{array}{rcl} -d_t - \nabla \cdot (D(x) \nabla d) & = & a_1 d - b_1 \beta^* d - b_2 \beta^* \eta \quad \text{em } Q, \\ -\eta_t - \mu \Delta \eta & = & a_2 \eta - b_2 c^* \eta - b_1 c^* d \quad \text{em } Q, \\ d(x, t) = \eta(x, t) & = & 0 \quad \text{sobre } \Sigma_D, \\ \frac{\partial c}{\partial \eta} = 0, \quad \frac{\partial \beta}{\partial \eta} & = & 0 \quad \text{sobre } \Sigma_N, \\ d(T) = c^*(T), \quad \eta(T) & = & 0 \quad \text{em } \Omega, \end{array} \right.$$

e a relação de otimalidade

$$v^* = P_1 \left(-\frac{d}{N} \right) \quad \text{e} \quad w^* = P_2 \left(-\frac{\eta}{N} \right),$$

onde

$$P_1 : L^2(\Omega \times (0, T)) \mapsto \{v \in L^\infty(\omega_1 \times (0, T)) \mid 0 \leq v \leq K\}$$

$$P_2 : L^2(\Omega \times (0, T)) \mapsto \{w \in L^\infty(\omega_2 \times (0, T)) \mid 0 \leq w \leq M\}$$

são as projeções ortogonais.

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EXISTÊNCIA E NÃO EXISTÊNCIA DE SOLUÇÃO GLOBAL PARA A EQUAÇÃO VISCOELÁSTICA DA ONDA NÃO LINEAR DE SEXTA ORDEM

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Trataremos aqui de estudar a existência e unicidade de solução fraca de um problema de Cauchy para um modelo viscoelástico da equação da onda não linear de sexta ordem. A seguir apresentaremos, através do método do poço potencial (*potential well*), um estudo acerca da existência e não existência de solução fraca global para tal problema de Cauchy.

1 Introdução

Vamos considerar o seguinte problema de Cauchy para a equação viscoelástica da onda não linear de sexta ordem

$$u_{tt} - au_{xx} + \int_0^t g(t-s)au_{xx}(s) ds + u_{xxxx} + u_{xxxxtt} = \varphi(u_x)_x \quad (1.1)$$

$$u(x, 0) = u_0, \quad u_t(x, 0) = u_1(x) \quad (1.2)$$

onde $a > 0$, $\varphi(z) = \alpha|z|^p$, $p > 1$, $\alpha \neq 0$ e a função de relaxação $g : [0, \infty) \rightarrow \mathbb{R}$ satifaz $g(t) \geq 0$, $g' \leq 0$, $0 < 1 - \int_0^\infty g(s) ds = l$. Além disso, consideramos os dados iniciais $u_0, u_1 \in H^s(\mathbb{R})$, $s > 0$.

Quando $g = 0$ a equação (1.1), foi introduzida por Roseneu em [1], e estudada, na forma do problema de Cauchy (1.1) e (1.2), em [2, 3].

Em nosso trabalho estudaremos, em primeiro lugar, a existência e unicidade de solução fraca local para o problema de Cauchy (1.1) e (1.2). Para este fim seguiremos as seguintes etapas: na primeira etapa usando o método de Faedo-Galerkin provaremos a existência de solução fraca local para a equação viscoelástica da onda linear:

$$u_{tt} - au_{xx} + \int_0^t g(t-s)au_{xx}(s) ds + u_{xxxx} + u_{xxxxtt} = f(x, t) \quad (1.3)$$

$$u(x, 0) = u_0, \quad u_t(x, 0) = u_1(x) \quad (1.4)$$

onde $f \in L^1(0, T; H^{s-2}(\mathbb{R}))$. Na segunda etapa, usando o Teorema do ponto fixo de Banach, com o auxilio do problema (1.3) e (1.4), mostraremos a existência de solução fraca local para o problema (1.1) e (1.2). Posteriormente, fazendo uso do método do poço potencial, abordaremos o problema de existência e não existência de solução fraca global.

Ao longo deste trabalho, usaremos as seguintes notações: H^s denota o espaço de sobolev de ordem s sobre \mathbb{R} com norma $\|f\|_{H^s} = \|(I - \partial_x^2)^{\frac{s}{2}} \hat{f}\| = \|(1 + \xi^2)^{\frac{s}{2}} \hat{f}\|$, onde s é um número real, I é um operador unitário e $\partial_x = \frac{\partial}{\partial x}$ denota a derivada com respeito a x .

2 Resultados principais

Teorema 2.1. *Suponha que $\frac{3}{2} < s < p + 1$, $u_0, u_1 \in H^s(\mathbb{R})$. Deste modo, o problema de Cauchy (1.1),(1.2) admite uma única solução local fraca $u(x, t)$, definida sobre um intervalo de tempo maximal $[0, T_0)$ com $u \in$*

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$C^1([0, T_0); H^s(\mathbb{R}))$. Além disso, se

$$\sup_{t \in [0, T_0)} (\|u(t)\|_{H^s(\mathbb{R})} + \|u_t(t)\|_{H^s(\mathbb{R})}) < \infty$$

então, $T_0 = \infty$.

Supondo $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$ e $[0, T_0)$ o intervalo de tempo maximal de existência da solução do problema de Cauchy (1.1) e (1.2) vamos considerar, para cada $t \in [0, T_0)$, o seguinte funcional de energia associado a este problema de Cauchy,

$$\begin{aligned} E(t) &= \frac{1}{2} \left(\|u_t(t)\|_{L^2(\mathbb{R})}^2 + a(1 - \int_0^t g(s) ds) \|u_x(t)\|_{L^2(\mathbb{R})}^2 + a(g \diamond u)(t) + \|u_{xx}(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xxt}(t)\|_{L^2(\mathbb{R})}^2 \right) \\ &+ \frac{\alpha}{p+1} \int_{\mathbb{R}} |u_x(t)|^p u_x(t) dx. \end{aligned}$$

onde,

$$(g \diamond u)(t) = \int_0^t g(t-s) \|u_x(s) - u_x(t)\|_{L^2(\mathbb{R})}^2 ds.$$

Denotando por,

$$C_0 = \sup_{0 \neq u \in H^2(\mathbb{R})} \frac{\|u_x(t)\|_{L^{p+1}}}{(al\|u_x(t)\|_{L^2}^2 + \|u_{xx}(t)\|_{L^2}^2)^{\frac{1}{2}}}$$

e

$$d = \frac{p-1}{2(p+1)} |\alpha|^{-\frac{2}{p-1}} C_0^{-\frac{2(p+1)}{p-1}}$$

podemos definir os conjuntos estável (poço potencial) e instável como,

$$\begin{aligned} W &= \left\{ u(t) \in H^2(\mathbb{R}); al\|u_x(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xx}(t)\|_{L^2(\mathbb{R})}^2 < \frac{2(p+1)}{p-1} d \right\} \\ V &= \left\{ u(t) \in H^2(\mathbb{R}); al\|u_x(t)\|_{L^2(\mathbb{R})}^2 + \|u_{xx}(t)\|_{L^2(\mathbb{R})}^2 > \frac{2(p+1)}{p-1} d \right\}. \end{aligned}$$

Teorema 2.2. Assuma que $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$. Se $E(0) < d$ e $u_0 = u(0) \in W$ então o problema de Cauchy (1.1) e (1.2) tem uma única solução global $u \in C^1([0, \infty); H^s(\mathbb{R}))$ e $u(t) \in W$ para todo $t \in [0, \infty)$

Teorema 2.3. Suponha que $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$. Se $E(0) \leq d$ e $al\|u_{0x}\|_{L^2(\mathbb{R})}^2 + \|u_{0xx}\|_{L^2(\mathbb{R})}^2 \leq \frac{2(p+1)}{p-1} d$, então o problema de Cauchy (1.1) e (1.2) admite uma única solução fraca global $u \in C^1([0, \infty); H^s(\mathbb{R}))$.

Teorema 2.4. Assuma que $2 \leq s < p+1$, $u_0, u_1 \in H^s(\mathbb{R})$. Se $E(0) \leq d$, $u_0 = u(0) \in V$ e $(u_0, u_1)_{L^2(\mathbb{R})} + (u_{0xx}, u_{1xx}) \geq 0$, quando $E(0) = d$, então a solução fraca do problema de Cauchy (1.1) e (1.2) explode em tempo finito.

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CONTROLABILIDADE NULA PARA EQUAÇÕES PARABÓLICAS DEGENERADAS COM TERMOS NÃO-LOCAIS

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1 Introdução

A teoria de controlabilidade exata para equações de evolução parabólica é uma das áreas mais desenvolvidas de controle ótimo. A história da teoria começa nos anos 60 com os trabalhos de Yu. Ergorov, D. Russel and H. Fattorini que consideraram o caso de equações lineares parabólicas e hiperbólicas unidimensionais. Durante os últimos anos avanços fundamentais foram feitos nesta área, veja [1],[2] e [3].

Neste trabalho estudamos a controlabilidade nula da equação do calor unidimensional

$$u_t - \left(b \left(x, \int_0^1 u \right) u_x \right)_x + f(t, x, u) = h\chi_\omega(x), \quad (t, x) \in (0, T) \times (0, 1), \quad (1)$$

onde b apresenta um termo não-local e uma degeneração em 0.

Diversos problemas relevantes são descritos por equações com termos degenerados ou não-locais como por exemplo no estudo de migração de populações de bactérias em um recipiente, sistemas de reação-difusão e na teoria de vibrações não lineares, veja por exemplo [4], [5] e [6].

Entretanto, poucos trabalhos foram feitos considerando equações com termos degenerados e não locais, veja [7].

2 Resultados

Neste trabalho estudamos a controlabilidade nula do seguinte problema não-linear

$$\begin{cases} u_t - \left(b \left(x, \int_0^1 u \right) u_x \right)_x + f(t, x, u) = h\chi_\omega, & (t, x) \in (0, T) \times (0, 1) \\ u(t, 1) = u(t, 0) = 0, & \forall t \in (0, T) \\ u(0, x) = u_0(x), & \forall x \in (0, 1), \end{cases} \quad (2.1)$$

onde $u_0 \in L^2(0, 1)$ e $h \in L^2((0, T) \times (0, 1))$ é um controle atuando na subintervalo $\omega = (\alpha, \beta) \subset \subset (0, 1)$. Além disso, consideramos as seguintes hipóteses sobre f e b :

A.1. Seja $g \in C^1(\mathbb{R})$, com derivada limitada, e suponhamos que $g(0) = 1$. Consideremos também $a \in C([0, 1]) \cap C^1((0, 1])$ satisfazendo $a(0) = 0$, $a > 0$ em $(0, 1]$ e

$$xa' \leq Ka, \quad \forall x \in [0, 1] \text{ e algum } K \in (0, 1].$$

Com estas notações, a função $b : [0, 1] \times \mathbb{R} \rightarrow \mathbb{R}$ é definição por

$$b(x, r) = g(r)a(x).$$

A.2. Seja $f := f(t, x, r)$ de classe C^1 com derivadas limitadas tal que $f(t, x, 0) = 0$.

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Por fim definimos o seguinte espaço de Sobolev com peso.

$$H_a^1 := \{u \in L^2(0, 1); \text{ } u \text{ é absolutamente contínua em } [0, 1], \sqrt{a}u_x \in L^2(0, 1) \text{ e } u(1) = u(0) = 0\}$$

com a norma $\|u\|_{H_a^1}^2 := \|u\|_{L^2(0,1)}^2 + \|\sqrt{a}u_x\|_{L^2(0,1)}^2$.

Sob essas condições obtivemos o seguinte resultado.

Teorema 2.1. *Admitindo as hipóteses A.1 e A.2, o sistema não-linear (2.1) é localmente nulamente controlável no tempo $T > 0$. Em outras palavras, existe $\varepsilon > 0$ tal que, se $u_0 \in H_a^1$ e $\|u_0\|_{H_a^1} \leq \varepsilon$, então existe um controle $h \in L^2((0, T) \times \omega)$ associado a uma solução u de (2.1) que satisfaz*

$$u(T, x) = 0, \text{ para todo } x \in [0, 1]$$

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ON THE CONVERGENCE RATE OF THE GRADE-TWO COMPLEX FLUID TO THE NAVIER-STOKES EQUATIONS

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Abstract

The equations for the flow of a viscoelastic, grade-two, non-Newtonian complex fluid can be written as

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t} - \nu \Delta \mathbf{u} - \alpha \frac{\partial}{\partial t} \Delta \mathbf{u} + \nabla \times (\mathbf{u} - \alpha \Delta \mathbf{u}) \times \mathbf{u} + \nabla p = \mathbf{f} & \text{in } \Omega \times (0, T), \\ \nabla \cdot \mathbf{u} = 0 & \text{in } \Omega \times (0, T), \end{cases} \quad (0.1)$$

with Ω being a two- or three-dimensional bounded domain. Moreover, \mathbf{f} is the external force density which acts on the system, $\nu > 0$ is the kinematic fluid viscosity and $\alpha > 0$ is a given positive constant. We complete this initial boundary value problem with homogeneous Dirichlet boundary and initial conditions. It is important to observe that when $\alpha = 0$ we recover the classical Navier-Stokes equations for the flow of a viscous, incompressible, Newtonian fluid. This system and related α -regularization are proposed as efficient subgrid scale turbulence models of the Navier-Stokes equations.

The first proof of the existence of solutions of system (0.1) was given via Galerkin approximation in [2, 3]. The core of such a proof lay in obtaining eigenfunctions for the operator $\nabla \times \nabla \times (\mathbf{u} - \alpha \Delta \mathbf{u})$ as a basis to construct the approximations. This basis allowed to split the original system of equations into two problems: a generalized Stokes problem and a transport equation for the vorticity of $\mathbf{u} - \alpha \Delta \mathbf{u}$. It led to prove the existence and uniqueness results for weak solutions global (in time) in two dimensions, and local (in time) in three dimensions. The disadvantages of these results stem from the energy estimates being dependent on α ; so, the convergence to solutions of the Navier-Stokes equations cannot be proven. The only work in this sense was done in [4], where the convergence to weak solutions of the Navier-Stokes equations was shown. Comparable results to those of this work have been established in [1] from the Leray- α and the two-dimensional Navier-Stokes- α equations to the Navier-Stokes equations in a periodic domain and in [5] from the Euler- α equations to the Euler equations in the whole space.

In this talk, we will first present energy estimates which are independent of α and then the convergence rate of the solutions of the viscoelastic, grade-two, non-Newtonian equations to the corresponding solutions of the Navier-Stokes equations as the regularization parameter α goes to zero. To be more precise, we will prove error estimates in the L^2 -norm of order $\alpha^{1/2}$.

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DECAIMENTO GERAL DE SOLUÇÕES PARA UM SISTEMA NÃO LINEAR DE EQUAÇÕES DE ONDAS VISCOELÁSTICO COM AMORTECIMENTO DEGENERADO E TERMOS DE SOURCE

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1 Introdução

Neste trabalho trataremos em resolver os sistemas (1.1) e (2.4) onde (1.1) é dado logo abaixo e (2.4) é dado na observação final. Usaremos um método diferente do que foi feito por Shun-Tang Wu para resolver (1.1), mas o nosso objetivo principal é resolver um novo problema dado pelo sistema (2.4) na observação final. Consideremos o seguinte sistema de equações de ondas

$$\left\{ \begin{array}{l} u_{tt} + \Delta u + \int_0^t g(t-s)\Delta u(s)ds + (|u|^k + |v|^q)|u_t|^{m-1}u_t = f_1(u, v) \quad em \quad \Omega \times (0, \infty) \\ v_{tt} + \Delta v + \int_0^t h(t-s)\Delta v(s)ds + (|v|^\theta + |u|^\rho)|v_t|^{r-1}v_t = f_2(u, v) \quad em \quad \Omega \times (0, \infty) \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) \quad em \quad \Omega \\ v(x, 0) = v_0(x), \quad v_t(x, 0) = v_1(x) \quad em \quad \Omega \\ u(x, t) = v(x, t) = 0, \quad em \quad \Gamma, \quad t > 0. \end{array} \right. \quad (1.1)$$

onde Ω é um domínio limitado do \mathbb{R}^n ($n = 1, 2, 3$) com fronteira regular Γ e ν representa o vetor normal unitário exterior à Γ . As funções g e h são de classes C^1 e satisfazendo, para $s \geq 0$,

$$\begin{aligned} g(s) &\geq 0, \quad g'(s) \leq 0, \quad 1 - \int_0^\infty g(s)ds > 0 \\ h(s) &\geq 0, \quad h'(s) \leq 0, \quad 1 - \int_0^\infty h(s)ds > 0, \end{aligned} \quad (1.2)$$

e denotamos

$$l = \min\{l_1, l_2\}.$$

Concernente as funções $f_1(u, v)$ e $f_2(u, v)$, definimos como

$$\begin{aligned} f_1(u, v) &= a|u + v|^{p-1}(u + v) + b|u|^{\frac{p-3}{2}}|v|^{\frac{p+1}{2}}u \\ f_2(u, v) &= a|u + v|^{p-1}(u + v) + b|v|^{\frac{p-3}{2}}|u|^{\frac{p+1}{2}}v, \end{aligned} \quad (1.3)$$

com $a, b > 0$ e $p \geq 3$ se $n = 1, 2$, ou $p = 3$ se $n = 3$. Suponhamos que $0 < m, r < 1$ e as constantes k, l, θ, ρ são maiores ou igual a um, e se $n = 3$, inferimos que

$$\max\{k, q\} \leq 3(1 - m) \quad e \quad \max\{\theta, \rho\} \leq 3(1 - r).$$

2 Existência de Soluções e Taxa de Decaimento

Temos por objetivo encontrar soluções para (1.1) em espaços de sobolev adequados, usando o método de Faedo-Galerkin, encontrar taxas de decaimento de energia pelo método de Lyapunov.

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Observação:(Objetivo principal) Nossa pesquisa recente consiste encontrar existência de soluções em espaços de sobolev adequados e taxas de decaimento para o seguinte sistema

$$\left\{ \begin{array}{l} u_{tt} + \Delta u + \int_0^t g(t-s)\Delta u(s)ds + (|u|^k + |v|^q)|u_t|^{m-1}u_t = f_1(u, v) \quad em \quad \Omega \times (0, \infty) \\ v_{tt} + \Delta v + \int_0^t h(t-s)\Delta v(s)ds + (|v|^\theta + |u|^\rho)|v_t|^{r-1}v_t = f_2(u, v) \quad em \quad \Omega \times (0, \infty) \\ \frac{\partial u}{\partial \nu} - \int_0^t g(t-s)\frac{\partial u(s)}{\partial \nu}ds = h_1(u, v) \quad em \quad \Gamma_1 \times (0, \infty) \\ \frac{\partial v}{\partial \nu} - \int_0^t h(t-s)\frac{\partial v(s)}{\partial \nu}ds = h_2(u, v) \quad em \quad \Gamma_1 \times (0, \infty) \\ u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) \quad em \quad \Omega \\ v(x, 0) = v_0(x), \quad v_t(x, 0) = v_1(x) \quad em \quad \Omega \\ u(x, t) = v(x, t) = 0, \quad em \quad \Gamma_0, \quad t > 0. \end{array} \right. \quad (2.4)$$

onde Ω é um domínio limitado do \mathbb{R}^n ($n = 1, 2, 3$) com fronteira regular $\Gamma = \Gamma_0 \cup \Gamma_1$, Γ_0 e Γ_1 são fechados, disjuntos e $h_i(x, s)$, $i = 1, 2$ definida em $x \in \Gamma_1$ e $s \in \mathbb{R}$. As funções g, h, f_1, f_2 são as mesma do problema (1.1), e as constantes k, q, m, r, θ, ρ estão nas mesmas hipóteses de (1.1).

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ON COMPUTING EIGENPAIRS OF THE p -LAPLACIAN IN ANNULI

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1 Introduction

In our work we consider the following eigenvalue problem

$$\begin{cases} -\Delta_p u = \lambda |u|^{p-2} u, & \text{in } \Omega_{a,b} \\ u = 0 & \text{on } \partial\Omega_{a,b} \end{cases} \quad (1.1)$$

where $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ is the p -Laplacian operator, $p > 1$, and $\Omega_{a,b}$ is the annulus

$$\Omega_{a,b} := \{x \in \mathbb{R}^N : 0 < a < |x| < b\}, \quad N > 1.$$

We propose a method for computing the first eigenpair of (1.1) in the annulus. When the domain Ω is a ball or an annulus, first eigenfunctions must be radially symmetric and we explore these radial properties to focus our study in this boundary value problem

$$\begin{cases} -(r^{N-1} |u'|^{p-2} u')' = \lambda_p r^{N-1} u^{p-1}, & r \in (a, b) \\ u(a) = 0 = u(b). \end{cases} \quad (1.2)$$

It is easy to check that the eigenfunction u_p has a unique critical point $\rho \in (a, b)$, where it attains its maximum value. Thus, $u_p(r)$ is strictly increasing if $r \in [a, \rho]$, strictly decreasing if $r \in (\rho, b]$ and $u_p(\rho) = 1 = \|u_p\|_{L^\infty([a,b])}$. For each $t \in (a, b)$ we use an inverse iteration method to solve two (mixed Dirichlet-Neumann) radial eigenvalue problems. One of them in the annulus $\Omega_{a,t}$, with corresponding eigenvalue $\lambda_-(t)$, and the other in the annulus $\Omega_{t,b}$, with corresponding eigenvalue $\lambda_+(t)$. Then, using a matching procedure we adjust the parameter t to make coincide $\lambda_-(t)$ with $\lambda_+(t)$, obtaining the first eigenvalue λ_p . Hence, by a simple splicing argument we obtain the positive, L^∞ -normalized, radial first eigenfunction u_p . The found matching parameter turns out to be the maximum point ρ of u_p .

In order to carry out this plan we use the variational characterization of $\lambda_-(t)$ and $\lambda_+(t)$ to prove that these eigenvalues are strictly monotone and locally Lipschitz continuous as function of the variable t . Moreover, we derive the following upper and lower estimates for the maximum point ρ

$$\frac{b + a \left[\left(\frac{b}{a} \right)^{N-1} + \left(\frac{b}{a} \right)^N + 1 \right]^{\frac{1}{p}}}{1 + \left[\left(\frac{b}{a} \right)^{N-1} + \left(\frac{b}{a} \right)^N + 1 \right]^{\frac{1}{p}}} < \rho < \frac{a + b}{2}$$

and use them not only in the matching procedure but also to present a direct proof that u_p converges to the L^∞ -normalized distance function to the boundary, as $p \rightarrow \infty$. We also present some tables and graphs with numerical results based on the method.

Our method can also be employed to compute higher radial eigenpairs of the annulus $\Omega_{a,b}$ or of a ball.

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OPTIMAL CONTROL OF A MATHEMATICAL MODEL FOR RADIOTHERAPY OF GLIOMAS

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1 Introduction

In the last years there has been a lot of activity on mathematical modelling and analysis of gliomas progression, see [3], [4] and [5]. This work deals with the optimal control of a mathematical model of glioma progression incorporating the basic facts of the evolution of this type of primary brain tumor. We will consider a model for the compartment one of tumor cells of the simplest possible kinds, the Fischer-Kolmogorov equations, using ideas from Pérez-García [3]. The control is the n-tuple (d_1, \dots, d_n) , where d_i is the i-th applied radiotherapy dose and appears at the initial conditions impose at some prescribed times t_1, \dots, t_n . We search for controls that maximizes, over the class of admissible controls, the time that the tumor mass reaches a critical value M_* .

Let $\Omega \subset \mathbb{R}^N$ be a bounded open connected set ($N = 1, 2$ or 3) and let us fix the number of radiotherapy doses (n), the specific irradiation times t_j with $0 \leq t_1 \leq \dots \leq t_n < +\infty$, the doses $d_j \in L^2(\Omega)$ and the initial cell density c_0 , with $c_0 \in L^\infty(\Omega)$, $0 \leq c_0 \leq 1$.

Let the functions, of the spatial position x and time t , $C_j = C_j(x, t)$ be defined as follows. First, C_0 is the solution to the *pre-therapy* system

$$\begin{cases} C_{0,t} = D\Delta C_0 + \rho(1 - C_0)C_0, & \text{in } Q_0 := \Omega \times (0, t_1), \\ \frac{\partial C_0}{\partial \eta} = 0, & \text{on } \Sigma_0 := \partial\Omega \times (0, t_1), \\ C_0(x, 0) = c_0(x), & x \in \Omega. \end{cases} \quad (1.1)$$

where the positive constant D is the diffusion coefficient accounting for cellular motility and the parameter ρ is the proliferation rate.

Then, for $j = 1, 2, \dots, n - 1$, the j-th cell density C_j (*during therapy*) is the solution to the system

$$\begin{cases} C_{j,t} = D\Delta C_j + \rho(1 - C_j)C_j, & \text{in } Q_j := \Omega \times (t_j, t_{j+1}), \\ \frac{\partial C_j}{\partial \eta} = 0, & \text{on } \Sigma_j := \partial\Omega \times (t_j, t_{j+1}), \\ C_j(x, t_j) = SF_{d_j}(x)C_{j-1}(x, t_j), & x \in \Omega, \end{cases} \quad (1.2)$$

where SF_{d_j} is the survival fraction, i.e., the fraction of cells that are not lethally damaged by a dose d_j , is given by

$$SF_{d_j} = e^{-\alpha_t d_j - \beta_t d_j^2}, \quad (1.3)$$

where α_t and β_t are respectively the linear and quadratic coefficients for tumor cell damage. Finally, C_n is the solution to the *post-therapy* system

$$\begin{cases} C_{n,t} = D\Delta C_n + \rho(1 - C_n)C_n, & \text{in } Q_n := \Omega \times (t_n, \infty), \\ \frac{\partial C_n}{\partial \eta} = 0, & \text{on } \Sigma_n := \partial\Omega \times (t_n, \infty) \\ C_n(x, t_n) = SF_n(x)C_{n-1}(x, t_n), & x \in \Omega. \end{cases} \quad (1.4)$$

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From the C_j , we can now define the global in time tumor cell concentration $C = C(x, t)$, with

$$C(x, t) \equiv C_j(x, t) \text{ for } t \in [t_j, t_{j+1}), \quad 0 \leq j \leq n, \quad (1.5)$$

with the convention that $t_0 = 0$ and $t_{n+1} = +\infty$.

Then, the optimization problem consists of finding specific irradiation times t_j and doses d_j such that the overall survival time $T_*(t_1, \dots, t_n; d_1, \dots, d_n)$ is maximum. By definition, $T_*(t_1, \dots, t_n; d_1, \dots, d_n)$ is the smallest time for which the total tumor mass

$$M(t) := \int_{\Omega} C(x, t) \, dx, \quad (1.6)$$

reaches the critical value M_* . In other words, we want to maximize the pay-off function

$$T_*(t_1, \dots, t_n; d_1, \dots, d_n) := \inf \left\{ T \in \mathbb{R}_+ : \int_{\Omega} C(x, T) \, dx > M_* \right\} \quad (1.7)$$

subject to the constraint $(t_1, \dots, t_n; d_1, \dots, d_n) \in \mathcal{U}_{ad}$, where the set of *admissible controls* \mathcal{U}_{ad} is given by

$$\mathcal{U}_{ad} := \left\{ (t_1, \dots, t_n, d_1, \dots, d_n) \in \mathcal{U} : 0 \leq t_1 \leq t_2 \leq \dots \leq t_n, 0 \leq d_j \leq d_* \text{ a.e., } \alpha_h \left(\sum_{j=1}^n d_j + \frac{1}{\alpha_h/\beta_h} \sum_{j=1}^n d_j^2 \right) \leq E_* \text{ a.e.} \right\} \quad (1.8)$$

with $\mathcal{U} = \mathbb{R}^n \times L^2(\Omega)^n$. The coefficients α_h, β_h are the parameters of the normal tissue.

2 Main Result - Existence of an Optimal Control

Theorem 2.1. *Let us assume that $0 < M_* < |\Omega|$. Then, there exists at least one solution to the optimal control problem*

$$\begin{cases} \text{Maximize } T_*(v) \\ \text{subject to } v \in \mathcal{U}_{ad}. \end{cases} \quad (2.9)$$

This is equivalent to minimize the functional $J : \mathcal{U}_{ad} \mapsto \mathbb{R} \cup \{-\infty\}$, with

$$J(v) := \sup \left\{ -T \in \mathbb{R}_- : \int_{\Omega} C(x, T) \, dx > M_* \right\}, \quad \forall v \in \mathcal{U}_{ad}. \quad (2.10)$$

From well known results (see [1]), we observe that the existence of a minimum of J in \mathcal{U}_{ad} will be ensured if we prove the following: J is well-defined; \mathcal{U}_{ad} is non-empty, bounded, convex and closed in \mathcal{U} ; J is lower semi-continuous (l.s.c.) for the weak convergence in \mathcal{U} .

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CONTROLABILIDADE EXATA NA FRONTEIRA PARA UM SISTEMA DE TIMOSHENKO

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1 Introdução

A intenção do presente trabalho é investigar algumas propriedades das equações de Timoshenko que modelam o movimento de vigas, a saber: existência, unicidade, regularidade e controlabilidade exata na fronteira, com condições de contorno tipo Dirichlet.

Para uma viga de comprimento $L > 0$, seu movimento é descrito pelo sistema de equações parciais acopladas:

$$\begin{cases} I_\rho y_{tt} = EIy + K(z_x - y) + v \\ \rho z_{tt} = K(z_x - y)_x + w \end{cases} \quad \text{em } Q \quad (1.1)$$

onde $Q = (0, L) \times (0, T)$. Assumindo as condições de fronteira do tipo Dirichlet, isto é,

$$y(0, t) = y(L, t) = z(0, t) = z(L, t) = 0, \quad t \in (0, T)$$

e condições iniciais

$$y(x, 0) = y^0(x), \quad y_t(x, 0) = y^1(x), \quad z(x, 0) = z^0(x), \quad z_t(x, 0) = z^1(x), \quad x \in (0, L).$$

Aqui, t é a variável tempo e x é a coordenada do espaço ao longo da viga; y_x significa a derivada da função em relação a x e y_t a derivada em relação a t . Além disso, z indica o deslocamento transversal, y a rotação angular da seção transversal, k , I , I_ρ , E e ρ são coeficientes positivos de classe C^2 . A energia do sistema é definida por

$$E(t) = \frac{1}{2} \int_0^L (\rho(z_t)^2 + I_\rho(y_t)^2 + EI(y_x)^2 + K(z_x - y)^2) dx \quad (1.2)$$

de onde segue que $E(t) = E(0)$, $\forall t \geq 0$.

Existem duas velocidades de propagação de ondas associada a (1.1),

$$v_1 = \sqrt{\frac{EI}{I_\rho}} \quad v_2 = \sqrt{\frac{K}{\rho}}.$$

Temos que T_1 e T_2 denotam o tempo requerido pelos dois tipos de ondas para percorrem todo o comprimento da viga. Especificamente

$$T_1 = \int_0^L \frac{1}{v_1(x)} dx \quad T_2 = \int_0^L \frac{1}{v_2(x)} dx.$$

Temos $T_0 = 2\max(T_1, T_2)$, supondo que $T > T_0$. Observa-se que, se considerarmos o problema homogêneo adjunto a (1.1), a partir de (1.2) obtemos $E(t) = E(0)$, quer dizer, a energia se conserva ao longo da trajetória. Assim, nem uma solução não-nula da equação homogênea atinge o estado de repouso em tempo algum. Logo o problema da controlabilidade exata consiste precisamente em conduzir todas as trajetórias ao equilíbrio em um tempo uniforme,

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mediante a ação de uma força externa o *controle*. De forma mais precisa, o problema de controlabilidade exata para o sistema de Timoshenko, pode ser formulado da seguinte maneira: Estudar a existência de um tempo $T > 0$ tal que para cada par de dados iniciais $\{y^o, y^1\}, \{z^o, z^1\}$ exista controles v e w tais que a solução de (1.1) verifique

$$y(T) = y_t(T) = z(T) = z_t(T) = 0. \quad (1.3)$$

2 Resultados

Suponhamos uma viga de comprimento $L = 1$, $Q = (0, 1) \times (0, T) \subset \mathbb{R}^2$ e $T > 0$. Então temos o seguinte sistema:

$$\begin{cases} y_{tt} = a(x)y_{xx} + z_x - y & em \quad Q \\ z_{tt} = b(x)z_{xx} - y_x & em \quad Q \\ y(0, t) = v(t), \quad y(1, t) = 0 & em \quad (0, T) \\ z(0, t) = w(t), \quad z(1, t) = 0 & em \quad (0, T) \end{cases} \quad (2.4)$$

com as seguintes condições iniciais

$$y(x, 0) = y^o(x), \quad y_t(x, 0) = y^1(x), \quad z(x, 0) = z^o(x), \quad z_t(x, 0) = z^1(x), \quad x \in (0, 1).$$

O resultado a seguir garante a controlabilidade exata para o sistema (2.4), com o controle atuando na fronteira do domínio.

Teorema 2.1. Suponha $a(x) \in W^{1,\infty}(0, 1)$ e

$$a(x) \geq a_0 > 1, \quad b(x) \geq b_0 > 1 \quad em \quad (0, 1). \quad (2.5)$$

Seja $T > 2\alpha$, onde $\alpha = \max(\frac{1}{\sqrt{a_0}}, \frac{1}{\sqrt{b_0}})$. Então, para cada conjunto de dados iniciais $\{y^o, y^1\}, \{z^o, z^1\}$ pertencentes a $L^2(0, 1) \times H^{-1}(0, 1)$, existem controles $v(t), w(t) \in L^2(0, T)$ tal que a solução ultrafraca $y = y(x, t), z = (x, t)$ de (2.4) satisfaça (1.3).

Com estas hipóteses, incluímos coeficientes variáveis $a(x)$ e $b(x)$ nos termos onde figuram duas derivadas em relação a variável x . Isso significa que, as velocidades de cada tipo de movimento são diferentes em cada ponto da viga. A demonstração da controlabilidade é baseada no método HUM (Hilbert Uniqueness Method). Para isso faz-se necessário demonstrar uma desigualdade de observabilidade dada por

$$C \left\| \{\phi^o, \phi^1, \psi^o, \psi^1\} \right\|_{[H_0^1(0,1) \times L^2(0,1)]^2}^2 \leq \int_0^T a(0) |\phi_x(0, t)|^2 dt + \int_0^T b(0) |\psi_x(0, t)|^2 dt \quad (2.6)$$

onde C é uma constante positiva, $T > T_0$ e $y = (x, t)$ e $z = (x, t)$ é solução de (2.4). Um problema técnico que surge na demonstração de tal desigualdade é a existência de coeficientes variáveis nas equações do sistema, os quais nos impedem de usar princípios de continuação única decorrentes do teorema de Holmgren.

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REGULARITY FOR ANISOTROPIC FULLY NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS

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1 Introduction

In this work we develop a regularity theory for elliptic fully nonlinear integro-differential equations of the type

$$Iu(x) := \inf_{\alpha} \sup_{\beta} L_{\alpha\beta} u(x) = 0, \quad (1.1)$$

where

$$L_{\alpha\beta} u(x) := \int_{\mathbb{R}^n} (u(x+y) - u(x) - \nabla u(x) \cdot y \chi_{B_1}(y)) K_{\alpha\beta}(y) dy,$$

and the kernels $K_{\alpha\beta}$ are symmetric and satisfy the anisotropic bounds

$$\frac{\lambda c_\sigma}{\sum_{i=1}^n |y_i|^{n+\sigma_i}} \leq K_{\alpha\beta}(y) \leq \frac{\Lambda c_\sigma}{\sum_{i=1}^n |y_i|^{n+\sigma_i}}, \quad \forall y \in \mathbb{R}^n,$$

for $0 < \lambda \leq \Lambda$, $0 < \sigma_i < 2$, and $c_\sigma = c(\sigma_1, \dots, \sigma_n) > 0$ a normalization constant.

2 Mathematical Results

Our aim is to prove the C^γ regularity for a solution u of the equation (1.1) and under additional assumptions to the kernel $K_{\alpha\beta}$ we also obtain $C^{1,\gamma}$ regularity for u , with $\gamma \in (0, 1)$. The key that gave access to this regularity theory was the anisotropic nonlocal version of the Aleksandrov–Bakel’man–Pucci’s estimate:

Theorem 2.1 (ABP Nonlocal theorem). *Let $u \leq 0$ in $\mathbb{R}^n \setminus B_1$ and Γ be its concave envelope. Suppose $M^+ u(x) \geq -f(x)$ in B_1 . There is a finite family of open rectangles $\{\mathcal{R}_j\}_{j \in \{1, \dots, m\}}$ with diameters d_j such that:*

1. Any two rectangles \mathcal{R}_i and \mathcal{R}_j in the family do not intersect.

2. $\{u = \Gamma\} \subset \bigcup_{j=1}^m \overline{\mathcal{R}}_j$.

3. $\{u = \Gamma\} \cap \overline{\mathcal{R}}_j \neq \emptyset$ for all \mathcal{R}_j .

4. $d_j \leq \sqrt{\sum_{i=1}^n \left(\rho_0 2^{-\frac{1}{q_{\max}}} \right)^{\frac{2}{n+\sigma_i}}}$.

5. $|\nabla \Gamma(\overline{\mathcal{R}}_j)| \leq C \left(\max_{\overline{\mathcal{R}}_j} f^+ \right)^n |\overline{\mathcal{R}}_j|$.

6. $\left| \left\{ y \in C\tilde{\mathcal{R}}_j : u(y) \geq \Gamma(y) - C \left(\max_{\overline{\mathcal{R}}_j} f \right) (\tilde{d}_j)^2 \right\} \right| \geq \varsigma |\tilde{\mathcal{R}}_j|$,

where $q_i := -1 + \frac{3}{n+\sigma_i} + \sum_{j \neq i} \frac{1}{n+\sigma_j}$, M^+ is the the extremal Pucci operator (nonlocal version) and \tilde{d}_j is the diameter of the rectangle $\tilde{\mathcal{R}}_j$ corresponding to \mathcal{R}_j . The constants $\varsigma > 0$ and $C > 0$ depend only on n , λ and Λ .

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THE HADAMARD PRODUCT IN THE SPACE OF LORCH ANALYTIC MAPPINGS

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For a commutative Banach algebra E with unit we say that $f : E \rightarrow E$ is Lorch analytic (or **(L)-analytic**) in E if and only if there exist unique elements $a_n \in E$ such that $f(w) = \sum_{n=0}^{\infty} a_n w^n$ for all $w \in E$ (see [2]). Consequently, $\lim \|a_n\|^{1/n} = 0$. We denote by $\mathcal{H}_L(E)$ the space of Lorch analytic mappings from E into E . It is easy to verify that $\mathcal{H}_L(E) \subset \mathcal{H}_b(E, E)$ where $\mathcal{H}_b(E, E)$ denotes the space of holomorphic mappings from E into E which are bounded on the bounded subsets of E . We refer to [1] and [6] for background on holomorphic mappings between Banach spaces.

With the pointwise product it is known that $(\mathcal{H}_L(E), \tau_b)$ is a commutative Fréchet algebra with unit where τ_b denotes the topology of uniform convergence on the bounded subsets of E . In [4] we give descriptions of its spectrum and also we study the spectra of other algebras of analytic mappings in the sense of Lorch.

In this work we describe the spectra of the algebra $\mathcal{H}_L(E)$ endowed with the Hadamard product and with the topology τ_b . We also study algebraic and topological properties of the space $\Gamma(E)$ of the sequences $(a_n)_n \subset E$ such that $\lim \|a_n\|^{1/n} = 0$. The proofs announced in this note can be found in [5].

1 The Results

We define the Hadamard product as the product $(f \cdot g)(w) = \sum_{n=0}^{\infty} a_n b_n w^n$ for every $w \in E$ if $f(w) = \sum_{n=0}^{\infty} a_n w^n$ and $g(w) = \sum_{n=0}^{\infty} b_n w^n$ for every $w \in E$ where $(a_n)_n, (b_n)_n \in \Gamma(E)$. Denote $H_L(E)$ the algebra of mappings from E into E that are analytic in the sense of Lorch, endowed with the Hadamard product and with the topology τ_b . So $H_L(E)$ is a commutative (m-convex) Fréchet algebra without unit.

Theorem 1.1. *Let E be a commutative Banach algebra with unit. The spectrum $\mathcal{M}(H_L(E))$ is homeomorphic to $\mathcal{M}(E) \times \mathbb{N}_0$.*

Corollary 1.1. *If \mathcal{I} is a closed maximal ideal in $H_L(E)$, then there exists $\varphi \in \mathcal{M}(H_L(E))$ such that $\mathcal{I} = \varphi^{-1}(0)$.*

Corollary 1.2. *The algebra $H_L(E)$ is semisimple whenever E is.*

One check easily that $\Gamma(E)$ endowed with the usual addition and scalar multiplication operations is a vector space. We define a topological structure in $\Gamma(E)$ and establish isomorphism between $\Gamma(E)$ and $H_L(E)$. For all $a = (a_n)_n, b = (b_n)_n \in \Gamma(E)$ define

$$d(a, b) = \sup\{\|a_0 - b_0\|; \|a_n - b_n\|^{1/n}, n \in \mathbb{N}\}.$$

Then d is a translation invariant metric in $\Gamma(E)$.

Theorem 1.2. *$(\Gamma(E), d)$ and $(H_L(E), \tau_b)$ are isomorphic as topological vector spaces.*

Proposition 1.1. *The Fréchet space $(\Gamma(E), d)$ is not normable.*

If we endow $\Gamma(E)$ with the usual product in spaces of sequences then $\Gamma(E)$ is a non-unital commutative algebra. Also the isomorphism established in Theorem 1.2 preserves the algebra structure. Hence $\Gamma(E)$ is a commutative Fréchet algebra without unit. So, the study of the algebra $H_L(E)$ leads to a better knowledge of the algebra $\Gamma(E)$.

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Proposition 1.2. *The spectrum $\mathcal{M}(\Gamma(E))$ is homeomorphic to $\mathcal{M}(E) \times \mathbb{N}_0$.*

Corollary 1.3. *If \mathcal{I} is a closed maximal ideal in $\Gamma(E)$, then there exists $\psi \in \mathcal{M}(\Gamma(E))$ such that $\mathcal{I} = \psi^{-1}(0)$.*

Corollary 1.4. *The algebra $\Gamma(E)$ is semi simple whenever E is.*

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PATTERNS IN A BALANCED BISTABLE EQUATION WITH HETEROGENEOUS ENVIRONMENTS ON SURFACES OF REVOLUTION

MAICON SÔNEGO *
 *

1 Introduction

In this work we study the following problem

$$\partial_t u_\epsilon = \epsilon^2 \Delta_g u_\epsilon + f(u_\epsilon, x), \quad (t, x) \in \mathbb{R}^+ \times \mathcal{M} \quad (1.1)$$

where $\epsilon > 0$ is a small parameter and $\mathcal{M} \subset \mathbb{R}^3$ is a surface of revolution without boundary with metric g . We consider

$$f(u, x) = -(u - a(x))(u - b(x))(u - c(x)), \quad (1.2)$$

where $a, b, c \in C^1(\mathcal{M})$ and $a(x) < b(x) = \frac{a(x)+c(x)}{2} < c(x)$ for all $x \in \mathcal{M}$. Such $f(u, x)$ is a typical example of the so-called bistable function.

We use the variational concept of Γ -convergence to obtain sufficient conditions that guarantee existence, stability and the geometric structure of four families of stationary solutions to the singularly perturbed parabolic equation (1.1). The conditions found relate the functions a, b, c and the geometry of the surface where such functions are defined.

For one-dimensional domains, i.e., when $\mathcal{M} = (0, 1)$ for instance, subjected to zero Neumann boundary condition there are several results. In [6] it was proved that if $c(x) - a(x)$ is C^2 and assume a nondegenerate local minimum at $x_0 \in (0, 1)$ then there exists a stable solution u_ϵ such that $u_\epsilon(x) \rightarrow c(x)$ on $(0, x_0)$ and $u_\epsilon(x) \rightarrow a(x)$ on (x_0, l) . In [5] this result was extended to a degenerate setting. In [1] this result was generalized to two-dimensional domains using essentially the same ideas used here, i.e., the variational concept of Γ -convergence. Indeed, our problem becomes simpler since (1.1) can be treated as a one-dimensional problem and so our conditions for the existence of patterns appear more naturally. There are some works regarding the effect of heterogenous environments under different aspects, we cite [3, 4, 2] and references therein.

2 Main Results

In order to mention our results consider a smooth curve C in \mathbb{R}^3 parametrized by $x = (x_1, x_2, x_3) = (\psi(s), 0, \chi(s))$, $s \in [0, l]$ with $\psi(0) = \psi(l) = 0$ and the borderless surface of revolution \mathcal{M} generated by C . We suppose that the functions $a(x)$, $b(x)$ and $c(x)$ does not depend on the angular variable θ , so that, abusing notation, we set $a(x(s, \theta)) = a(s)$, $b(x(s, \theta)) = b(s)$ and $c(x(s, \theta)) = c(s)$.

We found that a sufficient condition for existence of patterns to (1.1) is that the function $\psi(c - a)^3 : (0, l) \rightarrow \mathbb{R}$ has a isolated local minimum in $(0, l)$. In particular, if a and c are constant then the sufficient condition is satisfied as long as, roughly speaking, \mathcal{M} has a neck.

The geometric profile of these patterns are also given and more than that, we show that two of four families of patterns found develop internal transition layer as $\epsilon \rightarrow 0$, which are referred to as stable transition layers. Our approach provides convergence, as $\epsilon \rightarrow 0$, of the stable transition layers in $L^1(\Omega)$ rather than uniform convergence

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in compact sets outside the interface. All these results remain true for a surface of revolution with border under Neumann boundary condition and this case is also considered in this work.

Our result extend [6, 5] to surfaces of revolution. This can be seen by taking $\psi \equiv 1$, which would correspond to a finite right circular cylinder, and then the existence condition for patterns would be $c(x) - a(x)$ having an isolated local minimum in $(0, l)$, as found in [6, 5]. In the end, some simple examples are given to illustrate situations in which our results guarantee the existence of patterns.

Our main result is stated below. As usual χ_A denotes the characteristic function of a set A .

Theorem 2.1. *If the function $\psi(c - a)^3 : [0, l] \rightarrow \mathbb{R}$ assumes an isolated local minimum at $s_0 \in (0, l)$ then exists $\epsilon_0 > 0$ and four families of stable stationary solutions $\{u_\epsilon^j\}$, $j = 1, \dots, 4$, to (1.1) such that*

- $\|u_\epsilon^1 - u_0^1\|_{L^1(I)} \xrightarrow{\epsilon \rightarrow 0} 0$ where

$$u_0^1(s) = a(s)\chi_{(0, s_0)}(s) + c(s)\chi_{(s_0, l)}(s);$$

- $\|u_\epsilon^2 - u_0^2\|_{L^1(I)} \xrightarrow{\epsilon \rightarrow 0} 0$ where

$$u_0^2(s) = c(s)\chi_{(0, s_0)}(s) + a(s)\chi_{(s_0, l)}(s);$$

- $\|u_\epsilon^3 - u_0^3\|_{L^1(I)} \xrightarrow{\epsilon \rightarrow 0} 0$ where $u_0^3(s) = a(s)$ in $(0, l)$;

- $\|u_\epsilon^4 - u_0^4\|_{L^1(I)} \xrightarrow{\epsilon \rightarrow 0} 0$ where $u_0^4(s) = c(s)$ in $(0, l)$.

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THE 3-DIMENSIONAL CORED AND LOGARITHM POTENTIALS: PERIODIC ORBITS

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1 Introduction

In this paper we are interested in 3-degrees Hamiltonian systems of the form

$$H(x, y, z, p_x, p_y, p_z) = \frac{1}{2} (p_x^2 + p_y^2 + p_z^2) + V(x^2, y^2, z^2),$$

where V a smooth potential with an absolute minimum and a reflection symmetry with respect the three axes. The motivation for the choice of these symmetries becomes from the interest of these potentials in galactic dynamics. In particular, we considered the cored potential and the logarithm potential

$$V_C = \sqrt{1 + x^2 + y^2 + \frac{z^2}{q^2}}, \quad V_L = \frac{1}{2} \log \left(1 + x^2 + y^2 + \frac{z^2}{q^2} \right),$$

respectively, such potentials in 2-degrees of freedom have been studied by several authors, see for instance, [1], [2], [3], [4], [5].

Our goal is to study the periodic orbits of the corresponding Hamiltonian differential system using the averaging theory. In this paper, we find new families of periodic orbits parameterized by the energy and depending on the parameter q .

2 Mathematical Results

The cored Hamiltonian system is

$$\begin{aligned} \dot{x} &= p_x, & \dot{p}_x &= -\frac{x}{\sqrt{1 + x^2 + y^2 + \frac{z^2}{q^2}}}, \\ \dot{y} &= p_y, & \dot{p}_y &= -\frac{y}{\sqrt{1 + x^2 + y^2 + \frac{z^2}{q^2}}}, \\ \dot{z} &= p_z, & \dot{p}_z &= -\frac{z}{q^2 \sqrt{1 + x^2 + y^2 + \frac{z^2}{q^2}}}. \end{aligned} \tag{2.1}$$

After introducing a non-canonical scale transformation with a small parameter $\varepsilon > 0$, the Hamiltonian system

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2.1 can be reduced to study the differential system

$$\begin{aligned}\dot{x} &= p_x & \dot{p}_x &= -x + \varepsilon \frac{x(q^2(x^2 + y^2) + z^2)}{2q^2} + \mathcal{O}(\varepsilon^2), \\ \dot{y} &= p_y, & \dot{p}_y &= -\frac{y}{q^2} + \varepsilon \frac{x(q^2(x^2 + y^2) + z^2)}{2q^4} + \mathcal{O}(\varepsilon^2), \\ \dot{z} &= \frac{p_z}{q}, & \dot{p}_z &= -\frac{z}{q^2} + \varepsilon \frac{z(q^2(x^2 + y^2) + z^2)}{2q^4} + \mathcal{O}(\varepsilon^2).\end{aligned}\tag{2.2}$$

The logarithm Hamiltonian system has the small modification that, instead of ε , it has 2ε . Then we proceed the study of the system (2.2) which includes both Hamiltonian systems.

Theorem 2.1. *For $V > 0$ sufficiently small, at every energy level $H = h > 0$ the perturbed differential system 2.2 has at least 3 periodic solution*

$$\gamma^k(t, \varepsilon) = (x^k(t, \varepsilon), y^k(t, \varepsilon), z^k(t, \varepsilon), p_x^k(t, \varepsilon), p_y^k(t, \varepsilon), p_z^k(t, \varepsilon)),$$

for $k = 1, 2, 3$ such that

- (i) $\gamma^1(0, \varepsilon) \rightarrow (\sqrt{2h}, 0, 0, 0, 0, 0)$ when $\varepsilon \rightarrow 0$;
- (ii) $\gamma^2(0, \varepsilon) \rightarrow (0, \sqrt{2h}, 0, 0, 0, 0)$ when $\varepsilon \rightarrow 0$;
- (iii) $\gamma^3(0, \varepsilon) \rightarrow (0, 0, \sqrt{2h}q, 0, 0, 0)$ when $\varepsilon \rightarrow 0$.

Moreover, the families of periodic solutions $\gamma^1(t, \varepsilon)$, $\gamma^2(t, \varepsilon)$ and $\gamma^3(t, \varepsilon)$ bifurcate from planar periodic solutions of system 2.2 with $\varepsilon = 0$ living in the planes $(x, 0, 0, p_x, 0, 0)$, $(0, y, 0, 0, p_y, 0)$ and $(0, 0, z, 0, 0, p_z)$, respectively.

We observe that the families of periodic solutions γ^1 and γ^3 of Theorem 2.1 have already appeared in the work [1] but only when q was irrational where this problem was studied with only 2-degrees of freedom. Here we show that really they exist independent if q is irrational or rational.

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ON A CLASS FRACTIONAL SCHRÖDINGER EQUATIONS WITH CRITICAL GROWTH

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1 Introduction

This work we establish the existence and multiplicity of weak solutions for the following class of problems

$$(-\Delta)^{1/2}u + V(x)u = \lambda f(x, u) + h \quad \text{in } \mathbb{R}, \quad (1.1)$$

where $(-\Delta)^{1/2}$ is the operator $1/2$ -Laplacian, λ is a positive parameter, h is small perturbation and $V : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function which satisfies the following assumptions:

(V₁) There exists is a positive constant B such that

$$V(x) \geq -B \text{ for all } x \in \mathbb{R};$$

(V₂) The infimum $\lambda_1 := \inf_{\substack{u \in X \\ \|u\|_2=1}} \left(\frac{1}{2\pi} \int_{\mathbb{R}^2} \frac{(u(x) - u(y))^2}{|x - y|^2} dx dy + \int_{\mathbb{R}} V(x)u^2 dx \right)$ is positive.

(V₃) For any $r > 0$ and sequence $(x_n) \subset \mathbb{R}$ which goes to infinity, that is, $|x_n| \rightarrow \infty$,

$$\lim_{n \rightarrow \infty} \nu(B_n) = \infty,$$

where

$$\nu(B_n) = \inf_{\substack{u \in H_0^{1/2}(B_n) \\ \|u\|_{L^2(B_n)}=1}} \frac{1}{2\pi} \int_{B_n} \frac{(u(x) - u(y))^2}{|x - y|^2} dx dy + \int_{B_n} V(x)u^2 dx;$$

Here, motivated by an inequality of Trudinger-Moser type proved by T. Ozawa [3] we are interested in treat nonlinearities involving critical exponential growth which we define next. We say that $f(x, s)$ has *critical exponential growth* when for all $x \in \mathbb{R}$, there exists $\alpha_0 > 0$ such that

$$\lim_{|s| \rightarrow +\infty} f(x, s)e^{-\alpha|s|^2} = \begin{cases} 0, & \forall \alpha > \alpha_0, \\ +\infty, & \forall \alpha < \alpha_0. \end{cases}$$

For the nonlinearity $f(x, s)$, we assume the following hypotheses:

(f₁) $\limsup_{s \rightarrow 0} \frac{2F(x, s)}{s^2} < \lambda_1$ uniformly in x ;

(f₂) $f(x, s)$ is locally bounded in s , that is, for any bounded interval $J \subset \mathbb{R}$, there exists $C > 0$ such that $|f(x, s)| \leq C$ for every $(x, s) \in \mathbb{R} \times J$;

(f₃) There exists $\theta > 2$ such that, for all $(x, s) \in \mathbb{R} \times \mathbb{R} \setminus \{0\}$,

$$0 < \theta F(x, s) := \theta \int_0^s f(x, t) dt \leq s f(x, s);$$

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(f₄) There exist constants $s_0, M_0 > 0$ such that, for all $|s| \geq s_0$ and $x \in \mathbb{R}$,

$$0 < F(x, s) \leq M_0 |f(x, s)|;$$

(f₅) There exists $q > 2$ such that $\liminf_{s \rightarrow 0^+} \frac{f(x, s)}{s^{q-1}} > 0$ uniformly in x .

In order to apply variational methods we are consider a variational framework based in the subspace X of $H^{1/2}(\mathbb{R})$ given by

$$X = \left\{ u \in H^{1/2}(\mathbb{R}) : \int_{\mathbb{R}} V(x) u^2 dx < \infty \right\},$$

which will be a Hilbert space when endowed with the scalar product

$$\langle u, v \rangle := \frac{1}{2\pi} \left(\int_{\mathbb{R}^2} \frac{(u(x) - u(y))(v(x) - v(y))}{|x - y|^2} dx dy \right) + \int_{\mathbb{R}} V(x) uv dx, \quad (1.2)$$

to which we associate the standard norm $\|u\| = \langle u, u \rangle^{1/2}$.

In this context, we assume that $h \in X^*$ (dual space of X) and say that $u \in X$ is a weak solution for the problem (1.1) if the following equality holds:

$$\frac{1}{2\pi} \int_{\mathbb{R}^2} \frac{(u(x) - u(y))(v(x) - v(y))}{|x - y|^2} dx dy + \int_{\mathbb{R}} V(x) uv dx = \lambda \int_{\mathbb{R}} f(x, u)v dx + (h, v), \quad (1.3)$$

for all $v \in X$, where (\cdot, \cdot) denotes the duality pairing between X and X^* .

2 Mathematical Results

Theorem 2.1. *Suppose that (V₁) – (V₃) and (f₁) – (f₅) are satisfied. Then there exist $\delta_1 > 0$ and $\lambda_0 > 0$ such that for each $0 < \|h\|_* < \delta_1$ and $\lambda > \lambda_0$, problem (1.1) has at least two weak solutions. One of them with positive energy, while the other one with negative energy.*

Theorem 2.2. *Under the same hypotheses in Theorem 2.1, the problem without the perturbation has a nontrivial weak solution with positive energy.*

The proofs of our results rely on minimization methods in combination with the mountain-pass theorem. The main point of the proof of the Theorems 2.1 and 2.2 is to show that the associated functional satisfies the Palais-Smale compactness condition which allows us to find critical points for the functional. In our argument, it is crucial a version of the Trudinger-Moser inequality and a version of a lemma due to Lions. Our main difficulties are the lack of Palais-Smale compactness condition for certain energy levels due to critical exponential growth of the nonlinearity.

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ON SEMILINEAR WAVE EQUATIONS WITH NEGATIVE TERMS

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1 Introduction

Let Ω be a bounded open set of \mathbb{R}^n with a C^2 boundary Γ , Γ constituted of two disjoint closed parts Γ_0 and Γ_1 . By $\nu(x)$ is denoted the exterior unit normal at $x \in \Gamma_1$.

Motivated by the papers [1] and [2], we investigate the existence and decay of solutions of the following hyperbolic problem:

$$\begin{cases} u'' - \mu(t)\Delta u + g(x, u) = f & \text{in } \Omega \times (0, \infty); \\ u = 0 & \text{on } \Gamma_0 \times (0, \infty); \\ \frac{\partial u}{\partial \nu} + h(x, u') + q(x, u) = 0 & \text{on } \Gamma_1 \times (0, \infty); \\ u(x, 0) = u^0(x), \quad u'(x, 0) = u^1(x) & \text{, } x \in \Omega \end{cases}$$

where $\mu(t) \geq \mu_0 > 0$, $g(x, s)$, $q(x, s)$ behave as $|s|^\rho (\rho > 1)$, $|s|^\sigma (\sigma > 1)$, respectively, and $h(x, s)$ is just continuous and strongly monotone in $s \in \mathbb{R}$.

2 Main Results

The scalar product and norm of $L^2(\Omega)$ are denoted by (u, v) and $|u|$, respectively. By V is represented the Hilbert space

$$V = \{v \in H^1(\Omega); v = 0 \text{ on } \Gamma_0\}$$

equipped with the scalar product $((u, v)) = (\nabla u, \nabla v)$ and norm $\|u\| = ((u, u))^{1/2}$, respectively. Let $A = -\Delta$ be the self-adjoint operator of $L^2(\Omega)$ defined by the triplet $\{V, L^2(\Omega), ((u, v))\}$. Then its domain is given by

$$D(-\Delta) = \{v \in V \cap H^2(\Omega); \frac{\partial u}{\partial \nu} = 0 \text{ on } \Gamma_1\}.$$

We introduce the following hypotheses:

$$\mu \in W_{loc}^{1,1}(0, \infty); \quad 0 < \mu_0 \leq \mu(t) \leq \mu_1 < \infty, \quad \forall t \geq 0; \quad \mu' \in L^1(0, \infty). \quad (2.1)$$

Set $g \in W_{loc}^{1,1}(\mathbb{R}; L^\infty(\Omega))$ and $q \in W_{loc}^{1,1}(\mathbb{R}; L^\infty(\Gamma_1))$. Define $G(x, s) = \int_0^s g(x, \tau) d\tau$ and $Q(x, s) = \int_0^s q(x, \tau) d\tau$. Assume that there exist positive constants a_0, a_1, a_2 and b_0, b_1, b_2 such that

$$|G(x, s)| \leq a_0 |s|^{\rho+1}; \quad |g(x, s)| \leq a_1 |s|^\rho; \quad |\frac{\partial g}{\partial s}(x, s)| \leq a_2 |s|^{\rho-1}, \quad \forall s \in \mathbb{R}, \quad a.e. \quad x \in \Omega \quad (2.2)$$

and

$$|Q(x, s)| \leq b_0 |s|^{\sigma+1}; \quad |q(x, s)| \leq b_1 |s|^\sigma; \quad |\frac{\partial q}{\partial s}(x, s)| \leq b_2 |s|^{\sigma-1}; \quad q(x, 0) = 0, \quad \forall s \in \mathbb{R}, \quad a.e. \quad x \in \Gamma_1. \quad (2.3)$$

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The function h satisfies the conditions

$$h \in C^0(\mathbb{R}; L^\infty(\Gamma_1)) ; h(x, 0) = 0 , \text{ a.e. } x \in \Gamma_1 ; (h(x, s) - h(x, r))(s - r) \geq d_0(s - r)^2 , \forall s, r \in \mathbb{R} , \text{ a.e. } x \in \Gamma_1. \quad (2.4)$$

The real numbers ρ and σ satisfy the hypotheses

$$\rho > 1 , \sigma > 1 \text{ if } n = 1, 2 ; \frac{n+1}{n} \leq \rho \leq \frac{n}{n-2} , \frac{2n-1}{2(n-1)} \leq \sigma \leq \frac{n-1}{n-2} \text{ if } n \geq 3. \quad (2.5)$$

Theorem 2.1. *Assume that the above hypotheses (2.1)-(2.5) are satisfied. Consider*

$$u^0 \in D(-\Delta) \cap H_0^1(\Omega) , u^1 \in H_0^1(\Omega) , f \in L^1(0, \infty; L^2(\Omega)) , f' \in L_{loc}^1(0, \infty; L^2(\Omega)).$$

Then there exist two positive constants λ^ and N such that if*

$$\|u^0\| < \lambda^* , \left(\frac{2}{\mu_0} \right)^{1/2} ((2N)^{1/2} + \int_0^\infty |f(t)| dt) \exp \left(\frac{2}{\mu_0} \int_0^\infty |\mu'(t)| dt \right) < \lambda^*.$$

we have that there exists a function u in the class

$$\begin{cases} u \in L^\infty(0, \infty; V) , u' \in L^\infty(0, \infty; L^2(\Omega)) \cap L_{loc}^\infty(0, \infty; V) , u'' \in L_{loc}^\infty(0, \infty; L^2(\Omega)), \\ u' \in L^\infty(0, \infty; L^2(\Gamma_1)) , u'' \in L_{loc}^\infty(0, \infty; L^2(\Gamma_1)) \end{cases}$$

satisfying

$$\begin{cases} u'' - \mu \Delta u + g(., u) = f \text{ in } L_{loc}^2(0, \infty; L^2(\Omega)); \\ \frac{\partial u}{\partial \nu} + h(., u') + q(., u) = 0 \text{ in } L_{loc}^1(0, \infty; L^1(\Gamma_1)); \\ u(0) = u^0 , u'(0) = u^1. \end{cases}$$

Let $x^0 \in \mathbb{R}^n$ and $m(x) = x - x^0$, $x \in \mathbb{R}^n$. Assume that there exists x^0 such that

$$\Gamma_0 = \{x \in \Gamma; m(x) \cdot \nu(x) \leq 0\} , \Gamma_1 = \{x \in \Gamma; m(x) \cdot \nu(x) > 0\}. \quad (2.6)$$

Consider

$$\begin{cases} p \in C^0(\mathbb{R}) ; p(0) = 0 ; (p(s) - p(r))(s - r) \geq p_0(s - r)^2 , |p(s)| \leq p_1 |s| , \forall s, r \in \mathbb{R}; \\ a_0 \geq 1 ; \mu'(t) \leq 0 , \text{ a.e. } t \in (0, \infty) ; g(s) = |s|^\rho ; q(x, s) = 0 ; f = 0. \end{cases} \quad (2.7)$$

Set the energy

$$E(t) = \frac{1}{2}|u(t)| + \frac{\mu(t)}{2}\|u(t)\| + \frac{1}{\rho+1} \int_\Omega |u(x, t)|^\rho u(x, t) dx , \quad t \geq 0.$$

Theorem 2.2. *Let u be the solution obtained in Theorem 2.1 with the supplementary hypotheses (2.6) and (2.7). Then there exists a positive constant η such that*

$$E(t) \leq 3E(0)e^{-\frac{1}{3}\eta t} , \quad t \geq 0.$$

The existence of solutions follows by applying a special basis of $V \cap H^2(\Omega)$, using the Strauss'approximations of continuous functions and the Tartar's method. The decay of solutions is derived by the multiplier method.

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A QUASILINEAR PROBLEM WITH CONVECTIVE TERM

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1 Introduction

This work is concerned with the existence of solutions and estimates on the existence intervals concerning the parameters for the problem

$$\begin{cases} -\Delta_p u = a(x)f(u) + \lambda b(x)g(u) + \mu V(x, \nabla u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, $1 < p < \infty$, is the usual p -Laplacian operator; $\lambda > 0$; $\mu \geq 0$ are real parameters; $f, g : (0, \infty) \rightarrow [0, \infty)$; $a, b : \Omega \rightarrow [0, \infty)$; $a, b \neq 0$ and $V : \Omega \times \mathbb{R}^N \rightarrow [0, \infty)$ are continuous functions satisfying appropriate hypotheses and $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain.

For $\mu = 0$, problems like (1.1) have been studied intensively in recent years including nonlinearities that behave like sublinear and superlinear at zero and/or infinity and singular terms in zero. We quote [1, 3, 4, 5, 6] and references therein.

However, there are not many results in the case where the nonlinearities depend on the gradients of the solutions, that is, $\mu \neq 0$, with $p \neq 2$. In general, variational techniques are not suitable to handle (1.1). In the case $p = 2$, an interesting exception can be seen in [2].

Due principally to the difficulty in to apply standard comparison principles to the problem (1.1), because of the p -Laplacian operator and of the generalities on terms f and g permitted by us, it was not natural to hope that the lower and upper solutions standard method worked. Yet, after a careful sophisticate construction of many auxiliary functions for these terms, we were able to build an upper solution for problem (1.1) and to compare it with a lower solution built following pattern arguments.

The majority of papers dealing problems with dependence on the gradient are focused in the situations where the nonlinearities f and g have a sublinear behavior both at 0 and at $+\infty$. We solve (1.1) in the presence of the gradient term and for a larger class of functions f and g , including also nonlinearities that have a superlinear behavior at 0 and at ∞ or still an asymptotically linear behavior.

We emphasize that our results do not require any monotonicity condition and (or) singularity of the functions f and g , but we are particularly interested in the cases that f and g may have singularity at 0. We address the reader to [7].

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IMPULSIVE NEUTRAL FUNCTIONAL DYNAMIC EQUATIONS ON TIME SCALES

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1 Introduction

In the paper [3], the authors prove that the equations called *measure neutral functional differential equations* which integral form is given by

$$N(t)x_t - N(t_0)x_0 = \int_{t_0}^t f(x_s, s)dg(s)$$

can be regarded as generalized ordinary differential equations (generalized ODEs), with $N(t) : G([-r, 0], \mathbb{R}^n) \rightarrow \mathbb{R}^n$ given by $N(t)\varphi = \varphi(0) - \int_{-r}^0 d_\theta[\mu(t, \theta)]\varphi(\theta)$, where $G([-r, 0], \mathbb{R}^n)$ is the set of all regulated functions $\varphi : [-r, 0] \rightarrow \mathbb{R}^n$. Moreover μ is a left-continuous function in $\theta \in (-r, 0)$, of bounded variation on $\theta \in [-r, 0]$, and $\text{Var}_{[-s, 0]} \mu \rightarrow 0$, as $s \rightarrow 0$.

In this work we will assume that g is a left-continuous function and consider the possibility of adding impulses at preassigned times t_1, \dots, t_m , where $t_0 \leq t_1 < \dots < t_m < t_0 + \sigma$. For every $k \in \{1, \dots, m\}$, the impulse at t_k is described by the operator $I_k : \mathbb{R}^n \rightarrow \mathbb{R}^n$. In other words, the solution x should satisfy $\Delta^+ x(t_k) = I_k(x(t_k))$. This leads us to consider the impulsive measure neutral functional differential equation:

$$\begin{cases} D[N(x_t, t)] = f(x_t, t)Dg, \text{ whenever } u, v \in J_k \text{ for some } k \in \{0, \dots, m\}, \\ \Delta^+ x(t_k) = I_k(x(t_k)), \quad k \in \{1, \dots, m\}, \\ x_{t_0} = \phi, \end{cases} \quad (1.1)$$

where $J_0 = [t_0, t_1]$, $J_k = (t_k, t_{k+1}]$ for $k \in \{1, \dots, m-1\}$, and $J_m = (t_m, t_0 + \sigma]$. Without loss of generality, we can assume that g is such that $\Delta^+ g(t_k) = 0$ for every $k \in \{1, \dots, m\}$. Since g is a left-continuous function, it follows that g is continuous at t_1, \dots, t_m . Under this assumption, our problem can be rewritten as

$$\begin{cases} x(t) = x(t_0) + \int_{t_0}^t f(x_s, s)dg(s) + \int_{-r}^0 d[\mu(t, \theta)]x(t+\theta) - \int_{-r}^0 d[\mu(t_0, \theta)]\varphi(t_0+\theta) + \sum_{\substack{k \in \{1, \dots, m\}, \\ t_k < t}} I_k(x(t_k)), \\ x_{t_0} = \phi. \end{cases}$$

2 Integration on time scales

A time scale is a closed nonempty subset \mathbb{T} of the real line. For every $t \in \mathbb{T}$, we define the forward jump operator by $\sigma(t) = \inf\{s \in \mathbb{T}, s > t\}$ and the backward jump operator by $\rho(t) = \sup\{s \in \mathbb{T}, s < t\}$; we make the convention that $\inf \emptyset = \sup \mathbb{T}$ and $\sup \emptyset = \inf \mathbb{T}$. The graininess function is defined as $\mu(t) = \sigma(t) - t$.

A function $f : \mathbb{T} \rightarrow \mathbb{R}$ is called rd-continuous, if it is regulated on \mathbb{T} and continuous at right-dense points of \mathbb{T} .

For each pair of numbers $a, b \in \mathbb{T}$, $a \leq b$, let $[a, b]_{\mathbb{T}} = [a, b] \cap \mathbb{T}$. Given a set $B \subset \mathbb{R}^n$, the symbol $G([a, b]_{\mathbb{T}}, B)$ will be used to denote the set of all regulated functions $f : [a, b]_{\mathbb{T}} \rightarrow B$.

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In the time scale calculus, the usual derivative $f'(t)$ and integral $\int_a^b f(t) dt$ of a function $f : [a, b] \rightarrow \mathbb{R}$ are replaced by the Δ -derivative $f^\Delta(t)$ and Δ -integral $\int_a^b f(t) \Delta t$, where $f : [a, b]_{\mathbb{T}} \rightarrow \mathbb{R}$. Similarly to the classical case, there exist various definitions of the Δ -integral $\int_a^b f(t) \Delta t$, such as the Riemann Δ -integral or Lebesgue Δ -integral; these definitions as well as the definition of the Δ -derivative can be found in [1], [2]. The more general Kurzweil-Henstock Δ -integral was introduced in [4].

Given a real number $t \leq \sup \mathbb{T}$, let $t^* = \inf\{s \in \mathbb{T}; s \geq t\}$. Since \mathbb{T} is a closed set, we have $t^* \in \mathbb{T}$. Further, let

$$\mathbb{T}^* = \begin{cases} (-\infty, \sup \mathbb{T}] & \text{if } \sup \mathbb{T} < \infty, \\ (-\infty, \infty) & \text{otherwise.} \end{cases}$$

Finally, given a function $f : \mathbb{T} \rightarrow \mathbb{R}^n$, we consider its extension $f^* : \mathbb{T}^* \rightarrow \mathbb{R}^n$ given by $f^*(t) = f(t^*)$, $t \in \mathbb{T}^*$.

The following theorem from [5] describes the relation between the Δ -integral and the Kurzweil-Henstock-Stieltjes integral.

Theorem 2.1. *Let $f : \mathbb{T} \rightarrow \mathbb{R}^n$ be an rd-continuous function. Choose an arbitrary $a \in \mathbb{T}$ and define $F_1(t) = \int_a^t f(s) \Delta s$, $t \in \mathbb{T}$, and $F_2(t) = \int_a^t f^*(s) dg(s)$, $t \in \mathbb{T}^*$, where $g(s) = s^*$ for every $s \in \mathbb{T}^*$. Then $F_2 = F_1^*$.*

3 Mathematical Results

The aim of this work is to prove the following theorem which describes the relation between impulsive neutral functional dynamic equations and impulsive measure neutral functional differential equations.

Theorem 3.1. *Let $[t_0 - r, t_0 + \sigma]_{\mathbb{T}}$ be a time scale interval, $t_0 \in \mathbb{T}$, $B \subset \mathbb{R}^n$, $f : G([-r, 0], B) \times [t_0, t_0 + \sigma]_{\mathbb{T}} \rightarrow \mathbb{R}^n$, $\phi \in G([t_0 - r, t_0]_{\mathbb{T}}, B)$. Define $g(s) = s^*$ for every $s \in [t_0, t_0 + \sigma]$. Moreover, suppose the normalized function $\mu : \mathbb{T} \times \mathbb{T} \rightarrow \mathbb{R}^{n \times n}$ is such that $\mu(t, \cdot)$ is left-continuous on $(-r, 0)_{\mathbb{T}}$, of bounded variation on $[-r, 0]_{\mathbb{T}}$ and $\text{Var}_{[s, 0]} \mu(t, \cdot) \rightarrow 0$, as $s \rightarrow 0$. If $x : [t_0 - r, t_0 + \sigma]_{\mathbb{T}} \rightarrow B$ is a solution of the impulsive neutral functional dynamic equation*

$$x(t) = x(t_0) + \int_{t_0}^t f(x_s^*, s) \Delta s + \int_{-r}^0 \Delta_\theta [\mu(t, \theta)] x^*(t + \theta) - \int_{-r}^0 \Delta_\theta [\mu(t, \theta)] x^*(t_0 + \theta) + \sum_{\substack{k \in \{1, \dots, m\}, \\ t_k < t}} I_k(x(t_k)), \quad t \in [t_0, t_0 + \sigma]_{\mathbb{T}}, \quad (3.2)$$

$$x(t) = \phi(t), \quad t \in [t_0 - r, t_0]_{\mathbb{T}}, \quad (3.3)$$

then $x^* : [t_0 - r, t_0 + \sigma] \rightarrow B$ is a solution of the impulsive measure neutral functional differential equation

$$y(t) = y(t_0) + \int_{t_0}^t f(y_s, s^*) dg(s) + \int_{-r}^0 d_\theta [\mu^*(t, \theta)] y(t + \theta) - \int_{-r}^0 d_\theta [\mu^*(t, \theta)] y(t_0 + \theta) + \sum_{\substack{k \in \{1, \dots, m\}, \\ t_k < t}} I_k(y(t_k)), \quad t \in [t_0, t_0 + \sigma], \quad (3.4)$$

$$y_{t_0} = \phi^*. \quad (3.5)$$

Conversely, if $y : [t_0 - r, t_0 + \sigma] \rightarrow B$ satisfies (3.4) and (3.5), then it must have the form $y = x^*$, where $x : [t_0 - r, t_0 + \sigma]_{\mathbb{T}} \rightarrow B$ is a solution of (3.2) and (3.3).

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GENERAL RATES OF DECAY TO A CLASS OF VISCOELASTIC KIRCHHOFF PLATES

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1 Introduction

In the present work we discuss the well-posedness and the asymptotic behavior of energy to the following nonlinear viscoelastic Kirchhoff plate equation

$$u_{tt} - \Delta u_{tt} + \Delta^2 u - \operatorname{div} F(\nabla u) - \int_0^t g(t-s) \Delta^2 u(s) ds = 0 \quad \text{in } \Omega \times \mathbb{R}^+, \quad (1.1)$$

with simply supported boundary condition

$$u = \Delta u = 0 \quad \text{on } \partial\Omega \times \mathbb{R}^+, \quad (1.2)$$

and initial conditions

$$u(\cdot, 0) = u_0 \quad \text{and} \quad u_t(\cdot, 0) = u_1 \quad \text{in } \Omega, \quad (1.3)$$

where Ω is a bounded domain of \mathbb{R}^N with smooth boundary $\partial\Omega$. Here, $F : \mathbb{R}^N \rightarrow \mathbb{R}^N$ is a vector field and $g : [0, \infty) \rightarrow \mathbb{R}^+$ is a real function satisfying proper conditions. The term $\operatorname{div} F(\nabla u)$ constitutes a nonlinear perturbation (of non-locally Lipschitz type) and it contemplates the p -Laplacian operator as a particular case when $F(z) = |z|^p z$, $p \geq 0$. Besides, the only damping effect is given by the memory term and no additional weak or strong dissipation is necessary to show general decay rates of energy. See [4, 5] for viscoelastic wave equations. Our main results deal with the well-posedness and stability of energy by showing that its decay is similar to the memory kernel g . They improve those ones given in [1, 2, 3, 6].

2 Results

Let us first precise the hypotheses on g and F .

Assumption A1. $g : [0, \infty) \rightarrow \mathbb{R}^+$ is a C^1 -function such that $g(0) > 0$, $l := 1 - \int_0^\infty g(s) ds > 0$, and there exist a constant $\xi_0 > 0$ and a C^1 -function $\xi : [0, \infty) \rightarrow \mathbb{R}^+$ such that

$$g'(t) \leq -\xi(t)g(t), \quad \forall t > 0, \quad (2.4)$$

and

$$\xi(t) > 0, \quad \xi'(t) \leq 0, \quad \left| \frac{\xi'(t)}{\xi(t)} \right| \leq \xi_0, \quad \forall t \geq 0. \quad (2.5)$$

Assumption A2. $F : \mathbb{R}^N \rightarrow \mathbb{R}^N$ is a C^1 -vector field given by $F = (F_1, \dots, F_N)$ such that

$$|\nabla F_j(z)| \leq k_j(1 + |z|^{(p_j-1)/2}), \quad \forall z \in \mathbb{R}^N, \quad (2.6)$$

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where, for every $j = 1, \dots, N$, we consider $k_j > 0$ and p_j satisfying

$$p_j \geq 1 \quad \text{if } N = 1, 2 \quad \text{and} \quad 1 \leq p_j \leq \frac{N+2}{N-2} \quad \text{if } N \geq 3. \quad (2.7)$$

Moreover, F is a conservative vector field with $F = \nabla f$, where $f : \mathbb{R}^N \rightarrow \mathbb{R}$ is a real valued function satisfying

$$-\alpha_0 - \frac{\alpha l}{2} |z|^2 \leq f(z) \leq F(z) \cdot z + \frac{\alpha l}{2} |z|^2, \quad \forall z \in \mathbb{R}^N, \quad (2.8)$$

with $\alpha_0 \geq 0$ and $\alpha \in [0, \lambda)$, where λ is the embedding constant for $H^2(\Omega) \cap H_0^1(\Omega) \hookrightarrow H_0^1(\Omega)$.

Our first main result establishes the Hadamard well-posedness of (1.1)-(1.3) with respect to weak solutions.

Theorem 2.1 (Well-Posedness). *Under Assumptions A1 and A2 we have:*

(i) *If $(u_0, u_1) \in \mathcal{H} := (H^2(\Omega) \cap H_0^1(\Omega)) \times H_0^1(\Omega)$, then problem (1.1)-(1.3) has a weak solution in the class*

$$(u, u_t) \in L_{loc}^\infty(\mathbb{R}^+, (H^2(\Omega) \cap H_0^1(\Omega)) \times H_0^1(\Omega)). \quad (2.9)$$

(ii) *Given $U_0 = (u_0, u_1), V_0 = (v_0, v_1) \in \mathcal{H}$, let us consider the weak solutions $U = (u, u_t), V = (v, v_t)$ of the problem (1.1)-(1.3). Then*

$$\|U(t) - V(t)\|_{\mathcal{H}} \leq C \|U_0 - V_0\|_{\mathcal{H}}, \quad \forall t \in [0, T], \quad T > 0, \quad (2.10)$$

where $C = C(\|U_0\|_{\mathcal{H}}, \|V_0\|_{\mathcal{H}}, T) > 0$. In particular, problem (1.1)-(1.3) has a unique weak solution.

The energy corresponding to the problem with rotational inertia is defined as

$$E(t) = \frac{1}{2} \|u_t(t)\|_2^2 + \frac{1}{2} \|\nabla u_t(t)\|_2^2 + \frac{h(t)}{2} \|\Delta u(t)\|_2^2 + \frac{1}{2} (g \square \Delta u)(t) + \int_{\Omega} f(\nabla u(t)) dx, \quad (2.11)$$

where $h(t) = 1 - \int_0^t g(s) ds \geq l$ and $(g \square w)(t) := \int_0^t g(t-s) \|w(t) - w(s)\|_2^2 ds$.

Our second main result establishes the following general decay rate of the energy.

Theorem 2.2 (Stability). *Under the assumptions of Theorem 2.1, let (u, u_t) be the weak solution of problem (1.1)-(1.3) with $(u_0, u_1) \in \mathcal{H}$. Then there exist constants $K > 0$ and $\gamma > 0$ such that*

$$E(t) \leq K E(0) e^{-\gamma \int_0^t \xi(s) ds}, \quad \forall t \geq 0. \quad (2.12)$$

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DOMÍNIOS FINOS E REAÇÕES CONCENTRADAS NA FRONTEIRA

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Neste trabalho analizamos o comportamento assintótico de uma família de soluções de uma equação elíptica semilinear com condições de contorno homogênea de Neumann definida num domínio fino de \mathbb{R}^2 , que se degenera no intervalo unitário quando um parâmetro ϵ vai a zero. Além disso, assumimos que termos de reação na equação também estão concentrados numa vizinhança ϵ -oscilante de uma porção do contorno do domínio fino. Discutimos aqui a existência de um problema limite unidimensional, que aproxima o problema original, capturando a geometria do domínio fino, bem como o comportamento oscilatório da vizinhança onde as reações se concentram.

1 Introdução

Aqui discutimos o comportamento assintótico das soluções do seguinte problema elíptico semilinear:

$$\begin{aligned} -\Delta u^\epsilon + u^\epsilon &= f(u^\epsilon) + \frac{1}{\epsilon^\alpha} \chi_{\theta_\epsilon} g(u^\epsilon) && \text{em } R^\epsilon \\ \frac{\partial u^\epsilon}{\partial \nu^\epsilon} &= 0 && \text{sobre } \partial R^\epsilon. \end{aligned} \quad (1.1)$$

O domínio de definição das soluções R^ϵ é um domínio fino padrão dado por

$$R^\epsilon = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \in (0, 1), -\epsilon b(x_1) < x_2 < \epsilon G(x_1)\} \quad (1.2)$$

em que G e $b : (0, 1) \mapsto \mathbb{R}^+$ são funções suaves e positivas, uniformemente limitadas com $0 < G_0 \leq G(x) \leq G_1$ e $0 < b_0 \leq b(x) \leq b_1$ para todo $x \in (0, 1)$ e constantes positivas fixas G_0, G_1, b_0, b_1 . O vetor $\nu^\epsilon = (\nu_1^\epsilon, \nu_2^\epsilon)$ é normal a ∂R^ϵ , apontando para fora de R^ϵ , e $\frac{\partial}{\partial \nu^\epsilon}$ é a derivada normal. Note que as funções b e G , independentes de ϵ , definem o contorno inferior e superior do domínio fino respectivamente, e $R^\epsilon \subset (0, 1) \times (-\epsilon b_1, \epsilon G_1)$ se degenera no intervalo $(0, 1)$ quando $\epsilon \rightarrow 0$. As não-linearidades f e $g : \mathbb{R} \mapsto \mathbb{R}$ são de classe C^2 , e a função $\chi_{\theta_\epsilon} : \mathbb{R}^2 \mapsto \mathbb{R}$ é a função característica de uma vizinhança θ_ϵ do contorno superior de R^ϵ dada por

$$\theta_\epsilon = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \in (0, 1), \epsilon(G(x_1) - \epsilon^\alpha H_\epsilon(x_1)) < x_2 < \epsilon G(x_1)\}, \quad (1.3)$$

onde $\alpha > 0$ é um parâmetro fixo, $H_\epsilon : (0, 1) \mapsto \mathbb{R}^+$ é uma função suave não-negativa satisfazendo $0 \leq H_\epsilon(x) \leq G_0 + b_0$ para todo $x \in (0, 1)$ e $\epsilon > 0$. H_ϵ também pode oscilar quando $\epsilon \rightarrow 0$. Expressamos tal propriedade pela expressão:

$$H_\epsilon(x) = H(x, x/\epsilon^\beta), \quad \beta > 0, \quad (1.4)$$

com $H : (0, 1) \times \mathbb{R} \mapsto \mathbb{R}$ não-negativa e suave. Também assumimos que H é $l(x)$ -periódica na variável y para cada $x \in (0, 1)$, isto é, $H(x, y + l(x)) = H(x, y)$ para todo y , onde a função periódica l é positiva e uniformemente limitada com $0 < l_0 \leq l(x) \leq l_1$.

É fácil de ver que o conjunto aberto θ_ϵ é uma vizinhança do contorno superior de R^ϵ com espessura e comportamento oscilatório dependentes dos parâmetros positivos α and β respectivamente. α representa a ordem da espessura e β a ordem da oscilação da vizinhança θ_ϵ para $\epsilon \approx 0$. Note ainda que se H só depende da primeira variável x , então a função H_ϵ é independente de ϵ , e assim, a faixa estreita θ_ϵ não apresenta comportamento oscilatório. Procedemos como em [1, 2], usamos a função característica χ_ϵ e os parâmetros positivos ϵ e α para modelarmos a concentração da reação dada pela não-linearidade g em $\theta_\epsilon \subset R^\epsilon$, com $\epsilon \approx 0$, pelo termo $\frac{1}{\epsilon^\alpha} \chi_{\theta_\epsilon} \in L^\infty(R^\epsilon)$.

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Como R^ϵ é fino e se degenera no intervalo unitário quando $\epsilon \rightarrow 0$, é razoável esperar que a família de soluções u^ϵ se aproxime da solução de um problema unidimensional em $(0, 1)$ do mesmo tipo e com condições de contorno de Neumann capturando o perfil variável do domínio fino e o comportamento oscilatório da faixa estreita θ_ϵ . Com efeito, podemos mostrar que o problema limite para (1.1) é dado pela seguinte equação unidimensional:

$$-\frac{1}{p(x)}(p(x)u_x)_x + u = f(u) + \frac{\mu(x)}{p(x)}g(u) \quad \text{em } (0, 1) \quad (1.5)$$

com $u_x(0) = u_x(1) = 0$, onde p e $\mu : (0, 1) \mapsto (0, \infty)$ são definidas por

$$p(x) = G(x) + b(x), \quad \text{e} \quad \mu(x) = \frac{1}{l(x)} \int_0^{l(x)} H(x, y) dy. \quad (1.6)$$

A função p está associada a geometria do domínio fino R^ϵ , estabelecida pelas funções b e G . Já o coeficiente não-negativo $\mu \in L^\infty(0, 1)$ está relacionado a vizinhança oscilante θ_ϵ definida por H_ϵ . Logo obtemos um problema limite que captura o perfil variável de R^ϵ bem como o comportamento oscilante de θ_ϵ combinando resultados anteriormente obtidos em [3, 4]. Note que se H não depende da segunda variável, então a vizinhança θ_ϵ não apresenta comportamento oscilatório implicando $\mu(x) = H(x)$ em $(0, 1)$. Além disso, se supomos $H \equiv 0$, temos que o problema (1.1) não apresenta ‘reações concentradas’ coincidindo com problemas considerados em [3, 5].

2 Resultado principal

Para estudarmos (1.1) realizamos uma mudança de variável chegando ao seguinte problema

$$\begin{aligned} -\frac{\partial^2 u^\epsilon}{\partial x_1^2} - \frac{1}{\epsilon^2} \frac{\partial^2 u^\epsilon}{\partial x_2^2} + u^\epsilon &= f(u^\epsilon) + \frac{1}{\epsilon^\alpha} \chi_{o_\epsilon} g(u^\epsilon) \quad \text{em } \Omega \\ \frac{\partial u^\epsilon}{\partial x_1} N_1 + \frac{1}{\epsilon^2} \frac{\partial u^\epsilon}{\partial x_2} N_2 &= 0 \quad \text{sobre } \partial\Omega \end{aligned} \quad (2.7)$$

onde χ_{o_ϵ} é a função característica da faixa $o_\epsilon = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \in (0, 1), (G(x_1) - \epsilon^\alpha H_\epsilon(x_1)) < x_2 < G(x_1)\}$, $N = (N_1, N_2)$ é o vetor normal unitário de $\partial\Omega$ para $\Omega = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 \in (0, 1), -b(x_1) < x_2 < G(x_1)\}$.

A equivalência dos problemas (1.1) e (2.7) é obtida pela mudança de variável $(x, y) \rightarrow (x, \epsilon^{-1}y)$ que estica R^ϵ na direção y por um fator ϵ^{-1} , levando o problema a um domínio fixo Ω independente de ϵ . O fator ϵ^{-2} em frente a derivada na direção x_2 estabelece uma difusão muito alta quando $\epsilon \rightarrow 0$, homogeneizando a solução nessa direção, fazendo com que a solução no limite seja independente de x_2 , sendo então solução de um problema unidimensional.

Teorema 2.1. *Seja u^ϵ uma família de soluções dada por (2.7) com $\|u^\epsilon\|_{L^\infty(\Omega)} \leq R$. Então:*

- (i) *Existe sub-sequência, ainda denotada por u^ϵ , e uma função $u \in H^1(\Omega)$, $\|u\|_{L^\infty(\Omega)} \leq R$, $u(x_1, x_2) = u(x_1)$, solução de (1.5), tal que $\|u^\epsilon - u\|_{H^1(\Omega)} \rightarrow 0$ quando $\epsilon \rightarrow 0$.*
- (ii) *Se uma solução u de (1.5) pertencente a bola de raio R em $L^\infty(\Omega)$ é ainda hiperbólica, então, existe uma sequência u^ϵ de soluções do problema (2.7) com $\|u^\epsilon - u\|_{H^1(\Omega)} \rightarrow 0$.*

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ONDAS VIAJANTES PERIÓDICAS PARA VERSÕES GENERALIZADAS E NÃO HOMOGÊNEAS DAS EQUAÇÕES BBM E KDV

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1 Introdução

Neste trabalho estamos interessados em obter alguns resultados de existência de onda viajante periódica para problemas não homogêneos de versões generalizadas das equações de Benjamin-Bona-Mahony (BBM)

$$u_t + (f_0(u))_x - \epsilon u_{xx} - \delta u_{xxt} = h_0(x - \beta t), \quad x \in \mathbb{R}, t \geq 0, \quad (1.1)$$

e de Korteweg-de Vries-Burgers (KdVB)

$$u_t + (f_0(u))_x - \epsilon u_{xx} - \delta u_{xxx} = h_0(x - \beta t), \quad x \in \mathbb{R}, t \geq 0, \quad (1.2)$$

onde $\epsilon > 0$, $\delta > 0$ e $\beta > 0$ são constantes, $f_0 \in C^1(\mathbb{R})$ e h_0 é uma função contínua em \mathbb{R} , não identicamente nula, $2T$ -periódica para algum $T > 0$ e com a propriedade

$$\int_0^{2T} h_0(x) dx = 0. \quad (1.3)$$

As ondas viajantes periódicas que buscamos aqui são soluções da forma

$$u(x, t) = u(x - \beta t) = v(\eta), \quad \eta = x - \beta t, \quad (1.4)$$

onde v é periódica. Provamos que escolhendo β de forma apropriada, as equações (1.1) e (1.2) admitirão onda viajante periódica com velocidade de propagação igual a β .

2 Resultados

Fixando uma primitiva \bar{h}_0 para a função h_0 e tomando

$$M = \sup_{(w, \xi) \in [-1, 1] \times [0, 2T]} |f_0(w) - \bar{h}_0(\xi)|, \quad (2.5)$$

os resultados que obtemos neste trabalho, foram os seguintes:

Teorema 2.1. *Se $2M \leq \beta$, então a equação (1.1) admite onda viajante periódica.*

Teorema 2.2. *Para quaisquer dos três casos:*

- (i) $2M \leq \beta < \frac{\epsilon^2}{4\delta}$,
- (ii) $\left[\frac{3T^2\epsilon^2}{\delta^2(1 - e^{-\frac{\epsilon T}{\delta}})^2} + \frac{2T^2\epsilon^2}{\delta^2(1 - e^{-\frac{\epsilon T}{\delta}})} + 1 \right] M \leq \beta = \frac{\epsilon^2}{4\delta}$,
- (iii) $\left[\frac{8T}{\sqrt{4\beta\delta - \epsilon^2(1 - e^{-\frac{\epsilon T}{\delta}})^2}} + \frac{1}{\beta} \right] M \leq 1$ e $\beta > \frac{\epsilon^2}{4\delta}$,

a equação (1.2) admite onda viajante periódica.

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Fazendo uma breve discussão a respeito das provas dos Teoremas 2.1 e 2.2, começamos observando que pela caracterização das ondas viajantes em (1.4), as equações (1.1) e (1.2) são reduzidas respectivamente as EDO's de terceira ordem

$$v''' - \alpha v'' - \lambda^2 v' = -(f_1(v))' + h_1, \quad (2.6)$$

onde $h_1(v) = (\delta\beta)^{-1}h_0(v)$, $f_1(v) = (\delta\beta)^{-1}f_0(v)$, $\lambda^2 = \delta^{-1}$ e $\alpha = \epsilon/(\delta\beta)$;

$$v''' + \theta v'' + \rho^2 v' = (f_2(v))' - h_2, \quad (2.7)$$

onde $h_2(v) = \delta^{-1}h_0(v)$, $f_2(v) = \delta^{-1}f_0(v)$, $\rho^2 = \beta/\delta$ e $\theta = \epsilon/\delta$.

Daí, de posse das equações (2.6) e (2.7) podemos trabalhar no intervalo $[0, 2T]$ e adicionando a condição de fronteira

$$v^{(k)}(0) = v^{(k)}(2T), \quad k = 0, 1, 2, \quad (2.8)$$

verificar que quaisquer soluções dos problemas de fronteira (2.6)-(2.8) e (2.7)-(2.8) no intervalo $[0, 2T]$ podem ser estendidas em \mathbb{R} para funções que serão ondas viajantes periódicas das equações (1.1) e (1.2) respectivamente. Então utilizando o método da função de Green, veja [6], para os problemas (2.6)-(2.8) e (2.7)-(2.8), encontramos funções que nos servirão como núcleo de operadores integrais cujos pontos fixos serão soluções para os problemas (2.6)-(2.8) e (2.7)-(2.8). Concluindo com o teorema do ponto fixo de Schauder, veja [5], e as boas propriedades que obteremos das funções de Green, obtemos as provas dos Teorema 2.1 e 2.2.

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ANÁLISE NUMÉRICA DE UM MODELO DE MEMBRANAS ELÁSTICAS COM FRONTEIRA MÓVEL

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1 Introdução

Neste trabalho, estabeleceremos uma estimativa de erro para uma nova derivação da equação da onda para pequenas vibrações transversais de membranas elásticas.

Os modelos estudados foram desenvolvidos, por [5] e [6] como uma extensão do modelo de Kirchhoff-Carrier, uma vez que leva em conta a troca de tamanho durante a vibração e o comportamento não linear das membranas elásticas em geral. O operador que define este modelo matemático é dado por:

$$\hat{L}u(x, t) = \frac{\partial^2 u}{\partial t^2} - \left(a(t) + b(t) \int_{\Omega_t} |\nabla u|^2 dx \right) \Delta u = 0 \quad (1.1)$$

num domínio não cilíndrico \hat{Q} do \mathbb{R}^3 , onde as funções $a(t)$ e $b(t)$ são dadas por

$$a(t) = \frac{\tau_0}{m} + \frac{k}{m} \frac{K^2(t) - K_0^2}{K_0^2}, \quad b(t) = \frac{k}{2m\pi K^2(t)}, \quad (1.2)$$

Ω_t um domínio circular de raio $K(t)$, τ_0 a tensão inicial, m a massa, $k = \frac{E}{2(1-\sigma)}$ sendo E o módulo de Young do material e σ o coeficiente de Poisson.

As investigações sobre pequenas vibrações de um corpo elástico foi inicialmente proposto por D'Alembert, Euler e depois, considerando os pequenos deslocamentos verticais do corpo durante a vibração, temos os modelos de Kirchhoff e Carrier, sendo estes modelos com extremidades fixas. A análise de existência, unicidade e o comportamento assintótico de solução para a vibração da corda elástica com fronteira móvel em um dos extremos e com coeficiente constante foi analisado em [1] e [4], com o operador definido por;

$$Lu(x, t) = \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \quad \text{para} \quad 0 \leq x \leq vt \quad (1.3)$$

O problema da corda não linear, com fronteira móvel, que contém o modelo de Kirchhoff e do Carrier, foi desenvolvido [5] e [6], onde foi analisado a existência e unicidade local. Soluções numéricas do modelo unidimensional (1.3), usando o método das diferenças finitas, foi feita em [8].

Consideremos uma membrana elástica Ω identificada como um disco unitário de \mathbb{R}^2 com centro no origem e $K : [0, T] \rightarrow \mathbb{R}$ uma função C^2 . Representemos por Ω_t as deformações do disco unitário Ω por a função $K(t)$, isto é $\Omega_t = \{x = (x_1, x_2) \in \mathbb{R}^2 \mid x = K(t)y, \forall y = (y_1, y_2) \in \Omega, t \geq 0\}$. Representamos por Ω_0 um disco $K_0\Omega$ onde $K_0 = K(0)$. Consideremos o domínio não cilíndrico \hat{Q} do \mathbb{R}^3 definido por $\hat{Q} = \bigcup_{0 < t < T} \Omega_t \times \{t\}$ e sua fronteira lateral $\hat{\Sigma}$ é definida por $\hat{\Sigma} = \bigcup_{0 < t < T} \Gamma_t \times \{t\}$, onde Γ_t denota a fronteira $\partial\Omega_t$. Nesse trabalho propomos fazer uma estimativa de erro em Espaços de Sobolev para o problema

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$$(I) \quad \begin{cases} \hat{L}u(x, t) + \delta \left(\frac{K'(t)}{K(t)} x_i \frac{\partial u(x, t)}{\partial x_i} + u'(x, t) \right) = 0, & \forall (x, t) \in \hat{Q} \\ u(x, t) = 0, & \forall (x, t) \in \hat{\Sigma} \\ u(x, 0) = u_0(x), \quad u'(x, 0) = u_1(x), & \forall x \in \Omega_0 = K_0 \Omega, \end{cases}$$

onde $\delta > 0$. Considere as seguintes hipóteses sob $K(t)$:

(H1) $K \in C^2([0, T]; \mathbb{R})$, $K(t) \geq K_0 \geq 1$,

(H2) $|K''(t)| \leq C_0 \frac{(K')^2}{K}$,

(H3) $0 \leq K'(t) \leq K_1$, $\forall 0 \leq t \leq T$,

(H4) $\|u_h(0) - \tilde{u}_0\| \leq \bar{c}h\|u(0)\|$; $|u'_h(0) - \tilde{u}'_0| \leq \hat{c}h|u'(0)|$,

onde $(')$ denota a derivada no tempo e K_1 satisfaz, $K_1^2 \leq (\tau_0 - k)/2m$. Além disso \tilde{u}_0 e \tilde{u}'_0 são as interpolações das condições iniciais u_0 e u'_0 , \bar{c} e \hat{c} são constantes positivas independentes de h .

Representemos por $((,)), \|.\|; (,), |.|$ respectivamente o produto escalar e a norma em $H_0^1(\Omega)$ e $L^2(\Omega)$.

2 Teorema Principal

Teorema 2.1. Se u é solução do problema (I) com dados iniciais $u_0 \in H_0^1(\Omega_0) \cap H^2(\Omega_0)$, $u_1 \in H_0^1(\Omega_0)$, e suponhamos que $u \in L^\infty(0, T, H_0^1(\Omega_t) \cap H^2(\Omega_t))$, $u', u'' \in L^\infty(0, T, H^2(\Omega_t))$ sob as hipóteses (H1)-(H4), então existe uma constante positiva C dependente de u e independente de h tal que

$$|u' - u'_h|_{L^\infty((0, T); L^2(\Omega_t))} + \|u - u_h\|_{L^\infty((0, T); H_0^1(\Omega_t))} \leq Ch. \quad (2.4)$$

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ASYMPTOTIC BEHAVIOR FOR EVOLUTION EQUATIONS OF NEURAL FIELD TYPE

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1 Introduction

In this work we consider the non local evolution equation

$$\frac{\partial u}{\partial t}(x, t) = -u(x, t) + J * (f \circ u)(x, t) + h, \quad h > 0, \quad (1.1)$$

where $u(x, t)$ is a real-valued function on $\mathbb{R}^N \times \mathbb{R}_+$, h is a positive constant, $J \in C^1(\mathbb{R}^N)$ is a non negative even function supported in the ball with center at the origin and radius 1, and f is a non negative nondecreasing function. The $*$ above denotes convolution product in \mathbb{R}^N , namely: $(J * u)(x) = \int_{\mathbb{R}^N} J(x - y)u(y)dy$.

The function $u(x, t)$ denotes the mean membrane potential of a patch of tissue located at position $x \in \mathbb{R}^N$ at time $t \geq 0$. The connection function $J(x)$ determines the coupling between the elements at position x and position y . The non negative nondecreasing function $f(u)$ gives the neural firing rate, or averages rate at which spikes are generated, corresponding to an activity level u . The neurons at a point x are said to be active if $f(u(x, t)) > 0$. The parameter h denotes an external constant stimulus applied uniformly to the entire neural field.

In this work we summarize the results of [5], where we extend, for $L^p(\mathbb{R}^N, \rho)$, $N \geq 1$ and $1 < p < \infty$, the results (on global attractors) obtained in [4] in the phase space $L^2(\mathbb{R}, \rho)$. Furthermore, we exhibit a Lyapunov functional to the flow generated by (1.1).

We assume here the following hypotheses on the functions f and J :

(H1) the function $f : \mathbb{R} \rightarrow \mathbb{R}$ is globally Lipschitz, that is, there exists $k_1 > 0$ such that

$$|f(x) - f(y)| \leq k_1|x - y|, \quad \forall x, y \in \mathbb{R}; \quad (1.2)$$

(H2) there exists $a > 0$ such that $|f(x)| \leq a$, for all $x \in \mathbb{R}$;

(H3) the non negative, symmetric bounded function J has bounded derivative with

$$\sup_{x \in \mathbb{R}^N} \int_{\mathbb{R}^N} \partial_{x_i} J(x - y)dy \leq S \quad \text{and} \quad \sup_{y \in \mathbb{R}^N} \int_{\mathbb{R}^N} \partial_{x_i} J(x - y)dx \leq S,$$

for some constant $0 < S < \infty$ and $i = 1, \dots, N$.

2 Mathematical Results

We consider the flow generated by (1.1) in the phase space $L^p(\mathbb{R}^N, \rho) = \{u \in L^1_{loc}(\mathbb{R}^N) : \int_{\mathbb{R}^N} |u(x)|^p \rho(x)dx < \infty\}$.

Lemma 2.1. *Suppose that $\sup\{\rho(x) : x \in \mathbb{R}^N, |x - y| \leq 1\} \leq K\rho(y)$, for some constant K and all $y \in \mathbb{R}^N$. Then $\|J * u\|_{L^p(\mathbb{R}^N, \rho)} \leq K^{1/p} \|J\|_{L^1} \|u\|_{L^p(\mathbb{R}^N, \rho)}$.*

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The hypothesis (H1) and Lemma 2.1 are sufficient for the function $F(u) = -u + J * (f \circ u) + h$ to be globally Lipschitz in $L^p(\mathbb{R}^N, \rho)$, hence the Cauchy problem for (1.1) is well posed in this space with a unique global solution.

In the that follows, we denote by $S(t)$ the flow generated by (1.1), given by $[S(t)u](x) = u(x, t)$, where $u(x, t)$ is, by variation constant formula, given by

$$u(x, t) = e^{-t}u(x, 0) + \int_0^t e^{s-t}[J * (f \circ u)(x, s) + h]ds.$$

We recall that a set $\mathcal{B} \subset L^p(\mathbb{R}^N, \rho)$ is an absorbing set for the flow $S(t)$ in $L^p(\mathbb{R}^N, \rho)$ if, for any bounded set $B \subset L^p(\mathbb{R}^N, \rho)$, there is a $t_1 > 0$ such that $S(t)B \subset \mathcal{B}$ for any $t \geq t_1$, (see [6]).

Proceeding as in [4], we proof the following lemmas:

Lemma 2.2. *Suppose that the hypotheses (H1) and (H2) hold and let $R = aK^{1/p}\|J\|_{L^1} + h$. Then the ball with center at the origin and radius $R + \varepsilon$ is an absorbing set for the flow $S(t)$ in $L^p(\mathbb{R}^N, \rho)$ for any $\varepsilon > 0$.*

Lemma 2.3. *Suppose that the hypotheses (H1)-(H3) hold. Then, for any $\eta > 0$, there exists t_η such that $S(t_\eta)\mathcal{B}(0, R + \varepsilon)$ has a finite covering by balls of $L^p(\mathbb{R}^N, \rho)$ with radius smaller than η .*

We recall that a set $\mathcal{A} \subset L^p(\mathbb{R}^N, \rho)$ is a global attractor if \mathcal{A} is global maximal invariant compact set which attract each bounded set in $L^p(\mathbb{R}^N, \rho)$ unde the flow $S(t)$.

Using the Lemmas 2.2 and 2.3 we obtain the main results this work.

Theorem 2.1. *Assume the same hypotheses of Lemma 2.3. Then $\mathcal{A} = \omega(\mathcal{B}(0, R + \varepsilon))$, is a global attractor for the flow $S(t)$ generated by (1.1) in $L^p(\mathbb{R}^N, \rho)$ which is contained in the ball of radius R .*

To we exhibit an energy functional for the flow of (1.1), which decreases along of solutions of (1.1), we consider the following additional hypothesis:

(H4) the function f has positive derivative, it takes values between 0 and a and satisfies $|\int_0^a f^{-1}(r)dr| < L < \infty$.

Motivated by energy functionals from [2] and [3], we define the energy functional $F : L^p(\mathbb{R}^N, \rho) \rightarrow \mathbb{R}$ by

$$F(u) = \int_{\mathbb{R}^N} \left[-\frac{1}{2}f(u(x)) \int_{\mathbb{R}^N} J(x-y)f(u(y))dy + \int_0^{f(u(x))} f^{-1}(r)dr - hf(u(x)) \right] dx. \quad (2.3)$$

Using hypothesis (H4) we prove that functional given in (2.3) it is like a Lyapunov functional.

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EXPLOSÃO E EXISTÊNCIA GLOBAL DE SOLUÇÕES PARA UMA EQUAÇÃO PARABÓLICA NÃO LINEAR EM UM DOMÍNIO QUALQUER

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1 Introdução

Seja $\Omega \subset \mathbb{R}^N$ um domínio qualquer (limitado ou não ilimitado) com fronteira $\partial\Omega$ regular. Nós analisamos a existência de soluções globais e não globais (explodem em tempo finito) para a seguinte equação parabólica

$$u_t - \Delta u = h(t)f(u) \text{ em } \Omega \times (0, T), \quad (1.1)$$

$$u = 0 \text{ em } \partial\Omega \times (0, T), \quad (1.2)$$

$$u(0) = u_0 \geq 0 \text{ em } \Omega, \quad (1.3)$$

onde $h \in C[0, \infty)$, $f \in C[0, \infty)$ é uma função locamente Lipschitz e $u_0 \in C_0(\Omega)$.

Dada $u_0 \in C_0(\Omega)$, $u_0 \geq 0$, dizemos que $u \in C([0, T], C_0(\Omega))$ é uma solução de (1.1)-(1.3) no intervalo $[0, T]$ se satisfaz

$$u(t) = S(t)u_0 + \int_0^t S(t-\sigma)h(\sigma)f(u(\sigma))d\sigma, \quad (1.4)$$

para todo $t \in [0, T]$, onde $(S(t))_{t \geq 0}$ é o semigrupo analítico do calor.

Quando $h = 1$ e $f(u) = u^p$, Weissler encontrou uma solução global não-negativa de (1.1)-(1.3), veja [4,5]. Para isto, ele escolheu $a_0 > 0$ de modo que $\bar{u}(t) = a(t)S(t)u_0$, onde

$$a(t) = \left[a_0^{-(p-1)} - (p-1) \int_0^t h(\sigma) \|S(\sigma)u_0\|_\infty^{p-1} d\sigma \right]^{-\frac{1}{p-1}},$$

é uma supersolução do problema (1.1)-(1.3) definida para todo $t \geq 0$. Além disso, Weissler mostrou que se u é uma solução de (1.1)-(1.3), com $h = 1$ e $\Omega = \mathbb{R}^N$, definida no intervalo $(0, T)$, então $\|S(t)u_0\|^{p-1}t \leq (p-1)^{-1}$ para todo $t \geq 0$. Note que as condições encontradas por Weissler são determinadas pelo comportamento assíntotico de $\|S(t)u_0\|_\infty$.

Considerando um operador fortemente elíptico em lugar do Laplaciano na equação (1.1) e supondo as seguintes hipótese sobre a função f :

$$f \in C^1[0, \infty); \quad f(s) > 0 \quad \text{para } s > 0; \quad f(0) \geq 0; \quad f' \geq 0 \text{ e } G(w) = \int_w^\infty \frac{d\sigma}{f(\sigma)} < \infty \quad \text{se } w > 0,$$

Meier [2], mostrou o seguinte resultado.

Teorema 1.1. *Assuma que f satisfaz as condições acima e $h \in C[0, \infty)$.*

(i) *Seja f convexa com $f(0) = 0$. Então a solução u de (1.1)-(1.3) explode num tempo finito, se existe $\tau > 0$ tal que*

$$G(\|S(\tau)u_0\|_\infty) \leq \int_0^\tau h(\sigma)d\sigma.$$

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(ii) Seja $f(0) > 0$. Se existe $\tau > 0$ tal que

$$G(0) \leq \|S(\tau)u_0\|_\infty \int_0^\tau \frac{h(\sigma)}{\|S(\sigma)u_0\|_\infty} d\sigma,$$

então a solução de (1.1)-(1.3) explode num tempo finito.

2 Resultados

Nossos principais resultados são os seguintes: no Teorema 2.1 são dadas as condições para a existência global de uma solução do nosso problema, e no Teorema 2.2 damos algumas condições para que uma solução de nosso problema exploda em um tempo finito. Estes resultados estendem os resultados de [1,2].

Teorema 2.1. Assuma que f é localmente Lipschitz. Suponha que existe $a > 0$ tal que as funções f e $g : (0, \infty) \rightarrow [0, \infty)$, onde g é definida por $g(s) = f(s)/s$ são não-decrescentes em $(0, a]$. Se $v_0 \in C_0(\Omega)$, $v_0 \geq 0$, $v_0 \neq 0$, $\|v_0\|_\infty \leq a$ verifica

$$\int_0^\infty h(\sigma)g(\|S(\sigma)v_0\|_\infty) d\sigma < 1,$$

então existe $u_0^* \in C_0(\Omega)$, $0 \leq u_0^* \leq v_0$ tal que para todo $u_0 \in C_0(\Omega)$, $0 \leq u_0 \leq u_0^*$, $u_0 \neq 0$ a solução de (1.1)-(1.3) é global. Além disso, existe uma constante $\gamma > 0$ tal que $u(t) \leq \gamma \cdot S(t)u_0$ para todo $t \geq 0$. Em particular, $\lim_{t \rightarrow \infty} \|u(t)\|_\infty = 0$.

Teorema 2.2. Seja f uma função localmente Lipschitz, $f(0) = 0$, $f(s) > 0$ para todo $s > 0$ e G dada por $G(w) = \int_w^\infty \frac{d\sigma}{f(\sigma)}$. Assuma que as seguintes condições são satisfeitas:

(i) A função f é não-decrescente e verifica a seguinte propriedade

$$f(S(t)v_0) \leq S(t)f(v_0),$$

para todo $v_0 \in C_0(\Omega)$, $v_0 \geq 0$ e $t > 0$.

(ii) Existe $\tau > 0$ e $u_0 \in C_0(\Omega)$, $u_0 \geq 0$, $u_0 \neq 0$ tal que

$$G(\|S(\tau)u_0\|_\infty) \leq \int_0^\tau h(\sigma) d\sigma.$$

Então a solução do problema (1.1)-(1.3) explode num tempo finito $T_{max} \leq \tau$.

Para provar o Teorema 2.1 usamos o argumento de sequência monótona de Pinsky [3]. Já na prova do Teorema 2.2, utilizamos a formulação (1.4) para obter uma desigualdade diferencial ordinária da forma $\psi' \geq h(t)f(\psi)$.

Nossas principais referências seguem listadas abaixo.

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PEANO CURVES ON TOPOLOGICAL VECTOR SPACES

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1 Introduction

The existence of a Peano curve on the unit square, that is, a continuous surjection mapping the unit interval $[0, 1]$ onto $[0, 1]^2$, allows us to construct a continuous surjection from the real line \mathbb{R} to any Euclidean space \mathbb{R}^n . The algebraic structure of the set of these functions (as well as extensions to spaces with higher dimensions) is analyzed from the modern point of view of lineability, and large algebras are found within the families studied. We investigate topological vector spaces that are continuous image of the real line and provide an optimal lineability result, from which we conclude that the topological dual (endowed with the weak star topology) of any separable normed space is a continuous image of the real line.

2 Mathematical Results

Along this we will use the following notation for any topological space X :

$$\mathcal{CS}_\infty(\mathbb{R}^m, X) := \{f \in \mathcal{C}(\mathbb{R}^m, X) : f^{-1}(\{a\}) \text{ is unbounded for every } a \in X\}.$$

In [1] and [3], the following results provides maximal lineability and spaceability, respectively, when we deal with Euclidean spaces.

Theorem 2.1 (Albuquerque, 2014). *For every pair $m, n \in \mathbb{N}$, the set $\mathcal{CS}(\mathbb{R}^m, \mathbb{R}^n)$ is maximal lineable in the space $\mathcal{C}(\mathbb{R}^m, \mathbb{R}^n)$.*

Theorem 2.2 (Bernal and Ordóñez, 2014). *For each pair $m, n \in \mathbb{N}$, the set $\mathcal{CS}_\infty(\mathbb{R}^m, \mathbb{R}^n)$ is maximal dense-lineable and spaceable in $\mathcal{C}(\mathbb{R}^m, \mathbb{R}^n)$. In particular, it is maximal lineable in $\mathcal{C}(\mathbb{R}^m, \mathbb{R}^n)$.*

In order to look for algebrability of these spaces (in the complex frame), we need some tools related with the growth of an entire function: by $\mathcal{H}(\mathbb{C})$ we denote the space of all entire functions from \mathbb{C} to \mathbb{C} . For $r > 0$ and $f \in \mathcal{H}(\mathbb{C})$, we set $M(f, r) := \max_{|z|=r} |f(z)|$. The (growth) order $\rho(f)$ of an entire function $f \in \mathcal{H}(\mathbb{C})$ is defined as the infimum of all positive real numbers α with the following property: $M(f, r) < e^{r^\alpha}$ for all $r > r(\alpha) > 0$. Note that $\rho(f) \in [0, +\infty]$. Trivially, the order of a constant map is 0. If f is non-constant, we have

$$\rho(f) = \limsup_{r \rightarrow +\infty} \frac{\log \log M(f, r)}{\log r}.$$

For a non-constant polynomial in M complex variables $P \in \mathbb{C}[z_1, \dots, z_M]$, let $\mathcal{I}_P \subset \{1, \dots, M\}$ be the set of indexes k such that the variable z_k explicitly appears in some monomial (with non-zero coefficient) of P ; that is, $\mathcal{I}_P = \{n \in \{1, \dots, M\} : \frac{\partial P}{\partial z_n} \not\equiv 0\}$. The following result (of independent interest) concerns about the order of a polynomial of several variables evaluated on entire functions with different orders

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Lemma 2.1. Let $f_1, \dots, f_M \in \mathcal{H}(\mathbb{C})$ such that $\rho(f_i) \neq \rho(f_j)$ whenever $i \neq j$. Then

$$\rho(P(f_1, \dots, f_M)) = \max_{k \in \mathcal{I}_P} \rho(f_k),$$

for all non-constant polynomials $P \in \mathbb{C}[z_1, \dots, z_M]$. Moreover, $(f_k)_{k=1}^M$ is algebraically independent and generates a free algebra.

From this, we obtain an optimal algebrability result.

Theorem 2.3. For every $m \in \mathbb{N}$, the set $\mathcal{CS}_\infty(\mathbb{R}^m, \mathbb{C}^n)$ is maximal strongly algebrable in $\mathcal{C}(\mathbb{R}^m, \mathbb{C}^n)$.

The theorem of Hahn and Mazurkiewicz provides a topological characterization of Hausdorff topological spaces that are continuous image of the unit interval $[0, 1]$: these are precisely the compact, connected, locally connected metrizable topological spaces, which are called *Peano spaces*. We introduce a notion of the spaces that are continuous images of the unit interval, as guaranteed by the next result.

Definition 2.1. A topological space X is a σ -Peano space if there exists an increasing sequence of subsets $K_1 \subset K_2 \subset \dots \subset K_m \subset \dots \subset X$, such that each one of them is a Peano space (endowed with the topology inherited from X) and its union amounts to the whole space, that is, $\bigcup_{n \in \mathbb{N}} K_n = X$.

Proposition 2.1. Let X be a Hausdorff topological space. The following assertions are equivalent:

- (a) X is a σ -Peano space.
- (b) $\mathcal{CS}_\infty(\mathbb{R}, X) \neq \emptyset$.
- (c) $\mathcal{CS}(\mathbb{R}, X) \neq \emptyset$.

We provide a maximal lineability result when we deal with arbitrary topological spaces that are σ -Peano. Consequently, as earlier mentioned, one may easily conclude that the topological dual (endowed with the weak star topology) of any separable normed space is a continuous image of the real line.

Theorem 2.4. Let \mathcal{X} be a σ -Peano topological vector space. Then $\mathcal{CS}_\infty(\mathbb{R}^m, \mathcal{X})$ is maximal lineable in $\mathcal{C}(\mathbb{R}^m, \mathcal{X})$.

Corollary 2.1. Let \mathcal{N} be a separable normed space and \mathcal{N}' be its topological dual endowed with the weak*-topology. Then $\mathcal{CS}_\infty(\mathbb{R}^m, \mathcal{N}')$ is maximal lineable.

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DINÂMICA TOPOLOGÍCA PROBABILÍSTICA DE APLICAÇÕES GENÉRICAS DO ESPAÇO DE CANTOR

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1 Introdução

O estudo de propriedades genéricas é um tema clássico na área de sistemas dinâmicos. No contexto da dinâmica topológica, tal estudo tem sido desenvolvido nos últimos quarenta anos por diversos pesquisadores. Para o caso da dinâmica genérica de aplicações contínuas definidas sobre o intervalo unitário fechado, veja [1], por exemplo. Para o caso de aplicações contínuas e homeomorfismos em variedades compactas, veja [3] e [9], onde muitas outras referências podem ser encontradas. Finalmente, para a dinâmica genérica de aplicações do espaço de Cantor, veja [2], [3], [5], [7] e [8], por exemplo.

Por outro lado, o estudo da dinâmica induzida ao espaço das medidas de probabilidade também é um tema importante nas áreas de sistemas dinâmicos e teoria ergódica, uma vez que fornece exemplos não-triviais de comportamentos dinâmicos interessantes. Para uma visão clássica do assunto, recomendamos [4] e [10].

Portanto, é natural combinarmos ambos os temas e estudarmos a dinâmica induzida às probabilidades por aplicações genéricas. No presente trabalho desenvolvemos um tal estudo para aplicações contínuas e homeomorfismos do espaço de Cantor. Para tal, assim como fizemos no estudo da dinâmica coletiva em [6], utilizamos os resultados de estrutura de grafos das aplicações contínuas genéricas e dos homeomorfismos genéricos do espaço de Cantor estabelecidos em [5].

2 Resultados

Dado um espaço métrico compacto (M, d) , denotamos por $\mathcal{C}(M)$ (resp. $\mathcal{H}(M)$) o espaço de todas as aplicações contínuas de M em M (resp. de todos os homeomorfismos de M sobre M) munido da métrica do máximo:

$$\tilde{d}(f, g) := \max_{x \in M} d(f(x), g(x)).$$

Denotamos por $\mathcal{M}(M)$ o espaço de todas as medidas de Borel probabilísticas definidas sobre M e por \mathcal{B}_M o conjunto de todos os boreelianos de M .

Definimos a distância de Prohorov sobre $\mathcal{M}(M)$ por

$$d_P(\mu, \nu) := \inf\{\delta > 0 : \mu(X) \leq \nu(X^\delta) + \delta \text{ para todo } X \in \mathcal{B}_M\},$$

onde

$$X^\delta := \{x \in M : d(x, X) < \delta\}$$

é a δ -vizinhança de X ($X \subset M$). Dada $f \in \mathcal{C}(M)$, a aplicação induzida $\tilde{f} : \mathcal{M}(M) \rightarrow \mathcal{M}(M)$ é definida por

$$(\tilde{f}(\mu))(X) := \mu(f^{-1}(X)) \quad (\mu \in \mathcal{M}(M), X \in \mathcal{B}_M).$$

Note que $\tilde{f} \in \mathcal{C}(\mathcal{M}(M))$ e, além disso, se f é um homeomorfismo, então \tilde{f} também o é.

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Conforme mencionado na introdução, o objetivo do nosso trabalho é estudar a dinâmica induzida ao espaço $\mathcal{M}(M)$ pela aplicação contínua genérica e pelo homeomorfismo genérico no caso em que M é o espaço de Cantor. Nossa modelo para o espaço de Cantor é o espaço produto $\{0,1\}^{\mathbb{N}}$, onde $\{0,1\}$ é munido com a topologia discreta. Consideramos $\{0,1\}^{\mathbb{N}}$ munido com a métrica compatível d dada por $d(\sigma, \sigma) := 0$ e $d(\sigma, \tau) := \frac{1}{n}$, onde n é o menor inteiro positivo tal que $\sigma(n) \neq \tau(n)$ ($\sigma, \tau \in \{0,1\}^{\mathbb{N}}, \sigma \neq \tau$).

Como ilustração do tipo de resultado que obtemos, enunciaremos um dos nossos resultados envolvendo caos Li-Yorke. Para tal, vamos começar relembrando este conceito de caos.

Seja M um espaço métrico. Se $f : M \rightarrow M$ é uma aplicação contínua, um par $(x, y) \in M \times M$ é dito um par Li-Yorke para f se

$$\liminf_{n \rightarrow \infty} d(f^n(x), f^n(y)) = 0 \quad \text{e} \quad \limsup_{n \rightarrow \infty} d(f^n(x), f^n(y)) > 0.$$

A aplicação f é dita Li-Yorke caótica se existe um subconjunto não-enumerável S de M tal que (x, y) é um par Li-Yorke para f sempre que x e y são elementos distintos em S . É conhecido que das noções de caos mais importantes, essa é a mais fraca.

Foi provado pelos autores em [6] que a dinâmica coletiva do homeomorfismo genérico do espaço de Cantor apresenta comportamento Li-Yorke caótico. Na verdade, os autores mostram em [6] que o homeomorfismo genérico do espaço de Cantor apresenta, no sentido coletivo, comportamento distribucionalmente caótico uniforme; tal comportamento caótico é muito mais forte do que o caos Li-Yorke. Em grande contraste com esta situação, no presente trabalho temos o seguinte

Teorema 2.1. *Para $h \in \mathcal{H}(\{0,1\}^{\mathbb{N}})$ genérico, \tilde{h} não admite par Li-Yorke.*

Apresentamos também, ainda dentro desse contexto, respostas completas para questões envolvendo caos topológico, conjuntos recorrentes, pontos periódicos, continuidade em cadeia de aplicações contínuas e homeomorfismos e, por fim, sombreamento de homeomorfismos.

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REGULARIDADE DO TIPO GELFAND-SHILOV PARA PROBLEMAS DE CONTORNO ELÍPTICOS COM SÍMBOLOS SG NO SEMIPLANO

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O trabalho estuda a regularidade de problemas de contorno elípticos no semiplano, cujos coeficientes dos operadores diferenciais crescem polinomialmente (mais especificamente, são do tipo SG). Mostraremos que, exigindo-se uma certa de regularidade nos coeficientes, as soluções do problema de contorno pertencem ao espaço de Gelfand-Shilov. Com isto, concluímos que elas possuem um determinado decrescimento exponencial, além de satisfazerem uma regularidade do tipo Gevrey. Finalizaremos mostrando uma estratégia para lidar com problemas semi-lineares.

1 Introdução

O trabalho que apresentaremos estuda a regularidade de uma classe de problemas de contorno elípticos usando operadores pseudodiferenciais definidos globalmente.

Estaremos interessados em operadores diferenciais agindo em \mathbb{R}^n , chamados de SG, da forma

$$p(x, D)u = \sum_{\alpha \leq m_1} a_\alpha(x)D^\alpha u,$$

em que os coeficientes satisfazem $|\partial^\beta a_\alpha(x)| \leq C \langle x \rangle^{m_2 - |\beta|}$. Neste caso chamamos a função $p(x, \xi) = \sum_{\alpha \leq m_1} a_\alpha(x)\xi^\alpha$ de símbolo do operador.

A elipticidade é definida da seguinte forma: Existem constantes $R > 0$ e $C > 0$ tais que se $|(x, \xi)| \geq R$, então

$$|p(x, \xi)| \geq C \langle x \rangle^{m_2} \langle \xi \rangle^{m_1}$$

Esses operadores estão contidos na classe mais geral de operadores pseudodiferenciais com símbolo SG, cujo exemplo mais simples é a equação de uma partícula livre com energia fixa em mecânica quântica: $\left(-\frac{\hbar^2}{2m}\Delta - E\right)\psi = 0$. Outros exemplos apareceram recentemente com as generalizações de quinta e sétima ordem da equação de KdV, em trabalhos como os de Nicola, Rodino e Porubov. Equações diferenciais do tipo SG também foram usados por Melrose em trabalhos sobre teoria do espalhamento.

Há uma razoavelmente extensa literatura a respeito dos operadores SG. Entre os autores que os estudaram podemos citar também Parenti, Cordes, Schrohe, Kapanadze, Maniccia, Seiler, Erkip, Schulze, Lopes e Melo (na minha tese de doutorado), entre outros. Muitos desses aplicaram esses operadores também ao estudo de problemas elípticos de contorno em regiões não limitadas.

Recentemente, trabalhos de Capiello, Nicola, Gramchev e Rodino mostraram que, usando técnicas pseudodiferenciais e espaços do tipo Gelfand-Shilov, análogos aos espaços de Gevrey, é possível obter resultados muito mais precisos acerca da regularidade das soluções das equações elípticas lineares e semi-lineares do tipo SG. Tais resultados se mostraram bastante relevantes para a compreensão do decaimento exponencial de soluções de equações que aparecem no estudo de sólitos, em especial as generalizações de quinta e sétima ordem que aparecem nos trabalhos citados acima.

Os trabalhos de Capiello et al. levaram a uma questão bastante natural: Será que é possível obter uma regularidade como a obtida por eles também para os problemas elípticos de contorno? O que mostraremos é que sim! As mesmas técnicas pseudodiferenciais podem ser aplicadas em conjunto com resultados clássicos acerca dos

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projetores de Calderón, desenvolvidos por R. T. Seeley, para a obtenção de regularidade do tipo Gelfand-Shilov em problemas de contorno.

O objetivo do nosso trabalho é justamente mostrar como obter a regularidade acima. Dividiremos nossa apresentação em partes:

1) Mostraremos qual é a definição apropriada dos espaços de Gelfand-Shilov no semi-plano. Mostraremos que nossa definição coincide com a restrição das funções de Gelfand-Shilov definidas em todo \mathbb{R}^n .

2) Recordaremos os trabalhos de Seeley, Capiello et al.

3) Mostraremos como podemos juntar as técnicas dos trabalhos acima para obter a regularidade nos problemas de contorno. Para tanto, apresentaremos nossos resultados acerca do comportamento dos operadores pseudodiferenciais definidos por Capiello et al. no semiplano e como estes resultados levam a regularidade.

4) Esboçaremos uma estratégia para obter resultados para problemas semi-lineares. (Esta parte é a única com questões ainda em aberto).

2 Resultados Principais

O trabalho apresenta dois resultados principais. Vamos inicialmente definir os espaços de Gelfand-Shilov.

Definição 2.1. Sejam $\mu > 0$ e $\nu > 0$ constantes tais que $\mu + \nu \geq 1$. O espaço de Gelfand-Shilov $\mathcal{S}_\nu^\mu(\mathbb{R}_+^n)$ ($\mathcal{S}_\nu^\mu(\mathbb{R}^n)$) é definido como o espaço de funções $u \in C^\infty(\mathbb{R}_+^n)$ ($u \in C^\infty(\mathbb{R}^n)$) para as quais existem constantes $C > 0$ e $D > 0$ dependendo apenas de u tais que

$$|x^\alpha \partial_x^\beta u(x)| \leq CD^{|\alpha|+|\beta|} (\alpha!)^\nu (\beta!)^\mu, \quad \forall \alpha, \beta \in \mathbb{N}_0^n.$$

O primeiro resultado é a demonstração de que as funções $\mathcal{S}_\nu^\mu(\mathbb{R}_+^n)$ são restrições de funções $\mathcal{S}_\nu^\mu(\mathbb{R}^n)$ no semiplano. Este resultado é mais complicado do que se parece, já que construir uma extensão que satisfaça as estimativas de Gevrey requer bastante cuidado.

Teorema 2.1. Seja $f \in \mathcal{S}_\nu^\mu(\mathbb{R}_+^n)$, $\mu > 1$ e $\nu > 0$. Logo existe uma função $g \in \mathcal{S}_\nu^\mu(\mathbb{R}^n)$ tal que $g(x) = f(x)$ para todo $x \in \mathbb{R}_+^n$.

O segundo resultado é o teorema principal de regularidade. Usaremos a notação: $x \in \mathbb{R}^n$ pode ser escrito como $x = (x', x_n) \in \mathbb{R}^{n-1} \times \mathbb{R}$.

Teorema 2.2. Seja $\theta > 1$ e $\nu > 1$. Seja $u \in \mathcal{S}(\mathbb{R}_+^n)$ uma solução de

$$\left\{ \begin{array}{l} \sum_{|\alpha| \leq m_1} a_\alpha(x) D^\alpha u(x) = f(x), \quad x \in \mathbb{R}_+^n \\ \sum_{|\alpha| \leq m_1} b_{\alpha 1}(x') D^\alpha u(x', 0) = g_1(x'), \quad x' \in \mathbb{R}^{n-1} \\ \vdots \\ \sum_{|\alpha| \leq m_1} b_{\alpha r}(x') D^\alpha u(x', 0) = g_r(x'), \quad x' \in \mathbb{R}^{n-1} \end{array} \right..$$

Estamos supondo acima que o problema é elíptico (satisfaz condições do tipo Shapiro-Lopatinski adaptadas à nossa classe de símbolos). Se $f \in \mathcal{S}_\theta^\theta(\mathbb{R}_+^n)$, $g_1, \dots, g_r \in \mathcal{S}_\theta^\theta(\mathbb{R}^{n-1})$, a_α e $b_{\alpha j}$ satisfazem estimativas do tipo $|\partial^\sigma a_\alpha(x)| \leq CD^\sigma (\sigma!)^\nu \langle x \rangle^{m_2 - |\sigma|}$, então $u \in \mathcal{S}_\theta^\theta(\mathbb{R}_+^n)$, se $\theta > \nu$ e $u \in \mathcal{S}_{\tilde{\theta}}^{\tilde{\theta}}(\mathbb{R}^n)$ para qualquer $\tilde{\theta} > \theta$ se $\theta = \nu$.

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VARIATION-OF-CONSTANTS FORMULA FOR FDES VIA GENERALIZED ODES

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1 Introduction

We present a variation-of-constants formula for linear generalized ordinary differential equations in Banach spaces. More specifically, we are interested in establishing a relation between the solutions of the Cauchy problem for a linear generalized ordinary differential equation

$$\frac{dx}{d\tau} = D[A(t)x], \quad x(t_0) = \tilde{x}$$

and the solutions of the perturbed Cauchy problem

$$\frac{dx}{d\tau} = D[A(t)x + F(x, t)], \quad x(t_0) = \tilde{x},$$

where the functions involved are generalized Perron integrable and, hence, admit many discontinuities and oscillations. We also prove that there exists a one-to-one correspondence between the Cauchy problem for a linear functional differential equations of the form

$$\begin{cases} \dot{y} = \mathcal{L}(t)y_t, \\ y_{t_0} = \varphi, \end{cases}$$

where \mathcal{L} is a bounded linear operator and φ is a regulated function, and a certain class of linear generalized ordinary differential equations. As a consequence, we are able to obtain a variation-of-constants formula relating the solutions of the linear functional differential equation and the solutions of the perturbed problem

$$\begin{cases} \dot{y} = \mathcal{L}(t)y_t + f(y_t, t), \\ y_{t_0} = \varphi. \end{cases}$$

where the application $t \mapsto f(y_t, t)$ is Perron integrable, with t in an interval of \mathbb{R} , for each regulated function y .

2 Main Theorem

Let us consider the following initial value problem for a linear functional differential equation

$$\begin{cases} \dot{y} = \mathcal{L}(t)y_t, \\ y_{t_0} = \phi, \end{cases} \tag{2.1}$$

where $\phi \in G([-r, 0], \mathbb{R}^n)$, $\mathcal{L}: [t_0, t_0 + \sigma] \rightarrow L(G([-r, 0], \mathbb{R}^n), \mathbb{R}^n)$ and $\mathcal{L}(t): G([-r, 0], \mathbb{R}^n) \rightarrow \mathbb{R}^n$ is bounded and linear for every $t \in [t_0, t_0 + \sigma]$. Consider, also, the perturbed linear FDE

$$\begin{cases} \dot{y} = \mathcal{L}(t)y_t + f(y_t, t), \\ y_{t_0} = \phi, \end{cases} \tag{2.2}$$

where $f: G([-r, 0], \mathbb{R}^n) \times [t_0, t_0 + \sigma] \rightarrow \mathbb{R}^n$ and assume the following conditions:

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- (A) For every $y \in G([t_0 - r, t_0 + \sigma], \mathbb{R}^n)$, the application $t \mapsto \mathcal{L}(t)y_t$ is Kurzweil integrable over $[t_0, t_0 + \sigma]$;
- (B) There exists a Lebesgue integrable function $M: [t_0 - r, t_0 + \sigma] \rightarrow \mathbb{R}$ such that, for every $s_1, s_2 \in [t_0 - r, t_0 + \sigma]$ and $y, z \in G([t_0, t_0 + \sigma], \mathbb{R}^n)$,

$$\left| \int_{s_1}^{s_2} \mathcal{L}(s)(y_s - z_s) ds \right| \leq \int_{s_1}^{s_2} M(s) \|y_s - z_s\| ds.$$

Theorem 2.1. *Let y be the solution of the perturbed linear FDE (2.2), where we suppose the integrals involved are Kurzweil-Cauchy integrals. Let $T(t, s)$ be the solution operator of the linear FDE (2.1). Then, for $t_0 \leq t \leq t_0 + \sigma$, we have*

$$y(t) = T(t, t_0)\phi(0) + \int_{t_0}^t f(y_u, u) du - \int_{t_0}^t d_s[T(t, s)]h(s)(0).$$

where h is defined by

$$h(w)(\vartheta) = \begin{cases} 0, & t_0 - r \leq \vartheta \leq t_0, \\ \int_{t_0}^{\vartheta} f(y_u, u) du, & t_0 \leq \vartheta \leq w, \\ \int_{t_0}^w f(y_u, u) du, & w \leq \vartheta \leq t_0 + \sigma. \end{cases}$$

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UM ESQUEMA WENO COM UM NOVO TERMO ANTI-DISSIPATIVO PARA LEIS DE CONSERVAÇÃO HIPERBÓLICAS

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1 Introdução

Os esquemas WENO (do inglês “*weighted essentially non-oscillatory*”) são atualmente uma classe de métodos bastante popular para a resolução numérica de equações de leis de conservação hiperbólicas. Estes métodos não-lineares evitam realizar interpolações em regiões onde a solução é descontínua, através de uma fórmula que atribui pesos a estêncis de acordo com a suavidade da solução: quanto menos suave for a função em um dado estêncil, menor será a contribuição deste estêncil para a aproximação final. Desta forma, consegue-se gerar soluções numéricas com oscilações espúrias desprezíveis, isto é, com amplitude da ordem do comprimento da malha de pontos Δx [6]. Mais detalhes sobre os esquemas WENO podem ser encontrados em [6, 1] e referências ali contidas.

Em [2, 3] introduzimos o esquema WENO-Z, que é um aperfeiçoamento do esquema WENO clássico [5]. O WENO-Z mostrou-se menos dissipativo que o WENO clássico, capturando estruturas finas das soluções com uma melhor resolução. Ele também possui melhores propriedades de convergência nas vizinhanças de pontos críticos do que o esquema WENO clássico. Para mais detalhes, veja [4, 1] e referências ali contidas.

Neste trabalho, apresentamos o WENO-Z+, que é um novo esquema WENO que generaliza o WENO-Z com a inclusão de um termo anti-dissipativo [1]. Este termo é uma simples função dos indicadores de suavidade já existentes na fórmula do WENO-Z e permite alcançar resultados substancialmente mais precisos em regiões contendo choques e altos gradientes, sem alterar significativamente o custo computacional.

2 O esquema WENO-Z

A fórmula para os pesos ω_k^Z do esquema WENO-Z de ordem $2r - 1$ é dada por

$$\alpha_k^Z = d_k \left[1 + \left(\frac{\tau}{\beta_k + \varepsilon} \right)^p \right], \quad \omega_k^Z = \frac{\alpha_k^Z}{\sum_{j=0}^{r-1} \alpha_j^Z}, \quad k = 0, \dots, r-1, \quad (2.1)$$

onde r é o número de subestêncis; d_k são coeficientes denominados *pesos ideais*; β_k são os medidores de suavidade locais (i.e., que medem a suavidade em cada subestêncil); τ é o medidor de suavidade global (i.e., que mede a suavidade no estêncil completo de $2r - 1$ pontos); ε é o *parâmetro de sensibilidade* do esquema, e tipicamente possui valores pequenos (por exemplo, $\varepsilon = 10^{-16}$); e p é o *parâmetro de potência*. Para mais detalhes, consulte [2, 3].

3 O esquema WENO-Z+

A fórmula para os pesos ω_k^{Z+} do novo esquema WENO-Z+ de ordem $2r - 1$ é dada por

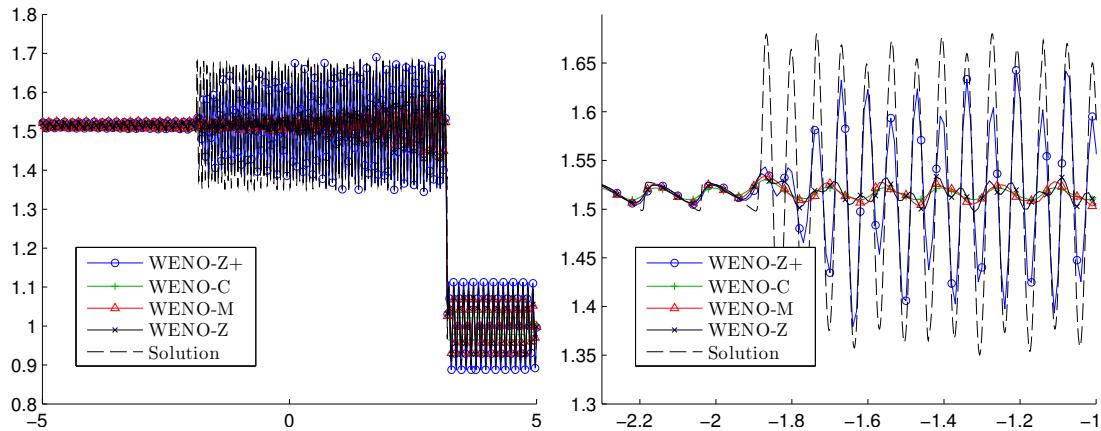
$$\alpha_k^{Z+} = d_k \left[1 + \left(\frac{\tau + \varepsilon}{\beta_k + \varepsilon} \right)^p + \lambda \left(\frac{\beta_k + \varepsilon}{\tau + \varepsilon} \right)^p \right], \quad \omega_k^{Z+} = \frac{\alpha_k^{Z+}}{\sum_{j=0}^{r-1} \alpha_j^{Z+}}, \quad k = 0, \dots, r-1, \quad (3.2)$$

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onde os termos d_k , β_k , τ , ε e p são os mesmos da fórmula do WENO-Z (2.1). A novidade é o parâmetro λ , que controla o tamanho do termo anti-dissipativo $\left(\frac{\beta_k + \varepsilon}{\tau + \varepsilon}\right)^p$: quanto maior o valor de λ , maior o peso dos subestêncis descontínuos e, portanto, menor a dissipação do esquema. Entretanto, valores altos de λ podem acarretar no aparecimento de oscilações espúrias na solução ou mesmo fazer com que o esquema fique instável. Nos testes que realizamos, encontramos que a escolha $\lambda = \Delta x^{2/3}$ diminui a dissipação do esquema sem acarretar em instabilidade.

4 Resultados numéricos



A figura mostra o resultado de um dos testes que realizamos, a interação de um choque com uma onda de entropia de Titarev–Toro para as equações de Euler em 1D, usando os esquemas WENO-Z+, WENO clássico (WENO-C), WENO mapeado (WENO-M) e WENO-Z, com uma malha de 1000 pontos. É possível notar que o WENO-Z+ possui uma resolução muito maior que a dos outros esquemas na região da solução que contém ondas de alta frequência. Para mais detalhes sobre o esquema WENO-Z+, incluindo o resultado de outros testes, consulte [1].

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OBSTACLE TYPE PROBLEMS IN ORLICZ-SOBOLEV SPACES

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We prove the Lewy-Stampacchia inequalities for obstacle problem in abstract form. As a consequence for a general class of quasi-linear elliptic operators of Ladyzhenskaya-Ural'tseva type, including $p(x)$ -Laplacian type operators, we derive $C^{1,\alpha}$ regularity for the solution.

Next, we extend basic regularity of the free boundary of the obstacle problem to some classes of heterogeneous quasilinear elliptic operators with variable growth that includes, in particular, the $p(x)$ -Laplacian. Under the assumption of Lipschitz continuity of the order of the power growth $p(x) > 1$, we use the growth rate of the solution near the free boundary to obtain its porosity, which implies that the free boundary is of Lebesgue measure zero for $p(x)$ -Laplacian type heterogeneous obstacle problems. Under additional assumptions on the operator heterogeneities and on data we show, in two different cases, that up to a negligible singular set of null perimeter the free boundary is the union of at most a countable family of C^1 hyper-surfaces:

- i) by extending directly the finiteness of the $(n-1)$ -dimensional Hausdorff measure of the free boundary to the case of heterogeneous p -Laplacian type operators with constant p , $1 < p < \infty$;
- ii) by proving the characteristic function of the coincidence set is of bounded variation in the case of non-degenerate or non singular operators with variable power growth $p(x) > 1$.

1 Introduction

Let Ω be a bounded open connected subset of \mathbb{R}^n , $n \geq 2$, $f \in L^\infty(\Omega)$ and $g \in W^{1,p(\cdot)}(\Omega) \cap L^\infty(\Omega)$, $g \geq 0$. We consider the quasilinear obstacle problem with a zero obstacle in its weak form:

$$(P) \left\{ \begin{array}{l} \text{Find } u \in K_g \text{ such that} \\ \int_{\Omega} \left(a(x, \nabla u) \cdot \nabla(v-u) + f(x)(v-u) \right) dx \geq 0, \quad \forall v \in K_g, \end{array} \right.$$

where $K_g = \{v \in W^{1,p(\cdot)}_0(\Omega) : v-g \in W^{1,p(\cdot)}_0(\Omega), v \geq 0 \text{ a.e. in } \Omega\}$, p is a measurable real valued function defined in Ω and satisfying for some positive numbers p_- and p_+

$$1 < p_- \leq p(x) \leq p_+ < \infty, \quad x \in \Omega.$$

The space $W^{1,p(\cdot)}(\Omega)$ is the Orlicz-Sobolev spaces: a generalization of usual Sobolev space.

We assume that the function $a : \Omega \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is such that $a(x, 0) = 0$ for a.e. $x \in \Omega$, and satisfies the structural assumptions with $\kappa \in [0, 1]$ and some positive constants c_0, c_1, c_2 , namely [4]

$$\sum_{i,j=1}^n \frac{\partial a_i}{\partial \eta_j}(x, \eta) \xi_i \xi_j \geq c_0 (\kappa + |\eta|^2)^{\frac{p(x)-2}{2}} |\xi|^2,$$

$$\sum_{i,j=1}^n \left| \frac{\partial a_i}{\partial \eta_j}(x, \eta) \right| \leq c_1 (\kappa + |\eta|^2)^{\frac{p(x)-2}{2}}$$

for a.e. $x \in \Omega$, a.e. $\eta = (\eta_1, \eta_2, \dots, \eta_n) \in \mathbb{R}^n \setminus \{0\}$ and for all $\xi = (\xi_1, \xi_2, \dots, \xi_n) \in \mathbb{R}^n$, and

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$$\begin{aligned} & |a(x_1, \eta) - a(x_2, \eta)| \\ & \leq c_2 |x_1 - x_2| \left[(\kappa + |\eta|^2)^{\frac{p(x_1)-1}{2}} + (\kappa + |\eta|^2)^{\frac{p(x_2)-1}{2}} \right] \left[1 + |\ln(\kappa + |\eta|^2)^{\frac{1}{2}}| \right], \end{aligned}$$

for $x_1, x_2 \in \Omega$, $\eta \in \mathbb{R}^n \setminus \{0\}$.

2 Mathematical Results

- By standard variational technics we prove the existence and uniqueness of the solution of (P) . Furthermore, we are able to provide a very short proof of its $C^{1,\alpha}$ regularity - as an immediate consequence of Lewy-Stampacchia inequalities (see [6]).
- We prove, that under the Lipschitz continuity assumption on $p(\cdot)$, the free boundary in (P) is a porous set (see [7]), and hence, it has Lebesgue measure zero.
- We show, that in the heterogeneous operators of p -Laplacian type, in case of $\kappa = 0$, the $(n-1)$ -dimensional Hausdorff measure of the free boundary is finite (Theorem 4.1 of [7]).
- When $\kappa > 0$, we are able to show the finiteness of $(n-1)$ dimensional Hausdorff measure of reduced free boundary.

In fact, it is known that the free boundary locally has finite \mathcal{H}^{n-1} -measure for several homogeneous operators: the p -Obstacle problem, [2] for $p = 2$ and [5] for $p > 2$, and for the A -Obstacle problem [3] that also includes the p -Laplacian ($1 < p < \infty$). It turns out, that the heterogeneous case is much more delicate in the $p(x)$ framework, but we are still able to show that at least the reduced free boundary has locally finite \mathcal{H}^{n-1} -measure. We use the bounded variation approach of Brézis and Kinderlehrer (see [1]) by showing that the set $\{u > 0\}$ has locally finite perimeter. Hence $\partial^* \{u > 0\}$ has locally finite \mathcal{H}^{n-1} -measure, where $\partial^* E$ is the essential boundary of E . As an important consequence, the free boundary may be written, up to a possible singular set of $\|\nabla \chi_{\{u>0\}}\|$ -measure zero, as a countable union of C^1 hyper-surfaces.

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EXISTÊNCIA DE SOLUÇÃO PARA UMA CLASSE DE EQUAÇÕES DE SCHRÖDINGER FRACIONÁRIAS

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1 Introdução

Utilizando um método variacional baseado na projeção sobre a variedade de Pohozaev, provamos a existência de soluções positivas para uma classe de equações de Schrödinger fracionárias com uma não-linearidade não-autônoma e não-homogênea.

Em termos gerais temos, para $\lambda > 0$, $s \in (0, 1)$ e $n > 2s$, a seguinte equação em \mathbb{R}^n

$$(-\Delta)^s u + \lambda u = a(x)f(u) \quad (1.1)$$

onde o operador laplaciano fracionário é definido para uma constante $C(n, s)$ adequada, pela expressão

$$(-\Delta)^s u(x) = C(n, s) \lim_{\epsilon \rightarrow 0^+} \int_{\mathbb{R}^n \setminus B_\epsilon(x)} \frac{u(x) - u(y)}{|x - y|^{n+2s}} dy.$$

e buscamos soluções no espaço de Sobolev $H^s(\mathbb{R}^n)$ (vide [2]). Assumimos que f satisfaz as seguintes condições:

$$(f1) \quad f \in C^1(\mathbb{R}, \mathbb{R}^+), \quad f(s) = 0 \text{ quando } s \leq 0 \text{ e } \lim_{s \rightarrow 0^+} \frac{f(s)}{s} = 0;$$

$$(f2) \quad \lim_{s \rightarrow +\infty} \frac{f(s)}{s} = 1;$$

$$(f3) \quad \text{se } F(s) := \int_0^s f(t)dt \text{ e } Q(s) := \frac{1}{2}f(s)s - F(s), \text{ então existe uma constante } D \geq 1 \text{ tal que } Q(s) \leq DQ(t) \text{ para todo } s \in [0, t] \text{ e } \lim_{s \rightarrow +\infty} Q(s) = +\infty.$$

Para a função $a : \mathbb{R}^n \rightarrow \mathbb{R}$, assumimos:

$$(A1) \quad a \in C^2(\mathbb{R}^n, \mathbb{R}^+), \text{ e } \inf_{\mathbb{R}^n} a > 0;$$

$$(A2) \quad \lim_{|x| \rightarrow +\infty} a(x) = a_\infty > \lambda;$$

(A3) $\nabla a(x) \cdot x \geq 0$, para todo $x \in \mathbb{R}^n$, com a desigualdade estrita num conjunto de medida não-nula;

$$(A4) \quad a(x) + \frac{\nabla a(x) \cdot x}{n} < a_\infty \text{ para todo } x \in \mathbb{R}^n;$$

$$(A5) \quad \nabla a(x) \cdot x + \frac{x \cdot \mathcal{H}_a(x) \cdot x}{n} \geq 0 \text{ para todo } x \in \mathbb{R}^n, \text{ onde } \mathcal{H}_a \text{ é a matriz hessiana de } a.$$

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2 Resultado principal

Considerando o problema limite em \mathbb{R}^n

$$(-\Delta)^s u + \lambda u = a_\infty f(u) \quad (2.2)$$

denotamos por $I_\infty : H^s(\mathbb{R}^n) \rightarrow \mathbb{R}$,

$$I_\infty(u) := \frac{1}{2} \int_{\mathbb{R}^n} |(-\Delta)^{s/2} u|^2 + \frac{\lambda}{2} \int_{\mathbb{R}^n} u^2 - \int_{\mathbb{R}^n} a_\infty F(u), \quad (2.3)$$

o seu funcional associado. Temos então o seguinte resultado:

Teorema 2.1. *Assuma que (A1)-(A5), (f1)-(f3) são válidas e que os seguintes fatos são válidos:*

1. $f \in C^1(\mathbb{R}) \cap \text{Lip}(\mathbb{R}, \mathbb{R}^+)$ e existe $\tau > 0$ tal que $\lim_{s \rightarrow 0^+} \frac{f'(s)}{s^\tau} = 0$;
2. $\|a_\infty - a\|_{L^\infty}$ é suficientemente pequeno;
3. o nível de energia mínima c_∞ de (??) é um nível crítico radial isolado de I_∞ ou a equação (??) admite uma solução radial positiva única.

Então a equação (??) admite uma solução não-trivial não-negativa $u \in H^s(\mathbb{R}^n)$.

Prova: A solução é obtida através do teorema de linking, juntamente com argumentos de concentração de compactade [1,3]. A variedade de Pohozaev [5] associada a equação (??), juntamente com a função *baricentro* [1], fornecem as ferramentas necessárias para a construção da estrutura de linking.

Os resultados aqui apresentados podem ser encontrados em [4].

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ESTIMATIVAS PARA NÚMEROS DE ENTROPIA DE CONJUNTOS DE FUNÇÕES SUAVES SOBRE O TORO \mathbb{T}^d

RÉGIS L. B. STÁBILE * & SÉRGIO A. TOZONI †

A teoria de entropia foi introduzida por Kolmogorov em meados da década de 1930. Desde então, muitos trabalhos têm visado obter estimativas assintóticas para números de entropia de diferentes classes de conjuntos.

Neste trabalho, obtemos estimativas inferiores e superiores para números de entropia de operadores multiplicadores associados a conjuntos de funções finitamente e infinitamente diferenciáveis sobre o toro d -dimensional \mathbb{T}^d . Demonstramos, em particular, que as estimativas obtidas são assintoticamente exatas em termos de ordem em várias situações.

1 Introdução

Seja A um subconjunto de um espaço de Banach X . Definimos o k -ésimo número de entropia do conjunto A por

$$e_k(A) = e_k(A, X) = \inf\{\epsilon > 0 : N(A, \epsilon B_X) \leq 2^{k-1}\},$$

onde B_X denota a bola unitária fechada do espaço X e $N(A, \epsilon B_X)$ denota o menor inteiro positivo N , caso exista, tal que existem pontos $x_1, x_2, \dots, x_N \in X$ satisfazendo

$$A \subset \bigcup_{i=1}^N (x_i + \epsilon B_X),$$

convencionando que $\inf \emptyset = +\infty$.

Se Y é um outro espaço de Banach e $T : X \rightarrow Y$ um operador limitado, definimos o k -ésimo número de entropia do operador T por $e_k(T(B_X), Y)$.

Dada $f \in L^1(\mathbb{T}^d)$, definimos a série de Fourier da função f por

$$\sum_{\mathbf{k} \in \mathbb{Z}^d} \widehat{f}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad \widehat{f}(\mathbf{k}) = \int_{\mathbb{T}^d} f(\mathbf{x}) e^{-i\mathbf{k} \cdot \mathbf{x}} d\nu(\mathbf{x}),$$

onde $\mathbf{k} \cdot \mathbf{x} = k_1 x_1 + k_2 x_2 + \dots + k_d x_d$, $\mathbf{k} = (k_1, k_2, \dots, k_d) \in \mathbb{Z}^d$, $\mathbf{x} = (x_1, x_2, \dots, x_d) \in \mathbb{T}^d$ e $d\nu$ denota a medida de Lebesgue normalizada sobre \mathbb{T}^d . Para $\mathbf{k} \in \mathbb{Z}^d$, denotamos também $|\mathbf{k}| = (k_1^2 + k_2^2 + \dots + k_d^2)^{1/2}$ e $|\mathbf{k}|_* = \max_{1 \leq j \leq d} |k_j|$.

Dados $l, N \in \mathbb{N}$, definimos $\mathcal{H}_l = [e^{i\mathbf{k} \cdot \mathbf{x}} : \mathbf{k} \in A_l \setminus A_{l-1}]$, $\mathcal{H}_l^* = [e^{i\mathbf{k} \cdot \mathbf{x}} : \mathbf{k} \in A_l^* \setminus A_{l-1}^*]$, $\mathcal{T}_N = \bigoplus_{l=0}^N \mathcal{H}_l$ e $\mathcal{T}_N^* = \bigoplus_{l=0}^N \mathcal{H}_l^*$, onde $A_l = \{\mathbf{k} \in \mathbb{Z}^d : |\mathbf{k}| \leq l\}$ e $A_l^* = \{\mathbf{k} \in \mathbb{Z}^d : |\mathbf{k}|_* \leq l\}$.

Sejam $\Lambda = \{\lambda_{\mathbf{k}}\}_{\mathbf{k} \in \mathbb{Z}^d}$, $\Lambda_* = \{\lambda_{\mathbf{k}}^*\}_{\mathbf{k} \in \mathbb{Z}^d}$, $\lambda_{\mathbf{k}}, \lambda_{\mathbf{k}}^* \in \mathbb{R}$ e sejam $1 \leq p, q \leq \infty$. Se para todo $\varphi \in L^p(\mathbb{T}^d)$ existirem funções $f = \Lambda \varphi \in L^q(\mathbb{T}^d)$ e $f^* = \Lambda_* \varphi \in L^q(\mathbb{T}^d)$ com expansões formais em série de Fourier dadas por

$$f \sim \sum_{l=1}^{\infty} \sum_{\mathbf{k} \in A_l \setminus A_{l-1}} \lambda_{\mathbf{k}} \widehat{f}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} \quad \text{e} \quad f^* \sim \sum_{l=1}^{\infty} \sum_{\mathbf{k} \in A_l^* \setminus A_{l-1}^*} \lambda_{\mathbf{k}}^* \widehat{f}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}},$$

respectivamente, tal que $\|\Lambda\|_{p,q} = \sup\{\|\Lambda \varphi\|_q : \varphi \in U_p\} < \infty$ e $\|\Lambda_*\|_{p,q} = \sup\{\|\Lambda_* \varphi\|_q : \varphi \in U_p\} < \infty$, dizemos que Λ e Λ_* são operadores multiplicadores limitados de L^p em L^q , com normas $\|\Lambda\|_{p,q}$ e $\|\Lambda_*\|_{p,q}$, respectivamente,

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onde U_p denota a bola unitária fechada do espaço $L^p(\mathbb{T}^d)$. Neste trabalho, consideramos operadores multiplicadores $\Lambda = \{\lambda_{\mathbf{k}}\}_{\mathbf{k} \in \mathbb{Z}^d}$ e $\Lambda_* = \{\lambda_{\mathbf{k}}^*\}_{\mathbf{k} \in \mathbb{Z}^d}$, onde $\lambda_{\mathbf{k}}$ e $\lambda_{\mathbf{k}}^*$ são da forma $\lambda(|\mathbf{k}|)$ e $\lambda(|\mathbf{k}|_*)$, respectivamente, para uma função real λ definida sobre $[0, \infty)$.

2 Resultados

Se $\Lambda^{(1)} = \{\lambda_{\mathbf{k}}\}_{\mathbf{k} \in \mathbb{Z}^d}$, $\Lambda_*^{(1)} = \{\lambda_{\mathbf{k}}^*\}_{\mathbf{k} \in \mathbb{Z}^d}$, onde a função $\lambda : [0, \infty) \rightarrow \mathbb{R}$ é definida por $\lambda(t) = t^{-\gamma}(\ln t)^{-\xi}$, $t > 1$ e $\lambda(t) = 0$ para $0 \leq t \leq 1$, $\gamma, \xi \in \mathbb{R}$, $\gamma > d/2$, $\xi \geq 0$, temos que $\Lambda^{(1)}U_p$ e $\Lambda_*^{(1)}U_p$ são conjuntos de funções finitamente diferenciáveis sobre \mathbb{T}^d , em particular, se $\xi = 0$ então $\Lambda^{(1)}U_p$ e $\Lambda_*^{(1)}U_p$ são classes do tipo Sobolev.

Se $\Lambda^{(2)} = \{\lambda_{\mathbf{k}}\}_{\mathbf{k} \in \mathbb{Z}^d}$, $\Lambda_*^{(2)} = \{\lambda_{\mathbf{k}}^*\}_{\mathbf{k} \in \mathbb{Z}^d}$, onde a função $\lambda : [0, \infty) \rightarrow \mathbb{R}$ é definida por $\lambda(t) = e^{-\gamma t^r}$, $\gamma, r > 0$, temos que $\Lambda^{(2)}U_p$ e $\Lambda_*^{(2)}U_p$ são conjuntos de funções infinitamente diferenciáveis ($0 < r < 1$), analíticas ($r = 1$) ou inteiras ($r > 1$) sobre o toro \mathbb{T}^d .

Para os resultados seguintes, usaremos as notações

$$\kappa_n = \begin{cases} 1, & p < \infty, q > 1, \\ (\ln n)^{-1/2}, & p < \infty, q = 1, \\ (\ln n)^{-1/2}, & p = \infty, q > 1, \\ (\ln n)^{-1}, & p = \infty, q = 1, \end{cases}, \quad \vartheta_n = \begin{cases} 1, & 2 \leq p \leq \infty, q < \infty, \\ (\ln n)^{1/2}, & 2 \leq p \leq \infty, q = \infty. \end{cases}$$

Escreveremos $a_n \gg b_n$ e $a_n \ll b_n$, quando existirem constantes positivas C_1 e C_2 tais que $a_n \geq C_1 b_n$ e $a_n \leq C_2 b_n$, respectivamente, para todo $n \in \mathbb{N}$. Se tivermos $a_n \gg b_n$ e $a_n \ll b_n$, escreveremos $a_n \asymp b_n$.

Teorema 2.1. *Seja $\Lambda^{(1)}$ o operador multiplicador definido acima. Então*

$$e_n(\Lambda^{(1)}U_p, L^q) \ll n^{-\gamma/d}(\ln n)^{-\xi}\vartheta_n \quad \text{e} \quad e_n(\Lambda^{(1)}U_p, L^q) \gg n^{-\gamma/d}(\ln n)^{-\xi}\kappa_n.$$

Os resultados permanecem válidos se considerarmos o operador $\Lambda_^{(1)}$ no lugar do operador $\Lambda^{(1)}$.*

Teorema 2.2. *Seja $\Lambda^{(2)}$ o operador multiplicador definido acima, com $0 < r \leq 1$. Então*

$$e_n(\Lambda^{(2)}U_p, L^q) \gg e^{-Cn^{r/(d+r)}}\kappa_n \quad \text{e} \quad e_n(\Lambda^{(2)}U_p, L^q) \ll e^{-Cn^{r/(d+r)}}\vartheta_n,$$

onde

$$C = \gamma^{d/(d+r)} \left(\frac{(d+r)d\Gamma(d/2)(\ln 2)}{2r\pi^{d/2}} \right)^{r/(d+r)}.$$

Os resultados permanecem válidos se considerarmos o operador $\Lambda_^{(2)}$ no lugar do operador $\Lambda^{(2)}$, trocando C por*

$$C_* = \gamma^{d/(d+r)} \left(\frac{(d+r)(\ln 2)}{2^d r} \right)^{r/(d+r)}.$$

Corolário 2.1. *Para $2 \leq p, q < \infty$ e $0 < r \leq 1$, temos*

$$e_k(\Lambda^{(1)}U_p, L^q) = e_k(\Lambda_*^{(1)}U_p, L^q) \asymp k^{-\gamma/d}(\ln k)^{-\xi}, \quad e_k(\Lambda^{(2)}U_p, L^q) \asymp e^{-Ck^{r/(d+r)}} \quad \text{e} \quad e_k(\Lambda_*^{(2)}U_p, L^q) \asymp e^{-C_*k^{r/(d+r)}}.$$

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ASYMPTOTIC DYNAMICS OF PARABOLIC EQUATIONS GOVERNED BY THE P-LAPLACIAN ON UNBOUNDED THIN DOMAINS

RICARDO P. SILVA *

1 Introduction

The systematic study of the asymptotic behavior of dissipative systems on thin domains started with the works [4, 5] by J. Hale and G. Raugel. The cern of the study is guided by the question: Is it possible to give some information on the dynamics of an evolution equation defined in a spatial domain which is *small* in some direction by mean a model on a lower dimensional spatial domain? If such systems possess global attractors then is possible to compare the asymptotic behavior of two semiflows in terms of the Hausdorff distance of their respective attractors.

There is an extensive bibliography on thin domain problems especially devoted on the reaction-diffusion model

$$\begin{aligned} u_t - \Delta u + \lambda u &= f(u), & \text{in } (0, \infty) \times \Omega^\epsilon, \\ \frac{\partial u}{\partial \eta_\epsilon} &= 0, & \text{on } (0, \infty) \times \partial \Omega^\epsilon, \end{aligned} \tag{1.1}$$

where Ω^ϵ is a family of bounded domains collapsing onto a lower dimensional subset.

In [4], Hale and Raugel considered the case of domain of the form

$$\Omega^\epsilon := \{(x, \epsilon y) \in \mathbb{R}^n \times \mathbb{R} : x \in \omega, 0 < y < g(x)\},$$

where ω is a bounded domain and g is a smooth positive function defined on ω . When ϵ is small, they compare the dynamics of (1.1) with the dynamics of the following equation defined in ω

$$\begin{aligned} u_t - \frac{1}{g} \operatorname{div}(g \nabla u) + \lambda u &= f(u), & \text{in } (0, \infty) \times \omega, \\ \frac{\partial u}{\partial \eta} &= 0, & \text{on } (0, \infty) \times \partial \omega. \end{aligned} \tag{1.2}$$

In particular they proof that the family of global attractors \mathcal{A}_ϵ associated to (1.1) is upper semicontinuous in $\epsilon = 0$.

M. Prizzi and K. Rybakowski in [6] treated a much more general class of thin domains, namely

$$\Omega^\epsilon := \{(x, \epsilon y) \in \mathbb{R}^m \times \mathbb{R}^n : (x, y) \in \omega\}, \tag{1.3}$$

where $\omega \subset \mathbb{R}^{m+n}$ is a bounded domain. They developed an abstract framework for the analysis of the problem (1.1) and they also shows the upper semicontinuity of the global attractors. In [1], F. António and M. Prizzi allowed in (1.3) the domain ω to be an unbounded set.

Associated with boundary oscillation (rough boundary) on thin structures, J. Arrieta et al. in [2] consider

$$\Omega^\epsilon := \{(x, \epsilon y) \in \mathbb{R} \times \mathbb{R} : x \in (0, 1), 0 < y < g(\epsilon^{-1}x)\},$$

where $g : \mathbb{R} \rightarrow \mathbb{R}$ is a L -periodic function. Combining methods from homogenization theory the authors showed that the limiting equation is

$$\begin{aligned} u_t - ru_{xx} + \lambda u &= f(u), & \text{in } (0, \infty) \times (0, 1), \\ u_x(t, 0) &= u_x(t, 1) = 0, & t > 0, \end{aligned}$$

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where $r > 0$ is called the *homogenized coefficient*. In particular the authors also show the upper semicontinuity of global attractors. For more references we refer the reader to the Montecatini lecture notes [7] by G. Raugel.

Despite the study of the asymptotic behavior for semilinear models be widely considered in the literature, the same is not true for the quasi-linear case. We consider an evolution equation governed by the p -laplacian operator as prototype of quasi-linear equations on an unbounded thin domain of the form

$$\Omega^\epsilon := \{(x, \epsilon y) \in \mathbb{R}^n \times \mathbb{R} : 0 < y < g(x)\}.$$

Considering in Ω^ϵ the family of quasi-linear evolution equations

$$\begin{aligned} u_t - \Delta_p u + a(x, \epsilon y)|u|^{p-2}u &= f(u), && \text{in } (0, \infty) \times \Omega^\epsilon, \\ \frac{\partial u}{\partial \eta^\epsilon} &= 0, && \text{on } (0, \infty) \times \partial \Omega^\epsilon, \end{aligned}$$

where $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ denotes the p -Laplacian operator, $2 < p < n$, we will compare the semiflow generated by them with the semiflow generated by the following equation (see [8])

$$u_t - \frac{1}{g} \operatorname{div}(g|\nabla u|^{p-2} \nabla u) + a(x, 0)|u|^{p-2}u = f(u), \quad \text{in } (0, \infty) \times \mathbb{R}^n.$$

Notice that in the case $p = 2$ the structure of the main part of the limiting problem agrees with Hale's and Raugel's limiting problem (1.2). Our aim is to prove existence of global attractors \mathcal{A}_ϵ by an auxiliary family of weighted Sobolev spaces.

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ATRATORES PULLBACK PARA PROBLEMAS PARABÓLICOS COM $p_\epsilon(x)$ -LAPLACIANO

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Neste artigo demonstramos a existência de atratores pullback para equações de evolução não-autônomas governadas por uma perturbação do operador maximal monótono.

1 Introdução

Neste trabalho analisamos, por meio da Teoria de Atratores Pullback ([2],[3]), o comportamento assintótico da seguinte família de problemas não-autônomos:

$$\begin{cases} u_t - \operatorname{div}(|\nabla u|^{p_\epsilon(x)-2}\nabla u) = B(t, u) \\ u(\tau) = u_0 \in H := L^2(\Omega), \end{cases} \quad (1.1)$$

onde $\Omega \subset \mathbb{R}^n$ é um domínio limitado com fronteira $\partial\Omega$ suave, $n \geq 1$, a função $p_\epsilon(x) := p(x) + \epsilon \in C(\bar{\Omega})$, com $\epsilon \in [0, 1]$, é tal que $2 + \delta \leq p_\epsilon(x) \leq 3 - \delta$, para $\delta > 0$ q.t.p. $x \in \Omega$ e $B : \mathbb{R} \times H \rightarrow H$ satisfaz:

- (i) Existe uma aplicação $L \in C(\mathbb{R}; L^\infty(\Omega))$ não decrescente e absolutamente contínua tal que

$$\|B(t, u_1) - B(t, u_2)\|_H \leq L(t, x)\|u_1 - u_2\|_H, \quad \forall t \in \mathbb{R}, \forall u_1, u_2 \in H;$$

- (ii) $B(t, 0) = 0$.

Como o operador principal $A_\epsilon u := -\operatorname{div}(|\nabla u|^{p_\epsilon(x)-2}\nabla u)$ é maximal monótono, do tipo subdiferencial, $\partial\varphi$, onde $\varphi : H \rightarrow [0, \infty]$ é uma função própria, convexa e semicontínua inferiormente (veja [7]), e com as hipóteses sobre B , podemos garantir, via Proposição 3.13 [1], que para todo dado inicial $u_0^\epsilon \in H$ existe uma única solução $u^\epsilon(\cdot) := u(\cdot, \tau)u_0^\epsilon \in C([\tau, \infty), H)$ do problema (1.1).

Antes de enunciarmos o resultado principal deste trabalho, que garante a existência de uma família de atratores pullback, consideremos o espaço generalizado de Lebesgue

$$L^{p_\epsilon(x)}(\Omega) = \left\{ u : \Omega \rightarrow \mathbb{R} : u \text{ é mensurável, } \int_{\Omega} |u(x)|^{p_\epsilon(x)} dx < \infty \right\},$$

sendo $p_\epsilon \in L^\infty(\Omega)$ e $p_\epsilon \geq 1$. Definimos $\rho(u) = \int_{\Omega} |u(x)|^{p_\epsilon(x)} dx$. Da mesma forma que em [4], [5] e [6], podemos garantir que $L^{p_\epsilon(x)}(\Omega)$ é um espaço de Banach com a norma $\|u\|_{L^{p_\epsilon(x)}(\Omega)} = \inf \{\lambda > 0; \rho(\frac{u}{\lambda}) \leq 1\}$ e $W^{1,p_\epsilon(x)}(\Omega)$ é um espaço de Banach com a norma $\|u\|_* := \|u\|_{L^{p_\epsilon(x)}(\Omega)} + \|\nabla u\|_{L^{p_\epsilon(x)}(\Omega)}$. Além disso, o espaço $\mathcal{X}_\epsilon := W_0^{1,p_\epsilon(x)}(\Omega)$ é definido como o fecho de $C_0^\infty(\Omega)$ em $W^{1,p_\epsilon(x)}(\Omega)$ e $\|\nabla u\|_{p_\epsilon(x)}$ e $\|u\|_*$ são normas equivalentes em $W_0^{1,p_\epsilon(x)}(\Omega)$.

Até onde conhecemos, [8] foi o primeiro trabalho a respeito de problemas parabólicos envolvendo o $p(x)$ -laplaciano usando a teoria de operadores monótonos.

2 Resultados

Por simplicidade denotaremos a solução de (1.1) por $u(t)$ no lugar de $u_\epsilon(t)$.

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Lema 1: Seja u uma solução global de (1.1) Então existe uma função não-decrescente $\beta : \mathbb{R} \rightarrow \mathbb{R}_+$ e $T > 0$ tal que

$$\|u(t)\|_H \leq \beta(t), \quad \forall t \geq T + \tau. \quad (2.2)$$

Lema 2: Seja u uma solução global de (1.1.) Então existe uma função não-decrescente $\omega : \mathbb{R} \rightarrow \mathbb{R}_+$ e $T_0 > 0$ tal que

$$\|u(t)\|_{\mathcal{X}} \leq \omega(t) + 1, \quad \forall t \geq T_0 + \tau. \quad (2.3)$$

Teorema 1: Suponha que o operador B satisfaça as hipóteses (i) e (ii). Para cada $\epsilon \in [0, 1]$ o problema (1.1) tem um processo associado $\{U_\epsilon(t, \tau)\}_{t \geq \tau}$ definido em H , o qual tem um atrator pullback $\{\mathcal{A}_\epsilon(t)\}_{t \in \mathbb{R}}$.

Observação 1: É importante observar que as estimativas obtidas nos Lemas 1 e 2 independem de ϵ e isso é essencial para que possamos garantir a existência de um atrator pullback para cada ϵ e assim de uma família de atratores.

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UNIFORM DECAY RATES FOR TERMODIFFUSION SYSTEM WITH SECOND SOUND AND LOCALIZED NONLINEAR DAMPING

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1 Introduction

Consider the thermodiffusion system given by

$$\begin{cases} \rho u_{tt} - (\lambda + 2\mu)u_{xx} + \gamma_1\theta_{1x} + \gamma_2\theta_{2x} + \alpha_3(x)g_3(u_t) = 0, \\ c\theta_{1t} + \sqrt{k}q_{1x} + \gamma_1u_{tx} + d\theta_{2t} = 0, \\ n\theta_{2t} + \sqrt{D}q_{2x} + \gamma_2u_{tx} + d\theta_{1t} = 0, \\ \tau_1 q_{1t} + \alpha_1(x)g_1(q_1) + \sqrt{k}\theta_{1x} = 0, \\ \tau_2 q_{2t} + \alpha_2(x)g_2(q_2) + \sqrt{D}\theta_{2x} = 0, \end{cases} \quad (1.1)$$

in $(0, L) \times (0, \infty)$, with initial conditions

$$\begin{cases} u(x, 0) = u_0(x), & u_t(x, 0) = u_1(x), \\ \theta_1(x, 0) = \theta_{10}(x), & \theta_2(x, 0) = \theta_{20}(x), \\ q_1(x, 0) = q_{10}(x), & q_2(x, 0) = q_{20}(x), \end{cases} \quad (1.2)$$

and boundary conditions

$$u(0, t) = u(L, t) = \theta_1(0, t) = \theta_1(L, t) = \theta_2(0, t) = \theta_2(L, t) = 0. \quad (1.3)$$

where u , θ_1 , q_1 are the displacement, temperature, and heat flux. The functions θ_2 and q_2 describe the chemical potentials and the associated flux. The coefficients λ and μ denotes the material constants, ρ is the density, γ_1 , γ_2 are the thermal and diffusion dilatation, k , D are the thermal conductivity. Furthermore n , c , d are the thermodiffusion coefficients and τ_1 , τ_2 are the relaxation time. All the coefficients above are positive and satisfy the condition $nc - d^2 > 0$ as in [4, 5, 7, 8].

The phenomenon of thermodiffusion is present many fields of science, for example, in problems of mixture, fracture mechanics and delamination. Advances in this theory includes new materials, especially in composites and thermodiffusion influence over ceramics, polymers and other contemporary materials.

2 Assumptions, Existence and Main Result

The following assumptions arround de parameters are made:

Hipotesis 2.1. Assume that $\alpha_i \in L^\infty(0, L)$, $i = 1, 2, 3$ are nonnegative functions such that

$$\alpha_i(x) \geq \alpha_{i0} > 0 \text{ in some interval } J_i \subset (0, L), \quad i = 1, 2, 3$$

and

$$\emptyset \neq (a_1, a_2) = I := \bigcap_{i=1}^3 J_i \subset (0, L).$$

As we can see, this functions can localize the damping mechanisms in a arbitrarily small region.

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Hypothesis 2.2. *The feedback function g_i , $i = 1, 2, 3$, is a continuous and monotone increasing, and, in addition, satisfies:*

- $g_i(s)s > 0$ for $s \neq 0$,
- $k_i s \leq g_i(s) \leq K_i s$, for $|s| > 1$, where k_i and K_i are positive constants.

The existence of solution comes from a combination of linear and nonlinear semigroup theory present in [1, 2, 6]. If $\{u, \theta_1, \theta_2, q_1, q_2\}$ is a solution of (1.1)-(1.3), we denote by $E(t)$, $t \geq 0$, the energy associated which is given by

$$E(t) = \frac{1}{2} \int_0^L \rho u_t^2 + (\lambda + 2\mu)u_x^2 + c\theta_1^2 + n\theta_2^2 + \tau_1 q_1^2 + \tau_2 q_2^2 + 2d\theta_1\theta_2 dx.$$

The decay rates of the energy are a consequence of an observability inequality and are given by the following result

Theorem 2.1. *Over the Assumption 2.2 and Assumption 2.1, if the initial data are bounded, there is $T_0 > 0$ such that the energy $E(t)$ of (1.1)-(1.3) satisfies*

$$E(t) \leq S \left(\frac{t}{T_0} - 1 \right), \quad \forall t > T_0,$$

with $\lim_{t \rightarrow \infty} S(t) = 0$, where $S(t)$ is the solution of the following differential equation

$$\begin{cases} \frac{d}{dt}S(t) + q(S(t)) = 0, \\ S(0) = E(0) \end{cases}$$

and q is given in [3].

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REGULARITY OF EXTREMAL SOLUTION WITH SINGULAR NONLINEARITY

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1 Introduction

We investigate a class of semilinear elliptic differential equations involving singular nonlinearities on Riemannian manifolds. More specifically, we consider the following class of semilinear elliptic problems

$$\begin{cases} -\Delta_g u = \lambda f(u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (P_\lambda)$$

where Ω is a smooth bounded domain of a Riemannian model (M, g) . This class of Riemannian manifolds includes the classical space forms, that is, the Euclidean, elliptic and hyperbolic spaces.

We prove the existence of $\lambda^* = \lambda^*(N, \Omega) > 0$ such that for $\lambda \in (0, \lambda^*)$ there exists a minimal classical solution u_λ , which satisfies $0 < u_\lambda < 1$ and are semistable. For $\lambda > \lambda^*$ there are no solutions of any kind. Furthermore, we obtain L^p -estimates for u_λ uniform in λ and as an application, we prove that the extremal solution $u^* := \lim_{\lambda \nearrow \lambda^*} u_\lambda$ is classical whenever $1 \leq N \leq 7$. In the case that Ω is a geodesic ball of M , we establish symmetry and monotonicity for the class of semistable solution.

According to the class of solutions which we consider, let us introduce the following values:

$$\begin{aligned} \lambda^* &:= \sup\{\lambda \geq 0 : (P_\lambda) \text{ has a classical solution}\} \\ \lambda_* &:= \sup\{\lambda \geq 0 : (P_\lambda) \text{ has a weak solution}\}. \end{aligned}$$

2 Results

Teorema 2.1. *The following holds:*

$$\lambda^* = \lambda_*.$$

In particular, for $\lambda > \lambda^$ there are no solutions, even in weak sense. Furthermore, the map $\lambda \rightarrow u_\lambda(x)$ is increasing on $(0, \lambda^*)$ for each $x \in \Omega$. This allows one to define*

$$u^*(x) := \lim_{\lambda \nearrow \lambda^*} u_\lambda(x),$$

which is a weak solution of (P_{λ^}) so-called extremal solution.*

An interesting question is whether the extremal solution u^* is a classical solution. We can infer regularity of extremal solution u^* when $N \leq 7$.

Teorema 2.2. *If $1 \leq N \leq 7$ then u^* is a classical solution of (P_{λ^*}) .*

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COMPLEX SYMMETRIC COMPOSITION OPERATORS

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1 Introduction

A bounded operator T on a complex Hilbert space \mathcal{H} is said to be *complex symmetric* if there exists an orthonormal basis for \mathcal{H} with respect to which T has a self-transpose matrix representation. An equivalent way to define complex symmetry is the following: if a *conjugation* is a conjugate-linear operator $C : \mathcal{H} \rightarrow \mathcal{H}$ that satisfies the conditions

- (a) C is isometric: $\langle Cf, Cg \rangle = \langle g, f \rangle \forall f, g \in \mathcal{H}$,
- (b) C is involutive: $C^2 = I$,

then we say that a bounded linear operator $T \in B(\mathcal{H})$ is *C-symmetric* if $T = CT^*C$ and complex symmetric if there exists a conjugation C with respect to which T is *C-symmetric*. Complex symmetric operators on Hilbert spaces are natural generalizations of complex symmetric matrices, and their general study was initiated by Garcia, Putinar and Wogen ([2][3][4][5]). The class of complex symmetric operators has a large number of concrete examples including all normal operators, binormal operators, Hankel operators, finite Toeplitz matrices, compressed shift operators and Volterra integral operators.

Let \mathbb{B}_n denote the open unit ball of \mathbb{C}^n . A linear fractional self-map ψ of \mathbb{B}_n is a map of the form

$$\psi(z) = \frac{Az + B}{\langle z, C \rangle + d} \quad (1.1)$$

where A is a linear operator on \mathbb{C}^n , with vectors $B, C \in \mathbb{B}_n$ and d a complex number. Fix a vector $a \in \mathbb{B}_n$. Let P_a be the orthogonal projection of \mathbb{C}^n onto the complex line generated by a and let $Q_a = I - P_a$. Setting $s_a = (1 - |a|^2)^{1/2}$, we denote by $\phi_a : \mathbb{B}_n \rightarrow \mathbb{B}_n$ the linear fractional map

$$\phi_a(z) = \frac{a - P_a z - s_a Q_a z}{1 - \langle z, a \rangle} \quad (1.2)$$

which by Rudin [7] is the involutive Moebius automorphism of \mathbb{B}_n that interchanges 0 and a . If f is a holomorphic function with domain \mathbb{B}_n , and $0 < r < 1$, then f_r denotes the dilated function defined by $f_r(z) = f(rz)$ for $|z| < 1/r$. Then f is in the Hardy-Hilbert space $H^2(\mathbb{B}_n)$ provided that

$$\sup_{0 < r < 1} \int_{S_n} |f_r|^2 d\sigma < \infty.$$

In particular, $H^2(\mathbb{B}_n)$ is a Hilbert space of analytic functions on \mathbb{B}_n such that the monomials $z^\alpha = z_1^{\alpha_1} \dots z_n^{\alpha_n}$ for $\alpha \in \mathbb{N}^n$ form an orthogonal basis.

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2 Main Result

If ψ is an analytic self-map of \mathbb{B}_n , then the composition operator C_ψ is defined by $C_\psi f = f \circ \psi$ for $f \in H^2(\mathbb{B}_n)$. The composition operator C_{ϕ_a} is bounded and invertible on $H^2(\mathbb{B}_n)$. It turns out that C_{ϕ_a} is complex symmetric, because $C_{\phi_a} \circ C_{\phi_a} = I$ and operators that are algebraic of degree 2 are complex symmetric [5, Thm. 2]. The main result of this talk will be the construction of an explicit conjugation \mathcal{J} on $H^2(\mathbb{B}_n)$ such that $\mathcal{J}C_{\phi_a}^*\mathcal{J} = C_{\phi_a}$ [6]. The particular case $n = 1$ resolves a problem of Garcia and Hammond [1].

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LOWER SEMICONTINUITY OF GLOBAL ATTRACTORS FOR EVOLUTION EQUATIONS OF NEURAL FIELD TYPE

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1 Introduction

We consider the nonlocal evolution equation proposed by Wilson and Cowan in [8], which is used to model neuronal activity, that is,

$$\frac{\partial u(w, t)}{\partial t} = -u(w, t) + \int_{S^1} J(wz^{-1})f(u(z, t))dz + h, \quad h > 0, \quad (1.1)$$

with $dz = \frac{\tau}{\pi}d\theta$, where $d\theta$ denotes integration with respect to arc length.

In (1.1), $u(w, t)$ is a real function on $S^1 \times \mathbb{R}_+$, $J \in C^1(S^1)$ is a non negative even function supported in the interval $[-1, 1]$, f is a non negative nondecreasing function and h is a positive constant. In this model, $u(w, t)$ denotes the mean membrane potential of a patch of tissue located at position w at time $t \geq 0$. The connection function J determines the coupling between the elements at position w with the element at position z . The non negative nondecreasing function $f(u)$ gives the neural firing rate, or averages rate at which spikes are generated, corresponding to an activity level u . The parameter h denotes a constant external stimulus applied uniformly to the entire neural field. We say that the neurons at point x is active if $S(w, t) > 0$, where $S(w, t) = f(u(w, t))$ is the firing rate of a neuron at position w at time t (see [4], [7] and [8]).

As proved in [6], assuming that the function $f \in C^1(\mathbb{R})$, f' locally Lipschitz with $0 < f'(r) < k_1, \forall r \in \mathbb{R}$, for some positive constant k_1 and f is a nondecreasing function taking values between 0 and $S_{max} > 0$, satisfying, for $0 \leq s \leq S_{max}$, $|\int_0^s f^{-1}(r)dr| < L < \infty$, the map $F(u, J) = -u + J * (f(u)) + h$ is continuously Frechet differentiable in $L^2(S^1)$ and, therefore, the equation

$$\frac{\partial u}{\partial t} = F(u, J) = -u + J * (f(u)) + h \quad (1.2)$$

generates a C^1 flow in $L^2(S^1)$ given, by the variation of constant formula, by

$$(T(t)u_0)(w) = e^{-t}u_0(w) + \int_0^t e^{-(t-s)}[J * (f \circ u)(w, s) + h]ds,$$

which we denote by $T_J(t)$ to make explicit dependence on the parameter J . Furthermore, in [4], was also proved the existence of global attractors, \mathcal{A}_J , for the flow $T_J(t)$ and that $\{\mathcal{A}_J\}$ is upper semicontinuous with respect to parameter J at $J_0 \in \mathcal{J}$, where $\mathcal{J} = \{J \in C^1(\mathbb{R}), \text{even non negative, supported in } [-1, 1], \|J\|_{L^1} = 1\}$.

We also assume that, for each $J_0 \in \mathcal{J}$, the set E , of the equilibria of $T_{J_0}(t)$, is such that $E = E_1 \cup E_2$, where
(a) the equilibria in E_1 are (constant) hyperbolic equilibria;
(b) the equilibria in E_2 are nonconstant and, for each $u_0 \in E_2$, zero is simple eigenvalue of the derivative of F , with respect to u , $DF_u(u_0, J_0) : L^2(S^1) \rightarrow L^2(S^1)$, given by $DF_u(u_0, J_0)v = -v + J_0 * (f'(u_0)v)$.

A simple computation shows that, if u_0 is a nonconstant equilibria of $T_{J_0}(t)$ then zero is always an eigenvalue of the operator $DF_u(u_0, J_0)v = -v + J_0 * (f'(u_0)v)$ with eigenfunction u'_0 . Therefore, the hypothesis (b) above says that we are in the ‘simplest’ possible situation for the linearization around nonconstant equilibria.

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2 Mathematical Results

Using the same techniques of [3], we prove the lower semicontinuity property of attractors, $\{\mathcal{A}_J\}$, at $J_0 \in \mathcal{J}$, where $\mathcal{J} = \{J \in C^1(\mathbb{R}), \text{ even non negative, supported in } [-1, 1], \|J\|_{L^1} = 1\}$, that is,

$$dist(A_{J_0}, A_J) = \sup_{x \in A_{J_0}} \inf_{y \in A_J} \|x - y\|_{L^2} \longrightarrow 0, \text{ as } J \rightarrow J_0.$$

The upper semicontinuity of the equilibria is a consequence of the upper semicontinuity of global attractors (see Theorem 11 of [4]). The lower semicontinuity of the *hyperbolic* equilibria is usually obtained via the Implicit Function Theorem. However, this approach fails here since the equilibria may appear in families. To overcome this difficulty, we need the concept of normal hyperbolicity, (see [1]).

Using normally hyperbolic Theorem (see [1], Theorem 12.5), we obtain the following result:

Theorem 2.1. *The set E_J of the equilibria of $T_J(t)$ is lower semi-continuous with respect to J at J_0 .*

Using results of [7] we show that the local unstable sets are actually Lipschitz manifolds in a sufficiently small neighborhood and vary continuously with J . More precisely, we have

Theorem 2.2. *If u_0 is a fixed equilibrium of (1.1) for $J = J_0$, then there is a $\delta > 0$ such that, if $\|J - J_0\|_{L^1} + \|u_0 - u_J\|_{L^2} < \delta$ and $U_J^\delta := \{u \in W_J^u(u_J) : \|u - u_J\|_{L^2} < \delta\}$ then U_J^δ is a Lipschitz manifold and*

$$dist(U_J^\delta, U_{J_0}^\delta) + dist(U_{J_0}^\delta, U_J^\delta) \rightarrow 0 \text{ as } \|J - J_0\|_{L^1} + \|u_0 - u_J\|_{L^2} \rightarrow 0.$$

Finally, using Theorem 2.1, Theorem 2.2 and Theorem 2.1 of [2] we obtain the main result of this work.

Theorem 2.3. *The family of attractors $\{\mathcal{A}_J\}$ is lower semicontinuous with respect to the parameter J at $J_0 \in \mathcal{J}$.*

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\mathcal{A} -COMPACT POLYNOMIALS

SONIA BERRIOS *

In this work we study the space of \mathcal{A} -compact polynomials in different contexts: composition ideal of polynomials, polynomial bidual, linearization. We characterize \mathcal{A} -compact polynomials in terms of the continuity and compactness of its adjoint.

1 Results

Let \mathcal{A} be an operator ideal and E a Banach space. Following [5], a subset A of E is said to be \mathcal{A} -compact if there exist a Banach space Z , an operator $T \in \mathcal{A}(Z; E)$ and a compact set $K \subset Z$ such that $A \subset T(K)$. Relying on this concept, the notion of \mathcal{A} -compact operator is defined in an obvious way: an operator $T \in \mathcal{L}(E; F)$ is said to be \mathcal{A} -compact if $T(B_E)$ is \mathcal{A} -compact in F . The set of all \mathcal{A} -compact operators between Banach spaces is denoted by $\mathcal{K}_{\mathcal{A}}$.

The following proposition gives a relation between \mathcal{A} -compact sets and \mathcal{I} -bounded sets to an appropriate operator ideal \mathcal{I} . Recall that a set $A \subset E$ is \mathcal{I} -bounded in E if $A \subset T(B_Z)$ for some Banach space Z and some $T \in \mathcal{I}(Z; E)$. (see [2]).

Proposition 1.1. *Let E be Banach space, and \mathcal{A} an operator ideal. Then A is \mathcal{A} -compact in E if and only if A is $K_{\mathcal{A}}$ - bounded in E .*

In [2, p. 965], Aron and Rueda introduce the following property: An operator ideal \mathcal{I} satisfies the *Condition Γ* if the closed absolutely convex hull of any \mathcal{I} -bounded set is \mathcal{I} -bounded. In relation to $K_{\mathcal{A}}$ we have

Proposition 1.2. *Let \mathcal{A} be an operator ideal. Then $K_{\mathcal{A}}$ satisfies the Condition Γ .*

For proof of this result it is sufficient to prove that the closed absolutely convex hull of any \mathcal{A} -compact set is \mathcal{A} -compact.

Let E and F be Banach spaces and let $x \in E$. An m-homogeneous polynomial $P \in \mathcal{P}(^m E; F)$ is said to be \mathcal{A} -compact if for every $x \in E$ there exists a neighborhood V_x of x such that $P(V_x)$ is \mathcal{A} -compact in F . If we now consider \mathcal{A} -bounded sets, we shall say $P \in \mathcal{P}(^m E; F)$ is \mathcal{A} -bounded if for every $x \in E$ there exists a neighborhood V_x of x such that $P(V_x)$ is \mathcal{A} -bounded in F (see [2]). The set of all \mathcal{A} -bounded m-homogeneous polynomials from E to F is denoted by $\mathcal{P}_{\mathcal{A}}(^m E; F)$. When $m = 1$ we write $\mathcal{L}_{\mathcal{A}}(E; F) = \mathcal{P}_{\mathcal{A}}(^1 E; F)$.

Using the fact $P \in \mathcal{P}(^m E; F)$ is \mathcal{A} -bounded if and only if $P(B_E)$ is \mathcal{A} -bounded in F (see [2, p. 962]), we have

Proposition 1.3. *Let E and F be Banach spaces, \mathcal{A} an operator ideal and $P \in \mathcal{P}(^m E; F)$. Then*

- a) $K_{\mathcal{A}} = \mathcal{L}_{K_{\mathcal{A}}}$.
- b) P is \mathcal{A} -compact if and only if P is $K_{\mathcal{A}}$ -bounded .
- c) P is \mathcal{A} -compact if and only if $P(B_E)$ is \mathcal{A} -compact in F .

Recall that the adjoint of $P \in \mathcal{P}(^m E; F)$ is the operator $P' : F' \rightarrow \mathcal{P}(^m E)$ such that $P'(\varphi)(x) = \varphi(P(x))$. In [4, p.166] is defined the polynomial bidual of an operator ideal \mathcal{I} as

$$\mathcal{I}^{\mathcal{P}-\text{bidual}}(E; F) = \{P \in \mathcal{P}(^m E; F) : P'' \in \mathcal{I}(\mathcal{P}(^m E)'; F'')\}.$$

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An m -homogeneous polynomial $P \in \mathcal{P}(^m E; F)$ belongs to $\mathcal{I} \circ \mathcal{P}$ if there are a Banach space G , a polynomial $Q \in \mathcal{P}(^m E; G)$ and an operator $u \in \mathcal{I}(G; F)$ such that $P = u \circ Q$. In this case we write $P \in \mathcal{I} \circ \mathcal{P}(^m E; F)$. If we now consider the space of all continuous m -linear mapping $\mathcal{L}(^m E; F)$, the composition ideal of multilinear mappings $\mathcal{I} \circ \mathcal{L}$ can be defined in a similar way. In this case we write $A \in \mathcal{I} \circ \mathcal{L}(^m E; F)$. For more details we refer to [3].

The following result generalize [6, Lemma 2.1] and [1, Theorem 3.1].

Theorem 1.1. *Let E and F be Banach spaces, $P \in \mathcal{P}(^m E; F)$ and \mathcal{A} an operator ideal. The following statements are equivalents:*

- a) P is \mathcal{A} -compact.
- b) $P \in K_{\mathcal{A}} \circ \mathcal{P}(^m E; F)$.
- c) $P_L \in K_{\mathcal{A}}(\hat{\otimes}_{\pi}^{m,s} E; F)$.
- d) $P^L \in K_{\mathcal{A}}(\hat{\otimes}_{\pi_s}^{m,s} E; F)$.
- e) $\dot{P} \in K_{\mathcal{A}} \circ \mathcal{L}(^m E; F)$.
- f) $P \in \mathcal{K}_{\mathcal{A}}^{\mathcal{P}-\text{bidual}}$

Corollary 1.1. *Let E and F be Banach spaces, $P \in \mathcal{P}(^m E; F)$ and \mathcal{A} an operator ideal. Then P is \mathcal{A} -compact if and only if P'' is \mathcal{A} -compact.*

We finish with a characterization of \mathcal{A} -compact polynomials in terms of the continuity and compactness of its adjoint. This result was studied for \mathcal{A} -compact operators in [7] and for p-compact polynomials in [6, Proposition 4.1]. We denote by $E'_{\mathcal{A}}$ the dual space of E considered with the topology of uniform convergence on \mathcal{A} -compact sets, and τ_c denote the topology of uniform convergence on compact sets.

Proposition 1.4. *Let E and F be Banach spaces, $P \in \mathcal{P}(^m E; F)$ and \mathcal{A} an operator ideal. The following statements are equivalents:*

- a) P is \mathcal{A} -compact.
- b) $P' : F_{\mathcal{A}} \rightarrow \mathcal{P}(^m E)$ is continuous .
- c) $P' : F_{\mathcal{A}} \rightarrow (\mathcal{P}(^m E), \tau_c)$ is compact.
- d) $P' : F_{\mathcal{A}} \rightarrow (\mathcal{P}(^m E), \tau_{\mathcal{B}})$ is compact for any Banach operator ideal \mathcal{B} .
- e) $P' : F_{\mathcal{A}} \rightarrow (\mathcal{P}(^m E), \tau_{\mathcal{B}})$ is compact for some Banach operator ideal such that \mathcal{B} .
- f) $P' : F_{\mathcal{A}} \rightarrow (\mathcal{P}(^m E), w^*)$ is compact

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SOBRE UMA CLASSE DE EQUAÇÕES DE SCHRÖDINGER QUASE LINEARES EM \mathbb{R}^n

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1 Introdução

Neste trabalho, estudamos a existência e multiplicidade de soluções fracas para algumas classes de equações elípticas quase lineares. Problemas deste tipo são bastante conhecidos e estudados na literatura. Consideremos a seguinte equação elíptica quase linear

$$-\Delta_p u + V(x)|u|^{p-2}u = f(x, u), \quad x \in \mathbb{R}^n, \quad (1.1)$$

com $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, $p \in (1, \infty)$ e $n \geq 1$, e as funções $V : \mathbb{R}^n \rightarrow \mathbb{R}$ e $f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$ são mensuráveis.

Equações deste tipo tem sido extensivamente estudada sob várias hipóteses sobre o potencial $V(x)$ e comportamentos diferentes da não linearidade $f(x, u)$. Alguns destes estudos são motivados pelo trabalho pioneiro de Rabinowitz [1] quando $p = 2$, $n \geq 3$ e a não linearidade $f(x, u)$ comporta-se no infinito como $|u|^{q-1}$ para algum $q \in [2, 2^*)$, isto é, $f(x, u)$ tem o crescimento subcrítico do tipo Sobolev. Para superar o problema da “perda de compacidade”, típico em problemas elípticos em domínio ilimitados, Rabinowitz em [1] considerou $V(x)$ um potencial coercivo e limitado inferiormente por uma constante positiva, isto é, $V(x) \rightarrow +\infty$ quando $|x| \rightarrow +\infty$ e $V(x) \geq V_0 > 0$ para todo $x \in \mathbb{R}^n$. Esta condição de coercividade foi melhorada por Bartsch e Wang em [2] ao assumir que para todo $L > 0$ a medida de Lebesgue do conjunto $\{x \in \mathbb{R}^n ; V(x) \leq L\}$ é finita. Bartsch e Wang usaram as mesmas hipóteses sobre a não linearidade $f(x, u)$ como em [1].

Ainda conseguindo preservar a compacidade do funcional, Sirakov em [4], considerou uma hipótese mais geral sobre o potencial $V(x)$,

$$\lim_{R \rightarrow \infty} \nu_t(\mathbb{R}^n \setminus \overline{B}_R) = \infty \quad \text{para algum } t \in [2, 2^*),$$

em que

$$\nu_t(\mathbb{R}^n \setminus \overline{B}_R) = \inf_{u \in H_0^1(\mathbb{R}^n \setminus \overline{B}_R) \setminus \{0\}} \frac{\int_{\mathbb{R}^n \setminus \overline{B}_R} (|\nabla u|^2 + V(x)|u|^2) dx}{\left(\int_{\mathbb{R}^n \setminus \overline{B}_R} |u|^t dx \right)^{2/t}}.$$

Além disso, Sirakov [4] abordou a situação em que a não linearidade pode ser ilimitada na variável x , situação que não foi abordada em [1,2].

Existem muitos resultados para equações do tipo (1.1) quando o domínio é limitado veja, por exemplo, [5] e suas referências. No entanto, quando o domínio é ilimitado e $1 < p < n$ não se encontram muitos artigos. Para o nosso conhecimento, um dos primeiros resultados de existência de soluções para estas equações foi abordado por Lyberopoulos em [3] na seguinte situação particular

$$-\Delta_p u + V(x)|u|^{p-2}u = Q_1(x)|u|^{t-2}u - Q_2(x)|u|^{s-2}u \quad \text{em } \mathbb{R}^n,$$

onde $1 < p < n$, V , Q_1 e Q_2 são funções não negativas e tais que Q_1 e Q_2 são dominadas por V quando $|x| \rightarrow +\infty$. Além disso, $t, s \in (1, p^*)$.

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Estendemos os resultados de Sirakov [4], no caso em que $1 < p < n$, provando a existência e multiplicidade de soluções para equação (1.1) com $V(x) = a(x) - b(x)$, em que a e b são funções mensuráveis não negativas. Além disso, analisamos a situação em que a não linearidade $f(x, u)$ pode ser ilimitada na variável x e possui o crescimento subcrítico do tipo Sobolev.

2 Resultados...

Descreveremos a seguir as hipóteses que usaremos ao longo deste trabalho:

(H₁) A função $a : \mathbb{R}^n \rightarrow [0, +\infty)$ é mensurável e

$$\lambda_1 := \inf_{u \in E \setminus \{0\}} \frac{\int_{\mathbb{R}^n} (|\nabla u|^p + a(x)|u|^p) dx}{\int_{\mathbb{R}^n} |u|^p dx} > 0,$$

em que E é um subespaço de $W^{1,p}(\mathbb{R}^n)$ dado da seguinte forma:

$$E = \left\{ u \in W^{1,p}(\mathbb{R}^n); \int_{\mathbb{R}^n} a(x)|u|^p dx < \infty \right\}.$$

Definimos para qualquer subconjunto aberto Ω de \mathbb{R}^n e para $s \in [p, p^*)$, $\nu_s(\Omega)$ por

$$\nu_s(\Omega) = \inf_{u \in W_0^{1,p}(\Omega) \setminus \{0\}} \frac{\int_{\Omega} (|\nabla u|^p + a(x)|u|^p) dx}{(\int_{\Omega} |u|^s dx)^{p/s}},$$

se $\Omega \neq \emptyset$ e $\nu_s(\Omega) = \infty$, se $\Omega = \emptyset$.

(H₂) Existe $s \in [p, p^*)$ tal que $\lim_{R \rightarrow \infty} \nu_s(\mathbb{R}^n \setminus \overline{B}_R) = \infty$, em que B_R é a bola em \mathbb{R}^n de raio R centrada na origem.

(H₃) Existem uma função $A \in L_{\text{loc}}^\infty(\mathbb{R}^n)$, com $A(x) \geq 1$, e constantes $\beta > 1$, $C_0 > 0$, $R_0 > 0$ tais que $A(x) \leq C_0 (1 + (a(x))^{1/\beta})$ para todo $|x| \geq R_0$.

(H₄) A função $b : \mathbb{R}^n \rightarrow [0, +\infty)$ é mensurável e $\|b\|_\sigma < S_{t_0}^p$ para algum $\sigma > 1$ onde $t_0 := \sigma p / (\sigma - 1) < p^\# + 1$, $p^\# := (p^* - 1) - p^2 / (\beta(n - p))$ e S_{t_0} é a melhor constante para a imersão de Sobolev $E \hookrightarrow L^{t_0}(\mathbb{R}^n)$.

(H₅) A função f é contínua e $A-$ superlinear na origem, isto é,

$$\lim_{s \rightarrow 0} \frac{f(x, s)}{A(x)|s|^{p-1}} = 0 \quad \text{uniformemente em } x \in \mathbb{R}^n.$$

(H₆) Existe $q \in [p - 1, p^\#)$ tal que $|f(x, s)| \leq C_0 A(x) (1 + |s|^q)$ para todo $(x, s) \in \mathbb{R}^n \times \mathbb{R}$.

(H₇) Existe $\mu > p$ tal que

$$0 < \mu F(x, t) = \mu \int_0^t f(x, s) ds \leq t f(x, t) \quad \text{para todo } (x, t) \in \mathbb{R}^n \times \mathbb{R} \setminus \{0\}. \quad (2.2)$$

Principais resultados:

Teorema 2.1. *Suponhamos que as condições (H₁) - (H₇) são satisfeitas. Então o problema (1.1) tem uma solução não trivial. Além disso, se $f(x, u)$ for ímpar em u , então (1.1) tem infinitas soluções.*

Teorema 2.2. *Suponhamos que as condições (H₁), (H₃) - (H₇) são satisfeitas. Além disso, assumindo que*

$$\lim_{k \rightarrow \infty} \nu_s(B(x_k, r)) = \infty \quad \text{para algum } s \in (2, p^*), \quad (2.3)$$

em que $B(x_k, r)$ é a bola de raio r centrada numa sequência $(x_k) \subset \mathbb{R}^n$ tal que $|x_k| \rightarrow \infty$. Então o problema (1.1) tem uma solução não trivial. Se, além disso, $f(x, u)$ é ímpar em u , então o problema (1.1) tem infinitas soluções.

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ESTIMATES FOR FOURIER SUMS AND EIGENVALUES OF INTEGRAL OPERATORS VIA MULTIPLIERS ON THE SPHERE

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1 Introduction

Let S^m denote the m -dimensional unit sphere in the euclidian space \mathbb{R}^{m+1} , endowed with the usual Lebesgue measure σ_m . We denote by ω_m the surface area of S^m . In this work, we will deal with the usual spaces $L^p(S^m) := L^p(S^m, \sigma_m)$, the norm of which we denote by $\|\cdot\|_p$.

We can represent a function $f \in L^p(S^m)$ by its spherical harmonics series $\sum_{k=0}^{\infty} \sum_{l=1}^{d_k} c_{k,l}(f) Y_{k,l}(x)$, in which $\{Y_{k,l} : l = 1, 2, \dots, d_k\}_{k \geq 0}$ is basis for the space of spherical harmonics of degree k in $m + 1$ variables and $c_{k,l}(f)$ are the Fourier coefficients of f defined by

$$c_{k,l}(f) := \frac{1}{\omega_m} \int_{S^m} f(y) \overline{Y_{k,l}(y)} d\sigma_m(y), \quad l = 1, 2, \dots, d_k, \quad k = 0, 1, \dots$$

For a fixed k , we denote by $\mathcal{Y}_k(f)$ the projection of f over the space $\text{span}\{Y_{k,l} : l = 1, 2, \dots, d_k\}$.

This work intends to provide decay rates for the sequence of eigenvalues of positive integral operators generated by kernels satisfying an abstract Hölder condition defined by a class of multipliers operators. For such purpose, the work involves the deduction of convenient estimates for the Fourier coefficients $c_{k,l}(f)$ through the rate of approximation of the class of multipliers operators that we will work with.

A multiplier operator refers to a linear operator T on $L^p(S^m)$ for which there exists a sequence $\{\eta_k\}$ of complex numbers (called the sequence of multipliers of T) such that $\mathcal{Y}_k(Tf) = \eta_k \mathcal{Y}_k(f)$, $f \in L^p(S^m)$ and $k = 0, 1, \dots$. An important category of bounded multiplier operators are those given by a convolution with a zonal measure. The class of bounded multiplier operators on $L^1(S^m)$ was characterized by C. Dunkl as that composed of operators which are convolutions with zonal measures on S^m . Among other things, this characterization reveals that the class of bounded multiplier operators on $L^2(S^m)$ is bigger than that of bounded multiplier operators on $L^1(S^m)$. Also, it is not hard to see that a multiplier operator on $L^2(S^m)$ is bounded if and only if its sequence of multipliers is bounded.

Since we will consider a family of multipliers operators $\{M_t : t \in (0, \pi)\}$ acting on $L^2(S^m)$, we can introduce a Hölder condition attached to it as follows. We say that a kernel K in $L^2(S^m \times S^m) := L^2(S^m \times S^m, \sigma_m \times \sigma_m)$ is $\{M_t : t \in (0, \pi)\}$ -Hölder if there exist a real number $\beta \in (0, 2]$ and a constant $B > 0$ so that

$$\int_{S^m} |M_t(K^y)(y) - K^y(y)| d\sigma_m(y) \leq B t^\beta, \quad t \in (0, \pi). \tag{1.1}$$

The above Hölder condition is implied by the more classical one which demands the existence of $\beta \in (0, 2]$ and a function B in $L^1(S^m)$ so that $\sup_x |M_t(K^y)(x) - K^y(x)| \leq B(y) t^\beta$, $y \in S^m$, $t \in (0, \pi)$.

Using a technique introduced in [2], the goal here is to deduce decay rates for certain positive integral operators on the sphere, those generated by a Mercer-like kernel satisfying a Hölder condition defined by a parameterized family of multipliers operators on $L^2(S^m)$, as that defined in (1.1). The main contribution here brings an important advance: the use of an abstract Hölder condition coupled with an abstract setting. In particular, many other settings can be putted into that of this note, and important classical results in the literature can be easily recovered ([2, 3]).

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2 Mathematical Results

An important intermediate step towards the results is the inequality in Theorem 2.1 below. It is an estimation for weighted sums of Fourier coefficients of integrable function, when the weights depend upon the sequence of multipliers of a multiplier operator. The result is an upgrade of those found in [1] which was obtained for particular multiplier operators only.

Theorem 2.1. *Let M be a multiplier operator on $L^p(S^m)$ with corresponding sequence of multipliers $\{\eta_k\}$. If $p \in (1, 2]$, then*

$$\left\{ \sum_{k=1}^{\infty} (d_k^m)^{(2-q)/2q} |\eta_k - 1|^q \left[\sum_{j=1}^{d_k^m} |\hat{f}(k, j)|^2 \right]^{q/2} \right\}^{1/q} \leq \omega_m^{(p-2)/2p} \|Mf - f\|_p, \quad f \in L^p(S^m),$$

in which q is the conjugate exponent of p . The inequality above becomes an equality in the case $p = 2$.

Our main result refers to linear integral operators $\mathcal{L}_K : L^2(S^m) \rightarrow L^2(S^m)$ of the form

$$\mathcal{L}_K(f)(x) = \int_{S^m} K(x, y) f(y) d\sigma_m(y), \quad x \in S^m, \quad f \in L^2(S^m),$$

in which $K : S^m \times S^m \rightarrow \mathbb{C}$ is a positive definite kernel belonging to $L^2(S^m \times S^m, \sigma_m \times \sigma_m)$ having a spherical harmonics expansion in the form

$$K(x, y) = \sum_{k=0}^{\infty} \sum_{j=1}^{d_k^m} a_{k,j} Y_{k,j}(x) \overline{Y_{k,j}(y)}, \quad x, y \in S^m, \quad (2.2)$$

where for every k , $a_{k,1} \geq a_{k,2} \geq \dots \geq a_{k,d_k^m}$. This operator has at most countably many nonnegative eigenvalues which can be ordered as $\alpha_1(\mathcal{L}_K) \geq \alpha_2(\mathcal{L}_K) \geq \dots \geq 0$, repetitions being included in accordance with algebraic multiplicities. In addition, $\mathcal{K}(Y_{k,j}) = a_{k,j}$, $j = 1, 2, \dots, d_k^m$, $k \in \mathbb{Z}_+$ and, consequently, the set $\{a_{k,j} : j = 1, 2, \dots, d_k^m; k = 0, 1, \dots\}$ is the set of eigenvalues of \mathcal{K} .

The result itself depends on a technical hypotheses which refers to a double indexed sequence $\{b_{k,n}\}$ of non-negative real numbers. It is *half-bounded away from 0* if $\lim_{n \rightarrow \infty} b_{k,n} = 0$, $k \in \mathbb{Z}_+$ and there exists a positive real number M so that $b_{k,n} \geq M$, $k \geq n$.

Theorem 2.2. *Let $\{M_t : t \in (0, \pi)\}$ be a uniformly bounded family of multiplier operators on $L^2(S^m)$, with corresponding multiplier sequences $\{\eta_k^t\}$. Let \mathcal{K} be a positive integral operator generated by a $\{M_t : t \in (0, \pi)\}$ -Hölder kernel K . If $\{|\eta_k^{1/n} - 1|\}$ is half-bounded away from 0, then $\lambda_n(\mathcal{K}) = O(n^{-1-\beta/m})$, as $n \rightarrow \infty$, in which β is the Hölder exponent of K with respect to $\{M_t : t \in (0, \pi)\}$.*

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PROPRIEDADE AHSP PARA ESPAÇOS FUNÇÃO MÓDULO

THIAGO GRANDO * & MARY LILIAN LOURENÇO †

Mostramos que dada uma família de espaços de Banach $(X_k)_{k \in K}$ com a propriedade AHSP, onde K é um espaço topológico Hausdorff não-vazio, a função módulo $(K, (X_k)_{k \in K}, X)$ satisfaz a AHSP e, como consequência, o par (l_1, X) satisfaz a BPBP.

1 Introdução

O estudo dos operadores que atingem a norma iniciou na década 50 com o trabalho de R. James, onde provou que um espaço de Banach X é reflexivo, se e somente se, todo funcional linear e contínuo definido em X atinge a norma. Em 1961, E. Bishop e R. R. Phelps começam a estudar classes de funcionais lineares contínuos definidos em espaços de Banach não reflexivos que atingem a norma, e provam que tal classe é densa em X^* . Mais tarde, B. Bollobás, provou uma “versão quantitativa” do teorema de Bishop-Phelps, conhecido como teorema de *Bishop-Phelps-Bollobás*:

Teorema 1.1 (B. Bollobás, [2]). *Seja $\epsilon > 0$ um número arbitrário. Se $x \in S_X$ e $x^* \in S_{X^*}$ são tais que $|1 - x^*(x)| < \frac{\epsilon^2}{4}$, então existem $y \in S_X$ e $y^* \in S_{X^*}$ tais que $y^*(y) = 1$, $\|y - x\| < \epsilon$ e $\|y^* - x^*\| < \epsilon$.*

Surgiu também a idéia de buscar o equivalente do Teorema de *Bishop-Phelps-Bollobás* para o caso de operadores. Assim, em 2008, M. D. Acosta, R. M. Aron, D. García e M. Maestre [1], introduziram a seguinte definição que chamaram de *propriedade de Bishop-Phelps-Bollobás(BPBP)*:

Definição 1.1. *Sejam X e Y espaços de Banach sobre um corpo \mathbb{K} . Dizemos que o par (X, Y) satisfaz a propriedade de *Bishop-Phelps-Bollobás*, se dado $\epsilon > 0$, existirem $\eta(\epsilon) > 0$ e $\beta(\epsilon) > 0$ com $\lim_{\epsilon \rightarrow 0} \beta(\epsilon) = 0$ tais que, para cada $T \in S_{\mathcal{L}(X, Y)}$, se $x \in S_X$ satisfaz $\|Tx\| > 1 - \eta(\epsilon)$, então existem $x_0 \in S_X$ e um operador $R \in S_{\mathcal{L}(X, Y)}$ tais que*

$$\|R(x_0)\| = 1, \quad \|x - x_0\| < \beta(\epsilon), \quad \|T - R\| < \epsilon.$$

Em [1] os autores mostraram alguns pares de espaços de Banach que possuem a BPBP. A partir daí, surgiram diversos trabalhos nessa direção. Assim, uma pergunta natural é: para quais espaços de Banach Y o par (l_1, Y) satisfaz a BPBP? Em [1], os autores definiram certas condições geométricas para Y de tal forma que o par (l_1, Y) tenha a BPBP. Essas condições receberam o nome de *Approximate Hyperplane Series Property* (AHSP):

Definição 1.2. *Um espaço de Banach X tem a propriedade AHSP se, e somente se, para todo $\epsilon > 0$ existirem $\gamma(\epsilon) > 0$ e $\eta(\epsilon) > 0$ com $\lim_{\epsilon \rightarrow 0^+} \gamma(\epsilon) = 0$, tais que, para cada sequência $(x_n)_n \subset B_X$ e cada série convexa $\sum_{n=1}^{\infty} \alpha_n x_n$ com*

$$\left\| \sum_{n=1}^{\infty} \alpha_n x_n \right\| > 1 - \eta(\epsilon),$$

existir um subconjunto $A \subset \mathbb{N}$, $\{z_n : n \in A\} \subset S_X$ e $x^ \in S_{X^*}$ satisfazendo:*

- (i) $\sum_{n \in A} \alpha_n > 1 - \gamma(\epsilon)$,
- (ii) $\|z_n - x_n\| < \epsilon$ para todo $n \in A$,

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(iii) $x^*(z_n) = 1$ para todo $n \in A$.

Eles apresentam alguns espaços que satisfazem essa propriedade, entre eles: os espaços normados de dimensão finita, $L^1(\mu)$ onde μ é uma medida σ -finita, $\mathcal{C}(K)$ onde K é um compacto Hausdorff e os espaços uniformemente convexos. Logo após, surgiram alguns resultados com outros espaços que possuem tal propriedade.

Neste trabalho mostramos sob determinadas condições, que o espaço função módulo satisfaz a propriedade AHSP e, como consequência, o par (l_1, X) satisfaz a BPBP.

2 Resultado

Definição 2.1. Uma função módulo é uma terna $(K, (X_k)_{k \in K}, X)$, onde K é um espaço topológico Hausdorff não-vazio, $(X_k)_{k \in K}$ uma família de espaços de Banach e X um $\mathcal{C}(K)$ -submódulo fechado do $\mathcal{C}(K)$ -módulo $\prod_{k \in K}^\infty X_k$, tal que as seguintes condições são satisfeitas:

- (1) Para todo $x \in X$, a função $k \mapsto \|x(k)\|$ de K em \mathbb{R} é semicontínua superior,
- (2) Para todo $k \in K$, temos $X_k = \{x(k) : x \in X\}$,
- (3) O conjunto $\{k \in K : X_k \neq 0\}$ é denso em K .

Teorema 2.1. Se $(X_k)_{k \in K}$ tem AHSP, para todo $k \in K$, então a função módulo $(K, (X_k)_{k \in K}, X)$ tem AHSP.

Corolário 2.1. Seja $(K, (X_k)_{k \in K}, X)$ uma função módulo, onde $(X_k)_{k \in K}$ tem AHSP, para todo $k \in K$. Então (l_1, X) satisfaz a BPBP.

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SPACEABILITY AND ALGEBRABILITY IN THE THEORY OF DOMAINS OF EXISTENCE IN BANACH SPACES

THIAGO R. ALVES *

1 Introduction

Let U be an open subset of an infinite-dimensional complex Banach space E . Let $\mathcal{H}(U)$ denote the algebra of all holomorphic functions on U . Let $\mathcal{E}(U)$ denote the set of all $f \in \mathcal{H}(U)$ such that U is the domain of existence of f . Throughout this work the algebra $\mathcal{H}(U)$ is equipped with the compact-open topology τ_c .

In this work we first show that if U is a connected domain of existence in a separable Banach space E , then the set $\mathcal{E}(U) \cup \{0\}$ contains a closed infinite dimensional subspace of $\mathcal{H}(U)$, that is $\mathcal{H}(U)$ is spaceable in the sense of [3]. Next we show that, under the same hypothesis, the set $\mathcal{E}(U) \cup \{0\}$ contains a closed subalgebra of $\mathcal{H}(U)$, which contains an infinite algebraically independent set. In particular $\mathcal{E}(U)$ is strongly algebrable in the sense of [5]. Finally we show that $\mathcal{E}(U) \cup \{0\}$ contains a dense infinite algebraically independent subalgebra in $\mathcal{H}(U)$, that is $\mathcal{E}(U)$ is densely strongly algebrable in the sense of [5].

Many authors have devoted their attention to the study of spaceable sets and algebrable sets during the last decade. We refer the reader to [6] for a survey on this recent trend in functional analysis. For a paper concerning spaceability and algebrability in the theory of domains of existence on finite-dimensional spaces we refer to [4]. We also refer to [1] for a work regarding algebrability in the theory of domains of existence on infinite-dimensional Banach spaces.

2 Mathematical Results

Given $x \in U$, let $d_U(x)$ denote the distance from x to the boundary of U . The next result concerning interpolation sequences involving holomorphic functions. To prove this result we use some ideas provided in [7, Section 2].

Proposition 2.1. *Let E be a separable Banach space, and let U be a domain of existence in E . Then:*

- (a) *For each sequence $(x_n)_{n=1}^{\infty}$ of distinct points of U such that $\lim_{n \rightarrow \infty} d_U(x_n) = 0$, there exists a function $f \in \mathcal{H}(U)$ such that $\lim_{n \rightarrow \infty} |f(x_n)| = \infty$ and $f(x_n) \neq f(x_m)$ whenever $n \neq m$.*
- (b) *For each sequence $(x_n)_{n=1}^{\infty}$ of distinct points of U such that $\lim_{n \rightarrow \infty} d_U(x_n) = 0$, and each sequence $(\alpha_n)_{n=1}^{\infty}$ in \mathbb{C} , there exists a function $f \in \mathcal{H}(U)$ such that $f(x_n) = \alpha_n$ for every $n \in \mathbb{N}$.*

Let us recall that a subset L of a complex Banach space E is said to be *locally determining* at zero if for every connected open neighborhood U of zero and every $f \in H(U)$, if $f = 0$ on $U \cap L$, then $f = 0$ on U . In [2], Ansemil and Dineen prove the following theorem:

Theorem 2.2. *Let E be a separable Banach space and let L be a locally determining set at zero. Then L contains a null sequence which is also locally determining at zero.*

The next proposition follows by applying the last theorem.

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Proposition 2.3. Let E be a separable Banach space and let W be an open subset of E . If $a \in \partial W$ and $\delta > 0$, then there exists a sequence $(x_p)_{p=1}^{\infty}$ in W such that

(a) $\sup_{p \in \mathbb{N}} \|a - x_p\| < \delta$ and $\lim_{p \rightarrow \infty} x_p = a$.

(b) If U is a connected open subset of E such that $a \in U$ and $f \in \mathcal{H}(U)$ satisfies $f(x_p) = 0$ for every p , whenever $x_p \in U$, then $f = 0$.

Proposition 2.4. Let X be an arbitrary set, and let \mathcal{A} be an algebra of functions $f : X \rightarrow \mathbb{C}$. If there exists a function $f \in \mathcal{A}$ and a sequence $(x_k)_{k=1}^{\infty}$ in X such that $\lim_{k \rightarrow \infty} |f(x_k)| = \infty$, then \mathcal{A} contains an infinite dimensional vector subspace.

Next theorem follows from Propositions 2.1, 2.3 and 2.4.

Theorem 2.5. Let E be a separable Banach space, and let U be a connected domain of existence in E . Then the set $\mathcal{E}(U)$ is spaceable.

Proposition 2.6. Let X be an arbitrary set, and let \mathcal{A} be an algebra of functions $f : X \rightarrow \mathbb{C}$. If there exists a sequence $(f_n)_{n=1}^{\infty}$ in \mathcal{A} and a sequence $(x_k)_{k=1}^{\infty}$ in X such that

$$|f_1(x_k)| \geq k \quad \text{and} \quad |f_n(x_k)| \geq \prod_{m < n} |f_m(x_k)|^k \quad (2.1)$$

for each $n \in \mathbb{N} \setminus \{1\}$ and $k \in \mathbb{N}$, then \mathcal{A} contains an infinite algebraically independent set of generators.

If \mathcal{A} is a complex commutative topological algebra, then a set $A \subset \mathcal{A}$ is said to be *closely strongly algebrable* if $A \cup \{0\}$ contains a closed subalgebra \mathcal{B} of \mathcal{A} which contains an infinite algebraically independent set of generators. Next theorem follows from Proposition 2.1, 2.3 and 2.6.

Theorem 2.7. Let E be a separable Banach space, and let U be a connected domain of existence in E . Then the set $\mathcal{E}(U)$ is closely strongly algebrable.

Theorem 2.8. Let E be a separable Banach space, and let U be a connected domain of existence in E . Then the set $\mathcal{E}(U)$ is densely strongly algebrable.

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LONG-TIME BEHAVIOR FOR A MODEL OF EXTENSIBLE BEAM WITH NONLOCAL NONLINEAR DAMPING

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1 Introduction

This work is concerned with the global existence and long-time behavior of solutions to the initial boundary value problem of an extensible beam equation with nonlocal nonlinear damping and source terms:

$$u_{tt} + \Delta^2 u - M(\|\nabla u\|_2^2) \Delta u + N(\|\nabla u\|_2^2) |u_t|^\gamma u_t + f(u) = h \quad \text{in } \Omega \times \mathbb{R}^+. \quad (1.1)$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\Gamma = \partial\Omega$. Corresponding to the displacement $u = u(x, t)$ we consider two different types of boundary conditions, namely, clamped or simply supported boundary conditions

$$u = \frac{\partial u}{\partial \nu} = 0 \quad \text{or} \quad u = \Delta u = 0 \quad \text{on } \Gamma \times \mathbb{R}^+, \quad (1.2)$$

respectively, where ν is the unit exterior normal to Γ . The initial conditions associated to (1.1) are given by

$$u(x, 0) = u_0(x) \quad \text{and} \quad u_t(x, 0) = u_1(x), \quad x \in \Omega. \quad (1.3)$$

Let $V_0 = L^2(\Omega)$ and $V_1 = H_0^1(\Omega)$, and to attend the two boundary conditions in (1.2) we define $V_2 = H_0^2(\Omega)$ or $V_2 = H^2(\Omega) \cap H_0^1(\Omega)$.

Let $\lambda_1 > 0$ the first eigenvalue of the bi-harmonic operator Δ^2 in V_2 . The function $M \in C^1(\mathbb{R}^+)$ and there exists constants $c_M > 0$ such that

$$\widehat{M}(\tau) \geq -\frac{1}{8\lambda_1^{1/2}}\tau - c_M \quad \text{and} \quad M(\tau)\tau - \frac{1}{2}\widehat{M}(\tau) \geq -\frac{1}{16\lambda_1^{1/2}}\tau - c_M, \quad \forall \tau \geq 0, \quad \widehat{M}(\tau) = \int_0^\tau M(s)ds. \quad (1.4)$$

The function $N \in C^1(\mathbb{R}^+)$ and exists constant $n_0 > 0$ such that

$$N(\tau) \geq n_0 > 0, \quad \forall \tau \geq 0. \quad (1.5)$$

The nonlinear function $f \in C^1(\mathbb{R})$, $f(0) = 0$, and there exist positive constants $c_f, c_{f'}$ such that

$$|f'(u)| \leq c_{f'}(1 + |u|^\rho), \quad \forall u \in \mathbb{R}, \quad (1.6)$$

$$\hat{f}(u) \geq -\frac{1}{16\lambda_1}|u|^2 - c_f \quad \text{and} \quad f(u)u - \hat{f}(u) \geq -\frac{1}{16\lambda_1}|u|^2 - c_f, \quad \forall u \in \mathbb{R}, \quad (1.7)$$

where

$$\hat{f}(u) = \int_0^u f(s)ds$$

with

$$\rho, \gamma > 0 \quad \text{if } 1 \leq n \leq 4 \quad \text{and} \quad 0 < \rho \leq \gamma < \frac{4}{n-4} \quad \text{if } n \geq 5. \quad (1.8)$$

Our analysis with respect to the global existence and long-time behavior of solutions is given on the phase space $\mathcal{H} = V_2 \times V_0$ equipped with norm

$$\|(u, v)\|_{\mathcal{H}}^2 = \|\Delta u\|_2^2 + \|v\|_2^2.$$

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2 Mathematical Results

The existence and uniqueness results of the global solutions in the space \mathcal{H} are given in the following theorem.

Theorem 2.1. *Let $h \in L^2(\Omega)$. Under hypotheses (1.4)-(1.8), if initial data $(u_0, u_1) \in \mathcal{H}$, then problem (1.1)-(1.3) has a unique weak solution*

$$(u, u_t) \in C([0, T], \mathcal{H}), \quad \forall T > 0, \quad (2.9)$$

satisfying

$$u \in L^\infty(0, T; V_2), \quad u_t \in L^\infty(0, T; V_0) \quad \text{and} \quad u_{tt} \in L^2(0, T; V'_2). \quad (2.10)$$

Proof The principle of the proof is classical. We using the Faedo-Galerkin method associated to compactness arguments (see, for instance [1], [2]) with minor changes on the nonlinear terms $f(u)$, $M(\|\nabla u(t)\|_2^2)|u_t|^\gamma u_t$ and on the external force $h \in L^2(\Omega)$.

The well-posedness of problem (1.1)-(1.3) given by Theorem 2.1 implies that the evolution operator $S(t) : \mathcal{H} \rightarrow \mathcal{H}$ defined by

$$S(t)(u_0, u_1) = (u(t), u_t(t)), \quad t \geq 0, \quad (2.11)$$

where (u, u_t) is the unique weak solution of the system (1.1)-(1.3), defines a nonlinear C_0 -semigroup which is locally Lipschitz continuous on the phase space \mathcal{H} . Therewith the dynamics of problem (1.1)-(1.3) can be studied through the continuous dynamical system $(\mathcal{H}, S(t))$.

Our main result in the present work is the following.

Theorem 2.2. *Assume that hypotheses of Theorem 2.1 hold. Then the dynamical system $(\mathcal{H}, S(t))$ associated to the problem (1.1)-(1.3) possesses a compact global attractor $\mathcal{A} \subset \mathcal{H}$. Moreover, the compact global attractor \mathcal{A} has finite fractal and Hausdorff dimensions.*

Proof The existence of a compact global attractor is granted once our dynamical system $(\mathcal{H}, S(t))$ is dissipative and satisfies an asymptotic smoothness property. Then to conclude the compact global attractor has finite fractal dimension we employ more recent results based on a quasi-stability property of Chueshov and Lasiecka [3, 4].

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GENERALIZED CONVOLUTION OF POSITIVE DEFINITE KERNELS ON COMPLEX SPHERES

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1 Introduction

Let Ω_{2q} be the unit sphere in \mathbb{C}^q and σ_q the usual normalized measure on it. A kernel K in $L^2(\Omega_{2q}, \sigma_q \times \sigma_q)$ is L^2 -positive definite if

$$\int_{\Omega_{2q}} \left(\int_{\Omega_{2q}} K(x, y) f(y) d\sigma_q(y) \right) \overline{f(x)} d\sigma_q(x) \geq 0, \quad f \in L^2(\Omega_{2q}, \sigma_q).$$

The convolution of two kernels K_1 and K_2 from $L^2(\Omega_{2q})$ is the kernel $K_1 * K_2$ given by the formula

$$(K_1 * K_2)(x, y) = \frac{1}{\omega_q} \int_{\Omega_{2q}} K_1(x, \xi) K_2(\xi, y) d\sigma_q(\xi), \quad x, y \in \Omega_{2q}, \quad (1.1)$$

in which $\omega_q := 2\pi^q/(q-1)!$ is the surface area of Ω_{2q} . This notion of convolution occurs more frequently in the literature in the case when at least one of the kernels involved is zonal and the other one is a function of one variable (see [2, 4]). The zonality of K corresponds to the existence of a function $K' : B[0, 1] \rightarrow \mathbb{C}$ so that

$$K(x, y) = K'(x \cdot y), \quad x, y \in \Omega_{2q}, \quad (1.2)$$

in which \cdot denotes the usual inner product in \mathbb{C}^q and $B[0, 1] := \{z \in \mathbb{C} : z\bar{z} \leq 1\}$. We will write

$$\mathcal{B}^2(\Omega_{2q}^2) := \{K \in L^2(\Omega_{2q}^2, \sigma_q \times \sigma_q) : K \text{ is zonal}\}.$$

Adapting arguments found in [3], one can see that, in the case $q \geq 2$, a kernel K on Ω_{2q} is L^2 -positive definite and zonal if and only if the generating function K' appearing in (1.2) have a double series representation of the form

$$K'(z) = \sum_{m,n=0}^{\infty} a_{m,n}^{q-2}(K') R_{m,n}^{q-2}(z), \quad z \in B[0, 1], \quad (1.3)$$

in which $a_{m,n}^{q-2}(K') \geq 0$, $m, n \in \mathbb{Z}_+$. The convergence of the series needs to be in $L^2(B[0, 1], \nu_{q-2})$, where

$$d\nu_{q-2}(z) = \frac{q-1}{\pi} (1 - |z|^2)^{q-2} dx dy, \quad z = x + iy \in B[0, 1].$$

The symbol $R_{m,n}^{q-2}$ stands for the disk or generalized Zernike polynomial of bi-degree (m, n) associated with the dimension q ([5]). In the case $q = 1$, (1.3) still holds but one needs to replace $R_{m,n}^{q-2}$ with $R_m(z) = z^m$, the double sum with $\sum_{m \in \mathbb{Z}_+}$ and $B[0, 1]$ with Ω_2 itself. In order to quote our main results, it is convenient to introduce the zonal kernels defined by the disk polynomials:

$$Z_{m,n}(x, y) := R_{m,n}^{q-2}(x \cdot y), \quad m, n \in \mathbb{Z}_+, \quad x, y \in \Omega_{2q}, \quad (q \geq 2)$$

and

$$Z_m(x, y) := R_m(x \cdot y) = x^m y^{-m}, \quad m \in \mathbb{Z}, \quad x, y \in \Omega_2.$$

Since q remains fixed, the omission of the dimension q in both notations introduced above should cause no confusion.

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2 Main Results

The disk polynomials are orthogonal in the following sense ($q \geq 2$)

$$\int_{B[0,1]} R_{m,n}^{q-2}(z) \overline{R_{k,l}^{q-2}(z)} d\nu_{q-2}(z) = \frac{\delta_{mk}\delta_{nl}}{h_{m,n}^{q-2}}$$

where

$$h_{m,n}^{q-2} = \frac{m+n+q-1}{q-1} \binom{m+q-2}{q-2} \binom{n+q-2}{q-2}.$$

On the other hand, $\{R_m : m \in \mathbb{Z}\}$ is an orthonormal basis of $L^2(\Omega_{2q}, \sigma_2)$. If we write $\langle \cdot, \cdot \rangle_2$ to denote the normalized inner product of $L^2(\Omega_{2q}^2, \sigma_q \times \sigma_q)$, that is,

$$\langle K_1, K_2 \rangle_2 := \frac{1}{\omega_q^2} \int_{\Omega_{2q}^2} K_1(x, y) \overline{K_2(x, y)} d(\sigma_q \times \sigma_q)(x, y), \quad K_1, K_2 \in L^2(\Omega_{2q}^2, \sigma_q \times \sigma_q),$$

the main results in this note can be stated as follows.

Theorem 2.1. ([1]) Let K belong to $L^2(\Omega_{2q}^2, \sigma_q \times \sigma_q)$. If $K = J * J$ for some J in $\mathcal{B}^2(\Omega_{2q}^2)$, then

$$\sum_{m,n=0}^{\infty} h_{m,n}^{q-2} |\langle K, Z_{m,n} \rangle_2| < \infty \quad (q \geq 2)$$

and

$$\sum_{m=-\infty}^{\infty} |\langle K, Z_m \rangle_2| < \infty \quad (q = 1).$$

Theorem 2.2. ([1]) Let K be a kernel in $L^2(\Omega_{2q}^2)$. If $q \geq 2$, assume that all the Fourier coefficients $\langle K, Z_{m,n} \rangle_2$ are nonnegative and that

$$\sum_{m,n=0}^{\infty} h_{m,n}^{q-2} \langle K, Z_{m,n} \rangle_2 < \infty.$$

Otherwise, assume that all Fourier coefficients $\langle K, Z_m \rangle_2$ are nonnegative and that

$$\sum_{m=-\infty}^{\infty} \langle K, Z_m \rangle_2 < \infty.$$

Then, there exists an L^2 -positive definite kernel P in $\mathcal{B}^2(\Omega_{2q}^2)$ such that $K = P * P$. In particular, K is an L^2 -positive definite element of $\mathcal{B}^2(\Omega_{2q}^2)$.

Theorem 2.3. ([1]) If K is a continuous, zonal and L^2 -positive definite kernel on Ω_{2q} , then there exists a positive definite kernel P in $\mathcal{B}^2(\Omega_{2q}^2)$ such that $K = P * P$.

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TIPOS DE HOLOMORFIA E OPERADORES DE CONVOLUÇÃO EM ESPAÇOS DE FUNÇÕES Θ -HOLOMORFAS DE UM DADO TIPO E UMA DADA ORDEM

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Neste trabalho caracterizamos os operadores de convolução nos espaços $Exp_{\Theta}^k(E)$, para $k \in [1, +\infty)$ e $Exp_{\Theta,0}^k(E)$, para $k \in [1, +\infty]$ de funções holomorfas definidas no espaço de Banach E a valores complexos.

A caracterização destes espaços é importante para se obter resultados de existência e aproximação de soluções para equações de convolução. Nesta caracterização utilizaremos o conceito de tipo de holomorfia definido por L. Nachbin em [3] e o conceito de π_1 -tipo de holomorfia, introduzido em [1] e explorado em [2].

1 Definições e Resultados

Definição 1.1. Seja $(\mathcal{P}_{\Theta}(^jE))_{j=0}^{\infty}$ um tipo de holomorfia de E em \mathbb{C} . Se $\rho > 0$ e $k \geq 1$, denotamos por $\mathcal{B}_{\Theta,\rho}^k(E)$ o espaço vetorial complexo de todas $f \in \mathcal{H}(E)$ tais que $\widehat{d^j f}(0) \in \mathcal{P}_{\Theta}(^jE)$, para todo $j \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$ e

$$\|f\|_{\Theta,k,\rho} = \sum_{j=0}^{\infty} \rho^{-j} \left(\frac{j}{ke} \right)^{\frac{j}{k}} \left\| \frac{1}{j!} \widehat{d^j f}(0) \right\|_{\Theta} < +\infty,$$

que é um espaço de Banach com a norma $\|\cdot\|_{\Theta,k,\rho}$.

Definição 1.2. Seja $(\mathcal{P}_{\Theta}(^jE))_{j=0}^{\infty}$ um tipo de holomorfia de E em \mathbb{C} e $k \geq 1$. Denotaremos por $Exp_{\Theta}^k(E)$ o espaço vetorial complexo $\bigcup_{\rho>0} \mathcal{B}_{\Theta,\rho}^k(E)$ com a topologia limite induutivo localmente convexa. Consideramos $Exp_{\Theta,0}^k(E) = \bigcap_{\rho>0} \mathcal{B}_{\Theta,\rho}^k(E)$ com a topologia limite projetivo localmente convexa.

Aqui estamos considerando as topologias limite induutivo e projetivo dadas pelas inclusões naturais.

Definição 1.3. Seja $k \geq 1$. Um operador de convolução em $Exp_{\Theta}^k(E)$ é uma aplicação linear contínua

$$\mathcal{O}: Exp_{\Theta}^k(E) \longrightarrow Exp_{\Theta}^k(E)$$

tal que $\tau_{-a}(\mathcal{O}(f)) = \mathcal{O}(\tau_{-a}f)$, para todo $a \in E$ e $f \in Exp_{\Theta}^k(E)$, onde $\tau_{-a}f(x) = f(x+a)$, para todo $x \in E$. Analogamente define-se um operador de convolução em $Exp_{\Theta,0}^k(E)$.

Denotaremos os conjuntos de todos os operadores de convolução em $Exp_{\Theta}^k(E)$ e $Exp_{\Theta,0}^k(E)$, respectivamente por \mathcal{A}_{Θ}^k e $\mathcal{A}_{\Theta,0}^k$.

Definição 1.4. Para $k \in [1, +\infty)$, $T \in [Exp_{\Theta}^k(E)]'$ e $f \in Exp_{\Theta}^k(E)$, definimos o *produto de convolução entre T e f* por $(T * f)(x) = T(\tau_{-x}f)$, para todo $x \in E$. Analogamente para $Exp_{\Theta,0}^k(E)$.

Nosso objetivo é mostrar que $T*$ define um operador de convolução nos espaços $Exp_{\Theta}^k(E)$ e $Exp_{\Theta,0}^k(E)$. Mais ainda, vamos mostrar que todo operador de convolução nestes espaços é desta forma, conforme o próximo resultado.

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Teorema 1.1. Seja $(\mathcal{P}_\Theta(^j E))_{j=0}^\infty$ um π_1 -tipo de holomorfia de E em \mathbb{C} , com $\|P\|_\Theta = \sum_{j=0}^q \|\psi_j^m\|_\Theta$ para todo $P = \sum_{j=0}^q \psi_j^m$, com $\psi_j \in E'$, $q, m \in \mathbb{N}$. Então, para $\mathcal{O} \in \mathcal{A}_\Theta^k$ (ou $\mathcal{O} \in \mathcal{A}_{\Theta,0}^k$) existe um único $T \in [Exp_\Theta^k(E)]'$ (ou $T \in [Exp_{\Theta,0}^k(E)]'$) tal que $\mathcal{O} = T *$.

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$L^p - L^q$ ESTIMATES FOR KLEIN-GORDON TYPE WAVE MODELS WITH NON-EFFECTIVE TIME-DEPENDENT POTENTIAL

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1 Introduction

We consider the Cauchy problem for Klein-Gordon type models,

$$u_{tt} - \Delta u + m(t)^2 u = 0, \quad u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), \quad (1.1)$$

with $tm(t) \rightarrow 0$ and $m \notin L^1$, i.e., $m(t)^2 u$ is non-effective time-dependent potential. The goal is apply a diagonalization procedure to Klein-Gordon problems (1.1) with sufficiently smooth time-dependent coefficient $m = m(t)$ aiming to find a representation for the solution by Fourier multipliers and then derive $L^p - L^q$ decay estimates on the conjugate line. This procedure is well-known as WKB analysis and was introduced by K. Yagdjian in [2] and M. Reissig - K. Yagdjian in [3].

A modified scattering result will complete our considerations.

The results presented in this abstract is a generalization of $L^2 - L^2$ estimates proved in the paper [1].

2 Mathematical Results

In order to get some feeling for the behavior of solutions to (1.1) we can transform the time-dependent potential to a time-dependent damping and a new potential. If we introduce the change of variables given by $u(t, x) = \psi(t)v(t, x)$, then the Cauchy problem (1.1) takes the form

$$v_{tt} - \Delta v + 2\frac{\psi'(t)}{\psi(t)}v_t + \left(\frac{\psi''(t)}{\psi(t)} + m(t)^2\right)v = 0, \quad v(0, x) = \frac{u_0(x)}{\psi(0)}, \quad v_t(0, x) = \frac{u_1(x)}{\psi(0)} \quad (2.2)$$

If we choose a suitable function ψ , then [4] gives us sufficient conditions in order to exclude contributions to the energy coming from the time-dependent potential. For the damping term that appears, we use some ideas of [5] about asymptotic properties of solutions to wave equations with time-dependent non-effective dissipation.

After this consideration let us consider the Cauchy problem (1.1) under the following conditions:

Hypothesis 2.1. Let $m(t) \in C^\ell(\mathbb{R}_+)$ satisfy

$$|m(t)| \lesssim \frac{1}{1+t}, \quad |m^{(k)}(t)| \lesssim \frac{m(t)}{(1+t)^k} \quad \text{for all } k \leq \ell. \quad (2.3)$$

Hypothesis 2.2. There exists a positive increasing function $\psi = \psi(t) \in C^\infty(\mathbb{R}_+)$, such that

$$\limsup_{t \rightarrow \infty} 2(1+t) \frac{\psi'(t)}{\psi(t)} < 1, \quad \left| \frac{\psi^{(k)}(t)}{\psi(t)} \right| \lesssim \frac{1}{(1+t)^k} \quad \text{for all } k \in \mathbb{N}. \quad (2.4)$$

And we assume the following relation between $m(t)$ and $\psi(t)$:

$$\int_0^\infty (1+\tau) \left| \frac{\psi''(\tau)}{\psi(\tau)} + m(\tau)^2 \right| d\tau \lesssim 1. \quad (2.5)$$

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Then we can prove the following $L^p - L^q$ estimates:

Theorem 2.1. *Assume Hypotheses 2.1 and 2.2. If the Cauchy data $u_0, u_1 \in \mathcal{S}(\mathbb{R}^n)$, then we have the $L^p - L^q$ estimates for the kinetic, elastic and potential energy as follows:*

$$\|(u_t(t, \cdot), \nabla_x u(t, \cdot), p(t)u(t, \cdot))\|_q \lesssim (1+t)^{-\frac{n-1}{2}(\frac{1}{p}-\frac{1}{q})} (\|u_0\|_{p,r+1} + \|u_1\|_{p,r})$$

for $p \in (1, 2]$, p and q on the conjugate line, $p(t) = (1+t)^{-1}\psi(t)$, and with regularity $r = n(\frac{1}{p} - \frac{1}{q})$.

To guarantee the optimality of our estimates we proved a modified scattering result to Cauchy problems for wave equation with scattering time-dependent mass term and non-effective time-dependent dissipation. In other words, consider the Cauchy problem for wave equations with time-dependent mass and dissipation

$$u_{tt} - \Delta u + b(t)u_t + m(t)u = 0, \quad u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), \quad (2.6)$$

where $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$, $b = b(t) \geq 0$ and $m = m(t) \geq 0$ under the following assumptions:

Hypothesis 2.3. *Suppose that $b(t)$ and $m(t)$ satisfy*

$$\left| \frac{d^k}{dt^k} b(t) \right| \leq C_k \left(\frac{1}{1+t} \right)^{k+1} \quad \text{for } k = 0, 1 \text{ and } m(t) \leq C \left(\frac{1}{1+t} \right)^2.$$

Hypothesis 2.4. *Suppose that $b(t)$ and $m(t)$ satisfy*

$$\limsup_{t \rightarrow \infty} tb(t) < 1 \quad \text{and} \quad (1+t)m(t) \in L^1.$$

Then if we consider v as the solution of the free wave equation, i.e.,

$$v_{tt} - \Delta v = 0, \quad v(0, x) = v_0(x), \quad v_t(0, x) = v_1(x), \quad (2.7)$$

we can prove that:

Theorem 2.2. *If Hypothesis 2.3 and Hypothesis 2.4 are satisfies, then there exists a bounded operator*

$$W_+ : (u_0, u_1) \in H^1(\mathbb{R}^n) \times L^2(\mathbb{R}^n) \rightarrow (v_0, v_1) \in H^1(\mathbb{R}^n) \times L^2(\mathbb{R}^n)$$

such that for Cauchy data (u_0, u_1) of (2.6) and associated data $(v_0, v_1) = W_+(u_0, u_1)$ to (2.7) the corresponding solutions $u = u(t, x)$ and $v = v(t, x)$ satisfy

$$\|\lambda(t)(u_t(t, \cdot), \nabla_x u(t, \cdot)) - (v_t(t, \cdot), \nabla_x v(t, \cdot))\|_2 \rightarrow 0$$

as $t \rightarrow \infty$.

The results presented here is part of the Nascimento's PhD-thesis.

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EXISTENCE OF SOLUTIONS FOR A CLASS OF $p(x)$ -KIRCHHOFF TYPE EQUATION VIA TOPOLOGICAL METHODS

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Abstract

The aim of this work is to obtain weak solutions for a class of $p(x)$ -Kirchhoff type problem subject to no flux boundary conditions. Our approach relies on the variable exponent theory of generalized Lebesgue-Sobolev spaces combined with a Fredholm-type result for a couple of nonlinear operators.

1 Introduction

In this paper we study the following problem

$$\begin{aligned} -M(A(x, \nabla u))\operatorname{div}(a(x, \nabla u)) &= f(x, u)|u|_{s(x)}^{t(x)} \quad \text{in } \Omega \\ u &= \text{constant} \quad \text{on } \partial\Omega \\ \int_{\partial\Omega} a(x, \nabla u) \cdot \nu d\Gamma &= 0. \quad \text{on } \partial\Omega \end{aligned} \tag{1.1}$$

where Ω is a bounded domain in \mathbb{R}^n with a smooth boundary $\partial\Omega$, and $N \geq 1$, $p, s, t \in C(\bar{\Omega})$ for any $x \in \bar{\Omega}$; $M : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a continuous function, f is a Caratheodory function and $\operatorname{div}(a(x, \nabla u))$ is a $p(x)$ -Laplacian type operator.

Recently, the studies of differential equations and variational problems with non standard $p(x)$ -growth conditions have received considerable attention (See[2] – [4]).

We consider (1.1) to study the existence of weak solutions. Employing a Fredholm type theorem (See Dinca[1]), we should be able to establish our results.

2 Mathematical Results

We need some theorems on $W^{1,p(x)}(\Omega)$ which we call a variable exponent Sobolev space.

Write: $C_+(\bar{\Omega}) = \{p(x) \in C(\bar{\Omega}) : p(x) > 1, \forall x \in \bar{\Omega}\}$; $p^+ = \max\{p(x); x \in C(\bar{\Omega})\}$; $p^- = \min\{p(x); x \in C(\bar{\Omega})\}$
 $M(\Omega) = \{u : u \text{ is a real-valued measurable function on } \Omega\}$,

$$L^{p(x)}(\Omega) = \{u \in M(\Omega) : \int_{\Omega} |u(x)|^{p(x)} dx < \infty\}$$

We can introduce a norm on $L^{p(x)}(\Omega)$

$$|u|_{p(x)} = \inf\{\lambda > 0 : \int_{\Omega} \left|\frac{u(x)}{\lambda}\right|^{p(x)} dx \leq 1\}$$

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and $(L^{p(x)}(\Omega), |u|_{p(x)})$ becomes a Banach Space. The space $W^{1,p(x)}(\Omega)$ is defined by

$$W^{1,p(x)}(\Omega) = \{u \in L^{p(x)}(\Omega) : |\nabla u| \in L^{p(x)}(\Omega)\}$$

with the norm

$$\|u\|_{p(x)} = |u|_{p(x)} + |\nabla u|_{p(x)} \quad \forall u \in W^{1,p(x)}(\Omega)$$

Let

$$V = \{u \in W^{1,p(x)}(\Omega) : u|_{\partial\Omega} = \text{constante}\}$$

Definition 2.1. A function $u \in V$ is said to be a weak solutions of (1.1) if

$$M\left(\int_{\Omega} A(x, \nabla u) dx\right) \int_{\Omega} (a(x, \nabla u)) \nabla v dx = |u|_{s(x)}^{t(x)} \int_{\Omega} f(x, u) v dx \quad , \quad \forall v \in V$$

Below, the function $f : \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ will be always assumed Caratheodory and

$$(f_0) \quad f(x, t)t \leq c_1|t|^{\alpha(x)-1} + c_2 \quad , \quad \forall (x, t) \in \Omega \times \mathbb{R}$$

where c_1, c_2 are positive constants, $\alpha \in C_+(\bar{\Omega})$ such that $1 < \alpha(x) < p^*(x)$, and $M : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a nondecreasing continuous function and satisfies $M(t) > m_0 > 0$. The $a : \bar{\Omega} \times \mathbb{R}^n \rightarrow \mathbb{R}$ is the continuous derivative with respect to ξ of the mapping $A : \bar{\Omega} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$, $A = A(x, \xi)$, that is $a(x, \xi) = D_\xi A(x, \xi)$, and suppose that the following conditions hold:

- a) $a(x, \xi) \leq c_0(1 + |\xi|^{p(x)-1})$, for all $x \in \Omega$ and $\xi \in \mathbb{R}^n$ for some constant $c_0 > 0$.
- b) A is $p(x)$ -uniformly convex: there exists a constant $k > 0$ such that

$$A(x, \frac{\xi + \psi}{2}) \leq \frac{1}{2}A(x, \xi) + \frac{1}{2}A(x, \psi) - k|\xi - \psi|^{p(x)}$$

for all $x \in \Omega$ and $\xi, \psi \in \mathbb{R}^n$

- c) The following inequalities hold true $a(x, \xi).\xi \leq p(x)A(x, \xi)$ for all $x \in \bar{\Omega}$ and $\xi \in \mathbb{R}^n$
- d) $A(x, 0) = 0$, for all $x \in \bar{\Omega}$
- e) $(a(x, \xi) - a(x, \eta)).(\xi - \eta) \geq \gamma|\xi - \eta|^{p(x)}$, some $\gamma \geq 1$

Our main result is as follows:

Theorem 2.1. Assume hypothesis (f_0) , (M_0) , $(a) - e)$ are fulfilled and $\frac{t^+ + \alpha^+}{2} < p^-$. The problem (1.1) has a weak solution.

Proof: We apply a Fredholm type result proved by Dinca [1]. \square

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ADVANCES IN FIRST ORDER STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS

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1 Introduction

We present some results concerning stochastic linear transport equations and quasilinear scalar conservation laws, where the additive noise is a perturbation of the drift. Due to the introduction of the stochastic term, we may prove for instance well-posedness for continuity equation (divergence-free), Cauchy problem, meanwhile uniqueness may fail for the deterministic case, see [1], [2], [3], [4] and [6]. Also for the transport equation, Dirichlet data, we established a better trace result by the introduction of the noise, see [7]. We introduce the study of stochastic hyperbolic conservation laws, in a different direction of [5], applying the kinetic-semigroup theory.

In particular, we establish wellposedness for stochastic continuity equation. Namely, we consider the following Cauchy problem: Given an initial-data u_0 , find $u(t, x; \omega) \in \mathbb{R}$, satisfying

$$\begin{aligned} \partial_t u(t, x; \omega) + \left(u(t, x; \omega) \left(b(t, x) + \frac{dB_t}{dt}(\omega) \right) \right) &= 0, \\ u|_{t=0} &= u_0, \end{aligned} \tag{1.1}$$

$((t, x) \in U_T, \omega \in \Omega)$, where $U_T = [0, T] \times \mathbb{R}^d$, for $T > 0$ be any fixed real number, $b : [0, T] \times \mathbb{R}^d \rightarrow \mathbb{R}^d$ is a given vector field, with $\operatorname{div} b(t, x) = 0$, $B_t = (B_t^1, \dots, B_t^d)$ is a standard Brownian motion in \mathbb{R}^d .

The Cauchy problem for the stochastic transport equation has taken great attention recently, see for instance [2], [4], [5], [6], and more recently the initial-boundary value problem in [8]. Concerning the deterministic case of the problem (1.1), also in a non-regular framework, the reader is mostly addressed to [3] and [1]. Those papers deal respectively with the Sobolev and the BV spatial regularity case, where the uniqueness proof relies on commutators. The main issue in this work is to prove uniqueness of weak L^∞ -solution of the Cauchy problem (1.1) for vector fields

$$\begin{aligned} b &\in L^q([0, T], (L^p(\mathbb{R}^d))^d), \quad p, q < \infty, \\ p &\geq 2, \quad q > 2, \quad \text{and} \quad \frac{d}{p} + \frac{2}{q} < 1. \end{aligned} \tag{1.2}$$

The last condition (1.2) is known in the fluid dynamic's literature as the Ladyzhenskaya-Prodi-Serrin condition, with \leq in place of $<$.

2 Mathematical Results

Theorem 2.1. *Assume conditions (1.2), and $\operatorname{div} b(t, x) \in L^1([0, T], L^\infty(\mathbb{R}))$. If $u, v \in L^\infty(U_T \times \Omega)$ are two weak L^∞ -solutions for the Cauchy problem (1.1), with the same initial data $u_0 \in L^\infty(\mathbb{R}^d)$, then $u = v$ almost everywhere in $[0, T] \times \mathbb{R}^d \times \Omega$.*

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PARABOLIC SYSTEMS WITH CROSS-DIFFUSION TYPE CONSTRAIN

VLADIMIR NEVES * & MIKHAIL VISHNEVSKII †

1 Introduction

In this paper we consider the parabolic system of the form

$$\partial_t u_\alpha = \frac{\partial}{\partial x_j} (a_{\alpha\beta}(\mathbf{x}, \mathbf{u}) \frac{\partial u_\beta}{\partial x_j}) + f_\alpha(\mathbf{x}, \mathbf{u}), \quad (t, \mathbf{x}) \in Q_T, \quad (1.1)$$

where for $T > 0$, $Q_T := (0, T) \times \Omega$, and $\Omega \subset \mathbb{R}^d$, ($d \in \mathbb{N}$ fixed), is an open bounded domain of class C^1 , which the unitary normal vector field on $\partial\Omega =: \Gamma$, is denoted by $\mathbf{n} = (n^1, \dots, n^d)$. The usual summation convention is assumed through the paper and, Latin, Greek indices ranges respectively from 1 to d and from 1 to N , for some $N \in \mathbb{N}$.

One of the main purposes here is to prove existence of global classical solution for (1.1). More precisely, assuming that

$$f_\alpha \in C^2(\overline{\Omega} \times \mathbb{R}^N), \quad (1.2)$$

also

$$\begin{aligned} a_{\alpha\beta} &\in C^2(\overline{\Omega} \times \mathbb{R}^N), \\ \inf \{a_{\alpha\beta}(\mathbf{x}, \mathbf{v}) \xi_\alpha \xi_\beta, \xi \in S^{N-1}, (\mathbf{x}, \mathbf{v}) \in \overline{\Omega} \times \mathbb{R}^N\} &=: \lambda_0 > 0, \end{aligned} \quad (1.3)$$

we seek for a vector function $\mathbf{u} : \overline{Q_T} \rightarrow \mathbb{R}^N$, which is continuously differentiable w.r.t. $t > 0$, twice continuously differentiable w.r.t. $x \in \Omega$, satisfying (1.1) and the following boundary-initial data:

(i) Initial condition. Given $\mathbf{u}_0(\mathbf{x}) \in C(\overline{\Omega})$, the vector value function \mathbf{u} must satisfy in $\{0\} \times \Omega$

$$\mathbf{u}(0, \mathbf{x}) = \mathbf{u}_0(\mathbf{x}). \quad (1.4)$$

(ii) Boundary condition. For $\delta^\alpha \in [0, 1]$, we consider on $\Gamma_T := (0, T) \times \Gamma$

$$\delta^\alpha \frac{\partial u_\alpha}{\partial \mathbf{n}} + (1 - \delta^\alpha) a_\alpha(\mathbf{x}) u_\alpha = 0, \quad (\text{here no sum in } \alpha) \quad (1.5)$$

where a_α is a smooth function, such that $a_\alpha(x) \geq \mu_0 > 0$ for all $\alpha = 1, \dots, N$, $x \in \mathbb{R}^d$. In particular, if $\delta^\alpha = 0$, ($\forall \alpha$), then we have Dirichlet data. On the other hand, for $\delta = 1$, ($\forall \alpha$), we have Neumann boundary condition, and in this case this type of parabolic systems describes the process of diffusion, and also cross-diffusion due to chemical transformations in closed systems.

We assume that the initial data \mathbf{u}_0 belongs to the space

$$E := \{u_{0\alpha}(\mathbf{x}) \in C(\overline{\Omega}) / u_{0\alpha}(\mathbf{x}) = 0 \text{ on } \partial\Omega \text{ when } \delta^\alpha = 0\}$$

and recall that, the local existence of solutions $\mathbf{u}(t, \mathbf{x}, \mathbf{u}_0)$ of problem (1.1), (1.4), (1.5) with initial data in E is proved in [1], see also [2].

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In this paper we apply some techniques and results from [3], and prove the global solvability of system (1.1), (1.4), (1.5). Albeit, we need some additional assumptions:

For $u_\alpha \geq 0$, $\alpha = 1, \dots, K-1, K+1, \dots, N$,

$$\begin{aligned} a_{\alpha K}(\mathbf{x}, u_1, \dots, u_{K-1}, 0, u_{K+1}, \dots, u_N) &= 0, \quad (\mathbf{x} \in \overline{\Omega}), \\ f_K(\mathbf{x}, u_1, \dots, u_{K-1}, 0, u_{K+1}, \dots, u_N) &\leq 0, \quad (\mathbf{x} \in \Omega). \end{aligned} \tag{1.6}$$

Moreover, there exists a smooth positive function $a(\mathbf{u}) \geq a_1 > 0$, and a nonnegative vector Λ , such that

$$\begin{aligned} (A^*(\mathbf{u}) - a(\mathbf{u})I_d)\Lambda &= 0, \\ \Lambda \cdot F(\mathbf{u}) &\leq 0, \end{aligned} \tag{1.7}$$

where $A^*(\mathbf{u})$ is the adjoint matrix of $A(\mathbf{u}) = a_{\alpha\beta}(\mathbf{x}, \mathbf{u})$, and $F(\mathbf{u}) = f_\alpha(\mathbf{x}, \mathbf{u})$. The first equation in (1.7) is usually called a conservation law.

Theorem 1.1 (Main Theorem). *Under conditions (1.2), (1.3), (1.6), and (1.7), the problem (1.1), (1.4), (1.5) has a classical solution $\mathbf{u} \in C_0^{2+\gamma}(\overline{Q_T})$ for every $T > 0$, $\mathbf{u}_0 \in E$, and some $\gamma > 0$.*

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ON WELL-POSEDNESS OF THE THIRD ORDER NLS EQUATION WITH TIME DEPENDENT COEFFICIENTS

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1 Introduction

We consider the initial value problem (IVP) associated to a third order nonlinear Schrödinger (tNLS) equation with variable coefficients

$$u_t + i\alpha(t)u_{xx} + \beta(t)u_{xxx} + i\gamma(t)|u|^2u = 0, \quad u(x, t_0) = u_0(x), \quad x, t, t_0 \in \mathbb{R}, \quad (1.1)$$

that arises in the context of high-speed soliton transmission in long-haul optical communication system and describes the evolution of the normalized complex envelope $u(x, t)$ of an optical pulse in a periodic dispersion map. The functions $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are real valued, $\alpha(t)$ represents the fiber dispersion that defines a dispersion map and is a periodic function with alternating values α^+ and α^- ; $\beta(t)$ models the third order dispersion and $\gamma(t)$ accounts for the effects of gain and loss.

In this work we are interested in studying the well-posedness issues for the IVP (??) for given data in the L^2 -based Sobolev spaces. Also, we will address the scaling limit to *fast dispersion management* as in [?].

Regarding the well-posedness issue to the IVP (??), we have the following:

- In the case when α, β, γ are constants, the local well-posedness for initial data in $H^s(\mathbb{R})$, $s > -\frac{1}{4}$ has been proved in [?].
- In the case when $\alpha, \beta \in C^1([-T_0 + t_0, T_0 + t_0])$, $t_0 \in \mathbb{R}$, $T_0 > 0$ with $\beta \neq 0$ for all $t \in [-T_0 + t_0, T_0 + t_0]$ and γ constant, the local well-posedness for initial data in $H^s(\mathbb{R})$, $s > \frac{1}{4}$ has been obtained in [?].
- Now the question arises: what about the local well-posedness in the case when $\alpha, \beta, \gamma \in C([-T_0 + t_0, T_0 + t_0])$ or are piecewise continuous on $[-T_0 + t_0, T_0 + t_0]$ or if one has $\alpha, \beta, \gamma \in L^\infty([-T_0 + t_0, T_0 + t_0])$?

The objective of this work is to provide answers to the questions posed above. If $\beta(t) = 0$ for all $t \in [-T_0 + t_0, t_0 + T_0]$, then the model (??) can be transformed to the dispersion management NLS studied in [?], where the local and global well-posedness of the associated Cauchy problem and possibility of finite time blow-up is investigated, see Theorem 3.1 and Lemma 3.2 in [?]. Throughout this work, we consider that the third order dispersion never vanish.

2 Mathematical Results

For $\alpha, \beta \in C([-T_0 + t_0, t_0 + T_0])$ and $\gamma \in L^\infty([-T_0 + t_0, t_0 + T_0])$ we prove

Theorem 2.1. *Let $u_0 \in H^s(\mathbb{R})$, $s \geq 0$. Let $\alpha, \beta \in C([-T_0 + t_0, t_0 + T_0])$ with $\beta(t) \neq 0$ for all $t \in [-T_0 + t_0, T_0 + t_0]$, $\gamma \in L^\infty([-T_0 + t_0, t_0 + T_0])$ with $\|\gamma\|_{L^\infty} = M$. Then there exist a time $T = T(\|u_0\|_{H^s}) < T_0$ and a unique solution u to the IVP (??) in $C([-T + t_0, T + t_0], H^s(\mathbb{R}))$. Moreover, the map $u_0 \mapsto u$ is smooth from $H^s(\mathbb{R})$ to $C([-T + t_0, T + t_0]; H^s(\mathbb{R})) \cap X_T^s$.*

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If the dispersion map α is 1-periodic and piecewise constant given by

$$\alpha(t) = \text{alpha}^+, \quad 0 < t \leq t^+, \quad \alpha(t) = -\alpha^-, \quad t^+ - 1 < t \leq 0, \quad (2.2)$$

where α^+ and α^- are positive constants, $t^+ \in (0, 1)$ and $\alpha(t+1) = \alpha(t)$ and $\beta(t)$, $\gamma(t)$ are constants, we can define Bourgain's type space $X^{s,b}$ to prove the following local well-posedness result.

Theorem 2.2. *Let $u_0 \in H^s(\mathbb{R})$, $s > -\frac{1}{4}$. Let α be a periodic function as defined in (??) and $\gamma, \beta \neq 0$ are constants. Then there exist a time $T = T(\|u_0\|_{H^s}) < T_0$ and a unique solution u to the IVP (??) in $C([-T+t_0, T+t_0], H^s(\mathbb{R}))$. Moreover, the map $u_0 \mapsto u$ is smooth from $H^s(\mathbb{R})$ to $C([-T+t_0, T+t_0], H^s(\mathbb{R})) \cap X^{s,b}$, $b > \frac{1}{2}$.*

We also prove the following global well-posedness result.

Theorem 2.3. *Let $u_0 \in H^s(\mathbb{R})$, $s \geq 0$ and $T > 0$ be any given time. Then the local solution to the IVP (??) obtained in Theorems ?? and ?? can be extended to the time interval $[-T+t_0, T+t_0]$.*

In what follows we put forward a study of the scaling limit of the fast dispersion management to the IVP (??) considering γ constant and α given by (??). For a small parameter $0 < \epsilon \ll 1$, we consider

$$u_t^\epsilon + i\alpha\left(\frac{t}{\epsilon}\right)u_{xx}^\epsilon + \beta\left(\frac{t}{\epsilon}\right)u_{xxx}^\epsilon + i\gamma|u^\epsilon|^2u^\epsilon = 0, \quad u^\epsilon(x, t_0) = u_0(x), \quad (2.3)$$

where γ is a constant.

We expect that the behavior of the limiting solution u^ϵ as $\epsilon \rightarrow 0^+$ to be close to the solution of the averaged equation

$$u_t^0 + i m(\alpha)u_{xx}^0 + m(\beta)u_{xxx}^0 + i\gamma|u^0|^2u^0 = 0, \quad u^0(x, t_0) = u_0(x), \quad (2.4)$$

where $m(\alpha)$ and $m(\beta)$ are averages given by

$$m(\alpha) := \int_0^1 \alpha(\tau)d\tau, \quad m(\beta) := \frac{1}{T} \int_0^T \beta(\tau)d\tau. \quad (2.5)$$

This expectation is in fact true as the conclusion of the following theorem shows.

Theorem 2.4. *Let $u_0 \in H^1(\mathbb{R})$ and $u^0 \in C(\mathbb{R}, H^1(\mathbb{R}))$ be the global solution to the averaged IVP (??). Furthermore, let $u^\epsilon \in C(\mathbb{R}, H^1(\mathbb{R}))$, be the global solution of (??). Then we have*

$$\lim_{\epsilon \rightarrow 0^+} \|u^\epsilon - u^0\|_{L_{[T-t_0, T+t_0]}^\infty H^1} = 0,$$

for all $T > 0$.

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