

# The control of evolution PDEs: some recent results and open problems

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These Lectures deal with the control of systems governed by some PDEs. I will mainly consider time-dependent problems. The aim is to present some fundamental results, some applications and some open problems related to the optimal control and the controllability properties of these systems.

In Lecture 1, I will review part of the existing theory for the optimal control of partial differential systems. This is a very broad subject and there have been so many contributions in this field over the last years that we will have to limit considerably the scope. In fact, I will only analyze a few questions concerning some very particular PDEs. We shall focus on the Laplace, the stationary Navier-Stokes and the heat equations. Of course, the existing theory allows to handle much more complex situations.

Lecture 2 is devoted to the controllability of some systems governed by linear time-dependent PDEs. I will consider the heat and the wave equations. I will try to explain which is the meaning of controllability and which kind of controllability properties can be expected to be satisfied by each of these PDEs. The main related results, together with the main ideas in their proofs, will be recalled.

Finally, Lecture 3 is devoted to present some controllability results for other time-dependent, mainly nonlinear, parabolic systems of PDEs. First, we will revisit the heat equation and some extensions. Then, some controllability results will be presented for systems governed by stochastic PDEs. Finally, I will consider several nonlinear systems from fluid mechanics: Burgers, Navier-Stokes, Boussinesq, micropolar, etc.

Along these Lectures, a set of questions (some of them easy, some of them more intricate or even difficult) will be stated. Also, several open problems will be mentioned. I hope that all this will help to understand the underlying concepts and results.