III ENAMA

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Resumo dos Trabalhos

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III ENAMA

O III ENAMA (Encontro nacional de análise matemática e aplicações) é uma realização do Departamento de Matemática da Universidade Estadual de Maringá, na cidade de Maringá, Paraná, no período de 18 a 20 de novembro de 2009.

O ENAMA é um evento na área de Matemática, mais especificamente, em Análise Funcional, Análise Numérica e Equações Diferenciais, criado para ser um fórum de debates e de intercâmbio de conhecimentos entre diversos especialistas, professores, pesquisadores e alunos de pós-graduação em Matemática do Brasil e do exterior. Nesta terceira edição, o evento contou com três mini-cursos, três palestras plenárias (conferências), sessenta e cinco comunicações orais e dez apresentações de pôsteres.

Os organizadores do III ENAMA desejam expressar sua gratidão aos órgãos e instituições que apoiaram e tornaram possível a realização deste evento: UEM, CNPq, CAPES, Fundação Araucária e SBMAC. Agradecem também a todos participantes do evento, bem como aos colaboradores pelo entusiasmo e esforço, que tanto contribuíram para o sucesso deste evento.

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SEMIDYNAMICAL SYSTEM FOR KURZWEIL EQUATIONS

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In this work, we consider an initial value problem for a class of generalized ODEs, also known as Kurzweil equations, and we prove the existence of a local semidynamical system there.

1 Introduction

We consider $\Omega = \mathcal{O} \times [0, +\infty)$, where $\mathcal{O} \subset \mathbb{R}^n$ is an open. Let $h : [0, +\infty) \to \mathbb{R}$ be a nondecreasing continuous function satisfying

$$|h(t_1 + s) - h(t_2 + s)| \leq |h(t_1) - h(t_2)|, \qquad t_1, t_2, s \in [0, +\infty).$$

We say that a function $G : \Omega \to \mathbb{R}^n$ belongs to the class $\mathcal{F}(\Omega, h)$, whenever $G(x, 0) = 0$ and, for all (x, s_2), (x, s_1), (y, s_2) and $(y, s_1) \in \Omega$, we have

$$\|G(x, s_2) - G(x, s_1)\| \leq |h(s_2) - h(s_1)| \qquad \text{and} \tag{1.1}$$

$$\|G(x, s_2) - G(x, s_1) - G(y, s_2) + G(y, s_1)\| \leq \|x - y\||h(s_2) - h(s_1)|. \tag{1.2}$$

Let $G \in \mathcal{F}(\Omega, h)$. A function $x : [\alpha, \beta] \to \mathbb{R}^n$ is a *solution of the generalized ordinary differential equation*

$$\frac{dx}{d\tau} = DG(x, t) \tag{1.3}$$

with the initial condition $x(t_0) = z_0$ *on the interval* $[\alpha, \beta] \subset [0, +\infty)$, if $t_0 \in [\alpha, \beta]$, $(x(t), t) \in \Omega$ for all $t \in [\alpha, \beta]$ and

$$x(v) - z_0 = \int_{t_0}^{v} DG(x(\tau), t), \qquad v \in [\alpha, \beta].$$

We say that $x : [t_0,\, t_0 + b) \to \mathbb{R}^n$ is the maximal solution of (1.3) with $x(t_0) = u \in \mathcal{O}$, if x is a solution of (1.3) on every interval $[t_0,\, t_0 + \beta]$, $\beta < b$, and it cannot be continued to $[t_0,\, t_0 + b]$. We denote $b = \omega(u,\, G)$ in this case and say that $[t_0,\, t_0 + \omega(u,\, G))$ is the maximal interval of definition of the solution x.

2 Existence of a local semidynamical system

Consider the generalized ODE (1.3), where $G : \Omega \to \mathbb{R}^n$ belongs to $\mathcal{F}(\Omega,\, h)$.

Now we introduce the notion of a local semidynamical system and we claim that initial value problems for the generalized ODE (1.3) generates a local semidynamical system.

For each $(v, G) \in \mathcal{O} \times \mathcal{F}(\Omega, h)$, let $I_{(v,G)}$ be an interval $[0,\, b) \subset \mathbb{R}$, with $b \in \mathbb{R}_+$ and define

$$S = \{(t, v, G) \in \mathbb{R}_+ \times \mathcal{O} \times \mathcal{F}(\Omega, h) :\, t \in I_{(v,G)}\}.$$

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A mapping

$$\pi : S \to \mathcal{O} \times \mathcal{F}(\Omega, h)$$

is called a *local semidynamical system* on $\mathcal{O} \times \mathcal{F}(\Omega, h)$, if the following properties hold:

i) $\pi(0, v, G) = (v, G)$, for every $(v, G) \in \mathcal{O} \times \mathcal{F}(\Omega, h)$;

ii) Given $(v, G) \in \mathcal{O} \times \mathcal{F}(\Omega, h)$, if $t \in I_{(v,G)}$ and $s \in I_{\pi(t,v,G)}$, then $t+s \in I_{(v,G)}$ and $\pi(s, \pi(t, v, G)) = \pi(t+s, v, G)$;

iii) For each $(v, G) \in \mathcal{O} \times \mathcal{F}(\Omega, h)$ fixed, $\pi(t, v, G)$ is continuous at every $t \in I_{(v,G)}$.

iv) $I_{(v,G)} = [0, b_{(v,G)})$ is maximal in the following sense: either $I_{(v,G)} = \mathbb{R}_+$ or, if $b_{(v,G)} \neq +\infty$, then the positive orbit

$$\{\pi(t, v, G) : t \in [0, b_{(v,G)})\} \subset \mathcal{O} \times \mathcal{F}(\Omega, h)$$

cannot be continued to a larger interval $[0, b_{(v,G)} + c)$, $c > 0$;

v) If $(v_k, G_k) \stackrel{k \to +\infty}{\longrightarrow} (v, G)$, where (v, G) and $(v_k, G_k) \in \mathcal{O} \times \mathcal{F}(\Omega, h)$, $k = 1, 2, ...$, then

$$I_{(v,G)} \subset \liminf I_{(v_k,G_k)}.$$

If the domain of π is $\mathbb{R}_+ \times \mathcal{O} \times \mathcal{F}(\Omega, h)$, then π is called a *global semidynamical system.*

Now, let $G \in \mathcal{F}(\Omega, h)$. For each $t \geq 0$, we define the translate G_t of G by

$$G_t(x, s) = G(x, t + s) - G(x, t), \tag{2.4}$$

where $(x, s) \in \Omega$. It is clear that the translates G_t of G belong to $\mathcal{F}(\Omega, h)$ for each $t \geq 0$.

Theorem 2.1. *Assume that for each $u \in \mathcal{O}$ and $G \in \mathcal{F}(\Omega, h)$, $x(t, u, G)$ is the unique maximal solution of the initial value problem*

$$\frac{dx}{d\tau} = DG(x, t), \qquad x(0) = u. \tag{2.5}$$

Let $[0, \omega(u, G))$, $\omega(u, G) > 0$, be the maximal interval of definition of $x(\cdot, u, G)$. Define $\pi : S \to \mathcal{O} \times \mathcal{F}(\Omega, h)$ by

$$\pi(t, u, G) = (x(t, u, G), G_t), \tag{2.6}$$

where $S = \{(t, u, G) \in \mathbb{R}_+ \times \mathcal{O} \times \mathcal{F}(\Omega, h) : t \in I_{(u,G)}\}$. Then π is a local semidynamical system on $\mathcal{O} \times \mathcal{F}(\Omega, h)$.

Note that the maximal interval $I_{(u,G)}$ of the semidynamical system given by (2.6) coincides with $[0, \omega(u, G))$ necessarily, since the second component G_t of the flow is defined for all $t \in [0, +\infty)$.

Proof. The proof of this Theorem is presented in [1], Theorem 4.4, and it will be discussed in the Congress.

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ON THE EXISTENCE OF ATTRACTORS FOR AUTONOMOUS ODE'S WITH DISCONTINUOUS RIGHTHAND SIDE

S. M. AFONSO*, M. FEDERSON* & E. TOON*

In this work, we prove the existence of a global attractor for an autonomous ODE with discontinuous righthand side. This is done by means of the theory of generalized ODE's also known as Kurzweil equations.

1 Introduction

Let X be a Banach space. We shall deal with semigroups $\{T(t),\, t \in \mathbb{R}^+\}$ of continous operators $T(t) : X \to X$ acting on X. We shall denote $\{T(t),\, t \in \mathbb{R}^+, X\}$ or simply $\{T(t)\}$. In what follows, the term semigroup refers to any family of single-valued continuous operators $T(t) : X \to X$ depending on the parameter $t \in \mathbb{R}^+$ and enjoying the semigroup property: $T(t_1 + t_2)x = T(t_1)T(t_2)x$ for all $t_1,\, t_2 \in \mathbb{R}^+$ and $x \in X$. A semigroup $\{T(t)\}$ is called continuous if the mapping $(t, x) \mapsto T(t)x$ from $\mathbb{R}^+ \times X$ to X is continuous.

Let $A,\, B \subset X$. We say that A attracts B under the action of the semigroup $\{T(t)\}$ if

$$\lim_{t\to\infty} dist_H(T(t)B, A) = 0,$$

where $dist_H(A, B) = \sup_{x\in A} \inf_{y\in B} d(x, y)$. We say that $\mathcal{A} \subset X$ is a global attractor for the semigroup $\{T(t)\}$ if $\mathcal{A}$ is compact, invariant and attracts bounded subsets of X.

We consider $\Omega = \mathcal{O} \times [0, +\infty)$, where $\mathcal{O} \subset BV([0, \infty), \mathbb{R}^n)$ is an open subset and $BV([0, \infty), \mathbb{R}^n)$ is the space of functions $x : [0, \infty) \to \mathbb{R}^n$ which are locally of bounded variation. We consider this space with the usual variation norm.

Let $h : [0, +\infty) \to \mathbb{R}$ be a nondecreasing function. We say that a function $G : \Omega \to BV([0, \infty), \mathbb{R}^n)$ belongs to the class $\mathcal{F}(\Omega, h)$, whenever $G(x, 0) = 0$ and, for all (z, s_2), (z, s_1), (y, s_2) and $(y, s_1) \in \Omega$, we have

$$\|G(z, s_2) - G(z, s_1)\| \leq |h(s_2) - h(s_1)| \qquad \text{and} \tag{1.1}$$

$$\|G(z, s_2) - G(z, s_1) - G(y, s_2) + G(y, s_1)\| \leq \|z - y\||h(s_2) - h(s_1)|, \tag{1.2}$$

where $\|\cdot\|$ denotes the norm in X.

Let $G \in \mathcal{F}(\Omega, h)$. A function $x : [\alpha, \beta] \to \mathbb{R}^n$ is a *solution of the generalized ordinary differential equation*

$$\frac{dx}{d\tau} = DG(x, t) \tag{1.3}$$

with the initial condition $x(t_0) = z_0$ *on the interval* $[\alpha, \beta] \subset [0, +\infty)$, if $t_0 \in [\alpha, \beta]$, $(x(t), t) \in \Omega$ for all $t \in [\alpha, \beta]$ and

$$x(v) - z_0 = \int_{t_0}^{v} DG(x(\tau), t), \qquad v \in [\alpha, \beta].$$

We say that $x : [t_0,\, t_0 + b) \to \mathbb{R}^n$ is the maximal solution of (1.3) with $x(t_0) = u \in \mathcal{O}$, if x is a solution of (1.3) on every interval $[t_0,\, t_0 + \beta]$, $\beta < b$, and it cannot be continued to $[t_0,\, t_0 + b]$. We denote $b = \omega(u,\, G)$ in this case and say that $[t_0,\, t_0 + \omega(u,\, G))$ is the maximal interval of definition of the solution x.

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2 Existence of a global attractor

Consider the initial value problem for an autonomous ODE

$$\begin{cases} \dfrac{dx}{dt} = f(x(t)) \\ x(0) = u, \end{cases} \tag{2.4}$$

where $f : \mathcal{D} \to \mathbb{R}^n$, $\mathcal{D} \subset \mathbb{R}^n$ is open, satisfies the following conditions:

- there exists a Lebesgue integrable function $M : [0, \infty) \to \mathbb{R}$ such that for all function $x : [0, \infty) \to \mathbb{R}^n$ locally of bounded variation and for all u_1, $u_2 \in [0, \infty)$, we have
$$\left|\left|\int_{u_1}^{u_2} f(x(s))ds\right|\right| \le \int_{u_1}^{u_2} M(s)ds;$$
- there exists a Lebesgue integrable function $L : [0, \infty) \to \mathbb{R}$ such that for all functions x, $y : [0, \infty) \to \mathbb{R}^n$ locally of bounded variation and for all u_1, $u_2 \in [0, \infty)$, we have
$$\left|\left|\int_{u_1}^{u_2} [f(x(s)) - f(y(s))]ds\right|\right| \le \int_{u_1}^{u_2} L(s)\|x(s) - y(s)\|ds.$$

Define $G : \mathcal{D} \times [0, \infty) \to \mathbb{R}^n$, where $\mathcal{D} \subset \mathbb{R}^n$ is an open set, by

$$G(z, t) = f(z)t, \tag{2.5}$$

It is easy to check that $G \in \mathcal{F}(\Omega, h)$, with $\Omega = \mathcal{D} \times [0, \infty)$, where

$$h(t) = \int_0^t [M(s) + L(s)]ds, \quad t \in [0, \infty).$$

Consider the initial value problem

$$\frac{dx}{d\tau} = DG(x, t), \qquad x(0) = u, \tag{2.6}$$

where G is given by (2.5), and let $x(t, 0, u)$ be the solution of (2.6) defined on its maximal interval $[0, \omega(u, G))$. Note that given the conditions above, we can easily check that the maximal interval of this solution is $[0, \infty)$.

Proposition 2.1. *The solution of the initial value problem (2.6) is a continuous semigroup.*

Theorem 2.1. *There exists a global attractor for the solution of (2.6).*

Theorem 2.2. *There exists a global attractor for the solution of (2.4).*

The proof of the above results will be discussed in the Congress. Some applications will also be presented.

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MULTIPLICIDADE DE SOLUÇÕES PARA UM PROBLEMA ENVOLVENDO O OPERADOR p-BIHARMÔNICO COM PESO

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Neste trabalho demonstramos a existência de três soluções para um problema envonvendo o operador p-biharmônico com peso. As duas primeiras soluções são obtidas usando o princípio variacional de Ekeland; a terceira solução é obtida através de uma variante do teorema do passo da montanha.

Especificamente, estudamos a classe de problemas elípticos quase lineares

$$\Delta(\rho(x)|\Delta u|^{p-2}\Delta u) + g(x,u) = \lambda_1 h(x)|u|^{p-2}u \quad \text{em} \quad \Omega,$$
$$u(x) = 0, \quad \Delta u(x) = 0 \quad \text{sobre} \quad \partial\Omega,$$

em que $1 < p < \infty$, $\Omega \subset \mathbb{R}^N$ (para $n \geq 1$) é um domínio limitado com fronteira diferenciável e $\rho \in \mathcal{C}(\bar{\Omega}, \mathbb{R})$ com $\inf_{\bar{\Omega}} \rho(x) > 0$. Além disso, usamos as seguintes hipóteses.

(G_1) $g : \Omega \times \mathbb{R} \to \mathbb{R}$ é uma função contínua e limitada com $g(x,0) = 0$.

(G_2) A primitiva $G(x,s) = \displaystyle\int_0^s g(x,t)dt$ é limitada.

Seja $X \equiv W^{2,p}(\Omega) \cap W_0^{1,p}(\Omega)$ um espaço de Sobolev munido com a norma dada por

$$\|u\| \equiv \left\{ \int_\Omega \rho(x)|\Delta u(x)|^p dx \right\}^{\frac{1}{p}}.$$

Definimos

$$\lambda_1 = \inf_N \left\{ \int_\Omega \rho|\Delta u|^p dx \right\} \qquad \text{em que} \qquad N = \left\{ u \in X \colon \int_\Omega h|u|^p dx = 1 \right\},$$

o primeiro autovalor do seguinte problema de autovalor com peso

$$\Delta(\rho(x)|\Delta u|^{p-2}\Delta u = \lambda_1 h(x)|u|^{p-2}u \qquad \text{in } \Omega,$$
$$u(x) = 0 \quad \Delta u(x) = 0 \quad \text{sobre} \quad \partial\Omega,$$

com a hipótese

(h) A função $h \in \mathcal{C}(\bar{\Omega}, \mathbb{R})$ é tal que $h \geq 0$ e $h > 0$ em um subconjunto de Ω com medida positiva.

Sabemos que o primeiro autovalor λ_1 é simples, isolado e positivo. Além disso, a autofunção ϕ_1 associada a λ_1 pode ser escolhida como sendo positiva. A notação $(\Delta(\rho(x)|\Delta u|^{p-2}\Delta)$ indica o operador de quarta ordem chamado de operador p-biharmônico com peso ρ. Este tipo de não linearidade fornece um modelo para o estudo de ondas viajantes em pontes suspensas no caso em que $p = 2$ e $\rho = 1$. Já o caso em que ρ não é constante aparece em problemas de elasticidade envolvendo a lei de Hooke não-linear. Além disso, o operador p-biharmônico pode ser usado para estudar sistemas hamiltonianos semilineares.

Definimos o funcional de energia $I : X \longrightarrow \mathbb{R}$ por

$$I(u) \equiv \frac{1}{p}\int_\Omega \rho(x)|\Delta u|^p dx + \int_\Omega G(x,u)dx - \frac{\lambda_1}{p}\int_\Omega h(x)|u|^p dx.$$

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Com as hipóteses (G_1) e (G_2), temos $I \in \mathcal{C}^1(\Omega, \mathbb{R})$ e sua derivada de Fréchet é dada por

$$I'(u) \cdot v = \int_\Omega \rho(x)|\Delta u|^{p-2}\Delta u \Delta v dx + \int_\Omega g(x,u)v dx - \lambda_1 \int_\Omega h(x)|u|^{p-2}uv dx.$$

Definimos $V = \langle \phi_1 \rangle$ e $Z = \left\{ u \in X\colon \int_\mathbb{R} hu|\phi_1|^{p-2}\phi_1 = 0 \right\}$. Observamos que Z é o subespaço complementar fechado de V e, portanto, temos a soma direta $X = V \oplus Z$. Definimos também

$$\lambda_2 = \inf_{u \in Z} \left\{ \int_\Omega \rho(x)|\Delta u|^p dx\colon \int_\Omega h(x)|u|^p dx = 1 \right\},$$

que verifica a relação $0 < \lambda_1 < \lambda_2$; dessa forma,

$$\int_\Omega h|w|^p dx \leq \frac{1}{\lambda_2}\int_\Omega \rho|\Delta w|^p dx, \quad \text{para todo} \quad w \in Z.$$

Além disso, usamos as hipóteses seguintes.

(G_3) $g(x,t) \to 0$ quando $|t| \to \infty$, para todo $x \in \Omega$.

(G_4) $G(x,t) \geq \dfrac{\lambda_1 - \lambda_2}{p} h(x)|t|^p$ para todo $x \in \Omega$ e para todo $t \in \mathbb{R}$.

(G_5) Existem $\delta > 0$ e $0 < m < \lambda_1$ tais que $G(x,t) \geq \dfrac{m}{p} h(x)|t|^p$ para todo $x \in \Omega$ e para todo $|t| < \delta$.

Definimos $T(x) = \liminf_{|t|\to\infty} G(x,t)$ e $S(x) = \limsup_{|t|\to\infty} G(x,t)$ para todo $x \in \Omega$.

(G_6) Existem $t^-, t^+ \in \mathbb{R}$ com $t^- < 0 < t^+$ tais que $\int_\Omega G(x,t^\pm)\phi_1 dx \leq \int_\Omega T(x)dx < 0$ e

(G_7) $\int_\Omega S(x)dx \leq 0.$

Definimos os subconjuntos

$$C^+ = \{t\phi_1 + z\colon t \geq 0 \quad \text{e} \quad z \in Z\} \qquad \text{and} \qquad C^- = \{t\phi_1 + z\colon t \leq 0 \quad \text{e} \quad z \in Z\}.$$

Observamos que $\partial C^+ = \partial C^- = Z$.

Nosso principal resultado é o seguinte.

Teorema 0.1. *1. Com as hipóteses (h), (G_1), (G_2), (G_4) e (G_6), existem $u \in C^+$ e $v \in C^-$ soluções do problema tais que $I(u) < 0$ e $I(v) < 0$.*

2. Com as hipóteses (h), (G_1)–(G_3), (G_5)–(G_7), o problema tem uma solução w tal que $I(w) > 0$.

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CONCENTRAÇÕES DAS SOLUÇÕES POSITIVAS DE UMA CLASSE DE PROBLEMAS QUASE LINEARES EM $\mathbb{R}$

C.O. ALVES *† O.H. MIYAGAKI ‡§ & S.H.M.SOARES ¶

Estuda-se as concentrações das soluções positivas para a seguinte classe de problemas quase lineares

$$(P) \qquad \epsilon^2 v'' - V(x)v + |v|^{q-1}v + \epsilon^2 k(|v|^2)''v = 0, \quad x \in \mathbb{R}.$$

A prova é feita aplicando o método variacional, usando diretamente o funcional de Euler-Lagrange associado ao problema num espaço de Sobolev adequado. Encontra-se uma família de soluções $\{u_\epsilon\}$ que se concentra na vizinhança do mínimo local de V quando ϵ tende a zero.
O nosso resultado principal é o que se segue:

1 Resultado

Teorema 1.1. *Suponha que V satisfaz (V_0) e (V_1), a saber*

$$(V_0) \qquad V(x) \geq \alpha > 0, \quad \forall x \in \mathbb{R}$$

e que exista um conjunto aberto e limitado Λ em $\mathbb{R}$ tal que

$$(V_1) \qquad \inf_{x \in \partial\Lambda} V(x) > \inf_{x \in \overline{\Lambda}} V(x) = V_0.$$

Então, existe $\epsilon_0 > 0$ tal que o problema (P) possui uma solução positiva $v_\epsilon \in H^1(\mathbb{R})$ para todo $\epsilon \in (0, \epsilon_0)$. Além disso, se y_ϵ é um ponto de máximo de v_ϵ, tem-se

$$y_\epsilon \to y \text{ quando } \epsilon \to 0$$

onde y é um número tal que o potencial V assume o valor mínimo em $\overline{\Lambda}$, ou seja, $V(y) = \inf_{x \in \overline{\Lambda}} V(x)$.

O Teorema estabelece resultado de concentrações das soluções de (P) quando ϵ é suficientemente pequeno, mostrando que resultado ainda vale no case $k > 0$ and $N = 1$, estendendo vários resultados no caso semilinear com $k = 0$ (veja e.g. [1,2, 3,4, 5,6]). O ponto chave no argumento da prova é um resultado de compacidade local em $H^1(\mathbb{R})$, e a dificuldade aparece não só na falta de compacidade de imersão de Sobolev, mas também na presença do termo $\int_{\mathbb{R}} v^2|v'|^2dx$ no funcional associado a (P).

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Estabilidade exponencial em misturas termoviscoelásticas de sólidos

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Neste trabalho nós consideramos o sistema

$$\begin{aligned}
&\rho_1\, u_{tt} - a_{11}\, u_{xx} - a_{12}\, w_{xx} - b_{11}\, u_{xxt} - b_{12}\, w_{xxt} + \alpha\,(u - w) + \alpha_1\,(u_t - w_t) + \beta_1\,\theta_x = 0,\\
&\rho_2\, w_{tt} - a_{12}\, u_{xx} - a_{22}\, w_{xx} - b_{12}\, u_{xxt} - b_{22}\, w_{xxt} - \alpha\,(u - w) - \alpha_1\,(u_t - w_t) + \beta_2\,\theta_x = 0,\\
&c\,\theta_t - \kappa\,\theta_{xx} + \beta_1\, u_{xt} + \beta_2\, w_{xt} = 0,
\end{aligned} \tag{0.1}$$

com $0 < x < L,\ t > 0$, onde $u = u(x, t)$, $w = w(x, t)$, são os deslocamentos de duas partículas no tempo t, $\theta = \theta(x, t)$ é a diferença de temperatura em cada ponto x no tempo t de uma viga unidimensional composta por uma mistura de dois sólidos termoviscoélasticos. A descrição deste modelo pode ser encontrada em Ieşan e Quintanilla [1] ou Ieşan e Nappa [2]. Assumimos que ρ_1, ρ_2, c, κ, e α são constantes positivas, $\alpha_1 \geq 0$ e $\beta_1^2 + \beta_2^2 \neq 0$. A matriz $A = (a_{ij})$ é simétrica e definida positiva e $B = (b_{ij}) \neq 0$ é simétrica e definida não negativa, isto é,

$$a_{11} > 0, \qquad a_{11}\, a_{22} - a_{12}^2 > 0,\ \ b_{11} \geq 0, \qquad b_{11}\, b_{22} - b_{12}^2 \geq 0.$$

Estudamos o sistema (0.1) com as condições iniciais

$$u(\,.\,,0) = u_0, \qquad u_t(\,.\,,0) = u_1, \qquad w(\,.\,,0) = w_0, \qquad w_t(\,.\,,0) = w_1, \qquad \theta(\,.\,,0) = \theta_0 \tag{0.2}$$

e as condições de fronteira

$$u(0, t) = u(L, t) = w(0, t) = w(L, t) = \theta_x(0, t) = \theta_x(L, t) = 0 \qquad \text{in} \qquad (0, \infty). \tag{0.3}$$

Nossa proposta neste trabalho é investigar a estabilidade exponencial do semigrupo associado ao sistema (0.1)-(0.3) e apresentar alguns exemplos numéricas para mostrar o comportamento assintótico de soluções. Indicamos o livro de Liu and Zheng [3] para uma pesquisa sobre os métodos e técnicas utilizados nas provas dos teoremas apresentados a seguir.

1 Principais Resultados

O problema (0.1)-(0.3) pode ser reduzido ao seguinte problema de valor inicial

$$\frac{d}{dt}U(t) = \mathcal{A}U(t), \qquad U(0) = U_0, \quad \forall\, t > 0 \tag{1.4}$$

com $U(t) = (u,\, w,\, u_t,\, w_t,\, \theta)^T$, $U_0 = (u_0,\, w_0,\, u_1,\, w_1,\, \theta_0)^T$, sendo $\mathcal{A} : \mathcal{D}(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ o operador, com domínio

$$\begin{aligned}
\mathcal{D}(\mathcal{A}) = \big\{U = (u,\, w,\, v,\, \eta,\, \theta) \in \mathcal{H} :\ & v,\ \eta \in H_0^1(0,\, L),\ a_{11}\, u + a_{12}\, w + b_{11}\, v + b_{12}\, \eta \in H^2(0,\, L),\\
& a_{12}\, u + a_{22}\, w + b_{12}\, v + b_{22}\, \eta \in H^2(0,\, L),\ \theta \in H^2(0, L),\ \theta_x \in H_0^1(0, L)\big\}
\end{aligned}$$

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denso em $\mathcal{H} = H_0^1(0,\,L) \times H_0^1(0,\,L) \times L^2(0,\,L) \times L^2(0,\,L) \times L_*^2(0,\,L)$, dado por

$$\mathcal{A}\begin{pmatrix} u \\ w \\ v \\ \eta \\ \theta \end{pmatrix} = \begin{pmatrix} v \\ \eta \\ \frac{1}{\rho_1}\,(a_{11}\,u + a_{12}\,w + b_{11}\,v + b_{12}\,\eta)_{xx} - \frac{\alpha}{\rho_1}\,(u-w) - \frac{\alpha_1}{\rho_1}\,(v-\eta) - \frac{\beta_1}{\rho_1}\,\theta_x \\ \frac{1}{\rho_2}\,(a_{12}\,u + a_{22}\,w + b_{12}\,v + b_{22}\,\eta)_{xx} + \frac{\alpha}{\rho_2}\,(u-w) + \frac{\alpha_1}{\rho_2}\,(v-\eta) - \frac{\beta_2}{\rho_2}\,\theta_x \\ -\frac{\beta_1}{c}\,v_x - \frac{\beta_2}{c}\,\eta_x + \frac{\kappa}{c}\,\theta_{xx} \end{pmatrix}. \tag{1.5}$$

O operador $\mathcal{A}$ gera um semigrupo de classe C_0 de contrações, $S_{\mathcal{A}}(t)$, no espaço $\mathcal{H}$ e nós provamos que

Teorema 1.1. *Assuma que*

(a) $\alpha_1 > 0$ *e*

  (a.1) $b_{11} \neq -b_{12}$ *ou* $b_{22} \neq -b_{12}$ *ou* $\beta_1 \neq -\beta_2$,

ou

  (a.2) $\rho_2(a_{11} + a_{12}) \neq \rho_1(a_{22} + a_{12})$.

(b) $\alpha_1 = 0$ *e*

  (b.1) $\{(b_{11}, b_{12}),\ (\beta_1, \beta_2)\}$ *ou* $\{(b_{12},\, b_{22}),\ (\beta_1,\, \beta_2)\}$ *é linearmente independente,*

ou

  (b.2) $\dfrac{n^2\,\pi^2}{L^2} \neq \dfrac{\alpha\,((\rho_1\,\beta_2^2 - \rho_2\,\beta_1^2) + \beta_1\,\beta_2\,(\rho_1 - \rho_2))}{\beta_1\,\beta_2\,(\rho_2\,a_{11} - a_{22}\,\rho_1) - a_{12}\,(\beta_1^2\,\rho_2 - \beta_2^2\,\rho_1)}$, *para todo* $n \in \mathbb{N}$.

Então o conjunto $i\,\mathbb{R} = \{i\,\lambda :\ \lambda \in \mathbb{R}\}$ *está contido em* $\rho(\mathcal{A})$ *e*

$$\limsup_{|\lambda| \to +\infty} \|(i\,\lambda\,I - \mathcal{A})^{-1}\|_{\mathcal{L}(\mathcal{H})} < \infty.$$

Portanto $S_{\mathcal{A}}(t)$ *é exponencialmente estável, isto é, existem constantes positivas* M *e* μ *tais que*

$$\|S_{\mathcal{A}}(t)\|_{\mathcal{L}(\mathcal{H})} \leq M \exp(-\,\mu\,t).$$

Observamos que para provar o próximo teorema é suficiente mostrar que existe uma sequência de números reais (λ_ν), com $\lambda_\nu \to \infty$, e uma sequência limitada (F_ν) em $\mathcal{H}$ tais que $\|(i\lambda_\nu I - \mathcal{A})^{-1} F_\nu\|_{\mathcal{H}} \to \infty$, $\nu \to \infty$.

Teorema 1.2. *Suponha que uma das condições abaixo ocorra*

(a) $\alpha_1 = 0$; $(b_{11},\, b_{12})$, $(b_{12},\, b_{22})$ *e* $(\beta_1,\, \beta_2)$ *são colineares e* $\beta_2\,(\beta_1\,\rho_2\,a_{11} + \beta_2\,\rho_1\,a_{12}) = \beta_1\,(\beta_2\,\rho_1\,a_{22} + \beta_1\,\rho_2\,a_{12})$.

(b) $\alpha_1 > 0$; $b_{11} = b_{22} = -b_{12}$ *e* $\beta_1 = -\beta_2$ *e* $\rho_2(a_{11} + a_{12}) = \rho_1(a_{22} + a_{12})$.

Então o semigrupo $S_{\mathcal{A}}(t)$ *não é exponencialmente estável .*

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Well-posedness and Stability of the Periodic Nonlinear Waves Interactions for the Benney System

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We establish local well-posedness results in weak periodic function spaces for the Cauchy problem of the Benney system. The Sobolev space $H^{1/2}\times L^2$ is the lowest regularity attained and also we cover the energy space $H^1\times L^2$, where global well-posedness follows from the conservation laws of the system. Moreover, we show the existence of smooth explicit family of periodic travelling waves of *dnoidal* type and we prove, under certain conditions, that this family is orbitally stable in the energy space.

1 Mathematical Results

We consider the system introduced by Benney in [4] which models the interaction between short and long waves, for example in the theory of resonant water wave interaction in nonlinear medium:

$$\begin{cases} iu_t+u_{xx}=uv+\beta|u|^2u, & (x,t)\in\mathbb{T}\times\triangle T\\ v_t=(|u|^2)_x, & \\ u(x,0)=u_0(x),\quad v(x,0)=v_0(x), & \end{cases}\tag{1.1}$$

where $u=u(x,t)$ is a complex valued function representing the enveloped of short waves, and $v=v(x,t)$ is a real valued function representing the long wave. Here β is a real parameter, $\triangle T$ is the time interval $[0,T]$ and $\mathbb{T}$ is the one dimensional torus $\mathbb{T}=\mathbb{R}/\mathbb{Z}$.

We obtain the following results concerning well-posedness for Cauchy problem (1.1):

Theorem 1.1 (Local Well-Posedness)**.** *For any $(u_0,v_0)\in H^r_{per}\times H^s_{per}$ provided the conditions:*

$$\max\{0,\, r-1\}\le s\le\min\{r,\, 2r-1\},\tag{1.2}$$

there exist a positive time $T=T\left(\|u_0\|_r,\|v_0\|_s\right)$ and a unique solution $(u(t),v(t))$ of the initial value problem (1.1), satisfying

$$(\eta_T(t)u,\eta_T(t)v)\in X^r_{per}\times Y^s_{per}\quad \textit{and}\quad (u,v)\in C\left(\triangle T;\ H^r_{per}\times H^s_{per}\right).$$

Moreover, the map $(u_0,v_0)\longmapsto(u(t),v(t))$ is locally uniformly continuous from $H^r_{per}\times H^s_{per}$ into $C\left(\triangle T;\ H^r_{per}\times H^s_{per}\right)$.

Theorem 1.2. *Let $\beta\neq 0$. Then for any $r<0$ and $s\in\mathbb{R}$, the initial value problem (1.1) is locally ill-posed for data in $H^r_{per}\times H^s_{per}$.*

On the other hand, we prove that there exist a smooth explicit family of profiles solutions of minimal period L,

$$(\omega,c)\in\mathcal{A}_\beta\to(\varphi_{\omega,c},n_{\omega,c})\in H^n_{per}([0,L])\times H^m_{per}([0,L]),$$

where $\mathcal{A}_\beta=\{(x,y): y>0, 1>\beta y,\ \ and\ \ x<-\frac{2\pi^2}{L^2}-\frac{y^2}{4}\}$ and which depends of the Jacobian elliptic function dn called *dnoidal*, more precisely,

$$\varphi_{\omega,c}(\xi)=\sqrt{\frac{c}{1-\beta c}}\,\eta_1 dn\left(\frac{\eta_1}{\sqrt{2}}\xi;\kappa\right)\quad\text{and}\quad n_{\omega,c}(\xi)=-\frac{\eta_1^2}{1-\beta c}dn^2\left(\frac{\eta_1}{\sqrt{2}}\xi;\kappa\right)\tag{1.3}$$

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with $\eta_1 = \eta_1(\omega, c)$ and $\kappa = \kappa(\omega, c)$, being smooth functions of ω and c. So, by following Angulo [1] and Grillakis *et al.* [8], [9], we obtain the following stability theorem.

Theorem 1.3 (Stability Theory)**.** *Let $(\omega, c) \in \mathcal{A}_\beta$ such that for $c > 0$ there is $q \in \mathbb{N}$ satisfying $4\pi q/c = L$. Define $\sigma \equiv -\omega - \frac{c^2}{4}$. Then $\Phi(\xi) = e^{ic\xi/2}\varphi_{\omega,c}(\xi)$, $\Psi(\xi) = n_{\omega,c}(\xi)$, with $\varphi_{\omega,c}, n_{\omega,c}$ given in (1.3), is orbitally stable in $H^1_{per}([0, L]) \times L^2_{per}([0, L])$ by the periodic flow generated by (1.1):*

(a) for $\beta \leq 0$,

(b) for $\beta > 0$ and $8\beta\sigma - 3c(1 - \beta c)^2 \leq 0$.

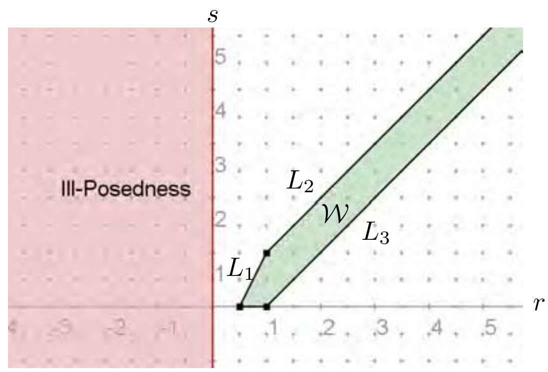

Figure 1: Well-posedness results for periodic Benney system. The region $\mathcal{W}$, limited by the lines $L_1 : s = 2r - 1$, $L_2 : s = r$ and $L_3 : s = r - 1$, contain the indices (r, s) where the local well-posedness is achieved in Theorem 1.1.

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SOME APPLICATIONS OF NONLINEAR FUNCTIONAL ANALYSIS TO THE THEORY OF ELECTRONS LINEAR ACCELERATORS

CARLOS C. ARANDA *

Particle accelerators are central in many applications: like electrons accelerators for cancer illness, protons accelerators for heating plasmas in research tokamaks, advanced spacial rockets like ions accelerator or even for generating strong radiations like light sincroton facilities. The central aspect of many of this kind of technology is the linear nature of employed functional analysis and the aprioristic method of the quatum mechanics theory. First we present and new method of calibration for the probabilistic wave given by Schrodringer equation based on Bayesian statistics. Secondly we present some problems of energy or variational nature related to particle accelertators with strong nonlinearities and posible detours like degree theory or conections with discontinuous maps. It is well kwnon from theoretics physics that the loss of the Palais-Smale condition is a indication of particle creation.

1 Mathematical Results

Let us consider the weighted eigenvalue problem

$$-\Delta u = \lambda m(x) u \text{ in } \Omega \quad u = 0 \text{ on } \partial\Omega \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$. Suppose $m = m^+ - m^-$ in $L^\infty(\Omega)$, where $m^+ = \max(m, 0)$, $m^- = -\min(m, 0)$. Denote

$$\Omega_+ = \{x \in \Omega : m(x) > 0\}$$

$$\Omega_- = \{x \in \Omega : m(x) < 0\}$$

and $|\Omega_+|$, $|\Omega_-|$ its Lebesgue measures. It is well known (see [2] for a nice survey) that if $|\Omega_+| > 0$ and $|\Omega_-| > 0$, then (1.1) has a double sequence of eigenvalues

$$\ldots \leq \lambda_{-2} < \lambda_{-1} < 0 < \lambda_1 < \lambda_2 \leq \ldots,$$

where λ_1 and λ_{-1} are simple and the associated eigenfunctions $\varphi_1 \in C(\overline{\Omega})$, $\varphi_{-1} \in C(\overline{\Omega})$ can be taken $\varphi_1 > 0$ on Ω, $\varphi_{-1} > 0$ on Ω. Where λ_1 and λ_{-1} are the principal eigenvalues of (1.1) φ_1 and φ_{-1} are the associated principal eigenfunctions.

Theorem 1.1 (Localization of the maximum principle.)**.** *Suppose $m = m^+ - m^-$ in $L^\infty(\Omega)$ such that $|\Omega^+| > 0$, $|\Omega^-| > 0$. Then the principal eigenfunctions $\varphi_1 > 0$, $\varphi_{-1} > 0$ of (1.1) satisfy*

$$\|\varphi_1\|_{L^\infty(\Omega)} = \|\varphi_1\|_{L^\infty(\Lambda_{m^+},\, m^+dx)} \tag{1.2}$$

$$\|\varphi_{-1}\|_{L^\infty(\Omega)} = \|\varphi_{-1}\|_{L^\infty(\Lambda_{m^-},\, m^-dx)} \tag{1.3}$$

where $\|\varphi_1\|_{L^\infty(\Lambda_{m^+},\, m^+dx)}$ (respectively $\|\varphi_{-1}\|_{L^\infty(\Lambda_{m^-},\, m^-dx)}$) is the essential supremum on Λm^+ with respect to the measure m^+dx (respectively on Λ_{m^-} w. r. t. m^-dx).

Here Λ_{m^+} is the support of the distribution m^+ in Ω.

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EXACT BOUNDARY CONTROLLABILITY FOR A BOUSSINESQ SYSTEM OF KdV-KdV TYPE

F. D. ARARUNA * G. G. DORONIN † & A. F. PAZOTO ‡

In recent work, Bona, Chen and Saut [1] have derived a family of Boussinesq systems which describe the two-way propagation of small amplitude gravity waves on the surface of water in a canal. These family of systems reads as follows:

$$\left| \begin{array}{l} \eta_t + w_x + (\eta w)_x + a w_{xxx} - b \eta_{xxt} = 0, \\ w_t + \eta_x + w w_x + c \eta_{xxx} - d w_{xxt} = 0. \end{array} \right. \tag{0.1}$$

Here η is the elevation from the equilibrium position and $w = w_\theta$ is the horizontal velocity in the flow at height θh, with h being the undisturbed depth of the liquid and θ a fixed constant in the interval $[0,1]$. The parameters a, b, c and d are assumed to satisfy the consistency conditions $2(a+b) = \theta^2 - 1/3$ and $2(c+d) = 1 - \theta^2 \geq 0$. Contrary to the classical Korteweg-de Vries equation which assumes that the waves travel only in one direction, system (0.1) is free of the presumption of unidirectionality and may have a wider range of applicability.

The present work concerns the exact boundary controllability of the nonlinear Boussinesq system of KdV-KdV type (i.e. $a = c > 0$ and $b = d = 0$) posed in a bounded domain:

$$\left| \begin{array}{lll} \eta_t + w_x + (\eta w)_x + w_{xxx} = 0 & \text{in} & (0,L) \times (0,T), \\ w_t + \eta_x + w w_x + \eta_{xxx} = 0 & \text{in} & (0,L) \times (0,T), \\ \eta(0,\cdot) = \eta(L,\cdot) = w(0,\cdot) = w(L,\cdot) = 0 & \text{on} & (0,T), \\ \eta_x(0,\cdot) - w_x(0,\cdot) = f & \text{on} & (0,T), \\ \eta_x(L,\cdot) + w_x(L,\cdot) = g & \text{on} & (0,T), \\ \eta(\cdot,0) = \eta_0, \quad w(\cdot,0) = w_0 & \text{in} & (0,L), \end{array} \right. \tag{0.2}$$

where f and g are boundary control inputs. Since the constants a and c are irrelevant in the arguments and results, we consider $a = c = 1$.

Many other control and stabilization problems for dispersive equations have been studied in last decades, see [2, 3, 6, 7, 8, 9, 10, 11] and the references therein. However, due to its one-way propagation properties, problems posed in a bounded domain for single dispersive equations make its physical sense doubtful; therefore, the study of control and related problems posed on a bounded interval for systems like (0.2) is ripe to development, [5].

The exact controllability problem for (0.2) is formulated as follows: given $T > 0$, the initial and final data $\{\eta_0, w_0\}, \{\eta_T, w_T\}$ from an appropriate space, to find controls f and g such that solution $\{\eta, w\} = \{\eta, w\}(x,t)$ of (0.2) satisfies the conditions

$$\eta(\cdot,T) = \eta_T \quad \text{and} \quad w(\cdot,T) = w_T \quad \text{in} \quad (0,L). \tag{0.3}$$

Our aim is to obtain the exact controllability of (0.2). For this, we combine a linear observability with a data smallness for nonlinear problem. More precisely, consider the linear system

$$\left| \begin{array}{lll} \eta_t + w_x + w_{xxx} = 0 & \text{in} & (0,L) \times (0,T), \\ w_t + \eta_x + \eta_{xxx} = 0 & \text{in} & (0,L) \times (0,T), \\ \eta(0,\cdot) = \eta(L,\cdot) = w(0,\cdot) = w(L,\cdot) = 0 & \text{on} & (0,T), \\ \eta_x(0,\cdot) - w_x(0,\cdot) = f & \text{on} & (0,T), \\ \eta_x(L,\cdot) + w_x(L,\cdot) = g & \text{on} & (0,T), \\ \eta(\cdot,0) = \eta_0, \quad w(\cdot,0) = w_0 & \text{in} & (0,L). \end{array} \right. \tag{0.4}$$

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According to the Hilbert uniqueness method (HUM) introduced by Lions (see [4]), the exact controllability of (0.4) is equivalent to a suitable observability inequality for the adjoint system. However, it is well-known that the observability result for a single linear KdV equation holds if and only if L is not a critical length in the sense of [7], that is

$$L \notin \mathcal{N} := \left\{ 2\pi \sqrt{\frac{k^2 + l^2 + kl}{3}} : k, l \in \mathbb{N} \right\}.$$

In this way, one can expect that (0.4) possesses the same kind of restriction. Our main results are the follows.

Theorem 0.1. *Let $T > 0$ and $L \in (0, \infty) \setminus \mathcal{N}$ be given. Then for every initial and final data $\{\eta_0, w_0\}, \{\eta_T, w_T\} \in \left[L^2(0, L)\right]^2$, there exists a pair of controls $\{f, g\} \in \left[L^2(0, T)\right]^2$ such that (0.3) holds.*

To prove this claim, instead of Rosier's technique based on the Fourier transform and Paley-Wiener's theorem, we provide quite simple algebraic approach which looks easier and more appropriate for other dispersive systems. As a consequence of this theorem and the Banach contraction principle, we get

Theorem 0.2. *Let $T > 0$ and $L \in (0, \infty) \setminus \mathcal{N}$ be given. Then there exists a real $r > 0$ such that for every initial and final data $\{\eta_0, w_0\}, \{\eta_T, w_T\} \in \left[L^2(0, L)\right]^2$ satisfying $\|\{\eta_0, w_0\}\|_{[L^2(0,L)]^2} < r$ and $\|\{\eta_T, w_T\}\|_{[L^2(0,L)]^2} < r$, there exists a pair of controls $\{f, g\} \in \left[L^2(0, T)\right]^2$ such that (0.2) is exactly controllable.*

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UPPER SEMICONTINUITY OF ATTRACTORS FOR A PARABOLIC PROBLEM ON A THIN DOMAIN WITH HIGHLY OSCILLATING BOUNDARY

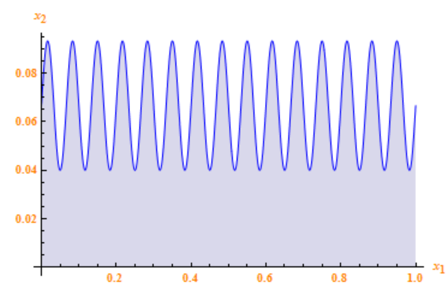
J. M. ARRIETA * A. N. CARVALHO † M. C. PEREIRA ‡ & R. P. SILVA §

In this work we study the continuity of the asymptotic dynamics of a dissipative reaction-diffusion equation in a thin domain with oscillating boundary. We consider the reaction-diffusion equation

$$\begin{cases} u^\epsilon_t - \Delta u^\epsilon + u^\epsilon = f(u^\epsilon) & \text{in } \Omega^\epsilon \\ \frac{\partial u^\epsilon}{\partial N^\epsilon} = 0 & \text{in } \partial\Omega^\epsilon, \end{cases} \tag{0.1}$$

where $\Omega^\epsilon = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \in (0,1) \text{ and } 0 < x_2 < \epsilon g(x_1/\epsilon)\}$, with g a positive, L-periodic C^1-function, $N^\epsilon = (N^\epsilon_1, N^\epsilon_2)$ is the unit outward normal field to $\partial\Omega^\epsilon$ and $\epsilon > 0$ is a small parameter. The nonlinearity $f : \mathbb{R} \mapsto \mathbb{R}$ is a $\mathcal{C}^2$-function which is bounded with bounded derivatives up to second order.

Observe that $\Omega^\epsilon \subset \mathbb{R}^2$ is a open set that degenerates to a line segment as the parameter ϵ goes to zero.



Under the above assumptions, we obtain for each $\epsilon > 0$ that the $\mathcal{C}^1$-semiflow generated by equation (0.1) has a global attractor $\mathcal{A}_\epsilon$[1] on $H^1(\Omega^\epsilon)$. We are interested in to investigate the continuity properties of the family of attractors $\{\mathcal{A}_\epsilon : \epsilon > 0\}$ as the parameter ϵ tends to 0.

To do this, we deal first the linear elliptic problem associated to (0.1). Using *homogenization methods*, we obtain formally the limit problem by the *multiple-scale method* and we proof its convergence following the idea of Tartar [6, 7] and Cioranescu & Saint Jean Paulin [3] that use an auxiliary problem together with extension operators.

Subsequently we work out an appropriate functional setting to prove the convergence of the resolvent operators given by the elliptic equations involved, to finally understand the relationship between the attractors $\mathcal{A}_\epsilon$ of (0.1) and the attractor $\mathcal{A}_0$ of the *homogenized limit*. We show that this family of attractors is upper semicontinuous at $\epsilon = 0$.

This functional setting make use of several concepts like the concept of convergence for a sequence $\{u^\epsilon\}_{\epsilon>0}$ where u^ϵ belongs to different spaces for each ϵ, an appropriate concept of compactness for families living in different spaces and the concept of *compact convergence* as the key concept to treat the behavior of compact operators in different spaces. This setting is developed mainly in [1, 2, 5].

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[1] See for example [1, 2].

1 Main results

In appropriate functional setting, we can see the problem (0.1) as an evolutionary equation

$$\begin{cases} u_t + A_\epsilon u = f(u) & t > 0 \\ u(0) \in L_\epsilon \end{cases}$$

for certain family of spaces L_ϵ. Also, we can see the homogenized limit problem also as evolutionary equation

$$\begin{cases} u_t + A_0 u = f(u) & t > 0 \\ u(0) \in L_0 \end{cases}$$

in a certain space L_0.

Since the operators A_ϵ and A_0 are defined in different spaces, we need a tool to compare them. To that end, consider a family $E_\epsilon \in \mathcal{L}(L_0, L_\epsilon)$, $\epsilon > 0$, with the property that $\|E_\epsilon u\|_{L_\epsilon} \to \|u\|_{L_0}$. We say that $L_\epsilon \ni u_\epsilon \xrightarrow{E} u_0 \in L_0$ if $\|u_\epsilon - E_\epsilon u_0\|_{L_\epsilon} \xrightarrow{\epsilon \to 0} 0$ (u_ϵ E-converges to u_0). We say that a family of compact operators $\{B_\epsilon \in \mathcal{L}(L_\epsilon) : \epsilon > 0\}$ converges compactly to B_0 if $\|u_\epsilon\|_{L_\epsilon} = 1$ implies that $\{B_\epsilon u_\epsilon\}$ has an E- convergent subsequence and $u_\epsilon \xrightarrow{E} u_0$ implies $B_\epsilon u_\epsilon \xrightarrow{E} B_0 u_0$.

One of our key results is the compact convergence of the resolvent operators.

Theorem 1.1. *The family of compact operators $\{A_\epsilon^{-1} \in \mathcal{L}(L_\epsilon)\}_{\epsilon>0}$ converges compactly to the compact operator $A_0^{-1} \in \mathcal{L}(L_0)$ as $\epsilon \to 0$.*

With the convergence of the resolvent operators, we show the convergence of the linear semigroups $\{e^{A_\epsilon t} : t \geq 0\}$ to $\{e^{A_0 t} : t \geq 0\}$. Thus, using the variation of constants formula, we prove the convergence of the semiflows. Once this is accomplished, the upper semicontinuity of attractors is easily obtained with an appropriate notion of convergence. Recall that the family $\{\mathcal{A}_\epsilon : \epsilon \in (0, \epsilon_0]\}$ is *upper semicontinuous* at $\epsilon = 0$ if $\sup_{u^\epsilon \mathcal{A}_\epsilon} \inf_{u \in \mathcal{A}_0} \|u^\epsilon - E_\epsilon u\|_{L_\epsilon} \xrightarrow{\epsilon \to 0} 0$.

Theorem 1.2. *The family of attractors $\{\mathcal{A}_\epsilon\}_{\epsilon \in [0,1]}$ is E-upper semicontinuous at $\epsilon = 0$ in H^s for all $s \in [0, 1)$.*

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CONTROLE NA FRONTEIRA PARA UM SISTEMA DE EQUAÇÕES DE ONDA

W. D. BASTOS*, A. SPEZAMIGLIO* & C. A. RAPOSO †

Recentemente, Rajaram e Najafi [6] estudaram controlabilidade exata na fronteira para o sistema de equações $u_{tt}-\Delta u+\alpha(u-v)+\beta(u_t-v_t)=0$, $v_{tt}-\Delta v+\alpha(v-u)+\beta(v_t-u_t)=0$ em que $\alpha>0$ e $\beta>0$. Em [6] considerou-se controle do tipo Dirichlet em domínios suaves do $\mathbb{R}^n$, $n\geq 2$, e o método HUM com a condição geométrica usual. Controlabilidade para tal sistema com controle do tipo Neuman, até onde pudemos observar, ainda não foi estudado. Neste trabalho nos propomos a examinar essa questão. Inicialmente estudamos controlabilidade exata na fronteira para o referido sistema com $\alpha>0$ e $\beta=0$. Obtemos controle do tipo Neuman para estados iniciais com energia finita, em domínios parcialmente suaves do plano. Em seguida examinamos o caso em que há fricção $\beta>0$.

Esses sistemas de equações descrevem vibrações transversais de duas membrans dispostas paralelamente e conectadas por uma camada de material elástico (veja, por exemplo,[5]). Estabilização na fronteira para tais sistemas, em várias dimensões, tem sido estudada extensivamente na última década. Veja por exemplo [1], [2], e respectivas referências.

Aqui usaremos as notações $\|\cdot\|_1$ e $\|\cdot\|_0$ para as normas dos espaços de Sobolev $H^1(U)$ e $H^0(U)=L^2(U)$ respectivamente, onde U é o domínio em questão. Definimos $\mathcal{H}(U)=H^1(U)\times L^2(U)\times H^1(U)\times L^2(U)$ e

$$|(u_1,u_2,v_1,v_2)|=(\|u_1\|_1^2+\|u_2\|_0^2+\|v_1\|_1^2+\|v_2\|_0^2)^{\frac{1}{2}}$$

para todo $(u_1,u_2,v_1,v_2)\in\mathcal{H}(U)$.

1 O resultado principal

Seja $\Omega\subset\mathbb{R}^2$ um polígono curvo, isto é, um domínio limitado, simplesmente conexo com fronteira Γ de classe C^∞ por partes e sem cúspides. Assumimos que Ω situa-se em um mesmo lado de Γ e denotamos η o seu vetor normal exterior, definido quase sempre em Γ. Considere o sistema

$$\begin{array}{ll} u_{tt}-\Delta u+\alpha(u-v)=0 & \text{em } \Omega\times]0,T[,\\ v_{tt}-\Delta v+\alpha(v-u)=0 & \text{em } \Omega\times]0,T[,\\ \frac{\partial u}{\partial\eta}=f,\quad \frac{\partial v}{\partial\eta}=g & \text{em } \Gamma\times]0,T[,\\ u(\cdot,0)=u_1,\quad u_t(\cdot,0)=u_2,\quad v(\cdot,0)=v_1,\quad v_t(\cdot,0)=v_2 & \text{em } \Omega. \end{array} \tag{1.1}$$

O resultado principal deste trabalho é o seguinte teorema:

Teorema 1.1. *Dado um polígono curvo $\Omega\subset\mathbb{R}^2$, existe $T_0>diam(\Omega)$ tal que, para cada $T>T_0$ e estado inicial $(u_1,u_2,v_1,v_2)\in\mathcal{H}(\Omega)$, existem controles f, $g\in L^2(\Gamma\times]0,T[)$ de forma que a solução de (1.1) satisfaz*

$$u(\cdot,T)=u_t(\cdot,T)=v(\cdot,T)=v_t(\cdot,T)=0 \quad em \quad \Omega.$$

Corolário 1.1. *Se $\beta>0$ e $\alpha\geq(\frac{\beta}{2})^2$ então o mesmo vale se as equações são substituídas por*

$$\begin{array}{ll} u_{tt}-\Delta u+\alpha(u-v)+\beta(u_t-v_t)=0 & em\ \Omega\times]0,T[,\\ v_{tt}-\Delta v+\alpha(v-u)+\beta(v_t-u_t)=0 & em\ \Omega\times]0,T[. \end{array}$$

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A demonstração do teorema é baseada no princípio "controlabilidade via estabilização" introduzido por D. L. Russell [7]. Para tanto, observamos o seguinte resultado de decaimento local de energia para uma equação hiperbólica:

Lema 1.1. *Se* $W \in H^1_{loc}(\mathbb{R}^2 \times \mathbb{R})$ *é solução do problema de Cauchy*

$$\begin{array}{ll} W_{tt} - \Delta W + \lambda W = 0 & em \ \ \mathbb{R}^2 \times \mathbb{R} \\ W(0) = W_1, \quad W_t(0) = W_2 & em \ \mathbb{R}^2 \end{array}$$

onde $\lambda \geq 0$, $W_1 \in H^1(\mathbb{R}^2)$ *e* $W_2 \in L^2(\mathbb{R}^2)$ *são funções com suporte compacto num domínio limitado* $U \subset \mathbb{R}^2$ *então, para cada* $T_0 >$ *diam*(U) *existe* $k = k(\lambda, T_0, U) > 0$ *tal que, para todo* $t \geq T_0$,

$$\|W_t(\cdot,t)\|_0^2 + \|W(\cdot,t)\|_1^2 \leq \frac{k}{t^2}\{\|W_2\|_0^2 + \|W_1\|_1^2\}. \tag{1.2}$$

Uma demonstração do lema pode ser vista em [3]. Agora considere o problema de Cauchy

$$\begin{array}{lll} u_{tt} - \Delta u + \alpha(u - v) = 0, & v_{tt} - \Delta v + \alpha(v - u) = 0 & \text{em } \mathbb{R}^2 \times \mathbb{R} \\ u(.,0) = u_1, \quad u_t(.,0) = u_2, & v(.,0) = v_1, \quad v_t(.,0) = v_2 & \text{em } \ \mathbb{R}^2 \end{array}$$

onde $\alpha > 0$ e $(u_1, v_1, u_2, v_2) \in \mathcal{H}(\mathbb{R}^2)$ tem suporte compacto. As funções $z = u + v$ e $w = u - v$ satisfazem

$$z_{tt} - \Delta z = 0, \qquad w_{tt} - \Delta w + 2\alpha w = 0 \qquad \text{em } \mathbb{R}^2 \times \mathbb{R},$$

respectivamente. Consequentemente a estimativa (1.2) se aplica a cada uma delas. Usando a definição da norma e a identidade do paralelogramo obtemos.

$$|(u(.,t), u_t(.,t), v(.,t), v_t(.,t))|^2 \leq \frac{const}{t^2}\,|(u_1, u_2, v_1, v_2)|$$

para todo t suficientemente grande. Assim, a parte "estabilização" do método de Russell fica verificada. A demonstração do teorema prossegue como em [7], [3] ou [4].

É possível considerar, para geometrias especiais, o caso em que parte da fronteira permanece fixa, como em [4]. Isto será considerado numa publicação mais completa.

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ANALYSIS OF A TWO-PHASE FIELD MODEL FOR THE SOLIDIFICATION OF AN ALLOY

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Among the possibilities to model phase change, phase field models are possibly the most successful in the sense that for them it is rather natural to incorporate several complex physical phenomena influencing phase change; they also allow occurrence of transition layer (*mushy* zones). For such models, numerical simulations are possible even in the case of formation of complex geometries, like dendrites, as interfaces separating different phases.

In this work the interest is a rigorous mathematical analysis of the phase field model for the solidification/melting of a metallic alloy with two different kinds of crystallization given by:

$$\tau_t - b\Delta\tau = l_1 u_t + l_2 v_t + f \quad \text{in} \quad Q \tag{0.1}$$

$$u_t - k_1\Delta u = -a_1 u(1-u-v)(1-2u-v+c_1\tau+d_1) \quad \text{in} \quad Q \tag{0.2}$$

$$v_t - k_2\Delta v = -a_2 v(1-v-u)(1-2v-u+c_2\tau+d_2) \quad \text{in} \quad Q \tag{0.3}$$

$$\partial\tau/\partial n = \partial u/\partial n = \partial v/\partial n = 0 \quad \text{on} \quad \partial\Omega\times(0,T), \tag{0.4}$$

$$\tau = \tau_0, \quad u = u_0, \quad v = v_0 \quad \text{in} \quad \Omega\times\{t=0\}, \tag{0.5}$$

Here $\Omega \subset \mathbb{R}^3$, $0 < T < +\infty$ and $Q = \Omega \times (0,T)$. The unknown function τ is associated to the temperature; the phase field unknown functions u and v represent solid fractions of two different kinds of crystallizations. In equations $(0.1)-(0.3)$, b, l_1, l_2, k_1, k_2, a_1, a_2, c_1, c_2, d_1 and d_2 are given constants depending on physical properties of the involved material. In particular, b is a thermal diffusion coefficient; l_1 and l_2 are related to the latent heat associated to each kind of material states; k_1 and k_2 are related to the width of the transitions layers. The given function f is related to the density of heat sources and sinks. Here $n = n(x)$ denotes the outwards unit normal to $\partial\Omega$; the initial data τ_0, u_0 and v_0 are suitable given functions.

The system (0.1)-(0.5) can be viewed as a generalization of the model treated in Hoffman & Jiang [1]. It is also related to a model for solidification of certain metallic alloys allowing two kinds of crystallizations derived and studied by Steinbach *et al.* in [2], [3]. In [2], [3] numerical simulations and comparisons are performed to support the proposed model, but no rigorous mathematical analysis is presented.

We remark that the fact that here we have more than one phase field function brings another mathematical difficult as compared to models with just one of them. In fact, in this last case the higher power nonlinearities have the right sign for the process of obtaining the weaker estimates. On the other hand, here we also have higher powers nonlinearities with are products of different phase fields and thus we have no control of their signs. We also remark that, differently of what occurs in the usual phase field models, in the present one, there are terms in which the temperature appears multiplying the phase fields, bringing nonlinearities that are harder to handle than the ones in the usual models. These difficulties demands that we be very careful even to find the weaker estimates.

In this work we obtained several theoretical results concerning (0.1)-(0.5): global existence and uniqueness of solutions; regularity; continuous dependence with respect to the given function f and initial data. These results are important for the considerations that may lead to the proper choice of algorithms for numerical simulation. As it is usual in this context of simulations, results holding for a simple case may also support the arguments for the proper choice of algorithms in the case of related but more general models.

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1 Mathematical Results

Let us consider the following hypotheses:

(i) $\Omega \subset \mathbb{R}^3$ is a bounded C^2-domain, $0 < T < +\infty$, $Q = \Omega \times (0, T)$;

(ii) τ_0, u_0, $v_0 \in L^\infty(\Omega) \cap W_2^2(\Omega)$, u_0, $v_0 \geq 0$ and $\partial \tau_0/\partial n|_{\partial\Omega} = \partial u_0/\partial n|_{\partial\Omega} = \partial v_0/\partial n|_{\partial\Omega} = 0$;

(iii) $f \in L^q(Q)$ with $q > 5/2$;

(iv) b, k_1, k_2, a_1, a_2 are positive constants; l_1, l_2, c_1, c_2, d_1, d_2 are real constants.

Theorem 1.1. *Let us assume that hypotheses* (**i**) – (**iv**) *hold. There exists* κ_0*, depending on* Ω*,* T*, the constants in* (0.1) – (0.3) *and the norms of* f*,* u_0 *and* v_0 *such that, if* $\max_i(|c_i|) \leq \kappa_0$*, then* (0.1) – (0.5) *possesses exactly one solution* $(\tau, u, v) \in W_{\overline{q}}^{2,1}(Q) \times W_{10/3}^{2,1}(Q) \times W_{10/3}^{2,1}(Q)$ *with* $\overline{q} = \min(10/3, q)$ *that satisfies the estimate*

$$\|\tau\|_{W_{\overline{q}}^{2,1}(Q)} + \|u\|_{W_{10/3}^{2,1}(Q)} + \|v\|_{W_{10/3}^{2,1}(Q)} \leq C \Big(\|\tau_0\|_{W_2^2(\Omega)} + \|u_0\|_{W_2^2(\Omega)} + \|v_0\|_{W_2^2(\Omega)} + \|f\|_{L^q(Q)} + \|\tau_0\|^3_{W_2^2(\Omega)} + \|u_0\|^3_{W_2^2(\Omega)} + \|v_0\|^3_{W_2^2(\Omega)} + \|f\|^3_{L^2(Q)} \Big),$$

where C *depends on* Ω*,* T *and the constants in* (0.1) – (0.3).

Furthermore, $0 \leq u, v \leq M := \max\left(\|u_0\|_{L^\infty}, \|v_0\|_{L^\infty}, \max_i |d_i| + 2\right)$.

Besides, if $0 \leq u_0, v_0 \leq 1$*, then there exists* κ_1*, depending on* Ω*,* T*, the constants in* (0.1) – (0.3) *and the norms of* f*,* u_0 *and* v_0 *such that, if* $\max_i(|c_i|, |d_i|) \leq \kappa_1$*, the solution of* (0.1) – (0.5) *given above satisfies* $0 \leq u, v \leq 1$.

The existence of solution in the last theorem is proved using a Leray-Schauder Fixed Point Theorem and the uniqueness is proved by using standard arguments.

By using bootstrapping arguments we prove the following result concerning the regularity of such solutions.

Theorem 1.2. *Let us assume that hypotheses* (**i**) – (**iv**) *hold and* $\max_i(|c_i|, |d_i|) \leq \kappa_0$*, where* κ_0 *is like in Theorem 1.1. If* τ_0*,* u_0*,* $v_0 \in W_{3p/5}^2(\Omega)$ *with* $2 \leq 3p/5 < +\infty$*, then* $(\tau, u, v) \in W_{\tilde{q}}^{2,1}(Q) \times W_p^{2,1}(Q) \times W_p^{2,1}(Q)$ *and*

$$\|\tau\|_{W_{\tilde{q}}^{2,1}(Q)} + \|u\|_{W_p^{2,1}(Q)} + \|v\|_{W_p^{2,1}(Q)} \leq C \Big(\|\tau_0\|_{W_{3p/5}^2(\Omega)} + \|u_0\|_{W_{3p/5}^2(\Omega)} + \|v_0\|_{W_{3p/5}^2(\Omega)} + \|f\|_{L^q(Q)} \Big).$$

where $\tilde{q} = \min(p, q)$ *and* C *only depends on* Ω*,* T*,* M *and the constants in* (0.1) – (0.3).

Theorem 1.3. *Let us assume that hypotheses* (**i**) *and* (**iv**) *hold and* $\max_i(|c_i|, |d_i|) \leq \kappa_0$*, where* κ_0 *is like in Theorem 1.1. Let us consider initial conditions* τ_0^i*,* u_0^i*,* $v_0^i \in W_{3p/5}^2(\Omega)$ *with* $2 \leq 3p/5 < +\infty$ *and given functions* f_i *satisfying* (**ii**) *and* (**iii**)*. Let* (τ_i, u_i, v_i) *be the solution of* (0.1) – (0.5) *associated to* $(f_i, \tau_0^i, u_0^i, v_0^i)$*. Then* $(\tau_i, u_i, v_i) \in W_{\tilde{q}}^{2,1}(Q) \times W_p^{2,1}(Q) \times W_p^{2,1}(Q)$ *with* $\tilde{q} = \min(p, q)$ *and*

$$\|\tau_1 - \tau_2\|_{W_{\tilde{q}}^{2,1}(Q)} + \|u_1 - u_2\|_{W_p^{2,1}(Q)} + \|v_1 - v_2\|_{W_p^{2,1}(Q)} \leq C \left[\|\tau_0^1 - \tau_0^2\|_{W_{3p/5}^2(\Omega)} + \|u_0^1 - u_0^2\|_{W_{3p/5}^2(\Omega)} + \|v_0^1 - v_0^2\|_{W_{3p/5}^2(\Omega)} + \|f_1 - f_2\|_{L^q(Q)} \right],$$

where C *is like in Theorem 1.2 with* $M = \max_i\{\|u_0^i\|_{L^\infty(\Omega)},\ \|v_0^i\|_{L^\infty(\Omega)},\ |d_i| + 1\}$.

This result also follows from standard arguments.

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TEOREMAS DE REPRESENTAÇÃO PARA ESPAÇOS DE SOBOLEV EM INTERVALOS E MULTIPLICIDADE DE SOLUÇÕES PARA EDOs NÃO LINEARES

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Uma aplicação direta do teorema do passo da montanha com simetria garante a existência de infinitas soluções para o problema de valor de contorno

$$-\Delta u = |u|^{p-1}u,\ x \in \Omega \ \text{ e } \ u = 0 \text{ sobre } \partial\Omega,$$

onde Ω representa um domínio limitado regular em $\mathbb{R}^N$ com $N \geq 1$, $p > 1$ se $N = 1, 2$ e $2 < p + 1 < 2^* := \frac{2N}{N-2}$ se $N \geq 3$.

A presença de um termo não-homogêneo quebra a simetria do funcional associado e impossibilita o emprego do terorema do passo da montanha com simetria. Assim, uma questão natural é se o problema

$$\begin{cases} -\Delta u &= |u|^{p-1}u + h(x) & x \in \Omega, \\ u &= 0 & \text{sobre } \partial\Omega, \end{cases} \tag{0.1}$$

possui infinitias soluções ou não.

O estudo de (0.1) iniciou-se em 57 com Ehrmann e mais tarde em 75 Fučik & Lovicar apresentaram algumas contribuições. Eles provaram que a EDO

$$\begin{cases} -u'' = |u|^{p-1}u + h(x) & x \in (0, 1), \\ u(0) = u(1) = 0, \end{cases} \tag{0.2}$$

possui infinitas soluções no caso em que $p > 1$.

O caso envolvendo EDPs (0.1) foi tratado por Bahri & Beresticky, Struwe, Rabinowitz, Tanaka e por Bahri & Lions nos anos 80. No entanto, até o momento, uma resposta completamente satisfatória para o problema ainda não foi fornecida e o melhor resultado existente, apresentado por Tanaka e Bahri & Lions, garante a existência de infinitas soluções desde que: $h \in L^2(\Omega)$, $p > 1$ se $N = 1, 2$; $2 < p + 1 < \frac{2N-2}{N-2}$ se $N \geq 3$. Observe que este resultado não cobre completamente o intervalo subcrítico $(1, 2^* - 1)$. Assumindo a restrição de crescimento "natural", $p \in (1, 2^* - 1)$, Bahri provou que existe um conjunto aberto e denso de funções $h \in H^{-1}(\Omega)$ para o qual (0.1) possui infinitas soluções, i.e. a existência de infinitas soluções é genericamente verdade.

O interesse em resultados de multiplicidade sobre perturbações de problemas simétricos cresceu consideravelmente nos últimos anos e foi estudado em vários contextos. Por exemplo, mencionamos o estudo de problemas com condições de contorno não-homogêneas de Bolle, Ghoussoub & Tehrani e o estudo de sistemas elípticos de Tarsi.

Em um trabalho publicado este ano, Bonheure & Ramos estenderam os resultados de Tarsi. Eles consideraram o sistema

$$\begin{cases} -\Delta u &= |v|^{p-1}\, v + f(x) & x \in \Omega, \\ -\Delta v &= |u|^{q-1}\, u + g(x) & x \in \Omega, \\ u, v &= 0 & \text{sobre } \partial\Omega, \end{cases} \tag{0.3}$$

sob as seguintes hipóteses:

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1. $p, q > 1$,

2. $\frac{N}{2}\left(1 - \frac{1}{p+1} - \frac{1}{q+1}\right) < \frac{p}{p+1}$ se $p \leq q$ e $\frac{N}{2}\left(1 - \frac{1}{p+1} - \frac{1}{q+1}\right) < \frac{q}{q+1}$ if $q \leq p$,

3. $f, g \in L^2(\Omega)$.

A variação admissível para p e q generaliza, em um certo sentido, a variação obtida por Tanaka e Bahri & Lions para tratar (0.1), uma vez que estas coincidem quando $p = q$ e $f = g$ (o que implica que $u = v$).

Através do procedimento adotado por Bonheure & Ramos a hipótese $p > 1$ e $q > 1$ não pode ser removida. No entanto, a precisa noção de superlinearidade para o sistema (0.3) é

(H1) $p, q > 0$ e $pq > 1$.

Isto sugere que os resultados de Bonheure & Ramos podem ser melhorados. No caso unidimensional, (0.3) torna-se

$$\begin{cases} -u'' = |v|^{p-1} v + f(x) & x \in (0,1), \\ -v'' = |u|^{q-1} u + g(x) & x \in (0,1), \\ u, v = 0 & \text{sobre } \{0,1\}. \end{cases} \tag{0.4}$$

Suponha

(H2) $f, g \in C^1([0,1])$.

Neste trabalho, nosso resultado principal a respeito de (0.4) é uma extensão parcial dos resultados de Bonheure & Ramos.

Teorema 0.1. *Suponha (H1)-(H2). Então (0.4) possui um número infinito de soluções clássicas.*

Para tratar (0.4) sob a hipótese mais geral (H1), reduzimos (0.4) a uma equação não linear de quarta ordem e adotamos o método de Rabinowitz, o qual tem sido aplicado em várias situações e em particular por Garcia Azorero & Peral Alonso para tratar perturbações de simetria envolvendo o operador p-Laplaciano. Um argumento crucial no método de Rabinowitz é o emprego de estimativas assintóticas para o comportamento dos autovalores do Laplaciano. Uma vez que o Laplaciano é um operador linear auto-adjunto, estas estimativas assintóticas induzem, de forma imediata, desigualdades de Poincaré no ortogonal ao espaço gerado pelas n-primeiras auto-funções. No entanto, quando um operador não linear está envolvido, este assunto é muito mais delicado.

Em nosso problema, realizamos este passo utilizando alguns resultados sobre bases de Schauder que são obtidos através da teoria de análise de Fourier e o seguinte isomorfismo topológico entre $W^{m,p}((0,1))$ e $L^p((0,1)) \times \mathbb{R}^m$.

Teorema 0.2. *Seja $1 \leq p \leq \infty$ e $m \geq 1$. Então $W^{m,p}((0,1))$ é topologicamente isomorfo a $L^p((0,1)) \times \mathbb{R}^m$ e a aplicação $T_m : W^{m,p}((0,1)) \to L^p((0,1)) \times \mathbb{R}^m$, definida por*

$$T_m(u) := \left(u^{(m)}, u(0), u'(0), \ldots, u^{(m-1)}(0)\right), \tag{0.5}$$

é um isomorfismo topológico.

Através do Teorema 0.2 apresentamos demonstrações imediatas para resultados bem conhecidos sobre os espaços de Sobolev $W^{m,p}((0,1))$. Além disso, também obtemos outros resultados sobre estes espaços que são, até onde sabemos, novos. Por exemplo, fornecemos uma caracterização do espaço dual de $W^{m,p}((0,1))$. Também aplicamos o Teorema 0.2 para apresentar bases de Schauder explícitas para alguns espaços de Sobolev e alguns de seus subespaços.

Neste trabalho apresentamos duas classes de isomorfismos topológicos entre $W^{m,p}((0,1))$ e $L^p((0,1)) \times \mathbb{R}^m$. Cada uma delas torna-se importante em nosssas aplicações. A motivação para a primeira classe de isomorfismos tem origem no problema de valor inicial para uma EDO de ordem m. A segunda classe é motivada pelas condições de contorno de Navier para uma equação poliharmônica. Vale dizer que dependendo do tipo de problema de EDO considerado, outras classes de isomorfismos podem ser obtidas.

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LIMIT SETS AND THE POINCARÉ-BENDIXSON THEOREM IN IMPULSIVE SEMIDYNAMICAL SYSTEMS

E.M.BONOTTO *

We consider semidynamical systems with impulse effects at variable times and we discuss some properties of the limit sets of orbits of these systems such as invariancy, compactness and connectedness. As a consequence we obtain a version of the Poincaré-Bendixson Theorem for impulsive semidynamical systems.

1 Introduction

The theory of impulsive semidynamical systems is an important and modern chapter of the theory of topological dynamical systems. Interesting and important results about this theory have been studied such as "minimality", "invariancy", "recurrence", "periodic orbits", "stability" and "flows of characteristc 0^+". For details of this theory, see [1], [2], [3], [4], [6], [7] and [8], for instance.

In [5], the author presents the theory of Poincaré-Bendixson for non-impulsive two-dimensional semiflows. A natural question that arises is how the theory of Poincaré can be described in impulsive semidynamical systems.

In the present paper, we give important results about limit sets for impulsive semidynamical systems of type $(\mathrm{X}, \pi; \Omega, \mathrm{M}, \mathrm{I})$, where X is a metric space, (X, π) is a semidynamical system, Ω is an open set in X, $\mathrm{M} = \partial\Omega$ denotes the impulsive set and $\mathrm{I} : \mathrm{M} \to \Omega$ is the impulse operator. Our goal, however, is to establish the Poincaré-Bendixson Theorem in this setting.

We deal with various properties about limit sets. An important fact here is that we consider the closure of the trajectories in X rather than in Ω as presented in [7]. Thus, our impulsive system encompasses the one presented in [7]. Indeed, some new phenomena can occur. We study the invariancy, compactness and connectedness of limit sets in impulsive semidynamical systems with a finite numbers of impulses. Then we consider the more general case when the system presents infinitely many impulses and we obtain analogous results. Also, we present an important theorem which concerns an impulsive semidynamical systems $(\mathrm{X}, \pi; \Omega, \mathrm{M}, \mathrm{I})$, where $\overline{\Omega}$ is compact and $x \in \Omega$, and it says that if a trajectory through x has infinitely many impulses, $\{x_n\}_{n\geq 1}$, with $x_n \overset{n\to+\infty}{\longrightarrow} p$, then the limit set of x in $(\mathrm{X}, \pi; \Omega, \mathrm{M}, \mathrm{I})$, $\widetilde{\mathrm{L}}^+(x)$, is the union of a periodic orbit and the point $\{\mathrm{p}\}$.

Finally, we discuss a version of the Poincaré-Bendixson Theorem for impulsive semidynamical systems. The main result states that given an impulsive semidynamical system $(\mathbb{R}^2, \pi; \Omega, \mathrm{M}, \mathrm{I})$ and $x \in \Omega$, if we suppose $\overline{\Omega}$ is compact and $\widetilde{\mathrm{L}}^+(x)$ admits neither rest points nor initial points, then $\widetilde{\mathrm{L}}^+(x)$ is a periodic orbit.

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AN ASYMPTOTICALLY CONSISTENT GALERKIN METHOD FOR THE REISSNER–MINDLIN PLATE MODEL

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Recently, a new Galerkin finite element method for the biharmonic equation was introduced and analysed by the first and third authors. We now extend such ideas in a nontrivial way for the Reissner–Mindlin plate model. The extension is such that, as the plate thickness tends to zero, we recover the method for the biharmonic problem, i.e., the method follows the asymptotic behaviour of the continuous PDE. We present here an error estimate that is uniform with respect to the plate thickness.

1 Extended summary

The Reissner–Mindlin system is not only a good model for linearly elastic three-dimensional plates, but also it brings in computational challenges that require ingenious numerical methods. The reason is that the system depends nontrivially on ϵ, the half-thickness of the plate. As the plate become thinner, the Reissner–Mindlin solution converges to the biharmonic solution, in the same way the exact three-dimensional solution does. Thus, for small ϵ, naive numerical schemes fail, since they do not approximate well solutions of fourth order problems. This is described as a *numerical locking*. Recently some authors started to take advantage of the flexibility of discontinuous Galerkin (DG) finite element methods to design new, locking free, plate models [2]. We follow the same philosophy.

A new Galerkin method for the biharmonic problem have been considered in [4]. Such scheme was our motivation in this present work. We propose here a method for the Reissner–Mindlin system that, as ϵ tends to zero, "converges" to the scheme for the biharmonic. We prove convergence in a natural energy norm, and provide numerical tests that confirm our predictions.

Let Ω be a convex and polygonal two-dimensional domain with boundary $\partial\Omega$. Consider a homogeneous and isotropic linearly elastic plate occupying the three-dimensional domain $\Omega \times (-\epsilon, \epsilon)$. Assume that this plate is clamped on its lateral side, and under a transverse load of density per unity area $\epsilon^3 g$ that is symmetric with respect to its middle surface. There are two popular two-dimension models for the plate's displacement.

In the biharmonic model, the displacement at $(\boldsymbol{x}, x_3) \in \Omega \times (-\epsilon, \epsilon)$ is $\big(-x_3 \boldsymbol{\nabla}\, \psi(\boldsymbol{x}), \psi(\boldsymbol{x})\big)$, where

$$\psi = \operatorname*{arg\,min}_{\nu \in H_0^2(\Omega)} a(\boldsymbol{\nabla}\, \nu, \boldsymbol{\nabla}\, \nu) - (g, \nu). \tag{1.1}$$

Here, $a(\boldsymbol{\theta}, \boldsymbol{\eta}) = \int_\Omega \mathcal{C}\, e(\boldsymbol{\theta}) : e(\boldsymbol{\eta})\, d\boldsymbol{x}$, $(\cdot,\cdot)$ denotes the inner product in $L^2(\Omega)$ and $e(\boldsymbol{\theta})$ is the symmetric part of the gradient of $\boldsymbol{\theta}$. Finally, $\mathcal{C}\, e(\boldsymbol{\theta}) = [\,2\mu\, e(\boldsymbol{\theta}) + \lambda^* \operatorname{div} \boldsymbol{\theta}\, I\,]/3$, μ and λ are the Lamé coefficients, $\lambda^* = 2\mu\lambda/(2\mu + \lambda)$, and I is the identity matrix.

The simplest Reissner–Mindlin model approximation [1] is given by $\big(-x_3\boldsymbol{\theta}(\boldsymbol{x}), \omega(\boldsymbol{x})\big)$, where

$$(\boldsymbol{\theta}, \omega) = \operatorname*{arg\,min}_{(\boldsymbol{\theta},\omega)\in \boldsymbol{H}_0^1(\Omega)\times H_0^1(\Omega)} a(\boldsymbol{\theta}, \boldsymbol{\theta}) + \epsilon^{-2}\mu(\boldsymbol{\theta} - \boldsymbol{\nabla}\, \omega, \boldsymbol{\theta} - \boldsymbol{\nabla}\, \omega) - (g, \omega) \tag{1.2}$$

The relation between the biharmonic and Reissner–Mindlin models becomes clear since, as $\epsilon \to 0$, the sequence of solutions $(\boldsymbol{\theta}, \omega)$ converges to $(\boldsymbol{\nabla}\, \psi, \psi)$. This is an instance of a more general result of [3].

We next propose a "discontinuous formulation." Assume a regular partition $\mathcal{K}_h$ of the domain Ω into elements (triangles or quadrilaterals) K. We denote the set of edges by $\mathcal{E}_h$. For the definitions below, we assume that the

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functions involved have traces with enough regularity. Let K^- and K^+ be two distinct elements of $\mathcal{K}_h$ sharing the edge $e = K^- \bigcap K^+$. We define the jump of a function ϕ by $[\phi] = \phi^- \mathbf{n}^- + \phi^+ \mathbf{n}^+$, where $\phi^\pm = \phi|_{K^\pm}$ and $\mathbf{n}^\pm = \mathbf{n}_{K^\pm}$. For an vector function $\boldsymbol{\theta}$, define

$$[\boldsymbol{\theta}] = \boldsymbol{\theta}^- \cdot \mathbf{n}^- + \boldsymbol{\theta}^+ \cdot \mathbf{n}^+, \qquad [\![\boldsymbol{\theta}]\!] = \boldsymbol{\theta}^- \odot \mathbf{n}^- + \boldsymbol{\theta}^+ \odot \mathbf{n}^+,$$

where $\boldsymbol{\theta} \odot \mathbf{n} = (\boldsymbol{\theta}\mathbf{n}^T + \mathbf{n}\boldsymbol{\theta}^T)/2$. The average of scalar or vector function χ is $\{\chi\} = \frac{1}{2}(\chi^- + \chi^+)$. On boundary faces ∂K with outer normal $\mathbf{n}$, define the jumps and averages as $[\phi] = \phi|_K \mathbf{n}$, $[\boldsymbol{\theta}] = \boldsymbol{\theta}|_K \cdot \mathbf{n}$, $[\![\boldsymbol{\theta}]\!] = \boldsymbol{\theta}|_K \odot \mathbf{n}$, $\{\chi\} = \chi|_K$.

Under the assumption that the exact solution for the biharmonic problem belong to "appropriate" Sobolev spaces within *each* element, it is possible to rewrite the energy functional of (1.1) as [4]

$$\frac{1}{2} a_h(\boldsymbol{\nabla}\,\nu, \boldsymbol{\nabla}\,\nu) + \sum_{e \in \mathcal{E}_h} \left[-(\{\mathcal{C}\, e(\boldsymbol{\nabla}\,\nu)\}, [\![\boldsymbol{\nabla}\,\nu]\!])_e + \frac{\beta_e}{2}([\![\boldsymbol{\nabla}\,\nu]\!], [\![\boldsymbol{\nabla}\,\nu]\!])_e + ([\nu], \{\mathbf{div}\, \mathcal{C}\, e(\boldsymbol{\nabla}\,\nu)\})_e + \frac{\alpha_e}{2}([\nu], [\nu])_e \right] - (g, \nu).$$

We use the subscript e whenever inner products are considered on the partition's boundaries, and the positive stabilization parameters α_e, β_e are fixed to weakly impose the boundary conditions and inter-element continuity, and to stabilize the method. Note that the jumps over the edges vanish for sufficiently regular functions.

The Reissner–Mindlin functional in (1.2) corresponds to the critical point of

$$\frac{1}{2} a_h(\boldsymbol{\eta}, \boldsymbol{\eta}) + \sum_{e \in \mathcal{E}_h} \left[-(\{\mathcal{C}\, e(\boldsymbol{\eta})\}, [\![\boldsymbol{\eta}]\!])_e + \frac{\beta_e}{2}([\![\boldsymbol{\eta}]\!], [\![\boldsymbol{\eta}]\!])_e + ([\nu], \{\mathbf{div}\, \mathcal{C}\, e(\boldsymbol{\eta})\})_e + \frac{\alpha_e}{2}([\nu], [\nu])_e \right]$$
$$+ \epsilon^{-2} \mu(\boldsymbol{\eta} - \boldsymbol{\nabla}\,\nu, \boldsymbol{\eta} - \boldsymbol{\nabla}\,\nu)_h - (g, \nu).$$

As $\epsilon \to 0$, the modified Reissner–Mindlin energy "converges" to the modified biharmonic energy, in a proper sense.

The numerical solutions for the biharmonic and Reissner–Mindlin problems are based on the broken versions of the energies. The main result of this paper is the following.

Theorem 1.1. *Under certain assumptions for $\mathcal{K}_h$ and the penalization parameters α and β, and if the Reissner–Mindlin solution is regular enough, then the solution of the discontinuous Galerkin method $(\boldsymbol{\theta}_h, \omega_h)$, satisfy*

$$|\!|\!| \boldsymbol{\theta} - \boldsymbol{\theta}_h; \omega - \omega_h |\!|\!| \le c h^{p-1} \left(\|\boldsymbol{\theta}\|_p + \|\omega\|_{p+1} + \epsilon \|\boldsymbol{\gamma}\|_{p-1} \right), \tag{1.3}$$

where c does not depend on h or ϵ. The norm $|\!|\!| \cdot |\!|\!|$ is an "appropriate" energy norm.

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On strictly singular polynomials

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Let E and F be Banach spaces. An operator $T \in \mathcal{L}(E;F)$ is *strictly singular* if for every infinite-dimensional subspace Y of E, the restriction $T|_Y$ is not an isomorphism onto its range. The space of all such operators will be denoted by $\mathcal{SS}(E;F)$.

The concept of *strictly singular polynomial* was introduced in [2]. A continuous n-homogeneous polynomial $P \in \mathcal{P}(^nE;F)$ is *strictly singular*, denoted by $P \in \mathcal{SS} \circ \mathcal{P}(^nE;F)$, if there are a Banach space G, a n-homogeneous polynomial $Q \in \mathcal{P}(^nE;G)$ and an operator $u \in \mathcal{SS}(G;F)$ such that $P = u \circ Q$. We denote by $\mathcal{P}_{\mathcal{W}}(^nE;F)$ the subspace of all n-homogeneous weakly compact polynomials from E to F.

Let K be a compact Hausdorff space. It is well known that weakly compact linear operators defined on $C(K)$ are strictly singular ([1, Theorem 5.5.1]). We shall see that if K is scattered, n-homogeneous weakly compact polynomials defined on $C(K)$ are strictly singular. More generally, in the present work we study conditions on E and F for which weakly compact homogeneous polynomials from E to F are strictly singular.

1 Results

Theorem 1.1. *Let E and F be Banach spaces. Suppose that $\widehat{\bigotimes}_\pi^{n,s} E$ has the Dunford-Pettis property. Then, for each $n \in N$ we have*

$$\mathcal{P}_{\mathcal{W}}(^nE;F) \subset \mathcal{SS} \circ \mathcal{P}(^nE;F).$$

Corollary 1.1. *(a) If K is a scattered compact Hausdorff space, then $\mathcal{P}_{\mathcal{W}}(^nC(K);F) \subset \mathcal{SS} \circ \mathcal{P}(^nC(K);F)$ for each $n \in N$ and each Banach space F.*

(b) $\mathcal{P}_{\mathcal{W}}(^n\ell_1;F) \subset \mathcal{SS} \circ \mathcal{P}(^n\ell_1;F)$ for each $n \in N$ and each Banach space F.

(c) $\mathcal{P}_{\mathcal{W}}(^nL_1(\mu);F) \subset \mathcal{SS} \circ \mathcal{P}(^nL_1(\mu);F)$ for each $n \in N$ and each Banach space F.

Remark 1.1. As to (a), there are a non-scattered infinite compact Hausdorff space K and a weakly compact non-strictly singular bilinear mapping from $C(K) \times C(K)$ to ℓ_2. This does not settle the polynomial case because we cannot assure that this bilinear mapping is symmetric. We conjecture that there are a non-scattered infinite compact Hausdorff space K, a Banach space F and a weakly compact non-strictly singular 2-homogeneous polynomial from $C(K)$ to F.

A Banach space E has the *hereditarily Dunford-Pettis property* if every closed subspace of E has the Dunford-Pettis property.

Theorem 1.2. *Let E and F be Banach spaces. Suppose that F has the hereditarily Dunford-Pettis property. Then, for each $n \in N$ we have*

$$\mathcal{P}_{\mathcal{W}}(^nE;F) \subset \mathcal{SS} \circ \mathcal{P}(^nE;F).$$

A Banach space E is *weakly sequentially complete* if weakly Cauchy sequences in E are weakly convergent.

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Theorem 1.3. *Let E and F be Banach spaces. Suppose that F is weakly sequentially complete. Then, for each $n \in N$ we have*

$$\mathcal{SS} \circ \mathcal{P}(^n E; F) \subset \mathcal{P}_{\mathcal{W}}(^n E; F).$$

From a result due to Pełczyński (see [1, Theorem 5.5.3]) we conclude, in particular, that $\mathcal{W}(C(K); F) = \mathcal{SS}(C(K); F)$ for every compact Hausdorff space K and every Banach space F. In this direction we have:

Corollary 1.2. *If K is a scattered compact Hausdorff space, then for each $n \in N$ we have*

$$\mathcal{P}_{\mathcal{W}}(^n C(K); \ell_1) = \mathcal{SS} \circ \mathcal{P}(^n C(K); \ell_1)$$

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A unified Pietsch domination theorem

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1 Introduction

Pietsch's domination theorem [3, Theorem 2.12] is a cornerstone in the theory of absolutely summing liner operators. As expected, this type of domination theorem turned out to be a basic result in the several (linear and non-linear) theories which generalize and extend the linear theory of absolutely summing operators. In this direction, several Pietsch-type domination theorems have been proved for different classes of (linear and non-linear) mappings between Banach spaces. Among these theorems, we mention: the Farmer and Johnson domination theorem for Lipschitz summing mappings between metric spaces [5, Theorem 1(2)], the Pietsch and Geiss domination theorem for dominated multilinear mappings ([10, Theorem 14], [6, Satz 3.2.3]), the Dimant domination theorem for strongly summing multilinear mappings and homogeneous polynomials [4, Proposition 1.2(ii) and Proposition 3.2(ii)], the domination theorem for α-subhomogeneous mappings [1, Theorem 2.4], the Martínez-Giménez and Sánchez Pérez domination theorem for (D,p)-summing operators [7, Theorem 3.11], the Çaliskan and Pellegrino domination theorem for semi-integral multilinear mappings and the X. Mujica domination theorem for $\tau(p)$-summing multilinear mappings. In this work we construct an abstract setting which yields a unified version of Pietsch's domination theorem which comprises all the above results as particular cases. Moreover, this unified Pietsch domination theorem does not depend on algebraic conditions of the underlying mappings, such as linearity, multilinearity, etc. In other words, we prove that Pietsch-type dominations are algebra-free.

2 Results

Let X, Y and E be (arbitrary) sets, $\mathcal{H}$ be a family of mappings from X to Y, G be a Banach space and K be a compact Hausdorff topological space. Let $R\colon K \times E \times G \longrightarrow [0,\infty)$ and $S\colon \mathcal{H} \times E \times G \longrightarrow [0,\infty)$ be mappings so that

• There is $x_0 \in E$ such that

$$R(\varphi, x_0, b) = S(f, x_0, b) = 0$$

for every $\varphi \in K$ and $b \in G$.

• The mapping

$$R_{x,b}\colon K \longrightarrow [0,\infty)\ ,\ \ R_{x,b}(\varphi) = R(\varphi, x, b)$$

is continuous for every $x \in E$ and $b \in G$.

• It holds that

$$R\left(\varphi, x, \eta b\right) \leq \eta R\left(\varphi, x, b\right) \text{ and } \eta S(f,x,b) \leq S(f,x,\eta b)$$

for every $\varphi \in K, x \in E, 0 \leq \eta \leq 1, b \in G$ and $f \in \mathcal{H}$.

Definition 2.1. Let R and S be as above and $0 < p < \infty$. A mapping $f \in \mathcal{H}$ is said to be *R-S-abstract p-summing* is there is a constant $C_1 > 0$ so that

$$\sum_{j=1}^{m} S(f, x_j, b_j)^p \leq C_1 \sup_{\varphi \in K} \sum_{j=1}^{m} R\left(\varphi, x_j, b_j\right)^p, \tag{2.1}$$

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for all $x_1, \ldots, x_m \in E$, $b_1, \ldots, b_m \in G$ and $m \in \mathbb{N}$. The infimum of such constants C_1 is denoted by $\pi_{RS,p}(f)$.

It is not difficult to show that the infimum of the constants above is attained, i.e., $\pi_{RS,p}(f)$ satisfies (2.1).

Theorem 2.1. *Let R and S be as above, $0 < p < \infty$ and $f \in \mathcal{H}$. Then f is R-S-abstract p-summing if and only if there is a constant $C > 0$ and a regular Borel probability measure μ on K such that*

$$S(f, x, b) \leq C \left(\int_K R(\varphi, x, b)^p \, d\mu(\varphi) \right)^{\frac{1}{p}} \tag{2.2}$$

for all $x \in E$ and $b \in G$. Moreover, the infimum of such constants C equals $\pi_{RS,p}(f)^{\frac{1}{p}}$.

For convenient choices of R, S and $\mathcal{H}$, we recover from Theorem 2.1 all the domination theorems mentioned in the introduction (including, of course, Pietsch's original theorem).

Definition 2.2. Let E and F be Banach spaces. An arbitrary mapping $f\colon E \longrightarrow F$ is *absolutely p-summing at* $a \in E$ if there is a $C \geq 0$ so that, for every natural number m and every $x_1, \ldots, x_m \in E$,

$$\sum_{j=1}^{m} \|f(a + x_j) - f(a)\|^p \leq C \sup_{\varphi \in B_{E'}} \sum_{j=1}^{m} |\varphi(x_j)|^p .$$

As [8, Theorem 3.5] makes clear, the above definition is actually an adaptation of [8, Definition 3.1]. We finish applying Theorem 2.1 to show that, even in the absence of algebraic conditions, absolutely p-summing mappings are exactly those which enjoy a Pietsch-type domination:

Theorem 2.2. *Let E and F be Banach spaces. An arbitrary mapping $f\colon E \longrightarrow F$ is absolutely p-summing at $a \in E$ if and only if there is a constant $C > 0$ and a regular Borel probability measure μ on $B_{E'}$ such that, for all $x \in E$,*

$$\|f(a + x) - f(a)\| \leq C \left(\int_{B_{E'}} |\varphi(x)|^p \, d\mu(\varphi) \right)^{\frac{1}{p}} . \tag{2.3}$$

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EIGENVALUE BOUNDS FOR MICROPOLAR COUETTE FLOW

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We derive some eigenvalue bounds for micropolar plane Couette flow.

1 Eigenvalue bounds for micropolar Couette flow

Consider the initial boundary value problem

$$\begin{cases} \mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \dfrac{1}{R}\Delta\mathbf{u} \\ \operatorname{div}\mathbf{u} = 0 \\ \mathbf{u}(x,0,t) = (0,0) \\ \mathbf{u}(x,1,t) = (1,0) \\ \mathbf{u}(x,y,t) = \mathbf{u}(x+1,y,t) \\ \mathbf{u}(x,y,0) = \mathbf{f}(x,y) \end{cases} \tag{1.1}$$

where $\mathbf{u} : \mathbb{R}\times[0,1]\times[0,\infty) \longrightarrow \mathbb{R}^2$ is the unknown function $\mathbf{u}(x,y,t) = (u(x,y,t), v(x,y,t))$, and the positive parameter R is the Reynolds number. The initial condition $\mathbf{f}(x,y)$ is assumed to be divergence free and compatible with the boundary conditions. It can be easily seen that $\mathbf{U}(x,y) = (y,0)$, $P = constant$ is a steady solution of problem (1.1). The vector field $\mathbf{U}(x,y) = (y,0)$ is known as plane Couette flow. This flow is linearly stable, that is, the eigenvalues of the linear operator associated with the system of differential equations governing perturbations of the flow have negative real part[5]. Even though this is the case, there are still fundamental issues not completely understood; for example, for the 3 dimensional Couette flow, transition to turbulence is observed in laboratory for Reynolds numbers as low as $R \approx 350$ (Lundbladh & Johansson[4] and Tillmark & Alfredsson[6]).

Micropolar fluids[3] are fluids with asymmetric stress tensor, which are governed by the equations

$$\begin{cases} \mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = (\nu+\nu_r)\Delta\mathbf{u} + 2\nu_r \operatorname{curl}\mathbf{w}, \\ \mathbf{w}_t + (\mathbf{u}\cdot\nabla)\mathbf{w} + 4\nu_r\mathbf{w} = (c_a+c_d)\Delta\mathbf{w} + (c_0+c_d-c_a)\nabla\operatorname{div}\mathbf{w} + 2\nu_r\operatorname{curl}\mathbf{u}, \\ \operatorname{div}\mathbf{u} = 0, \end{cases} \tag{1.2}$$

The unknowns $\mathbf{u}$, $\mathbf{w}$, and p are, respectively, the linear velocity, the angular velocity of rotation of fluid particles, and the pressure distribution of the fluid. The positive constants ν, ν_r, c_0, c_a, c_d are related with viscosity properties of the fluid, and satisfy $c_0 + c_d > c_a$. We are interested in studying stability of plane Couette flow for such fluids. To this end, the first step is to consider general shear flows

$$\mathbf{U} = (U(y),0,0),\ \ \mathbf{W} = (0,0,W(y)),\ \ P = constant,\ \ y\in(0,1),$$

and perturbations $\mathbf{u} = (u(x,y), v(x,y), 0)$, $\mathbf{w} = (0,0,w(x,y))$ of the base flow. The linearized equations for such perturbations are

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{U} + (\mathbf{U}\cdot\nabla)\mathbf{u} + \nabla p = (\nu+\nu_r)\Delta\mathbf{u} + 2\nu_r\operatorname{curl}\mathbf{w} \tag{1.3}$$

$$\mathbf{w}_t + (\mathbf{U}\cdot\nabla)\mathbf{w} + (\mathbf{u}\cdot\nabla)\mathbf{W} = 2\nu_r\operatorname{curl}\mathbf{u} - 4\nu_r\mathbf{w} + (c_0+c_d-c_a)\nabla\operatorname{div}\mathbf{w} + (c_a+c_d)\Delta\mathbf{w} \tag{1.4}$$

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under periodic condtions in x, $\mathbf{u}(t,x,y) = \mathbf{u}(t,x+1,y)$, $\mathbf{w}(t,x,y) = \mathbf{w}(t,x+1,y)$, and initial and boundary conditions

$$\mathbf{u}(0,x,y) = 0,\ \mathbf{w}(0,x,y) = 0$$
$$\mathbf{u}(t,x,0) = 0 = \mathbf{u}(t,x,1),\ \mathbf{w}(t,x,0) = 0 = \mathbf{w}(t,x,1)$$

Introducing the dimensionless streamfunction ψ by $\psi_y = -u$, $\psi_x = v$, and considering specific disturbances

$$\begin{aligned} \psi &= \widetilde{\psi}(y)e^{i\alpha(x-ct)}, \\ w &= \widetilde{w}(y)e^{i\alpha(x-ct)}, \end{aligned}$$

onde gets the dimensionless equations[2]

$$\begin{aligned} i\alpha\left[(U-c)(D^2-\alpha^2)-U''\right]\widetilde{\psi} &= \left(\frac{1}{R_\mu}+\frac{1}{2R_k}\right)(D^2-\alpha^2)^2\widetilde{\psi} - \frac{R_0}{R_k}(D^2-\alpha^2)\widetilde{w}, \\ i\alpha\left[(U-c)\widetilde{w} - W'\widetilde{\psi}\right] &= \frac{1}{R_\gamma}(D^2-\alpha^2)\widetilde{w} - \frac{2R_0}{R_\nu}\widetilde{w} + \frac{1}{R_\nu}(D^2-\alpha^2)\widetilde{\psi}, \end{aligned} \tag{1.5}$$

where R_γ, R_μ, R_ν, R_k, and R_0 are dimensionless parameters and $D := \frac{d}{dy}$. The growth of the disturbances with time depends on the sign of c_i, the imaginary part of $c = c_r + ic_i$: If c_i is positive, the disturbance grows with time. If it is negative, it decays with time. In case c is real, one tipically has an oscillatory behavior. Here, it is important to note that c_i has the same sign as the real part of the eigenvalues of the linear operator associated with the differential equations governing perturbations of the flow, so that c_i being negative assures linear stability of the flow. Define $R_1 := \max\{\frac{R_\mu}{2}, R_k\}$, $R_2 := \min\{R_\nu, \frac{R_k}{R_0}\}$, $R_3 := \max\{\frac{R_\nu}{2R_0}, R\gamma\}$. We prove the following theorem, assuring bounds for c_i.

Theorem 1.1. *Let $c = c_r + ic_i$ be any eigenvalue of 1.5. If $R_1 < R_2$, and $R_3 < \frac{R_2}{2}$, then*

$$c_i \leq \frac{q_1+q_2}{2\alpha} - \frac{\pi^2+\alpha^2}{\alpha R},$$

where $\frac{1}{R} := min\{\frac{1}{R_1} - \frac{1}{R_2}, \frac{1}{R_3} - \frac{2}{R_2}\}$, $q_1 := max_{y\in[0,1]}|U'(y)|$, $q_2 := max_{y\in[0,1]}|W'(y)|$. Moreover, there are no amplified disturbances if

$$\begin{cases} \alpha R q_1 < \dfrac{(4,73)^2\pi}{2} + 2^{\frac{3}{2}}\alpha^3, \\ \quad\text{and} \\ \alpha R q_2 < \sqrt{2(\pi^2+\alpha^2)}(4,73)^2, \end{cases} \quad or \quad \begin{cases} \alpha R q_1 < (4,73)^2\pi + 2\alpha^2\pi, \\ \quad\text{and} \\ \alpha R q_2 < 2\alpha^2\sqrt{\pi^2+\alpha^2}. \end{cases}$$

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Upper semicontinuity of global attractors for p-laplacian parabolic problems

Simone M. Bruschi * & Cláudia B. Gentile † & Marcos R. T. Primo ‡

In 1974 N. Chafee and E. F. Infante completely described the set of stationary solutions of a semilinear parabolic problem like

$$\begin{cases} u_t = \lambda u_{xx} + u - u^3, & (x,t) \in (0,1) \times (0,+\infty) \\ u(0,t) = u(1,t) = 0, \ 0 \le t < +\infty \\ u(x,0) = u_0(x), & x \in (0,1), \end{cases} \tag{1}$$

where λ is a positive parameter and the initial data are sufficiently smooth. The set of equilibrium states, E_λ, is taken as function of λ and, roughly speaking, the authors obtain that, for large values of λ, the only stationary solution is zero, and all nonconstant equilibria bifurcate from zero, two by two, while λ cross the values of a sequence λ_n, obtained from the eigenvalues of the linearized problem. For details see [3].

A similar problem, involving the p-Laplacian operator was studied by Takeuchi and Yamada in 2000. They consider the problem

$$\begin{cases} u_t = \lambda(|u_x|^{p-2}u_x)_x + |u|^{q-2}u(1-|u|^r), & (x,t) \in (0,1) \times (0,+\infty) \\ u(0,t) = u(1,t) = 0, & 0 \le t < +\infty \\ u(x,0) = u_0(x), & x \in (0,1), \end{cases} \tag{2}$$

where $p > 2$, $q \ge 2$, $r > 0$ and $\lambda > 0$. In this case the set of equilibrium points, E_λ, is always infinity if $p > q$ and, if $p = q$ or $p < q$, E_λ is a finite set only for large values of λ. However, in each of the three cases, there is the possibility of the existence of continuum equilibrium sets, which does not happen in the semilinear case, $p = 2$. Notice that problem (1) can be seen as a limit problem of (2) taking $p = q = r = 2$.

If we consider only the case $p = q$ in (2), there are several similarities between this problem and (1). In fact, although there is the possibility of bifurcation of a continuum equilibrium set in (2), the numbers of connected components of E_λ is always finite for fixed values of λ, and the scheme of bifurcation of this components is the same of (1). The stability properties of equilibria are the same, that is, in both cases the trivial solution is asymptotically stable for large values of the diffusion parameter λ and became unstable when appears the first pair of nontrivial stationary solutions, which are asymptotically stable as long as they exist. Any other stationary solution is unstable, for any p and q.

It is well known that problems (1) and (2) are globally well-posed in $L^2(0,1)$ and there is a global attractor $\{\mathcal{A}_2\}$ for (1), [3, 10, 5, 7]. The existence of a global attractor $\{\mathcal{A}_p\}$ for (2) is easily obtained from the uniform estimates that we will obtain in this work. Furthermore, the problems (1) and (2) generate gradient systems in $C_0^1([0,1])$ and $W_0^{1,p}(0,1)$ respectively and therefore, the global attractors are characterized as the union of the unstable set of equilibrium points, [5, 11]. Another interesting similarity we can point out is that, in both problems, the lap-number does not increase through orbits, if the initial conditions are continuous. With this information we can determinate which equilibrium points can belong to the ω-limit set of any initial data. The non-increasing property of lap-number was obtained for (1) and (2) by Matano in 1982 and by Gentile and Bruschi in 2005 respectively, [8, 4].

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With all of this it is interesting to investigate in which way the parameter $p \geq 2$ affects the dynamic of (2), analyzing the continuity properties of the flows and the global attractors $\mathcal{A}_p$, with respect to parameter $p \geq 2$.

In this work we obtain some uniform estimates, with respect to parameter $p \geq 2$, for solutions of (2) on $L^2(0,1)$ and $W_0^{1,p}(0,1)$, then we will use these uniform estimates to prove the following compactness result:

Theorem 1. *The set $M^p := \{u_p\,;\ p \in (2,3],\ u_p$ is a solution of $(2)\}$, is relatively compact in $C([0,T]; L^2(0,1))$.*

With help of this result and the uniform estimates we can prove the continuity of the flows in $C([0,T] : L^2(0,1))$ for each $T > 0$ and finally we can prove the following result:

Theorem 2. *The family of global attractors*

$$\{\mathcal{A}_p \subset L^2(\Omega);\ 2 \leq p \leq 3\}$$

of the problem (2) is upper semicontinuity in $p = 2$, on $L^2(0,1)$ topology.

The proofs of the results presented here can be found in [2].

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EXISTENCE OF POSITIVE SOLUTIONS FOR THE p-LAPLACIAN WITH DEPENDENCE ON THE GRADIENT

H. BUENO [*], G. ERCOLE [†], W. FERREIRA [‡] & A. ZUMPANO [§]

In this paper we consider the Dirichlet problem

$$\begin{cases} -\Delta_p u &= \omega(x) f(u, |\nabla u|) \quad \text{in } \Omega, \\ u &= 0 \quad \text{on } \partial\Omega \end{cases} \tag{0.1}$$

where $\Omega \subset \mathbb{R}^N$ $(N > 1)$ is a smooth, bounded domain, $\Delta_p u = \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is the p-Laplacian, $1 < p < +\infty$, $\omega\colon \overline{\Omega} \to \mathbb{R}$ is a continuous, nonnegative function with isolated zeros (which we will call *weight function*) and the C^1-nonlinearity $f\colon [0, \infty) \times \mathbb{R}^N \to [0, \infty)$ satisfy simple hypotheses.

Adapting methods and techniques developed in Ercole and Zumpano [?], where the nonlinearity f does not depend on $|\nabla u|$, we start by obtaining radial, positive solutions for the problem

$$\begin{cases} -\Delta_p u &= \omega_R f(u, |\nabla u|) \quad \text{in } B_R, \\ u &= 0 \quad \text{on } \partial B_R, \end{cases} \tag{0.2}$$

where B_R is the ball with radius R centered at x_0 and ω_R a *radial* weight function. For this, no asymptotic behavior on f is assumed but, instead, simple local hypotheses on the nonlinearity f. (See hypothesis (H2) in the sequence.) The application of the Schauder Fixed Point Theorem yields a radial solution u of (**??**).

When the nonlinearity f does not depend on the gradient, the same technique was generalized in Bueno, Ercole and Zumpano [?] to smooth, bounded domains $\Omega \subset \mathbb{R}^N$. However, if Ω is not a ball, the dependence of f on $|\nabla u|$ prevents controlling $\|\nabla u\|_\infty$ in Ω and thus the application of Schauder's Fixed Point Theorem. To cope with the general case of a smooth, bounded domain Ω, we apply the method of sub- and super-solution as developed in [?], which imposes an assumption on f related to the growth of $|\nabla u|$:

(H1) $f(u, v) \le C(|u|)(1 + |v|^p)$ for all (x, u, v), where $C\colon [0, \infty) \to [0, \infty)$ is increasing.

Since the same assumption is also related to the regularity of a weak solution, hypotheses like (H1) are found in papers that do not apply the sub- and super-solution method (see Boccardo, Murat and Puel [?] and Ruiz [?]).

By considering a ball $B_\rho \subset \Omega$, radial symmetrization of the weight function ω permits us to consider a problem in the radial form (**??**) in the subdomain B_ρ. As a consequence of our study, (**??**) has a radial solution u_ρ defined in $\overline{B_\rho}$. Defining the extension $\underline{u}$ of u_ρ by $\underline{u}(x) = 0$, if $x \in \Omega \setminus \overline{B_\rho}$, we prove that $\underline{u}$ is a sub-solution of problem (**??**).

A super-solution of (**??**) turns out to be a consequence of our hypothesis on the nonlinearity f. As we said before, no asymptotic behavior is assumed on f but *local* hypotheses of the type

(H2) $\{0 \le f(u, |v|) \le bM^{p-1}, \text{ if } 0 \le u \le M, \ |v| \le \gamma M\}$

(H3) $\{a\delta^{p-1} \le f(u, |v|), \text{ if } \delta < u < M, \ |v| \le \gamma M\}$,

where δ, M, a, b and γ are constants defined in the paper. This type of hypothesis will be considered in different scenarios: always with Dirichlet boundary data, solving the radial problem (**??**) and solving problem (**??**).

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In order to obtain a super-solution for (**??**), we consider the problem (of the torsional creep type)

$$\begin{cases} -\Delta_p \phi_{\Omega_2} &= \omega \quad \text{in } \Omega_2, \\ \phi_{\Omega_2} &= 0 \quad \text{on } \partial\Omega_2, \end{cases} \tag{0.3}$$

where Ω_2 is a smooth domain such that $\Omega \subset \Omega_2$. If $\|\cdot\|$ denotes the sup-norm, it turned out that we had to control the quotient

$$\frac{\|\nabla \phi_{\Omega_2}\|}{\|\phi_{\Omega_2}\|} \tag{0.4}$$

in order to obtain a super-solution for problem (**??**).

Choosing $\Omega_2 = B_R$ (a ball such that $\Omega \subset B_R$) and denoting the solution of (**??**) by ϕ_R, that is

$$\begin{cases} -\Delta_p \phi_R &= \omega_R \quad \text{in } B_R, \\ \phi_R &= 0 \quad \text{on } \partial B_R, \end{cases} \tag{0.5}$$

(the notation of (**??**) is applied here), a super-solution for (**??**) will depend on ϕ_R and M and is a consequence of radial symmetrization of the weight function ω. The quotient (**??**) is controlled thanks to the radial symmetry of ϕ_R.

In the special case $\omega \equiv 1$, a second super-solution is produced by solving

$$\begin{cases} -\Delta_p \phi_1 &= 1 \quad \text{in } \Omega, \\ \phi_1 &= 0 \quad \text{on } \partial\Omega. \end{cases} \tag{0.6}$$

To control the quotient (**??**) – that is, to control $\frac{\|\nabla \phi_1\|}{\|\phi_1\|}$ – we supposed Ω to be convex and applied regularization methods (as in Kawohl [**?**] and Sakagushi [**?**]) and results of Payne and Philippin [**?**].

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Existence of positive solution for a quasilinear problem depending on the gradient*

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We prove the existence of a positive solution for the quasilinear Dirichlet problem

$$\begin{cases} -\Delta_p u &= f(x,u,\nabla u) \quad \text{in } \Omega \\ \quad u &= 0 \qquad\qquad\quad \text{on } \partial\Omega, \end{cases} \tag{0.1}$$

where $\Delta_p u := \text{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is the p-Laplacian operator for $p > 1$, f is a nonnegative, continuous function satisfying simple hypotheses and $\Omega \subset \mathbb{R}^N$ is a bounded, smooth domain.

In general, this problem is not suitable for variational techniques and thus topological methods (as fixed-point or degree results) and/or blow-up arguments are normally employed to solve it ([5, 7]).

In the case of the Laplacian (i.e., $p = 2$) an interesting combination of variational and topological techniques (precisely, a combination of the mountain pass geometry with the contraction lemma) was first used in [4] and has motivated some works (e.g., [2]).

Our proof of existence of a positive solution for (0.1) is a immediate consequence of the sub- and super-solution method for quasilinear equations involving dependence on the gradient ([1, 6]).

Let $\omega \neq 0$ be a continuous, nonnegative function and λ_1 be the first eigenvalue of the Dirichlet problem for $-\Delta_p$ with weight ω in the domain Ω, that is, λ_1 is the least positive number such that

$$\begin{cases} -\Delta_p u &= \lambda_1 \omega u^{p-1} \quad \text{in } \Omega \\ \quad u &= 0 \qquad\qquad \text{on } \partial\Omega, \end{cases} \tag{0.2}$$

for some $u \in W_0^{1,p}(\Omega)$, $u > 0$ in Ω. Our assumptions on the nonlinearity f depend on the chosen weight function ω.

The first one is standard and also presented in [4]: near $u = 0^+$ the values of the nonlinearity $f(x,u,v)$ must be greater than $\lambda_1 u^{p-1}\omega(x)$. We show that for ϵ small enough this assumption produces a positive sub-solution u_ϵ of (0.1) such that $\|u_\epsilon\|_\infty = \epsilon$. (This is a well known fact, if the nonlinearity f depends only on (x,u). In the case of dependence on $(x,u,\nabla u)$, this fact was overlooked in previous papers.)

The second assumption is that f, restricted to a suitable compact set, is bounded from above by a special multiple of the weight ω. This approach follows [3], where (0.1) was also independent of the gradient. We show that this hypothesis produces a super-solution U for (0.1), with $u_\epsilon \leq U$ for ϵ small enough.

The third and last assumption on f is related to the growth of $|\nabla u|$. It is merely technical and can be chosen as any hypothesis that guarantees the existence of a solution of (0.1) from an ordered sub- and super-solution pair. We have taken for granted the growth condition stated in [1].

1 The main result

For a chosen (continuous) weight function $\omega \neq 0$, let $\phi \in C^{1,\alpha}(\bar{\Omega}) \cap W_0^{1,p}(\Omega)$ be such that $-\Delta_p \phi = \omega$ in Ω. It is well known that $\phi > 0$ in Ω.

By setting $\alpha = (\|\phi\|_\infty)^{-(p-1)}$, $\mu = \frac{\|\nabla\phi\|_\infty}{\|\phi\|_\infty}$ and $B_{\mu M} := \left\{v \in \mathbb{R}^N : |v| \leq \mu M\right\}$ we assume that the *continuous* nonlinearity f satisfies, for some arbitrary positive constant M:

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$(H1)$ $0 \leq f(x,u,v) \leq \alpha\omega(x)M^{p-1}, \qquad (x,u,v) \in \overline{\Omega} \times [0,M] \times B_{\mu M}$

$(H2)$ $\lim\limits_{u \to 0^+} \dfrac{f(x,u,v)}{u^{p-1}} \geq \lambda_1 \omega(x), \quad (x,v) \in \overline{\Omega} \times B_{\mu M} \quad$ (uniformly)

$(H3)$ $f(x,u,v) \leq C(|u|)(1+|v|^p)$ for all (x,u,v), where $C\colon [0,\infty) \to [0,\infty)$ is increasing.

In Figure 1, hypotheses $(H1)$ and $(H2)$ are interpreted in a particular situation.

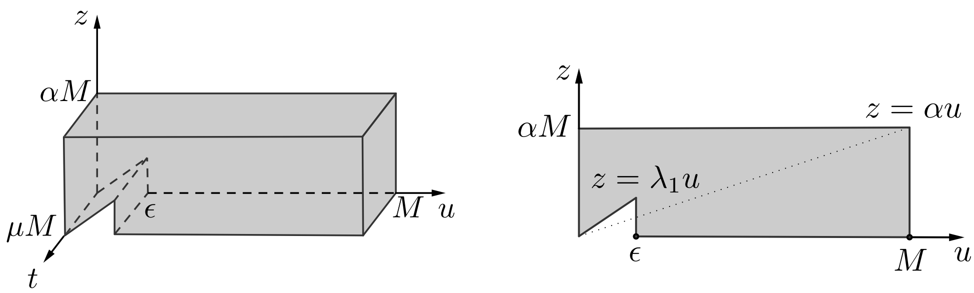

Figure 1: For $f(x,u,v) = \omega(x)g(u,|v|)$ and $p = 2$: (a) the graph of $g(u,t)$ passes through a "box with a small step" in its floor; (b) for each $c \in [0,\mu M]$, the graph of $g(u,c)$ passes through the gray area.

We state the main result, where u_1 denotes the positive first eigenfunction of the Dirichlet problem (0.2), normalized such that $\|u_1\|_\infty = 1$:

Theorem 1.1. *If the nonlinearity f satisfies $(H1)-(H3)$, the Dirichlet problem* (0.1) *has at least one positive solution $u \in C^{1,\alpha}(\bar{\Omega}) \cap W_0^{1,p}(\Omega)$ satisfying the bounds*

$$0 < \epsilon u_1 \leq u \leq \frac{M\phi}{\|\phi\|_\infty} \quad , \tag{1.3}$$

for all $\epsilon > 0$ sufficiently small.

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EXISTENCE RESULTS FOR THE KLEIN-GORDON-MAXWELL EQUATIONS IN HIGHER DIMENSIONS WITH CRITICAL EXPONENTS

P. C. CARRIÃO *, P. L. CUNHA † & O. H. MIYAGAKI ‡

This article concerns the existence of solutions for the Klein-Gordon-Maxwell ($\mathcal{KGM}$) system in $\mathbb{R}^N$ with critical Sobolev exponents

$$-\Delta u + [m_0^2 - (\omega + \phi)^2]u = \mu|u|^{q-2}u + |u|^{2^*-2}u \quad \texttt{in } \mathbb{R}^N, \tag{0.1}$$

$$\Delta\phi = (\omega + \phi)u^2 \quad \texttt{in } \mathbb{R}^N \tag{0.2}$$

where $2 < q < 2^* = 2N/(N-2)$, $\mu > 0$, $m_0 > 0$ and $\omega \neq 0$ are real constants and also $u, \phi : \mathbb{R}^N \to \mathbb{R}$.

Such system has been first introduced by Benci and Fortunato [1] as a model which describes nonlinear Klein-Gordon fields in three-dimensional space interacting with the eletromagnetic field. Further, in the quoted paper [2] they proved existence of solitary waves of the couplement Klein-Gordon-Maxwell equations when the nonlinearity has subcritical behaviour.

Some recent works have treated this problem still in the subcritical case and we cite a couple of them.

D'Aprile and Mugnai [4] estabilished the existence of infinitely many radially symmetric solutions for the subcritical ($\mathcal{KGM}$) system in $\mathbb{R}^3$. They extended the interval of definition of the power in the nonlinearity exhibited in [2]. For related works, see [8] and [10].

Non-existence results and a treatment of the ($\mathcal{KGM}$) system in bounded domains can be found in ([3], [5], [6], [7] and references therein).

With this *Ansatz* Cassani [3] proved the existence of nontrivial radially symmetric solutions in $\mathbb{R}^3$ for the critical case. He was able to show that

- if $|m_0| > |\omega|$ and $4 < q < 2^*$, then for each $\mu > 0$ there exists at least a radially symmetric solution for system (0.1)-(0.2).
- if $|m_0| > |\omega|$ and $q = 4$, then system (0.1)-(0.2) also has at least a radially symmetric solution by supposing μ sufficiently large.

The goal of this paper is to complement Theorem 1.2 from Cassani in [3] and also extend it in higher dimensions.

1 Mathematical Results

Theorem 1.1. *Assume either $|m_0| > |\omega|$ and $4 \leq q < 2^*$ or $|m_0|\sqrt{q-2} > |\omega|\sqrt{2}$ and $2 < q < 4$.*

Then system (0.1)-(0.2) has at least one radially symmetric (nontrivial) solution (u, ϕ) with $u \in H^1(\mathbb{R}^N)$ and $\phi \in \mathcal{D}^{1,2}(\mathbb{R}^N)$ provided that

i) $N \geq 6$, $N = 5$ for $2 < q < \frac{8}{3}$, $N = 4$ and $N = 3$ for $4 < q < 2^*$, if $\mu > 0$

ii) $N = 5$ for $\frac{8}{3} \leq q < 2^*$ and $N = 3$ for $2 < q \leq 4$, if μ is sufficiently large.

Proof In order to get this result we will explore the Brézis and Nirenberg technique and some of its variants. See e.g. [9]. □

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Campos Quadráticos no Plano com ligações de selas em linha reta.

P. C. Carrião *, M.E.S. Gomes † & A.A. Gaspar Ruas ‡

Apresentamos uma classificação dos retratos de fase dos campos de vetores quadráticos no plano que possuem uma reta invariante contendo duas selas e as singularidades finitas formam um quadrilátero.

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Perturbation Theory for Second Order Evolution Equation in discrete time

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This work deals with the existence, uniqueness and stability of solutions for semilinear discrete second order evolution equations in Banach spaces by using recent characterization of well-posedness for second order evolution equations in terms of $R-$boundedness and ℓ_p-multipliers.

1 Introduction

Let X be a Banach space and let A be a bounded linear operator. In Ref. [1], Castro et al. have characterized the well-posedness in weighted spaces $\ell_p^r(\mathbb{Z}_+;X) := \{(x_n) : (r^{-n}x_n) \in \ell_p(\mathbb{Z}_+;X)\}$ $(r > 0)$ for the following discrete second order evolution equation:

$$\Delta^2 u_n - Au_n = f_n, \quad n \in \mathbb{Z}_+, \tag{1.1}$$

with zero initial condition, $f \in \ell_p^r(\mathbb{Z}_+;X)$ and $A \in \mathcal{B}(X)$. The well-posedness of equation (1.1) in weighted spaces $\ell_p^r(\mathbb{Z}_+;X)$ is equivalent to the well-posedness of the evolution equation

$$\Delta_r^2 x_n - r^2 Ax_n = f_n, \quad \text{for all } n \in \mathbb{Z}_+, \tag{1.2}$$

in the usual spaces $\ell_p(\mathbb{Z}_+;X)$. The authors shown how $R-$boundedness properties of the resolvent operator A and ℓ_p-multipliers can be used to obtain a characterization of well-posedness of equation (1.2) in UMD spaces, i.e. Banach spaces with unconditional martingale difference . The probabilistic characterization of UMD turns out to be equivalent to the L^p-boundedness of the Hilbert transform. Classical theorems on L^p-multiplier are no longer valid for operator-valued functions unless the underlying space is isomorphic to a Hilbert space. However, recent work of Clément et al. [2], Weis [6, 7] and Clément and Prüss [3], show that the right notion in this context is $R-$boundedness of the sets operators.

In this article, we are concerned with the study of existence and stability for the semilinear evolution problem

$$\Delta_r^2 x_n - r^2 Ax_n = G(n, x_n, \Delta_r x_n), \quad \text{for all } n \in \mathbb{Z}_+, \quad x_0 = x_1 = 0, \tag{1.3}$$

this is accomplished by using the well-posedness properties of the vector-valued evolution equation (1.2). A motivation for this study is the recent articles by Cuevas and Lizama Ref. [4] and Cuevas and De Souza Ref. [5], where the authors have treated discrete semilinear problems for certain evolution equations of second order. We generalize several results presented in the previous papers as qualitatively as extending to a more general class of equations.

2 Results

We make the following assumption:
Assumption (A): Suppose that the following conditions hold:

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(i) The function $G : \mathbb{Z}_+ \times X \times X \longrightarrow X$ satisfies a Lipschitz condition in $X \times X$, i.e. for all $z, w \in X \times X$ and $n \in \mathbb{Z}_+$, we get $||G(n,z) - G(n,w)||_X \leq \rho_n ||z - w||_{X\times X}$, where $\rho := (\rho_n)$ is a positive sequence such that $\sum_{n=0}^{\infty} \rho_n \gamma(r,n) < +\infty$.
(ii) $G(\bullet, 0, 0) \in l_1(\mathbb{Z}_+; X)$.

Teorema 2.1. *Assume that $\alpha = 1 + \sqrt{2}$, $r \geq r_0$, $1/(1+\sqrt{2}) < r_0$. Let X be a UMD space and $T \in \mathcal{B}(X)$ be an analytic operator and assume that the following conditions hold:*
(a) Assumption (A) is fulfilled.
(b) The set $\mathcal{M} := \{(z-r)^2((z-r)^2 - r^2(I-T))^{-1} : \ |z| = \alpha r, \ z \neq \alpha r\}$ is R-bounded.
Then, there is an unique solution $x = (x_n)$ of equation (1.3) such that $(\Delta_r^2 x_n), ((I-T)x_n)$ belong to $\ell_p(\mathbb{Z}_+; X)$. Moreover, one has the following a priori estimates for the solution:

$$\sup_{n\in\mathbb{Z}_+} [\frac{1}{\gamma(r,n)} \big(||x_n||_X + ||\Delta_r x_n||_X\big)] \leq ||G(\bullet,0,0)||_1 ||\rho_\bullet \gamma(r,\bullet)||_1^{-1} e^{2A(r)}, \tag{2.4}$$

and

$$||\Delta_r^2 x||_p + ||r^2(I-T)x||_p \leq C(r)||G(\bullet,0,0)||_1 e^{2A(r)}, \quad 1 < p < +\infty, \tag{2.5}$$

where $C(r)$ is a constant depending on r and $A(r) := 2(1 + 2||\mathcal{K}^r||_{\mathcal{B}(l_p(\mathbb{Z}_+;X))})\Theta(r)||\rho_\bullet \gamma(r,\bullet)||_1$ and

$$\Theta(r) = \begin{cases} \dfrac{2-r}{(1-r)^{2-\frac{1}{p}}}, & \text{for } r < 1, \\ \dfrac{r}{(r-1)^{2-\frac{1}{p}}}, & \text{for } r > 1, \\ 1, & \text{for } r = 1, \end{cases} \tag{2.6}$$

Teorema 2.2. *Let X be a UMD space. Assume that $\alpha = 1+\sqrt{2}$, $r \geq r_0$, $1/(1+\sqrt{2}) < r_0$ and (A) hold. In addition, suppose that $T \in \mathcal{B}(X)$ is an analytic operator with $1 \in \rho(T)$ and the set $\mathcal{M}$ in (b) of Theorem 2.1 is $R-$bounded. Then, the system (1.3) is stable, that is the solution $x = (x_n)$ of (1.3) is such that $x_n \to 0$ as $n \to \infty$.*

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RELAÇÃO ENTRE HOMOTOPIA MONOTÔNICA DE TRAJETÓRIAS DE UM SISTEMA DE YOUNG E CONJUGAÇÃO DE SISTEMAS DE YOUNG

P. J. CATUOGNO * & M. G. O. VIEIRA †

O conceito de homotopia monotônica de trajetórias foi explorado por F. Colonius, E. Kizil e L. A. San Martin [2] no contexto de sistemas de controles associados a equações diferenciais ordinárias. No presente trabalho, introduzimos a noção de homotopia monotônica de trajetórias no contexto de sistemas de Young e obtemos resultados que permitem relacionar homotopia monotônica e conjugação de sistemas de Young.

Considere $T > 0$. Denotamos o intervalo $[0, T]$ por J e dado um espaço de Banach E, denotamos o conjunto de todos os caminhos contínuos de J em E por $C(J, E)$.

Definição 0.1. Sejam $p \in [1, \infty)$ e E um espaço de Banach. A *p-variação* de um caminho $X\colon J \to E$ é definida por

$$\|X\|_{p,J} = (\sup_{D\in\mathcal{P}(J)} \sum_{t_i\in D} \left\|X_{t_{i+1}} - X_{t_i}\right\|^p)^{\frac{1}{p}} \tag{0.1}$$

onde $\mathcal{P}(J)$ denota o conjunto de todas as partições $D = \{0 = t_0 < ... < t_{k-1} < t_k = T\}$ do intervalo J.

Dado um espaço de Banach E, o subconjunto de $C(J, E)$ constituído de todos os caminhos contínuos de J em E com p-variação finita é denotado por $\mathcal{V}^p(J, E)$ e tal subconjunto é um espaço de Banach quando munido com a *norma da p-variação* dada por

$$\|X\|_{\mathcal{V}^p(J,E)} = \|X\|_{p,J} + \sup_{t\in J} \|X_t\|\,. \tag{0.2}$$

Definição 0.2. Sejam E_1 e E_2 espaços de Banach, $1 \le p < \gamma$, $f \in Lip^\gamma(E_2, \mathcal{L}(E_1, E_2))$, $\Delta \subset \mathcal{V}^p(J, E_1)$ um conjunto fechado por reparametrizações positivas e por concatenações e $M \subset E_2$. Dizemos que a lista

$$(f, \Delta, M) \tag{0.3}$$

é um *sistema de Young com p-variação* para o qual J é *o intervalo de definição*, E_1 e E_2 são os *espaços associados*, f é o *campo*, Δ é o *conjunto dos controles de integração* e M é o *espaço de estados*.

Definição 0.3. Seja $\Sigma = (f, \Delta, M)$ um sistema de Young com p-variação, definido no intervalo J e com E_1 e E_2 espaços associados. Os elementos do conjunto

$$T(\Sigma) = \{\alpha \in \mathcal{V}^p(J, E_2) : \alpha_{\cdot} = u + \textstyle\int_0^{\cdot} f(\alpha_s)\, dX_s,\ X \in \Delta \ \ e\ \ \alpha(J) \subset M\} \tag{0.4}$$

são chamados de *trajetórias do sistema de Young* Σ.

Dado $\Sigma = (f, \Delta, M)$ um sistema de Young com p-variação, definido no intervalo J e com E_1 e E_2 espaços associados, denotamos uma trajetória $\alpha \in T(\Sigma)$ por $I^u_\Sigma(X)$ para indicar que $\alpha_t = u + \int_0^t f(\alpha_s)\, dX_s$, para algum $X \in \Delta$ e para todo $t \in J$. Dados $u, v \in M$, o conjunto das trajetórias do sistema Σ que iniciam em u e terminam em v é denotado por $T(\Sigma, u, v)$ e tal conjunto será munido com a topologia $\mathcal{T}^p$ induzida pela norma da p-variação em $\mathcal{V}^p(J, E_2)$.

Definição 0.4. Sejam $p \in [1, \infty)$, Σ um sistema Young com p-variação e $\alpha, \beta \in T(\Sigma, u, v)$. Dizemos que α *é p-monotonicamente homotópica a* β, com respeito ao sistema Σ, se existe um caminho contínuo $H\colon [0, 1] \to T(\Sigma, u, v)$ em relação a topologia $\mathcal{T}^p$ tal que $H(0) = \alpha$ e $H(1) = \beta$. O caminho H é chamado *homotopia p-monotônica*.

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Usamos a notação $\alpha \simeq_p \beta$ para dizer que α é p-monotonicamente homotópica a β e a notação $H\colon \alpha \simeq_p \beta$ para indicar que H é a homotopia p-monotônica entre α e β.

Proposição 0.1. *Sejam $p \in [1,\infty)$, Σ um sistema de Young com p-variação e $\alpha, \beta \in T(\Sigma, u, v)$. Se $\alpha \simeq_p \beta$ (com respeito a Σ), então existe uma aplicação contínua $L\colon J \times [0,1] \to M$ satisfazendo:*

i) $L(t,0) = \alpha_t$ e $L(t,1) = \beta_t$, para todo $t \in J$.
ii) $L^s \in T(\Sigma, u, v)$, para todo $s \in [0,1]$, onde $L^s(t) = L(t,s)$.

Decorre da proposição acima que se $\alpha \simeq_p \beta$, então $\alpha \simeq \beta$, onde $\simeq$ denota a relação de equivalência dada pela homotopia clássica entre caminhos contínuos. Entretanto, existem sistemas de Young Σ, com $\alpha, \beta \in T(\Sigma, u, v)$, nos quais $\alpha \simeq \beta$ e não existe $H\colon [0,1] \to T(\Sigma, u, v)$ tal que $H\colon \alpha \simeq_p \beta$.

No estudo dos sistemas de Young um problema que pode ser considerado é o de comparar dois sistemas, identificando-os se eles tiverem as mesmas propriedades essenciais de estrutura. Nos sistemas de Young as trajetórias são os elementos mais relevantes, portanto, é de se esperar que qualquer noção de equivalência entre sistemas de Young preserve de alguma forma as trajetórias.

Definição 0.5. Sejam $\Sigma = (f, \Delta, M)$ e $\Sigma' = (g, \Delta, N)$ dois sistemas de Young com p-variação e definidos no intervalo J. Dizemos que Σ é *topologicamente conjugado* a Σ' se existe um homeomorfismo $h\colon M \to N$, chamado de *conjugação*, tal que

$$h(I^u_\Sigma(X)(t)) = I^{h(u)}_{\Sigma'}(X)(t) \tag{0.5}$$

para todo $(t, u, X) \in J \times M \times \Delta$.

A seguir apresentamos o principal resultado deste trabalho.

Teorema 0.1. *Sejam $L\colon J \times [0,1] \to M$ uma aplicação contínua e $\Sigma = (f, \Delta, M)$ um sistema de Young com p-variação, definido no intervalo J e com E_1 e E_2 espaços associados. Se $H(s) \in T(\Sigma, u, v)$, para todo $s \in [0,1]$, onde*

$$H(s)(t) = L(t,s) \tag{0.6}$$

então as seguintes afirmações são válidas:

i) Existe uma sequência $\{H^n\}_{n\in\mathbb{N}} \subset C([0,1], \mathcal{V}^1(J, E_2))$ equicontínua tal que $\lim_{n\to\infty} \|H^n(s) - H(s)\|_{\mathcal{V}^q(J,E_2)} = 0$, para todo $s \in [0,1]$ e para todo $q > p$.
ii) $H : [0,1] \to T(\Sigma, u, v)$ é uma homotopia q-monotônica, para todo $q > p$.
iii) Se $\sup_{s\in[0,1]} \|H(s)\|_{p,J} < \infty$ então $H : [0,1] \to T(\Sigma, u, v)$ é uma homotopia p-monotônica.

Com o teorema acima obtemos o próximo resultado, o qual permite relacionar a noção de homotopia monotônica de trajetórias de um sistema de Young com a noção conjugação entre dois sistemas de Young.

Teorema 0.2. *Sejam Σ e Σ' sistemas de Young com p-variação e $\alpha, \beta \in T(\Sigma, u, v)$. Se $\alpha \simeq_p \beta$ e h é uma conjugação entre Σ e Σ' então $h \circ \alpha \simeq_q h \circ \beta$, para todo $q > p$.*

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EXISTENCE OF SOLUTIONS IN WEIGHTED SOBOLEV SPACES FOR DIRICHLET PROBLEM OF SOME DEGENERATE SEMILINEAR ELLIPTIC EQUATIONS

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In this paper we prove the existence of (weak) solutions in the weighted Sobolev spaces $W_0^{1,2}(\Omega,\omega)$ for the Dirichlet problem

$$(P)\begin{cases} Lu(x)-\mu\, u(x)g(x)=-f(x,u(x)), & \text{on } \Omega \\ u(x)=0, & \text{on } \partial\Omega \end{cases}$$

where L is the partial differential operator

$$Lu(x)=-\sum_{i,j=1}^{n} D_j(a_{ij}(x)D_iu(x)), \tag{0.1}$$

with $D_j=\partial/\partial x_j$ $(j=1,...,n)$, Ω is a bounded open set in $\mathbb{R}^n$, where the coefficients a_{ij} are measurable, real-valued functions whose coefficient matrix $\mathcal{A}=(a_{ij})$ is symmetric and satisfies the degenerate elliptic condition

$$\lambda|\xi|^2\omega(x)\leq \sum_{i,j=1}^{n} a_{ij}(x)\,\xi_i\,\xi_j\leq \Lambda|\xi|^2\omega(x), \tag{0.2}$$

for all $\xi\in\mathbb{R}^n$ and a.e. $x\in\Omega$, ω is weight functions (i.e., locally integrable, nonnegative function on $\mathbb{R}^n$) and $\mu\in\mathbb{R}$.

Let ω be a locally integrable nonnegative function in $\mathbb{R}^n$ and assume that $0<\omega<\infty$ almost everywhere. We say that ω belongs to the Muckenhoupt class A_p, $1<p<\infty$, or that ω is an A_p-weight, if there is a constant $C=C_{p,\omega}$ such that

$$\left(\frac{1}{|B|}\int_B \omega(x)dx\right)\left(\frac{1}{|B|}\int_B \omega^{1/(1-p)}(x)dx\right)^{p-1}\leq C$$

for all balls $B\subset\mathbb{R}^n$, where $|.|$ denotes the n-dimensional Lebesgue measure in $\mathbb{R}^n$.

Theorem 1 *Let ω be a weight function, $\omega\in A_2$. Suppose that*
(H1) *The function $f:\Omega\times\mathbb{R}\rightarrow\mathbb{R}$ satisfies the Carathéodory condition, that is, $x\mapsto f(x,t)$ is on Ω for all $t\in\mathbb{R}$ and $t\mapsto f(x,t)$ is continuous on $\mathbb{R}$ for almost all $x\in\Omega$;*
(H2) *There exist two nonnegative functions $g_1\in L^2(\Omega,\omega)$ and $g_2\in L^\infty(\Omega)$ such that $|f(x,t)|\leq g_1(x)+g_2(x)|t|$;*
(H3) *The function $t\mapsto f(x,t)$ is monotone increasing on $\mathbb{R}$ for all $x\in\Omega$;*
(H4) $|g(x)|\leq C_1\,\omega(x)$ *a.e.* $x\in\Omega$.
Then the problem (P) has exactly one solution $u\in W_0^{1,2}(\Omega,\omega)$. Moreover, if $g_1\in L^2(\Omega,\omega)\cap L^2(\Omega,\omega^{-1})$ and $g_2/\omega\in L^\infty(\Omega)$, then

$$\|u\|_{W_0^{1,2}(\Omega,\omega)}\leq C_2\,\|g_1\|_{L^2(\Omega,\omega^{-1})}.$$

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Example Let $\Omega = \{(x,y)\in\mathbb{R}^2 \ : x^2+y^2<1\}$, and consider the weight $\omega(x,y)=(x^2+y^2)^{-1/2}$ $(\omega\in A_2)$, and the function $f\ :\ \Omega\times\mathbb{R}\to\mathbb{R}$, $f((x,y),t)=\dfrac{\cos(xy)}{(x^2+y^2)^{1/2}}+t\mathrm{e}^{-(x^2+y^2)}$. Let us consider the partial differential operator

$$Lu(x,y) = -\frac{\partial}{\partial x}\left(a(x^2+y^2)^{-1/2}\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(b(x^2+y^2)^{-1/2}\frac{\partial u}{\partial y}\right)$$

with $0<a<b$, $g(x,y)=\sin(xy)/(x^2+y^2)^{1/2}$, $g_1(x,y)=(x^2+y^2)^{-1/2}$ and $g_2(x,y)=\mathrm{e}^{-(x^2+y^2)}$. By Theorem 1 the problem

$$\left\{\begin{array}{l} Lu(x,y)-\mu\, u(x,y)\, g(x,y) = -f((x,y),u(x,y)), \quad \text{on} \;\; \Omega \\ u(x,y)=0, \quad \text{on} \;\; \partial\Omega \end{array}\right.$$

has a solution $u\in W_0^{1,2}(\Omega,\omega)$.

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BIFURCATION IN A MECHANICAL SYSTEM WITH RELAXATION OSCILLATIONS

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Consider the system:

$$\begin{cases} x'' + \omega_1^2 x = dy^2 + \mu\left(1-x^2\right)x', \ \mu \gg 1 \\ y'' + \omega_2^2 y = exy - kx'\left(y'\right)^3. \end{cases} \tag{0.1}$$

This equation appears as a simplified model of the action of a wind field on suspended cables. For details see [1]. The progressive damping, x^2x', has been included to ensure a limited vibration amplitude. Notice that (0.1) is an autoparametric system. Indeed, if $y=0$ there is a van der Pol oscillator with a relaxation oscillation because of the condition $\mu \gg 1$. In this note, it is shown that, under adequate conditions on the coefficients $\omega_1, \omega_2, d, e, k$, (0.1) has a periodic orbit which stability depends on some inequalities satisfied by those parameters. If not , one has unstable periodic orbit. In other words, it is obtained a bifurcation result for the system (0.1) . It would be emphasized that this bifurcation result is new in the literature in this field. See, for example, [2] and [3].

1 Preliminary Results

The system (0.1) can be rewritten as a singularly perturbed system. Hence, one gets

$$\begin{cases} x' = u, \\ y' = v, \\ v' = -\omega_2^2 y + exy - kuv^3, \\ \varepsilon u' = \varepsilon\left(-\omega_1^2 x + dy^2\right) + \left(1-x^2\right)u \end{cases} \tag{1.2}$$

where $\varepsilon = 1/\mu$. For (1.2) can be used the result of [4] on Invariant Manifolds. For that, it is necessary to assume the following inequality

$$1 - x^2 \leqslant -2\beta < 0, \tag{1.3}$$

for some $\beta > 0$. Hence there is an invariant manifold $\mathcal{M}_\varepsilon$ given by the graph of

$$u = \varepsilon\frac{\omega_1^2 x - dy^2}{1-x^2} + O\left(\varepsilon^2\right). \tag{1.4}$$

By substituting (1.4) in $(1.2)_{1,2,3}$ the following reduced system is obtained

$$\begin{cases} x' = \varepsilon\frac{\omega_1^2 x - dy^2}{1-x^2} + O\left(\varepsilon^2\right), \\ y' = v, \\ v' = -\omega_2^2 y + exy - \varepsilon k\frac{\omega_1^2 x - dy^2}{1-x^2}v^3 + O\left(\varepsilon^2\right), \end{cases} \tag{1.5}$$

Now, assume that

$$\omega_2^2 - ex > 0, \tag{1.6}$$

and define $\omega = \omega(x) = \sqrt{\omega_2^2 - ex}$. Then, (1.5) can be written as

$$\begin{cases} x' = \varepsilon\frac{\omega_1^2 x - dy^2}{1-x^2} + O\left(\varepsilon^2\right), \\ y' = v, \\ v' = -\omega^2 y - \varepsilon k\frac{\omega_1^2 x - dy^2}{1-x^2}v^3 + O\left(\varepsilon^2\right), \end{cases} \tag{1.7}$$

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Now, consider the following change of variables $y = rsin(\theta)$, $v = \omega rcos(\theta)$. Applying it at (1.7) one gets

$$\begin{cases} x' &= \varepsilon\frac{\omega_1^2 x - dr^2 sin^2(\theta)}{1-x^2} + O(\varepsilon^2), \\ r' &= \varepsilon\frac{\omega_1^2 x - dr^2 sin^2(\theta)}{1-x^2}\left(\frac{(\omega^3 r^3 cos^4(\theta))}{\omega} + \frac{ercos^2(\theta)}{2\omega^2}\right) + O(\varepsilon^2) \\ \theta' &= \omega - \varepsilon\frac{\omega_1^2 x - dr^2 sin^2(\theta)}{1-x^2}\left(\frac{\omega^3 r^3 cos^3(\theta)sin(\theta)}{\omega} + \frac{ersin(\theta)cos(\theta)}{2\omega^2}\right) + O(\varepsilon^2). \end{cases} \tag{1.8}$$

We have

$$\begin{cases} \frac{dx}{d\theta} &= \frac{\varepsilon}{\omega}\left(\frac{\omega_1^2 x - dr^2 sin^2(\theta)}{1-x^2}\right) + O(\varepsilon^2), \\ \frac{dr}{d\theta} &= \frac{\varepsilon}{\omega}\left(\frac{\omega_1^2 x - dr^2 sin^2(\theta)}{1-x^2}\right)\left(-k\omega^2 r^3 cos^4(\theta) + \frac{ercos^2(\theta)}{2\omega^2}\right) + O(\varepsilon^2) \end{cases} \tag{1.9}$$

Now, we shall use the Averaging Method in order to study the dynamics of (1.9). The averaged system is given by

$$\begin{cases} \frac{dx}{d\theta} &= \frac{\varepsilon}{2\pi}\frac{2\pi\,{\omega_1}^2\,x - \pi\,d\,r^2}{\omega\,(1-x^2)} + O(\varepsilon^2) \\ \frac{dr}{d\theta} &= \frac{\varepsilon}{2\pi}\frac{\pi\,r\left(6\,\omega^4\,k\,r^2\,{\omega_1}^2\,x - 4\,e\,{\omega_1}^2\,x - \omega^4\,d\,k\,r^4 + d\,e\,r^2\right)}{8\,\omega^3\,(x^2-1)} + O(\varepsilon^2) \end{cases} \tag{1.10}$$

2 Main Result

Using (1.10), the results on Invariant Manifolds given at [4] and taking into account the inequalities (1.3), (1.6) one obtains the following result.

Suppose that the following inequalities hold

$$\frac{\sqrt{22}}{4}\sqrt{e} < \omega_2, \quad 72\omega_2^6 < 1331e\left(\omega_2^2 - e\right)^2. \tag{2.11}$$

Let k be such that

$$k_{inf} = \max\left\{\frac{d\,e}{4\,{\omega_1}^2\,({\omega_2}^2 - e)^2}, \frac{3\,d\,e^2}{2\,{\omega_1}^2\,{\omega_2}^6}\right\} < k < \frac{1331\,d\,e^2}{288\,{\omega_1}^2\,{\omega_2}^6} = k_{sup}. \tag{2.12}$$

From (2.11), (2.12), [5], pg.301, Theorem 6.6.2 , pg.304 and [4] one obtains that (1.2) has a periodic solution. It can be concluded if k is adequately near of k_{sup}, $k < k_{sup}$ and $\omega_2 < \frac{18\,\sqrt{2}\,\sqrt{11}\,\sqrt{e}}{121-\sqrt{11}\,\sqrt{899}}$ such orbit is orbitally stable, see [5]. Suppose that $(2.11)_1$ and $k > \frac{1331\,d\,e^2}{288\,{\omega_1}^2\,{\omega_2}^6}$ hold. Thus one gets that the above periodic orbit is unstable.

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S-ASYMPTOTICALLY ω-PERIODIC AND ASYMPTOTICALLY ALMOST AUTOMORPHIC SOLUTIONS FOR A CLASS OF PARTIAL INTEGRODIFFERENTIAL EQUATIONS

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Resumo

In this work, we study the existence of S-asymptotically ω-periodic and asymptotically almost automorphic solutions for a class of partial integrodifferential equations.

1 Introduction

The study of existence of almost periodic, asymptotically almost periodic, almost automorphic, asymptotically almost automorphic, pseudo almost periodic and pseudo almost automorphic solutions is one the most attracting topics in the qualitative theory of differential equations. However, the literature concerning S-asymptotically ω-periodic functions with values in Banach spaces is very new (cf. [2,3,5]). We investigate existence of S-asymptotically ω-periodic and asymptotically almost automorphic solutions for a class of integrodifferential equations (cf. [4]).

2 Existence Results

In this work, we study the existence of S-asymptotically ω-periodic and asymptotically almost automorphic solutions for a class of abstract partial integrodifferential equations of the form:

$$u'(t) = Au(t) + \int_0^t B(t-s)u(s)ds + g(t,u(t)),\ t \geq 0, \tag{2.1}$$

$$u(0) = x_0, \tag{2.2}$$

where $A : D(A) \subset X \to X$, $B(t) : D(B(t)) \subset X \to X$, $t \geq 0$, are densely defined, closed linear operators on a Banach space X; $D(A) \subset D(B(t))$ for every $t \geq 0$ and $g(\cdot)$ is a continuous function, $x_0 \in X$.

We will assume the following condition:

(A) The resolvent operator $(R(t))_{t\geq 0}$ is uniformly exponentially stable, i.e., $\|R(t)\| \leq Me^{-\delta t}$ for all $t \geq 0$ and some constant $M, \delta > 0$.

Teorema 2.1. *Assume that* **(A)** *is fulfilled. Let $g : [0,\infty) \times X \to X$ be a continuous function such that $g(\cdot, 0)$ is integrable on $[0,\infty)$ and there is an integrable function $L : [0,\infty) \to [0,\infty)$ such that*

$$\|g(t,x) - g(t,y)\| \leq L(t)\|x - y\|, \tag{2.3}$$

for all $x, y \in X$ and $t \geq 0$. Then the problem (2.1)-(2.2) has a unique S-asymptotically ω-periodic mild solution for all $\omega > 0$.

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To establish our next result, we consider functions $g : [0, \infty) \times X \to X$ that satisfies the following boundedness condition.

(B) There exists a continuous nondecreasing function $W : [0, \infty) \to [0, \infty)$ such that $\|g(t, x)\| \le W(\|x\|)$ for all $t \in [0, \infty)$ and $x \in X$.

Teorema 2.2. *Assume that* **(A)** *is fulfilled. Let $g : [0, \infty) \times X \to X$ be an uniformly S-asymptotically ω-periodic on bounded sets and asymptotically uniformly continuous on bounded sets that satisfies assumption* (**B**)*, and the following conditions:*

(a) For each $\nu \ge 0$, $\lim_{t\to\infty} \frac{1}{h(t)} \int_0^t e^{-\delta(t-s)} W(\nu h(s))ds = 0$. We set

$$\beta(\nu) := \left\| \|R(\cdot)x_0\| + M \int_0^{\cdot} e^{-\delta(\cdot - s)} W(\nu h(s))ds \right\|_h,$$

where M is the constant given in Condition **(A)**.

(b) For each $\varepsilon > 0$ there is $r > 0$ such that for every $u, v \in C_h(X)$, $\|v - u\|_h \le r$ implies that $M \int_0^t e^{-\delta(t-s)} \|g(s, v(s)) - g(s, u(s))\|ds \le \varepsilon$, for all $t \in [0, \infty)$.

(c) For all $a, b \in [0, \infty)$, $a \le b$, and $r > 0$, the set $\{g(s, h(s)x) : a \le s \le b,\ x \in X, \|x\| \le r\}$ is relatively compact in X.

(d) $\liminf_{\xi\to\infty} \frac{\xi}{\beta(\xi)} > 1$.

Then the problem (2.1)-(2.2) has an S-asymptotically ω-periodic mild solution.

Teorema 2.3. *Let $g \in AAA([0, \infty) \times X; X)$ such that satisfies assumptions* **(B)***, (a), (b), (c) and (d) of Theorem 2.2 and the following condition:*

(a)* $g(t, \cdot)$ is uniformly continuous on bounded sets uniformly for $t \in [0, \infty)$, that is, for every $\epsilon > 0$ and every bounded subset K of X, there exists $\delta_{\epsilon,K} > 0$ such that $\|g(t, x) - g(t, y)\| \le \epsilon$ for all $t \ge 0$ and all $x, y \in K$, with $\|x - y\| \le \delta_{\epsilon,K}$.

Then there exists an asymptotically almost automorphic mild solution to Eq. (2.1)-(2.2).

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Almost automorphic and pseudo-almost automorphic solutions to semilinear evolution equations with nondense domain

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Resumo

In this work we study the existence and uniqueness of almost automorphic (resp. pseudo-almost automorphic) solutions to a first-order differential equation with linear part dominated by a Hille-Yosida type operator with non dense domain.

1 Introduction

In recent years, the theory of almost automorphic functions has been developed extensively (see, e.g., Bugajewski and N'guérékata [2] and the references therein). However, the literature concerning pseudo almost automorphic functions is very new. Recently an interesting article has appeared by Liang et al. [5] concerning the composition of pseudo almost automorphic functions. The same authors in [6] have applied the results to obtain pseudo-almost automorphic solutions to semilinear differentail equations (see also [7]).

2 Existence Results

In this work, we study the existence and uniqueness of almost automorphic and pseudo-almost automorphic solutions for a class of abstract differential equations described in the form

$$x'(t) \quad = \quad Ax(t) + f(t, x(t)),\, t \in \mathbb{R}, \tag{2.1}$$

where A is an unbounded linear operator, assumed to be Hille-Yosida of negative type, having the domain $D(A)$, not necessarily dense, on some Banach space X; and $f : \mathbb{R} \times X_0 \to X$ is a continuous function, where $X_0 = \overline{D(A)}$. The regularity of solutions for (2.1) in the space of pseudo-almost periodic solutions was considered in Cuevas and Pinto [3] (see [4]).

Teorema 2.1. *Assume that $f : \mathbb{R} \times X_0 \to X$ be an almost automorphic function in $t \in \mathbb{R}$ for each $x \in X_0$ and assume that satisfies a $L-$Lipschitz condition in $x \in X_0$ uniformly in $t \in \mathbb{R}$. If $CL < -\omega$, where $C > 0$ is the constant in Lemma 2.5 in [1], then (2.1) has a unique almost automorphic mild solution which is given by*

$$y(t) = \int_{-\infty}^{t} T_{-1}(t-s) f(s, y(s)) ds, t \in \mathbb{R}. \tag{2.2}$$

Teorema 2.2. *Assume that $f : \mathbb{R} \times X_0 \to X$ be a pseudo-almost automorphic function and that there exists a bounded integrable function $L_f : \mathbb{R} \to [0, \infty)$ satisfying*

$$||f(t, x) - f(t, y)|| \leq L_f(t)||x - y||, t \in \mathbb{R},\, x,\, y \in X_0. \tag{2.3}$$

Then (2.1) has a unique pseudo-almost automorphic (mild) solution.

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Weak and Periodical solution for the Equation of Motion of Oldroyd Fluid with Variable Viscosity

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It is well know that, the motion of an incompressible fluid is described by the system of Cauchy equations

$$\frac{\partial u}{\partial t} + u_i \frac{\partial u}{\partial x_i} + \nabla p = \text{div } \sigma + f, \quad \text{div } u = 0, \tag{0.1}$$

where $u = (u_1, ..., u_n)$ is the velocity, p is the pressure in the fluid, f is the density of external forces and σ is the deviator of the stress tensor. The Hooke's Law establishes a relationship between the stress tensor σ and the deformation tensor $D_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ and their derivatives. For example, for an incompressible Stokes fluid the relationship has the form $\sigma = \alpha D + \beta D^2$ where α and β are scalar functions. If $\alpha \equiv$ constant $\equiv 2\nu > 0$ and $\beta \equiv 0$ we have the Newton's Law $\sigma = 2\nu D$, which substituting in (0.1) we obtain the equations of motion of Newtonian fluid, which is called the Navier-Stokes equations:

$$u' - \nu\Delta u + (u.\nabla)u + \nabla p = f, \quad \text{div}\, u = 0.$$

Oldroyd proposed a model of a viscous incompressible fluid whose defining equations has the form Its follows that

$$\lambda \frac{\partial \sigma}{\partial t} + \sigma = 2\nu\left(D + k\nu^{-1}\frac{\partial D}{\partial t}\right). \tag{0.2}$$

Multiplying (0.2) by $e^{\frac{t}{\lambda}}$, integrating and assuming that $\sigma(x,0) = D(x,0) = 0$, we obtain

$$\sigma(x,t) = 2k\lambda^{-1}D(x,t) + 2\lambda^{-1}(\nu - k\lambda^{-1})\int_0^t e^{-\frac{(t-\xi)}{\lambda}} D(x,\xi)d\xi \tag{0.3}$$

where λ, ν, k are positive constants with $\nu - \frac{k}{\lambda} > 0$. Thus, substituting (0.3) in (0.1), the equation for the motion of Oldroyd fluid can be written by the system of integro-differential equations

$$\frac{\partial u}{\partial t} + (u.\nabla)\, u - \mu\Delta u - \int_0^t \beta(t-\xi)\Delta u(x,\xi)d\xi + \nabla p = f, \; x \in \Omega, \, t > 0 \tag{0.4}$$

and the incompressible condition $\text{div}\, u = 0, \; x \in \Omega, \, t > 0$, with initial and boundary conditions $u(x,0) = u_0, \; x \in \Omega$, and $u(x,t) = 0 \; x \in \Gamma, \; t \geq 0$. Here, $\mu = k\lambda^{-1} > 0$ and $\beta(t) = \gamma e^{-\delta t}$, where $\gamma = \lambda^{-1}\left(\nu - k\lambda^{-1}\right)$ with $\delta = \lambda^{-1}$. In Lions [5] we find investigation for a mixed problem for the case of the Navier-Stokes with viscosity of the type $\nu = \nu_0 + \nu_1\|u(t)\|^2$, $\nu_0 > 0$ and $\nu_1 > 0$ are positives constants.

In the present work we consider a mixed problem similar to Lions [5], adding a memory term, that is $-\int_0^t g(t-\sigma)\Delta u(\sigma)d\sigma$. More precisely, in this paper we study the mixed problem

$$\frac{\partial u}{\partial t} + (u.\nabla)\, u - (\mu_0 + \mu_1\|u\|^2)\Delta u - \int_0^t \beta(t-\xi)\Delta u(x,\xi)d\xi + \nabla p - f \tag{0.5}$$

with the incompressible condition $\text{div}\, u = 0, \; x \in \Omega, \, t > 0$ and initial and boundary conditions $u(x,0) = u_0, \; x \in \Omega$, and $u(x,t) = 0 \; x \in \Gamma, \; t \geq 0$ under standard hypothesis on f and u_0. Making use of the Galerkin's approximations, we establish existence of weak solutions. Uniqueness and periodical solutions are also analyzed. We consider

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$g : [0, \infty) \to [0, \infty)$ is a function of $W^{1,1}(0, \infty)$ such that $g(t) = \gamma e^{-\delta t}$, with $\gamma = \frac{\mu_0}{6}$ and δ positive constants.
We define the following spaces $\mathcal{V} = \{\varphi \in \mathcal{D}(\Omega)^n; \quad \operatorname{div} \varphi = 0\}$, $V = V(\Omega)$ with inner product and norm denoted respectively by $((u, z)) = \displaystyle\sum_{i,j=1}^{n} \int_\Omega \frac{\partial u_i}{\partial x_j}(x) \frac{\partial z_i}{\partial x_j}(x)\, dx$, $\|u\|^2 = \displaystyle\sum_{i,j=1}^{n} \int_\Omega \left(\frac{\partial u_i}{\partial x_j}(x)\right)^2 dx$, $H = H(\Omega)$ with inner product and norm defined, respectively, by $(u, v) = \displaystyle\sum_{i=1}^{n} \int_\Omega u_i(x) v_i(x)\, dx$, $|u|^2 = \displaystyle\sum_{i=1}^{n} \int_\Omega |u_i(x)|^2\, dx$ and V_2 with inner product and norm denoted, respectively by $((u, z))_{V_2} = \displaystyle\sum_{i=1}^{n} (u_i, v_i)_{H^2(\Omega)}$, $\|u\|^2_{V_2} = ((u, u))_{V_2}$,
We introduce the following bilinear and the trilinear form

$$a(u, v) = \sum_{i,j=1}^{n} \int_\Omega \frac{\partial u_i}{\partial x_j}(x) \frac{\partial v_i}{\partial x_j}(x)\, dx = ((u, v)) \text{ and } b(u, v, w) = \sum_{i,j=1}^{n} \int_\Omega u_i(x) \frac{\partial v_j}{\partial x_i}(x) w_j(x)\, dx.$$

Next we shall state the main results of this work.

Definition 0.1. *A weak solution to the boundary value problem (0.5) is a function $u : Q \to \mathbb{R}^n$, such that $u \in L^4(0, T; V) \cap L^\infty(0, T; H)$, for $T > 0$, satisfying the identity*

$$\left| \begin{array}{l} (u', v) + \mu a(u, v) + b(u, u, v) + \langle \mathcal{A}u, v \rangle \quad - \left(\displaystyle\int_0^t g(t - \sigma) \Delta u(\sigma) d\sigma, v \right) dt = (f, v)\, dt, \ \forall v \in V. \\ \\ u(x, 0) = u_0(x). \end{array} \right. \tag{0.6}$$

Remark 0.1. *We denote by $\mathcal{A}$ the monotonic and hemicontinous operator $\mathcal{A} : V \longrightarrow V'$, $\langle \mathcal{A}u, v \rangle = \|u\|^2 a(u, v)$ (see, for example, Lions [5], p. 218). We have that $\mathcal{A}u = -\|u\|^2 \Delta u$.*

Theorem 0.1. *If $n \leq 4, f \in L^{4/3}(0, T; V')$ and $u_0 \in V$, then there exists a function $u = u(x, t)$ defined for $(x, t) \in Q$, solution to the boundary value problem (0.5) in the sense of Definition 0.1.*

Theorem 0.2. *We suppose that $n = 2, 3$, then there exists a unique function u solution to the boundary value problem (0.5) in the sense of Definition 0.1.*

Theorem 0.3. *Under the assumptions of Theorem 0.1, there exists a function $u : Q \to \mathbb{R}^n$, solution to Problem (0.5), in the sense of Definition 0.1, such that $u(0) = u(T)$.*

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Numerical Analysis of an explicit finite element scheme for the convection-diffusion equation

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This work deals with an explicit scheme for solving time-dependent convection-diffusion problems in N-dimensional regions of arbitrary shape, for $N \geq 1$. The scheme is based on a piecewise linear finite element approximation. This is combined with a suitable weighted mass matrix, but the technique can also be applied to other types of space discretization. Convenient bounds for the time step in terms of both the weights and the mesh step size, leads to the stability of the scheme in the maximum norm, in both space and time. Convergence in the same sense is also proven to hold under the classical acute angle condition.

The time-dependent convection-diffusion equation is expressed as follows:

Find a scalar valued function $u(\mathbf{x}, t)$ defined in Ω, a bounded open subset of $\Re^N$ with boundary $\partial\Omega$, for $N = 1, 2$ or 3, for every time t, $0 < t < +\infty$, such that,

$$\left\{ \begin{array}{l} \dfrac{\partial u}{\partial t} + \mathbf{a} \cdot \nabla u - \nu \Delta u = f \ ; \ \text{in } \Omega \times (0, \infty) \\ u = g \ ; \ \text{on } \partial\Omega \times (0, \infty) \\ u = u^0 \ ; \ \text{ in } \Omega \text{ for t} = 0 \end{array} \right. \quad (0.1)$$

where ν is a positive constant and $\mathbf{a}$ is a given solenoidal velocity. The data f and g are respectively, a given forcing function belonging to $L^\infty[\Omega \times (0, \infty)]$, and a prescribed boundary value in $L^\infty[\partial\Omega \times (0, \infty)]$. We assume that $u^0 \in L^\infty(\Omega)$, and that for every $t \in (0, \infty)$, $\mathbf{a}(\cdot, t) \in L^\infty(\Omega)$.

The numerical solution of equation (0.1) is known to be delicate in the case where convection is largely dominant, that is, the case where $\nu << ||\mathbf{a}(\cdot, t)||_{\infty,\Omega}$, $\forall t$.

The efficient solution of problems combining both diffusive and advective phenomena, is a crucial step to handle numerical simulations in countless technological applications, in particular those involving fluid flow. In the framework of numerical simulations with finite elements, one of the first techniques employed to model convection was the so-called Lesaint-Raviart method (cf. [6]). However a little later relevant contributions to the time-dependent case incorporating diffusion arose, as it is well reported in [4]. In this respect we should quote the pioneer upwinding schemes due to Tabata and collaborators (see e.g. [8] and [1]).

Since the mid-eighties, the most widespread manner to deal with dominant convection in finite element codes has been the use of stabilizing procedures based on the space mesh parameter, among which the streamline upwind Petrov-Galerkin (SUPG) technique introduced by Brooks & Hughes [2] is one of the most popular. However, as far as time time-dependent problems are concerned, it turns out that the time step plays a better stabilizing role, provided a formulation well suited to the equations to be solved is employed. A good illustration of this assertion in the case of the time-dependent Navier-Stokes equations can be found in [3].

Ruas, Brasil Jr. and Trales gave in [7] a contribution in this direction, in the case where convection-diffusion equation (0.1) is discretized in space with piecewise linear finite elements, combined with a non standard explicit forward Euler scheme for the time integration, and a standard Galerkin approach. However contrary to most methods employed in the same context, this method uses a classical lumped mass matrix on the left hand side and a suitably weighted mass matrix on the right hand side. In so doing it turns out to be simple to implement and well performing, even in the case of largely dominant convection.

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A main theoretical result proven in [7] states that the numerical solution is stable in the maximum norm in both space and time, and even convergent under a classical angle condition, provided that roughly the time step is bounded by the space mesh parameter multiplied by a mesh-independent constant that we specify. As the authors should clarify, the scheme studied in this paper follows similar principles to the one long exploited by Kawahara and collaborators, for simulating convection dominated phenomena (see e.g. [5] among several other papers published by them before and later on). As shown in those papers their scheme generates stable approximations under non restrictive conditions on the time-step. However the strength of the present scheme as compared to theirs, relies on the fact that it is consistent even for non uniform meshes in any space dimension. Actually rigorous conditions are exhibited here for it to generate first order convergent approximations in the sense of the maximum norm.

An outine of this work's presentation is as follows: First we describe the type of approximation that characterizes the method under study for solving equation (0.1). More particularly we focus on the weighted manner to deal with the finite element mass matrix on the right hand side at every time step, in order to obtain a stable and consistent scheme independent of the choice of stabilizing numerical parameters. Next we recall the stability results that hold for this method in the sense of the space and time maximum norm, applying to non restrictive sets of weights. Then conditions to be satisfied by the weights, allowing for optimal error estimates derived in [7] are specified. Finally we address the main contribution of this work, that is, a straightforward and systematic way to determine a set of weights satisfying the above conditions, for each inner node of the mesh. Illustrative numerical results obtained with this methodology complete the presentation.

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O PROBLEMA DE RIEMANN PARA UM SISTEMA DE LEIS DE CONSERVAÇÃO COM DADOS ESPECIFICADOS

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Neste trabalho estamos interessados na resolução do problema de Riemann para um sistema de leis de conservação, proveniente da modelagem matemática de um escoamento trifásico (água, óleo, gás) em um meio poroso, dado por

$$\frac{\partial U(x,t)}{\partial t} + \frac{\partial F(U(x,t))}{\partial x} = 0\,, \qquad x \in I\!R \quad t > 0\,, \tag{0.1}$$

$$U(x,t=0) = \begin{cases} U_-\,, & se \quad x < 0\,, \\ U_+\,, & se \quad x > 0\,, \end{cases} \tag{0.2}$$

em que as variável dependente $U(x,t) = (s_w, s_g, s_o)$ representa as saturações das fases água, óleo e gás, respect., e $F(U) = (f_w(U), f_o(U), f_g(U))$ representa a função de fluxo do sistema. No problema específico que estamos tratando aqui o estado *à esquerda* U_- representa uma mistura do tipo gás-água a ser injetada num reservatório petrolífero com o intuito de deslocar uma mistura tipo água-óleo, inicialmente residente no mesmo, representada pelo estado *à direita* U_+.

As saturações s_w, s_o e s_g assumem valores no espaço de estados, chamado de triângulo de saturações, dado por:

$$\Delta = \{(s_w,\, s_o,\, s_g)\,|\; 0 \le s_w \le 1,\; 0 \le s_o \le 1,\; 0 \le s_g \le 1,\; s_w + s_o + s_g = 1\}.$$

As funções de fluxo fracionário do sistema (0.1) obtidas para o modelo de Corey, [2], com permeabilidades relativas quadráticas, são dadas por

$$f_j(s_w, s_o, s_g) = \frac{s_j^2/\mu_j}{\lambda(s_w, s_o, s_g)}, \quad i = w, o, g,$$

com,

$$\lambda(s_w, s_o, s_g) = \frac{s_w^2}{\mu_w} + \frac{s_o^2}{\mu_o} + \frac{s_g^2}{\mu_g}\,, \quad \text{em que} \quad \mu_w,\, \mu_o \text{ e } \mu_g \text{ são constantes.}$$

O sistema (0.1) tem a peculiaridade de possuir quatro pontos umbílicos isolados (ponto com velocidades características coincidentes e matriz jacobiana múltipla da matriz identidade), sendo que três destes pontos correspondem aos vértices do triângulo de saturações e o outro é interior ao triângulo dado por $\bar{U} = (\mu_w/\mu_t,\, \mu_o/\mu_t, \mu_g/\mu_t)$, com $\mu_t = \mu_w + \mu_o + \mu_g$.

Em Isaacson *et al* [3] foi considerado o caso em que $\mu_w \equiv \mu_o \equiv \mu_g$. Com isto o sistema (0.1) possuia uma simetria tripla com relação às medianas do triângulo das saturações fazendo com que o ponto umbílico $\bar{U}$ coincidisse com o baricentro do triângulo. Isto permitiu uma redução considerável no número de casos a serem considerados na descrição da solução do problema de Riemann. Em seguida um segundo passo foi dado em Souza [5], onde foi resolvido o problema de Riemann para dados iniciais arbitrários no triângulo das saturações, considerando $\mu_w > \mu_g \equiv \mu_o$. No caso duas simetrias foram quebradas sendo que, com relação ao caso de [3], o ponto umbílico foi deslocado ao longo de uma das medianas do triângulo de saturações numa determinada direção. Com isto o número

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de casos a serem considerados na descrição da solução aumentou substancialmente. Recentemente Azevedo *et al* [1] foi considerado o caso da quebra total da simetria com $\mu_w \neq \mu_o \neq \mu_g \neq \mu_w$, em que o ponto umbílico $\bar{U}$ não está mais sobre nenhuma das medianas do triângulo das saturações, mas para o caso particular de o estado à direita U_+ coincidir com o vértice do triângulo das saturações correspondendo a composição inicial do reservatório de apenas óleo, e o estado à esquerda U_- ao longo do lado que representa misturas bifásicas do tipo gás-água. Neste trabalho estamos considerando uma variação tanto do trabalho em [1] como em [5]. Fixamos a relação $\mu_w \equiv \mu_o > \mu_g$, em que o deslocamento do ponto umbílico $\bar{U}$ se dá ao longo de uma mediana do triângulo das saturações distinta daquela considerada em [5]. Com relação ao trabalho em [1], cujo estado à direita era fixado como um dos vértices, passamos a considerar um caso de interesse mais prático variando U_+ ao longo do lado do triângulo das saturações que representa a composição inicial do reservatório como sendo uma mistura do tipo água-óleo. Este lado é denotado por WO. O estado U_- é considerado como em [5] representando uma mistura do tipo gás-água ao longo de outro lado do triângulo das saturações, o qual é denotado por GW.

A metodologia utilizada para a construção da solução do problema de Riemann (0.1)-(0.2) consistiu, inicialmente, em descrever as curvas de Hugoniot baseadas em estados U_+ ao longo do lado WO. Genericamente provamos que estas curvas consistem do próprio lado WO conjuntamente com dois ramos de hipérboles interiores ao triângulo das saturações. Em seguida analisando os gráficos das velocidades características, bem como o gráfico da velocidade de choque, ao longo destes ramos de curvas de Hugoniot obtivemos três estados especiais U_+ determinando quatro segmentos disjuntos ao longo do lado WO inferindo sequências de ondas com velocidades rápidas distintas na solução do problema. De maneira análoga, para cada estado à direita U_+ fixado num destes quatro segmentos, obtivemos também um número finito de estados especiais U_- separando segmentos disjuntos ao longo do lado GW para os quais a estrutura da sequência de ondas de velocidades lentas são distintas. Feito isto passamos a construir a solução do problema de Riemann em si a partir das várias sequências de ondas possíveis descritas nos dois passos iniciais.

Construída a solução para todos os estados à direita U_+, ao longo do lado WO, e para todos os estados à esquerda, ao longo do lado GW, mostramos a dependência contínua da solução no sentido $\mathcal{L}^1_{Loc}$ com relação aos dados iniciais e obtivemos também que para cada estado U_+ fixado existe um único estado à esquerda U_-, chamado de estado crítico, para o qual a estratégia de injeção seja ótima, isto é, a recuperação seja máxima. Para este estado crítico mostramos que a solução do problema de Riemann pode ser descrita por até três sequências de ondas com trajetórias distintas ao longo do triângulo das saturações, mas que consistem exatamente da mesma solução no espaço físico xt. Para U_+ em dois segmentos extremos ao longo do lado WO a solução consiste de duas ondas distintas, independentemente do dado à esquerda U_-. Para U_+ nos outros dois segmentos internos ao lado WO a solução, para alguns estados à esquerda U_-, pode consistir de até três ondas, sendo a onda de velocidade intermediária não clássica no sentido de Lax, [4].

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On solitary waves for the generalized Benjamin–Ono–Zakharov–Kuznetsov equation

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We study the generalized Benjamin–Ono–Zakharov–Kuznetsov (BO–ZK) equation in two space dimensions,

$$u_t + \alpha H u_{xx} + \varepsilon u_{xyy} + u^p u_x = 0, \quad (x, y) \in \mathbb{R}^2, \ \ t \in \mathbb{R}^+.$$

Here $p > 0$ is a real constant, the constant ε measures the transverse dispersion effects and is normalized to ± 1, the constant α is a real parameter and H is the Hilbert transform defined by

$$Hu(x, y, t) = \text{p.v.} \frac{1}{\pi} \int_{\mathbb{R}} \frac{u(z, y, t)}{x - z}\, dz,$$

where p.v. denotes the Cauchy principal value. When $p = 1$, the BO–ZK equation appears in electromigration and the interaction of the nanoconductor with the surrounding medium, by considering Benjamin-Ono dispersive term with the anisotropic effects included via weak dispersion of ZK-type (see [4]). In fact, the BO–ZK equation can be viewed as a generalization of the well known one-dimensional Benjamin-Ono equation.

The solitary waves we are interested in are of the form

$$u(x, y, t) = \varphi_c(x - ct, y).$$

By using some Pohojaev type identities and the concentration-compactness principle, we classify the existence and non-existence of solitary waves depending on the sign of the dispersions and on the nonlinearity. In the framework introduced by Cazenave and Lions [1] we study the nonlinear stability of solitary waves. More precisely, we prove the following results.

Theorem 0.1. *The BO–ZK equation do not admit any nontrivial solitary wave solution if none of the following cases occur:*

(i) $\varepsilon = 1,\ c > 0,\ \alpha < 0,\ p < 4$;

(ii) $\varepsilon = -1,\ c < 0,\ \alpha > 0,\ p < 4$;

(iii) $\varepsilon = 1,\ c < 0,\ \alpha < 0,\ p > 4$;

(iv) $\varepsilon = -1,\ c > 0,\ \alpha > 0,\ p > 4$.

Theorem 0.2. *Let $c > 0$ and $0 < p < 4/3$. Then the solitary wave φ_c is Z-stable with regard to the flow of the BO-ZK equation, that is, for all positive ϵ, there is a positive δ such that if $u_0 \in H^s$, $s > 2$, and $\|u_0 - \varphi_c\|_Z \le \delta$, then the solution $u(t)$ of BO–ZK equation with $u(0) = u_0$ satisfies*

$$\sup_{t \ge 0} \inf_{\psi \in N_c} \|u(t) - \psi\|_Z \le \epsilon,$$

where Z is the closure of $C_0^\infty(\mathbb{R}^2)$ for the norm

$$\|\varphi\|_Z^2 = \|\varphi\|_{L^2(\mathbb{R}^2)}^2 + \|\varphi_y\|_{L^2(\mathbb{R}^2)}^2 + \left\| D_x^{1/2} \varphi \right\|_{L^2(\mathbb{R}^2)}^2,$$

and N_c is the set of the minimizers.

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Solução Geral da Equação de Hamilton-Jacobi Unidimensional

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1 Introdução

A importância prática e conceitual da equação de Hamilton-Jacobi pode ser apontada de maneira extensiva: como um conceito fundamental em Mecânica Clássica[1]; como uma ferramenta prática para resolver equações diferenciais[2]; como uma base para a quantização[3]; como uma aproximação de ordem zero no método WKB[4]; etc...

As soluções da equação de Hamilton-Jacobi são usualmente determinadas como soluções integrais pelo método de separação de variáveis. Mas soluções gerais destas equações são mais importantes, não somente por seu significado conceitual[5], mas porque geram uma infinidade de soluções integrais.

O caráter desta equação - uma equação diferencial parcial não linear - faz com que a procura por uma solução geral seja quase sempre uma tarefa insuperável[6, 7], estando indisponível até agora, algum procedimento que aplicado a essas equações resulte numa solução geral. É o propósito deste artigo apresentar uma solução para este problema centenário no caso unidimensional.

2 Solução Geral da Equação de Hamilton-Jacobi

Considere a equação mais geral de Hamilton-Jacobi para um sistema conservativo não relativístico unidimensional

$$ap^2 + V - q = 0, \tag{2.1}$$

onde $p = \partial S/\partial x$ e $q = \partial S/\partial t$.

Portanto

$$dS = pdx + qdt = d(px + qt) - xdp - tdq, \tag{2.2}$$

onde utilizamos uma transformação semelhante a de Legendre.

Substituindo p obtido a partir de (2.1) obtemos

$$dS = d\left(\frac{x\sqrt{a(q-V)}}{a} + qt\right) - \frac{x(a'V - aV' - qa')}{2\sqrt{a(q-V)}}dx - \left(t + \frac{x}{2\sqrt{a(q-V)}}\right)dq, \tag{2.3}$$

onde $a' = da/dx$ e $V' = dv/dx$. Integrando

$$S(x,t) = x\sqrt{(q-V)/a} + qt - F(x,q), \tag{2.4}$$

sendo F tal que

$$\frac{\partial F}{\partial q} = t + \frac{x}{2\sqrt{a(q-V)}}, \tag{2.5}$$

$$\frac{\partial F}{\partial x} = \frac{x(a'V - aV' - qa')}{2\sqrt{a(q-V)}} \equiv H(x,q). \tag{2.6}$$

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A integração da Eq. (2.6) conduz a $F = \int H(x,q)dx + G(q)$, onde G é uma função arbitrária. Usando este resultado em (2.5) obtemos a equação que define a variável $q = q(x,t)$, para cada escolha arbitrária da função G:

$$\int \frac{\partial H}{\partial q} dx + G'(q) = y + \frac{x}{2\sqrt{a(q-V)}}. \tag{2.7}$$

Então $S = S(x,t)$, fornecido por (2.4) é uma solução geral.

É interessante ressaltar que no método de separação de variáveis aplicados a equação de Hamilton-Jacobi as soluções usuais são obtidas fazendo a hipótese de que $q = constante$ (i.e., $dq = 0$, $S(x,t) = W(x) + C(t)$).

3 Exemplo

Como exemplo vamos considerar uma partícula livre descrita pela equação de Hamilton-Jacobi como $ap^2 - q = 0$ ($a = constante$). A partir de (2.4) se obtém a solução

$$S = x\sqrt{q/a} + qt - F.$$

onde a função F é determinada pela solução do sistema obtido de (2.5) e (2.6)

$$F'(q) = t + \frac{x}{2\sqrt{aq}}.$$

Esta equação fornece a cada escolha da função arbitrária F a variável $q = q(x,t)$. Por exemplo, se $F = Cq$ então $q = x^2/4a(C-t)^2$, logo $S(x,t) = x^2/4a(C-t)$. Esta solução foi previamente obtida utilizando dados do movimento da partícula[8], o que é desnecessário no nosso método.

A solução $x\sqrt{C/a} + Ct$ obtida pelo método de separação de variáveis é obtida substituindo $dq = 0$ em (2.3).

4 Final Remarks

O procedimento apresentado para resolver a equação de Hamilton-Jacobi é uma extensão do apresentado no II ENAMA[9]. A condição de integrabilidade da forma Pfaffiana (2.3) resulta nas equações (2.5) e (2.6).

A extensão do método para outros equações e a formulação geral do método para este tipo de equações está dentro das possíveis abordagens posteriores.

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Differentiability, Analyticity and Optimal Rates of decay to Damped wave equation

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This work is concerned with analyticity, differentiability and asymptotic stability of the C_0 semigroups associated with the following initial value problem

$$u_{tt} + Au + Bu_t = 0$$
$$u(0) = u_0, \quad u_t(0) = u_1$$

where A, and B are a self-adjoint positive definite operators with domain $D(A^\alpha) = D(B)$ dense in a Hilbert space H and satisfying the following hypotheses

(H1) There exists positive constants C_1 and C_2 such that

$$C_1 A^\alpha \leq B \leq C_2 A^\alpha.$$

which means

$$C_1(A^\alpha u, u) \leq (Bu, u) \leq C_2(A^\alpha u, u)$$

for any $u \in D(A^\alpha)$.

(H2) The bilinear form $b(u, w) = (B^{1/2}u, B^{1/2}w)$ is continuous on $D(A^{\alpha/2}) \times D(A^{\alpha/2})$. By the Riesz representation theorem, assumption (H2) implies that there exists an operator $S \in \mathcal{L}(D(A^{\alpha/2}))$ such that

$$(Bu, w) = (A^{\alpha/2}Su, A^{\alpha/2}w)$$

for any $u, w \in D(A^{\alpha/2})$.

There exists a large literature about the above problem dealing with asymptotic behaviour of the solutions to the damped wave equation see for example [7, 8, 9] and the references therein. In contrast to this results, there exists only a few literature dealing with regularity properties of the damped wave equation, like analyticity and differentiability of the corresponding semigroup. Here we mention two references. First, in [6] the authors proved that the semigroup associated to the damped wave equation is analytic if $1/2 \leq \alpha \leq 1$. This result established a fortiori the conjetures of Goon Chen and David L. Russel on structural damping for elastic systems, which referred to the case $\alpha = 1/2$. Second, in K. Liu and Z. Liu [10], the authors proved also the analyticity of the corresponding semigroup when $\alpha \in [1/2, 1]$ and the differentiability of the semigroup provides $\alpha \in]0, 1/2]$.

In the two above cited papers there are no information about the behaviour of the semigroup for $-1 \leq \alpha \leq \frac{1}{2}$, which frequently appears in applications. In this work we also show a class of operator A, and B, for which the above equation is analytic, differentiable and exponentially stable. Here we develop a simpler proof than in [6, 10], without using contradictions arguments. In addition, we show in case that the semigroup is not exponentially stable, that the solution of problem decays polynomially to zero as time goes to infinity. We show the our rate decay is optimal. To do so, we show for any contraction semigroup, a necessary condition to get the polynomial rate of decay. That is to say, the main result of this work is to get a fully characterization of the damping term for $-1 \leq \alpha \leq 1$. We show as in [6, 10] that the semigroup is analytic if and only if $1/2 \leq \alpha \leq 1$, it is differentiable

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when $\alpha \in]0,1[$ and that it is exponentially stable if and only if $\alpha \in [0,1]$. Finally, in case of $\alpha = -\gamma < 0$ we show that the corresponding semigroup decays polynomially to zero as $t^{-1/\gamma+\epsilon}$ ($\epsilon \ll 1/\gamma$), and we show that this rate of decay is optimal in $D(A)$ in the sense that is not possible to improve the rate $t^{-1/\gamma}$ with initial data over the domain of the operator A.

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A TRANSFORMADA DE FOURIER-BOREL ENTRE ESPAÇOS DE FUNÇÕES Θ-HOLOMORFAS DE UM DADO TIPO E UMA DADA ORDEM

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Neste trabalho, usamos o conceito de π_1-tipo holomorfia, introduzido em [2], para obter resultados de dualidade via transformada de Fourier-Borel entre o dual do espaço vetorial complexo $Exp^k_{\Theta,A}(E)$ de todas as funções Θ-holomorfas definidas no espaço de Banach E de ordem k e tipo estritamente menor que A, e o espaço vetorial complexo $Exp^{k'}_{\Theta',0,(\lambda(k)A)^{-1}}(E')$ de todas as funções Θ'-holomorfas em E' de ordem k' e tipo menor ou igual a $(\lambda(k)A)^{-1}$. A transformada de Fourier-Borel identifica algebricamente e topologicamente estes dois espaços se considerarmos a topologia forte no dual. Provamos também que a transformada de Fourier-Borel identifica algebricamente o dual do espaço vetorial complexo $Exp^k_{\Theta,0,A}(E)$ de todas as funções Θ-holomorfas definidas em E de ordem k e tipo menor ou igual a A, com o espaço vetorial complexo $Exp^{k'}_{\Theta',(\lambda(k)A)^{-1}}(E')$ de todas as funções Θ'-holomorfas em E' de ordem k' e tipo estritamente menor que $(\lambda(k)A)^{-1}$.

Os resultados que provamos generalizam resultados deste tipo obtidos por V. V. Fávaro [1], M. C. Matos [6], A. Martineau [5] e contém como casos particulares os resultados obtidos por [2], C. Gupta [3], B. Malgrange [4], M. C. Matos [7] e X. Mujica [8].

1 Definições e Resultados

Introduziremos abaixo os espaços $Exp^k_{\Theta,A}(E)$ e $Exp^k_{\Theta',0,A}(E')$.

Definição 1.1. *Seja* $(\mathcal{P}_\Theta(^jE))_{j=0}^\infty$ *um tipo de holomorfia de* E *em* $\mathbb{C}$. *Se* $\rho > 0$ *e* $k \geq 1$, *denotamos por* $\mathcal{B}^k_{\Theta,\rho}(E)$ *o espaço vetorial complexo de todas* $f \in \mathcal{H}(E)$ *tais que* $\widehat{d^j}f(0) \in \mathcal{P}_\Theta(^jE)$, *para todo* $j \in \mathbb{N}_0 = \{0,1,2,\ldots\}$ *e*

$$\|f\|_{\Theta,k,\rho} = \sum_{j=0}^{\infty} \rho^{-j}\left(\frac{j}{ke}\right)^{\frac{j}{k}} \left\|\frac{1}{j!}\widehat{d^j}f(0)\right\|_\Theta < +\infty,$$

que é um espaço de Banach com a norma $\|\cdot\|_{\Theta,k,\rho}$.

Definição 1.2. *Seja* $(\mathcal{P}_\Theta(^jE))_{j=0}^\infty$ *um tipo de holomorfia de* E *em* $\mathbb{C}$. *Se* $A \in (0,+\infty)$ *e* $k \geq 1$, *denotamos por* $Exp^k_{\Theta,A}(E)$ *o espaço vetorial complexo* $\bigcup_{\rho<A} \mathcal{B}^k_{\Theta,\rho}(E)$ *com a topologia limite indutivo localmente convexa. Consideramos o espaço vetorial complexo* $Exp^k_{\Theta,0,A}(E) = \bigcap_{\rho>A} \mathcal{B}^k_{\Theta,\rho}(E)$ *com a topologia limite projetivo localmente convexa. Se* $A = +\infty$ *e* $k \geq 1$, *consideramos o espaço vetorial complexo* $Exp^k_{\Theta,\infty}(E) = \bigcup_{\rho>0} \mathcal{B}^k_{\Theta,\rho}(E)$ *com a topologia limite indutivo localmente convexa e se* $A = 0$ *e* $k \geq 1$, *consideramos o espaço vetorial complexo* $Exp^k_{\Theta,0}(E) = Exp^k_{\Theta,0,0}(E) = \bigcap_{\rho>0} \mathcal{B}^k_{\Theta,\rho}(E)$ *com a topologia limite projetivo localmente convexa.*
Estamos considerando as topologias limite indutivo e projetivo dadas pelas inclusões naturais.

Proposição 1.1. *Seja* $(\mathcal{P}_\Theta(^jE))_{j=0}^\infty$ *um tipo de holomorfia de* E *em* $\mathbb{C}$.

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(a) Para cada $A \in (0, +\infty]$ e $k > 1$, $Exp^k_{\Theta,A}(E)$ é uma espaço DF.

(b) Para cada $A \in [0, +\infty)$ e $k > 1$, $Exp^k_{\Theta,0,A}(E)$ é um espaço de Fréchet.

Seja $(\mathcal{P}_\Theta(^jE))_{j=0}^\infty$ é um π_1-tipo de holomorfia de E em $\mathbb{C}$. Definimos a transformada de Borel

$$\mathcal{B}_\Theta \colon \left[\mathcal{P}_\Theta(^jE)\right]' \to \mathcal{P}(^jE')$$

por $\mathcal{B}_\Theta T(\varphi) = T(\varphi^j)$, para todo $T \in \left[\mathcal{P}_\Theta(^jE)\right]'$ e $\varphi \in E'$. Denotamos a imagem de $\mathcal{B}_\Theta$ por $\mathcal{P}_{\Theta'}(^jE')$ com a norma definida em $\mathcal{P}_{\Theta'}(^jE')$ dada por $\|\mathcal{B}_\Theta T\|_{\Theta'} = \|T\|$.

Considerando a sequência de espaços de Banach $(\mathcal{P}_{\Theta'}(^jE'))_{j=0}^\infty$ é possível definir de maneira análoga a Definição 1.2, os espaços vetoriais complexos $Exp^k_{\Theta',A}(E')$ e $Exp^k_{\Theta',0,A}(E')$ para todo $A \in (0, +\infty]$ e $k \geq 1$.

Enunciaremos agora os resultados principais deste trabalho:

Teorema 1.1. *Se $(\mathcal{P}_\Theta(^jE))_{j=0}^\infty$ é um π_1-tipo de holomorfia de E em $\mathbb{C}$, então a transformada de Fourier-Borel*

$$F \colon \left[Exp^k_{\Theta,0,A}(E)\right]' \longrightarrow Exp^{k'}_{\Theta',(\lambda(k)A)^{-1}}(E'),$$

dada por $FT(\varphi) = T(e^\varphi)$, é um isomorfismo algébrico entre estes espaços, para $k \in (1, +\infty)$ e $A \in (0, +\infty]$. Aqui $\lambda(k) = \frac{k}{(k-1)^{\frac{k-1}{k}}}$, para $k \in (1, +\infty)$ e k' denota o conjugado de k.

Teorema 1.2. *Se $(\mathcal{P}_\Theta(^jE))_{j=0}^\infty$ é um π_1-tipo de holomorfia de E em $\mathbb{C}$, então a transformada de Fourier-Borel*

$$F \colon \left[Exp^k_{\Theta,A}(E)\right]'_\beta \longrightarrow Exp^{k'}_{\Theta',0,(\lambda(k)A)^{-1}}(E'),$$

dada por $FT(\varphi) = T(e^\varphi)$, é um isomorfismo topológico entre estes espaços, para $k \in (1, +\infty)$ e $A \in (0, +\infty]$. A letra β indica que estamos considerando a topologia forte no dual.

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POLINÔMIOS LORENTZ SOMANTES, LORENTZ NUCLEARES E RESULTADOS DE DUALIDADE

V. V. Fávaro *, M. C. Matos † & D. Pellegrino‡

O estudo da teoria de dualidade de polinômios em espaços de Banach desempenha um papel importante em Análise Funcional. Nessa linha, vários trabalhos foram feitos e aplicados na teoria de equações de convolução como, por exemplo, os trabalhos citados na presente bibliografia.

Neste trabalho, trataremos das noções de polinômios homogêneos Lorentz somantes (introduzida em [13]) e polinômios Lorentz nucleares (introduzida em [5]) e caracterizaremos, via transformada de Borel, o dual da classe de polinômios Lorentz nucleares definidos no espaço de Banach E com uma respectiva classe de polinômios Lorentz somantes definidos em E'.

Resultados deste tipo contribuirão para o estudo de funções holomorfas definidas a partir destas classes de polinômios e, posteriormente, na investigação de resultados de existência e aproximação de soluções para equações de convolução definidas sobre os espaços de tais funções holomorfas.

1 Definições e Resultado Principal

Introduziremos abaixo os espaços de polinômios Lorentz somantes e nucleares e enunciaremos o resultado de dualidade. E e F denotarão espaços de Banach definidos sobre $\mathbb{K} = \mathbb{R}$ ou $\mathbb{C}$.

Definição 1.1. Sejam $x = (x_j)_{j=1}^{\infty} \in l_{\infty}(E)$ e

$$a_n(x) := \inf \left\{ \|x-u\|_{\infty} ; u \in c_{00}(E) \text{ e } card(u) < n \right\}.$$

Para $0 < r, q < +\infty$, o *espaço de sequências Lorentz* $l_{(r,q)}(E)$ é o conjunto de todas as sequências $x = (x_j)_{j=1}^{\infty} \in l_{\infty}(E)$ tais que

$$\left(n^{\frac{1}{r}-\frac{1}{q}} a_n(x) \right)_{n=1}^{\infty} \in l_q.$$

Definição 1.2. Para $0 < p, q, r, s < \infty$, dizemos que um polinômio n-homogêneo $P \in \mathcal{P}(^nE;F)$ é *Lorentz* $((s,p);(r,q))$*-somante* se $(P(x_j))_{j=1}^{\infty} \in l_{(s,p)}(F)$ para cada $(x_j)_{j=1}^{\infty} \in l_{(r,q)}^{w}(E)$.

Aqui, $l_{(r,q)}^{w}(E)$ denota o espaço de todas as sequências em E fracamente Lorentz (r,q)-somantes.

O espaço vetorial de todos os polinômios n-homogêneos Lorentz $((s,p);(r,q))$-somantes de E em F é denotado por $\mathcal{P}_{as((s,p);(r,q))}(^nE;F)$.

Definição 1.3. Sejam $n \in \mathbb{N}$ e $r, q, s, p \in [1, \infty[$ tais que $r \leq q$, $s' \leq p'$ e

$$1 \leq \frac{1}{q} + \frac{n}{p'}.$$

Um polinômio n-homogêneo $P : E \to F$ é *Lorentz* $((r,q);(s,p))$*-nuclear* se

$$P(x) = \sum_{j=1}^{\infty} \lambda_j (\varphi_j(x))^n y_j,$$

com $(\lambda_j)_{j=1}^{\infty} \in l_{(r,q)}$, $(\varphi_j)_{j=1}^{\infty} \in l_{(s',p')}^{w}(E')$ e $(y_j)_{j=1}^{\infty} \in l_{\infty}(F)$.

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Denotamos por $\mathcal{P}_{N,((r,q);(s,p))}({}^nE;F)$ o conjunto de todos os polinômios n-homogêneos Lorentz $((r,q);(s,p))$-nucleares.

É possível definir em $\mathcal{P}_{N,((r,q);(s,p))}({}^nE;F)$ e $\mathcal{P}_{as((s,p);(r,q))}({}^nE;F)$ quase-normas que tornam os espaços completos.

Enunciaremos agora o resultado principal deste trabalho:

Teorema 1.1. *Se E' tem a propriedade da aproximação limitada então a aplicação linear*

$$\Psi: \mathcal{P}_{N,((r,q);(s,p))}({}^nE;F)' \to \mathcal{P}_{as((r',q');(s',p'))}({}^nE';F'),$$

dada por $\Psi(T) = P_T$ é um isomorfismo topológico. O polinômio $P_T : E' \to F'$ é dado por

$$P_T(\varphi)(y) = T(\varphi^n y)$$

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EXISTENCE OF PERIODIC SOLUTIONS FOR A CLASS OF IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS

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In this work, we establish conditions for the existence of periodic solutions for a class of non-autonomous functional differential equations subject to self-supporting conditions in the frame of Henstock-Kurzweil integrable functions.

1 Introduction

We consider retarded functional differential equations (we write RFDEs for short) subjected to changes of state in short periods of time and we use the Henstock-Kurtzweil integration theory to treat them. These problems are related to impulsive RFDE's. In this work, we denote by $H([a,b])$ the space of Henstock-Kurzweil integrable functions $f: [a,b] \longrightarrow \mathbb{R}$, with integral $(K) \int_{[a,b]} f(t)dt$. Two functions $f, g \in H([a,b])$ are called *equivalent* whenever $\widetilde{f} = \widetilde{g}$, were $\widetilde{f}$ and $\widetilde{g}$ are the indefinite integral of the functions f and g, respectively. We consider the space $H([a,b])_A$ of equivalence classes of functions of $H([a,b])$ endowed with the Alexiewicz norm $\|\cdot\|_A$

$$f \in H([a,b]) \implies \|f\|_A = \|\widetilde{f}\|_\infty = \sup_{t\in[a,b]} \left|(K)\int_{[a,b]} f(s)ds\right|.$$

Let $r,\ \sigma,\ a$ be n on-negative numbers. By $PC(\sigma,\ r,\ a)$ we mean the space, endowed with the supremum norm, of piecewise continuous functions from $[\sigma - r,\ \sigma + a]$ to $\mathbb{R}$ which are left continuous. Let $\mathcal{O} \subset PC(\sigma,\ r,\ a)$ be a open set and let $H \subset H([-r,0],\mathbb{R})$ be such that if $x \in \mathcal{O}$, then $x_t \in H,\ t \in [\sigma, \sigma + a]$. We identify the set H with a subset H_A of $H([-r,0])_A$.

Let x be a real-valued function defined $a.e$ on $[\sigma - r, \sigma + a]$ and continuous on $[\sigma, \sigma + a]$. We denote by ${}^t x,\ t \in [\sigma, \sigma + a]$, the pair $(x_t, x(t)) \in H_A \times \mathbb{R}$, where x_t is defined $a.e$ on $[-r, 0]$. Then we consider the non-autonomous retarded differential equation

$$\begin{cases} \dot{x} = f(t, x_t) \\ {}^\sigma x = (\phi, x^0) \end{cases} \tag{1.1}$$

where $f : G \subset [\sigma, \sigma + a] \times H_A \mapsto \mathbb{R}$, with G open, and for every $x \in \mathcal{O}_{\phi,x^0} = \{x \in \mathcal{O};\ x_\sigma = \phi,\ x(\sigma) = x^0\}$, such that the function $t \in [\sigma, \sigma + a] \mapsto f(t, x_t) \in \mathbb{R}$ is Henstock-Kurzweil integrable, for some $\sigma \geqslant 0,\ a > 0$.

Given ${}^\sigma x = (\phi, x^0)$, we say that $x = x(\cdot\ ; \phi, x^0)$ is a *Henstock solution* or simply a *solution* of problem (1.1) through (ϕ, x^0) if x is defined a.e. in $[\sigma - r, \sigma + a]$, continuous on $[\sigma, \sigma + a]$, with $x_\sigma = \phi,\ x(\sigma) = x^0 \in \mathbb{R}$, and for every interval $[t_1, t_2] \subset [\sigma, \sigma + a]$, the integral equation

$$x(t_2) = x(t_1) + (K) \int_{[t_1,t_2]} f(s, x_s) ds$$

is satisfied.

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2 Existence of periodic solutions

Let f a real-valued function defined on $\mathbb{R}\times\mathcal{G}$, where $\mathcal{G}$ is an open subset of H_A, $-\Omega \le f(t,\psi) \le -\omega$, for all $(t,\psi)\in\mathbb{R}\times\mathcal{G}$, $0<\omega\le\Omega$. Suppose for every $x\in\mathcal{O}_{\phi,x^0}$, the map $t\in[\sigma,\sigma+a]\mapsto f(t,x_t)\in\mathbb{R}$ is Henstock-Kurzweil integrable. Assume, in addition, that there is a locally Lebesgue integrable function $L=L(t)$, $t\in[\sigma,\sigma+a]$, such that $x_1,x_2\in\mathcal{O}$ implies

$$\left|(K)\int_{t_1,t_2}[f(s,(x_1)_s)-f(s,(x_2)_s)]ds\right| \le \int_{[t_1,t_2]} L(s)||(x_1)_s-(x_2)_s||_A ds,$$

with $\int_{[t,t+a]} L(s)ds \le N$ for every t and a given N with $aN<1$. Under these conditions, consider the initial value problem (1.1) subjected to the self-supporting conditions

$$x(t-)=0 \implies x(t+)=c, \tag{2.2}$$

where $c>0$ is fixed.

Consider a piecewise continuous function $\eta:[-r,0]\to\mathbb{R}$ such that η is left continuous and satisfies the conditions:

(N1) $\eta(0)=0$;

(N2) the set of all zeros of η in $[-r,0[$ coincides with the set of its discontinuities and $\eta(t)=0$, $t\in[-r,0[$ implies $\eta(t+)=c$;

(N3) $-\Omega(t_2-t_1)\le\eta(t_2)-\eta(t_1)\le-\omega(t_2-t_1)$, whenever η is continuous in $[t_1,t_2]$.

Thus η is strictly decreasing in any interval where it is continuous. If we denote by $Z_\eta=\{s_i;\ i=0,1,2,\dots,m\}$ the set of all zeros of η, with $-r\le s_m<s_{m-1}<\dots<s_0=0$, then

$$\begin{cases} \dfrac{c}{\Omega}\le s_{i-1}-s_i\le\dfrac{c}{\omega},\ i=1,2,\dots,m\\ s_m+r\le\dfrac{c}{\omega}. \end{cases} \tag{2.3}$$

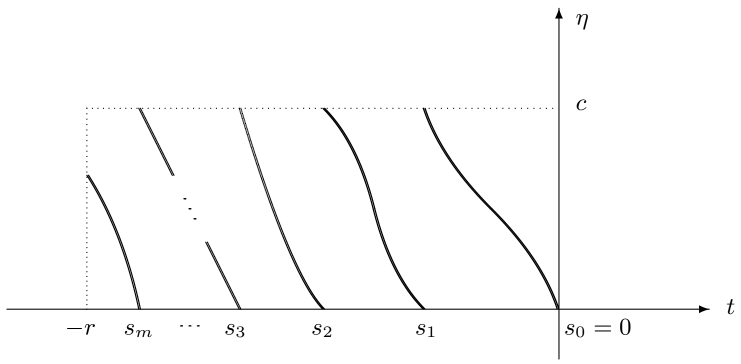

Figure 1: A typical function η.

Let K be the set of all functions $\eta:[-r,0]\to\mathbb{R}$ defined as above.

Theorem 2.1. *There exists a $\phi\in K$ for which problem (1.1), (2.2) admits a periodic solution $x(\cdot\,;\phi,x^0)$.*

Proof. The proof of this theorem is presented in [1].

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SOME CONTRIBUTIONS OF THE KURZWEIL INTEGRATION THEORY TO RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

M. FEDERSON * & Š SCHWABIK †

We present new results for retarded functional differential equations (RFDEs) concerning the stability of solutions. These results are converse-type Lyapunov theorems for RFDEs (see [2]) which were obtained by means of the relation between RFDEs and a certain class of generalized ODEs ([1]). The concept of non-absolute integration introduced by J. Kurzweil is the heart of generalized ODEs and it allows one to deal with highly oscillating functions. Other implications of Kurzweil integration theory to RFDEs can be found in [3], for instance.

Converse Lyapunov theorems for RFDEs

Let $G^{-}(I,\mathbb{R}^n)$ denote the space of left continuous regulated functions from an interval $I \subset \mathbb{R}$ to $\mathbb{R}^n$ with the topology of local uniform convergence. Consider the initial value problem for a RFDE

$$\begin{cases} \dot{y}(t) = f(y_t, t), \\ y_{t_0} = \phi, \end{cases}$$

where

- $\phi \in G^{-}([-r,0],\mathbb{R}^n)$, $r \geq 0$,
- $f(\phi,t) : G^{-}([-r,0],\mathbb{R}^n) \times [0,+\infty) \to \mathbb{R}^n$;
- $t \mapsto f(y_t,t)$ belongs to $L^1_{loc}([t_0,+\infty),\mathbb{R}^n)$, for every $y \in G^{-}([-r,\infty),\mathbb{R}^n)$.

We refer to the retarded system above by RFDE(f). It is clear that RFDE(f) is equivalent to the system

$$\begin{cases} y(t) = y(t_0) + \displaystyle\int_{t_0}^{t} f(y_s,s)\,ds, \quad t \in [t_0,+\infty), \\ y_{t_0} = \phi, \end{cases}$$

Assume further that $f(0,t) = 0$, $\forall t \in \mathbb{R}$, that is, $y \equiv 0$ is a solution of the RFDE(f) and suppose the following additional properties hold:

(A) $\exists\ M \in L^1_{loc}([t_0,+\infty),\mathbb{R})$ s.t. for $x \in G^{-}([-r,0],\mathbb{R}^n)$ and $u_1,u_2 \in [t_0,+\infty)$,

$$\left| \int_{u_1}^{u_2} f(x_s,s)\,ds \right| \leq \int_{u_1}^{u_2} M(s)\,ds;$$

(B) $\exists\ L \in L^1_{loc}([t_0,+\infty),\mathbb{R})$ s.t. for $x,y \in G^{-}([-r,0],\mathbb{R}^n)$ and $u_1,u_2 \in [t_0,+\infty)$,

$$\left| \int_{u_1}^{u_2} [f(x_s,s) - f(y_s,s)]\,ds \right| \leq \int_{u_1}^{u_2} L(s)\,\|x_s - y_s\|\,ds.$$

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Our main results are as follows. The notation and terminology will be specified in the Congress.

Theorem 1. *If the trivial solution* $y \equiv 0$ *of RFDE*(f) *is variationally stable, then for every* $0 < a < c$*, there exists a function* $U : [t_0 - r, +\infty) \times E_a \to \mathbb{R}$*, where* $\overline{E_a} = \{\psi \in G^-([-r, 0], \mathbb{R}^n); \|\psi\| < a\}$*, such that for every* $x \in E_a$*, the function* $U(\cdot, \psi)$ *belongs to* $BV^-([t_0 - r, +\infty), \mathbb{R})$ *and the following conditions hold:*

(i) $U(t, 0) = 0,\ t \in [t_0 - r, +\infty)$*;*

(ii) $|U(t, \psi) - U(t, \overline{\psi})| \leq \|\psi - \overline{\psi}\|,\ t \in [t_0 - r, +\infty),\ \psi, \overline{\psi} \in E_a$*.*

(iii) U *is positive definite along every solution* $y(t)$ *of RFDE*(f)*, that is, there is a function* $b : [0, +\infty) \to \mathbb{R}$ *of Kamke class such that*

$$U(t, y_t) \geq b(\|y_t\|), \quad (t, y_t) \in [t_0 - r, +\infty) \times E_a;$$

(iv) for all solutions $y(t)$ *of RFDE*(f)*,*

$$\dot{U}(t, y_t) = \limsup_{\eta \to 0_+} \frac{U(t + \eta, y_{t+\eta}) - U(t, y_t))}{\eta} \leq 0,$$

that is, the right derivative of U *along every solution* $y(t)$ *of RFDE*(f) *is non-positive.*

Theorem 2. *If the trivial solution* $y \equiv 0$ *of RFDE*(f) *is variationally asymptotically stable, then for every* $0 < a < c$*, there exists a function* $U : [t_0 - r, +\infty) \times E_a \to \mathbb{R}$ *such that for every* $x \in B_a$*, the function* $U(\cdot, x)$ *belongs to* $BV^-([t_0 - r, +\infty), \mathbb{R})$ *and the following conditions hold:*

(i) $U(t, 0) = 0,\ t \in [t_0 - r, +\infty)$*;*

(ii) $|U(t, \psi) - U(t, \overline{\psi})| \leq \|\psi - \overline{\psi}\|,\ t \in [t_0 - r, +\infty),\ \psi, \overline{\psi} \in E_a$*.*

(iii) U *is positive definite along every solution* $y(t)$ *of the retarded equation RFDE*(f)*, that is, there is a function* $b : [0, +\infty) \to \mathbb{R}$ *of Kamke class such that*

$$U(t, y_t) \geq b(\|y_t\|), \quad (t, y_t) \in [t_0 - r, +\infty) \times E_a;$$

(iv) for all solutions $y(s)$ *of RFDE*(f) *defined for* $s \geq t$*, where* $y(t) = \psi \in E_a$*, the following relation holds*

$$\dot{U}(t, y_t) = \limsup_{\eta \to 0_+} \frac{U(t + \eta, y_{t+\eta}) - U(t, y_t)}{\eta} \leq U(t, \psi).$$

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PERIODIC ORBITS OF THE KALDOR-KALECKI MODEL WITH DELAY

MARTA C. GADOTTI *

M.V.S. Frasson, S.H. J. Nicola and P.Z. Táboas jointly contributed to the development of this work.

N. Kaldor [4] published in 1940 a macroeconomic trade cycle model which describes the interaction of the national gross product and the capital stock by the following nonlinear system:

$$\dot{Y} = \alpha\left[I(Y,K) - S(Y,K)\right], \qquad \dot{K} = I(Y,K) - \delta K. \tag{0.1}$$

The dot denotes derivative with respect to t, I and S are the investment and the savings functions, respectively, Y is the national income, K is the capital stock, α is the adjustment coefficient in the goods market, usually referred as speed of adjustment, and δ is the depreciation rate of the capital stock.

In [6] a delay $T > 0$ is incorporated in the investment in the second equation of (0.1) as a reflexion of the Kalecki's hypothesis of gestation lag:

$$\dot{Y}(t) = \alpha[I\big(Y(t), K(t)\big) - S\big(Y(t), K(t)\big)], \quad \dot{K}(t) = I\big(Y(t-T), K(t)\big) - \delta K(t). \tag{0.2}$$

We investigate the case where savings and investment have the same rate with respect to the income at $Y = 0$. Varying the speed of adjustment α one obtains a sequence (α_k), $\alpha_k \to \infty$ as $k \to \infty$, such that a branch Γ_k of small amplitude periodic solutions of (0.2), whose frequencies approach ∞ as $k \to \infty$, emanates from the equilibrium at $\alpha = \alpha_k$, $k = 0, 1, \ldots$. These bifurcations hold interest by themselves, but from the economic viewpoint the lower terms of (α_k) are the most important.

Let us keep the variables Y, K, the functions I, S, and the parameters α, δ with the meaning they have in Equation (0.1). The investment function $I(Y,K)$ is supposed to be separated, that is,

$$I(Y,K) = J(Y) + N(K)$$

where $J(0) = N(0) = 0$, $[dJ/dY]_{Y=0} = \eta$, $[dN/dK]_{K=0} = \beta$, with $\beta < 0 < \eta$.

The savings depends only on the income, $S(Y,K) = S(Y)$, and $[dS/dY]_{Y=0} = \gamma \in (0,1)$.

Let us assume the Kalecki's statement of the existence of a gestation lag between the investment decision and its implementation. By the time re-scaling, the Kaldor-Kalecki model will be represented in the form

$$\dot{Y}(t) = \alpha\left[J\big(Y(t)\big) + N\big(K(t)\big) - S\big(Y(t)\big)\right],$$

$$\dot{K}(t) = J\big(Y(t-1)\big) + N\big(K(t)\big) - \delta K(t). \tag{0.3}$$

The linearization of (0.3) near $(0,0)$ leads to the system

$$\dot{Y}(t) = \alpha(\eta - \gamma)Y(t) + \alpha\beta K(t),$$

$$\dot{K}(t) = \eta Y(t-1) - (\delta - \beta)K(t), \tag{0.4}$$

whose characteristic equation is

$$\lambda^2 + A\lambda + B + De^{-\lambda} = 0, \tag{0.5}$$

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where $A = \alpha(\gamma - \eta) + (\delta - \beta)$, $B = \alpha(\gamma - \eta)(\delta - \beta)$ e $D = -\alpha\beta\eta$.

Let us assume that the parameters α, β, γ, δ, and η are related in such a way that A, D are positive and $B \geqslant 0$.

The following theorem provides sufficient conditions for the origin of the YK-plane to be an asymptotically stable equilibrium of the Kaldor-Kalecki model (0.3).

Theorem 0.1. *Let $D > 0$ be sufficiently small in such a way that*

$$H \geqslant 2De^{A/2} \tag{0.6}$$

Then all roots λ of the characteristic equation (0.5) satisfy $\Re(\lambda) < 0$.

From now on we are concerned with the case β, γ, δ, η fixed and $\gamma = \eta$. Letting $\alpha > 0$ to vary, a sequence of Hopf bifurcations will be obtained at values $\alpha_0 < \alpha_1 < \cdots \to +\infty$ of α for Kaldor-Kalecki system (0.3).

We need the two hypotheses below:

(H1) The Linear System (0.4) for $\alpha = \alpha_0$ has a simple pure imaginary characteristic root $\lambda_0 = ib_0 \neq 0$ and all characteristic roots $\lambda \neq \lambda_0, \overline{\lambda}_0$ satisfy $\lambda \neq m\lambda_0$ for any integer m.

Assuming the hypothesis (H1) is satisfied, if $\alpha \in (\alpha_0 - \sigma, \alpha_0 + \sigma)$ for some $\sigma > 0$, it is known that the corresponding Linear System (0.4) has a simple characteristic root $\lambda(\alpha)$ with continuous derivative $\lambda'(\alpha)$ and $\lambda(\alpha_0) = \lambda_0$. See [3, Section 7.10, Lemma 10.1].

(H2) $\Re(\lambda'(0)) \neq 0$.

Theorem 0.2. *Suppose the hypotheses (H1) and (H2) are satisfied. Then there exist constants a_0, σ, $\delta_0 > 0$ and C^1 functions $\alpha = \alpha(a)$, $\omega = \omega(a)$, for $a < |a_0|$, with $\alpha(0) = \alpha_0$, $\omega(0) = 2\pi/b_0$ and a nonconstant $\omega(a)$-periodic solution $X^*(a) = \big(Y^*(a), K^*(a)\big)$ of System (0.3) for $\alpha = \alpha(a)$. Moreover, if $|\alpha - \alpha_0| < \sigma$ and $X(t) = \big(Y(t), K(t)\big)$ is a nonconstant ω-periodic solution of (0.3), with $|X(t)| < \delta_0$, $t \in \mathbb{R}$, and $|\omega - 2\pi/b_0| < \delta_0$, then $X(t)$ is one of the solutions $X^*(a)$, $a < |a_0|$, except for phase shift.*

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Uniqueness of the extension of 2-homogeneous polynomials

P. Galindo * & M. L. Lourenço †

Homogeneous polynomials of degree 2 on the complex Banach space $c_0(\ell_n^2)$ are shown to have unique norm-preserving extension to the bidual space. This is done by using M-projections and extends the analogous result for c_0 proved by P-K. Lin.

1 Introduction

R. Aron and P. Berner [1] showed that on any complex Banach space every n-homogeneous continuous polynomial has an extension to the bidual space. Later A. Davie and T. Gamelin [4] proved that such extension is norm-preserving. The uniqueness of the extension fails in general. However, R. Aron, C. Boyd and Y. S. Choi [2] proved that norm-attaining 2-homogeneous continuous polynomials on c_0 have unique norm-preserving extension to the bidual space ℓ_∞. Following such trend Y. S. Choi, K. H. Han and H. G. Song [3] got the same result for $d_*(w,1)$, the predual of a Lorentz space, while M. L. Lourenço and L. Pellegrini did the same with the c_0 sum of Hilbert spaces [7].

Recently, P-K. Lin [5] has proved that on c_0 the norm-attaining condition can be removed and still obtain uniqueness. This note deals with the same topic and relying on Lin's technique and M-projections, we are able to get, under certain conditions, the uniqueness of the extension. As a consequence it turns out that 2-homogeneous continuous polynomials on $c_0(\ell_n^2)$ have unique norm-preserving extension, regardless they attain the norm. We also show that if a Hilbert space is an M-summand in E, and a 2-homogeneous continuous polynomial on E attains its norm at some point in the Hilbert space, then it factors through the Hilbert space.

2 Results

Lemma 2.1. *Let E be a Banach space and Q an M-projection. If A is a 2-homogeneous polynomial on E, we have for any unit vector $z \in Q(E)$ that*

$$\|A \circ (Id - Q)\| \leq \|A\| - \Re e A(z).$$

In particular, $\|A \circ (Id - Q)\| + \|A \circ Q\| \leq \|A\|$.

Proposition 2.1. *Let E be a Banach space and $\{E_m\}_{m\in\mathfrak{M}}$ a directed family of Banach spaces which are M-summands in E^{**} with $\bigcup_j E_{m_j}$ dense in E for any cofinal set $\{m_j\}$ in $\mathfrak{M}$. Assume that for some basis in E_m, the norm is invariant when rotating the coordinates and that $\|\Re e(u) + i\Re e(v)\|^2 \leq \|u\|^2 + \|v\|^2$ for $u, v \in E_m$.*

If P is a 2-homogeneous polynomial on E such that for all m $\|P_{|_{E_m}}\| < \|P\|$ and $\|P_{|_{E_m}}\|$ is attained at a point whose coordinates are nonvanishing real numbers, then P has a unique norm-preserving extension to E^{**}.

For a sequence (X_n) of Banach spaces, recall that the c_0-direct sum is the Banach space

$$c_0(X_n) = (\{(x_n)_n : x_n \in X_n \ \wedge \ \lim_n \|x_n\| = 0, \text{ endowed with } |||(x_n)||| = \sup_n \|x_n\|\}).$$

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Example 2.1. *If $\{X_n\}$ are Hilbert spaces, the space direct sum $c_0(X_n)$ fulfills the conditions on E in Proposition 2.6. In this case $E_m = \oplus_1^m X_n$ and Q_m is the projection over the first m-sequences,*

Consider $P((x_j^n)) = \sum_{n=1}^{\infty} \frac{1}{2^n} \sum_j (x_j^n)^2$ where $(x_j^n) \in X_n$. Clearly, $\|P\| = 1$ and P is not norm-attaining. Also $P((x_n^j)) = \sum_{n=1}^{m} \frac{1}{2^n} \sum_j (x_j^n)^2$ for $((x_n^j)) \in E_m$ and $\|P_{|E_m}\| = 1 - \frac{1}{2^m} < 1$. Further $\|P_{|E_m}\|$ is attained at a point whose coordinates can be chosen real and non vanishing. Proposition 2.6 assures that P has unique norm preserving extension to the bidual space $\ell_\infty(X_n)$.

Corollary 2.1. *All homogeneous polynomials of degree 2 on the complex Banach space $c_0(\ell_n^2)$ have unique norm-preserving extension to the bidual space.*

Proof. In case the polynomial is norm-attaining, the conclusion holds by Corollary 2.2 in [6]. Otherwise, we may apply Proposition 2.1 since the corresponding subspaces E_m, as in Example 2.1, are finite dimensional and, thus, the polynomial attains its norm in each of them, and by a suitable change of basis on E_m, the coordinates of the point of attainment can be chosen real and not vanishing. □

Corollary 2.2. *All homogeneous polynomials P of degree 2 on the complex Banach space $c_0(I)$, for any index set I, have unique norm-preserving extension to the bidual space.*

Proof. If P attains its norm, any possible norm-preserving extension A to $\ell_\infty(I)$ depends, according to Lemma 2.1, on a finite number of variables that is, $A((x_i)) = A(x_1, \dots, x_m, 0 \dots) = P(x_1, \dots, x_m, 0 \dots)$. So the extension is unique.

For the case that P does not attain its norm, we consider the family $\{E_m\}_{m \in \mathfrak{M}}$ of the spaces generated by a finite number of elements of the canonical basis in $c_0(I)$. The assumptions on the space in Proposition 2.1. as well as on the polynomial are fulfilled, thus P has unique norm-preserving extension. □

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AVERAGING FOR IMPULSIVE RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS VIA GENERALIZED ORDINARY DIFFERENTIAL EQUATION

J. B. GODOY * & M. FEDERSON *

In this work, we consider the following initial value problem for a retarded functional differential equation with impulses

$$\begin{cases} \dot{x} = \varepsilon f(t, x_t), \quad t \neq t_k \\ \Delta x(t_k) = \varepsilon I_k(x(t_k)), \; k = 0, 1, 2, \ldots \\ x_{t_0} = \phi, \end{cases} \tag{0.1}$$

where f is defined in a open set $\Omega \subset \mathbb{R} \times G^-([-r,0], \mathbb{R}^n)$, $r > 0$, and takes values in $\mathbb{R}^n$, $\varepsilon > 0$ is a small parameter and $\phi \in G^-([-r,0], \mathbb{R}^n)$, where $G^-([-r,0], \mathbb{R}^n)$ denotes the space of regulated functions from $[-r,0]$ to $\mathbb{R}^n$ which are left continuous. Furthermore, $t_0 < t_1 < \ldots < t_k < \ldots$ are pre-assigned moments of impulse effects such that $\lim_{k \to +\infty} t_k = +\infty$ and $\Delta x(t_k) = x(t_k^+) - x(t_k)$. The impulse operators I_k, $k = 0, 1, \ldots$, are continuous mappings from $\mathbb{R}^n$ to $\mathbb{R}^n$. For each $x \in G^-([-r,\infty), \mathbb{R}^n)$, $t \mapsto f(t, x_t)$ is locally Lebesgue integrable and its indefinite integral satisfies a Carathéodory-type condition. Moreover, f is Lipschitzian with respect to the second variable. We define

$$f_0(\psi) = \lim_{T \to \infty} \frac{1}{T} \int_{t_0}^{T} f(t, \psi) dt \quad \text{and} \quad I^0(x) = \lim_{T \to \infty} \frac{1}{T} \sum_{0 \le t_i < T} I_i(x), \tag{0.2}$$

where $\psi \in G^-([-r,0], \mathbb{R}^n)$ and $x \in \mathbb{R}^n$, and we consider the "averaged" autonomous functional differential equation

$$\begin{cases} \dot{y} = \varepsilon[f_0(y_t) + I^0(y(t))] \\ y_{t_0} = \phi. \end{cases} \tag{0.3}$$

Then we prove that, under certain conditions, the solution $x(t)$ of (0.1) approximates the solution $y(t)$ of (0.3) in an asymptotically large time interval.

1 Mathematical Results

Theorem 1.1. *Let y be solution of RFDE with impulses*

$$\begin{cases} \dot{y} = f(y_t, t), \; t \neq t_i \\ \Delta x(t_i) = I_i(x(t_i)), \quad i = 1, 2, \ldots \\ y_{t_0} = \phi \end{cases} \tag{1.4}$$

and y^ε be solution of RFDE with impulses given by (0.1), where $\phi \in G^-([-r,0], \mathbb{R}^n)$ and $f : G^-([-r,0], \mathbb{R}^n) \times [0,\infty) \to \mathbb{R}^n$ satisfies the conditions

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(A) There is a function locally Lebesgue integrable, $M_1 : \mathbb{R} \to \mathbb{R}$*, such that for* $x \in PC_1$ *and* $u_1, u_2 \in [0, +\infty)$,

$$\left| \int_{u_1}^{u_2} f\left(x_s, s\right) ds \right| \leq \int_{u_1}^{u_2} M_1\left(s\right) ds;$$

(K) There is a positive constant K *such that, for* $\psi, \varphi \in G^-([-r, 0], \mathbb{R}^n)$ *and* $u \in [0, +\infty)$,

$$|f(\psi, u) - f(\varphi, u)| \leq K \|\psi - \varphi\|.$$

Assume that

$$\lim_{T\to\infty} \frac{1}{T} \int_{\alpha}^{T+\alpha} M(s)ds \leq c, \quad c = constant, \quad \alpha \geq 0, \tag{1.5}$$

Let $0 \leq t_1 < t_2 < ... < t_k < \dots$ *be a sequence of points such that* $t_k \to \infty$ *when* $k \to \infty$ *and*

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{\alpha \leq t_i \leq \alpha+T} 1 \leq d, \quad \alpha \geq 0, \tag{1.6}$$

and assume that the sequence of impulse operators $I_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 0, 1, 2, \dots$, *is such that*

(A′) There is a constant $K_1 > 0$ *such that for* $i = 1, 2, \dots$ *, and for all* $x \in \mathbb{R}^n$

$$\|I_i(x)\| \leq K_1,$$

(B′) There is a constant $K_2 > 0$ *such that for* $i = 1, 2, \dots$ *and for all* $x, y \in \mathbb{R}^n$

$$\|I_i(x) - I_i(y)\| \leq K_2 \|x - y\|.$$

Further, assume that I^0 *and* f_0 *are given by* (0.2)*. Then for all* $\mu > 0$ *and* $L > 0$*, there is a* $\varepsilon_0 > 0$ *such that for* $\varepsilon \in (0, \varepsilon_0)$*, the inequality*

$$\|(y^\varepsilon)_t - (\overline{y}^\varepsilon)_t\| < \mu$$

holds for all $t \in [0, L/\varepsilon]$*, where* $\overline{y}^\varepsilon$ *is a solution of RFDE given by* (0.3) *on* $[0, L/\varepsilon]$.

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UMA CARACTERIZAÇÃO DO ESPAÇO DE HARDY H^1 SOBRE PRODUTO DE SEMI-PLANOS

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1 Introdução

Apresentamos inicialmente algumas definições. Se E é um espaço de Banach e $P = (p_1, p_2)$ com $0 < p_1, p_2 \leq \infty$, $L^P(\mathbb{R}^2, E)$ é o espaço de todas as funções f definidas sobre $\mathbb{R}^2$ a valores em E tais que $||f||_E$ é Lebesgue mensurável e

$$||f||_{L^P(\mathbb{R}^2,E)} = \left(\int_{\mathbb{R}} \left(\int_{\mathbb{R}} ||f||_E^{p_1} dx_1 \right)^{p_2/p_1} dx_2 \right)^{1/p_2} < \infty \,,$$

com as modificações usuais quando algum dos p_i é igual a ∞. Analogamente, dado $Q = (q_1, q_2)$ com $0 < q_1, q_2 \leq \infty$, indicamos por $\ell^Q(\mathbb{Z}^2, E)$ ($\ell^Q(\mathbb{Z}^2)$ quando $E = \mathbf{C}$) o espaço de todas as (multi)sequências $(c_N)_{N \in \mathbb{Z}^2}$ em E tal que

$$||(c_N)_{N\in\mathbb{Z}^2}||_{\ell^Q(\mathbb{Z}^2,E)} = \left(\sum_{j=-\infty}^{\infty} \left(\sum_{-\infty}^{\infty} ||c_N||_E^{q_1} \right)^{q_2/q_1} \right)^{1/q_2} < \infty$$

com as modificações usuais quando algum dos q_i for igual a ∞.

A transformada de Fourier de uma função $f \in L^1(\mathbb{R}^2, E)$ é definida por

$$\mathcal{F}f(x) = \hat{f}(x) = \int\int_{\mathbb{R}^2} e^{-2\pi i x.y} f(y) dy \,,$$

onde $x.y = x_1y_1 + x_2y_2$. Usaremos a seguinte notação: $\square = \{(0,0),\ (1,0),\ (0,1),\ (1,1)\}$.

Definição 1.1. *Seja E um espaço de Hilbert e $f \in L^1(\mathbb{R}^2, E)$. Suas transformadas de Hilbert $H_k f$, $k \in \square$, são os elementos de $\mathcal{S}'(\mathbb{R}^2, E)$ definidos por:*

(1) $\mathcal{F}(H_{10}f) = -\mathrm{i}\ sgx\ \mathcal{F}(f)(x,y)$,

(2) $\mathcal{F}(H_{01}f) = -\mathrm{i}\ sgy\ \mathcal{F}(f)(x,y)$,

(3) $\mathcal{F}(H_{11}f) = (-\mathrm{i}\ sgx)(-\mathrm{i}\ sgy)\mathcal{F}(f)(x,y)$,

(4) $(H_{00}f) = f$.

Definição 1.2. *Seja E um espaço de Hilbert. $H^1(\mathbb{R} \times \mathbb{R}, E)$ é o espaço vetorial das funções f em $L^1(\mathbb{R}^2, E)$ tais que suas transformadas de Hilbert, $H_k f$, $k \in \square \setminus \{(0,0)\}$, pertencem a $L^1(\mathbb{R}^2, E)$. Munimos o espaço $H^1(\mathbb{R} \times \mathbb{R}, E)$ com a norma $\|f\|_{H^1(\mathbb{R}\times\mathbb{R},E)} = \sum_{k\in\square} \|H_k f\|_{L^1(\mathbb{R}^2,E)}$, onde $H_{00}f = f$.*

Definição 1.3. *Seja E um espaço de Hilbert. Uma função g de $\mathbb{R}^2$ em E pertence a $BMO(\mathbb{R} \times \mathbb{R}, E)$, se ela pode ser representada como $g = \sum_{k\in\square} H_k g_k$, onde $H_{00}g_{00} = g_{00}$ e $\sum_{k\in\square} ||g_k||_{L^\infty(\mathbb{R}^2,E)} < \infty$. Munimos o espaço $BMO(\mathbb{R} \times \mathbb{R}, E)$ com a norma $||g||_{BMO(\mathbb{R}\times\mathbb{R},E)} = \inf\{\sum_{k\in\square} ||g_k||_{L^\infty(\mathbb{R}^2,E)}\}$, onde o ínfimo é tomado sobre todas as representações de g na forma $g = \sum_{k\in\square} H_k g_k$.*

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Chang-Fefferman demonstraram em [2] que $(H^1(\mathbb{R}\times\mathbb{R}))^* = BMO(\mathbb{R}\times\mathbb{R})$. Este resultado também vale para o caso vetorial $BMO(\mathbb{R}\times\mathbb{R}, E)$, onde E é um espaço de Hilbert. Isto é utilizado na demonstração do Teorema 2.1.

Lema 1.1. *(Berg-Lofstron [1]) Existe $\phi \in S(\mathbb{R})$, such that*

(1) *supp* $\mathcal{F}\phi = \{t \in \mathbb{R} \ : \ 2^{-1} \le |t| \le 2\}$;

(2) $|\mathcal{F}\phi(t)| > 0$ *se* $2^{-1} < |t| < 2$;

(3) $\sum_{i=-\infty}^{\infty} \mathcal{F}\phi(2^{-i}t) = 1$ *se* $t \neq 0$.

Definição 1.4. *(*Sistema de Funções Testes*). Seja ϕ como no Lema 1.1 e para cada $i \in \mathbb{Z}$ seja ϕ_i a função dada por $\phi_i(t) = 2^i\phi(2^i t)$. A família $(\phi_i)_{i\in\mathcal{Z}}$ é chamada de um sistema de funções testes sobre $\mathbb{R}$. Observamos que valem as seguintes condições:*

(1) *supp* $\mathcal{F}\phi_i = \{t \in \mathbb{R} \ : \ 2^{i-1} \le |t| \le 2^{i+1}\}$; $i \in \mathcal{Z}$;

(2) $|\mathcal{F}\phi_i(t)| > 0$ *se* $2^{i-1} < |t| < 2^{i+1}$;

(3) $\sum_{i=-\infty}^{\infty} \mathcal{F}\phi_i(t) = 1$ *se* $t \neq 0$.

Definição 1.5. *Sejam $(\phi_i)_{i\in\mathcal{Z}}$ e $(\psi_j)_{j\in\mathcal{Z}}$ sistemas de funções testes como na Definição 1.4. Então, $H_0^{1,2}(\mathbb{R}\times\mathbb{R})$ é o espaço vetorial das funções $f \in L^1(\mathbb{R}^2)\cap S'(\mathbb{R}^2)$, a valores reais, que satisfazem $(\phi_i\psi_j * f)_{ij} \in L^1(\ell^2(\mathcal{Z}^2))$. Munimos o espaço $H_0^{1,2}(\mathbb{R}\times\mathbb{R})$ com a norma $||f||^{\phi,\psi}_{H_0^{1,2}} = ||(\phi_i\psi_j * f)_{ij}||_{L^1(\ell^2(\mathcal{Z}^2))}$. Esta norma independe dos sistemas de funções testes utilizados, como pode ser visto em Schmeisser-Tribel [4].*

2 Resultado Obtido

Neste trabalho obtemos a seguinte caracterização do espaço de Hardy $H^1(\mathbb{R}\times\mathbb{R})$ a dois parâmetro:

Teorema 2.1. *Uma função f em $L^1(\mathbb{R}^2)$ pertence a $H^1(\mathbb{R}\times\mathbb{R})$ se, e somente se, f pertence a $H_0^{1,2}(\mathbb{R}\times\mathbb{R})$. Além disso, existe constante $C > 0$ tal que*

$$C^{-1}||f||_{H^1(\mathbb{R}\times\mathbb{R})} \le ||f||_{H_0^{1,2}(\mathbb{R}\times\mathbb{R})} \le C||f||_{H^1(\mathbb{R}\times\mathbb{R})} .$$

Na demonstração deste teorema utilizamos, como uma das ferramentas essenciais, resultados obtidos por Gomes-Silva em [3]. Esses resultados se referem a ação de certos operadores integrais singulares vetoriais com núcleo produto sobre os espaços $H^1(\mathbb{R}\times\mathbb{R}, E)$ e $BMO(\mathbb{R}\times\mathbb{R}, E)$, onde E é um espaço de Hilbert.

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ON THE PROBLEM OF KOLMOGOROV ON HOMOGENEOUS MANIFOLDS

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We give the solution of a well-known problem of Kolmogorov on sharp asymptotic of the rates of convergence of Fourier sums on sets of smooth functions on homogeneous manifolds. In 1935 A. N. Kolmogorov [1] established a famous result on convergence of Fourier series on sets of differentiable functions on the circle (one dimensional sphere). Namely,

$$\sup_{f\in W_\infty^\gamma} \|f - S_n(f)\|_\infty = \frac{4}{\pi} n^{-\gamma}\log n + O\left(n^{-\gamma}\right), \gamma \in \mathbb{N}.$$

This discovery has established an entire epoch in Analysis and Approximation Theory. Since that time a significant progress has been made in this area and there are hundreds of results which are worthy to be mentioned. Therefore, it is important to consider the problem of A. N. Kolmogorov on the d-dimensional sphere and more general manifolds, the two-point homogeneous spaces, the spheres $\mathbb{S}^d, d = 1, 2, 3, \ldots$ the real projective spaces $\mathrm{P}^d(\mathbb{R}), d = 2, 3, 4, \ldots$ the complex projective spaces $\mathrm{P}^d(\mathbb{C}), 4, 6, 8, \ldots$, the quaternionic projective spaces $\mathrm{P}^d(\mathbb{H}), d = 8, 12, \ldots$ and the Cayley elliptic plane $\mathrm{P}^{16}(\mathrm{Cay})$.

1 The Results

Let $\mathbb{M}^d$ be a compact globally symmetric space of rank 1, ν its normalized invariant volume element, Δ its Laplace-Beltrami operator. The eigenvalues of Δ are discrete, nonnegative and form an increasing sequence θ_k, $\lim_{k\to\infty}\theta_k = \infty$. Corresponding eigenspaces H_k, $k \geq 0$ are finite dimensional and $L_2\left(\mathbb{M}^d, \nu\right) = \oplus_{k=0}^{\infty} H_k$. Let $\dim H_k = d_k$. Denote by $\{Y_j^k\}_{j=1}^{d_k}$ an orthonormal basis of H_k. For any $f \in L_1\left(\mathbb{M}^d, \nu\right)$ consider the sequence of Fourier sums

$$S_n(f) = \sum_{k=0}^{n}\sum_{j=1}^{d_k} c_{k,j}(f) Y_j^k, \;\; c_{k,j}(f) = \int_{\mathbb{M}^d} f\overline{Y_j^k} d\nu.$$

Let

$\mathbb{M}^d$	$\mathbb{S}^d$	$\mathrm{P}^d(\mathbb{R})$	$\mathrm{P}^d(\mathbb{C})$	$\mathrm{P}^d(\mathbb{H})$	$\mathrm{P}^{16}(\mathrm{Cay})$
$\mathcal{K}(\mathbb{M}^d)$	$\frac{2\Gamma(\frac{d-1}{4})\Gamma(\frac{d+1}{4})}{\pi^{3/2}\left(\Gamma(\frac{d}{2})\right)^2}$	$\frac{2\Gamma(\frac{d-1}{4})}{\pi\Gamma(\frac{d}{2})\Gamma(\frac{d+1}{4})}$	$\frac{2\Gamma(\frac{d-1}{4})\Gamma(\frac{3}{4})}{\pi^{3/2}\Gamma(\frac{d}{2})\Gamma(\frac{d+2}{4})}$	$\frac{2\Gamma(\frac{d-1}{4})\Gamma(\frac{3}{4})}{\pi\Gamma(\frac{d}{2})\Gamma(\frac{d+5}{4})}$	$\frac{11\cdot 2^{1/2}}{2949120\ \pi^{1/2}}$

(see [2], [3] for details).

Theorem 1. *Let $W_p^\gamma(\mathbb{S}^d)$ be the Sobolev's class. For any $\gamma > 0$ and $p = 1, \infty$ we have*

$$\sup_{f\in W_p^\gamma(\mathbb{M}^d)} \|f - S_n(f)\|_p$$

$$= \mathcal{K}(\mathbb{M}^d)\, n^{-\gamma+(d-1)/2} + O\left(n^{-\gamma} \left\{ \begin{array}{ll} 1, & d=2 \\ \ln n, & d=3 \\ n^{(d-3)/2}, & d\geq 4 \end{array} \right\} \right),$$

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where $\mathbb{M}^d = \mathrm{P}(\mathbb{C}^d),\ \mathrm{P}(\mathbb{C}^d),\ \mathrm{P}(\mathbb{H}^d),\ \mathrm{P}^{16}(\mathrm{Cay})$. *If* $\mathbb{M}^d = \mathrm{P}(\mathbb{R}^d)$, *then*

$$\sup_{f\in W_p^{\gamma}(\mathbb{M}^d)} \|f - S_{2n}(f)\|_p$$

$$= \frac{2\Gamma(\frac{d-1}{4})}{\pi\Gamma(\frac{d}{2})\Gamma(\frac{d+1}{4})}\, n^{-\gamma+(d-1)/2} + O\left(n^{-\gamma} \left\{ \begin{array}{ll} 1, & d=2 \\ \ln n, & d=3 \\ n^{(d-3)/2}, & d\geq 4 \end{array} \right\} \right).$$

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ENTROPY AND WIDTHS OF SETS OF INFINITE DIFFERENTIABLE AND ANALYTIC FUNCTIONS ON HOMOGENEOUS SPACES

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1 Introduction

In this work we are continuing to develop methods to estimate n-widths and entropy numbers of multiplier operators begun in [1] - [3]. Our main aim is to give an unified treatment for a wide range of multiplier operators Λ on symmetric manifolds. Namely, we investigate entropy numbers and n-widths of decaying multiplier sequences of real numbers $\Lambda = \{\lambda_k\}_{k\in\mathbb{N}}$, $\Lambda : L_p(\mathbb{M}^d) \to L_q(\mathbb{M}^d)$ on two-point homogeneous spaces $\mathbb{M}^d$: $\mathbb{S}^d$, $\mathbb{P}^d(\mathbb{R})$, $\mathbb{P}^d(\mathbb{C})$, $\mathbb{P}^d(\mathbb{H})$, $\mathbb{P}^{16}(\mathrm{Cay})$. Suppose that A is a convex, compact, centrally symmetric subset of a Banach space X with closed unit ball B_X. The Kolmogorov n-width of A in X is defined by

$$d_n(A,X) := \inf_{X_n} \sup_{f\in A} \inf_{g\in X_n} \|f-g\|,$$

where X_n runs over all subspaces of X of dimension n. The nth entropy number $e_n(A,X)$ is defined as the infimum of all $\epsilon > 0$ such that there exist $x_1, ..., x_{2^{n-1}}$ in X satisfying

$$A \subset \bigcup_{k=1}^{2^{n-1}} (x_k + \epsilon B_X).$$

On the circle $\mathbb{T}^1 = \mathbb{S}^1$, unlike of the Kolmogorov n-widths, the entropy numbers of Sobolev's classes $W_p^\gamma(\mathbb{S}^1)$, $\gamma > 0$, have the same order for all $1 \le p, q \le \infty$, i.e., $n^{-\gamma} \asymp e_n(W_p^\gamma(\mathbb{S}^1), L_q(\mathbb{S}^1)) \ll d_n(W_p^\gamma(\mathbb{S}^1), L_q(\mathbb{S}^1))$. We show that for the multiplier sequences $\Lambda = \{\lambda_k\}_{k\in\mathbb{N}}$ which decay to zero exponentially fast, the n-widths and entropy numbers are essentially different. If $\Lambda = \{\lambda_k\}_{k\in\mathbb{N}}$, $\lambda_k = e^{-\gamma k^r}$, $\gamma > 0$, $0 < r \le 1$, $2 \le p, q < \infty$, $U_p(\mathbb{S}^d)$ denotes the closed unit ball of $L_p(\mathbb{S}^d)$, then we show that $d_n(\Lambda U_p(\mathbb{S}^d), L_q(\mathbb{S}^d)) \ll e_n(\Lambda U_p(\mathbb{S}^d), L_q(\mathbb{S}^d))$. The results we derive are apparently new even in the one dimensional case.

For easy of notation we will write $a_n \gg b_n$ for two sequences, if $a_n \ge C b_n$ for $n \in \mathbb{N}$ and $a_n \asymp b_n$ if $C_1 b_n \le a_n \le C_2 b_n$ for all $n \in \mathbb{N}$ and some constants C, C_1 and C_2.

2 The results

For us, Λ will be the multiplier operator from $L_p(\mathbb{M}^d)$ to $L_q(\mathbb{M}^d)$, $1 \le p, q \le \infty$, on a two-point homogeneous space

$$M^d \in \left\{\mathbb{S}^d, \mathbb{P}^d(\mathbb{R}), \mathbb{P}^d(\mathbb{C}), \mathbb{P}^d(\mathbb{H}), \mathbb{P}^{16}(\mathrm{Cay})\right\},$$

defined by the sequence $\{\lambda_k\}_{k\in\mathbb{N}}$, $\lambda_k = e^{-\gamma k^r}$, $\gamma > 0$, $0 < r \le 1$. Remark that $\Lambda U_p(\mathbb{M}^d)$ is a set of infinite differentiable functions or analytic functions on the manifold $\mathbb{M}^d$ if $0 < r < 1$ or $r = 1$, respectively. Each two-point homogeneous space $\mathbb{M}^d$ of dimension d is associated with parameters $\alpha, \beta \in \mathbb{R}_+$ as follows:

	$\mathbb{S}^d$	$\mathbb{P}^d(\mathbb{R})$	$\mathbb{P}^d(\mathbb{C})$	$\mathbb{P}^d(\mathbb{H})$	$\mathbb{P}^{16}(Cay)$
α	(d-2)/2	(d-2)/2	(d-2)/2	(d-2)/2	7
β	(d-2)/2	(d-2)/2	0	1	3

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Consider the constants:

$$\mathcal{R} := \gamma \left(\frac{\Gamma(\alpha+2)\Gamma(\alpha+\beta+2)}{\Gamma(\beta+1)} \right)^{r/d}, \ \ \mathbb{M}^d \in \{\mathbb{S}^d, \mathbb{P}^d(\mathbb{C}), \mathbb{P}^d(\mathbb{H}), \mathbb{P}^{16}(\mathrm{Cay})\},$$

$$\mathcal{R} := \gamma \, (d!)^{r/d}, \ \ \mathbb{M}^d = \mathbb{P}^d(\mathbb{R}),$$

$$\mathcal{C} := \gamma^{d/(d+r)} \left(\frac{d(\ln 2)(d+r)\Gamma(\alpha+1)\Gamma(\alpha+\beta+2)}{2r\Gamma(\beta+1)} \right)^{r/(d+r)}, \ \ \mathbb{M}^d \in \{\mathbb{S}^d, \mathbb{P}^d(\mathbb{C}), \mathbb{P}^d(\mathbb{H}), \mathbb{P}^{16}(\mathrm{Cay})\},$$

$$\mathcal{C} := \gamma^{d/(d+r)} \left(\frac{d!(\ln 2)(d+r)}{r} \right)^{r/(d+r)}, \ \ \mathbb{M}^d = \mathbb{P}^d(\mathbb{R}).$$

Theorem 2.1. *We have that*

$$d_n(\Lambda U_p, L_q) \gg e^{-\mathcal{R} n^{r/d}} \begin{cases} 1, & 1 \le p \le 2, \ 1 < q \le 2, \\ 1, & 2 \le p < \infty, 2 \le q \le \infty, \\ 1, & 1 \le p \le 2 \le q < \infty, \\ (\log n)^{-1/2}, & 1 \le p \le 2, \ q = 1, \\ (\log n)^{-1/2}, & p = \infty, \ 2 \le q \le \infty, \\ (\log n)^{-1/2}, & 1 \le p \le 2, \ q = \infty, \end{cases}$$

and

$$d_n(\Lambda U_p, L_q) \ll e^{-\mathcal{R} n^{r/d}} n^{(1-r/d)(1/p-1/2)} \begin{cases} q^{1/2}, & 1 \le p \le 2 \le q < \infty, \\ (\log n)^{1/2}, & 1 \le p \le 2, \ q = \infty. \end{cases}$$

Theorem 2.2. *We have that*

$$e_k(\Lambda U_p, L_q) \gg \exp\left(-\mathcal{C} k^{r/(d+r)}\right) \begin{cases} 1, & p < \infty, \ q > 1, \\ (\log k)^{-1/2}, & p < \infty, \ q = 1, \\ (\log k)^{-1/2}, & p = \infty, \ q > 1, \\ (\log k)^{-1}, & p = \infty, \ q = 1, \end{cases}$$

and

$$e_k \left(\Lambda U_p, L_q\right) \ll \exp\left(-\mathcal{C} k^{r/(d+r)}\right) \begin{cases} 1, & p \ge 2, 1 \le q < \infty, \\ \log k, & p \ge 2, q = \infty. \end{cases}$$

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PROBLEMA MISTO GERAL PARA A EQUAÇÃO KDV POSTO NA SEMI-RETA

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Há muitos estudos sobre a equação KdV em várias formas. A ela foram dedicados vários trabalhos sobre problemas de valores iniciais e de fronteira, como os de Bona [2] e outros pesquisadores. Um problema misto, com condições mais gerais na fronteira para a equação KdV, em um domínio limitado foi considerado no trabalho de Bubnov [1]. Em nosso trabalho consideramos um problema misto para a equação KdV posto na semi-reta ($x > 0$) com condições gerais na fronteira, a saber:

$$u_t + D^3u + uDu + Du = 0, \quad \text{em } \mathbb{R}^+ \times (0,T); \tag{0.1}$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R}^+; \tag{0.2}$$

$$D^2u(0,t) + \alpha Du(0,t) + \beta u(0,t) = 0, \quad t \in (0,T); \tag{0.3}$$

onde os coeficientes α e β são escalares tais que para qualquer número real $d > 0$

$$\Delta = \beta - \alpha d^{\frac{1}{3}} + d^{\frac{2}{3}} \neq 0,$$
$$\beta > 0 \quad \text{e} \quad |\,\alpha| < min\{2\beta, 1\}, \tag{0.4}$$

onde $T > 0$, $\mathbb{R}^+ = \{x \in \mathbb{R};\ x > 0\}$, $u : \mathbb{R}^+ \times (0,T) \to \mathbb{R}$ é uma função desconhecida, u_t denota sua derivada parcial com respeito a $t > 0$ e D^j significa derivada de ordem $j \in \mathbb{N}$ com respeito a variável espacial x. Como resultado, provamos a existência e unicidade de solução regular deste problema.

1 Problema e resultados principais

O resultado principal deste trabalho é o seguinte:

Teorema 1.1. *Sejam $u_0 \in H^3(\mathbb{R}^+)$, α e β satisfazendo (0.4) e existe um real $k > 0$ tal que*

$$\left(e^{kx}, \left[\sum_{i=0}^{3} |D^i u_0|^2 + |u_0 Du_0|^2\right]\right) < \infty.$$

Então existe $T > 0$ tal que o problema (0.1)-(0.3) tem uma única solução regular:

$$u \in L^\infty(0,T; H^3(\mathbb{R}^+)) \cap L^2(0,T; H^4(\mathbb{R}^+)),$$

$$u_t \in L^\infty(0,T; L^2(\mathbb{R}^+)) \cap L^2(0,T; H^1(\mathbb{R}^+)).$$

Além disso, vale a seguinte estimativa:

$$\sum_{i=0}^{3} \left(e^{kx}, |D^i u|^2\right)(t) + \left(e^{kx}, u_t^2\right)(t) + \int_0^t \left(e^{kx}, |Du_\tau|^2\right)(\tau)d\tau$$

$$\leq \left(e^{kx}, \left[\sum_{i=0}^{3} |D^i u_0|^2 + |u_0 Du_0|^2\right]\right) \textit{para q.t. } t \in (0,T).$$

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Prova: *Existência e Unicidade* - Para mostrar a existência de soluções regulares desse problema usamos o método de semi-discretização com respeito a t. Mais ainda, usamos a função peso exponencial e^{kx} (onde $k > 0$ é a taxa de decaimento dos dados iniciais) para estimar a taxa de decaimento da solução quando $x \to \infty$. Finalmente, depois de fazer algumas estimativas a priori, construimos a solução usando o Teorema do ponto fixo de Banach. ∎

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On an evolution equation with acoustic boundary conditions

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In this paper we analyze from the mathematical point of view a mixed problem for a nonlinear wave equation of Carrier type (see [2] and [3]) with weak internal damping, coupled with acoustic boundary conditions on a portion of the boundary and homogeneous Dirichlet boundary condition on the rest. Our goal is to extend some of the results of Frota-Goldstein [1] in the sense of considering a weaker internal damping and a quadratic non-linearity in the Carrier equation. Precisely, let Ω be an open, bounded and connected set of $\mathbb{R}^n$, $n \leq 4$, with smooth boundary Γ. Suppose Γ is divided into two disjoint portion of positive measure $\Gamma = \Gamma_0 \cup \Gamma_1$ and ν is the outward unit normal vector on Γ. By $Q = \Omega \times (0,T)$, for $T > 0$ a real number, one denotes the cylinder of $\mathbb{R}^{n+1}$ with lateral boundary $\Sigma = \Gamma \times (0,T) = \Sigma_0 \cup \Sigma_1$, being $\Sigma_0 = \Gamma_0 \times (0,T)$ and $\Sigma_1 = \Gamma_1 \times (0,T)$. We study the following initial boundary value problem

$$\left| \begin{array}{l} u''(x,t) - M\Big(\displaystyle\int_\Omega |u(x,t)|^2_{\mathbb{R}}\, dx\Big)\Delta u(x,t) + u^2(x,t) + \beta u'(x,t) = 0 \quad \text{in} \quad Q, \\ u(x,t) = 0 \quad \text{on} \quad \Sigma_0, \\ \rho u'(x,t) + f(x)\delta''(x,t) + g(x)\delta'(x,t) + h(x)\delta(x,t) = 0 \quad \text{on} \quad \Sigma_1, \\ \dfrac{\partial u}{\partial \nu}(x,t) - \delta'(x,t) = 0 \quad \text{on} \quad \Sigma_1, \\ u(x,0) = u_0(x), \;\; u'(x,0) = u_1(x), \qquad x \in \Omega, \\ \delta(x,0) = \delta_0(x), \;\; \delta'(x,0) = \dfrac{\partial u_0}{\partial \nu}(x), \qquad x \in \Gamma, \end{array} \right. \tag{1}$$

where $\beta\,,\rho$ are positive constants, $f, g, h : \overline{\Gamma_1} \to \mathbb{R}$ are given functions and $M(\lambda) = a + b\lambda$, for all $\lambda \geq 0$, with $a\,,\,b$ positive real constants.

Let $\gamma_0 : H^1(\Omega) \to H^{\frac{1}{2}}(\Gamma)$ be the trace map of order zero and let

$$V = \{u \in H^1(\Omega)\,;\, \gamma_0(u) = 0, \text{ a.e. in } \Gamma_0\}$$

equipped with inner product and norm

$$((u,v)) = \sum_{i=1}^{n} \int_\Omega \left(\frac{\partial u}{\partial x_i}\right)\left(\frac{\partial v}{\partial x_i}\right) dx\;,\;\; \|u\| = \left(\int_\Omega |\nabla u|^2_{\mathbb{R}^n}\, dx\right)^{\frac{1}{2}},$$

respectively. The inner product and norm in $L^2(\Omega)$ and $L^2(\Gamma)$ are denoted by $(\cdot,\cdot)$, $|\cdot|$ and $(\cdot,\cdot)_\Gamma$, $|\cdot|_\Gamma$, respectively. Let C_0, C_1, C_2 be constants such that

$$|\gamma_0(\phi)|_\Gamma \leq C_0\|\phi\|\,,\;\; |\phi| \leq C_1\|\phi\| \text{ and } |\phi|_{L^3(\Omega)} \leq C_2\|\phi\|\,, \text{ for all } \phi \in V.$$

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On functions f, g and h we assume the following assumptions:

$$f,\, g,\, h \in \mathrm{C}^0(\overline{\Gamma_1}); \tag{2}$$

$$0 < f_1 = \min_{x\in\overline{\Gamma_1}} f(x) \qquad \text{and} \qquad 0 < h_1 = \min_{x\in\overline{\Gamma_1}} h(x)\,; \tag{3}$$

$$0 < \frac{C_0^2\,\beta\,\rho}{2} \leq g_1 = \min_{x\in\overline{\Gamma_1}} g(x)\,; \tag{4}$$

Now we are in conditions to state:

Theorem 1. *Let (2)-(4) hold. Suppose that $u_0 \in V \cap H^2(\Omega)$, $u_1 \in V$ and $\delta_0 \in L^2(\Gamma)$ satisfy*

$$\frac{2b^2C_1^2}{\rho\beta a^2}\Lambda_0^2 + \frac{4bC_1}{\sqrt{a}}\Lambda_0 + \frac{\sqrt{\rho}\beta C_2^3}{\sqrt{2a}}\Lambda_0^{1/2} < \frac{\rho\beta a}{4}, \tag{5}$$

where

$$\Lambda_0 = \frac{3\rho}{2}|u_1|^2 + \frac{3\rho}{8}\,|u_0|^2 + \left(a + b\,|u_0|^2\right)\left[\rho\,\|u_0\|^2 + \left(\left|f^{1/2}\frac{\partial u_0}{\partial \nu}\right|_{\Gamma_1}^2 + \left|h^{1/2}\delta_0\right|_{\Gamma_1}^2\right)\right] + \frac{2}{3}\rho C_2^3\,\|u_0\|^2\,. \tag{6}$$

Then there exists a unique global weak solution (u, δ) to (1).

Our proof to Theorem 1 is based on the Faedo-Galerkin method following the idea contained in Tartar [4].

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On a Coupled System in Banach Space

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Is this work we study the existence of solutions of the following coupled system in Banach spaces.

$$(*)\quad \left| \begin{array}{l} u'' + M(\|u(t)\|_W^2, \|v(t)\|_W^2)Au + \delta u' = 0 \ in\ L^\infty(0,\infty; D(A^{\frac{1}{2}})) \\ v'' + M(\|u(t)\|_W^2, \|v(t)\|_W^2)Av + \delta v' = 0 \ in\ L^\infty(0,\infty; D(A^{\frac{1}{2}})) \\ u(0) = u^0, v(0) = v^0 \\ u'(0) = u^1,\ \ v'(0) = v^1, \end{array} \right.$$

with $\delta > 0$. Here W is a Banach space, A an unbounded positive self-adjoint linear operator of a Hilbert space H, $M(s,r)$ a smooth function defined in $[0,\infty)^2$ and satisfying $M(s,r) \geq m_0 > 0$.

Problem (*) was motivated by the works of Kirchhoff [12], Carrier [1] and Nayfeh and Mook [11]. An extensive list of references on the Kirchhoff equation can be seen in Medeiros *et al.* [10]

Consider the small vibrations of an elastic stretched string of length L and denote by $(u(x,t), v(x,t), w(x,t)$ the displacement of the point x of the string at the instant t. In [11] is showed that u and v are solutions of Problem (*) with $M(\|u(t)\|_W^2, \|v(t)\|_W^2) = \int_0^L u_x^2 dx + \int_0^L v_x^2 dx$

Recently the second author and *et al* [6] study the existence and decay solutions of the initial value problem for the equation $u''(t) + M(\|u(t)\|_W^2)Au(t) + \delta u'(t) = 0, t > 0$

Let V and H be real separable Hilbert spaces with V densely and continuously embedding in H. Denote by $|u|$ the norm of H. Consider the operator A defined by the triplet $\{V, H, (u,v)_V\}$

We assume that

$(H1)$ W' is strictly convex and W is continuously embedding in D(A)

where W' represents the dual of the space W and D(A), the domain of A. Thus there exist positive constants k_0 and k_1 such that

$\|u\|_W \leq k_0|Au|, \forall u \in D(A)\ \ and\ \ \|u\|_W \leq k_1|A^{3/2}u|, \forall u \in D(A^{3/2})$

We also assume that

$$(H2)\quad \left| \begin{array}{l} M \in C^1([0,\infty) \times [0,\infty)), \\ M(s,r) \geq m_0 > 0,\ \forall s,r \geq 0 \\ M_s(s,r) \geq 0,\ M_r(s,r) \geq 0,\ \forall s,r \geq 0 \\ |M_s(s,r)|s^{\frac{1}{2}} \leq c_0 M(s,r),\ \forall s,r \geq 0 \\ |M_r(s,r)|r^{\frac{1}{2}} \leq c_0 M(s,r),\ \forall s,r \geq 0 \end{array} \right.$$

Theorem 0.1. *Assume hypotheses* $(H1), (H2)$. *Consider*

$(H3)$ $\{u^0, v^0\} \in [D(A^{\frac{3}{2}})]^2,\ \{u^1, v^1\} \in [D(A)]^2$

satisfying

$(H4)$ $2\ c_0 k_0 M^{1/2}(k_1^2\varphi(0), k_1^2\varphi(0))\varphi^{1/2}(0) < \delta.$

where

$\varphi(0) = \frac{|Au^1|^2 + |Av^1|^2}{M(\|u^0\|_W^2, \|v^0\|_W^2)} + |A^{3/2}u^0|^2 + |A^{3/2}v^0|^2$

Then there exists a unique pair of functions $\{u,v\}$ *in the class*

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$\{u, v\} \in [L^\infty(0, \infty; D(A^{\frac{3}{2}}))]^2;$
$\{u', v'\} \in [L^\infty(0, \infty; D(A))]^2;$
$\{u'', v''\} \in [L^\infty(0, \infty; D(A^{\frac{1}{2}}))]^2,$

satisfying

$$(*) \quad \left| \begin{array}{l} u'' + M(\|u(t)\|_W^2, \|v(t)\|_W^2)Au + \delta u' = 0 \ in \ L^\infty(0, \infty; D(A^{\frac{1}{2}})) \\ v'' + M(\|u(t)\|_W^2, \|v(t)\|_W^2)Av + \delta v' = 0 \ in \ L^\infty(0, \infty; D(A^{\frac{1}{2}})) \\ u(0) = u^0, \ v(0) = v^0 \\ u'(0) = u^1, \ v'(0) = v^1 \end{array} \right.$$

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NONCONSTANT STABLE EQUILIBRIA INDUCED BY SPATIAL DEPENDENCE IN NONLINEAR BOUNDARY CONDITIONS*

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An important question in problems modeling time-dependent phenomena is stability of its stationary solutions (also called equilibrium solutions or equilibria, for short). It is well known that for some semilinear parabolic equations with gradient-like structure, equilibrium solutions play a fundamental role since understanding the long term dynamics depends heavily on its existence and stability. See, for instance, [1, 2, 8] and the references therein.

In this work we consider the problem

$$\begin{cases} \partial_t u = \Delta u \qquad \text{in} \quad \Omega \times \mathbb{R}^+ \\ \\ \dfrac{\partial u}{\partial \nu} = \varepsilon \eta(x) g(u) \qquad \text{on} \quad \partial\Omega \times \mathbb{R}^+ \end{cases} \tag{0.1}$$

where $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ is a bounded smooth domain, ν the outward pointing normal vector, $\varepsilon > 0$ a parameter and $g(u) = u - u^3$ is the prototype balanced bistable function. We suppose that $\eta \in C^{1,\gamma}(\partial\Omega)$, for some $0 < \gamma < 1$, satisfies

(H) $\quad \eta$ is a indefinite weight and $\displaystyle\int_{\partial\Omega} \eta(x)\, d\mathcal{H}^{n-1} \neq 0$

where $\mathcal{H}^{n-1}$ denotes the $(n-1)$-dimensional Hausdorff measure and an indefinite weight means a function which assume both positive and negative values. Our main goal is, following [7], to prove existence of two nonconstant exponentially stable equilibria to (0.1)—stability in Lyapunov sense—one of them positive and the other one negative, if $\varepsilon > 0$ is sufficiently large in any n-dimensional smooth domain $(n \geq 2)$.

Related to the question of existence of nonconstant stable equilibria to parabolic problems under linear or nonlinear boundary conditions, the geometry of the domain plays a fundamental role in answering many of them. For instance, when $\eta \equiv$ const, that is, when there is no explicit spatial dependence in the boundary condition, the geometry of the domain determines the existence of nonconstant stable equilibrium solutions to (0.1). Indeed, if $\Omega = B_R(0)$ (the n-dimensional ball with radius R and centered at the origin) then there is no nonconstant stable equilibrium solution to (0.1) whereas such solutions can be created for dumbbell type, thus non-convex, domains. See [4, 5]. Still for the case $\eta \equiv$ const, existence of such solutions in three-dimensional convex domains has been established in [6].

When $n = 2$, $\eta \equiv 1$ and $g(x, u) = u(1-u)(c(x)-u)$, where $0 < c(x) < 1$, existence of nonconstant stable equilibrium solutions to (0.1) was proved in [3] for smooth planar domains. Our work corroborates the assertion that explicit spatial dependence on boundary condition for reaction-diffusion equations can be a mechanism to create nonconstant stable equilibrium solutions. We emphasize that, under our hypotheses, problem (0.1) exemplifies this property can holds true in arbitrary smooth domains $\Omega \subset \mathbb{R}^n$, $n \geq 2$.

The condition that the parameter is large is actually necessary. Indeed, there is no nonconstant stable equilibria to (0.1) when the parameter is small because we prove that there are no equilibria to (0.1) besides the constant ones if $\varepsilon > 0$ is small enough. It is also proved, via Implicit Function Theorem, that nonconstant stable equilibria obtained are isolated and depend smoothly on the parameter.

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Application on stability of differential equations with piecewise constant argument using dichotomic map

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Abstract

The asymptotic stability of the equation $x'(t) = -bx(t) + f(x([t]))$ with argument $[t]$, where $[t]$ designates the greatest integer function, is studied by means of dichotomic maps.

Summary

The study of differential equations with piecewise continuous argument has been subject of recent investigations [3] and in the stability study of this type of equation using dichotomic maps some literature can be cited [1, 2, 4, 5, 6].

We proved that the null solution of the equation $x^{'}(t) = -bx(t)+cx([t])$ is asymptotically stable since $b \geq \delta > 0$, $|c| < k\delta$ and $k \in (0, 1)$ [4]. The same result was proved to the equation $x^{'}(t) = -b(t)x(t) + c(t)x([t])$ since $b : \mathbb{R}_+ \to \mathbb{R}_+^*$ and $c : \mathbb{R}_+ \to \mathbb{R}$ be a continuous maps such that $0 < d \leq b(t) < +\infty$ and $| c(t) | \leq b < \mu.d$ for all t and for some $0 < \mu < 1$ [5].

The aim of this work is to extend the result to the equation

$$x'(t) = -bx(t) + f(x([t])) \qquad (1.1)$$

using dichotomic maps, with imposed conditions about the function f and the parameter b. This equation is a particular case of the equation

$$x'(t) = f(t, x(t), x([t])) \qquad (1.2)$$

where $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous map with $f(t, 0, 0) = 0$ for all $t \in \mathbb{R}$.

We denote by $x(., t_o, \psi)$ the solution of (1.2) with $x_{t_o}(., t_o, \psi) = \psi$ and $x_t(., t_o, \psi)(\theta) = x(t+\theta, t_o, \psi), \theta \in [-1, 0]$, $\psi \in C$, where C denotes Banach space of the continuous maps from [-1,0] into $\mathbb{R}^n$. The solution through $\psi \equiv 0$, that is, $x(., t_o, 0)$, is the null solution.

If $V : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is a continuous map, roughly speaking, we say that V is dichotomic with respect to (1.2) if, for all points where the derivative of V with respect t along (1.2) is nonnegative at time t, then there exists a previous instant $\bar{t}$, $\bar{t} < t$ such that $V(t, x(t)) \leq V(\bar{t}, x(\bar{t}))$.

V is strictly dichotomic with respect to (1.2) when (i) if V is as above, then we must have $p(V(t, x(t)) < V(\bar{t}, x(\bar{t}))$, with p a continuous and nondecreasing map satisfying $p(y) > y$, $y \in (0, \delta)$ for some $\delta > 0$ and $t - \bar{t} \leq M < \infty$, and (ii) if the derivative of V with respect to t along a solution tends to zero as $t \to \infty$ and if V tends to a constant function as $t \to \infty$, it must imply that this solution tends to the null solution as $t \to \infty$. We will use the following results to prove the desired result [2].

Theorem 0.1. *: Let $u, v : \mathbb{R}_+ \to \mathbb{R}_+$ be continuous, nondecreasing functions which are positive for $s > 0$ and $u(0) = v(0) = 0$. If there exists a positive definite dichotomic map with respect to (1.2), $V : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ such that $u(|x|) \leq V(t, x) \leq v(|x|)$, $t \in \mathbb{R}$, $x \in \mathbb{R}^n$, then the null solution of (1.2) is stable.*

Theorem 0.2. *Let V be a continuously differentiable strictly dichotomic map with respect to (1.2) in Theorem (0.1). Then the null solution of (1.2) is asymptotically stable.*

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Now we present the main result.

Theorem 0.3. *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map and $b \in \mathbb{R}$, $b > 1$. If $f(x) \le x$ for $x \ge 0$, $f(x) \ge x$ for $x \le 0$ and $xf(x) > 0$ for $x \ne 0$, then the null solution of (1.1) is stable. Moreover, if f is continuously differentiable at zero and $0 < f'(0) < 1$, then the null solution of (1.1) is asymptotically stable.*

Proof The idea of the proof is to suppose $V'(x_t) \ge 0$ for some t with $V(x) = \frac{x^2}{2}$, that is,

$$V'(x_t) = x(t).x'(t) = x(t)[-bx(t) + f(x([t]))] \ge 0$$

and to analyze the cases $x(t) \ge 0$ and $x(t) < 0$. If $x(t) \ge 0$, then $-bx(t) + f(x([t])) \ge 0$, that is, $f(x([t])) \ge bx(t)$. By hypothesis, x and $f(x)$ have the same signal and, since $f(x([t])) \ge bx(t) \ge 0$, then $x([t]) \ge f(x([t])) \ge bx(t)$. Therefore $x^2([t])/2 \ge b^2x^2(t)/2 > x^2(t)/2$, that is, $V(x([t])) > V(x(t))$. The case $x(t) \le 0$ is analogous. Hence, in both cases, we have that if $V'(x_t) \ge 0$, then there exists a previous instant $[t]$, $[t] \le t$ such that $V(x(t)) \le V(x([t]))$. So, V is dichotomic with respect to (1.1).
Taking $v(x) = x^2$ and $u(x) = x^2/4$, by Theorem (0.1), we have that the null solution of (1.1) is stable. Now, we will prove that V is asymptotically dichotomic with respect to (1.1). Since $0 < f'(0) < 1$, there is a ball $B(0, \epsilon_o)$ centered at zero with radius $\epsilon_o > 0$ such that, for $x \in B(0, \epsilon_o)$, $0 < f'(x) < 1$. Since the null solution of (1.1) is stable, we can take $\delta_o > 0$ such that if $|\psi| < \delta_o$, then $x(t, 0, \psi) \in B(0, \epsilon_o)$ and $0 < f'(x(t, 0, \psi)) < 1$ for all $t \ge 0$ and all $|\psi| < \delta_o$. Suppose now $V'(x(t)) \ge 0$ and $x(t) \ge 0$; the case $x(t) \le 0$ is similar. It follows that $f(x([t])) \ge bx(t)$. By Mean Value Theorem, $f(x([t])) - f(0) = f'(w)x([t]) \ge bx(t)$. Since $w \in B(0, \epsilon_o)$, $f'(w) = K < 1$, we have that $Kx([t]) \ge bx(t)$. We choose L, $0 < K < L < 1$ and so $x([t]) > \frac{b}{L}x(t)$, that is, $\frac{x^2([t])}{2} > \frac{b^2}{2L^2}x^2(t), 0 < L < 1$. By using $p(y) = \frac{b^2}{L^2}y$ for $y > 0$, we obtain that whenever $V'(x_t) \ge 0$ for some t, we have an anterior instant $[t]$ such that $p(V(x(t))) < V(x([t]))$. Now, let $V'(x_t) \to 0$ and $x(t) \to w$ when $t \to \infty$.
Then $0 = \lim_{t\to\infty} x(t)[-bx(t) + f(x([t]))] = w[-bw + f(w)]$, that is, $w = 0$ or $f(w) = bw$.
If $w > 0$, by hypothesis $f(w) \le w$, it follows that $bw \le w$. So, $(b - 1)w \le 0$ and since $w > 0$ we have $b \le 1$, a contradiction. If $w < 0$, we obtain similar contradiction. Therefore $w = 0$ and V is a strictly dichotomic map with respect to (1.1). By theorem (0.2), the null solution of (1.1) is asymptotically stable.

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DECAY RATES FOR EIGENVALUES OF POSITIVE INTEGRAL OPERATORS ON THE SPHERE

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1 Introduction

Let m be a positive integer at least 2, S^m the unit sphere in $\mathbb{R}^{m+1}$ and $d\sigma_m$ the surface element of S^m. Let K be a positive definite kernel on S^m ([1]), that is, a function $K : S^m \times S^m \to \mathbb{C}$ from $L^2(S^m \times S^m, \sigma_m \times \sigma_m)$ satisfying

$$\sum_{\mu=1}^{n}\sum_{\nu=1}^{n} c_\mu \overline{c_\nu} K(x_\mu, x_\nu) \geq 0, \tag{1.1}$$

for all positive integer n, complex numbers $c_1, c_2, \ldots, c_n$ and points $x_1, x_2, \ldots, x_n$ in S^m, and consider the integral operator given by the formula

$$\mathcal{K}(f)(x) = \int_{S^m} K(x,y) f(y)\, d\sigma_m(y), \quad x \in S^m. \tag{1.2}$$

Since K is hermitian a.e., the formula (1.2) defines a self-adjoint compact operator in $L^2(S^m, \sigma_m) := L^2(S^m)$. As so, the spectrum of $\mathcal{K}$ consists of at most countably many nonnegative eigenvalues which we assume arranged in decreasing order

$$\lambda_1(\mathcal{K}) \geq \lambda_2(\mathcal{K}) \geq \ldots \geq 0, \tag{1.3}$$

repeating each one according to its algebraic multiplicity.

The main result here will describe decay rates for the sequence $\{\lambda_n(\mathcal{K})\}$, assuming a smoothness assumption of Lipschitz type on the kernel K. To explain that, denote by d_m the usual geodesic distance on S^m. One of its closed forms is

$$d_m(x,y) = \arccos(1 - 2^{-1}\|x-y\|^2), \quad x, y \in S^m, \tag{1.4}$$

in which $\|\cdot\|$ stands for the usual norm in $\mathbb{R}^{m+1}$. If $\beta > 0$ and $B \in L^1(S^m)$ then a kernel K is said to be (B,β)-*Lipschitz* (with respect to d_m) when

$$|K(w,x) - K(w,y)| \leq B(w) d_m(x,y)^\beta, \quad x, y, w \in S^m. \tag{1.5}$$

2 Main Result

The main achievement in this note is Theorem 2.1 described at the end. The approach we adopt to prove the theorem is based upon an interesting result from operator theory which provides an alternative interpretation for the singular values of an operator acting on a Hilbert space. We will write $\mathcal{B}(H)$ to denote the vector space of all bounded operators on a Hilbert space H.

Proposition 2.1. *([3], p. 51) Let T be a compact operator on a Hilbert space H. Arrange the singular values of T in decreasing order taking into account multiplicities, say $s_1(T) \geq s_2(T) \geq \cdots 0$. Then*

$$s_n(T) = \min\{\|T - U\|_H : U \in \mathcal{B}(H) \text{ and } \operatorname{rank} U \leq n-1\}, \quad n = 1, 2, \ldots. \tag{2.1}$$

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The number on the right-hand side of the above equation is usually called the *n-approximation number* of the operator T.

Returning to the integral operator $\mathcal{K}$, basic functional analysis implies that it has a unique square root $\sqrt{\mathcal{K}}$ obeying the formula

$$\sqrt{\mathcal{K}}(f)(x) = \int_{S^m} K^{1/2}(x,y)f(y)\, d\sigma_m(y), \quad f \in L^2(S^m), \quad x \in S^m. \tag{2.2}$$

In addition to that, the eigenvalues of $\sqrt{\mathcal{K}}$ and $\mathcal{K}$ are related to each other via the formula $\lambda_n(\mathcal{K})^{1/2} = \lambda_n(\sqrt{\mathcal{K}})$, $n = 1, 2, \ldots$. Then, it is quite clear that ([4, 7])

$$\lambda_n(\sqrt{\mathcal{K}}) = \min\{\|\sqrt{\mathcal{K}} - U\|_2 : U \in \mathcal{B}(L^2(S^m)) \text{ and rank } U \leq n-1\}, \quad n = 1, 2, \ldots. \tag{2.3}$$

Therefore, our strategy will be to obtain a sharp bound on the n-approximation number of $\sqrt{\mathcal{K}}$. That will be done by computing the expression $\|\sqrt{\mathcal{K}} - U\|_2$ for a specially chosen operator U of rank at most $n-1$ on $\mathcal{B}(L^2(S^m))$, namely, a composition of a convenient spherical convolution operator with $\sqrt{\mathcal{K}}$ itself. The convenient convolution we use is based on the so-called *generalized Jackson kernel* ([5, 6]). It depends upon two positive integers l and $\mu \geq 2$ and is given by the formula

$$\mathcal{D}_\nu(t) = \frac{1}{k_\nu}\left(\frac{\sin(\mu t/2)}{\sin(t/2)}\right)^{2l}, \quad t \in [0, \pi], \quad \nu = l(\mu - 1), \tag{2.4}$$

where the constant k_ν is chosen so that

$$\int_{S^m} \mathcal{D}_\nu(d_m(x,y))d\sigma_m(y) = \sigma_{m-1}, \quad x \in S^m. \tag{2.5}$$

Since $\mathcal{D}_\nu$ is an even n-order trigonometric polynomial in the variable t, the formula

$$R_\nu(f)(x) = \frac{1}{\sigma_{m-1}} \int_{S^m} \mathcal{D}_\nu(d_m(x,y))f(y)d\sigma_m(y), \quad f \in L^2(S^m), \quad x \in S^m, \tag{2.6}$$

defines a spherical polynomial $R_\nu(f)$ of degree ν in the variable x. Thus $R_\nu(\sqrt{\mathcal{K}})$ is a linear operator of rank at most ν and we estimate the expression $\|\sqrt{\mathcal{K}} - R_\nu(\sqrt{\mathcal{K}})\|_2$ in order to obtain an upper bound for $\lambda_\nu(\sqrt{\mathcal{K}})$. Since $\nu = l(\mu - 1)$, a convenient choice for l and μ then provides the desired upper bound for $\lambda_n(\sqrt{\mathcal{K}})$ when n is large enough.

Theorem 2.1. *If K is (B, β)-Lipschitz, then there exist a constant C depending on m and β only such that $\lambda_n(\mathcal{K}) \leq Cn^{-\beta}$, as long as $n > m + \beta/2$.*

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MULTIPLICIDADE DE SOLUÇÕES PARA UM PROBLEMA ELÍPTICO EM $\mathbb{R}^N$ ENVOLVENDO EXPOENTE CRÍTICO E FUNÇÃO PESO

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Estabelecemos a existência e multiplicidade de soluções não triviais do seguinte problema

$$(P_{\lambda,f}) \qquad \begin{cases} -\Delta u = \lambda f(x) u^{q-1} + u^{2^*-1}, & \text{em} \quad \mathbb{R}^N \\ 0 \le u \in \mathcal{D}^{1,2}(\mathbb{R}^N), \end{cases}$$

onde $1 < q < 2 < 2^* = \frac{2N}{N-2}$, $N \ge 3$, λ um parâmetro positivo e a função f satisfaz as seguintes condições:

(H) $f \doteq f_+ + f_-$ ($f_+ = \max\{f, 0\}$, $f_- = \min\{f, 0\}$) é uma função mensurável, localmente limitada sobre $\mathbb{R}^N \backslash \{0\}$, com $0 \not\equiv f_+ \in C(\mathbb{R}^N \backslash \{0\})$ e

$$f(x) = \begin{cases} O(|x|^b), & \text{quando} \quad |x| \to 0 \\ O(|x|^a), & \text{quando} \quad |x| \to \infty, \end{cases}$$

para números reais a, b satisfazendo

$$a < \frac{N}{2^*}(q - 2^*) < b.$$

Tais restrições nas condições de crescimento da função peso f são necessárias para que tenhamos uma condição de compacidade. Condições semelhantes foram utilizadas por Szulkin e Willem [11]. Ainda Egnell [9], bem como Noussair, Swanson e Yang [10] demonstraram que tais restrições nas condições de crescimento são necessárias para que haja solução não trivial.

Nos últimos anos, inúmeros trabalhos apresentaram resultados de existência e multiplicidade de soluções fracas para equações elípticas semilineares com crescimento crítico. Citamos o trabalho pioneiro de Ambrosetti, Brézis e Cerami [2] que, dentre outros problemas, analisou a existência e multiplicidade de soluções positivas de

$$(P_{\lambda,f,\Omega}) \qquad \begin{cases} -\Delta u = \lambda f(x) u^{q-1} + u^{p-1}, & \text{em} \quad \Omega \\ u = 0, & \text{sobre} \quad \partial\Omega \end{cases}$$

onde $1 < q < 2 < p \le 2^*$ e Ω é domínio limitado de $\mathbb{R}^N$. Considerando o caso particular em que $f \equiv 1$, eles provaram a existência de $\lambda_0 > 0$, tal que o problema $(P_{\lambda,f,\Omega})$, admite ao menos duas soluções positivas se $\lambda \in (0, \lambda_0)$, possui uma solução positiva para $\lambda = \lambda_0$ e não possui solução para $\lambda > \lambda_0$. Recentemente, ainda com $f \equiv 1$ e para o caso em que $\Omega = B_N(0,1)$, ou seja, Ω é a bola unitária, os autores Adimurthi, Pacella e Yadava [1], Damascelli, Grossi e Pacella [6] e Tang [12] provaram que existem exatamente duas soluções positivas para $\lambda \in (0, \lambda_0)$ e apenas uma solução positiva para $\lambda = \lambda_0$. Ainda no caso de Ω ser um domínio limitado qualquer, Wu em [13], considerou o problema $(P_{\lambda,f,\Omega})$ sob a hipótese que $f \in C(\overline{\Omega})$, com $f_+ \not\equiv 0$. Sob tais condições ele garantiu, através de métodos variacionais sobre a variedade de Nehari, a existência de $\lambda_0 > 0$ de modo que o problema $(P_{\lambda,f,\Omega})$ admite ao menos duas soluções positivas para $\lambda \in (0, \lambda_0)$. Para outros resultados similares sobre domínios limitados citamos dentre outros os artigos [4], [7] e suas referências.

Em todo o espaço, ou seja, quando Ω é igual a $\mathbb{R}^N$, os autores Garcia e Peral [3], além de outros resultados, provaram através de métodos variacionais, a existência de $\lambda_0 > 0$ de modo que o problema $(P_{\lambda,f})$ admite ao menos

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duas soluções não negativas para $\lambda \in (0, \lambda_0)$, sob as condições que $f \in L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ e $f_+ \not\equiv 0$. Para mais resultados relacionados em todo o espaço fazemos menção à [5], [8], bem como suas referências.

A seguir enunciamos o nosso resultado:

Teorema 0.1. *Suponha que f é uma função mensurável em $\mathbb{R}^N$ satisfazendo (H). Então existe uma constante positiva $\Lambda = \Lambda(q, f, N)$, onde para todo $\lambda \in (0, \Lambda)$ o problema $(P_{\lambda,f})$ possui ao menos duas soluções não triviais.*

O nosso teorema pode ser visto como uma generalização dos resultados acima mencionados, principalmente dos resultados obtidos em [3] e [13], pois uma classe mais geral de funções peso é considerada. Por exemplo, a função

$$f(x) = -|x|^{-2}\chi_A(|x|) + \chi_B(|x|)|3x - 6|^{-N} + |x|^{-N}\chi_D(|x|),$$

onde $A = (0, 1)$, $B = [2, 3)$ e $D = [3, \infty)$, satisfaz a hipótese (H), mas não está em $L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ e também não pertence à $C(\mathbb{R}^N)$, condições estas que a função peso f deve satisfazer em [3] e [13] respectivamente. Do mesmo modo que [3], usamos métodos variacionais, mais precisamente, combinamos técnicas de minimização e uma versão do Teorema do passo da montanha sem a condição de Palais-Smale, além disso obtemos um comportamento "preciso" dos níveis de Palais-Smale.

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PROBLEMA DO TIPO AMBROSETTI-PRODI PARA UM SISTEMA ENVOLVENDO O OPERADOR $p-$LAPLACIANO

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Este trabalho destina-se ao estudo de um problema do tipo Ambrosetti-Prodi para um sistema elítico envolvendo o operador $p-$Laplaciano. Problemas do tipo Ambrosetti-Prodi são amplamente estudados por diversos autores e receberam essa terminologia devido aos primeiros matemáticos que o resolveram em 1972. Ambrosetti e Prodi em [1] mostraram, utilizando teoremas de inversão para aplicações diferenciáveis com singularidades em espaços de Banach, a existência de uma variedade $\mathcal{M}$ de classe C^1 em $C^{0,\alpha}(\Omega)$ que divide o espaço em duas componentes conexas $\mathcal{N}$ e $\mathcal{O}$ de modo que o problema

$$\begin{cases} -\Delta u &= f(u) + g(x), \quad \text{em} \quad \Omega \\ u &= 0, \quad \text{sobre} \quad \partial\Omega \end{cases} \tag{0.1}$$

não possui soluções se $g \in \mathcal{N}$, possui exatamente uma solução se $g \in \mathcal{M}$ e possui exatamente duas soluções se $g \in \mathcal{O}$. Nesse caso f é uma função a valores reais de classe C^2 satisfazendo $f''(s) > 0$, para todo $s \in \mathbb{R}$ e $0 < \lim_{s\to-\infty} f'(s) < \lambda_1 < \lim_{s\to+\infty} f'(s) < \lambda_2$, com λ_1, λ_2 o primeiro e o segundo autovalor de $(-\Delta, W_0^{1,2}(\Omega))$ respectivamente. Com diferentes variantes e formulações, vários autores vem estudando problemas do tipo Ambrosetti-Prodi para operadores lineares de segunda ordem.

Mais recentemente problemas do tipo Ambrosetti-Prodi envolvendo o operador $p-$Laplaciano tem sido estudados por Arcoya e Ruiz [2], bem como Koizumi e Schmidt [7], entre outros, utilizando principalmente a teoria do grau de Leray-Schauder.

Para o caso de sistemas existem resultados para o caso de operadores lineares de segunda ordem uniformemente elíticos, onde podemos citar [3,4,5], entre outros. A respeito de sistemas que envolvem o operador $p-$Laplaciano, com $p \neq 2$, não conhecemos nenhuma referência ou resultado. Motivados pelos trabalhos de [3] e [5] estudaremos o seguinte sistema

$$(S_t) \qquad \begin{cases} -\Delta_p u_1 = f_1(x, u_1, u_2) + t_1\phi_1 + h_1, & \text{em} \quad \Omega \\ -\Delta_p u_2 = f_2(x, u_1, u_2) + t_2\phi_2 + h_2, & \text{em} \quad \Omega \\ u_1 = u_2 = 0, & \text{sobre} \quad \partial\Omega, \end{cases}$$

onde $h_i, \phi_i \in L^\infty(\Omega)$, com $\phi_i \succ 0$ em Ω, para $i = 1,2$ e $t = (t_1, t_2) \in \mathbb{R}^2$ é um parâmetro.

O sistema (S_t) pode ser reescrito na forma matricial

$$-\Delta_p u = f(x,u) + t\phi + h, \quad \text{em} \;\; \Omega \quad u = 0 \;\; \text{sobre} \;\; \partial\Omega,$$

onde $u = (u_1, u_2)^T$, $h = (h_1, h_2)^T$, $f(x,u) = (f_1(x,u_1,u_2), f_2(x,u_1,u_2))^T$ e $t\phi = (t_1\phi_1, t_2\phi_2)^T$.

Além disso, $f_i : \overline{\Omega} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ é uma função contínua satisfazendo as seguintes condições:

(H1) $|f_i(x, s_1, s_2)| \leq C(1 + |s_1|^{q_i} + |s_2|^{r_i})$, $1 < r_i, q_i \leq p-1$, $i = 1,2$,

(H2) existe $\sigma > 0$ tal que $f_i(x,s_1,s_2) + \sigma|s_i|^{p-2}s_i$ é não decrescente em s_i, para todo $(x, s_j) \in \overline{\Omega} \times \mathbb{R}^2$, $i = 1,2$,

(H3) $f_i(x,0,0) = 0$ e f_i é quase-monótona para todo $(x,s_i) \in \overline{\Omega} \times \mathbb{R}$, isto é, para cada $i \neq j$, $i = 1,2$, $f_i(x,s_1,s_2)$ é não decrescente em u_j,

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(H4) existem matrizes estritamente cooperativas ($a_{ij}, c_{ij} > 0$ para $i \neq j$),

$$A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad A_2 = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

com $a_{11} + a_{12} \leq 0$, $a_{21} + a_{22} \leq 0$ e existe $\delta \in [0,1]$ tal que $\lambda_1 - \delta c_{11} - (1-\delta)c_{22} < 0$, sendo λ_1 o primeiro autovalor de $(-\Delta_p, W_0^{1,p}(\Omega))$, tais que $\lambda_1(\Delta_p + A_1) > 0, \quad \lambda_1(\Delta_p + A_2) < 0$, onde

$$\lambda_1(\Delta_p + A_i) = \sup \mathcal{E}(A_i),$$

$$\mathcal{E}(A_i) = \{\lambda \in \mathbb{R} : \exists \varphi \in (W_0^{1,p}(\Omega))^2 : \varphi > 0, \Delta_p \varphi + (A_i + \lambda I)\Psi_p(\varphi) \leq 0\},$$

com $\Psi_p(s) = (\psi_p(s_1), \psi_p(s_2))^T$ e $\psi_p(s_i) = |s_i|^{p-2}s_i$. Além disso, existem constantes $b_1, b_2 > 0$ tais que

$$f(x,s) \geq A_1\Psi_p(s) - b_1\bar{e}, \forall s \leq 0, \quad f(x,s) \geq A_2\Psi_p(s) - b_2\bar{e}, \forall s \geq 0,$$

para $s = (s_1, s_2) \in \mathbb{R}^2$, onde $\bar{e} = (1,1)^T = e^T$,

(H5) para cada sequência $\{s_n\} \subset \mathbb{R}^2$ tal que $\|s_n^-\|$ é limitado e $\|s_n^+\| \to \infty$ quando $n \to \infty$,

$$\liminf_{n \to \infty} \frac{f(x, s_n) - f(x, s_n^+)}{\|s_n^+\|^{p-1}} \geq 0.$$

A seguir enunciaremos o nosso resultado:

Teorema 0.1. *Suponha que $(H1) - (H5)$ ocorrem. Então existem curvas Lipschitzianas Γ^* e Γ_* que dividem $\mathbb{R}^2$ em três conjuntos disjuntos $\mathcal{M}$, $\mathcal{N}$ e $\mathcal{O}$ tais que o problema (S_t):*

i) *possui ao menos duas soluções para $t \in \mathcal{M}$,*

ii) *possui ao menos uma solução para $t \in \Gamma^* \cup \Gamma_* \cup \mathcal{O}$,*

iii) *não possui solução para $t \in \mathcal{N}$.*

Uma das soluções é encontrada através do método de sub e supersolução. Para a segunda solução vamos utilizar a teoria do grau de Leray-Schauder. Como é conhecido, para aplicarmos essa teoria precisamos obter estimativas a priori das eventuais soluções. Para obter tais estimativas fez-se necessário o uso da teoria das soluções de viscosidade e uma adaptação das estimativas ABP obtidas em [6].

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UNIFORM CONVERGENCE THEOREM FOR THE KURZWEIL INTEGRAL FOR RIESZ SPACE-VALUED FUNCTIONS

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We prove that the uniform convergence of a sequence of Kurzweil integrable functions imply the convergence of the sequence formed by its corresponding integrals.

1 Introduction

In [4], it was proved that:

If X is a Dedekind σ-complete weakly σ-distributive Riesz space, $f \in X^{[a,b]}$ is a bounded function and $(f_n)_{n\in N}$ is a sequence of Kurzweil integrable functions on $[a,b]$ such that $f_n \xrightarrow{u} f$, then

$$f \text{ is Kurzweil integrable on } [a,b] \text{ and } \int_a^b f_n \xrightarrow{o} \int_a^b f\,. \tag{1.1}$$

Our purpose here is to obtain (1.1) without the assumption that f is bounded. In the sequel follows some defintions.

A Riesz space X is a real vector space on which a partial ordering $\le$ compatible with its vector space structure is defined, that is, $(X,\le)$ is a lattice. A sequence $(x_n)_{n\in N}$ of elements of X is said to be (o)-convergent to $x \in X$, and we write $x_n \xrightarrow{o} x$, if there exists a nonincreasing sequence $(a_n)_{n\in N}$ of elements of X such that

$$a_n \searrow 0 \quad \text{and} \quad |x_k - x| \le a_k\,, \quad \text{for all } k \in \mathbb{N}\,.$$

A sequence $(f_n)_{n\in N}$ of elements of $X^{[a,b]}$ is said to be uniformly convergent to $f \in X^{[a,b]}$, and we write $f_n \xrightarrow{u} f$, if there exists a nonincreasing sequence $(u_n)_{n\in N}$ of elements of X such that

$$u_n \searrow 0 \quad \text{and} \quad |f_k(x) - f(x)| \le u_k\,, \quad \text{for all } x \in [a,b] \text{ and } k \in \mathbb{N}. \tag{1.2}$$

A bounded double sequence $(a_{ij})_{ij}$ of elements of X is said to be a (D)-sequence if, for each $i \in \mathbb{N}$, the sequence $(a_{ij})_{j\in N}$ is nonincreasing and $a_{ij} \searrow 0$ as $j \to \infty$. The Riesz space X is said to be a Dedekind σ-complete weakly σ-distributive Riesz space, if it is Dedekind σ-complete Riesz space (i.e., every non-empty, enumerable subset that has upper bound admits a least upper bound or supremum) and

$$\bigwedge_{\varphi\in N^N}\left(\bigvee_{i=1}^{\infty} a_{i\varphi(i)}\right) = 0\,, \quad \text{for all } (D)\text{-sequence } (a_{ij})_{ij} \text{ of elements of } X.$$

A function $f \in X^{[a,b]}$, where X is a Dedekind σ-complete weakly σ-distributive Riesz space, is said to be Kurzweil integrable on [a,b] if there exists $L := \displaystyle\int_a^b f \in X$, satisfying :
there is a (D)-sequence $(a_{ij})_{ij}$ of elements of X such that, for every $\varphi \in \mathbb{N}^N$, there is $\delta = \delta(\varphi) \in \mathbb{R}_+{}^{[a,b]}$ such that

$$\left|\sum_{\Pi} f - \int_a^b f\right| \le \bigvee_{i=1}^{\infty} a_{i\varphi(i)}\,, \text{ for all } \Pi \in \mathcal{A}_{[a,b]}(\delta)\,. \tag{1.3}$$

The set $\mathcal{A}_{[a,b]}(\delta)$ consists of all δ-fine partitions Π of $[a,b]$, that is, $\Pi := \{(I_k,\alpha_k)\}_{k\in\Lambda}$, $\Lambda \subset \mathbb{N}$ finite and $I_k \subset\,]\alpha_k - \delta(\alpha_k)\,,\, \alpha_k + \delta(\alpha_k)[$, for all $k \in \Lambda$.

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2 Uniform convergence theorem

The proof of Theorem 2.1 follows closely the ideas presented by Riečan and Vrábelová in [4].

Theorem 2.1 (Uniform convergence theorem). *Let X be a Dedekind σ-complete weakly σ-distributive Riesz space and $(f_n)_{n\in\mathbb{N}}$ be a sequence of Kurzweil integrable functions on $[a,b]$. If $f_n \xrightarrow{u} f$, then*

$$f \text{ is Kurzweil integrable on } [a,b] \quad \text{and} \quad \int_a^b f_n \xrightarrow{o} \int_a^b f$$

Proof. Since $f_n \xrightarrow{u} f$, there exists a sequence $(u_n)_{n\in\mathbb{N}}$ as in (1.2). Observing that a sequence which is uniformly convergent is also Cauchy uniformly, by Proposition 3.2.1 in [1], there exists $L \in X$ such that $\int_a^b f_n \xrightarrow{o} L$. Thus, there exists a nonincreasing sequence $(w_n)_{n\in\mathbb{N}}$ of elements of X such that

$$w_n \searrow 0 \quad \text{and} \quad \left|\int_a^b f_k - L\right| \le w_k, \quad \text{for all } k \in \mathbb{N}\,.$$

For each $n \in \mathbb{N}$, there is a (D)-sequence $\left(a_{ij}^{(n)}\right)_{ij}$ satisfying the Kurzweil integrability condition (1.3) for f_n. Given $\varphi \in \mathbb{N}^{\mathbb{N}}$ and considering $p := \min_{i\in\mathbb{N}} \varphi(i+1)$, for $\Pi \in \mathcal{A}_{[a,b]}(\delta_p)$, we have

$$\left|\sum_{\Pi} f - L\right| \le u_p\,(b-a) + w_p + \bigvee_{i=1}^{\infty} a_{i\varphi(i+p+1)}^{(p)}\,, \text{ for all } \Pi \in \mathcal{A}_{[a,b]}(\delta_n)\,. \tag{2.4}$$

Let $S := u_1\,(b-a) + w_1 + \bigvee_{i=1}^{\infty} a_{ii}^{(1)}$ and a triple sequence $(c_{nij})_{nij}$ of elements of X defined by

$$c_{nij} := \begin{cases} w_j + u_j\,(b-a) & \text{if} \quad n=1 \\ a_{ij}^{(n-1)} & \text{if} \quad n \ge 2 \end{cases}, \quad i,j \in \mathbb{N}\,.$$

By the Lemma 2 in [3], there exists a double sequence $(b_{ij})_{ij}$ on X such that $(S \wedge b_{ij})_{ij}$ is a (D)-sequence and

$$S \wedge \left(\sum_{r=1}^{n} \bigvee_{i=1}^{\infty} c_{ri\psi(i+r)}\right) \le \bigvee_{j=1}^{\infty} (S \wedge b_{j\psi(j)})\,, \quad \text{for all } n \in \mathbb{N} \text{ and } \psi \in \mathbb{N}^{\mathbb{N}}. \tag{2.5}$$

Rewriting the inequality (2.4), we obtain

$$\left|\sum_{\Pi} f - L\right| \le \bigvee_{j=1}^{\infty} c_{1j\varphi(j+1)} + \bigvee_{i=1}^{\infty} c_{(p+1)i\varphi(i+p+1)} \le \sum_{r=1}^{p+1} \bigvee_{i=1}^{\infty} c_{ri\varphi(i+r)}. \tag{2.6}$$

On the other hand, $\left|\sum_{\Pi} f - L\right| \le u_1\,(b-a) + w_1 + \bigvee_{i=1}^{\infty} a_{ii}^{(1)} = S$. Therefore, for a $\delta \in {\mathbb{R}_+}^{[a,b]}$ sufficiently fine, applying (2.6) and (2.5), we conclude the proof. ∎

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APPROXIMATION OF COMPACT HOLOMORPHIC MAPPINGS IN RIEMANN DOMAINS OVER BANACH SPACES

JORGE MUJICA *

1 Introduction

Let $\mathcal{H}(U)$ denote the vector space of all complex-valued holomorphic functions on a nonempty open subset U of a complex Banach space E. Let τ_0, τ_ω and τ_δ respectively denote the compact-open topology, the compact-ported topology and the bornological topology on $\mathcal{H}(U)$. We refer to the books of Dineen [4] or Mujica [9] for background information on infinite dimensional complex analysis.

The study of the approximation property for the spaces $(\mathcal{H}(U), \tau_0)$, $(\mathcal{H}(U), \tau_\omega)$ and $(\mathcal{H}(U), \tau_\delta)$ was initiated by Aron and Schottenloher [2]. More recently Boyd, Dineen and Rueda [3] studied the approximation property for the space of holomorphic functions on U which are weakly uniformly continuous on U-bounded sets, with its natural topology. In those papers the authors obtained their best results when $U = E$ or when U is a balanced open subset of E.

In a more recent paper Dineen and Mujica [5] extended some of the results of Aron and Schottenloher on $(\mathcal{H}(U), \tau_0)$ to the case of arbitrary open sets, and promised to devote a subsequent paper to the study of $(\mathcal{H}(U), \tau_\omega)$.

A forthcoming paper of Dineen and Mujica [6] is devoted to the study of the approximation property for the spaces $(\mathcal{H}(U), \tau_\omega)$ and $(\mathcal{H}(U), \tau_\delta)$. This note is an announcement of the main results in [6]. Dineen and Mujica [6] observe at the outset that the spaces $(\mathcal{H}(U), \tau_\omega)$ and $(\mathcal{H}(U), \tau_\delta)$ behave very differently from the space $(\mathcal{H}(U), \tau_0)$. Indeed among other results Dineen and Mujica [5] proved that if E is a Banach space with a Schauder basis, then $(\mathcal{H}(U), \tau_0)$ has the approximation property for every open subset U of E. On the other hand, it follows from a remark of Floret [7] that if U is any open subset of the Hilbert space ℓ_2, then neither $(\mathcal{H}(U), \tau_\omega)$ nor $(\mathcal{H}(U), \tau_\delta)$ has the approximation property.

In their main result Dineen and Mujica [6] show that if E is a Banach space with a shrinking Schauder basis, and with the property that every continuous complex-valued polynomial on E is weakly continuous on bounded sets, then $(\mathcal{H}(U), \tau_\omega)$ and $(\mathcal{H}(U), \tau_\delta)$ have the approximation property for every open subset U of E. The classical space c_0, the original Tsirelson space T^* and the Tsirelson*-James space T_J^* are examples of Banach spaces which satisfy the hypotheses of our main theorem.

Even though Dineen and Mujica [6] are mainly interested in the study of holomorphic functions defined on open subsets of Banach spaces, they deal more generally with holomorphic functions defined on Riemann domains over Banach spaces. The reason is that their proofs rely heavily on results and techniques from the theory of holomorphic approximation in pseudoconvex Riemann domains over Banach spaces with a Schauder basis, and the fact, established by Alexander [1] and Hirschowitz [8] that every space of the form $(\mathcal{H}(U), \tau_\delta)$, with $U \subset E$ open, is topologically isomorphic to a space of the form $(\mathcal{H}(\widehat{U}), \tau_\delta)$, where $\widehat{U}$ is a pseudoconvex Riemann domain over E.

2 Main results

Let E and F be Banach spaces, let (X, ξ) be a Riemann domain over E, and let $\mathcal{H}_k(X; F)$ denote the vector space of all compact holomorphic mappings from X into F. With this notation the main results in [6] are the following:

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Theorem 2.1. *Let E be a Banach space, and let (X,ξ) be a Riemann domain over E. Then:*

(a) If $\mathcal{H}(X)\otimes F$ is dense in $(\mathcal{H}_k(X;F),\tau_\omega)$ for every Banach space F, then $(\mathcal{H}(X),\tau_\omega)$ has the approximation property.

(b) If E is separable, (X,ξ) is connected, and $\mathcal{H}(X)\otimes F$ is dense in $(\mathcal{H}_k(X;F),\tau_\delta)$ for every Banach space F, then $(\mathcal{H}(X),\tau_\delta)$ has the approximation property.

Theorem 2.2. *Let E be a Banach space with a shrinking Schauder basis, and with the property that every continuous complex-valued polynomial on E is weakly continuous on bounded sets. Let (X,ξ) be a connected Riemann domain over E. Then:*

(a) $\mathcal{H}(X)\otimes F$ is sequentially dense in $(\mathcal{H}_k(X;F),\tau_\delta)$ for every Banach space F.

(b) $(\mathcal{H}(X),\tau_\omega)$ and $(\mathcal{H}(X),\tau_\delta)$ have the approximation property.

Theorem 2.1 is obtained with the aid of the ε-product of Laurent Schwartz. In Theorem 2.2, (b) follows from (a) by Theorem 2.1. Theorem 2.2(a) is obtained with the aid of results and techniques from the first paper of Dineen and Mujica [5] and from a recent paper of Mujica and Vieira [11]. A preliminary version of Theorem 2.2, when X is a pseudoconvex open subset of E, was announced at the Second ENAMA, in João Pessoa, in November 2008. To delete the hypothesis of pseudoconvexity it was necessary to consider the case of Riemann domains. Versions of some of the results in [2] and [5], and preliminary versions of some of the results in [6], with complete proofs, were presented by Mujica [10] at a five-lecture course at the Universidad Complutense de Madrid, in June 2009. I am indebted to José M. Ansemil for his kind invitation to deliver that course.

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Operadores de composição entre álgebras de Fréchet uniformes

C. NACHTIGALL *

Este trabalho tem por objetivo principal estudar as relações entre operadores de composição da forma $T_g : A \to A$ e as respectivas aplicações $g : M_A \to M_A$, onde M_A é o espectro da uF-álgebra A. Os resultados apresentados fazem parte da tese de doutorado que está sendo desenvolvida junto ao IMECC - UNICAMP, sob a orientação da professora Daniela Mariz Vieira e co-orientação do professor Jorge Mujica.

Definição 0.1. Seja A uma F-álgebra com unidade e. Denotaremos por M_A o conjunto de todos os homomorfismos complexos contínuos e não nulos da álgebra A. Dizemos que M_A é o *espectro* de A. Para cada $f \in A$, definimos a função $\hat{f} : M_A \to \mathbb{C}$ tal que $\hat{f}(\phi) = \phi(f)$, para toda $\phi \in M_A$, onde $\hat{f}$ é chamada de transformação de Gelfand de f, e denotamos $\hat{A} = \{\hat{f}|_{M_A}; f \in A\}$. Chamamos de *topologia de Gelfand* em M_A, a menor topologia que torna cada transformação de Gelfand contínua em M_A. Desta forma, a aplicação $f \to \hat{f}$ é um isomorfismo topológico entre A e $\hat{A}$, e podemos considerar $A \subset C(M_A)$. Denotemos por β a topologia forte em M_A, induzida por (A', β).

Definição 0.2. Uma F-álgebra A é chamada de *uF-álgebra* (*álgebra de Fréchet uniforme*) se $p_n(f^2) = p_n(f)^2$, para toda $f \in A$, onde $(p_n)_{n\in\mathbb{N}}$ é uma sequência de seminormas que gera a topologia de A.

Toda uB-álgebra (álgebra de Banach uniforme) é uma álgebra de uF-álgebra. O espaço da funções contínuas em um $K-$espaço hemicompacto $X = \cup_{n\in\mathbb{N}} K_n$, denotado por $C(X)$, com a topologia dada pelas seminormas $p_n(f) = \sup_{y\in K_n} |f(x)|$, assim como o espaço das funções holomorfas do tipo limitado, denotado por $H_b(U)$, onde U é um aberto em um espaço de Banach, munido da topologia da convergência uniforme sobre os $U-$limitados, são exemplos de uF-álgebras.

Definição 0.3. Sejam A uma uF-álgebra e $T : A \longrightarrow A$ um operador linear. Dizemos que T é um *Operador de Composição* se existe uma aplicação contínua $g : M_A \longrightarrow M_A$ tal que $T(f) = \hat{f} \circ g$, para toda $f \in A$. Neste caso denotamos $T = T_g$.

Na verdade, vale que todo homomorfismo unitário $T : A \to A$ (i.e., T é contínuo, linear, multiplicativo e $T(e) = e$) é um operador de composição, onde $g : M_A \longrightarrow M_A$ é dada por $T'|_{M_A}$ e T' é o operador adjunto de T.

Definição 0.4. Sejam A uma uF-álgebra e $T : A \longrightarrow A$ um operador contínuo. Dizemos que T é *pontualmente limitado* se existe uma vizinhança de zero $V \subset A$ tal que $T(V) \subset A \subset C(M_A)$ é pontualmente limitado, isto é, para cada $x \in M_A$, o conjunto $C_x = \{T(f)(x) : f \in V\}$ é limitado em $\mathbb{C}$.

A seguir, apresentamos os principais resultados obtidos:

Proposição 0.1. Sejam X um K-espaço hemicompacto e $T : A \subset C(X) \longrightarrow C(X)$ um operador de composição pontualmente limitado, onde A é uma subálgebra fechada de $C(X)$. Então $T(A) \subset C_b(X)$ e T é contínuo, onde $C_b(X)$ denota a uB-álgebra formada pelas funções de $C(X)$ que são limitadas em X, com a norma do supremo.

Corolário 0.1. Sejam A uma uF-álgebra e $T : A \longrightarrow A$ um operador pontualmente limitado. Então $T(A) \subset B$, onde B é uma uB-álgebra. Consequentemente, T é contínuo.

O corolário 0.1 generaliza o teorema 2.1 de [1]. Em [1], o resultado é obtido para o caso particular em que $A = H_b(U)$ e $B = H^\infty(U)$ (funções holomorfas e limitadas em U).

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Proposição 0.2. Sejam A uma uF-álgebra e $T_g : A \longrightarrow A$ um operador de composição compacto. Se $L \subset M_A$ é um conjunto β-limitado em M_A então $g(L)$ é β-relativamente compacto em M_A.

A proposição 0.2 generaliza parte da proposição 2.12 de [1], onde o resultado é obtido no caso particular em que $A = H_b(U)$ e L é um conjunto $U-$limitado.

Seja Y um conjunto qualquer e suponhamos que $Y = \bigcup_{n\in\mathbb{N}} Y_n$, onde $Y_n \subset Y_{n+1}$, para todo $n \in \mathbb{N}$. Vamos dizer que um subconjunto $A \subset Y^{\mathbb{C}} = \{f : Y \longrightarrow \mathbb{C}\}$ é uma *álgebra uniforme no conjunto* Y se A é uma sub-álgebra das funções em Y que são limitadas em cada Y_n, separam pontos de Y e contém as constantes. Neste caso, dizemos que A é uma Y*-álgebra* e denotamos $A = A(Y)$. Vamos considerar em A a topologia gerada pelas semi-normas $p_n(f) = \sup_{y\in Y_n} |f(y)|$, $\forall f \in A$. Com esta topologia, A é uma uF-álgebra. Os espaços $C(X)$ e $H_b(U)$ citados acima são exemplos de uF-álgebras que são $Y-$álgebras. Notemos que o conjunto Y pode ser identificado com um subconjunto do espectro de $A(Y)$, através da aplicação avaliação $\delta : Y \rightarrow M_A$, onde $\delta_y(f) = f(y)$, para cada $f \in A = A(Y)$. No que segue, não distinguiremos Y de $\delta(Y) \subset M_A$. Definimos em Y a topologia mais fraca que torna todas as funções de A contínuas em Y. A seguir, temos uma aplicação da proposição 0.2 para $A = A(Y)$:

Corolário 0.2. Sejam $A = A(Y)$ uma uF-álgebra e $T_g : A \longrightarrow A$ um operador de composição compacto. Então $g(Y_n)$ é β-relativamente compacto em M_A.

Um homomorfismo unitário $T : A(Y) \rightarrow A(Y)$ é um *operador de composição em* Y quando $g|_Y : Y \rightarrow Y$. Estamos interessados em obter condições para que um homomorfismo unitário $T : A(Y) \rightarrow A(Y)$ seja um operador de composição em Y. Dizemos que um sunconjunto $L \subset Y$ é $Y-$*limitado* se existe $n \in \mathbb{N}$ tal que $L \subset Y_n$. Dizemos que Y é $A-$*convexo* se $(\hat{Y_n})_A = \{y \in Y : |f(y)| \leq \sup_{z\in Y_n} |f(z)|, \forall f \in A\}$ é $Y-$limitado, para cada $n \in \mathbb{N}$.

Proposição 0.3. Sejam $A = A(Y)$ uma uF-álgebra e $T = T_g : A \rightarrow A$ um homomorfismo unitário. Então T é um operador de composição em Y se e somente se para cada $n \in \mathbb{N}$, existe $n_k \in \mathbb{N}$ tal que $g(Y_n) \subset (\hat{Y_{n_k}})_A$.

Corolário 0.3. Sejam $A = A(Y)$ uma uF-álgebra e $T = T_g : A \rightarrow A$ um homomorfismo unitário, onde Y é A-convexo. Então T é um operador de composição em Y se e somente se para cada $n \in \mathbb{N}$, existe $n_k \in \mathbb{N}$ tal que $g(Y_n) \subset (Y_{n_k})_A$.

O corolário 0.3 generaliza a proposição 3.1.1 de [2], onde o resultado é obtido para a uF-álgebra $A(Y) = H_b(U)$.

Proposição 0.4. Sejam $A = A(Y)$ uma uF-álgebra e $T = T_g : A(Y) \rightarrow A(Y)$ um operador de composição em Y. Então T_g é pontualmente limitado se e somente se existe $n_0 \in \mathbb{N}$ tal que $g(Y) \subset (\hat{Y_{n_0}})_A$.

Corolário 0.4. Sejam $A = A(Y)$ uma uF-álgebra e $T = T_g : A(Y) \rightarrow A(Y)$ um operador de composição em Y, onde Y é um conjunto $A-$convexo. Então T_g é pontualmente limitado se e somente se existe $n_0 \in \mathbb{N}$ tal que $g(Y) \subset Y_{n_0}$.

Proposição 0.5. Sejam E um espaço de Banach com a propriedade de aproximação e $U \subset E$ um conjunto $H_b(U)-$convexo e equilibrado. Seja $T : H_b(U) \rightarrow H_b(U)$ um homomorfismo unitário que é τ_0 pontualmente contínuo em conjuntos limitados de $H_b(U)$. Então T é um operador de composição em U, onde τ_0 indica a topologia compacto aberta em $H_b(U)$.

O corolário 0.4 e a proposição 0.5 generalizam 2.3-b e 2.3-a de [1], respectivamente, onde os resultados são obtidos para um operador de composição $T_g : H_b(U) \rightarrow H_b(U)$, onde U é um aberto absolutamente convexo de E.

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Exponential attractor for a nonlinear dissipative beam equation of Kirchhoff type

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In this paper we prove the existence and uniqueness of global solution and existence of a exponential attractor for the following Kirchhoff type beam equation

$$\begin{cases} u_{tt} + \Delta^2 u - M(\|\nabla u\|^2)\Delta u + g(u_t) + f(u) = 0 \text{ in } \Omega \times \mathbb{R}^+, \\ u = \frac{\partial u}{\partial \nu} = 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) \end{cases}$$

where β is a positive constant, M is a continuously differentiable real value function with $M(s) \geq 0$, for all $s \geq 0$, Ω is a bounded open set of $\mathbb{R}^n$ with smooth boundary Γ, and the functions

$$g(u_t) \approx \beta u_t \text{ and } f(u) \approx |u|^\rho u$$

are, respectively, linear dissipation and forcing term.

The results of existence and uniqueness of local solution are obtained using the theory of semigroups [16]. We show that the solution exists globally obtaining uniform estimates. Finally, we show that the dynamical system associated to this equation posses a global attractor and a finite dimensional exponentially attracting set, employing the methods given in [20] and [8].

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A CRITERION FOR NUCLEARITY OF POSITIVE INTEGRAL OPERATORS

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We investigate the nuclearity of positive integral operators on $L^2(X,\mu)$ when X is a Hausdorff locally compact second countable (LCH) space and μ is a non-degenerate and finite Borel measure. This setting includes the case in which X is a compact metric space and μ is a special finite measure. The results apply to spheres, tori and other important subspaces of the usual space $\mathbb{R}^m$.

The main result in this note can be seen as a generalization of another one proved in [8] for the case $X = [a, b]$. The proof there used in a key manner the so-called Steklov's smoothing operator to construct an averaging process to generate a convenient approximation to the integral operator $\mathcal{K}$ generated by a kernel $K : X \times X \to \mathbb{C}$. The upgrade to the case in which X is a subspace of $\mathbb{R}^n$ was discussed in [6] and references therein. By assuming that the Lebesgue measure of nonempty intersections of X with open balls of $\mathbb{R}^n$ was positive and using auxiliary approximation integral operators generated by an averaging process constructed via the Hardy-Littlewood theory, the main result in [6] described necessary and sufficient conditions for the traceability of the integral operator, under the assumption of positive definiteness of the kernel. Unfortunately, this result excluded some important cases such as spheres and tori. Thus, one of the achievements in the present note is the filling in of such gap, still using a similar average process but dropping the restrictive assumption of finiteness on the measure.

Since our spaces are no longer metric, the Hardy-Littlewood theory in the average arguments need to be replaced. We will rely on techniques involving martingales following very closely the development of Brislawn in [1]. In other words, we will define auxiliary integral operators based on a martingale constructed from special partitions of X. The main difference between the construction delineated here and that in [1] is that in the present one we need to guarantee that the elements in the partitions belong to the topology of X.

1 Main result

Let X be a (LCH) topological space endowed with a non-degenerate and finite Borel measure μ. We shall investigate the nuclearity of integral operators $\mathcal{K} : L^2(X,\mu) \to L^2(X,\mu)$ generated by a suitable kernel $K : X \times X \to \mathbb{C}$ from $L^2(X \times X, \mu \times \mu)$. The setting just described allows the space $L^2(X,\mu)$ to have a countable complete orthonormal subset ([5, p.92]) while the operator $\mathcal{K}$, which is given by the formula

$$\mathcal{K}(f) := \int_X K(\cdot, y) f(y)\, d\mu(y), \quad f \in L^2(X,\mu), \tag{1.1}$$

becomes compact. As so, the spectral theorem for compact operators is applicable and $\mathcal{K}$ can be represented in the form

$$\mathcal{K}(f) = \sum_{n=1}^{\infty} \lambda_n \langle f, f_n \rangle f_n, \quad f \in L^2(X,\mu), \tag{1.2}$$

in which $\{\lambda_n\}$ is a sequence of real numbers (possibly finite) converging to 0 and $\{f_n\}$ is a complete orthonormal sequence in $L^2(X,\mu)$. The symbol $\langle \cdot, \cdot \rangle$ will stand for the usual inner product of $L^2(X,\mu)$.

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The basic requirement on the kernel K will be its positive definiteness. A kernel K from $L^2(X \times X, \mu \times \mu)$ is $L^2(X,\mu)$-*positive definite* when the corresponding integral operator $\mathcal{K}$ is positive:

$$\langle \mathcal{K}(f), f\rangle \geq 0, \quad f \in L^2(X,\mu). \tag{1.3}$$

Fubini's theorem is all that is need in order to show that a $L^2(X,\mu)$-positive definite kernel is hermitian $\mu \times \mu$-a.e.. As so, $\mathcal{K}$ is self-adjoint with respect to $\langle \cdot, \cdot\rangle$. In particular, the sequence $\{\lambda_n\}$ mentioned in the previous paragraph needs to be entirely composed of nonnegative numbers.

Nuclearity of $\mathcal{K}$ refers to the property

$$\sum_{f\in\mathfrak{B}} \langle \mathcal{K}^*\mathcal{K}(f), f\rangle^{1/2} < \infty \tag{1.4}$$

for every orthonormal basis $\mathfrak{B}$ of $L^2(X,\mu)$. In the formula above, $\mathcal{K}^*$ is the adjoint of $\mathcal{K}$. The main result establishes a necessary and sufficient condition on K in order that $\mathcal{K}$ be nuclear. Its description depends upon a special martingale $\{E_n(f)\}$ defined via the formula

$$E_n(f)(x) = \frac{1}{\mu(O_n(x))} \int_{O_n(x)} f\, d\mu, \tag{1.5}$$

in which $\{O_n(x)\}$ is a suitable family of open subsets of X.

The main result is as follows.

Theorem *Let K be $L^2(X,\mu)$-positive definite. The integral operator $\mathcal{K}$ is nuclear if and only if*

$$\limsup_{n\to\infty} \int_X E_n(K)(x,x)\, d\mu(x) < \infty. \tag{1.6}$$

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UNIQUENESS THEOREM FOR THE MODIFIED HELMHOLTZ EQUATION INVERSE SOURCE PROBLEM.

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If we consider the uniqueness problem of reconstruction of an unknown characteristic source inside a domain modeled by Poisson equation with a source given by a non homogeneous characteristic star-shape function, a well known result, back to 1938, by Novikov , see [5], says that this kind of source can be reconstructed uniquely from the Cauchy boundary data. In this work we assume that the model is giving by a modified Helmholtz equation, in which the Laplacian operator is perturbed by an absorption term, and proof's a uniqueness result for the case of characteristic convex domain. The main application are in stationary and transient inverse problems modeled with partial differential equations.

Let $\Omega \subset \mathbb{R}^N$ an bounded regular domain. Let $\omega \subset \Omega$ open with C^2 boundary. Let χ_ω the characteristic function of the set ω. The direct problem with the modified Helmholtz operator: to find a regular field u that satisfy the system

$$\begin{cases} -\Delta u + \kappa^2 u = & f\chi_\omega \quad \Omega \\ u = g & \partial\Omega \\ \kappa \in L^\infty(\Omega) & f \in L^2(\Omega). \end{cases} \tag{0.1}$$

is well posed and has a unique solution $u \in H^1(\Omega)$.

The inverse source problem consists in by knowing the Cauchy data in the boundary $\partial\Omega$, that is the Dirichlet to Neumann map in at least one Dirichlet datum g, to recover the source $f\chi_\omega$. This problem has been studied for generic sources by [1] who shown that it is useless to change the input Dirichlet data g. The unique information available is given by only one measurement, say, that Neumann boundary measurements

$$\partial_\nu u = g_\nu. \tag{0.2}$$

1 Mathematical Results

Theorem 1.1. *Consider the direct problem (0.1) and its associated inverse problem (0.2) with two sources $f\chi_{\omega_1}$ and $f\chi_{\omega_2}$. Suppose that $f(x) > 0$ for $x \in \omega$. Let ω_1 , $\omega_2 \subset \Omega$ domains with C^2 boundary and $\omega_1 \backslash \omega_2$, $\omega_2 \backslash \omega_1$, $\omega_1 \cap \omega_2$ convex. If the Cauchy data for the two problems are the same, then $\omega_1 = \omega_2$.*

Proof. Suppose that $\omega_1 \neq \omega_2$. Let the u_1 e u_2 respective solutions for (0.1) for the same Cauchy data. Then, $w = u_1 - u_2$ satisfies

$$\begin{cases} -\Delta w + \kappa^2 w = & f(\chi_{\omega_1} - \chi_{\omega_2}) = F \quad \Omega \\ w = 0 & \partial\Omega \\ \frac{\partial w}{\partial \nu} = 0 & \partial\Omega \end{cases} \tag{1.3}$$

where, $w \in H^2(\Omega) \cap H^1_0(\Omega)$ is the unique solution of (1.3). We also note that $F = 0$ if $x \in \Omega \backslash (\omega_1 \cup \omega_2)$; $F = f$ if $x \in \omega_1 \backslash \omega_2$; $F = -f$ if $x \in \omega_2 \backslash \omega_1$ and $F = 0$ if $x \in \omega_1 \cap \omega_2$.

From now on, for simplicity, we omit the zero Cauchy data for w, $(w, \frac{\partial w}{\partial \nu}) = (0, 0)$ on $\partial\Omega$, $\partial\omega_2 \backslash \omega_1$, $\partial\omega_1 \backslash \omega_2$. This is a direct consequence of Homesgren's theorem applied to the regular region $\Omega \backslash (\omega_1 \cup \omega_2)$.

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Now we applied the Maximum Modulo Principle to the region $\omega_2 \backslash \omega_1$ for which

$$-\Delta w + \kappa^2 w = -f < 0 \tag{1.4}$$

to obtain that there exist $x_1 \in \partial\omega_1 \cap \omega_2$ with $w(x_1) > 0$ such that $w(x_1) \geq w(x)$, $\forall\ x \in \omega_1 \backslash \omega_2$, or there exist $x_3 \in (\omega_2 \backslash \omega_1)^{\circ}$ such that $w(x_3) < 0$.

The results above assures that w is not constant for all interior points of $\omega_2 \backslash \omega_1$. The Strong Maximum Principle established that the second condition above and the normal derivative of w at the boundary $\omega_2 \backslash \omega_1$can not be zero simultaneously, which implies the occurrence of first above condition. In an analogous way, for region $\omega_1 \backslash \omega_2$,

$$-\Delta w + \kappa^2 w = f > 0 \tag{1.5}$$

and there exist $x_2 \in \partial\omega_2 \cap \omega_1$ with $w(x_2) < 0$ such that $w(x_2) \leq w(x)$, $\forall\ x \in \omega_2 \backslash \omega_1$.

Also, for region $\omega_1 \cap \omega_2$

$$-\Delta w + \kappa^2 w = 0. \tag{1.6}$$

and it follows from the Maximum Modulus Principle that there exist x_1', $x_2' \in \omega_1 \cap \omega_2$ such that $w(x_1') \geq w(x) \geq w(x_2')$ $\forall\ x \in \omega_1 \cap \omega_2$. It can be proved that $x_1' = x_1$ e $x_2' = x_2$. Note that also w can not be constant in regions $\omega_2 \backslash \omega_1$, $\omega_1 \backslash \omega_2$.

Now suppose that w is constant in $\omega_1 \cap \omega_2$. This will imply that $w \equiv 0$, and by continuity we have that $w(x_1) = 0$ e $w(x_2) = 0$. Applying the Maximum Modulus principle to region (1.5) and (1.4)we obtain:

i. There exist $x_3 \in (\omega_2 \backslash \omega_1)^{\circ}$ such that $w(x_3) < 0$ and $\frac{\partial w}{\partial \nu} = 0$ on $\partial\omega_2 \backslash \omega_1$.

ii. There exist $x_4 \in (\omega_1 \backslash \omega_2)^{\circ}$ such that $w(x_4) > 0$ and $\frac{\partial w}{\partial \nu} = 0$ on $\partial\omega_1 \backslash \omega_2$.

Again by the Strong Maximum Principle we obtain that (i) e (ii) cannot occurs simultaneously. This contradiction shows that w is not constant inside $\omega_1 \cap \omega_2$

Suppose now that w also is not constant in $\omega_1 \cap \omega_2$. By the Strong Maximum Principle applied to equation (1.6) it follows that the exterior normal derivative of w in x_2 is positive and consequently the normal interior derivative is negative. If we again applies the Strong Maximum Principle to the region of equation (1.5) we obtain that the exterior normal derivative of w in x_2 is positive, and since $w \in H^2(\Omega) \cap H_0^1(\Omega)$ we have that the normal interior in (1.6) and exterior in (1.5) must coincides. Since one is positive and the other is negative, we obtain a contradiction with the hypotheses that $\omega_1 \neq \omega_2$.

□

We also present numerical results from computational experiments.

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Método de Camadas de Potencial para o Problema Compressível de Navier-Stokes

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Considere as equações de Navier Stokes Compressíveis:

$$\begin{cases} \rho \dfrac{D\boldsymbol{v}}{Dt} - \mu\rho\Delta\boldsymbol{v} - \nu\rho\nabla \operatorname{div} \boldsymbol{v} + \nabla P(\rho) & = & \rho \boldsymbol{f} \\ \rho_t + \operatorname{div}(\rho \boldsymbol{v}) & = & 0 \end{cases} \tag{0.1}$$

com dados iniciais de fronteira:

$$\begin{cases} \boldsymbol{v} = 0 \quad sobre \quad \partial D \times [0,T] \\ \boldsymbol{v}\big|_{t=0} = \boldsymbol{v}_0 \ , \quad \rho\big|_{t=0} = \rho_0 \quad sobre \quad D \end{cases} \tag{0.2}$$

definido em $(\boldsymbol{x}, t) \in D_T = D \times (0,T)$, onde $D \subset \mathbb{R}^3$, é um domínio limitado do plano com fronteira Lipschitz ∂D (veja também Brown [1]), $\boldsymbol{v} = (v^1, v^2, v^3)$ é a velocidade do fluido, ρ é a densidade, $P = P(\rho)$ a pressão e $\boldsymbol{f}$ a força externa. Os termos da difusão μ e da convecção ν, são tais que $\mu < \nu$.

1 Equações Lineais

Considere o sistema Linearizado

$$\begin{cases} \boldsymbol{u}_t - \mu\Delta\boldsymbol{u} - \nu\nabla \operatorname{div} \boldsymbol{u} + \dfrac{c^2}{\rho}\nabla\eta & = & \boldsymbol{f} \ , & em \quad D_T \\ \eta_t + \rho \operatorname{div} \boldsymbol{u} & = & g \ , & em \quad D_T \\ \boldsymbol{u} & = & 0 \ , & sobre \quad S_T \\ \boldsymbol{u}\big|_{t=0}, \ \eta\big|_{t=0} & = & 0 \ , & sobre \quad D \end{cases} \tag{1.3}$$

Usando a segunda equação do sistema (1.3) na primeira, temos

$$\frac{\partial \boldsymbol{u}}{\partial t} = \mu\Delta\boldsymbol{u} + \nu\nabla \operatorname{div} \boldsymbol{u} + c^2 \int_0^t (\nabla \operatorname{div} \boldsymbol{u})(\boldsymbol{x}, s)\, ds + \boldsymbol{F}(\boldsymbol{x}, t) \tag{1.4}$$

onde $\boldsymbol{F} = \boldsymbol{f} - \dfrac{c^2}{\rho} \displaystyle\int_0^t (\nabla g)(\boldsymbol{x}, s)\, ds.$

2 Solução Fundamental

Seja $\mathscr{X} = \operatorname{div} \boldsymbol{u}$, $\boldsymbol{\mathscr{W}} = \nabla \times \boldsymbol{u}$. Então temos

$$\begin{aligned} \frac{\partial \mathscr{X}}{\partial t} &= (\mu+\nu)\Delta\mathscr{X} + c^2 \int_0^t \Delta\mathscr{X}\, ds + \operatorname{div} \boldsymbol{F} \\ \frac{\partial \boldsymbol{\mathscr{W}}}{\partial t} &= \mu\Delta\boldsymbol{\mathscr{W}} + \nabla \times \boldsymbol{F} \end{aligned}$$

A solução fundamental da primeira equação pode ser escrita como,

$$\Gamma(\boldsymbol{x}, t) = \boldsymbol{G}_{\mu+\nu}(\boldsymbol{x}, t) + \Gamma_1^+(\boldsymbol{x}, t) + \Gamma_1^-(\boldsymbol{x}, t) + \Gamma_2(\boldsymbol{x}, t) \tag{2.5}$$

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tal que

$$\mathscr{W} = \boldsymbol{G}_{\mu} \circledast (\nabla \times \boldsymbol{F}) \qquad , \qquad \mathscr{X} = \Gamma \circledast \operatorname{div} \boldsymbol{F}$$

Então

$$\boldsymbol{u}(\boldsymbol{x}, t) \quad = \quad [\boldsymbol{G}_{\mu} + \mathcal{A}\, \boldsymbol{G}_{\mu} - \mathcal{A}\, \boldsymbol{\Gamma}] \circledast \boldsymbol{F}$$

onde $\mathcal{A} = (-\Delta)^{-1}\nabla \operatorname{div}$. Então a solução do sistema (1.4), em D, tem a forma

$$\boldsymbol{u}(\boldsymbol{x}, t) \quad = \quad \int_0^t \int_D \mathbb{G}(\boldsymbol{x} - \boldsymbol{y}, t - \tau) \boldsymbol{F}(\boldsymbol{y}, \tau)\, d\boldsymbol{y}\; d\tau$$

onde

$$\mathbb{G}(\boldsymbol{x}, t) = \boldsymbol{G}_{\mu}(\boldsymbol{x}, t) + (-\Delta)^{-1}\nabla \operatorname{div} \boldsymbol{G}_{\mu}(\boldsymbol{x}, t) - (-\Delta)^{-1}\nabla \operatorname{div} \boldsymbol{\Gamma}(\boldsymbol{x}, t) \tag{2.6}$$

Substituindo a equação (2.5), temos

$$\boxed{\mathbb{G}_{ij}(\boldsymbol{x}, t) = \Gamma_{ij}(\boldsymbol{x}, t) - \mathscr{R}_i \mathscr{R}_j \left(\Gamma_1^+ + \Gamma_1^- + \Gamma_2\right)(\boldsymbol{x}, t)} \tag{2.7}$$

onde $i,\ j = 1, 2, 3.$ $\Gamma_{ij}(\boldsymbol{x}, t) = \delta_{ij} G_{\mu}(\boldsymbol{x}, t) + \displaystyle\int_{\mu t}^{(\mu+\nu)t} \frac{\partial^2 G}{\partial x_i \partial x_j}(\boldsymbol{x}, \theta)\, d\theta$ é o núcleo achado por Brown e Shen [2].

3 Resultados

Potências de camada dupla são estabelecidos no *Problema de Navier Stokes Compressível*. Mediante o calculo da condição de salto na fronteira é possível estabelecer existência e unicidade da solução como potencial de camada. De maneira geral temos

$$\mathscr{D}(\boldsymbol{g})(\boldsymbol{p}, t) \quad = \quad \frac{1}{2}\mathscr{D}_0(\boldsymbol{p}, t)\boldsymbol{g}(\boldsymbol{p}, t) + K(\boldsymbol{g})(\boldsymbol{p}, t)$$

onde

$$\mathscr{D}_0(\boldsymbol{p}, t) \quad = \quad \int_0^t h(t - s)\, ds + \frac{1}{\mu} g_j(\boldsymbol{p}, t) - 2AN_j(\boldsymbol{p}) \langle \boldsymbol{N}(\boldsymbol{p}), \boldsymbol{g}(\boldsymbol{p}, t)\rangle$$

$$K(\boldsymbol{g}(\boldsymbol{p}, t)) \quad = \quad p.v. \int_0^t \int_{\partial D} \frac{\partial\, \mathbb{G}}{\partial \boldsymbol{N}(\boldsymbol{p})}(\boldsymbol{p} - \boldsymbol{q}, t - s) g_j(\boldsymbol{q}, s)\, d\boldsymbol{q}\; ds$$

e

$$K^*(\boldsymbol{g}(\boldsymbol{p}, t)) \quad = \quad p.v. \int_0^t \int_{\partial D} \frac{\partial\, \mathbb{G}}{\partial \boldsymbol{N}(\boldsymbol{q})}(\boldsymbol{p} - \boldsymbol{q}, t - s) g_j(\boldsymbol{q}, s)\, d\boldsymbol{q}\; ds$$

Lema 3.1. *Seja D um domínio Lipschitz em $\mathbb{R}^3$, com fronteira conexa. Então para todo $\boldsymbol{f} \in L^2(S_T)$*

$$\left\| \left(\frac{1}{2}\mathscr{D}_0 - K^*\right) \boldsymbol{f} \right\| \leq C \left\{ \left\| \left(\frac{1}{2}\mathscr{D}_0 + K^*\right) \boldsymbol{f} \right\| + \left| \int_{S_T} \mathscr{S}(\boldsymbol{f}) d\boldsymbol{x}\, dt \right| \right\}$$

onde C depende somente sobre a constante de Lipschitz para D.

Teorema 3.1. $\frac{1}{2}\mathscr{D}_0 + K : L^2(S_T) \to L^2(S_T)$ *é invertível.*

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ANALYSIS OF PERTURBATIONS IN A BOOST CONVERTER

R. P. ROMERO * & R. P. PAZOS †

The main goal of this work is the study of perturbations in a boost converter. For that, it is applied a method envolving discrete wavelet transform technique, with grouping statistical methods, such as Principal Component Analysis, in order to generate a database of subsignals a_k. So, for another perturbed signal whose output voltage is measured, the correspondent a_k is compared with the data base. For signal of great size it is employeed the power cepstrum of the subsignal a_k, and then there are applied the statistical tecniques.

1 Introduction

One of the more used circuits in power electronics is the *boost converter*. This is a very well known step-up converter topology and widely used for low power switching power supplies. This topology includes voltage source E connected to an inductor L with a paralell circuit envolving a capacitor C and a resistor R, with a controlled switch, in order to grew the output voltage V_{out}. The key principle that drives the boost converter is the tendency of an inductor to resist changes in current. When being charged it acts as a load and absorbs energy (somewhat like a resistor), when being discharged, it acts as an energy source (somewhat like a battery).

Changes in parameters R and E represent load disturbances and fluctuations of the tension source, respectively. In this work, the main focus is on some pertubations in the source.

2 Mathematical Background

2.1 Instantaneous model of the boost converter

Tne circuit analysis of the basic boost converter, operating in a continuous conduction way, allows to deduce the ideal instantaneous model:

$$L\,\frac{d\,x_1}{d\,t} = -u\,x_2 + E \tag{2.1}$$

$$C\,\frac{d\,x_2}{d\,t} = u\,x_1 - \frac{1}{R}\,x_2 \tag{2.2}$$

where $x_1 = i_L$ represents the intensity of electric current in the inductor, $x_2 = v_C$ is the tension in the capacitor, $u = 1 - q(t\,,\ x)$ is the control and $q(t\,,\ x)$ is the function representing the discret state of the electronic switch. Furthermore, the output voltage V_{out} must be regulated in order that $V_{out} = v_C > E$. The system (2.1 - 2.2) is known as *instantaneous model of the boost converter*. This system describes the dynamic of the variables $x_1(t)$ and $x_2(t)$, including the high frequency components generated by the high frequency of commutation of the switch, characterizing the ripple of this signal type.

2.2 Discrete wavelets transforms

Definition 2.1. *Let be $\psi : D \to \mathbb{R}$ a function of type $\mathcal{L}_{\in}(\mathbb{R})$, denominated generating function* [1]*, such that the functions $\psi_{a\,,\,b}$ definided by scaling and translation transformations from ψ*

$$\psi_{a\,,\,b}(\,t\,) = \frac{1}{|\,a\,|^{1/p}}\,\psi\left(\frac{t-b}{a}\right)\,,\ \textit{with}\ \ p>0\,,\ a\,,\ b \in \mathbb{R}\,,\ a \neq 0 \tag{2.3}$$

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[1] Historically named as *mother function*

generate a functional base. The wavelet family is the set of functions $\psi_{a\,,\,b}$, being frequently the value $p = 2$ the most used. [Bach 00]

Certanly, there are many wavelet transforms, but the most remarkable can be the discrete wavelet transform (DWT) such as Haar [Haar 10], Daubechies [Daub 92], Coiflet, Meyer, among the main of those. These DWT can be applied to a function $f(t)$ using filter banks. More technically, a wavelet is a mathematical function used to divide a given function or continuous-time signal into different scale components.

With multirresolution analysis, it is possible to model signals with abrupt variation [Bach 00].

3 Methodology

1. For a signals $S = [\, s_1 \,,\, s_2 \,,\,\ ,\, \ldots \,,\, s_n \,]$ it is applied a DWT [Pazos 07], up to level k , such that the k-th level decomposition seems as:

$$\mathcal{DWT}(\,\mathcal{S}\,) = \langle\, a_k \,|\, d_k \,|\, d_{k-1} \,|\, \cdots \,|\, d_2 \,|\, d_1 \,\rangle \tag{3.4}$$

2. If the subsignals a_k corresponding to output voltages including perturbations in the source have high amount of energy of the original signals, then can be serve for the database.

3. Applying PCA (or HCA) grouping statistical techniques, tha database can be classified.

4. For each signal obtained by direct measurement, the DWT chosen is applied to the k-th level in order to establish comparision with the database. So it can be identified, and then reconstructed.

5. For subsignals a_k of great size, there can be applied the power cepstrum.

4 Main Results

The dynamics for states 1 and 2 are established. An important database of pairs $[\, V_{in} \,,\, V_{out} \,]$ and their correspondent pair of subsignals a_k are generated. Some a_k of output tension associated to perturbation for identification are measured. The grouping statistical tecniques were used, so the identification was efficient. In some cases the use of the power cepstrum, was robust.

5 Conclusion

The method to study perturbations in a boost converter allows the combination of different techniques envolving discrete wavelets transforms, power electronics, dynamical systems, grouping statistical techniques. There exists an approach for signals of short size and for signals of great size; in the last case it is applied the power cepstrum for the subsignal a_k . Applications for other areas is a straightforward task.

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QUASI-LINEAR ELLIPTIC PROBLEMS UNDER STRONG RESONANCE CONDITIONS

E. D. Silva *

Abstract : In this notes we establish existence and multiplicity of solutions for an quasi-linear elliptic problem which has strong resonance at the first eigenvalue.

Keywords : Quasilinear Elliptic Equation, Strong Resonance, Variational Methods, Morse Theory.

1 Introduction

In this notes we discuss the existence and multiple solutions of the Dirichlet boundary value problem

$$\begin{cases} -\Delta_p u = \lambda_1 |u|^{p-2}u + f(x,u) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded open domain with smooth boundary $\partial\Omega$, $1 < p < N$ and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function such that

$$\lim_{|t|\to\infty} \frac{f(x,t)}{|t|^{p-1}} = 0. \tag{1.2}$$

Here $\triangle_p$ denotes the p-Laplacian operator, that is, $\triangle_p u = div(|\nabla u|^{p-2}\nabla u)$. When $p = 2$, it is the usual Laplacian operator.

From a variational stand point of view, finding solutions of (1.1) in $W_0^{1,p}(\Omega)$ is equivalent to finding critical points of the C^1 functional J given by

$$J(u) = \frac{1}{p}\int_\Omega |\nabla u|^p dx - \frac{\lambda_1}{p}\int_\Omega |u|^p dx - \int_\Omega F(x,u)dx, \; \forall\, u \in W_0^{1,p}(\Omega), \tag{1.3}$$

where $F(x,t) = \int_0^t f(x,s)ds$ and the Sobolev space $W_0^{1,p}(\Omega)$ is a Banach space endowed with the norm $\|u\| = (\int_\Omega |\nabla u|^p dx)^{\frac{1}{p}}$.

It is well known that the p-homogeneous boundary value problem

$$\begin{cases} -\Delta_p u = \lambda_1 |u|^{p-2}u \;\; in \; \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{1.4}$$

has the first eigenvalue $\lambda_1 > 0$ that is simple and has an associated eigenfunction denoted by Φ_1 which is positive in Ω, see [4]. It is also known that λ_1 is a isolated point of $\sigma(-\triangle_p)$, the spectrum of $-\triangle_p$, which contains at least an increasing eigenvalue sequence obtained by the Lusternik-Schnirlaman theory.

Therefore, by (1.2), the problem (1.1) presents the resonance phenomena at the first eigenvalue. These problems are very interesting and they have a vast literature which starts by celebrated work [2].

The main goal of this notes is find existence and multiple solutions of problem (1.1) assuming strong resonance conditions at infinity. These problems has been studied since the appearance of work [1]. More specifically, we consider the following restrict situations

$$\lim_{|t|\to\infty} f(x,t) = 0, \text{ and } |F(x,t)| \le C, \; \forall\, (x,t) \in \Omega \times \mathbb{R}. \tag{1.5}$$

Moreover, we make some conditions which are weaker than the non-quadricity condition at infinity introduced by [3]. More specifically, we introduce the following hypothesis

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$(H0)$ There are functions $a, b \in L^1(\Omega)$ such that

$$\limsup_{|t|\to\infty} tf(x,t) \leq a(x) \preceq 0, \;\forall\; x \in \Omega, \tag{1.6}$$

or

$$\liminf_{|t|\to\infty} tf(x,t) \geq b(x) \succeq 0, \;\forall\; x \in \Omega. \tag{1.7}$$

Here the inequality $a(x) \preceq 0$ means that $a(x) \leq 0$, $\forall\; x \in \Omega$ with strict inequality holding on some subset $\overline{\Omega} \subseteq \Omega$ which has positive Lebesgue measure.

2 Mathematical Results

In this section we presents the main results. Here, we will always use the Variational Methods and Morse Theory. First, we can prove the following result

Theorem 2.1. *(Existence) Suppose* $(SR), (H0)$*. Then the problem (1.1) has at least one solution* $u_0 \in W_0^{1,p}(\Omega)$*.*

Now, we take $F(x,0) \equiv 0, f(x,0) \equiv 0$ which implies that $u = 0$ is a trivial solution of problem (1.1). In this case the key point is assure the existence of nontrivial solutions. We need some additional hypothesis

$(H1)$ There are $\delta > 0$ and $\alpha \in (0, \lambda_1)$ such that

$$F(x,t) \leq \frac{\alpha - \lambda_1}{p}|t|^p, \;\forall\; |t| \leq \delta, \forall x \in \Omega.$$

$(H2)$ There is $t_\star \in \mathbb{R}\backslash\{0\}$ such that

$$\int_\Omega F(x, t_\star \Phi_1(x))dx > 0.$$

Thus, combining Ekeland's Variational Principle and Mountain Pass Theorem, we can prove the following multiplicity result

Theorem 2.2. *Suppose* $(SR), (H0), (H1), (H2)$*. Then the problem (1.1) has at least two nontrivial solutions* $u_0, u_1 \in W_0^{1,p}(\Omega)$*.*

Next, we consider the following hypothesis

$(H3)$ There are $r > 0$ and $\epsilon \in (0, \lambda_2 - \lambda_1)$ such that

$$0 \leq F(x,t) \leq \frac{\lambda_2 - \lambda_1 - \epsilon}{p}|t|^p, \;\forall\; |t| \leq r, \forall x \in \Omega.$$

Then, using the Three-Critical Point Theorem, we can show the following result

Theorem 2.3. *Suppose* $(SR), (H0), (H3)$*. Then the problem (1.1) has at least two nontrivial solutions.*

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EXISTÊNCIA DE SOLUÇÕES PARA UMA EQUAÇÃO ABSTRATA DO TIPO KIRCHHOFF

M. A. J. SILVA *

Neste trabalho estudamos uma equação de operadores do tipo Kirchhoff

$$M\big(||A^{\alpha}u||_H^2\big)Au = Nu, \qquad 0 \leq \alpha < 1, \tag{1}$$

onde $M : \mathbb{R}^+ \to \mathbb{R}^+$ é uma função contínua, $A : D(A) \subset H \to H$ é um operador linear auto-adjunto e $N : H \to H$ é um operador contínuo, possivelmente não linear.

O problema (1) corresponde à forma estacionária da equação de ondas de Kirchhoff [4]. Sobre esse assunto, várias referências podem ser encontradas, como por exemplo, em [1,2,3,5]. Na forma abstrata (1), envolvendo operadores com operadores com potência fracionária, foi considerado em [2].

Seja Ω é um domínio de $\mathbb{R}^N$. No caso em que $H = L^2(\Omega)$, $A = -\Delta$, e $\alpha = 1/2$, a equação (1) se reduz à equação elíptica do tipo Kirchhoff

$$-M\big(||\nabla u||_2^2\big)\Delta u = f(x,u), \tag{2}$$

com condições de fronteira do tipo Dirichlet. Notemos que nesse exemplo, o operador não linear N corresponderá ao operador de Nemytskii associado a f, definido por $(Nu)(x) = f(x,u(x))$. Problemas do tipo (2) são equações elípticas não localmente definidas, e diversos resultados podem ser vistos em [1].

Nosso objetivo é apresentar uma extensão do resultado principal de [2]. Com efeito, em [2], o resultado de existência de soluções para o problema (1) é baseado na hipótese

$$\|Nu\|_H \leq a\|u\|_H + b,$$

onde a, b são constantes positivas convenientemente escolhidas. Utilizando o mesmo método apresentado em [2,3,5], a saber, o Método de Galerkin, mostramos como a sublinearidade de N pode ser removida em certos casos.

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ON THE STEADY VISCOUS FLOW OF A NON-HOMOGENEOUS ASYMMETRIC FLUID

FÁBIO V. SILVA *

We are concerned with the solvability of the following system of equations

$$\left.\begin{aligned} -(\mu+\mu_r)\Delta v + \rho(v\cdot\nabla)v + \nabla p = 2\mu_r \operatorname{curl} w + \rho f \\ \nabla\cdot(\rho v) = 0, \qquad \nabla\cdot v = 0, \\ -(c_a+c_d)\Delta w + \rho\jmath(v\cdot\nabla)w - (c_0-c_a+c_d)\nabla(\nabla\cdot w) + 4\mu_r w = 2\mu_r \operatorname{curl} v + \rho g \end{aligned}\right\} \tag{0.1}$$

in a bounded planar domain Ω, having a C^2 boundary, subject to the following boundary conditions

$$\rho = \rho_0 \quad \text{on } \Gamma, \qquad v = v_0,\ w = w_0 \quad \text{on } \partial\Omega, \quad \text{with } \int_{\partial\Omega} v_0\cdot n = 0. \tag{0.2}$$

This system governs steady motions of a class of fluids having a non-symmetric stress tensor and whose particles undergo translations and rotations as well. In (0.1) the unknowns are ρ, the density, v, w, the fields of velocity and rotation of particles and p, the pressure. The fields f and g are, respectively, given external sources of linear and angular momenta densities whereas $\jmath, \mu, \mu_r, c_0, c_a, c_d$ are positive constants characterizing the medium and also satisfying $c_0 > c_a+c_d$. $\Gamma\subset\partial\Omega$ is a connected arc on which $v_0\cdot n < 0$. Such model, which contains the incompressible, density dependent Navier-Stokes system as a particular case ($w\equiv 0$), is named the non-homogeneous micropolar fluid model and was introduced in [1]. Details on the physical meaning of the several parameters above may be found in [5].

Frolov addressed in [2], the solvability of the above boundary value problem in the case $w\equiv 0$. Loosely speaking Frolov's technique may be summarized as follows: let ψ be the stream-function of the velocity field *i.e.,* $v = (-\psi_{x_2}, \psi_{x_1}) = \nabla^\perp\psi$. For a given smooth scalar function η, letting $\rho = \eta(\psi)$ we have $\operatorname{div}(\rho v) = \eta'(\psi)\nabla\psi\cdot\nabla^\perp\psi \equiv 0$. Thus, $\eta > 0$ is fixed from before so that we have $\rho\mid_\Gamma = \rho_0$ and equations $(0.1)_{2,3}$ may be dropped. The solution is found as a fixed point of a certain operator which fulfills the assumptions of Leray-Schauder theorem.

We managed to prove existence of solution to the above system by coupling ideas of Frolov [2] and those of Łukaszewicz [5]. Owing to the presence of the equation $(0.1)_4$, we first solve an auxiliary problem for w and next we plug it into the right-hand-side of $(0.1)_1$ and follow Frolov's scheme for obtaining v.

As the author points out [2], some minor changes on the arguments are needed so that they apply to the case of Γ consisting of a finite union of connected arcs, $\Gamma = \cup\gamma_j$, with $\rho|_{\gamma_j} = \rho_{0_j}$ and each ρ_{0_j} being continuous. In [6] with a similar approach, the harder problem of mixing two fluids with different, and discontinuous, densities in a bounded planar domain is considered. More recently Frolov's result was reobtained in a boundary control problem for non-homogeneous incompressible fluids [3]. It is worth of notice that in this latter work the author does not require Hölder continuity of neither ρ_0 or η: they are assumed to be merely continuous.

Uniqueness of the solution is not tackled in the above mentioned papers. Some instances in [4] show that one should not expect this problem to be uniquely solvable. Nevertheless, this will be object of further investigation.

1 Notations and main result

Before stating our main result we introduce some notations and clarify what is meant by (0.1) to hold in a planar domain. By $L^p(\Omega)$ we denote the Lebesgue spaces and $W^{k,p}(\Omega), k\geq 0, p>1$, denotes the standard Sobolev spaces

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modelled in $L^p(\Omega)$. We endow $W^{k,p}$ with the norm $\|\xi\|_{k,p}^p = \int_\Omega \sum_{|\alpha|\le k} |D^\alpha \xi|^p$. As usual, $W^{0,p}(\Omega) = L^p(\Omega)$ and $W^{k,2}(\Omega) = H^k(\Omega),\ k \ge 0$. In the same vein, $W^{k-1/p,p}(\partial\Omega), k \ge 0, p > 1$, denote the trace spaces and $\|\cdot\|_{k-1/p,p}$ their norms. $\mathcal{V}$ is the set of divergence-free vector fields $\varphi = (\varphi_1, \varphi_2)$ such that $\varphi \in C_0^\infty(\Omega)$ and V is the closure of $\mathcal{V}$ in the H^1-norm and $\mathbf{H} = \{\varphi \in H^1 \mid \operatorname{div}\varphi = 0 \text{ in } \Omega, \int_{\partial\Omega} \varphi \cdot n = 0\}$. By $C^{m,\beta}(\Omega)$ we denote the set of all m times continuously differentiable functions in Ω whose m-th order derivatives are Hölder continuous with exponent $\beta \in (0,1]$.

We regard $v = (v_1(x_1,x_2), v_2(x_1,x_2), 0), w = (0,0,w(x_1,x_2))$ and write $\nabla^\perp\psi = (-\partial_{x_2}\psi, \partial_{x_1}\psi)$ and $\operatorname{curl}(\phi_1,\phi_2) = \partial_{x_1}\phi_2 - \partial_{x_2}\phi_1$. Given u, v, a pair of vector fields, we denote $[(u\cdot\nabla)v]_j = \sum_k u_k \partial_{x_k} v_j$. This way equations (0.1) may be written componentwise

$$\left.\begin{array}{c} -(\mu+\mu_r)\Delta v_j + \rho v\cdot\nabla v_j + \partial_{x_j} p = (-1)^{j-1} 2\mu_r \partial_{x_j} w + \rho f_j, \quad j = 1,2 \\ \nabla\cdot(\rho v) = 0, \qquad \nabla\cdot v = 0 \\ -(c_a + c_d)\Delta w + \rho j v\cdot\nabla w + 4\mu_r w = 2\mu_r \operatorname{curl} v + \rho g \end{array}\right\} \text{ in } \Omega. \tag{1.3}$$

Assuming $f, g \in L^2(\Omega), w_0 \in H^{1/2}(\partial\Omega), v_0 \in H^{1/2}(\partial\Omega)$ with $\int_{\partial\Omega} u_0 \cdot n = 0, \rho_0 \in C^{0,\beta}(\Gamma)$, we homogenize boundary conditions as usual and consider a perturbed problem obtained by plugging $v = u + a,\ w = \underline{w} + b$ into (1.3). Here $u \in V, \underline{w} \in H_0^1(\Omega)$ are new unknowns whereas a, b are suitable extensions of v_0, w_0 to the whole Ω. We then resort to a weak formulation, testing the first two equations with divergence-free vector fields of compact support, which eliminates the pressure gradient. It is recovered afterwards as a consequence of de Rham's lemma. The steps mentioned above in the introduction provide us with a weak solution of this perturbed problem, with which we form a weak solution corresponding to the original problem.

Our main result reads

Theorem 1.1. *Given $w_0 \in H^{1/2}(\partial\Omega), v_0 \in H^{1/2}(\partial\Omega)$ with $\int_{\partial\Omega} u_0 \cdot n = 0, \rho_0 \in C^{0,\beta}(\Gamma), f, g \in L^2(\Omega)$, the system (0.1) has a weak solution $\rho \in C^{0,\gamma}(\overline{\Omega}), \gamma < \beta, v \in \mathbf{H}, w \in H^1(\Omega)$.*

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EQUAÇÕES TOTALMENTE NÃO LINEARES COM FRONTEIRAS LIVRES: TEORIA DE EXISTÊNCIA E DE REGULARIDADE

Eduardo V. Teixeira * & Gleydson C. Ricarte †

Seja $\partial\Omega$ uma hipersuperfície compacta suave em $\mathbb{R}^n$, $\varphi : \partial\Omega \to \mathbb{R}$ uma função não-negativa, $g : \bar{\Omega} \to \mathbb{R}$ uma função positiva. Uma questão importante na matemática aplicada é a de saber se podemos encontrar uma outra hipersuperfície compacta $\Gamma = \partial\Omega' \subset \Omega$ tal que seja possível resolver o problema superdeterminado

$$\left\{ \begin{array}{rcl} F(x, D^2 u) & = & 0 \;\text{ em }\; \Omega \setminus \Omega' \\ u & = & \varphi \;\text{ em }\; \partial\Omega \\ u = 0 & , & u_\nu = g \text{ em } \Gamma \end{array} \right. \tag{0.1}$$

onde ν é o vetor normal interior a Γ e $F : \mathcal{S}(n) \times \Omega \to \mathbb{R}$ é um operador totalmente não linear aonde $\mathcal{S}(n)$ denota o espaço das matrizes simétricas de ordem n.

Versões variacionais do problema acima estão em conexão com o trabalho monumental de Alt e Caffarelli [1]. De fato, para problemas regidos pelo Laplaciano, soluções do problema (0.1) podem ser obtidas como mínimos do funcional

$$J(v) := \int_\Omega |\nabla v|^2 + g^2(X)\chi_{\{v>0\}} dX, \tag{0.2}$$

dentre funções $H^1(\Omega)$ com $v = \varphi$ em $\partial\Omega$. De fato é possível mostrar (vide [1]) que um mínimo do functional em (0.2) satisfaz $u_\nu = g$ na $\partial\{u > 0\}$. Inicialmente a equação é entendida em um sentido bastante fraco. Isto se deve ao fato de não ser possível a princípio guarantir regularidade suficiente para u (observe que o potencial em J é descontínuo; portanto a teoria do Cálculo das Variações não se aplica) muito menos suavidade da fronteira livre $\partial\{u > 0\}$. Revolucionárias ferramentas com embasamento na teoria geométrica da medida foram então desenvolvidas em [1] para mostrar que a menos de um conjunto de medida nula, $\partial\{u >\}$ é uma hipersuperfície de classe $C^{1,\alpha}$ e portanto a equação $u_\nu = g$ é satisfeita classicamente, módulo um possível (e inevitável) conjunto singular.

Infelismente, para problemas governados por operadores que não admitem um functional de Euler-Lagrange, por exemplo operadores da forma não divergente ou totalmente não lineares ou ainda equações com termos de transporte, $\Delta u + \mathbf{b}(x) \cdot \nabla u$, a teoria variacional de Alt-Caffarelli não pode ser empregada e novas estratégias precisam ser desenvolvidas para solucionar o problema superdeterminado (0.1).

Uma possível abordagem para tais problemas na ausência de caracterizações variacionais está baseado em técnicas de perurbações singulares. A idéia básica é a seguinte: uma possível solução para o problema (0.1) será obtida como limite de soluções do problema regularizado

$$\left\{ \begin{array}{rcl} F(x, D^2 u_\epsilon) & = & g^2(x).\beta_\epsilon(u_\epsilon) \text{ in } \Omega \\ u_\varepsilon & = & \varphi \text{ on } \partial\Omega. \end{array} \right. \tag{0.3}$$

aonde β_ϵ é uma aproximação apropriada da função δ_0 de Dirac. Para cada $\varepsilon > 0$ fixado, (0.3) modela problemas de difusão com ativação de alta energia. É importante para este campo de pesquisa, estimativas e propriedades geométricas uniformes em ε. Com relação ao problema superdeterminado acima, (0.1), estimativas uniformes em ε são então tranportadas para o problema de fronteira livre original.

Problemas de fronteira livre resultante da passagem ao limite de problemas regularizados têm recebido grande atenção dos últimos anos. Versões simplificada deste problema foram vastamente investigadas nos anos 80 por

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Lewy-Stampacchia, Caffarelli, Kinderlehrer e Nirenberg, Alt e Phillips, dentre outros. Problemas em meios não-homogêneos com hipóteses mínimas de regularidade nos coeficientes foram estudados em [2]. Em [3], Teixeira fornce uma princípio variacional para a abordagem de equações não-variacionais com termos de transporte, $Lu = \sum_{i,j} \partial_j(a_{ij}(x)D_i u) + \sum_i b_i(x)D_i u + c(x)u = \beta_\epsilon(u)$.

Embora o trabalho [3] contemple uma variedade de problemas elípticos não-variacionais de relevância física, as técnicas desenvolvidas neste artigo ainda são de cunho variacional e dificilmente seriam adaptáveis à problemas totalmente não lineares e não variacionais como $F(x, D^2u)$.

De fato a análise de problemas inevitavelmente não variacionais exigem novas abordagens em praticamente todos as etapas do projeto. O desenvolvimento destas soluções é o tema de minha tese de doutorado supervisionada por Eduardo Teixeira na UFC.

1 Resultados...

Em seguida listo em um único teorema todos os resultados obtidos até o presente momento do projeto.

Teorema 1.1. *A respeito do problema regularizado 0.3:*

- ✓ *Para cada $\varepsilon > 0$ fixado, a equação 0.3 possui uma solução maximal u_ε.*
- ✓ *Fixado um subdomínio $\widetilde{\Omega} \Subset \Omega$, existe uma constante $C = C(\widetilde{\Omega}) > 0$, independente de $\varepsilon > 0$, tal que $|\nabla u_\varepsilon| < C$ em $\widetilde{\Omega}$. Ou seja u_ε é localmente uniformemente Lipschitz contínuo. Esta regularidade é sharp.*
- ✓ *A família u_ε é uniformemente fortemente não degenerada, ou seja, $\sup_{B_r} u_\varepsilon \geq cr$. Tal degenerecência é ótima.*
- ✓ *A menos de subsequência, u_ε converge local uniformemente para uma função Lipschitz contínua u_0 que satisfaz $F(x, D^2u_0) = 0$ em $\{u_0 > 0\}$ no sentido da viscosidade. Ademais, $u_0(x) \gtrsim dist(x, \partial\{u_0 > 0\})$, para todo $x \in \{u_0 > 0\}$.*
- ✓ *A dimensão de Hausdorff da fronteira livre $\partial\{u_0 > 0\}$ é $n-1$. Para qualquer bola centrada na fronteira livre, vale $\mathcal{H}^{n-1}(B \cap \partial\{u_0 > 0\}) \sim r^{n-1}$.*
- ✓ *A fronteira livre reduzida tem medida total.*

A listagem dos resultados acima (propriedades da geometria fraca relevantes ao problema) nos coloca em posição favorável para a investigação da regularidade da fronteira livre. Neste tocante, destacamos a necessidade de provarmos condições de fronteira livre adequadas. Até o momento obtemos condições de fronteira livre apropriadas para operadores lineares (da forma não divergente) $Lv = a_ij(x)D_ijv$, com coeficientes Lipschitz contínuos. Neste caso demostramos com sucesso a regularidade $C^{1,\alpha}$ da fronteira livre $\partial\{u_0 > 0\}$ a menos de um conjunto de medida de Hausdorff $n-1$ nulo.

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WAVE EQUATION WITH ACOUSTIC/MEMORY BOUNDARY CONDITIONS

A. VICENTE * & C. L. FROTA †

Let $\Omega \subset \mathbb{R}^n$ be an open, bounded and connected set with smooth boundary Γ. Suppose Γ is divided into two portion of positive measure $\Gamma = \Gamma_0 \cup \Gamma_1$ such that $\Gamma_0 \cap \Gamma_1 = \emptyset$. Let ν be the outward unit normal vector on Γ. In this work we consider the mixed problem for the wave equation with acoustic/memory boundary conditions

$$u'' - \Delta u = F \qquad \text{in } \Omega \times (0,T), \tag{1}$$

$$u + \int_0^t \beta(t-s)\frac{\partial u}{\partial \nu}(s)\,ds = 0 \qquad \text{on } \Gamma_0 \times (0,T), \tag{2}$$

$$\frac{\partial u}{\partial \nu} = \delta' \qquad \text{on } \Gamma_1 \times (0,T), \tag{3}$$

$$u' + f\delta'' + g\delta' + h\delta = 0 \qquad \text{on } \Gamma_1 \times (0,T), \tag{4}$$

$$u(x,0) = u_0(x),\ u'(x,0) = u_1(x)\,, \qquad x \in \Omega, \tag{5}$$

$$\delta(x,0) = \delta_0(x),\ \delta'(x,0) = \frac{\partial u_0}{\partial \nu}(x)\,, \qquad x \in \Gamma_1, \tag{6}$$

where $' = \dfrac{\partial}{\partial t}$; $\Delta = \displaystyle\sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is the Laplacian operator; $F : \Omega \times (0,T) \to \mathbb{R}$; $f, g, h \in C(\overline{\Gamma_1})$ such that $f(x), h(x) > 0$ and $g(x) \geq 0$, for all $x \in \overline{\Gamma_1}$; $\beta : \mathbb{R}^+ \to \mathbb{R}$, $u_0, u_1 : \Omega \to \mathbb{R}$ and $\delta_0 : \Gamma_1 \to \mathbb{R}$ are given functions.

Mixed problems for wave equations with homogeneous boundary conditions have been studied for a long time. However, time-dependent boundary conditions seems to be more suitable to model concrete applications.

In this direction boundary conditions of memory type, as equation (2), imposed on a portion of the boundary and Dirichlet condition on the rest of the boundary, have been considered, see for instance [1, 4, 11, 12, 13]. Equation (2) means that the portion Γ_0 is clamped in a body with viscoelastic properties. On the other hand, wave equations equipped with time-dependent acoustic boundary conditions have been considered also. For locally reacting boundaries, conditions (3) and (4), were introduced by Beale-Rosecrans [2] and studied in [3, 5, 6, 7, 8, 9, 10]. In these cases, the solution u of the wave equation (1) is the velocity potential of a fluid undergoing acoustic wave motion and $\delta(x,t)$ is the normal displacement to the boundary at time t with the boundary point x. Similarly, acoustic boundary conditions have been coupled with homogeneous Dirichlet condition on a portion of the boundary, excepted in [10, 14] where the acoustic boundary condition were imposed in the whole boundary Γ.

The main purpose of this paper is to study the combination of acoustic and memory boundary conditions. We prove the existence and uniqueness of global solution to the problem (1)-(6).

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ZEROS DE POLINÔMIOS EM ESPAÇOS DE BANACH REAIS

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Responder se zeros de um polinômio n-homogêneo sobre um espaço de Banach *real* de dimensão infinita contém algum subespaço de dimensão infinita, é um problema não resolvido completamente. Estre trabalho tem como objetivo apresentar uma solução parcial para este problema para um particular tipo de polinômio em um particular tipo de espaço. Os resultados aqui apresentados foram obtidos por J. Ferrer em [3] e [4].
Em seu artigo [3], J. Ferrer demonstra construtivamente que todo polinômio n-homogêneo fracamente contínuo sobre os limitados de um espaço de Banach, cujo o dual não seja w^*-separável, se anula em um subespaço também de dual não w^*-separável. Aplicando estas técnicas em seu artigo [4], J. Ferrer demonstra que se K é um espaço topológico compacto não satisfazendo a *Condição de Cadeia Contável* então, todo polinômio n-homogêneo contínuo sobre $\mathcal{C}(K)$ se anula em um subespaço não separável de $\mathcal{C}(K)$.

1 Resultados

Teorema 1.1. *Seja X um espço de Banach e $\{P_k : k \in \mathbb{N}\}$ uma coleção de polinômios homogêneos fracamente contínuos sobre os subconjuntos limitados de X . Se X^* não for w^*-separável então X admite um subespaço fechado Z de dual não w^*-separável e tal que $Z \subset \bigcap_{k\in\mathbb{N}} P_k^{-1}(0)$.*

Um aplicação interessante deste teorema pode ser obtida em conjunto com o próximo resultado bem conhecido.

Teorema 1.2. *Se X é um espaço de Banach com a propriedade de Dunford-Pettis e não contendo subespaço isomorfo a l_1 então todo polinômio homogêneo contínuo sobre X é fracamente contínuo sobre os subconjuntos limitados de X.*

Então se X é um espaço de Banach com a propriedade de Dunford-Pettis de dual não w^*-separável e não contendo subespaço isomorfo a l_1, para toda coleção $\{P_k : k \in \mathbb{N}\}$ de polinômios homogêneos contínuos sobre X, existe um subespaço fechado $Z \subset \bigcap_{k\in\mathbb{N}} P_k^{-1}(0)$, mais geralmente basta apenas que X contenha um subespaço satisafendo essa condições.
Como exemplo de espaço satisfazendo essas condições temos $c_0(\Gamma)$ onde Γ é um conjunto não contável. Utilizando esse fato pode se demonstrar o seguinte resultado, devido a J. Ferrer.

Teorema 1.3. *Seja $\{P_k : k \in \mathbb{N}\}$ uma coleção de polinômios homogêneos contínuos definidos sobre $c_0(\Gamma)$ onde Γ é um conjunto não contável. Então existe um subsepaço Z, tal que $Z \subset \bigcup_{k\in\mathbb{N}} P_k^{-1}(0)$ e $Z \cong c_0(\Gamma)$.*

O próximo teorema nos dá uma classe de espaços contendo subespaço isomorfo a algum $c_0(\Gamma)$ onde γ é um conjunto não contável. Lembremos que um espaço topológico satisfaz a Condição de Cadeia Contável ou CCC se qualquer coleção de abertos disjuntos for contável.

Teorema 1.4. *Se K é um espaço de Hausdorff compacto não satisfazendo a condição CCC então $\mathcal{C}(K)$ contém um subespaço isomorfo a $c_0(\Gamma)$ onde Γ é um conjunto não contável.*

Então obtemos o seguinte resultado, devido a J. Ferrer em [4]

Corolário 1.1. *Seja K um espaço de Hausdorff compacto não satisfazendo a condição CCC. Então todo polinômio n-homogêneo contínuo definido sobre $\mathcal{C}(K)$, se anula em um subespaço isomorfo a $c_0(\Gamma)$, para algum Γ não contável.*

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SOBRE UMA EQUAÇÃO DO TIPO BENJAMIN-BONA-MAHONY EM DOMÍNIO NÃO CILÍNDRICO

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O objetivos deste trabalho é estabelecer a existência e unicidade de soluções de um problema misto associado à uma equação do tipo Benjamin-Bona-Mahony em um domínio não cilíndrico. Assim, denota-se por O um conjunto limitado não vazio contido em $\mathbb{R}^n \times \mathbb{R}$. Suponha que $\Omega_s = O \cap \{t = s\}$, com $s \in \mathbb{R}$, sejam conjuntos abertos não vazios e limitados com fronteiras Γ_s. Para $T > 0$ define-se o domínio não cilíndrico

$$\widehat{Q} = \bigcup_{0 \le s \le t} \Omega \times \{s\}.$$

Nestas condições, investiga-se o seguinte problema misto

$$\left| \begin{array}{l} u_t(x,t) + A(u_t(x,t)) + \operatorname{div}(\phi(u(x,t))) = 0 \text{ em } \widehat{Q}, \\ u(x,t) = 0 \text{ sobre } \widehat{\Sigma} = \bigcup_{0 \le s \le t} \Gamma_s \times \{s\}, \\ u(x,0) = u_0(x) \text{ em } \Omega_0, \end{array} \right. \tag{0.1}$$

onde $Au = -\sum\limits_{i,j=1}^{n} \frac{\partial}{\partial x_i}(a_{i,j}\frac{\partial u}{\partial x_j})$ é um operador de segunda ordem e $a_{i,j} \in C^1(\overline{\Omega} \times \mathbb{R}, \mathbb{R})$ é tal que

$$\sum_{i,j=1}^{n} a_{i,j}(x,t)\xi_i\xi_j \ge c_0(\xi_1^2 + ... + \xi_n^2) \text{ para todo } \xi_1, ..., \xi_n \in \mathbb{R}.$$

Definição 0.1. *Uma solução fraca do problema (0.1) é uma função* $u : \widehat{Q} \to \mathbb{R}$ *com a regularidade*

$$u, u_t \in L^\infty(0,T; H_0^1(\Omega_t)),$$

satisfazendo a identidade integral

$$\int_{\widehat{Q}} u_t(x,t)\theta(x,t)dxdt + \int_{\widehat{Q}} a_{i,j}(x,t)\frac{\partial u_t(x,t)}{\partial x_i}\frac{\partial \theta(x,t)}{\partial x_j}dxdt - \int_{\widehat{Q}} \phi(u(x,t))\nabla\theta(x,t)dxdt = 0,$$

para todo $\theta \in L^2(0,T; H_0^1(\Omega_t))$ *e tal que* $u(x,0) = u_0(x)$ *em* Ω_0.

Para estabelecer a existência e unicidade de soluções do problema misto (0.1) no sentido da definição 0.1 supõe-se as seguintes hipótese:
i) $\phi \in C^2(\mathbb{R}^n \times \mathbb{R})$ com $\phi(s) = (\phi_1(s), ..., \phi_n(s))$ e $\phi(0) = 0$;
ii) $|\phi_i(s)| \le \zeta(|s| + |s|^2)$;
iii) $|\phi_i'(s)| \le \Lambda(1 + |s|)$;
iv) Se $t_1 \le t_2$ então $\text{proj}|_{t=0}\Omega_{t_1} \subseteq \text{proj}|_{t=0}\Omega_{t_2}$. Isto significa que a familia $(\Omega_t)_{0 \le t \le T}$ é não-decrescente;
v) Para qualquer $t \in (0,T)$, Se $v \in H_0^1(\Omega)$ e $v(x,t) = 0$ q.s. em $x \in \Omega - \Omega_t$, então $v \in H_0^1(\Omega_t)$ para todo $t \in [0,T]$.

Teorema 0.1. *Suponha* $u_0 \in H_0^1(\Omega_0)$ *e que as hipóteses (i)-(v) sejam satisfeitas. Então existe uma única função real* u *definida em* $\widehat{Q}$ *solução no sentido da Definição 0.1 do problema misto (0.1).*

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Prova: *Existência* - A existência de soluções é obtida por meio do método de penalização. A idéia da demonstração é dada como segue.

Problema Penalizado - Suponha $Q \subset \widehat{Q}$ onde $Q = \Omega \times]0, T[$ e Ω um aberto limitado de $\mathbb{R}^n$ com bordo regular tal que $\Omega_0 \in \Omega$. Seja $M : Q \to \mathbb{R}$ o termo penalizante dado por

$$M(x,t) = 1 \text{ em } Q - \widehat{Q} \cup \{\Omega_0 \times 0\} \text{ e } M(x,t) = 0 \text{ em } \widehat{Q} \cup \{\Omega_0 \times 0\}.$$

O problema penalizado consiste em dado $\epsilon > 0$, determinar uma família de funções $u^\epsilon : Q \longrightarrow \mathbb{R}$ tal que

$$u^\epsilon, u^\epsilon_t \in L^\infty(0, T; H^1_0(\Omega)),$$

para $T > 0$, satisfazendo

$$\int_Q u^\epsilon_t(x,t)\theta(x,t)dxdt + \int_Q a_{i,j}(x,t)\frac{\partial u^\epsilon_t(x,t)}{\partial x_i}\frac{\partial \theta(x,t)}{\partial x_j}dxdt +$$
$$\frac{1}{\epsilon}\int_Q M(x,t)u^\epsilon_t(x,t)\theta(x,t)dxdt - \int_Q \phi(u^\epsilon(x,t))\nabla\theta(x,t)dxdt = 0,$$

para todo $\theta \in L^2(0, T; H^1_0(\Omega))$. Além disso, $u^\epsilon(x, 0) = \widetilde{u}_0(x)$ em Ω, onde $\widetilde{u}_0$ é a extensão de u_0 a Ω pondo zero em $\Omega - \Omega_0$.

Para obter-se u^ϵ aplica-se o método de Faedo-Galerkin . Observa-se que a penalização possibilita investigar o problema misto em um cilíndro, cuja técnica de resolução é conhecida obtendo o caso não cilíndrico quando $\epsilon \to 0$.

Unicidade - Suponha que u e $\widehat{u}$ são duas soluções do problema misto não cilíndrico (0.1). Definindo $\omega = u - \widehat{u}$, tem-se que $\omega, \omega_t \in L^\infty(0, T; H^1_0(\Omega))$ e satisfaz

$$\int_{\widehat{Q}} \omega_t(x,t)\theta(x,t)dxdt + \int_{\widehat{Q}} a_{i,j}(x,t)\frac{\partial \omega_t(x,t)}{\partial x_i}\frac{\partial \theta(x,t)}{\partial x_j}dxdt = \int_{\widehat{Q}} [\phi(u) - \phi(\widehat{u})]\nabla\theta(x,t)dxdt, \;\; \omega(x,0) = 0 \text{ em } \Omega_0$$

para todo $\theta \in L^2(0, T; H^1_0(\Omega_t))$.

Na demonstração da unicidade é fundamental o seguinte resultado

Lema 0.1. *Se ω é uma solução fraca do problema (0.1) com $u_0 = 0$ e $0 \leq \rho < t$, então*

$$\int_{\widehat{Q_\rho}} |\omega|^2 dxdt \leq \rho^2 \int_{\widehat{Q_\rho}} |\omega_t|^2 dxdt.$$

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LINEARIZAÇÃO DE APLICAÇÕES MULTILINEARES CONTÍNUAS

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Neste trabalho mostraremos como se faz a linearização de aplicações multilineares contínuas entre espaços de Banach. Para tanto introduziremos a norma projetiva no produto tensorial e mostraremos que as aplicações multilineares contínuas entre espaços de Banach estão em correspondência biunívoca com os operadores lineares contínuos definidos no produto tensorial projetivo. Além disso, apresentaremos duas propriedades importantes da norma projetiva. Primeiro mostraremos que ela não respeita subespaços e segundo que respeita quocientes.

1 Resultados...

Sejam $X_1, \ldots, X_n$ espaços vetoriais sobre o corpo $\mathbb{K}$, onde $\mathbb{K} = \mathbb{R}$ ou $\mathbb{C}$. Denotaremos por $X_1 \otimes \cdots \otimes X_n$ o produto tensorial algébrico dos espaços $X_1, \ldots, X_n$.

Definição 1.1. Sejam $E_1, \ldots, E_n, F$ e G espaços vetoriais normados. Denotaremos por $\mathcal{L}(E_1, \ldots, E_n; F)$ o espaço de Banach de todas as aplicações n-lineares contínuas de $E_1 \times \cdots \times E_n$ em F com a norma do sup. Quando $n = 1$ e $F = \mathbb{K}$, denotaremos $\mathcal{L}(E_1, \mathbb{K}) = E_1'$. E $\mathcal{L}(G; F)$ denota o espaço de Banach de todos operadores lineares contínuos de G em F com a norma do sup.

Definição 1.2. Sejam $E_1, \ldots, E_n$ espaços vetoriais normados. Para cada tensor $u \in E_1 \otimes \cdots \otimes E_n$ define-se:

$$\pi(u) = \inf \left\{ \sum_{j=1}^{k} ||x_1^j|| \cdots \left\| x_n^j \right\| : u = \sum_{j=1}^{k} x_1^j \otimes \cdots \otimes x_n^j \right\}.$$

Denota-se por $E_1 \otimes_\pi \cdots \otimes_\pi E_n$ o produto tensorial de $E_1, \ldots, E_n$ dotado com a norma π. Esta norma é conhecida como a norma projetiva. O completamento do espaço $E_1 \otimes_\pi \cdots \otimes_\pi E_n$ será denotado por $E_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi E_n$. O espaço de Banach $E_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi E_n$ será chamado de produto tensorial projetivo dos espaços vetoriais normados $E_1, \ldots, E_n$.

O Teorema abaixo mostra em que sentido o produto tensorial projetivo realiza a linearização de aplicações multilineares contínuas. No entanto, primeiro consideremos a seguinte aplicação n-linear

$$\sigma_n \colon E_1 \times \cdots \times E_n \longrightarrow E_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi E_n$$

dada por $\sigma_n(x_1, \ldots, x_n) = x_1 \otimes \cdots \otimes x_n$.

Teorema 1.1. *Sejam $E_1, \ldots, E_n$ e F espaços vetoriais normados. Se $B \colon E_1 \times \cdots \times E_n \longrightarrow F$ é uma aplicação n-linear contínua então existe um único operador linear contínuo $B_L \colon E_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi E_n \longrightarrow F$ satisfazendo $B_L(x_1 \otimes \cdots \otimes x_n) = B(x_1, \ldots, x_n)$ para quaisquer $x_j \in E_j$ com $j = 1, \ldots, n$, ou seja, o diagrama abaixo é comutativo:*

$$\begin{array}{ccc} E_1 \times \cdots \times E_n & \xrightarrow{\quad B \quad} & F \\ & \searrow^{\sigma_n} \quad \nearrow_{B_L} & \\ & E_1 \hat{\otimes}_\pi \cdots \hat{\otimes}_\pi E_n & \end{array}$$

A correspondência $B \longleftrightarrow B_L$ é um isomorfismo isométrico entre os espaços de Banach $\mathcal{L}(E_1, \ldots, E_n; F)$ e $\mathcal{L}(E_1 \widehat{\otimes}_\pi \cdots \widehat{\otimes}_\pi E_n; F)$.

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Abaixo mostraremos como a norma projetiva não respeita subespaços.

Observação 1.1. Sejam E e F espaços vetoriais normados e $G \subseteq E$ um subespaço. Suponha que $G \otimes_\pi F$ seja um subespaço de $E \otimes_\pi F$. Assim, pelo Teorema de Hahn Banach temos que todo elemento de $(G \otimes_\pi F)'$ se estende a um elemento de $(E \otimes_\pi F)'$. Seja $u \in \mathcal{L}(G; F')$. Definindo $A\colon G \times F \longrightarrow \mathbb{K}$ por $A(x, y) = u(x)(y)$ e usando a identificação natural entre os espaços de Banach $\mathcal{L}(G; F')$ e $\mathcal{L}(G, F; \mathbb{K})$ temos que $A \in \mathcal{L}(G, F; \mathbb{K})$. Do Teorema 1.1 segue que $A_L \in (G \widehat{\otimes}_\pi F)' = (G \otimes_\pi F)'$, e portanto existe $\varphi \in (E \otimes_\pi F)' = (E \widehat{\otimes}_\pi F)'$ tal que $\varphi|_{G \widehat{\otimes}_\pi F} = A_L$. Novamente, pelo Teorema 1.1, existe $B \in \mathcal{L}(E, F; \mathbb{K})$ tal que $B_L = \varphi$. Agora usando a identificação natural entre os espaços de Banach $\mathcal{L}(E, F; \mathbb{K})$ e $\mathcal{L}(E; F')$ e definindo $u'\colon E \longrightarrow F'$ por $u'(x)(y) = B(x, y)$ temos que $u' \in \mathcal{L}(E; F')$. Seja $x \in G$. Então

$$u(x)(y) = A(x, y) = A_L(x \otimes y) = \varphi(x \otimes y) = B_L(x \otimes y) = B(x, y) = u'(x)(y),$$

para todo $y \in F$. Então $u(x) = u'(x)$ para todo $x \in G$. Logo, u' é extensão de u a E. Em resumo, todo elemento de $\mathcal{L}(G; F')$ se estende a um elemento de $\mathcal{L}(E; F')$. Agora, vejamos um exemplo que nem sempre a última afirmação acima acontece.

Exemplo 1.1. Seja $G \subsetneq E$ um subespaço não complementado do espaço normado E. Considere $I_G\colon G \longrightarrow G$. Sabemos que $I_G \in \mathcal{L}(G; G)$. Suponha por absurdo que I_G se estenda a E, ou seja, existe $u \in \mathcal{L}(E; G)$ tal que $u|_G = I_G$. Por outro lado, dado $x \in E$ observe que

$$u^2(x) = u(u(x)) = I_G(u(x)) = u(x),$$

isto é, $u^2 = u$. Assim, concluimos que u é uma projeção de E sobre G e consequentemente temos que G é subespaço complementado de E. Absurdo, logo I_G não se estende a E, ou seja, existe um elemento de $\mathcal{L}(G; G)$ que não se estende a um elemento de $\mathcal{L}(E; G)$. Assim, fazendo $G = F'$ concluimos da observação 1.1 que mesmo G sendo um subespaço de E isto não implica que $G \otimes_\pi F$ seja um subespaço de $E \otimes_\pi F$. É nesse sentido que dizemos que a norma projetiva não respeita subespaços. Um exemplo mais concreto deste fato pode ser visto quando $E = L_1[0, 1]$ e $F = \ell_2$, pois segue de [3, Teorema 1.12] que $L_1[0, 1]$ possui um subespaço G não complementado em $L_1[0, 1]$ e isomorfo a ℓ_2. Logo, de acordo com o que foi visto acima e sabendo que $G = F' = (\ell_2)' = \ell_2$ tem-se que $\ell_2 \otimes_\pi \ell_2$ não é um subespaço de $L_1[0, 1] \otimes_\pi \ell_2$.

Definição 1.3. Sejam E e F espaços vetoriais normados. Dizemos que E é um quociente de W se existe um subespaço fechado $Y \subseteq W$ tal que $E = W/Y$.

Definição 1.4. Sejam Z e Y espaços vetoriais normados. Dizemos que o operador linear contínuo $Q\colon Z \longrightarrow Y$ é um operador quociente se Q é sobrejetor e $\|y\| = \inf\{\|z\| : z \in Z, Q(z) = y\}$ para todo $y \in Y$.

A proposição abaixo mostra como a norma projetiva respeita quocientes.

Proposição 1.1. *Sejam $E_1, \ldots, E_n$ e $W_1, \ldots, W_n$ espaços vetoriais normados. Se E_i é um quociente de W_i para todo $i = 1, \ldots, n$ então $E_1 \otimes_\pi \cdots \otimes_\pi E_n$ é um quociente de $W_1 \otimes_\pi \cdots \otimes_\pi W_n$.*

Nas referências abaixo todos os resultados deste trabalho são provados para o produto tensorial de dois espaços de Banach. Um dos objetivos é apresentar demonstrações quando o produto tensorial possui n espaços de Banach.

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CÁLCULO DE VARIAÇÕES E AS EQUAÇÕES DE EULER - O PROBLEMA DE SUPERFÍCIE MÍNIMA

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Resolver um problema de otimização, significa, como o próprio nome diz, buscar o melhor resultado. Para exemplificar a aplicabilidade desses problemas, escuta-se frequentemente os termos: lucro máximo, custo mínimo, tempo mínimo, tamanho ótimo e caminho mais breve. Uma área da matemática que é muito útil na solução de problemas de otimização é o Cálculo de Variações.

Foi a partir do século XVII, na Europa Ocidental, que o Cálculo de Variações teve seu progresso substancial, contando com a colaboração de cientistas renomados, tais como Lagrange, Euler, Isaac Newton e os irmãos Jackes e Jean Bernoulli. Jean por ter proposto o problema da braquistócrona, que consiste em encontrar a forma (curva) de um fio que leva um corpo a se mover de um extremo fixo A até outro B em tempo mínimo, apenas sujeito à ação da gravidade e Jackes por propor e discutir o problema das figuras isoperimétricas (caminhos planos fechados de uma dada espécie e perímetro fixo que cercam uma área máxima) [5].

Neste trabalho, estudamos a teoria do Cálculo de Variações, procurando destacar as diversas semelhanças com o Cálculo de uma variável real. Além disso, alguns problemas clássicos de otimização, como a braquistócrona e as figuras isoperimétricas são analisados, com destaque especial para o problema de superfície mínima.

Definição 1. *Um funcional J é uma regra que associa a cada função x em Ω um único número real.*

Definição 2. *Se x e $x+\delta x$ são funções para os quais o funcional J está definido, então o incremento de J, denotado por ΔJ é dado por*

$$\Delta J = J(x+\delta x) - J(x).$$

Definição 3. *O incremento de um funcional pode ser escrito como:*

$$\Delta J(x,\delta x) = \delta J(x,\delta x) + g(x,\delta x).\,\|\delta x\|$$

onde δJ é linear em δx. Se $\lim_{\|\delta x\|\to 0} g(x,\delta x) = 0$ então J é diferenciável em x e δJ é a variação de J em x.

Lema Fundamental do Cálculo de Variações

Se a função $h(t)$ é contínua em $[t_0, t_f]$ e

$$\int_{t_0}^{t_f} h(t)\delta x(t)dt = 0$$

para toda função $\delta x(t)$ que é contínua no intervalo $[t_0, t_f]$, então $h(t)$ deve ser zero em todo intervalo $[t_0, t_f]$.

Dem. ver Elsgolts [2].

Teorema Fundamental do Cálculo de Variações

Seja x uma função em Ω e $J(x)$ um funcional diferenciável em x. Suponha que as funções em Ω não sejam limitadas. Se x^* é um extremo, a variação de J deve se anular em x^*, isto é, $\delta J(x^*,\delta x) = 0$ para todo δx admissível.

Dem. ver Kirk [1].

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Considere agora x uma função escalar de classe C^1 e g de classe C^2. O problema clássico do Cálculo de Variações consiste em encontrar uma função x^* para a qual o funcional

$$J(x) = \int_{t_0}^{t_f} g(x(t), x'(t), t)dt \tag{1}$$

tenha um extremo relativo, onde t_0 e t_f são fixos e $x(t_0)$ e $x(t_f)$ conhecidos (problemas com fronteiras fixas).

Considerando x^* uma curva extremal, após a aplicação do Teorema Fundamental do Cálculo de Variações e algumas manipulações algébricas, obtemos que, se x^* é um extremal então:

$$\delta J(x^*, \delta x) = 0 = \int_{t_0}^{t_f} \left\{ \frac{\partial g}{\partial x}(x^*(t), x'^*(t), t) - \frac{d}{dt}\left[\frac{\partial g}{\partial x'}(x^*(t), x'^*(t), t)\right]\right\} \delta x(t)dt.$$

Aplicando o Lema Fundamental obtemos que uma condição necessária para x^* ser um um extremo é:

$$\frac{\partial g}{\partial x}(x^*(t), x'^*(t), t) - \frac{d}{dt}\left[\frac{\partial g}{\partial x'}(x^*(t), x'^*(t), t)\right] = 0$$

que é chamada **Equação de Euler**, que em geral é uma equação diferencial não linear.

O problema de superfície mínima consiste em encontrar uma curva com fronteiras fixas, cuja rotação em torno do eixo das abscissas gera uma superfície de área mínima. Dessa forma, o funcional a ser minimizado [4] é dado por

$$J(x) = 2\pi \int_{t_0}^{t_f} x\sqrt{1 + x'^2}dt$$

A equação diferencial obtida, aplicando a equação de Euler para o problema é dada por:

$$x\sqrt{1 + x'^2} - \frac{xx'^2}{1 + x'^2} = C_1$$

cuja solução, obtida por uma mudança de variável é,

$$x = C_1 \cosh \frac{t - C_2}{C_1}.$$

As curvas acima são conhecidas como uma família de catenárias, onde C_1 e C_2 são constantes a serem determinadas a partir das condições iniciais.

No trabalho de dissertação abordamos também outros tipos de problemas como os de fronteiras móveis, onde critérios adicionais devem ser satisfeitos [3].

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Estabilidade de soluções tipo Ondas Viajantes para a Equação KdV

Isnaldo Isaac B. *

Este trabalho está baseado no artigo de *Jonh P. Albert* [1]. Temos com objetivo demonstrar um resultado de estabilidade orbital para soluções Tipo Ondas Viajantes para a Equação KdV no espaço de Energia $\mathbf{H}^1(\mathbb{R})$. Para obter tal resultado fazemos uso do **Método de Compacidade Concentrada** introduzido na literatura por *Lions* em [2] e [3]. Aplicamos esse método tendo em mente duas leis de conservação da KdV, a saber:

$$Q(u) = \frac{1}{2}\int_{\mathbb{R}} u^2 dx \quad \text{e} \quad E(u) = \frac{1}{2}\int_{\mathbb{R}} \left[(u_x)^2 - \frac{1}{3}u^3\right] dx.$$

Um resultado essencial para provar a estabilidade foi o uso da **Boa Colocação Global para a Equação KdV** obtido em [4] por *C. Kenig*, *G. Ponce* e *V. Vega*.

1 Resultados

Lembramos o problema de valor inicial para a KdV:

$$(PVI) \qquad \begin{cases} \dfrac{\partial u}{\partial t} + \dfrac{\partial^3 u}{\partial x^3} + u\dfrac{\partial u}{\partial x} = 0 \\ u(x,0) = u_0(x) \end{cases} . \tag{1.1}$$

Estabilidade aqui significa que a solução do (PVI) está tão próximo quanto se queira da solução tipo onda viajante desde que a distância, na norma $\mathbf{H}^1(\mathbb{R})$, do dado inicial a onda viajante seja suficientemente pequena.

Segue o resultado principal deste trabalho.

Teorema 1.1. *Para cada $\varepsilon > 0$, existe $\delta > 0$ tal que se*

$$\|u_0 - \phi_c\|_{\boldsymbol{H}^1(\mathbb{R})} < \delta, \tag{1.2}$$

então a solução $u(x,t)$ do problema (1.1) com $u(x,0) = u_0$ satisfaz

$$\inf_{y\in\mathbb{R}} \|u(\cdot,t) - \phi_c(\cdot + y)\|_{\boldsymbol{H}^1(\mathbb{R})} < \varepsilon, \ \forall t \in \mathbb{R}. \tag{1.3}$$

O esquema da demonstração é o seguinte: Aplicamos o Lema de Compacidade Concentrada:

Lema 1.1. *Seja (ρ_n) uma sequência de funções em $L^1(\mathbb{R}^n)$ tal que:*

$$\rho_n \geq 0 \ \text{em } \mathbb{R}^n, \ \int_{\mathbb{R}^n} \rho_n dx = \lambda \tag{1.4}$$

com $\lambda > 0$ fixado. Então existe uma subsequência (ρ_{n_k}) satisfazendo uma das três propriedades abaixo:

(i) ***(Compacidade)*** *Existe $y_k \in \mathbb{R}^n$ tal que $\rho_{n_k}(\cdot + y_k)$ acumula-se, ou seja,*

$$\forall \varepsilon > 0, \exists R < \infty, \ \int_{y_k + B_R} \rho_{n_k} dx \geq \lambda - \varepsilon; \tag{1.5}$$

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(ii) ***(Nulidade)***

$$\lim_{k \longrightarrow \infty} \sup_{y \in \mathbb{R}^n} \int_{y+B_R} \rho_{n_k} dx = 0, \ \forall R < \infty; \tag{1.6}$$

(iii) ***(Dicotomia)*** *Existe* $\alpha \in (0, \lambda)$ *tal que, para todo* $\varepsilon > 0$*, existe* $k_0 \geq 1$ *e* $\rho_k^1, \rho_k^2 \in L^1(\mathbb{R}^n)$ *com* $\rho_k^1, \rho_k^2 > 0$ *satisfazendo para* $k \geq k_0$

$$\begin{cases} \left\| \rho_{n_k} - (\rho_k^1 + \rho_k^2) \right\|_{L^1(\mathbb{R}^n)} < \varepsilon, \ \left| \int_{\mathbb{R}^n} \rho_k^1 dx - \alpha \right| < \varepsilon, \ \left| \int_{\mathbb{R}^n} \rho_k^2 dx - (\lambda - \alpha) \right| < \varepsilon \\ dist\left(Supp \ \rho_k^1, Supp \ \rho_k^2 \right) \stackrel{k}{\longrightarrow} \infty. \end{cases}$$

observando que o único caso possível é o da Compacidade.

Usando a Compacidade e o seguinte resultado

Teorema 1.2. *Se* $s \geq 1$ *então* ***PVI*** *é globalmente bem posto, ou seja,*

$$u \in L^\infty(\mathbb{R}, \boldsymbol{H}^{[s]}(\mathbb{R})).$$

Resta-nos organizar nos hipóteses, fixando $q = Q(\phi_c)$, onde ϕ_c é a solução tipo onda viajante para a KdV, e definindo

$$I_q = \inf \left\{ E(\psi); \ \psi \in \mathbf{H}^1 \text{ e } Q(\psi) = q \right\},$$

consigimos provar que a órbita

$$G_q = \left\{ \psi \in \mathbf{H}^1 : \ E(\psi) = I_q \text{ e } Q(\psi) = q \right\}$$

é estável, para a Equação KdV.

Por fim, prova-se que

Proposição 1.1. *Se* G_q *é não vazio então*

$$G_q = \{ \phi(\cdot + x_0); \ x_0 \in \mathbb{R} \}.$$

Para provar este resultado usamos a teoria de **Multiplicardores de Lagrange** em espaço de dimensão infinita e **Derivada de Fréchet.**

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BIFURCAÇÕES DE PONTOS DE EQUILÍBRIO

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Neste trabalho, apresentamos o estudo de bifurcações, com respeito ao parametro $\lambda > 0$, de pontos de equilíbrio para o problema parabólico quasi-linear governado pelo p-Laplaciano, $p > 2$, dado por

$$\begin{cases} u_t = \lambda(|u_x|^{p-2}u_x)_x + |u|^{q-2}u(1-|u|^r), \quad (x,t) \in (0,1)\times(0,+\infty) \\ u(0,t) = u(1,t) = 0, \quad 0 < t < +\infty \\ u(x,0) = u_0(x), \quad x \in (0,1). \end{cases} \tag{0.1}$$

e também do problema semilinear dado por

$$\begin{cases} u_t = \lambda u_{xx} + f(u), \quad (x,t) \in (0,\pi)\times(0,+\infty) \\ u(0,t) = u(\pi,t) = 0, \quad 0 \le t < +\infty \\ u(x,0) = u_0(x), \quad x \in (0,\pi). \end{cases} \tag{0.2}$$

onde $f(u) = au(1-u^2)$.

Desde que estudamos pontos de equilíbrio de (0.1) e (0.2), trabalhamos com as equações diferenciais ordinárias

$$\begin{cases} \lambda(|u_x|^{p-2}u_x)_x + |u|^{q-2}u(1-|u|^r) = 0, \quad x \in (0,1) \\ u(0) = u(1) = 0. \end{cases} \tag{0.3}$$

e

$$\begin{cases} \lambda u_{xx} + f(u) = 0, \quad 0 \le x \le \pi \\ u(0) = u(\pi) = 0. \end{cases} \tag{0.4}$$

Este estudo foi baseado nos artigos [1] e [2]. Estudamos o caso $p = q$ na equação (0.3) devido a existencia de algumas similaridades com o problema (0.4).

1 Descrição do conjunto de equilíbrios

A seguir, enunciamos os teoremas que descrevem o conjunto dos pontos de equilíbrio para os problemas (0.2) e (0.1), para cada $\lambda > 0$.

Teorema 1.1. *Seja* $\lambda_n = \dfrac{n^2}{f'(0)}$, *onde* $n \in \mathbb{N}^*$. *Então para cada* $n \in \mathbb{N}^*$ *e* $\lambda \in [\lambda_n, +\infty)$, *o problema* (1.2) *tem dois pontos de equilíbrio* $u_n^{\pm}(\lambda) \in B_0(1)$ *que possuem as seguintes propriedades*

i) $u_n^{\pm}(\lambda_n) = 0$.

ii) Para cada $\lambda \in (\lambda_n, +\infty)$, $u_n^{\pm}(\lambda)$ *tem exatamente* $n+1$ *zeros em* $[0,\pi]$.

Denotando esses zeros por $x_q^{\pm}(\lambda)$, $q = 0, 1, 2, ..., n$, *com*

$$0 = x_0^{\pm}(\lambda) < x_1^{\pm}(\lambda) < ... < x_n^{\pm}(\lambda) = \pi,$$

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temos $(-1)^q u_n^+(x;\lambda) > 0$ *para* $x_q^+(\lambda) < x < x_{q+1}^+(\lambda)$, $q = 0, 1, ..., n-1$ *e* $(-1)^q u_n^-(x;\lambda) < 0$ *para* $x_q^-(\lambda) < x < x_{q+1}^-(\lambda)$, $q = 0, 1, ..., n-1$.

Finalmente, para cada $\lambda \in [0, +\infty)$, *o problema* (1.2) *não tem pontos de equilíbrios além da origem* $u_0 = 0$ *e dos elementos* $u_n^\pm(\lambda)$, $n \geq 1$, *para os quais* $\lambda_n \leq \lambda$.

Denotaremos por E_λ o conjunto dos pontos de equilíbrio de (0.1), para cada $\lambda > 0$.

Teorema 1.2. *Seja* $p = q$. *Definimos para cada* $k \in \mathbb{N}$

$$\lambda_k = \frac{p}{p-1}(2(k+1)I_0)^{-p},$$

onde $I_0 = p^{\frac{1}{p}} \int_0^1 (1-t^p)^{-\frac{1}{p}} dt$. *Então temos que*

i) Se $\lambda_0 \leq \lambda$, *então* $E_\lambda = \{0\}$.

ii) Se $\lambda_{k+1} \leq \lambda < \lambda_k$, *então* $E_\lambda = \{0\} \bigcup_{l=0}^{k} \{\pm E_\lambda^l\}$ *onde* E_λ^l *possui as seguintes propriedades*

a) $E_\lambda^0 = \{u_\lambda^0\}$ *para* $\lambda > 0$.

b) Se $\lambda_l(1) \leq \lambda$, *para* $l = 1, 2, ...,$ *então* $E_\lambda^l = \{u_\lambda^l\}$.

c) Se $0 < \lambda < \lambda_l(1)$, *para* $l = 1, 2, ...,$ *então existe uma relação bijetora entre* E_λ^l *e* $[0, 1]^l$.

Em particular, o problema (1.3) *possui uma única solução positiva se, e somente se,* $\lambda < \lambda_0$.

Os pontos de equilíbrio u_λ^l *no caso (a), apresentam o mesmo comportamento do caso de (ii) no Teorema 1.1. Os pontos de equilíbrio* u_λ^l *no caso (b), apresentam patamares na altura* $u = 1$ *ou* $u = -1$

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VERSÕES NÃO-LINEARES DO TEOREMA DA DOMINAÇÃO DE PIETSCH

Antonio Gomes Nunes*

Resumo

Este trabalho é um resumo de nossa Dissertação de Mestrado, defendida na UFPB, sob orientação do Professor Daniel Pellegrino. Apresentamos algumas versões recentes do Teorema da Dominação de Pietsch (TDP): o TDP para aplicações subhomogêneas e uma versão abstrata do TDP. O caso linear e o trabalho [1], de certa forma, servem como modelo para as generalizações não lineares exibidas no presente trabalho. Apresentamos ainda uma demonstração do TDP linear baseada na demonstração encontrada no livro [5], embora tenhamos feito algumas modificações para evitar o uso de medidas com sinal. Em relação ao caso de aplicações subhomogêneas, obtemos ainda uma versão não usual do TDP para aplicações multilineares, que está essencialmente contida em [4]. A versão abstrata do TDP, também devida a Botelho, Pellegrino e Rueda (veja [2, 3]), engloba várias versões do TDP existentes na literatura, como o TDP para aplicações semi-integrais, fortemente somantes, assim como o caso linear e o de aplicações subhmogêneas.

1 Resultados

O Teorema da Dominação de Pietsch, em sua versão original, caracteriza os operadores lineares absolutamente somantes por intermédio de uma desigualdade:

Teorema 1.1. *(Teorema da Dominação de Pietsch - Caso Linear) Sejam $1 \leq p < \infty$ e $u : E \longrightarrow F$ um operador linear contínuo. Então u é p-somante se, e somente se, existem uma constante $C > 0$ e uma medida de probabilidade μ nos borelianos de B_{E^*}, com a topologia fraca estrela, tais que*

$$\|u(x)\| \leq C \left(\int_{B_{E^*}} |\varphi(x)|^p \, d\mu(\varphi) \right)^{\frac{1}{p}} \tag{1.1}$$

para cada $x \in E$. Neste caso, $\pi_p(u)$ é a menor de todas as constantes C tais que (1.1) ocorre.

O próximo resultado, encontrado em [4], é essencial para a demonstração das versões não lineares que apresentamos do TDP.

Teorema 1.2. *Sejam $1 \leq p < \infty$ e $\alpha > 0$. Se $f : E \longrightarrow F$ é α-subhomogênea, são equivalentes:*

(a) Existem constantes $C_\alpha \geq 0$ e $\varrho > 0$ tais que

$$\sum_{j=1}^{m} \|f(x_j)\|^{\frac{p}{\alpha}} \leq C_\alpha \sup_{\varphi \in B_{E^*}} \sum_{j=1}^{m} |\varphi(x_j)|^p$$

para todos $x_1, ..., x_m \in E$ e $m \in \mathbb{N}$ com $\left\| (x_j)_{j=1}^m \right\|_{w,p} \leq \varrho$.

(b) Existe uma constante $C_b \geq 0$ tal que

$$\left(\sum_{j=1}^{m} \|f(x_j)\|^{\frac{p}{\alpha}} \right)^{\frac{\alpha}{p}} \leq C_b \sup_{\varphi \in B_{E^*}} \left(\sum_{j=1}^{m} |\varphi(x_j)|^p \right)^{\frac{\alpha}{p}}$$

para todos $x_1, ..., x_m \in E$ e $m = 1, 2, ...$

(c) A função f é absolutamente $\left(\frac{p}{\alpha}; p\right)$-somante.

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Teorema 1.3. *(Botelho-Pellegrino-Rueda [4]) Sejam $1 \leq p < \infty$ e $\alpha > 0$. Então uma aplicação α-subhomogênea $f : E \longrightarrow F$ é absolutamente $\left(\frac{p}{\alpha}, p\right)$-somante se, e somente se, existem uma constante $C \geq 0$ e uma medida de probabilidade μ nos borelianos de B_{E^*} (com a topologia fraca estrela) tais que*

$$\|f(x)\| \leq C \left(\int_{B_{E^*}} |\varphi(x)|^p \, d\mu(\varphi) \right)^{\frac{\alpha}{p}} \tag{1.2}$$

para todo x em E.

O próximo resultado, essencialmente contido em [4], é consequência do anterior, e é diferente da versão usual do TDP para aplicações multilineares.

Teorema 1.4. *Seja $1 \leq p < \infty$. Uma aplicação $A \in \mathcal{L}(E_1, ..., E_n; F)$ é absolutamente $\left(\frac{p}{n}; p, ..., p\right)$-somante se, e somente se, existem uma constante $C > 0$ e uma medida de probabilidade μ nos borelianos de $B_{(E_1 \times \cdots \times E_n)^*}$ com a topologia fraca-estrela tais que*

$$\|A(x_1, ..., x_n)\| \leq C \left(\int_{B_{(E_1 \times \cdots \times E_n)^*}} |\varphi(x_1, ..., x_n)|^p \, d\mu(\varphi) \right)^{\frac{n}{p}}.$$

Finalmente, enunciamos a versão abstrata do TDP:

Teorema 1.5. *(Botelho-Pellegrino-Rueda) Sejam $1 \leq p < \infty$ e $f : E \longrightarrow F$ α-subhomogênea. Então f é R^{β}-abstrata p-dominada (com constante C_1) se, e somente se, existem uma constante $C > 0$ e uma medida de probabilidade μ nos borelianos em K tais que*

$$\|f(x)\| \leq C \left(\int_K R(\varphi, x)^p \, d\mu(\varphi) \right)^{\frac{\alpha}{p\beta}} \tag{1.3}$$

para todo $x \in E$. Além disso $C = C_1^{\frac{\alpha}{p\beta}}$.

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ESTUDO DO MODELO DE RONALD ROSS PARA PREVENÇÃO DA MALÁRIA

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1 Introdução

O objetivo deste trabalho é fazer uma análise qualitativa do modelo proposto por Ronald Ross no artigo "Contribution to the Analysis of Malaria Epidemiology" de Alfred J. Lotka, publicado na revista *The American Journal of Hygiene* da John Hopkins University, em 1923.

O modelo de Ross trata da propagação da malária em uma comunidade e o artigo de Lotka tornou-se célebre por ser uma das primeiras tentativas de modelagem matemática de uma epidemia e também porque foi muito usado pela Organização Mundial de Saúde para fazer avaliações durante as tentativas de erradicação da malária em várias partes do mundo.

O modelo é dado por um sistema não linear de duas equações diferenciais ordinárias que será analisado através do estudo de estabilidade dos seus pontos de equilíbrio, indicando condições para a extinção da epidemia. Considerando os seguintes parâmetros:
p - população humana, z - população humana afetada com malária, fz - parcela dos humanos afetados (pela picada), r - taxa de cura dos humanos, b - número de picadas que o homem recebe por unidade de tempo, N - taxa de natalidade, M - taxa de mortalidade, t - tempo, p', z', b', $f'z'$, r', M', N' são as quantidades referentes à população de mosquitos, e desprezando emigração e imigração na comunidade, temos que *a taxa de crescimento de indivíduos afetados se dá pelo número de novas infecções por unidade de tempo, subtraído do número de mortes por unidade de tempo e também do número de curas por unidade de tempo.*

Observemos que, se o mosquito picar um humano, em média, b' vezes por unidade de tempo, então $f'.z'$ mosquitos infectados terão $b'(f'.z')$ picadas infectadas (sobre humanos) por unidade de tempo e $\dfrac{p-z}{p}$ dessas picadas cairão sobre pessoas sadias.

Assumindo que toda pessoa picada torna-se afetada, então o número de novas infecções por unidade de tempo, na população humana, será $b'.f'.z'.\dfrac{p-z}{p}$.

Similarmente, se um humano for picado b vezes por unidade de tempo, o número de novas infecções entre os mosquitos será $b.f.z.\dfrac{p'-z'}{p'}$.

Com estas considerações, o modelo de Ross é dado por:

$$\frac{dz}{dt} = \frac{b'f'z'}{p}(p-z) - Mz - rz = f(z,z') = a_{11}z + a_{12}z' + a_{112}zz'$$

$$\frac{dz'}{dt} = \frac{bfz}{p'}(p'-z') - M'z' - r'z' = g(z,z') = a_{21}z + a_{22}z' + a_{212}zz'. \tag{1.1}$$

O sistema (1.1) pode ser visto como uma perturbação do sistema linear

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$$\dot{x} = Df(\bar{x})x \tag{1.2}$$

onde $Df(\bar{x})$ é a Matriz Jacobiana de f no ponto de equilíbrio $\bar{x}$.

De acordo com Figueiredo [3] e Coddington [4], temos o seguinte resultado:

Teorema 1.1. *Se um ponto de equilíbrio para o sitema linear (1.2) for assintoticamente estável, isto é, se os autovalores da matriz Jacobiana forem reais negativos, então também será para o sistema (1.1).*

2 Resultados

O sistema (1.1) admite dois pontos de equilíbrio dados por $(0,0)$ e $(p,q) = \left(\dfrac{a_{21}a_{12} - a_{11}a_{22}}{a_{11}a_{212} - a_{21}a_{112}}, \dfrac{a_{21}a_{12} - a_{22}a_{11}}{a_{22}a_{112} - a_{12}a_{212}}\right)$ e a análise dos autovalores da matriz Jacobiana associada nos remete ao seguinte resultado:

Teorema 2.1. *Se $M'r > b'^2 ff'p'$, isto é, se a mortabilidade vezes a taxa de cura for maior que o produto (número de picadas ao quadrado)*x*(taxa de infecção humana)*x*(taxa de infecção do mosquito)*x*(população de mosquito), então a origem para o sistema (1.1) será assintoticamente estável (extinção da epidemia).*
Se $N'r < b'ff'p'$, isto é, se a origem for instável, então (p,q) será assintoticamente estável.
E se a origem for estável, os valores estimados para p e q serão menores que zero (sem significado biológico).

Prova: Ao estudar a estabilidade do ponto de equilíbrio $(0,0)$, obtemos a equação característica $\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0$ que possui as raízes $\lambda = \dfrac{(a_{11} + a_{22}) \pm \sqrt{\triangle}}{2} = \dfrac{(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2}$. Note que $\triangle = (a_{11} - a_{22})^2 + 4a_{12}a_{21} > 0$, pois a_{12}, a_{21} são positivos, logo os autovalores são reais. Analisando o caso $(a_{11}a_{22} - a_{12}a_{21}) < 0$ teremos autovalores distintos com sinais diferentes (e $(0,0)$ será instável). De fato, se $(a_{11}a_{22} - a_{12}a_{21}) < 0$, então $-4(a_{11}a_{22} - a_{12}a_{21}) > 0$ o que implica $\sqrt{\triangle} > |a_{11} + a_{22}|$, ou seja, $(a_{11} + a_{22}) + \sqrt{\triangle} > 0$, pois a_{11} e a_{22} são negativos. De modo análogo mostra-se que $(a_{11} + a_{22}) - \sqrt{\triangle} < 0$. Agora, se $(a_{11}a_{22} - a_{12}a_{21}) > 0$ teremos autovalores distintos negativos, pois $\sqrt{\triangle} < |a_{11} + a_{22}|$, e pelo teorema (1.1) o ponto de equilíbrio $(0,0)$ será assintoticamente estável. Como $M'r > b'^2 ff'p'$ é equivalente à $a_{11}a_{22} - a_{12}a_{21} > 0$, concluímos que, com esta condição, $(0,0)$ é assintoticamente estável.

Para o ponto (p,q) a equação característica $\lambda'^2 + (a_{12}\frac{q}{p} + a_{21}\frac{p}{q})\lambda' - (a_{11}a_{22} - a_{12}a_{21}) = 0$ admite raízes $\lambda' = \dfrac{-(a_{12}\frac{q}{p} + a_{21}\frac{p}{q}) \pm \sqrt{\triangle}}{2} = \dfrac{-(a_{12}\frac{q}{p} + a_{21}\frac{p}{q}) \pm \sqrt{(a_{12}\frac{q}{p} + a_{21}\frac{p}{q})^2 + 4(a_{11}a_{22} - a_{12}a_{21})}}{2}$. Observe que $\triangle = (a_{12}\frac{q}{p} - a_{21}\frac{p}{q})^2 + 4a_{11}a_{22} > 0$, logo os autovalores são reais. Se $N'r < b'ff'p'$ que é equivalente à $(a_{11}a_{22} - a_{12}a_{21}) < 0$, temos que a origem é instável e $\sqrt{\triangle} < \left|a_{12}\dfrac{q}{p} + a_{21}\dfrac{p}{q}\right|$, desse modo os autovalores serão negativos, logo (p,q) será assintoticamente estável. Se a origem for estável, isto é, $(a_{11}a_{22} - a_{12}a_{21}) > 0$, $\dfrac{-(a_{11}a_{22} - a_{21}a_{12})}{a_{11}a_{212} - a_{21}a_{112}} < 0$ e $\dfrac{-(a_{11}a_{22} - a_{21}a_{12})}{a_{22}a_{112} - a_{12}a_{212}} < 0$ e portanto, os valores estimados para p e q serão menores que zero, que não tem significado biológico.

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25 ANOS DE HOMOGENEIZAÇÃO

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1 Introdução

Nosso objetivo neste trabalho é apresentar resultados de homogeneização obtidos em 25 anos de trabalho, bem como a evolução de seu tratamento matemático. A modelagem matemática de inúmeros fenômenos que ocorrem na natureza necessita de um conhecimento preciso do sistema físico envolvido. Nesse trabalho, mostraremos problemas onde o tratamento matemático desses sistemas faz uso do método sequencial, devido a *F. Murat*, que adaptou as idéias de *L. Tartar* ao construir um quadro abstrato de hipóteses, admitindo a existência de uma família adequada de funções testes, que permite obter o comportamento assintótico da sequência de soluções, ou seja, que permite passar o limite quando ε tende para zero e caracterizar esse limite. Apresentamos resumidamente, abaixo, dois exemplos de trabalhos nessa linha, um de 1982, e outro de 2004.

2 Problema elíptico estacionário

Em 1982, *D. Cioranescu* e *F. Murat* [1] consideraram o problema elíptico: Encontrar u_ε solução de

$$\begin{cases} -\Delta u_\varepsilon &= f, \quad \text{em } \Omega_\varepsilon \\ u_\varepsilon &= 0, \quad \text{sobre } \partial\Omega_\varepsilon, \end{cases}$$

onde f é dada em $H^{-1}(\Omega)$ e Ω_ε é um domínio "perfurado" do $\mathbb{R}^N$, aberto e limitado, obtido de Ω pela extração de um conjunto S_ε de buracos S_i^ε distribuídos periodicamente com período $\varepsilon > 0$ na direção de cada eixo coordenado. Em vez de hipóteses geométricas diretas sobre os buracos S_i^ε, admitiu-se a existência de uma família adequada de funções testes, dada pelo seguinte quadro funcional abstrato:

$$\begin{cases} \text{Existe uma sequência de funções } (w_\varepsilon, \mu_\varepsilon, \gamma_\varepsilon) \text{ tais que:} \\ \quad (i) \quad w_\varepsilon \in H^1(\Omega) \cap L^\infty(\Omega), \ \|w_\varepsilon\|_{L^\infty(\Omega)} \leq M_0; \\ \quad (ii) \quad w_\varepsilon = 0, \ \text{em } \ S_\varepsilon, \\ \quad (iii) \quad w_\varepsilon \rightharpoonup 1, \ \text{fraco em } H^1(\Omega), \ \text{e q.s. em } \Omega \\ \quad (iv) \quad \mu_k \in [W^{-1,\infty}(\Omega)]^N, \\ \quad (v) \quad -\Delta w_\varepsilon = \mu_\varepsilon - \gamma_\varepsilon \ \text{ onde } \mu_\varepsilon, \gamma_\varepsilon \in H^{-1}(\Omega) \mu_\varepsilon \to \mu, \ \text{forte em } H^{-1}(\Omega), \ \text{e} \\ \qquad\quad \left\langle \gamma_\varepsilon - \nu_\varepsilon \right\rangle_\Omega = 0 \text{ para todo } \nu_\varepsilon \in H_0^1(\Omega) \text{ tal que } \nu_\varepsilon = 0 \text{ em } S_\varepsilon. \end{cases}$$

Utilizando a extensão $\widetilde{u}_\varepsilon$ de u_ε a todo Ω definida por zero em S_ε, demonstrou-se que $\widetilde{u}_\varepsilon \rightharpoonup u$, fraco em $H_0^1(\Omega)$, onde $\widetilde{u}_\varepsilon$ é a única solução do problema original para cada $\varepsilon > 0$ fixado, e u é a única solução do problema homogeneizado

$$\begin{cases} -\Delta u + \mu u = f, \quad \text{em } \Omega \\ u = 0, \quad \text{sobre } \partial\Omega, \end{cases}$$

onde μ é uma medida de Radon, não negativa, pertencente a $H^{-1}(\Omega)$. Obteu-se ainda o resultado de correção

$$\widetilde{u}_\varepsilon = w_\varepsilon u + r_\varepsilon, \quad \text{com } r_\varepsilon \to 0, \ \text{forte em } H_0^1(\Omega),$$

que diz que $w_\varepsilon u$ é uma boa aproximação para a solução do problema original.

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3 Problemas relaxados

Em 2004, *G. Dal Maso* e *F. Murat* [2] estudaram o problema de Dirichlet

$$-Au = f,$$

onde A é um operador elíptico linear de segunda ordem com coeficientes mensuráveis limitados em Ω. Foi considerada uma sequência de problemas de evolução com condições de Dirichlet lineares da forma

$$\begin{cases} -\text{div}(A^\varepsilon Du_\varepsilon) = f \text{ em } \Omega^\varepsilon \\ u_\varepsilon \in H_0^1(\Omega_\varepsilon), \end{cases} \tag{3.1}$$

onde as matrizes A^ε e os domínios variáveis Ω^ε dependem do parâmetro ε. Os conjuntos Ω^ε, abertos, são todos contidos em um conjunto $\Omega \subset \mathbb{R}^n$, fixo, aberto e limitado, e as matrizes A^ε, definidas sobre Ω com coeficientes mensuráveis, são coercivas e limitadas. O processo de homogeneização consiste em estudar o comportamento das soluções u^ε quando ε tende para zero. No caso $\Omega^\varepsilon = \Omega$, existe uma subsequência ainda denotada por (A^ε) e uma matriz A^0, chamada de H-limite de (A^ε), tal que para cada $f \in L^\infty(0,T;H^{-1}(\Omega))$, as soluções v^ε dos problemas

$$\begin{cases} v^\varepsilon \in H_0^1(\Omega), \\ -\text{div}(A^\varepsilon Dv^\varepsilon) = f, \text{ em } \mathcal{D}'(\Omega), \end{cases}$$

convergem fracamente em $H_0^1(\Omega)$ para a solução v^0 de

$$\begin{cases} v^0 \in H_0^1(\Omega), \\ -\text{div}(A^0 Dv^0) = f, \text{ em } \mathcal{D}'(\Omega), \end{cases}$$

e satisfazem também $A^\varepsilon Dv^\varepsilon \rightharpoonup A^0 Dv^0$, fraco em $L^2(\Omega, \mathbb{R}^N)$.

4 Comentário final

O problema apresentado na seção 2 tem como restrição o fato de o domínio Ω_ε ser obtido de Ω através da extração de buracos distribuídos periodicamente com período $\varepsilon > 0$. Já o problema relaxado apresentado na seção 3 apresenta uma dependência simultânea de ε nos coeficientes e nos domínios perfurados, que agora podem ser mais gerais. O problema de Dirichlet relaxado é também mais geral. O desafio que se apresenta agora é obter soluções para problemas semelhantes ao da seção 2, utilizando a técnica aplicada na solução do problema da seção 3.

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Alguns aspectos teóricos sobre um sistema termo-elástico não linear

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1 Introdução

O objetivo deste trabalho é fazer uma análise qualitativa sobre as soluções do sistema misto termo-elástico não linear acoplado

$$\left|\begin{array}{l} u_{tt}(x,t) - M\Big(\displaystyle\int_0^L |u_x(t)|^2_{\mathbb{R}} dx\Big) u_{xx}(x,t) + u_{xxxx}(x,t) + \theta_x(x,t) + u_t(x,t) = 0 \text{ em } Q, \\ \theta_t(x,t) - \theta_{xx}(x,t) + u_{xt}(x,t) = 0 \text{ em } Q, \\ u(0,t) = u(L,t) = u_x(0,t) = u_x(L,t) = 0 \quad \text{para} \quad t \geq 0, \\ \theta(0,t) = \theta(L,t) = 0 \quad \text{para} \quad t \geq 0, \\ u(x,0) = u_0(x), \;\; u_t(x,0) = u_1(x) \quad \text{e} \quad \theta(x,0) = \theta_0(x) \quad \text{em} \quad]0,L[, \end{array}\right. \tag{1.1}$$

onde $Q =]0, L[\times]0, T[$ com $L > 0$ e $T > 0$. De modo preciso será estabelecido:

- a existência de soluções globais fraca e a estabelização assintótica da energia associada as soluções;
- a existência e a unicidade de soluções fortes não locais.

2 Os principais resultados

Mostra-se que o sistema (1.1) têm soluções fracas e globais $\{u, \theta\}$ desde que

$$u_0 \in H_0^2(0,L), \;\; u_1 \in L^2(0,L) \;\text{ e }\; \theta_0 \in H_0^1(0,L); \qquad M \in C^0(\mathbb{R}^+) \;\text{ e }\; M(\lambda) \geq 0. \tag{2.1}$$

A estabilização assintótica da energia associada as soluções fracas $\{u, \theta\}$ é obtida supondo-se, além da hipótese (2.1) sobre a função M, que ela satisfaça:

$$M(\lambda)\lambda \geq \widehat{M}(\lambda) \;\text{ para todo }\; \lambda \geq 0 \;\text{ onde }\; \widehat{M}(\lambda) = \int_0^\lambda M(s)ds. \tag{2.2}$$

Definição 2.1. *Uma solução fraca e global do problema (0.1) é um par de funções* $\{u, \theta\}$ *definido em* $Q_\infty =]0, L[\times[0, \infty[$ *com valores reais tal que*

$$u \in L^\infty_{loc}\left(0,\infty; H_0^2(0,L)\right), \quad u_t \in L^\infty_{loc}\left(0,\infty; L^2(0,L)\right), \quad \theta \in L^\infty_{loc}\left(0,\infty; L^2(0,L)\right) \cap L^2_{loc}\left(0,\infty; H_0^1(0,L)\right),$$

o par $\{u, \theta\}$ *satisfaz as identidades integrais*

$$-\int_0^T\int_0^L u_t(x,t)\gamma_t(x,t)dxdt + \int_0^T \left[M\left(|u_x(t)|^2\right)\int_0^L u_x(x,t)\gamma_x(x,t)dx\right]dt +$$
$$\int_0^T\int_0^L u_{xx}(x,t)\gamma_{xx}(x,t)dxdt + \int_0^T\int_0^L \theta_x(x,t)\gamma(x,t)dxdt + \int_0^T\int_0^L u_t(x,t)\gamma(x,t)dxdt = 0,$$
$$-\int_0^T\int_0^L \theta(x,t)\eta_t(x,t)dxdt + \int_0^T\int_0^L \theta_x(x,t)\eta_x(x,t)dxdt + \int_0^T\int_0^L u_{xt}(x,t)\eta(x,t)dxdt = 0,$$

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para $\gamma \in L^2(0,T;H_0^2(0,L))$, $\gamma_t \in L^2(0,T;L^2(0,L))$, $\gamma(0) = \gamma(T) = 0$, *e para* $\eta \in L^2(0,T;H_0^1(0,L))$ *com* $\eta(0) = \eta(T) = 0$. *Além disso,* $\{u,\theta\}$ *satisfaz as condições iniciais (1.1)*$_5$.

Teorema 2.1. *Supondo-se que as hipóteses (2.1) são válidas, então existe pelo menos uma solução* $\{u,\,\theta\}$ *do sistema (1.1) no sentido da Definição 2.1.*

A demonstração do Teorema 2.1 é baseada nos *Métodos de Faedo-Galerkin* e de *Compacidade*.

Teorema 2.2. *As soluções* $\{u,\,\theta\}$ *do sitema (1.1), garantidas pelo Teorema 2.1 são assintóticamente estáveis, desde que a hipótese estabelecida em (2.2) seja válida. Ou seja, a energia total do sistema (1.1) dada por*

$$E(t) = \frac{1}{2}\big\{|u_t(t)|^2 + \widehat{M}\left(|u_x(t)|^2\right) + |u_{xx}(t)|^2 + |\theta(t)|^2\big\},$$

satisfaz $E(t) \leq C\Lambda \exp(-\mu t)$ *para todo* $t \geq 0$, *onde* μ, C *and* Λ *são contantes reais positivas.*

Mostra-se que o sistema (1.1) têm uma única solução forte não global $\{u,\,\theta\}$ desde que

$$u_0 \in H_0^2(0,L) \cap H^4(0,L)\,,\;\; u_1 \in H_0^2(0,L) \;\text{ e }\; \theta_0 \in H_0^2(0,L)\,; \qquad M \in C^1(\mathbb{R}^+;\mathbb{R}) \;\text{ e }\; M(\lambda) \geq 0. \tag{2.3}$$

O conceito de soluções forte não locais para o problema (1.1) é dado por:

Definição 2.2. *Uma solução forte não local do problema (1.1) é um par de funções* $\{u,\,\theta\}$ *definido em* $Q =]0,L[\times[0,T[$ *com valores reais tal que*

$$u \in L^\infty(0,T;H_0^2(0,L) \cap H^4(0,L)),\;\; u_t \in L^\infty(0,T;H_0^2(0,L)),\;\; u_{tt} \in L^\infty(0,T;L^2(0,L)),$$
$$\theta \in L^\infty(0,T;H_0^2(0,L)),\;\; \theta_t \in L^\infty(0,T;L^2(0,L)),$$

para $T > 0$ *fixado, e satisfaz o sistema (1.1) q. s. em* $Q =]0,L[\times]0,T[$.

Teorema 2.3. *Se as hipóteses em (2.3) são válidas, então existe uma única solução* $\{u,\,\theta\}$ *de (1.1) no sentido da Definição 1.2.*

A demonstração do Teorema 2.3 é, também, baseada nos *Métodos de Faedo-Galerkin* e de *Compacidade* ■

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