

Conditionally Positive Definite Functions on Spheres and Their Applications

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Abstract: In 1942, Schoenberg published the seminal paper entitled “Positive Definite Functions on Spheres” in which he completely characterized all the zonal positive definite functions on spheres. However, the usefulness of these functions has not been fully realized until almost a half century later. With the help of modern computing technologies, we can employ them as spherical basis functions to model a variety of problems stemming from large quantities of data obtained from the Earth’s surface and atmosphere, such as in weather forecasting and global positioning systems. Numerous practical needs have motivated mathematicians to further study and generalize these functions and broaden the scope of their applications. In this lecture series, we will introduce the notions of “conditionally positive definite functions on spheres”, and study their properties. We will utilize them as basis functions to carry out certain interpolation and quasi-interpolation schemes. We will derive both deterministic and stochastic error estimates of the above approximation methods. The stochastic estimates are closely related to several machine learning problems on spherical domains.