Initial–boundary value problems for quasilinear evolution equations of an odd order

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Quasilinear evolution equations of an odd order (not less than three) are the class of the so-called dispersive equations. Such equations describe propagation of nonlinear waves in dispersive media. The considered equations can be written in such a form:

$$u_t - P_{2l+1}(\partial_x)u + \operatorname{div}_x g(u) = f(t,x), \tag{1}$$

where $u = u(t,x)$, $x = (x_1, \dots, x_n)$, $g = (g_1, \dots, g_n)$, $P_{2l+1}(\partial_x)$ is a linear differential operator with constant real coefficients of the order $2l+1$, where $l \in \mathbb{N}$. This class of equations generalizes such well-known equations as the Korteweg–de Vries equation (KdV)

$$u_t + u_{xxx} + uu_x = 0, \quad x \in \mathbb{R},$$

the Korteweg–de Vries–Burgers equation

$$u_t + u_{xxx} - \delta u_{xx} + uu_x = 0, \quad x \in \mathbb{R}, \quad \delta = \text{const} > 0,$$

the Kawahara equation

$$u_t - u_{xxxxx} + au_{xxx} + uu_x = 0, \quad x \in \mathbb{R}, \quad a = \text{const} \in \mathbb{R},$$

the Zakharov–Kuznetsov equation in two and three spatial dimensions

$$u_t + u_{xxx} + u_{xyy} + uu_x = 0, \quad (x,y) \in \mathbb{R}^2,$$
$$u_t + u_{xxx} + (u_{yy} + u_{zz})_x + uu_x = 0, \quad (x,y,z) \in \mathbb{R}^3.$$

Main assumptions on the operator $P_{2l+1}$ and the vector-function $g$ provide properties of solutions similar to the ones for KdV, namely the conservation law in the space $L_2$ and the local smoothing effect.

Inital value and initial-boundary value problems in various domains are considered for equations (1). Results on global existence and uniqueness of weak solutions are established for initial profiles lying in the space $L_2$. Here essential part of the study consists of construction and investigation of special solutions of the "boundary potential" type to linear analogues of equations (1).

In particular cases of equations (1), namely for $n = 1$ or for the Zakharov–Kuznetsov equation for $n = 2$, global well-posedness is established for initial and initial-boundary value problems in a scale of functional spaces of Sobolev type for initial functions from the spaces $H^k$, $k \in \mathbb{N}$. Here the use of solutions to corresponding linearized equations of the "boundary potential" type gives an opportunity to obtain these results under natural assumptions on boundary data.

Finally, more advanced results are established for the KdV equation itself (which is the most well-known equation of the considered type). Here theorems on global well-posedness for initial-boundary value problems are proved in the Bourgain type spaces for initial functions from the spaces $H^s$, $s \geq 0$.